COLOR THEORY

TOPICS TO BE COVERED

The color problem

Cone response and Tristimulus Vectors

Solution to the Color problem – using CMFs

Chromaticity Diagrams

Color Spaces and Examples

Displays

Color Quantization

THE COLOR PROBLEM

How do we perceive an object?

How do we capture the image of an object?

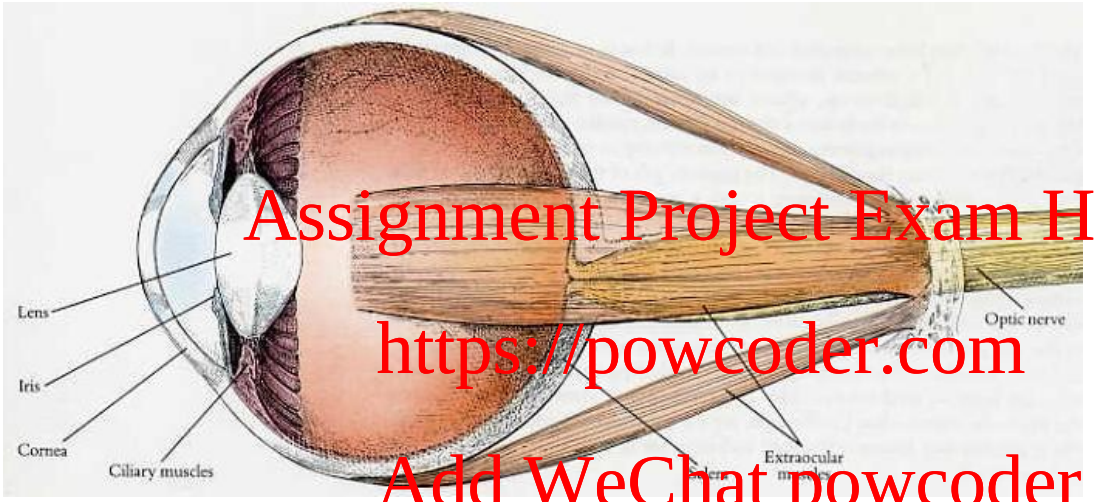
- **Digital Still Camera**
- **Camcorders**
- **Film processing/scanning**

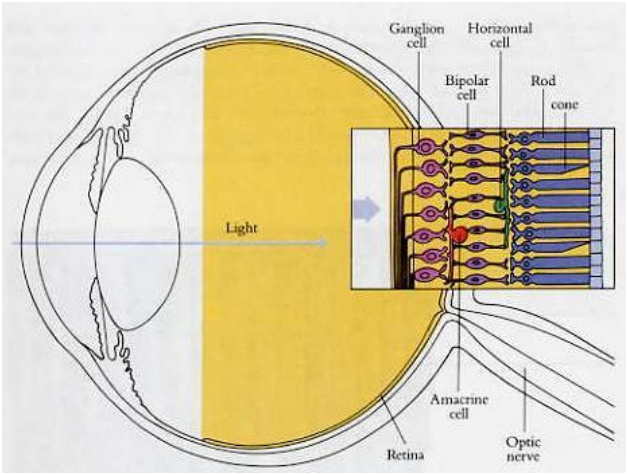
How do render an object image

- **Storage/Transfer issues**
- **Rendering on Client**
- **Printing**

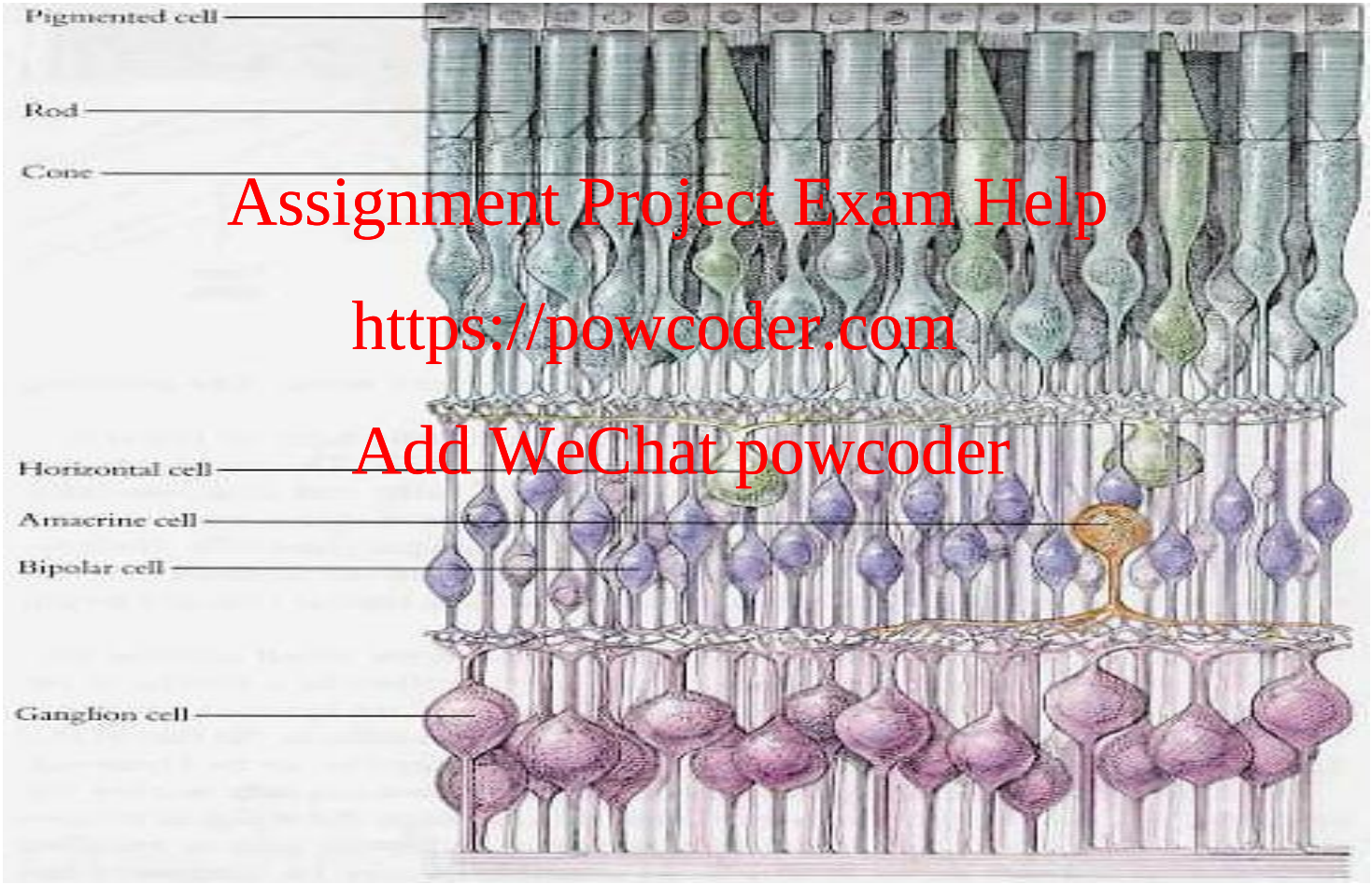
The Color Problem – How do you ensure that the colors in the image of a rendered object look the same as the original colors of that object?

EYE STRUCTURE

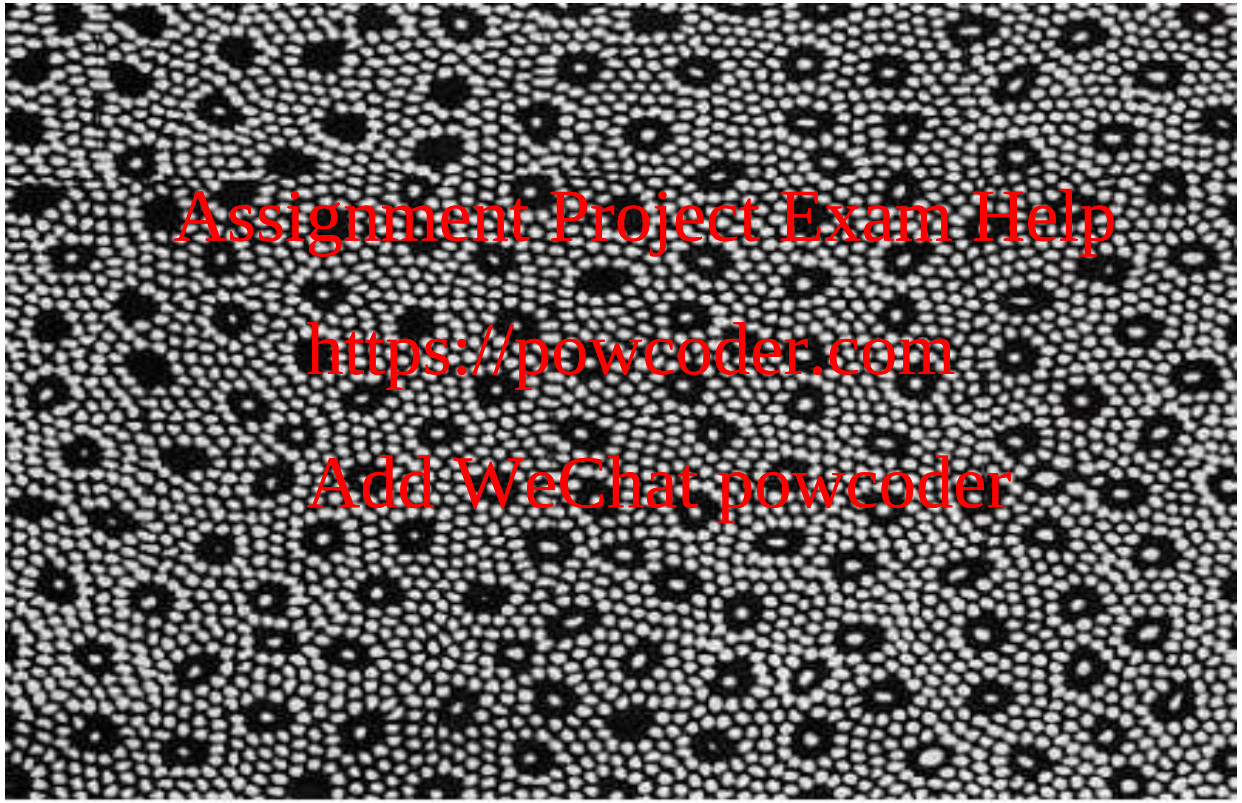




RETINA



RETINA



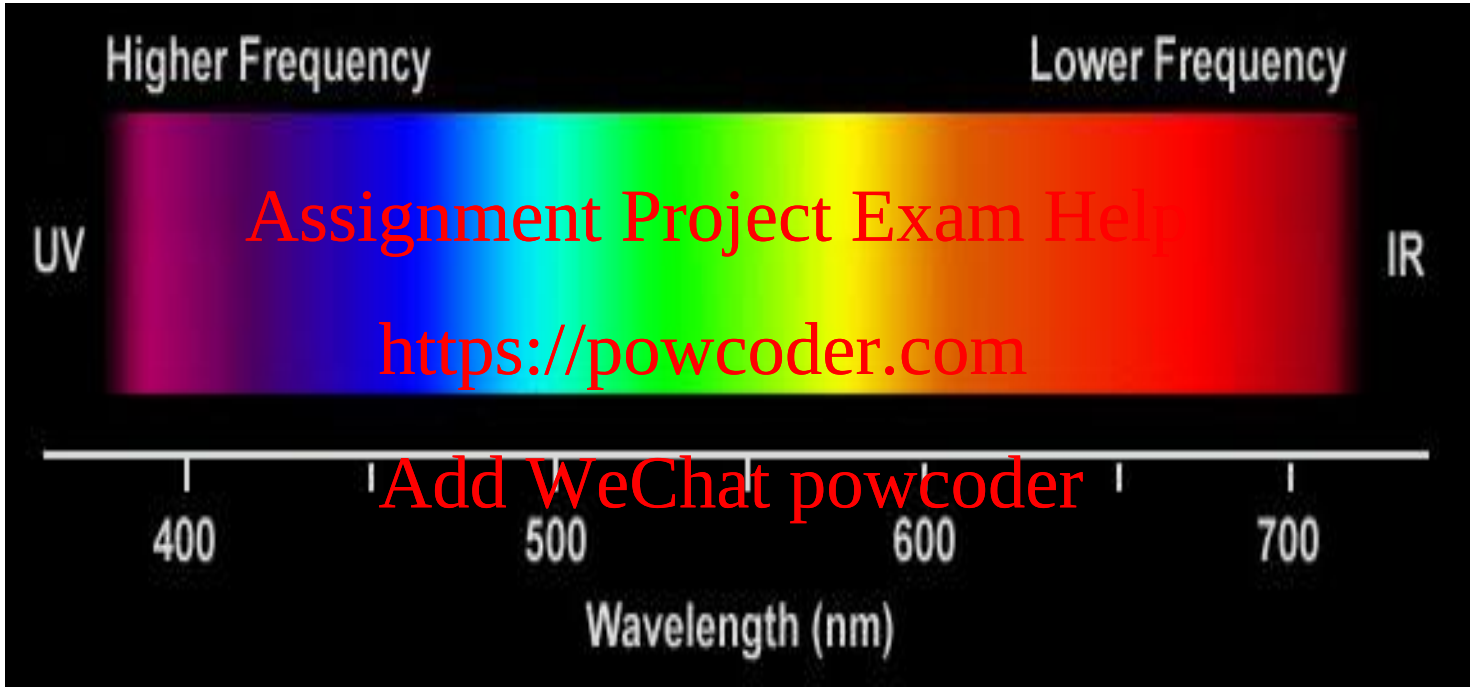
HOW DO WE PERCEIVE COLORS?

Spectrum of visible light *f(* λ **): λ =360-830 nm
(λ = wavelength)

In the *retina*: 3 types of cones with different spectral absorptance (or sensitivity). The response *c i* of a cone of type *i* is modeled as

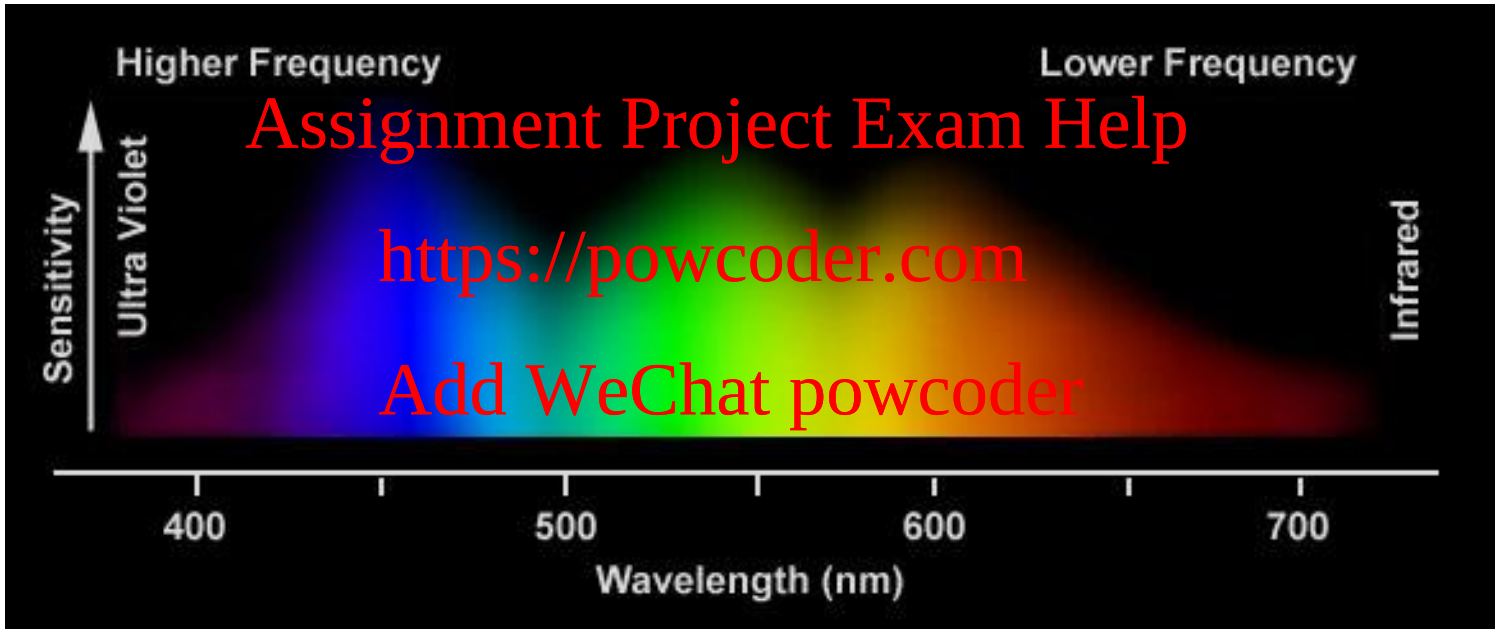
$$c_i = \int_{\lambda_{\min}}^{\lambda_{\max}} s_i(\lambda) f(\lambda) d\lambda$$

SPECTRAL DISTRIBUTION

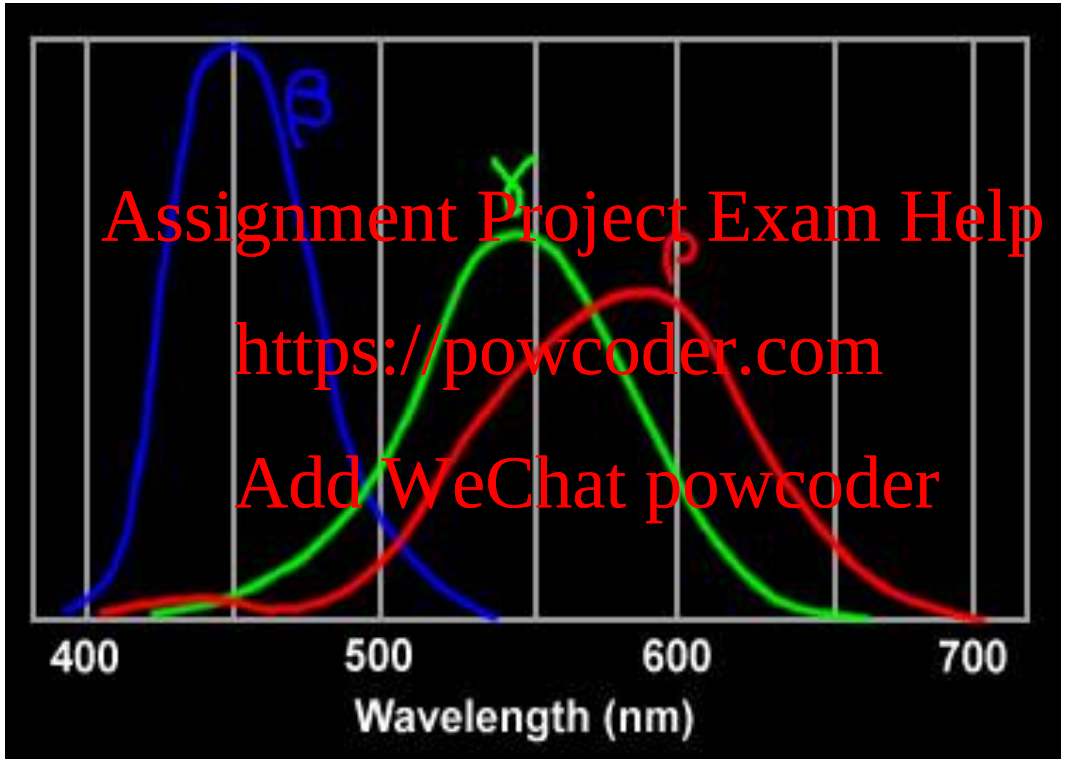


SPECTRAL SENSITIVITY

Sensitivity for Entire Spectrum



SPECTRAL SENSITIVITY OF CONES



DISCRETE FUNCTION EVALUATION OF CONE RESPONSE

Instead of evaluating the cone response as a continuous integral function, we will use discrete quantities with summations

- **Represent the light spectral stimulus $f(\lambda)$ as a vector f on N different wavelength (with a spectral interval of 10 nm)**
- **Represent each sensitivity $s_i(\lambda)$ as vector s_i as *N* different wavelength**

Then the cone response is represented by a vector (called tristimulus vector):

$$C = S^T f$$

TRISTIMULUS VECTORS

$C = S^T f$ is a 3-vector (called *color*) which represents how we *perceive* a light with spectrum f (which is a N-vector).

If two different spectra f_1 and f_2 produce the same color $C = S^T f_1 = S^T f_2$, then they *look the same*, that is, they are undistinguishable by the human eye.

Thus, it is *not* important that after processing, transmission and reproduction we recreate the same light spectrum f as the original, but that we reproduce a light spectrum that has the same perceived color c.

Thus, 3 values (the entries of C) are sufficient to represent the color of a pixel (instead of *N* values)

ISSUES WITH GENERATING COLOR

The tristimulus or color equation is $C = S^T f$

- **We need to know S**
- **We need to know f**

We don't actually know S; we will show soon that it doesn't matter

To understand f we need to know

- **How should we capture a light spectrum? (How do we take a color picture?)**
- **How do we reproduce a light spectrum? (How do we render a color?)**

HOW DOES A 3-CCD CAMERA WORK?

Light is refracted onto 3 CCDs (through a prism). In front of each CCD there is a *color filter* with frequency response $m_i(\lambda)$. Thus, a pixel of the i-th CCD gives an output

$$a_i = \int_{\lambda_{min}}^{\lambda_{max}} m_i(\lambda) f(\lambda) d\lambda$$

In discrete form: $a = M^T f$ where the i-th column of M contains the samples of $m_i(\lambda)$

Note that M can only have positive entries!

HOW DOES A COLOR PROJECTOR WORK?

Color Projectors work using Additive Synthesis: there are actually 3 light projectors focusing light onto the same spot.

Let the light spectra produced by each be: p_1, p_2, p_3. The intensity of each projector can be controlled at each pixel via a *gain* a_i. Thus, the actual light spectrum on the screen is $g = p_1a_1 + p_2a_2 + p_3a_3 = Pa$

where $P=[p_1, p_2, p_3]$ (size: Nx3) and $a=[a_1, a_2, a_3]^T$ (size:3x1)

Are 3 projectors able to create all the *light spectra* that we want to reproduce? No! But we just need that they create the *colors* we need to reproduce.

COLOR PROJECTOR (2)

Let { p_1, p_2, p_3 } by three "colorimetrically independent" light sources (called *primaries*).

Then, given any spectrum f, we can always find gains a such that the light g = Pa generated by additive synthesis has the same *color* of f.

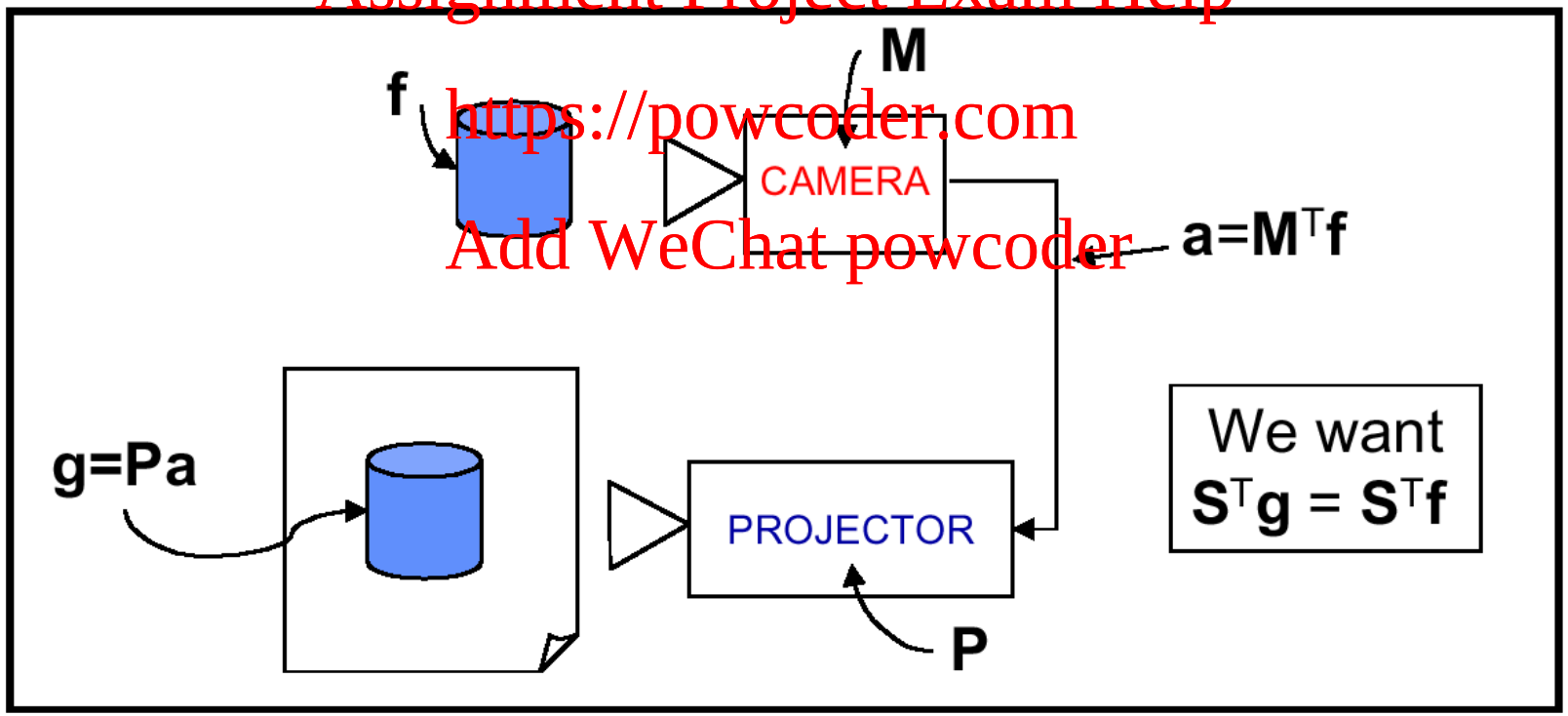
***Proof*: We need to find a vector a such that**

$$S^T g = S^T P a = C.$$ **Just choose** $a=(S^T P)^{-1} C$.

***Problem*: a may have negative components, which makes no physical sense (gains and spectra can only have positive values!)**

THE PROBLEM

Given a set of primaries P, how can we be sure that for any light spectrum f, the projector reproduces a light spectrum with the same color of f?



THE PROBLEM (2)

Given a set of primaries P, design the "filters" M of a color camera such that for any input spectrum f, the light g generated by the projector has the same color of f.

In other words: we want that the projectors, using as gains the output a of the camera, create a light g = Pa such that $S^T g = S^T P a = S^T f = C$.

And we want to do it without knowing S (which is difficult to compute)

PROBLEM SOLUTION

The solution is based on color matching experiments

The columns { m_1, m_2, m_3 } of M (size: *Nx*3) are called the color matching functions (CMF) associated with the primaries P.

Problem (again): CMF can take on negative values.

WHAT ARE COLOR MATCHING FUNCTIONS?

Like we said before: the three CMFs { m_1, m_2, m_3 } correspond to the " filters" in a color camera to generate the three color channels a = { a_1, a_2, a_3 }.

Each color signal can be used to control the intensities of the primaries (projectors) in order to generate a spectrum g = Pa that has the same color as f

$$S^T g = S^T P a$$

Note: Given a set or primaries P, there is just one set of CMFs M associated to it, and vice-versa

Remember, C is the "perceptual" color vector; a is the color vector associated to the chosen primaries

CHANGING PRIMARIES

What happens when we change primaries? (e.g., we use a different projector, or we use a CRT screen instead of a projector)

Suppose we use primaries Q instead of P. We want to find a new color vector b **such that**

$$S^T P a = S^T Q b$$

The solution is b=Ta with $T=(S^T Q)^{-1}(S^T P)$

(T is a 3x3 matrix which is independent of a)

Thus T is a linear operator that maps from one color space into a new one.

CIE CMFS

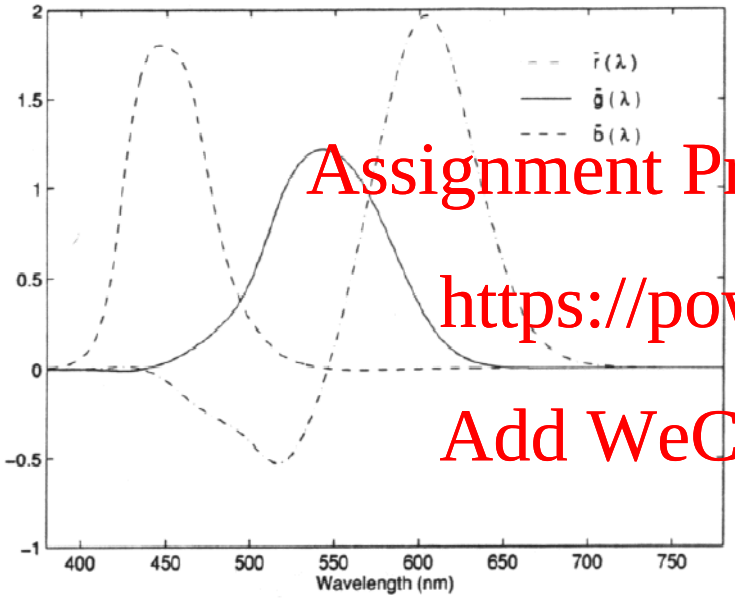
The International Commission on Illumination (CIE) has standardized two sets of CMFs: *RGB* and *XYZ*

RGB: The CMF's are associated with *monochromatic primaries* at wavelengths of 700, 546 and 435 nm. (Note that one RGB CMF takes on -ve values.)

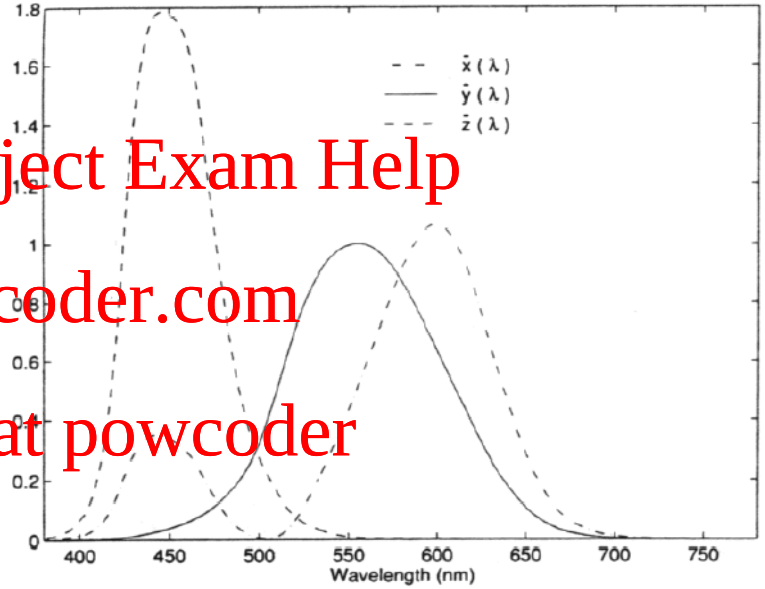
XYZ: One of the 3 CMFs, y(λ), is the *luminous efficient functio*n, which gives the relative sensitivity of the eye to each wavelength. *Y* is called *luminanc*e. (Note that all *XYZ* CMFs are positive; however, it can be shown that the corresponding primaries are not physically realizable)

Note: *{r(λ),g(λ),b(λ)}* ({*x(λ),y(λ),z(λ)*}) are the CMFs (corresponding to the columns of M); the values *{R,G,B}* {*X,Y,Z}* are the color channels (corresponding to a)

CIE CMFS



CIE RGB CMFs

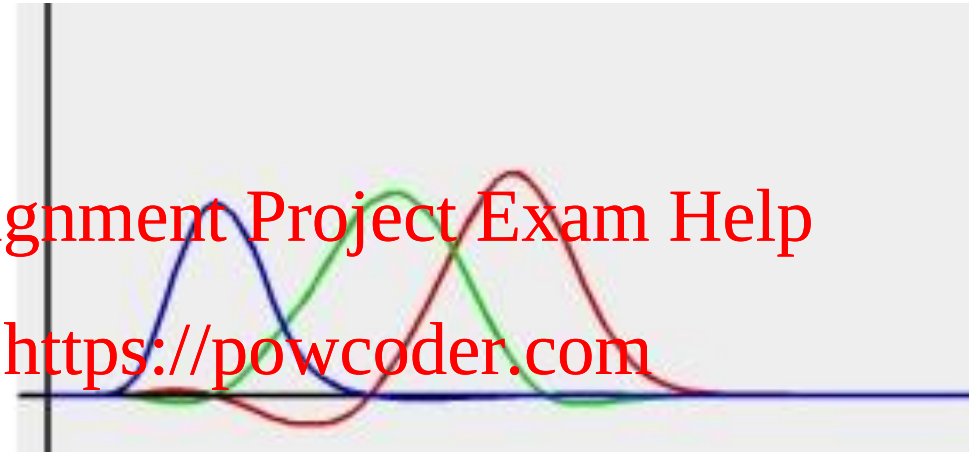


CIE XYZ CMFs

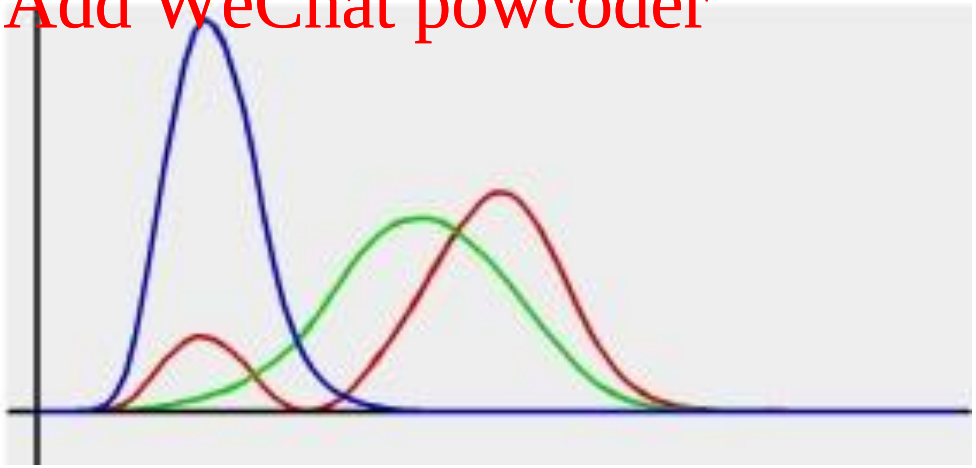
CIE CMFS

R, G, B primaries
(pure wavelength)

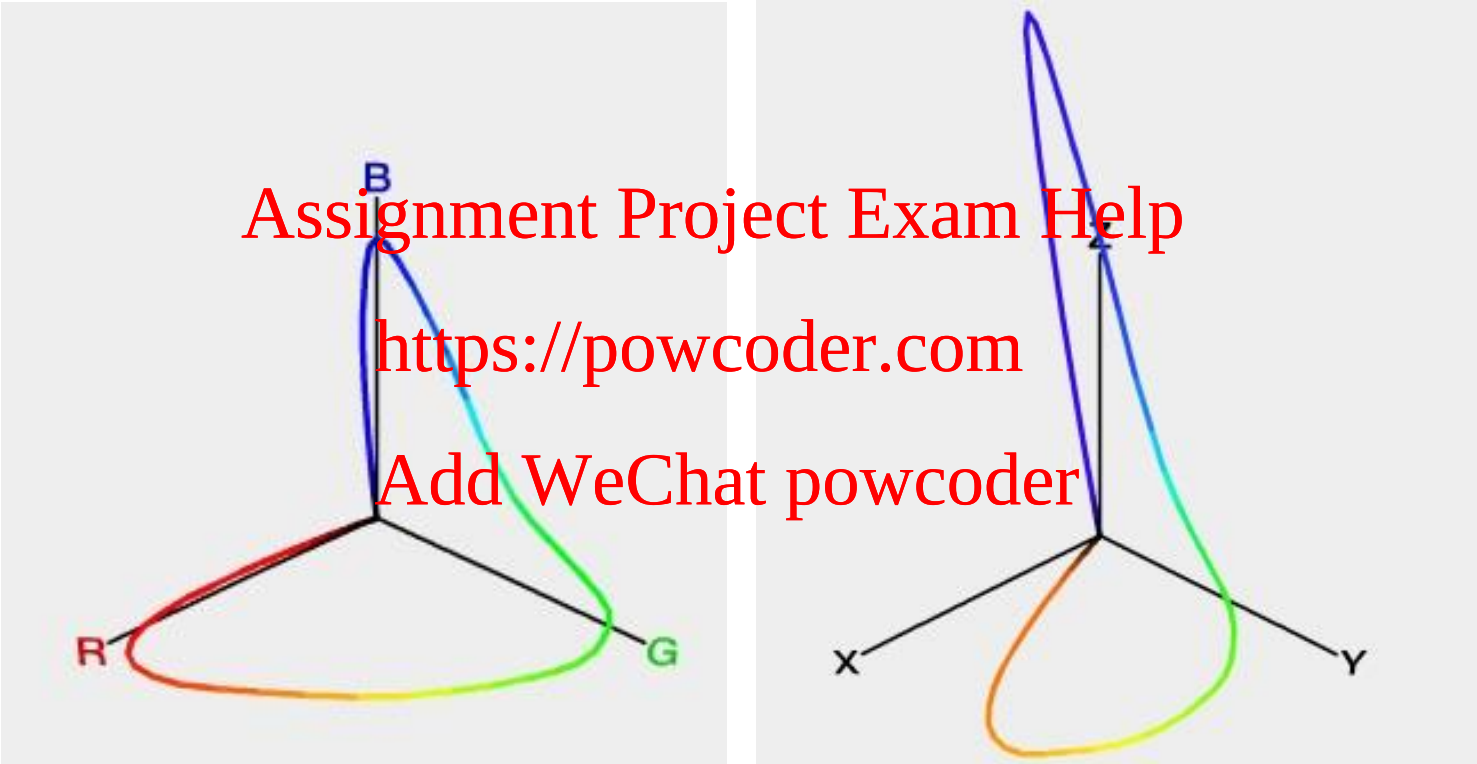
R = 615 nm
G = 525 nm
B = 445 nm



X, Y, Z primaries
(physically impossible but theoretically useful)



RGB & XYZ COLOR SPACE



CHROMATICITY DIAGRAMS

Can we represent all colors by additive synthesis of 3 primaries? No! **That's because CMFs can assume negative values, and the frequency responses of the color filters can only be positive.**

***Problem*: represent all achievable colors on a 2-D plane (color channels $a_1 a_2 a_3$ live in a 3-D space!)**

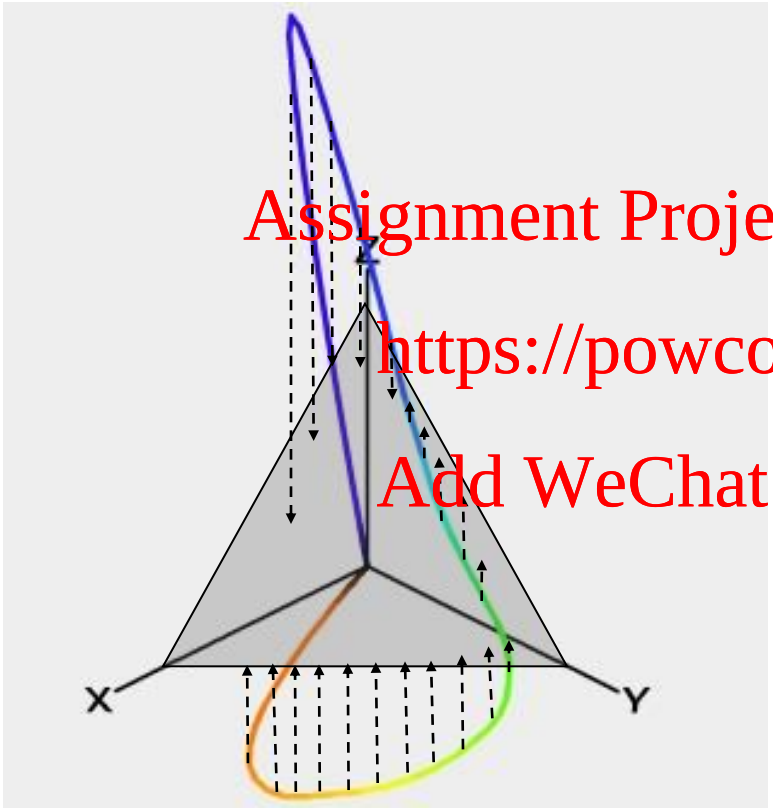
***Solution*: *normalize* colors to live in 2-D**

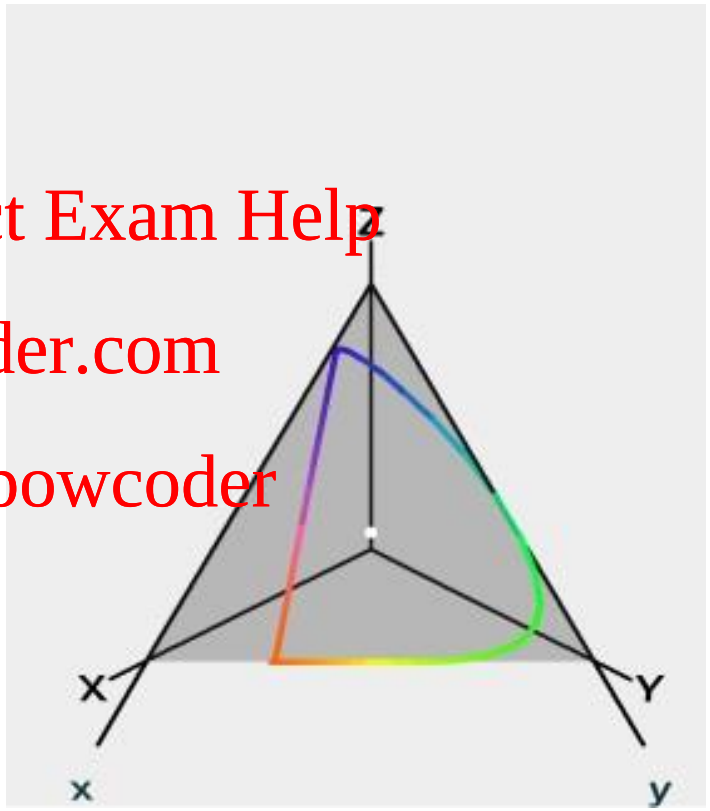
***Example*: CIE xy chromaticity diagram. Given a color value (X,Y,Z), compute the normalized values**

x=X/(X+Y+Z), y=Y/(X+Y+Z), z=Z/(X+Y+Z).

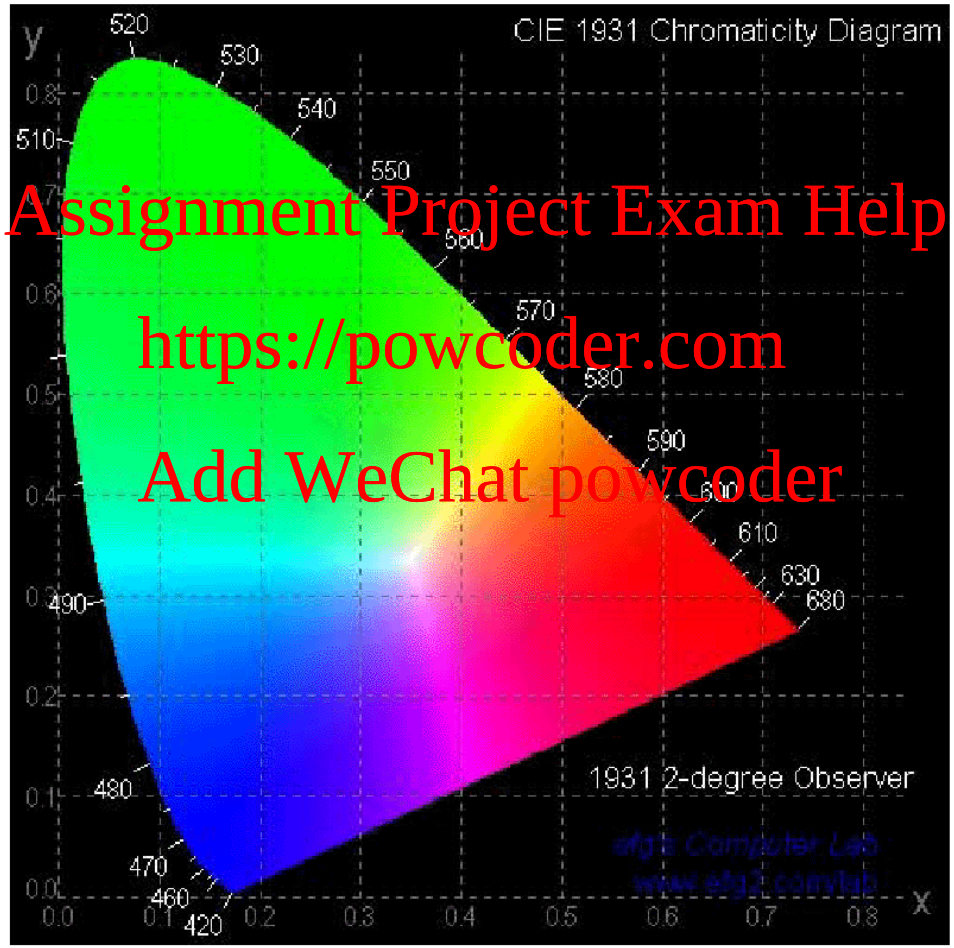
Then, plot only the colors corresponding to 2 chromaticity coordinates (x, y)

NORMALIZED XYZ SPACE



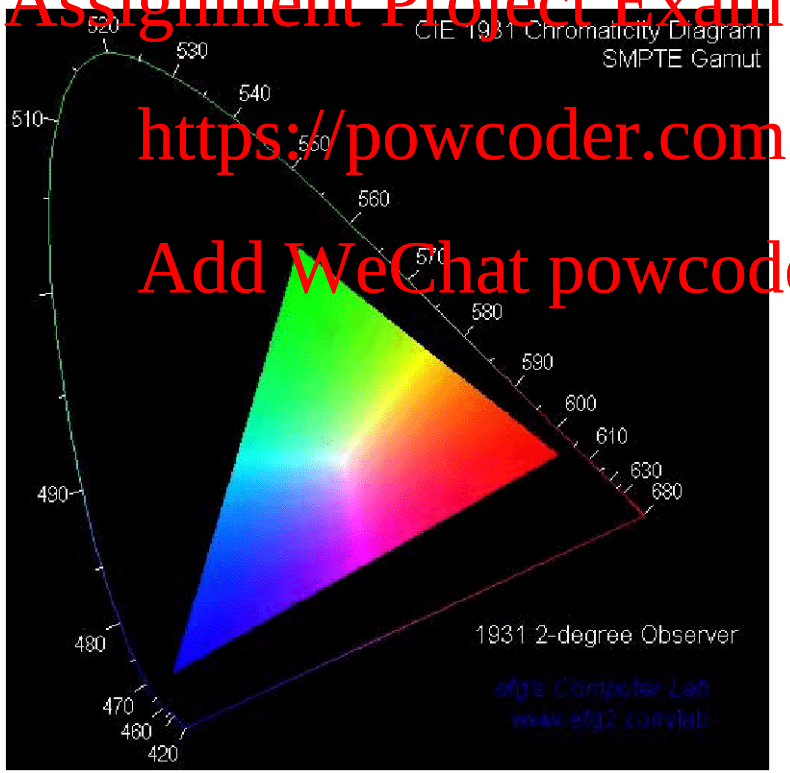


CIE *XY* CHROMATICITY DIAGRAMS



COLOR GAMUTS

Color gamut **of a color synthesis device = set of obtainable colors. Represented by a *polygonal* on the chromaticity diagram, example shown here**



VISUALIZATIONS

Color Applets

Visualization of CIE XYZ and RGB

Visualization of Color Gamuts

LINEAR TRANSFORMATIONS OF *RGB* COLOR SPACES

***RGB*: used in CRT monitors (additive synthesis)**

***YIQ* (used in NTSC TV). T=**
$$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & 0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix}$$

***YUV* (used in PAL, SECAM). T=**
$$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.1 \end{pmatrix}$$

***YCrCb* (for JPEG, MPEG). T=**
$$\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.500 & -0.4187 & -0.0813 \\ -0.1687 & -0.3313 & 0.5 \end{pmatrix}$$

2/7/2001

WHY IS *RGB* NOT USED?

Similarly to the processing in the human visual system, transformed color spaces such as *YIQ, YUV, YCrCb* represent color with *luminance (*Y) and *chrominance* (the other 2 channels). Advantages:

- **We can subsample the chrominance channels (e.g., 4:2:2, 4:2:0 subsampling schemes)**
- **We can quantize the chrominance channels more coarsely (with fewer bits)**
- **The chrominance channels are rather *uncorrelated* with the luminance channel, which yields better compression.**

OTHER TRANSFORMATIONS

***CMY*: Cyan, magenta, yellow – complementary to red, green and blue respectively. Used for *subtractive synthesis* from white in color printers.**

(C, M, Y)* = (1, 1, 1) - *(R, G, B)

***CMYK*: like *CMY*, uses black *(*K) as fourth color.**

Given *(C, M, Y)*:

K=min(*C, M, Y*); *C=C-K; M=M-K; Y=Y-K*

UNIFORM COLOR SPACES

CIE XYZ is not perceptually uniform because perceptual differences between colors do not correspond to equal distance in color space

CIE defined two uniform color spaces:

CIE Luv and CIE Lab:

- **L depends only on Y (luminance)**
- **Radial distance (e.g. sqrt(u^2+ v^2)) correlates to chroma (or saturation)**

- **Angular position (e.g. $\tan^{-1}(u/v)$) correlates to hue**

HSV COLOR SPACE

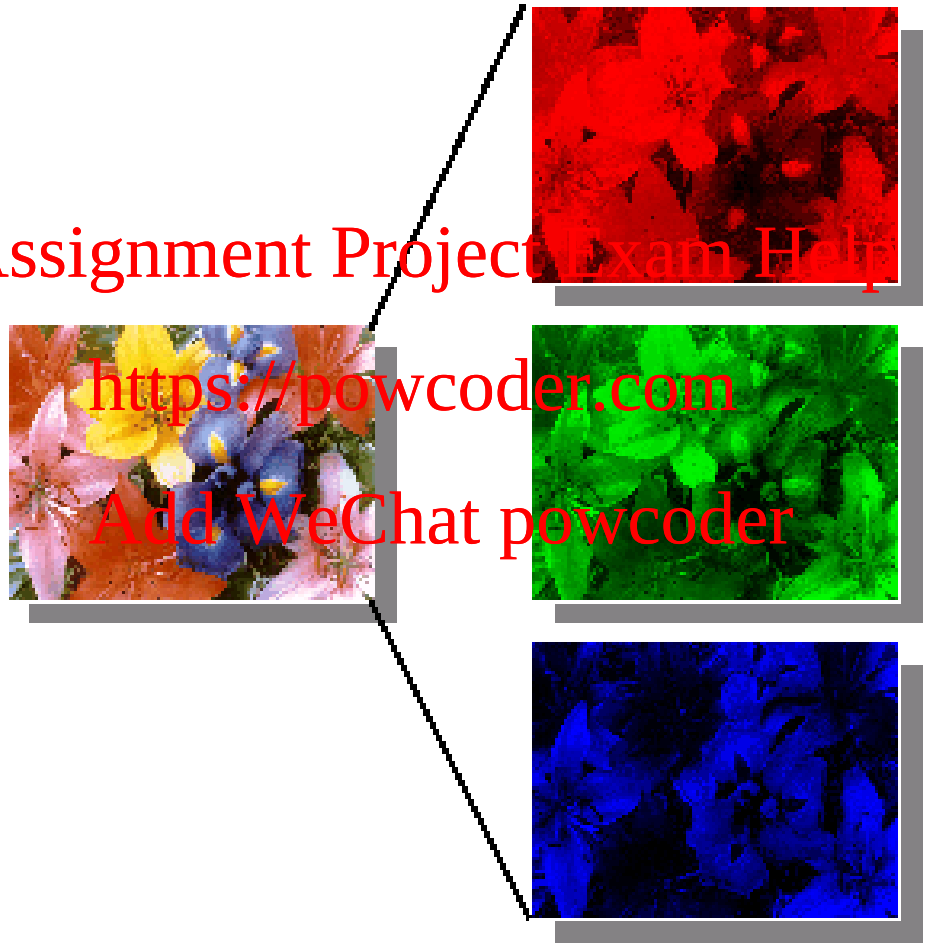
***HSV* is based on a cylindrical coordinate system**

***H (hue*) is measured in angles with respect to the *V (value*) axis. *S (saturation*) goes from 0 to 1.**

Complementary colors in HSV are 180 degrees from each other.

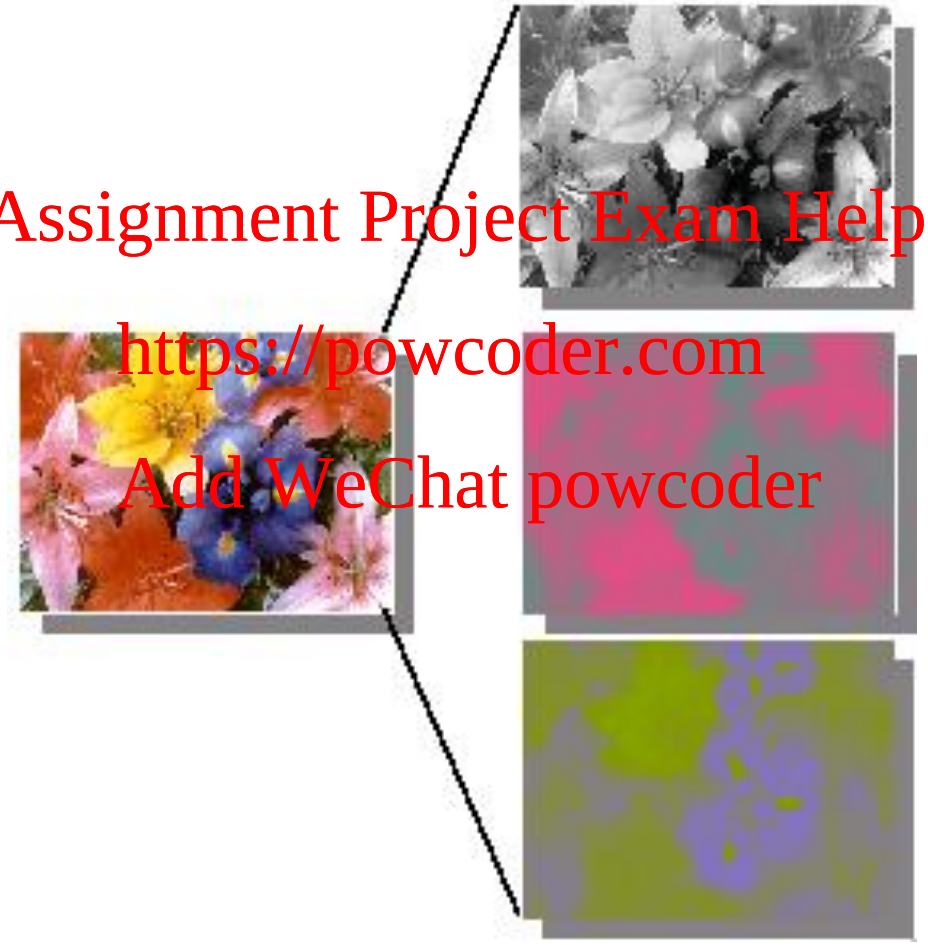
***Hue* distinguishes among colors such as red, green, purple, and yellow. *Saturation* refers to how far color is from a gray of equal intensity. *Value* represents the lightness (or brightness)**

COLOR SPACE EXAMPLES – RGB



COLOR SPACE EXAMPLES – YIQ

COLOR SPACE EXAMPLES – YUV



COLOR DISPLAYS

Visible light is produce by bombardment of a thin layer of phosphor by a beam of electrons.

Shadow-mask color CRT: mosaic of *r,g,b* light emitting phosphors illuminated by 3 independent electron beams.

Color is controlled through application of different voltage to *r, g, b* guns.

How can we take into account non-linearities?

GAMMA CORRECTION

For each gun, the power of the emitted light $S_i(\lambda)$ is a function of the *control voltage* v_i. Ideally, the relationship would be linear: $S_i(\lambda) = v_i p_i(\lambda)$ (so we would set $v_i = a_i$.) In practice, the relation is *not linear*. A better approximation is

$$S_i(\lambda) = (v_i / v)^{\gamma} p_i(\lambda)$$

where v is the maximum value of the voltage and γ is the exponential parameter. The monitor γ is usually approximately equal to 2.2

Thus, given a value of a_i, instead of controlling the light with voltage $v_i = a_i$, we should use $v_i = v\, a_i^{1/\gamma}$

This transformation is called *gamma correction*.

CCD CAMERAS

3 CCD

- **Light is refracted into each CCD using a prism**
- **In front of each CCD there is a color filter (corresponding to CMF functions *r*, *g*, *b*)**
- **More expensive (need 3 CCDs per pixel) – but better resolution**

1 CCD

- **There is 1 filter (*r*, *g* or *b*) on top of each pixel. The filters are chosen according to a "mosaic" layout. E.g.**

  r g b r g b r g b
  g b r g b r g b r
  b r g b r g b r g

- **Cheaper (1 CCD) but with less color resolution (e.g.: for an *r* pixel, the other colors must be interpolated from nearby pixels.**

COLOR QUANTIZATION

What is color quantization and why is it required?

Frame buffer architecture: Image is stored in a video memory from which controllers constantly refresh the display screen. How many bits per pixel should we use? (8-12-16-24…)

The more pixels you use, the longer it takes to read and longer to refresh your monitor screen.

Palletized image: Uses a finite set of colors indexed in a table (*palette* or *color m*ap or LUT). You may choose this color mapping in two ways

- **Pre designed or fixed**
- **Adaptively - "quantized" version is similar to the original**

ADAPTIVE COLOR QUANTIZATION METHODS

The process occurs in stages

- **Sampling the original image for color statistics – (Histogram)**
- **Choosing a color map based on those statistics**
- **Mapping colors to their representative colors to get new image**

Different algorithms proposed and used depend on how the second step above is done

- **Uniform Quantization**
- **Median Cut – (Heckbert)**
- **Octree – (Gervautz & Purgathofer)**
- **Peano Curve Mapping**

QUESTIONS

The chromaticity diagram in (*x*, *y*) represents the normalized color matching functions X, Y and Z. Prove that

$$Z = [(1 - x - y)/y]\,Y$$

QUESTIONS

The chromaticity space is a 2D space obtained by projecting the 3D conical XYZ space onto a plane.

- **What is the equation of this plane**
- **Why is there no black color in the chromaticity space?**
- **Where do the ClE primaries lie in the chromaticity space?**