Computational Linguistics

CSC 2501 / 485
Fall 2018

10. Maximum Entropy Models

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(slides borrowed from Chris Manning and Dan Klein)

Introduction

- Much of what we've looked at has been "generative"
  - PCFGs, Naive Bayes for WSD

- In recent years there has been extensive use of *conditional* or *discriminative* probabilistic models in NLP, IR, Speech (and ML generally)

- Because:
  - They give high accuracy performance
  - They make it easy to incorporate lots of linguistically important features
  - They allow automatic building of language independent, retargetable NLP modules

Joint vs. Conditional Models

- We have some data $\{(d, c)\}$ of paired observations d and hidden classes c.
- Joint (generative) models place probabilities over both observed data and the hidden stuff (generate the observed data from hidden stuff):
  - All the best known StatNLP models:
    - *n*-gram models, Naive Bayes classifiers, hidden Markov models, probabilistic context-free grammars

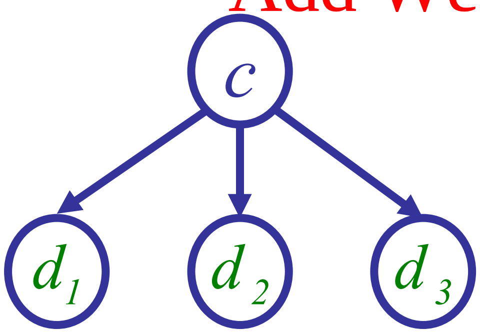
P(c,d)

- Discriminative (conditional) models take the data as given, and put a probability over hidden structure given the data:
    - Logistic regression, conditional log-linear or maximum entropy models, conditional random fields, (SVMs, …)

P(c|d)

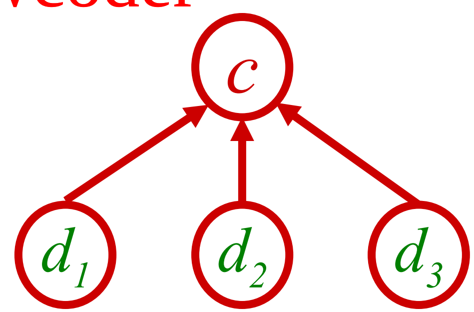
Bayes Net/Graphical Models

- Bayes net diagrams draw circles for random variables, and lines for direct dependencies
- Some variables are observed; some are hidden
- Each node is a little classifier (conditional probability table) based on incoming arcs



Naive Bayes

Generative



Logistic Regression

Discriminative

Conditional models work well: Word Sense Disambiguation

Training Set	
Objective	Accuracy
Joint Like.	86.8
Cond. Like.	98.5

Test Set	
Objective	Accuracy
Joint Like.	73.6
Cond. Like.	76.1

- Even with *exactly the same features*, changing from joint to conditional estimation increases performance
- That is, we use the same smoothing, and the same *word-class* features, we just change the numbers (parameters)

(Klein and Manning 2002, using Senseval-1 Data)

Features

- In these slides and most MaxEnt work: *features* are elementary pieces of evidence that link aspects of what we observe d with a category c that we want to predict.

- A feature has a (bounded) real value: $f\!: C \times D \rightarrow \mathbf{R}$
- Usually features specify an indicator function of properties of the input and a particular class (*every one we present is*). They pick out a subset.

  - $f_i(c, d) \equiv [\Phi(d) \wedge c = c_j]$ [Value is 0 or 1]
- We will freely say that $\Phi(d)$ is a feature of the data d, when, for each c_j, the conjunction $\Phi(d) \wedge c = c_j$ is a feature of the data-class pair (c, d).

Features

- For example:
  - $f_1(c, w_i t_i) \equiv [c = \text{"NN"} \wedge \text{islower}(w_0) \wedge \text{ends}(w_0, \text{"d"})]$
  - $f_2(c, w_i t_i) \equiv [c = \text{"NN"} \wedge w_{-1} = \text{"to"} \wedge t_{-1} = \text{"TO"}]$
  - $f_3(c, w_i t_i) \equiv [c = \text{"VB"} \wedge \text{islower}(w_0)]$

IN NN	TO NN	TO VB	IN JJ
in bed	*to aid*	*to aid*	*in blue*

- Models will assign each feature a *weight*
- Empirical count (expectation) of a feature:

$$\text{empirical } E(f_i) = \sum_{(c,d) \in \text{observed}(C,D)} f_i(c,d)$$

- Model expectation of a feature:

$$E(f_i) = \sum_{(c,d) \in (C,D)} P(c,d) f_i(c,d)$$

Feature-Based Models

- The decision about a data point is based only on the features active at that point.

Text Categorization	Word-Sense Disambiguation	POS Tagging
Data	Data	Data
BUSINESS: Stocks hit a yearly low …	… to restructure bank:MONEY debt.	DT JJ NN … The previous fall …
Label	Label	Label
BUSINESS	MONEY	NN
Features	Features	Features
{…, stocks, hit, a, yearly, low, …}	{…, P=restructure, N=debt, L=12, …}	{W=fall, PT=JJ PW=previous}

Example: Text Categorization

(Zhang and Oles 2001)

- Features are a word in document and class (they do feature selection to use reliable indicators)
- Tests on classic Reuters data set (and others)
  - Naïve Bayes: 77.0% F_1
  - Linear regression: 86.0%
  - Logistic regression: 86.4%
  - Support vector machine: 86.5%
- Emphasizes the importance of *regularization* (smoothing) for successful use of discriminative methods (not used in most early NLP/IR work)

Example: POS Tagging

- Features can include:
  - Current, previous, next words in isolation or together.
  - Previous (or next) one, two, three tags.
  - Word-internal features: word types, suffixes, dashes, etc.

Decision Point

Local Context

-3	-2	-1	0	+1
DT	NNP	VBD	???	???
The	Dow	fell	22.6	%

Features

W_0	22.6
W_{+1}	%
W_{-1}	fell
T_{-1}	VBD
T_{-1}-T_{-2}	NNP-VBD
hasDigit?	true
…	…

(Ratnaparkhi 1996; Toutanova et al. 2003, etc.)

Other MaxEnt Examples

- Sentence boundary detection (Mikheev 2000)
  - Is period end of sentence or abbreviation?
- PP attachment (Ratnaparkhi 1998)
  - Features of head noun, preposition, etc.
- Language models (Rosenfeld 1996)
  - $P(w_0|w_{-n},\ldots,w_{-1})$. Features are word n-gram features, and trigger features which model repetitions of the same word.
- Parsing (Ratnaparkhi 1997; Johnson et al. 1999, etc.)
  - Either: Local classifications decide parser actions or feature counts choose a parse.

Conditional vs. Joint Likelihood

- A *joint* model gives probabilities $P(c,d)$ and tries to maximize this joint likelihood.
  - It turns out to be trivial to choose weights: just relative frequencies.
- A *conditional* model gives probabilities $P(c|d)$. It takes the data as given and models only the conditional probability of the class.
  - We seek to maximize conditional likelihood.
  - Harder to do (as we'll see…)
  - More closely related to classification error.

Feature-Based Classifiers

- "Linear" classifiers:
  - Classify from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - For a pair (c,d), features vote with their weights:
    - $\text{vote}(c) = \Sigma \lambda_i f_i(c,d)$

1.2 –1.8 TO NN *to aid* TO VB *to aid* 0.3

  - Choose the class c which maximizes $\Sigma \lambda_i f_i(c,d) = \text{VB}$
  - There are many ways to chose weights
    - Perceptron: find a currently misclassified example, and nudge weights in the direction of a correct classification

Feature-Based Classifiers

- Exponential (log-linear, maxent, logistic, Gibbs) models:
  - Use the linear combination $\Sigma\lambda_i f_i(c,d)$ to produce a probabilistic model:

$$P(c|d,\lambda)=\frac{\exp\sum_i \lambda_i f_i(c,d)}{\sum_{c'}\exp\sum_i \lambda_i f_i(c',d)}$$

exp is smooth and positive but see also below

Normalizes votes.

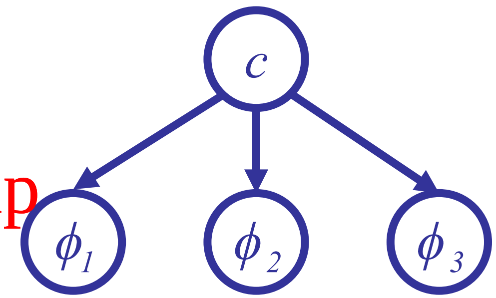
    - P(NN|*to, aid,* TO) = $e^{1.2}e^{-1.8}/(e^{1.2}e^{-1.8} + e^{0.3}) = 0.29$
    - P(VB|*to, aid,* TO) = $e^{0.3}/(e^{1.2}e^{-1.8} + e^{0.3}) = 0.71$
  - The weights are the parameters of the probability model, combined via a "soft max" function
  - Given this model form, we will choose parameters $\{\lambda_i\}$ that *maximize the conditional likelihood* of the data according to this model.

Other Feature-Based Classifiers

- The exponential model approach is one way of deciding how to weight features, given data.
- It constructs not only classifications, but probability distributions over classifications.
- There are other (good!) ways of discriminating classes: SVMs, boosting, even perceptrons – though these methods are not as trivial to interpret as distributions over classes.

Comparison to Naïve-Bayes

- Naïve-Bayes is another tool for classification:
  - We have a bunch of random variables (data features) which we would like to use to predict another variable (the class):



  - The Naïve-Bayes likelihood over classes is:

$$\frac{P(c)\prod_i P(\varphi_i|c)}{\sum_{c'} P(c')\prod_i P(\varphi_i|c')} \Rightarrow \frac{\exp\left[\log P(c)+\sum_i \log P(\varphi_i|c)\right]}{\sum_{c'}\exp\left[\log P(c')+\sum_i \log P(\varphi_i|c')\right]}$$

$$\Rightarrow \frac{\exp\left[\sum_i \lambda_{ic} f_{ic}(d,c)\right]}{\sum_{c'}\exp\left[\sum_i \lambda_{ic'} f_{ic'}(d,c')\right]}$$

Naïve-Bayes is just an exponential model.

Comparison to Naïve-Bayes

- The primary differences between Naïve-Bayes and maxent models are:

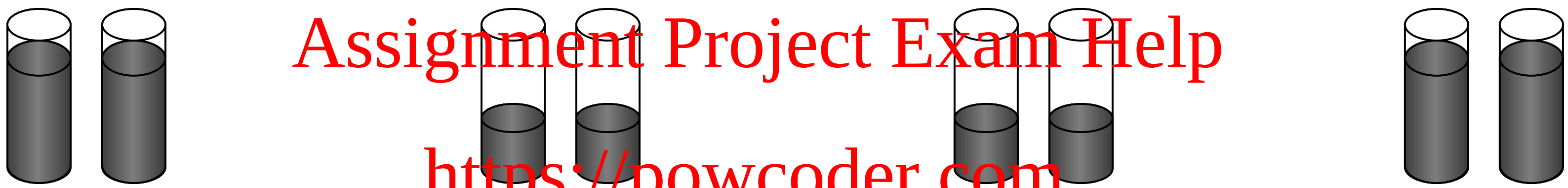
Naïve-Bayes	Maxent
Trained to maximize joint likelihood of data and classes.	Trained to maximize the conditional likelihood of classes.
Features assumed to supply independent evidence.	Features weights take feature dependence into account.
Feature weights can be set independently.	Feature weights must be mutually estimated.
Features must be of the conjunctive $\Phi(d) \wedge c = c_i$ form.	Features need not be of this conjunctive form (but usually are).

Example: Sensors

Reality

Raining

Sunny



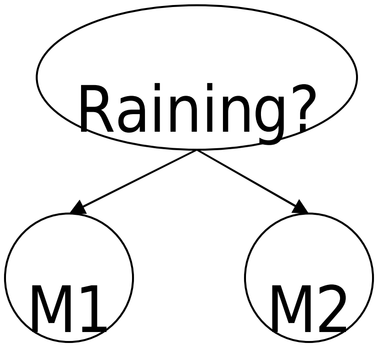
P(+,+,r) = 3/8

P(-,-,r) = 1/8

P(-,-,s) = 3/8

P(+,+,s) = 1/8

NB Model



NB FACTORS:

- P(s) = 1/2
- P(+|s) = 1/4
- P(+|r) = 3/4

PREDICTIONS:

- P(r,+,+) = (½)(¾)(¾)
- P(s,+,+) = (½)(¼)(¼)
- P(r|+,+) = 9/10
- P(s|+,+) = 1/10

Example: Sensors

- Problem: NB multi-counts the evidence.

$$\frac{P(r|+\ldots+)}{P(s|+\ldots+)} = \frac{P(r)}{P(s)}\frac{P(+|r)}{P(+|s)}\cdots\frac{P(+|r)}{P(+|s)}$$

- Maxent behavior:

  - Take a model over $(M_1, \ldots M_n, R)$ with features:
    - f_{ri}: M_i=+, R=r weight: λ_{ri}

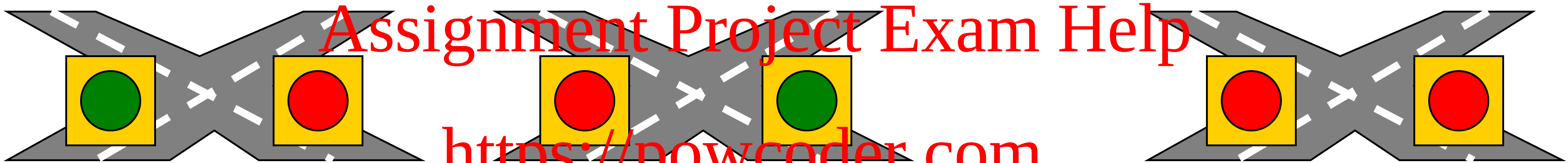
    - f_{si}: M_i=+, R=s weight: λ_{si}
  - $\exp(\lambda_{ri}-\lambda_{si})$ is the factor analogous to $P(+|r)/P(+|s)$
  - … but instead of being 3, it will be $3^{1/n}$
  - … because if it were 3, $E[f_{ri}]$ would be far higher than the target of 3/8!
- NLP problem: we often have overlapping features….

Example: Stoplights

Reality

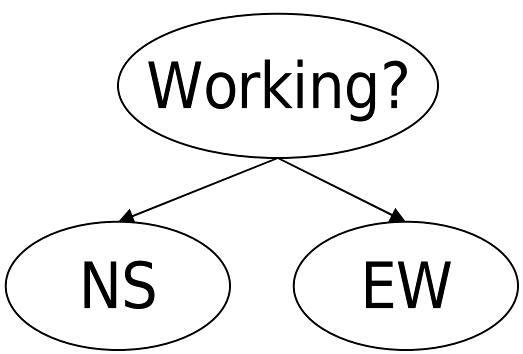
Lights Working

Lights Broken



P(g,r,w) = 3/7 P(r,g,w) = 3/7 P(r,r,b) = 1/7

NB Model



NB FACTORS:

- P(w) = 6/7
- P(r|w) = 1/2
- P(g|w) = 1/2
- P(b) = 1/7
- P(r|b) = 1
- P(g|b) = 0

Example: Stoplights

- What does the model say when both lights are red?
  - P(b,r,r) = (1/7)(1)(1) = 1/7 = 4/28
  - P(w,r,r) = (6/7)(1/2)(1/2) = 6/28 = 6/28
  - P(w|r,r) = 6/10!
- We'll guess that (r,r) indicates lights are working!
- Imagine if P(b) were boosted higher, to 1/2:
  - P(b,r,r) = (1/2)(1)(1) = 1/2 = 4/8
  - P(w,r,r) = (1/2)(1/2)(1/2) = 1/8 = 1/8
  - P(w|r,r) = 1/5!
- Changing the parameters bought conditional accuracy at the expense of data likelihood!

Exponential Model Likelihood

- Maximum Likelihood (Conditional) Models :
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.

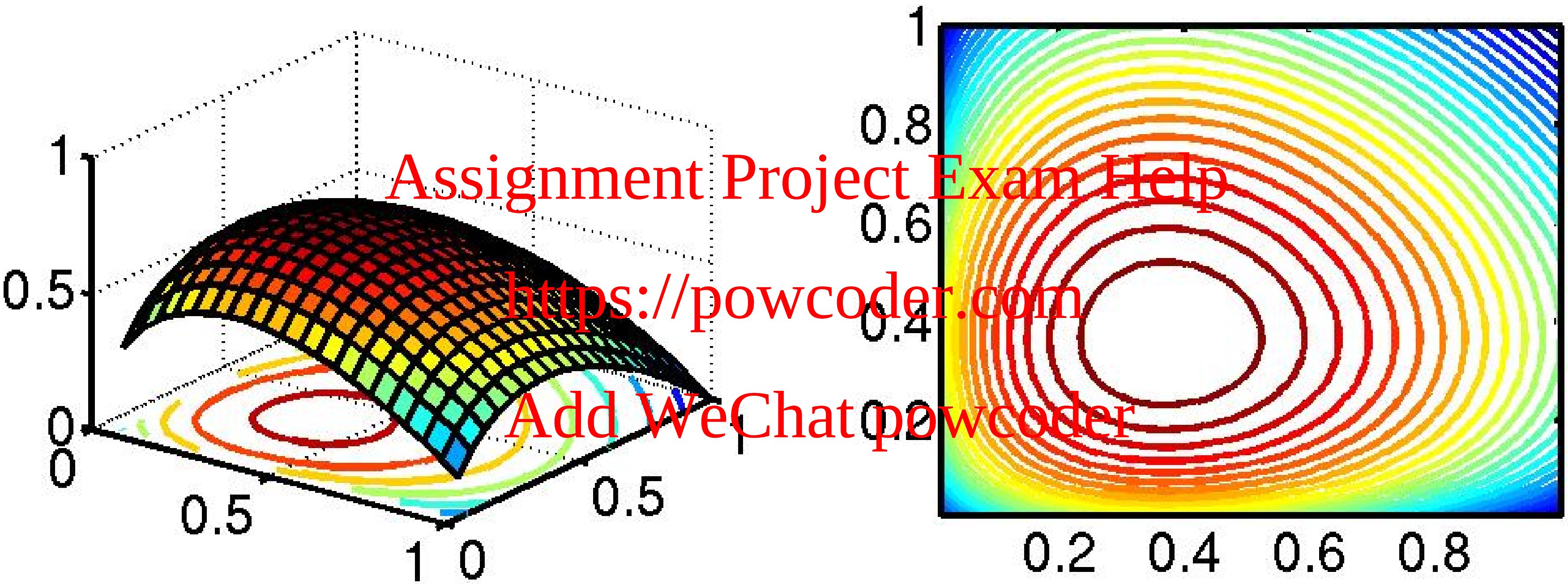
- Exponential model form, for a data set (C,D):

$$\log P(C|D,\lambda) = \sum_{(c,d)\in(C,D)} \log P(c|d,\lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Building a Maxent Model

- Define features (indicator functions) over data points.
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
    - Words, but also "word contains number", "word ends with *ing*"
  - Usually features are added incrementally to "target" errors.
- For any given feature weights, we want to be able to calculate:
  - Data (conditional) likelihood
  - Derivative of the likelihood wrt each feature weight
    - Use expectations of each feature according to the model
- Find the optimum feature weights (MaxEnt).

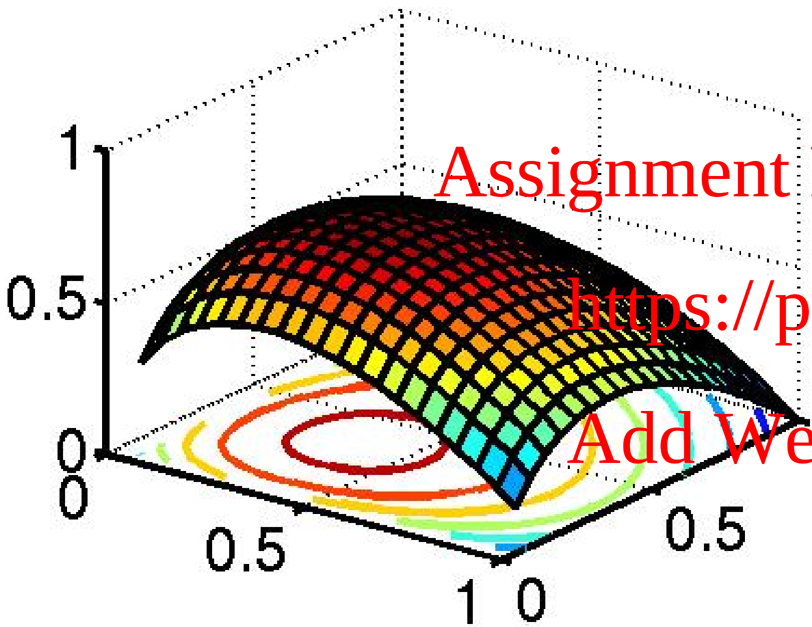
Digression: Lagrange's Method

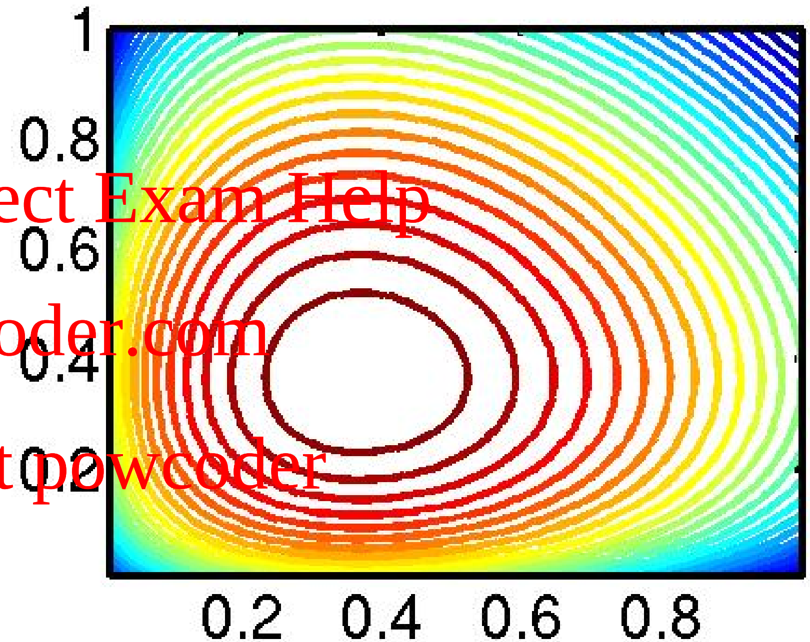


Task: find the highest *yellow* point.

This is “constrained optimization.”

Digression: Lagrange's Method



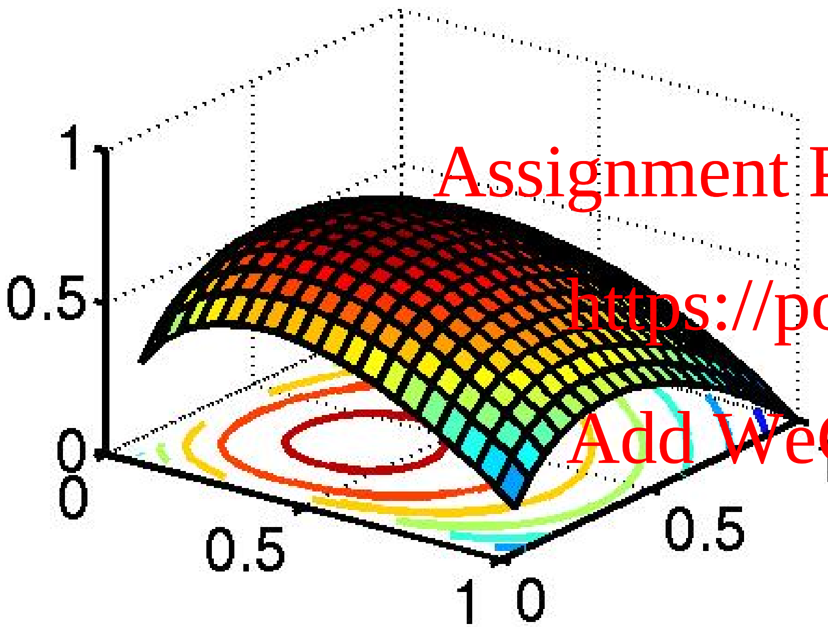


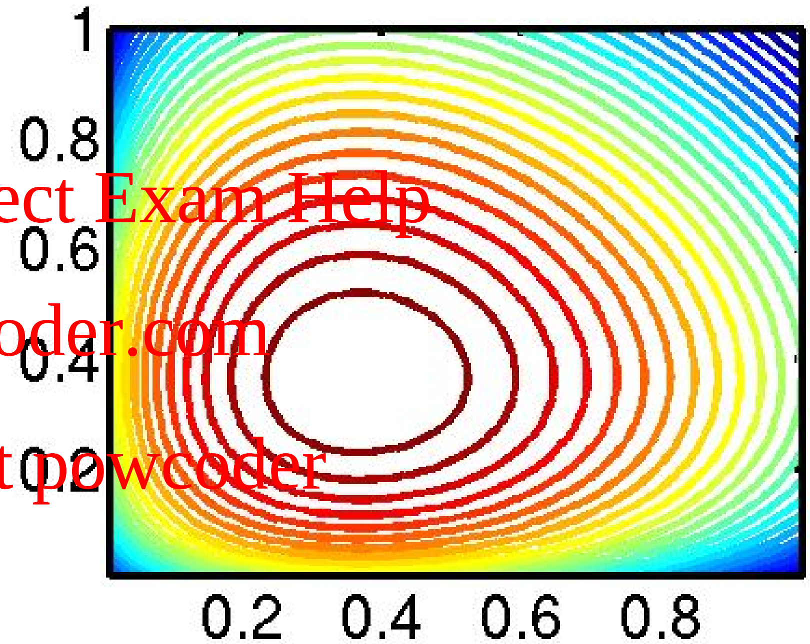
F(x,y): height of (x,y) on surface.

G(x,y): color of (x,y) on surface.

Maximize F(x,y) subject to constraint: G(x,y)=k.

Digression: Lagrange's Method





Suppose $G(x,y)-k=0$ is given by an implicit function $y=f(x)$.

(We're allowed to change coordinate systems, too.)

So we really want to maximize $u(x)=F(x,f(x))$.

Digression: Lagrange's Method

Maximize F(x,f(x)) so we want du/dx = 0:

$$\frac{du}{dx} = 0 = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{df}{dx}$$

We also know G(x,f(x)) - k = 0:

$$\frac{\partial G}{\partial x} + \frac{\partial G}{\partial y}\frac{df}{dx} = 0 \Longrightarrow \frac{df}{dx} = \frac{-\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial y}}$$

So: $$\frac{du}{dx} = \frac{\frac{\partial F}{\partial x}\frac{\partial G}{\partial y} - \frac{\partial F}{\partial y}\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial y}} = 0$$

Let: $$-\lambda := \frac{\frac{\partial F}{\partial x}}{\frac{\partial G}{\partial x}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial G}{\partial y}}$$

Lagrange Multipliers

$$-\lambda := \frac{\frac{\partial F}{\partial x}}{\frac{\partial G}{\partial x}} = \frac{\frac{\partial F}{\partial y}}{\frac{\partial G}{\partial y}}$$

These constants are called *Lagrange Multipliers*.

They allow us to convert constraint optimization problems into unconstrained optimization problems:

$$\Lambda(x, y; \lambda) = F(x, y) + \lambda G(x, y)$$

We don't actually care about Λ - we want its derivatives to be 0:

$$0 = \frac{\partial F}{\partial x_i} + \lambda \frac{\partial G}{\partial x_i} \text{ for all } i$$

So what is/are G?

$$\Lambda(x,y;\lambda) = F(x,y) + \sum_j \lambda_j G_j(x,y)$$

This generalizes to having multiple constraints - use one Lagrange multiplier for each.

We'll be searching over probability distributions p instead of (x,y).
But what should our constraints be? Answer:

$$E_p(f_j) - E_{\tilde{p}}(f_j) = 0$$

Up to the sensitivity of our feature representation, p acts like what we see in our training data.

So what is F?

$$\Lambda(x,y;\lambda) = F(x,y) + \sum_j \lambda_j G_j(x,y)$$

This generalizes to having multiple constraints - use one Lagrange multiplier for each.

We'll be searching over probability distributions p instead of (x,y).

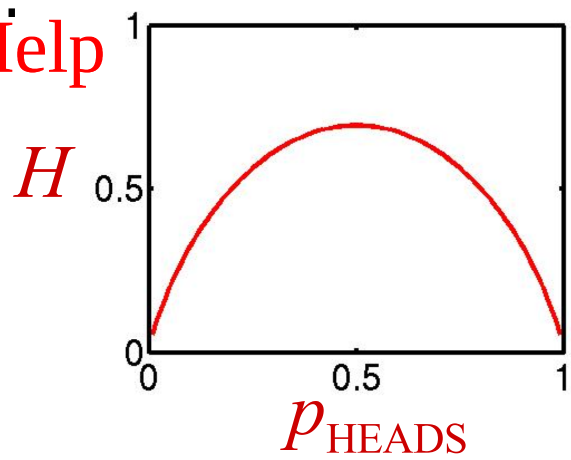
But what should we maximize as a function of p? Answer...

Maximize Entropy!

- Entropy: the uncertainty of a distribution.
- Quantifying uncertainty ("surprise"):
  - Event x
  - Probability p_x
  - "Surprise" $\log(1/p_x)$
- Entropy: expected surprise (over p):

$$H(p) = E_p\left[\log_2 \frac{1}{p_x}\right]$$

$$H(p) = -\sum_x p_x \log_2 p_x$$



A coin-flip is most uncertain for a fair coin.

Maximum Entropy Models

- Lots of distributions out there, most of them very spiked, specific, overfit.
- We want a distribution which is uniform except in specific ways we require.
- Uniformity means high entropy – we can search for distributions which have properties we desire, but also have high entropy.

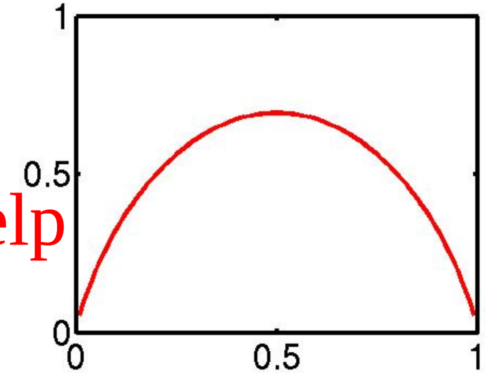
Ignorance is preferable to error and he is less remote from the truth who believes nothing than he who believes what is wrong – Thomas Jefferson (1781)

Maxent Examples I

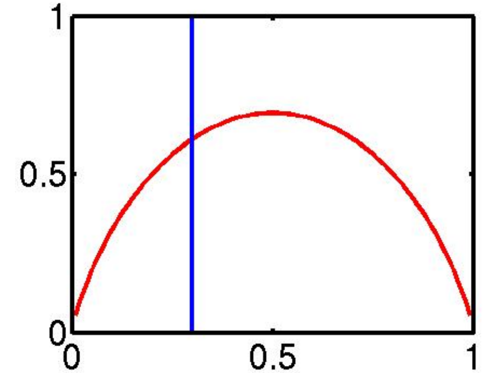
- What do we want from a distribution?
  - Minimize commitment = maximize entropy.
  - Resemble some reference distribution (data).
- Solution: maximize entropy H, subject to feature-based constraints:

$$E_p\left[f_i\right] = E_{\tilde{p}}\left[f_i\right]$$

- Adding constraints (features):
  - Lowers maximum entropy
  - Raises maximum likelihood of data
  - Brings the distribution further from uniform
  - Brings the distribution closer to data

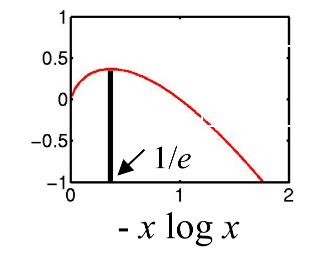


Unconstrained, max at 0.5

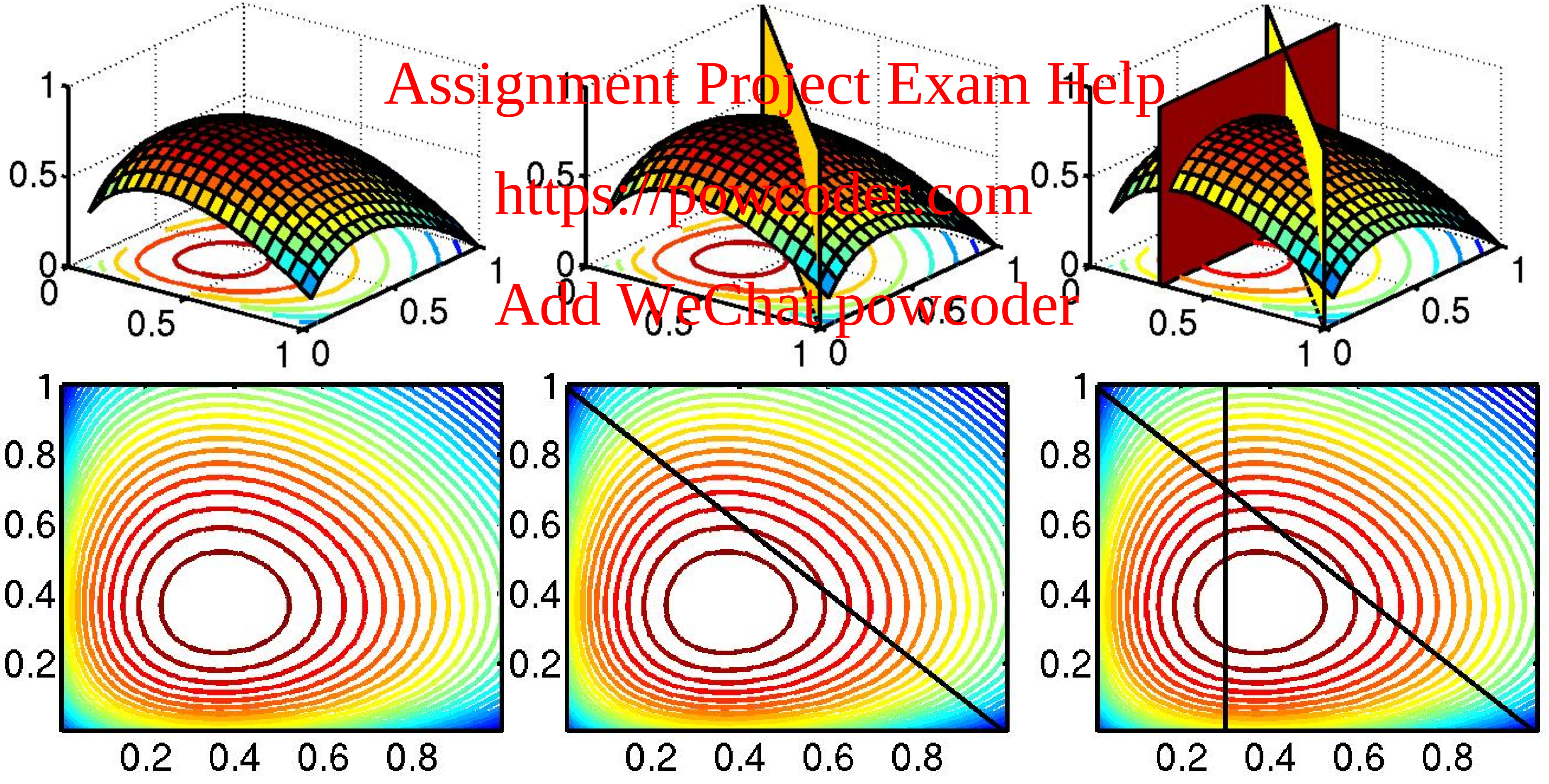


Constraint that $p_{\mathrm{HEADS}} = 0.3$

Maxent Examples II



$H(p_H p_T,)$ $\quad$ $p_H + p_T = 1$ $\quad$ $p_H = 0.3$



Maxent Examples III

- Lets say we have the following event space:

NN	NNS	NNP	NNPS	VBZ	VBD

- … and the following empirical data:

3	5	11	13	3	1

- Maximize H:

$1/e$	$1/e$	$1/e$	$1/e$	$1/e$	$1/e$

- … want probabilities: E[NN,NNS,NNP,NNPS,VBZ,VBD] = 1

1/6	1/6	1/6	1/6	1/6	1/6

Maxent Examples IV

- Too uniform!
- N* are more common than V*, so we add the feature f_N = {NN, NNS, NNP, NNPS}, with E[f_N] =32/36

NN	NNS	NNP	NNPS	VBZ	VBD
8/36	8/36	8/36	8/36	2/36	2/36

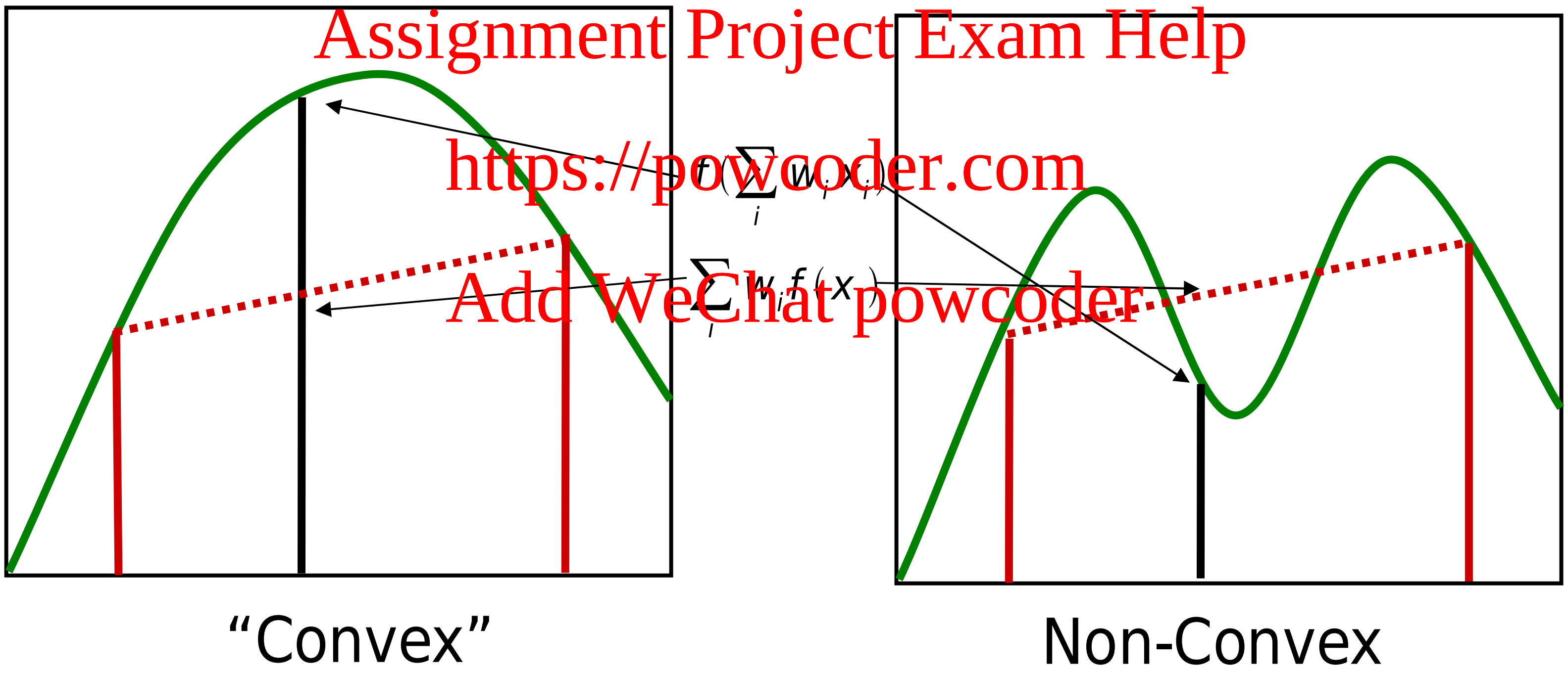
- … and proper nouns are more frequent than common nouns, so we add f_P = {NNP, NNPS}, with E[f_P] =24/36

4/36	4/36	12/36	12/36	2/36	2/36

- … we could keep refining the models, e.g. by adding a feature to distinguish singular vs. plural nouns, or verb types.

Digression: Jensen's Inequality

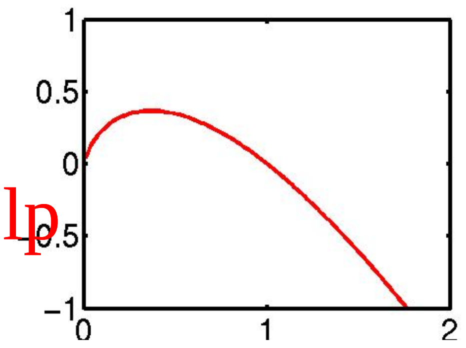
$$f\left(\sum_i w_i x_i\right) \geq \sum_i w_i f(x_i) \quad \text{where} \quad \sum_i w_i = 1$$

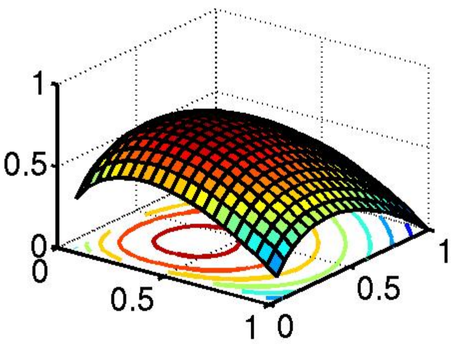


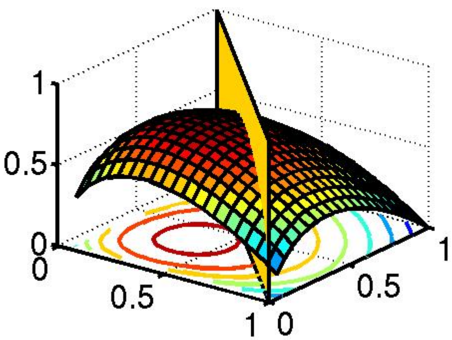
Convexity guarantees a single, global maximum because any higher points are greedily reachable.

Convexity

- Constrained $H(p) = -\sum x \log x$ is convex:
  - $-x \log x$ is convex
  - $-\sum x \log x$ is convex (sum of convex functions is convex).
  - The feasible region of constrained H is a linear subspace (which is convex)
  - The constrained entropy surface is therefore convex.
- The maximum likelihood exponential model (dual) formulation is also convex.







The Kuhn-Tucker Theorem

$$\Lambda(p;\lambda) = H(p) + \sum_j \lambda_j \left(E_p f_j - E_{\tilde{p}} f_j \right)$$

When the components of this are convex, we can find the optimal p and λ by first calculating:

$$p_\lambda = \underset{p}{argmax}\, \Lambda(p;\lambda)$$

with λ held constant, then solving the “dual:”

$$\bar{\lambda} = \underset{\lambda}{argmax}\, \Lambda(p_\lambda, \lambda).$$

The optimal p is then $p_{\bar{\lambda}}$.

The Kuhn-Tucker Theorem

$$\Lambda(p;\lambda) = H(p) + \sum_j \lambda_j (E_p f_j - E_{\tilde{p}} f_j)$$

For us, there is an analytic solution to the first part:

$$p_\lambda(c|d) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

So the only thing we have to do is find λ, given this.

Digression: Log-Likelihoods

- The (log) conditional likelihood is a function of the iid data (C,D) and the parameters λ:

$$\log P(C|D,\lambda) = \log \prod_{(c,d)\in(C,D)} P(c|d,\lambda) = \sum_{(c,d)\in(C,D)} \log P(c|d,\lambda)$$

- If there aren't many values of c, it's easy to calculate:

$$\log P_{\lambda}(C|D,\lambda) = \sum_{(c,d)\in(C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

- We can separate this into two components:

$$\log P_{\lambda}(C|D,\lambda) = \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)$$

$$\log P(C|D,\lambda) = N(\lambda) - M(\lambda)$$

- The derivative is the difference between the derivatives of each component

LL Derivative I: Numerator

$$\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{\partial \sum_i \lambda_i f_i(c,d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} f_i(c,d)$$

Derivative of the numerator is: the empirical count(f_i, c)

LL Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial \sum_{(c,d)\in(C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \frac{1}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{1} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} \frac{\exp \sum_i \lambda_i f_i(c',d)}{\sum_{c''} \exp \sum_i \lambda_i f_i(c'',d)} \frac{\partial \sum_i \lambda_i f_i(c',d)}{\partial \lambda_i}$$

$$= \sum_{(c,d)\in(C,D)} \sum_{c'} P(c'|d,\lambda)\, f_i(c',d)$$

= predicted count(f_i, λ)

LL Derivative III

$$\frac{\partial \log P_{\lambda}(C|D,\lambda)}{\partial \lambda_i} = E_{p}(f_i) - E_{\tilde{p}}(f_i)$$

- Our choice of constraint is vindicated: with our choice of p_λ, these correspond to the stable equilibrium points of the log conditional likelihood with respect to λ.

- The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if feature counts are from actual data).

Fitting the Model

- To find the parameters $\lambda_1, \lambda_2, \lambda_3 \ldots$

write out the conditional log-likelihood of the training data and maximize it

$$CLogLik(D) = \sum_{i=1}^{n} \log P(c_i | d_i)$$

- The log-likelihood is concave and has a single maximum; use your favorite numerical optimization package
- Good large scale techniques: conjugate gradient or limited memory quasi-Newton

Fitting the Model
Generalized Iterative Scaling

- A simple optimization algorithm which works when the features are non-negative
- We need to define a slack feature to make the features sum to a constant over all considered pairs from $D \times C$.
- Define

$$M = \max_{i,c} \sum_{j=1}^{m} f_j(d_i, c)$$

- Add new feature

$$f_{m+1}(d,c) = M - \sum_{j=1}^{m} f_j(d,c)$$

Generalized Iterative Scaling

- Compute empirical expectation for all features:

$$E_{\tilde{p}}(f_j)=\frac{1}{N}\sum_{i=1}^{n} f_j(d_i,c_i)$$

- Initialize $\lambda_j=0, j=1\ldots m+1$

- Repeat
  - Compute feature expectations according to current model

$$E_{p^t}(f_j)=\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{K} P(c_k|d_i) f_j(d_i,c_k)$$

  - Update parameters:

$$\lambda_{j(t+1)}=\lambda_{j(t)}+\frac{1}{M}\log\left(\frac{E_{\tilde{p}}(f_j)}{E_{p^t}(f_j)}\right)$$

- Until converged

Feature Overlap

- Maxent models handle overlapping features well.
- Unlike a NB model, there is no double counting!

Empirical

	A	a
B	2	1
b	2	1

	A	a
B		
b		

All = 1

	A	a
B		
b		

A = 2/3

	A	a
B		
b		

A = 2/3

	A	a
B	1/4	1/4
b	1/4	1/4

	A	a
B	1/3	1/6
b	1/3	1/6

	A	a
B	1/3	1/6
b	1/3	1/6

	A	a
B		
b		

	A	a
B	λ_A	
b	λ_A	

	A	a
B	$\lambda'_A+\lambda''_A$	
b	$\lambda'_A+\lambda''_A$	

Example: NER Overlap

Grace is correlated with PERSON, but does not add much evidence on top of already knowing prefix features.

Feature Weights

Feature Type	Feature	PERS	LOC
Previous word	*at*	-0.73	0.94
Current word	*Grace*	0.03	0.00
Beginning bigram	*<G*	0.45	-0.04
Current POS tag	NNP	0.47	0.45
Prev and cur tags	IN NNP	-0.10	0.14
Previous state	Other	-0.70	-0.92
Current signature	Xx	0.80	0.46
Prev state, cur sig	O-Xx	0.68	0.37
Prev-cur-next sig	x-Xx-Xx	-0.69	0.37
P. state - p-cur sig	O-x-Xx	-0.20	0.82
…			
Total:		**-0.58**	**2.68**

Local Context

	Prev	Cur	Next
State	Other	???	???
Word	at	Grace	Road
Tag	IN	NNP	NNP
Sig	x	Xx	Xx

Feature Interaction

- Maxent models handle overlapping features well, but do not automatically model feature interactions.

Empirical

	A	a
B	1	1
b	1	0

	A	a
B		
b		

All = 1

	A	a
B		
b		

A = 2/3

	A	a
B		
b		

B = 2/3

	A	a
B	1/4	1/4
b	1/4	1/4

	A	a
B	1/3	1/6
b	1/3	1/6

	A	a
B	4/9	2/9
b	2/9	1/9

	A	a
B	0	0
b	0	0

	A	a
B	λ_A	
b	λ_A	

	A	a
B	$\lambda_A+\lambda_B$	λ_B
b	λ_A	

Feature Interaction

- If you want interaction terms, you have to add them:

Empirical

	A	a
B	1	1
b	1	0

	A	a
B		
b		

A = 2/3

	A	a
B	1/3	1/6
b	1/3	1/6

	A	a
B		
b		

B = 2/3

	A	a
B	4/9	2/9
b	2/9	1/9

	A	a
B		
b		

AB = 1/3

	A	a
B	1/3	1/3
b	1/3	0

- A disjunctive feature would also have done it (alone):

	A	a
B		
b		

	A	a
B	1/3	1/3
b	1/3	0

Feature Interaction

- For log-linear/logistic regression models in statistics, it is standard to do a greedy stepwise search over the space of all possible interaction terms.
- This combinatorial space is exponential in size, but that's okay as most statistics models only have 4–8 features.
- In NLP, our models commonly use hundreds of thousands of features, so that's not okay.
- Commonly, interaction terms are added by hand based on linguistic intuitions.

Example: NER Interaction

Previous-state and current-signature have interactions, e.g. P=PERS-C=Xx indicates C=PERS much more strongly than C=Xx and P=PERS independently.

This feature type allows the model to capture this interaction.

Local Context

	Prev	Cur	Next
State	Other	???	???
Word	at	Grace	Road
Tag	IN	NNP	NNP
Sig	x	Xx	Xx

Feature Weights

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…			
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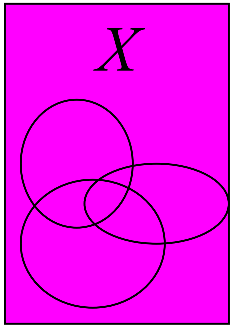
Classification

- What do these joint models of $\mathrm{P}(X)$ have to do with conditional models $\mathrm{P}(C|D)$?
- Think of the space $C \times D$ as a complex X.
  - C is generally small (e.g., 2-100 topic classes)
  - D is generally huge (e.g., space of documents)
- We can, in principle, build models over $\mathrm{P}(C,D)$.
- This will involve calculating expectations of features (over $C \times D$):

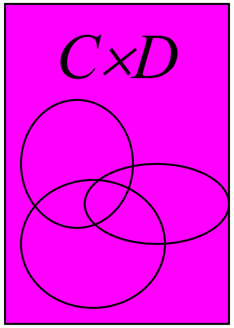
$$E(f_i) = \sum_{(c,d) \in (C,D)} P(c,d) f_i(c,d)$$

- Generally impractical: can't enumerate d efficiently.



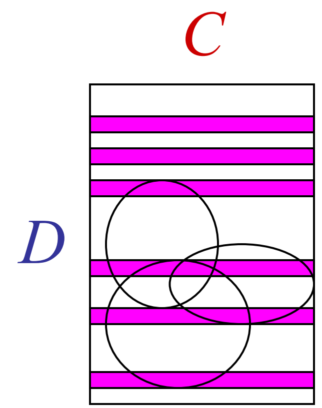
C

D



Classification II

- D may be huge or infinite, but only a few d occur in our data.
- What if we add one feature for each d and constrain its expectation to match our empirical data?



$$\forall (d) \in D \quad P(d) = \tilde{P}(d)$$

- Now, most entries of $\mathrm{P}(c,d)$ will be zero.
- We can therefore use the much easier sum:

$$E(f_i) = \sum_{(c,d)\in(C,D)} P(c,d) f_i(c,d)$$
$$= \sum_{(c,d)\in(C,D) \wedge \hat{P}(d) > 0} P(c,d) f_i(c,d)$$

Classification III

- But if we've constrained the D marginals

$$\forall (d) \in D \quad P(d) = \tilde{P}(d)$$

then the only thing that can vary is the conditional distributions:

$$P(c,d) = P(c|d)P(d)$$

$$= P(c|d)\tilde{P}(d)$$

- This is the connection between joint and conditional maxent / exponential models:
  - Conditional models can be thought of as joint models with marginal constraints.
- Maximizing joint likelihood and conditional likelihood of the data in this model are equivalent!