Machine Learning: Lecture 2

Concept Learning

and

Version Spaces

thanks to Brian Pardo (http://bryanpardo.com) for the illustrations on slides 17, 22, 25 and 26

What is a Concept? (1)

- A Concept is a a subset of objects or events defined over a larger set [Example: The concept of a _bird_ is the subset of all objects (i.e., the set of all _things_ or all _animals_) that belong to the category of bird.]

- Alternatively, a concept is a boolean-valued function defined over this larger set [Example: a function defined over all _animals_ whose value is _true_ for birds and _false_ for every other animal].

What is a Concept? (2)

- Given X the set of all examples.

- A ***concept*** C is a subset of X.

- A training example T is a subset of X such that some examples of T are elements of C (the positive examples) and some examples are not elements of C (the negative examples)

What is Concept-Learning? (1)

Given a set of examples labeled as members or non-members of a concept, *concept-learning* consists of automatically inferring the general definition of this concept.

In other words, *concept-learning* consists of approximating a boolean-valued function from training examples of its input and output.

What is Concept Learning? (2)

- **Learning:**

  {<xi,yi>} ➔ Learning system ➔ f: X→ Y

  with i=1..n,

  · xi ∈T, yi ∈ Y (={0,1})

  · yi= 1, if x1 is positive (∈ C)

  · yi= 0, if xi is negative (∉ C)

- **Goals of learning:**

  f must be such that for all xj ∈ X (not only ∈ T) - f(xj) =1 if xj ∈ C

  - f(xj) = 0, if xj ∉ C

The Problem of Induction: Computer Science's Answer

- **Problem:** As previously noted by philosophers, the task of induction is not well formulated. In computer science the problem can be thought of as follows: there exists an infinite number of functions that satisfy the goal ➔ It is necessary to find a way to constrain the search space of *f*.

- **Definitions:**
  - The set of all *f*s that satisfy the goal is called ***hypothesis space.***
  - The constraints on the hypothesis space is called the ***inductive bias***.
  - There are two types of inductive bias:
    - The ***hypothesis space restriction bias***
    - The ***preference bias***

Inductive Biases (1)

- **<u>Hypothesis space restriction bias</u>** ➔ We restrain the language of the hypothesis space. Examples:
- k-DNF: We restrict f to the set of Disjunctive Normal form formulas having an arbitrary number of disjunctions but at most, k conjunctive in each conjunctions.

- K-CNF: We restrict f to the set of Conjunctive Normal Form formulas having an arbitrary number of conjunctions but with at most, k disjunctive in each disjunction.

- <u>Properties of that type of bias:</u>
  - Positive: Learning will by simplified (Computationally)
  - Negative: The language can exclude the “good” hypothesis.

Inductive Biases (2)

- **Preference Bias:** It is an order or unit of measure that serves as a base to a relation of preference in the hypothesis space.

- Examples:

- **Occam's razor:** We prefer a simple formula for f.

- **Principle of minimal description length** (An extension of Occam's Razor): The best hypothesis is the one that minimise the total length of the hypothesis **and** the description of the exceptions to this hypothesis.

Using Biases for Learning (1)

- How to implement learning with these bias?

- <u>**Hypothesis space restriction bias:**</u>

  - Given:

    - A set S of training examples
    - A set of restricted hypothesis, H
  - Find: An hypothesis $f \in H$ that minimizes the number of incorrectly classified training examples of S.

Using Biases for Learning (2)

- **Preference Bias:**
  - Given:
    - A set S of training examples
    - An order of preference better(f1, f2) for all the hypothesis space (H) functions.
  - Find: the best hypothesis f ∈ H (using the “better” relation) that minimises the number of training examples S incorrectly classified.
- **Search techniques:**
  - Heuristic search
  - Hill Climbing
  - Simulated Annealing et Genetic Algorithm

Example of a Concept Learning task

- **Concept:** Good Days for Water Sports (values: Yes, No)
- **Attributes/Features:**
  - Sky (values: Sunny, Cloudy, Rainy)
  - AirTemp (values: Warm, Cold)
  - Humidity (values: Normal, High)
  - Wind (values: Strong, Weak)
  - Water (Warm, Cool)
  - Forecast (values: Same, Change)
- **Example of a Training Point:**

<Sunny, Warm, High, Strong, Warm, Same, Yes>

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Example of a Concept Learning task

Database:

Day	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*WaterSport*
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
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Chosen Hypothesis Representation:

Conjunction of constraints on each attribute where:

- "?" means "any value is acceptable"
- "0" means "no value is acceptable"

Example of a hypothesis: <?,Cold,High,?,?,?>
(If the air temperature is cold and the humidity high then it is a good day for water sports)

Example of a Concept Learning task

- **Goal:** To infer the "best" concept-description from the set of all possible hypotheses ("best" means "which best generalizes to all (known or unknown) elements of the instance space" . concept-learning is an ill-defined task)

- **Most General Hypothesis:** Everyday is a good day for water sports <?,?,?,?,?,?>
- **Most Specific Hypothesis:** No day is a good day for water sports <0,0,0,0,0,0>

Terminology and Notation

- The set of items over which the concept is defined is called the set of ***instances*** (denoted by X)
- The concept to be learned is called the ***Target Concept*** (denoted by c: X--> {0,1})
- The set of ***Training Examples*** is a set of instances, x, along with their target concept value c(x).
- Members of the concept (instances for which c(x)=1) are called ***positive examples***.
- Nonmembers of the concept (instances for which c(x)=0) are called ***negative examples***.
- H represents the set of ***all possible hypotheses***. H is determined by the human designer's choice of a hypothesis representation.
- **The goal of concept-learning is to find a hypothesis h:X --> {0,1} such that h(x)=c(x) for all x in X.**

Concept Learning as Search

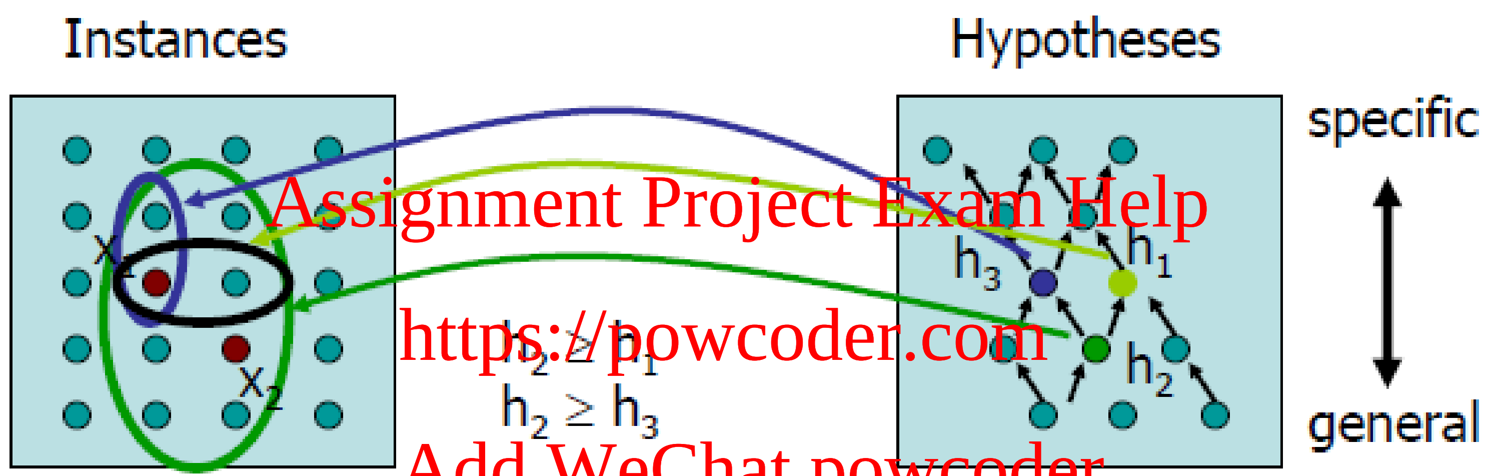
- Concept Learning can be viewed as the task of searching through a large space of hypotheses implicitly defined by the hypothesis representation.
- Selecting a Hypothesis Representation is an important step since it restricts (or *biases*) the space that can be searched. [For example, the hypothesis "If the air temperature is cold <u>or</u> the humidity high then it is a good day for water sports" cannot be expressed in our chosen representation.]

General to Specific Ordering of Hypotheses I

- **Definition:** Let *hj* and *hk* be boolean-valued functions defined over X. Then *hj* is **more-general-than-or-equal-to** *hk* iff For all x in X, [(*hj*(x) = 1) --> (*hk*(x)=1)]
- **Example:**
  - *h1* = <Sunny,?,?,Strong,?,?>
  - *h2* = <Sunny,?,?,?,?,?>

Every instance that is classified as positive by *h2* will also be classified as positive by *h1* in our example data set. Therefore *h2* is more general than *h1*.

General to Specific Ordering of Hypotheses II



x_1=< Sunny,Warm,High,Strong,Cool,Same>

x_2=< Sunny,Warm,High,Light,Warm,Same>

h_1=< Sunny,?,?,Strong,?,?>

h_2=< Sunny,?,?,?,?,?>

h_3=< Sunny,?,?,?,Cool,?>

from Bryan Pardo, EECS 349, Machine Learning, Fall 2009

Find-S, a Maximally Specific Hypothesis Learning Algorithm

- Initialize *h* to the most specific hypothesis in *H*
- For each positive training instance *x*
  - For each attribute constraint *ai* in *h*

    If the constraint *ai* is satisfied by *x*

    then do nothing

    else replace *ai* in h by the next more general constraint that is satisfied by *x*
- Output hypothesis *h*

Shortcomings of Find-S

- Although Find-S finds a hypothesis consistent with the training data, it does not indicate whether that is the only one available
- Is it a good strategy to prefer the most specific hypothesis?
- What if the training set is inconsistent (*noisy*)?
- What if there are several maximally specific consistent hypotheses? Find-S cannot backtrack!

Version Spaces and the Candidate-Elimination Algorithm

- **Definition:** A hypothesis *h* is **consistent** with a set of training examples *D* iff $h(x) = c(x)$ for each example $<x,c(x)>$ in *D*.
- **Definition:** The **version space**, denoted VS_H,D, with respect to hypothesis space *H* and training examples *D*, is the subset of hypotheses from *H* consistent with the training examples in *D*.

- **NB:** While a Version Space can be exhaustively enumerated, a more compact representation is preferred.

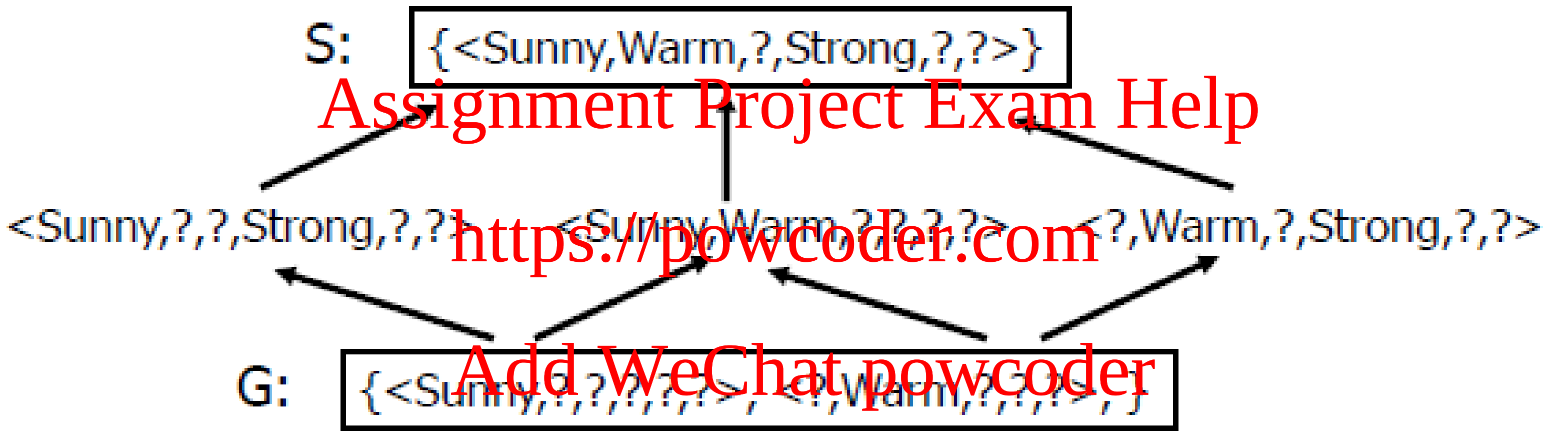
A Compact Representation for Version Spaces

- Instead of enumerating all the hypotheses consistent with a training set, we can represent its **most specific** and **most general** boundaries. The hypotheses included in-between these two boundaries can be generated as needed.

- **Definition:** The **general boundary** *G*, with respect to hypothesis space *H* and training data *D*, is the set of maximally general members of *H* consistent with *D*.

- **Definition:** The **specific boundary** *S*, with respect to hypothesis space *H* and training data *D*, is the set of minimally general (i.e., maximally specific) members of *H* consistent with *D*.

A Compact Representation for Version Spaces: An example



x_1 = <Sunny Warm Normal Strong Warm Same> +
x_2 = <Sunny Warm High Strong Warm Same> +
x_3 = <Rainy Cold High Strong Warm Change> -
x_4 = <Sunny Warm High Strong Cool Change> +

Version Spaces: Definitions

- Given C1 and C2, two concepts represented by sets of examples. If C1 ⊂ C2, then C1 is a ***specialisation*** of C2 and C2 is a ***generalisation*** of C1.
- C1 is also considered ***more specific*** than C2

- Example: The set of all blue triangles is more specific than the set of all triangles.

- C1 is an ***immediate specialisation*** of C2 if there is no concept that are a specialisation of C2 and a generalisation of C1.

- A ***version space*** define a graph where the nodes are concepts and the arcs specify that a concept is an immediate specialisation of another one.

Candidate-Elimination Learning Algorithm

- The candidate-Elimination algorithm computes the version space containing all (and only those) hypotheses from *H* that are consistent with an observed sequence of training examples.

Version Space Example

S: ~~{<∅, ∅, ∅, ∅, ∅, ∅ >}~~

G: {<?, ?, ?, ?, ?, ?>}

x_1 = <Sunny Warm Normal Strong Warm Same> +

S: ~~{< Sunny Warm Normal Strong Warm Same >}~~

G: {<?, ?, ?, ?, ?, ?>}

x_2 = <Sunny Warm High Strong Warm Same> +

S: {< Sunny Warm ? Strong Warm Same >}

G: {<?, ?, ?, ?, ?, ?>}

Version Space Example (cont'd)

S: {< Sunny Warm ? Strong Warm Same >}

G: ~~{<?, ?, ?, ?, ?, ?>}~~

x_3 = <Rainy Cold High Strong Warm Change> -

S: {< Sunny Warm ~~?~~ Strong Warm Same >}

G: {<Sunny,?,?,?,?,?>, <?,Warm,?,?,?,?>, ~~<?,?,?,?,?,Same>~~}

x_4 = <Sunny Warm High Strong Cool Change> +

S: {< Sunny Warm ? Strong ? ? >}

G: {<Sunny,?,?,?,?,?>, <?,Warm,?,?,?> }

Remarks on Version Spaces and Candidate-Elimination

- The version space learned by the Candidate-Elimination Algorithm will converge toward the hypothesis that correctly describes the target concept provided: (1) There are no errors in the training examples; (2) There is some hypothesis in *H* that correctly describes the target concept.

- Convergence can be speeded up by presenting the data in a strategic order. The best examples are those that satisfy exactly half of the hypotheses in the current version space.

- Version-Spaces can be used to assign certainty scores to the classification of new examples

Inductive Bias I: A Biased Hypothesis Space

Database:

Day	*Sky*	*AirTemp*	*Humidity*	*Wind*	*Water*	*Forecast*	*WaterSport*
1	Sunny	Warm	Normal	Strong	Cool	Change	Yes
2	Cloudy	Warm	Normal	Strong	Cool	Change	Yes
3	Rainy	Warm	Normal	Strong	Cool	Change	No

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- Given our previous choice of the hypothesis space representation, **no hypothesis is consistent with the above database**: we have ***BIASED*** the learner to consider only conjunctive hypotheses

Inductive Bias II: An Unbiased Learner

- In order to solve the problem caused by the bias of the hypothesis space, we can remove this bias and allow the hypotheses to represent every possible subset of instances. The previous database could then be expressed as: <Sunny, ?,?,?,?,?> v <Cloudy,?,?,?,?,?>
- ***<u>However, such an unbiased learner is not able to generalize beyond the observed examples!!!!</u>*** All the non-observed examples will be well-classified by half the hypotheses of the version space and misclassified by the other half.

Inductive Bias III: The Futility of Bias-Free Learning

- **Fundamental Property of Inductive Learning** A learner that makes no a priori assumptions regarding the identity of the target concept has no rational basis for classifying any unseen instances.
- **We constantly have recourse to inductive biases** *Example:* we all know that the sun will rise tomorrow. Although we cannot *deduce* that it will do so based on the fact that it rose today, yesterday, the day before, etc. (see the philosophical basis of induction) , we do take this **leap of faith** or use this **inductive bias**, naturally!

Inductive Bias IV: A Definition

- Consider a concept-learning algorithm *L* for the set of instances *X*. Let *c* be an arbitrary concept defined over *X*, and let $Dc = \{\langle x,c(x)\rangle\}$ be an arbitrary set of training examples of *c*. Let *L*(*xi*,*Dc*) denote the classification assigned to the instance *xi* by *L* after training on the data *Dc*. The inductive bias of *L* is any minimal set of assertions *B* such that for any target concept *c* and corresponding training examples *Dc*

(For all *xi* in *X*) [(*B* ^*Dc*^*xi*) |-- *L*(*xi*,*Dc*)]

Ranking Inductive Learners according to their Biases

Weak

Bias Strength

Strong

- **Rote-Learner:** This system simply memorizes the training data and their classification--- No generalization is involved.
- **Candidate-Elimination:** New instances are classified only if all the hypotheses in the version space agree on the classification
- **Find-S:** New instances are classified using the most specific hypothesis consistent with the training data