1 Game with a biased coin

Consider a game that is played with a biased coin that lands on heads with a probability p. Consider the procedure of the game as shown below:

1. Flip the coin
2. Flip the coin again
3. If both flips land on heads or both lands on tails, return to step 1.
4. Let the result of the last flip be the result of the experiment.

Prove that the result is equally likely to be either heads or tails.

2 Spam Filters

Suppose the university has designed an email spam filter that attempts to identify spam emails by looking for commonly occurring phrases in spam. Analysis has shown that 80% of email is spam. Suppose that 10% of the spam email contain the phrase "Large inheritance", whereas this phrase is only used in 1% of non-spam emails. Suppose a new email is received with the phase "Large inheritance", what is the probability that it is spam?

3 Telephone Call Requests

Suppose call requests arrive at a telephone switching office following a Poisson process with rate $\lambda = 4$ calls/sec.

1. Find the probability that at least 5 calls will arrive during the next 4 seconds.
2. Find the probability and cumulative distribution of the time, starting from now, until the arrival of the next call request.

4 Rainfall Analysis

The annual rainfall (in inches) in a certain region is normally distributed with mean $\mu = 40$ and $\sigma = 4$. What is the probability that starting with this year, it will take more than 10 years before a year occurs having a rainfall of more than 50 inches? What assumption are you making?

5 Markov Chains

A consumer in an specific market buys brand A with probability 0.8 if their last purchase was brand A and with probability 0.3 if their last purchase was brand B. We can model this stochastic process by a Markov Chain.

1. Draw the transition diagram, ie: draw the nodes, edges, and weights.
2. Convert your transition diagram to the one step probability transition matrix, $\mathbf{P}$.
3. Is the Markov chain ergodic? Why or why not?
4. Determine the steady state probabilities.
5. Compare your result from part 3 with $\mathbf{P}^{12}$.