Base Conversion

Three cases:

I. From any base *b* to base 10
II. From base 10 to any base *b*
III. From any base *b* to any other base *c*

From Base *b* to Base 10

- Base (radix): b
- Digits (symbols): 0 … (b – 1)
- $S_{n-1}S_{n-2}\cdots S_2S_1S_0$

$$\text{Value} = \sum_{i=0}^{n-1} (S_i b^i)$$

Use summation to transform any base to decimal

From Base *b* to Base 10

- Example: $1234_5 = ?_{10}$

  $= 1 * 5^3 + 2 * 5^2 + 3 * 5^1 + 4 * 5^0$

  $= 125 + 50 + 15 + 4$

  $= 194$

- Example: $201_5 = ?_{10}$

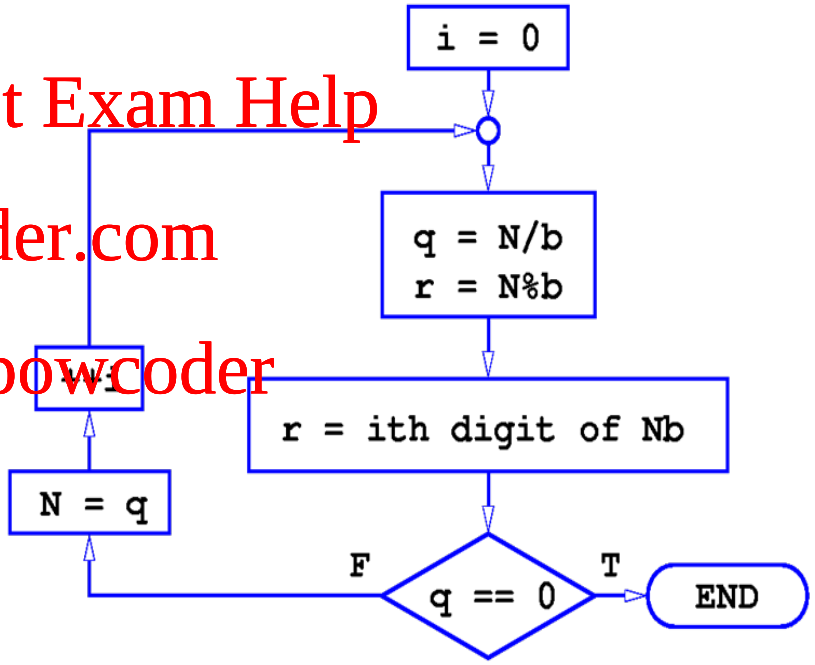
  $= 2 * 5^2 + 0 * 5^1 + 1 * 5^0$

  $= 50 + 1$

  $= 51$

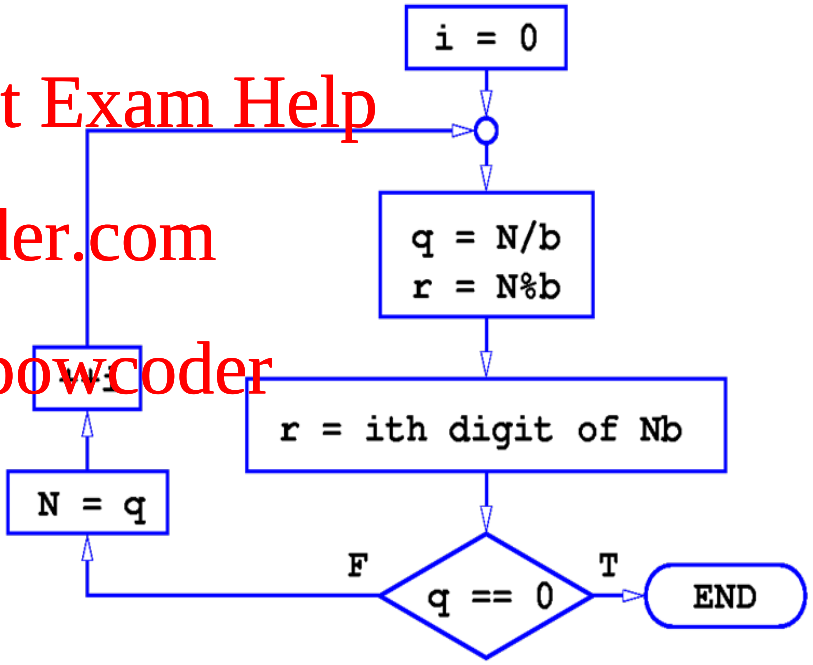
From Base 10 to Base *b*

- Use successive divisions
- Remember the remainders
- Divide again with the quotients



From Base 10 to Base *b*

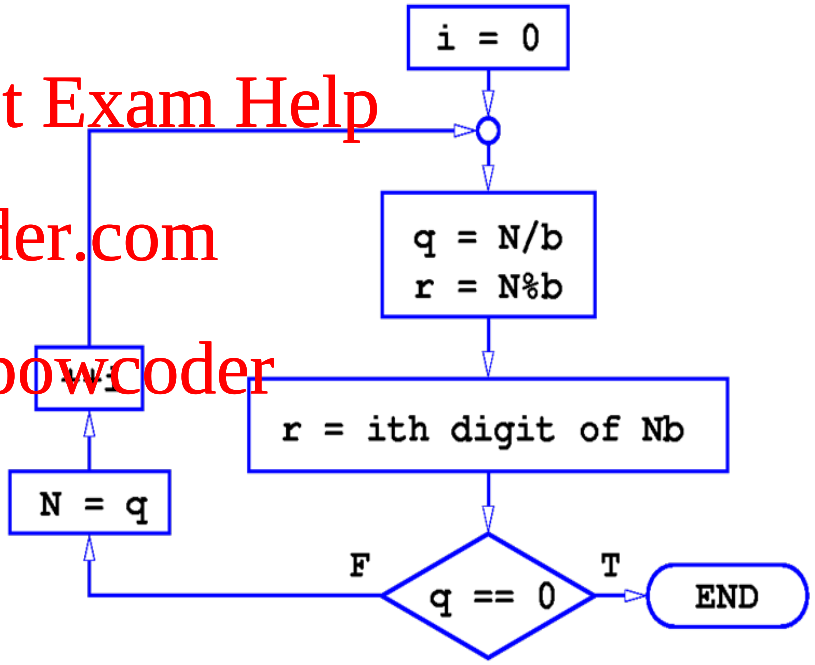
Example: $123_{10} = ??????_5$



From Base 10 to Base *b*

Example: $123_{10} = ?????_5$

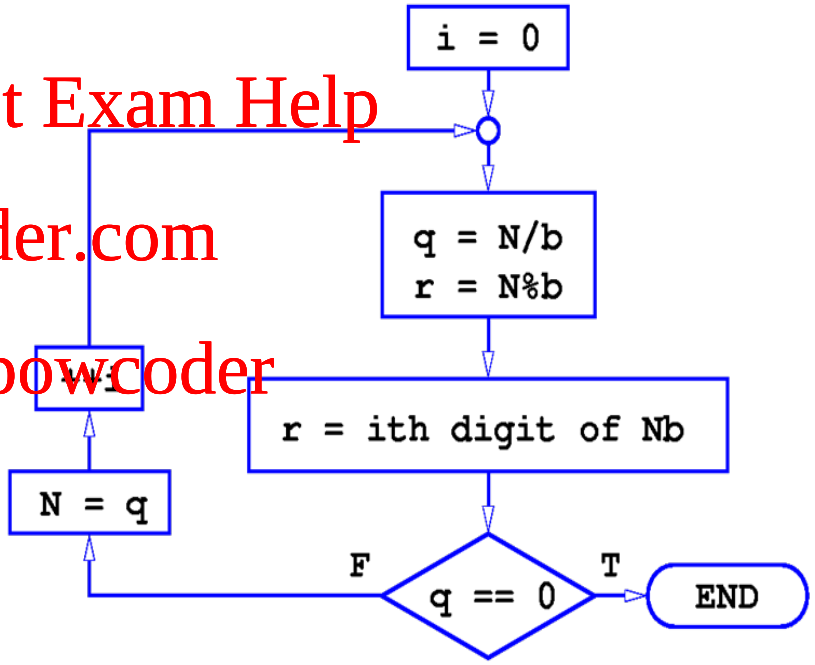
- N = 123, i = 0, b = 5
- q = N/b = 123/5 = 24
- r = 123 % 5 = 3
- $123_{10} = \_\ \_\ \_\ 3_5$



From Base 10 to Base *b*

Example: $123_{10} = \_\ \_\ \_\ \underline{3}_5$

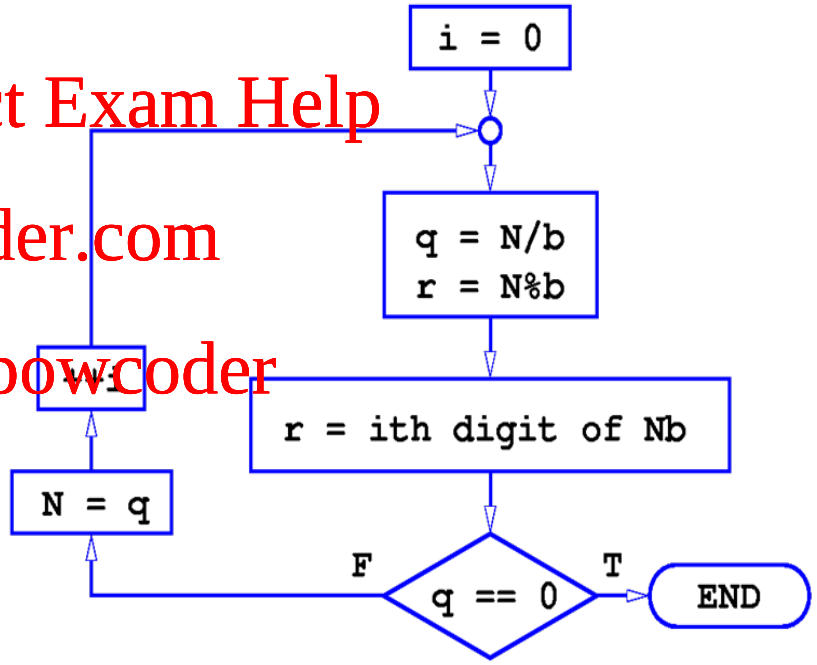
- N = 24, i = 1, b = 5
- q = N/b = 24/5 = 4
- r = 24 % 5 = 4
- $123_{10} = \_\ \_\ 4\ 3_5$



From Base 10 to Base *b*

Example: $123_{10} = \_\ \_\ 4\ 3_5$

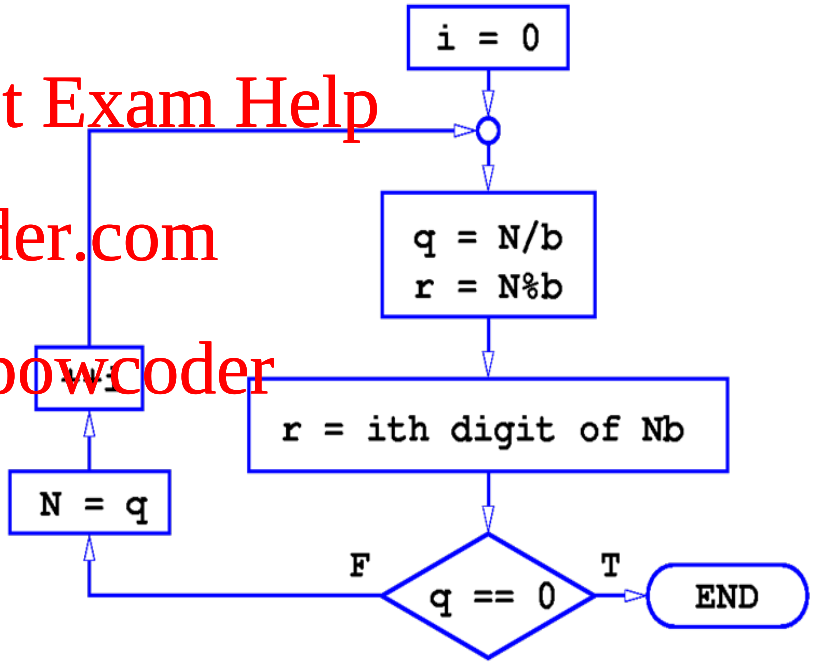
- N = 4, i = 2, b = 5
- q = N/b = 4/5 = 0
- r = 4 % 5 = 4
- $123_{10} = \_\ 4\ 4\ 3_5$



From Base 10 to Base *b*

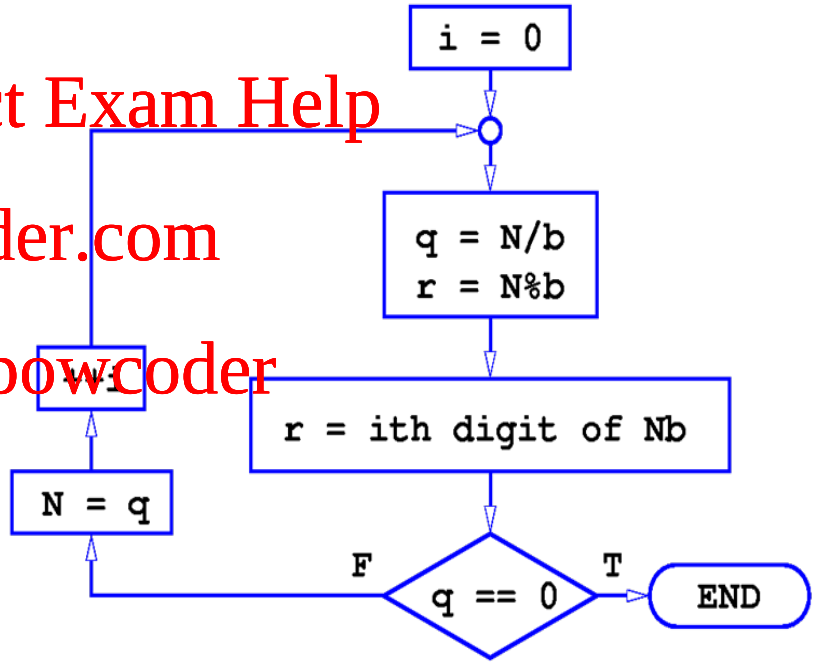
Example: $123_{10} = 443_5$

- q = 0, algorithm ends!



From Base 10 to Base *b*

Example: $2010_{10} = ?_5$

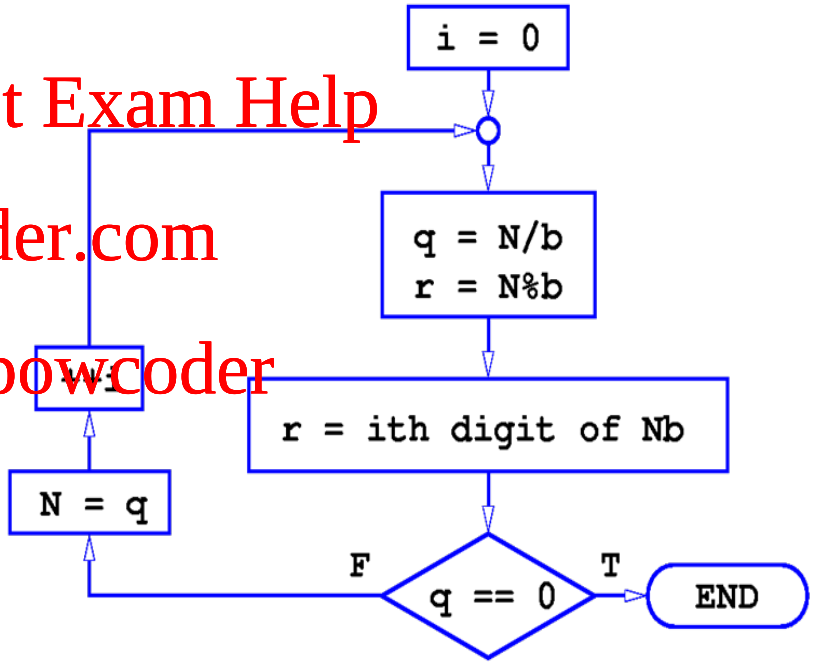


From Base 10 to Base *b*

Example: $2010_{10} = ??????_5$

Do it at home!!!

(ans = 31020)



From Base *b* to Base *c*

- Use a known intermediate base
- The easiest way is to convert from base *b* to base 10 first, and then from 10 to *c*
- Or, in some cases, it is easier to use base 2 as the intermediate base (we'll see them soon)

Binary Number System

- Base (radix): 2
- Digits (symbols): 0, 1
- Binary Digits, or bits
- Example:
  - $1001_2 = 1*2^3 + 0*2^2 + 0*2^1 + 1*2^0$
    $= 8 + 0 + 0 + 1$
    $= 9_{10}$
  - $11000_2 = 1*2^4 + 1*2^3 + 0*2^2 + 0*2^1 + 0*2^0$
    $= 16 + 8$
    $= 24_{10}$

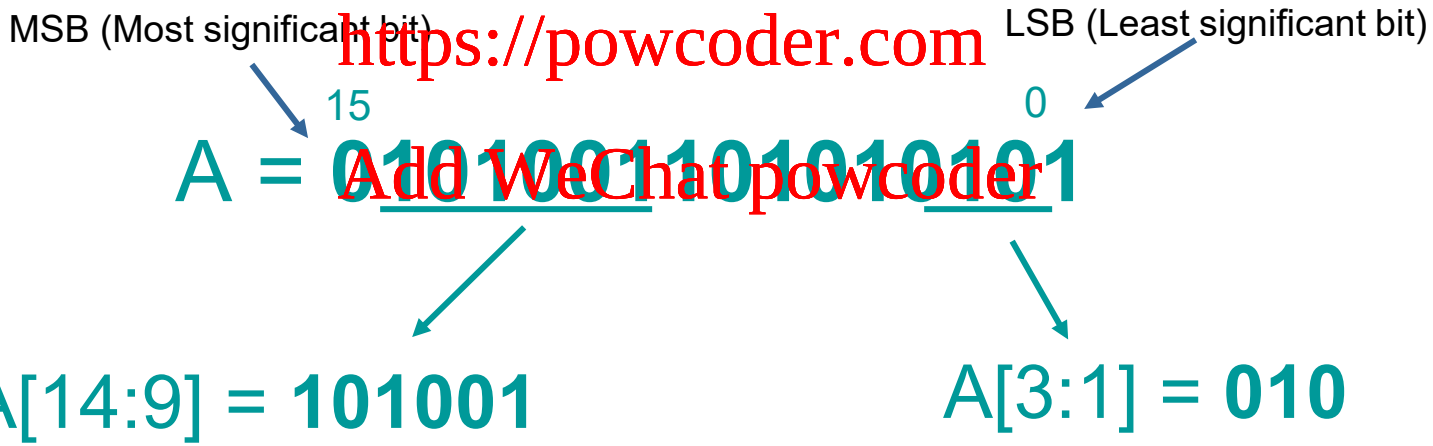
Octal Number System

- Base (radix): 8
- Digits (symbols): 0, 1, 2, 3, 4, 5, 6, 7
- $345_8 = 3*8^2 + 4*8^1 + 5*8^0$
  $= 192 + 32 + 5$
  $= 229_{10}$
- $1001_8 = 1*8^3 + 0*8^2 + 0*8^1 + 1*8^0$
  $= 512 + 1$
  $= 513_{10}$
- In C, octal numbers are represented with a leading 0 (0345 or 01001).

Representing Multi-bit Values

- Number bits from right (0) to left (n-1)

- Use brackets to denote range:
D[l:r] denotes bit **l** to bit **r**, from *left* to *right*



May also see A**<14:9>**, especially in hardware block diagrams.

Hexadecimal Number System

- Base (radix): 16
- Digits (symbols): 0-9, A–F (a-f)
- In C/MIPS: leading “0x” (e.g., 0xA3)
- Sometimes: leading “x” (e.g., “x3000”)
- Hexadecimal is also known as “hex” for short

Hex	Decimal
A	10
B	11
C	12
D	13
E	14
F	15

Examples of Converting Hex to Decimal

- $0xA3_{16} = A*16^1 + 3*16^0$

  $= 10*16 + 3*1$

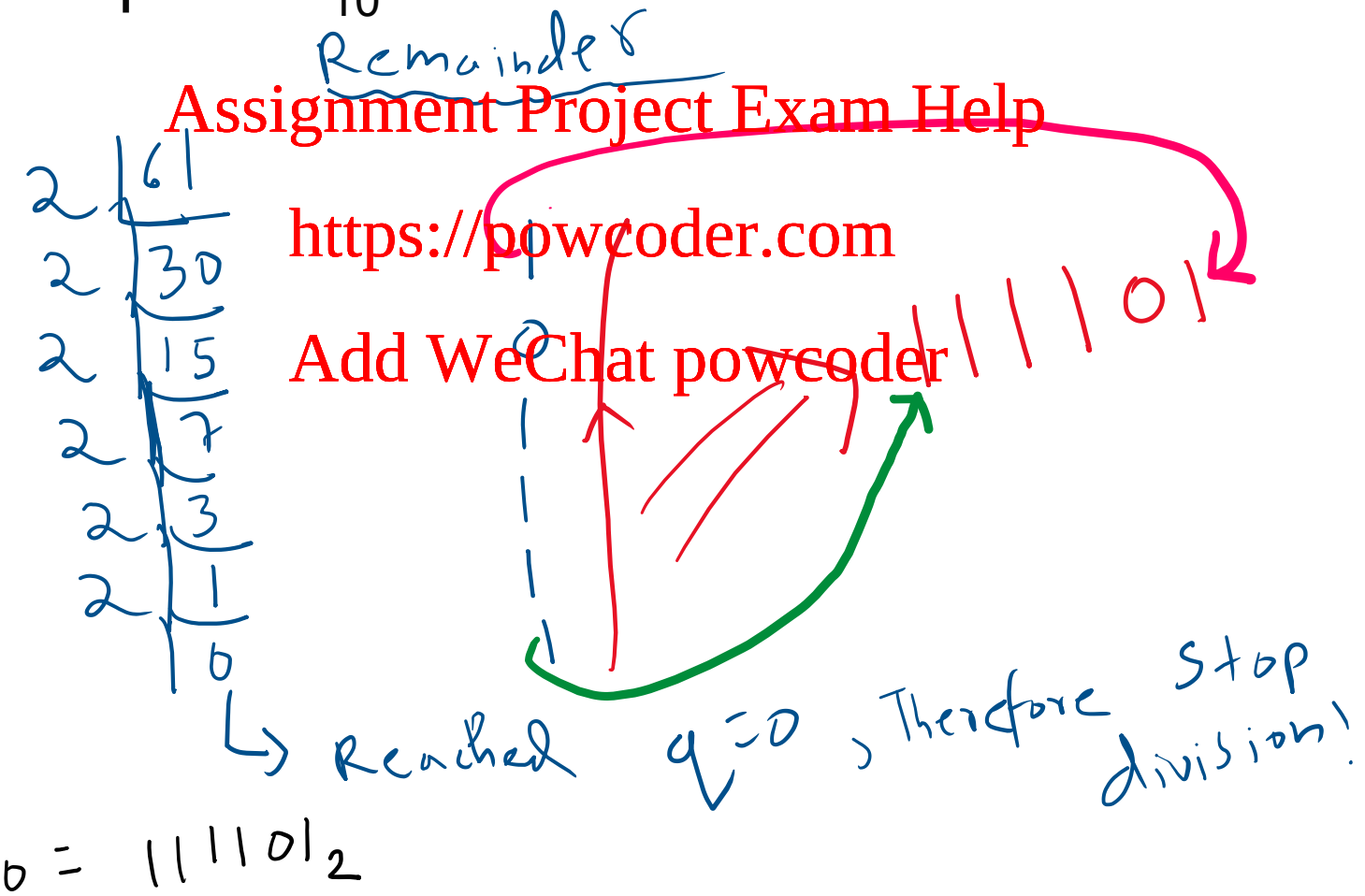
  $= 160 + 3$

  $= 163$

- $0x3E8_{16}$ = Do it at home! (answer is 1000)

Decimal To Binary Conversion: Method 1

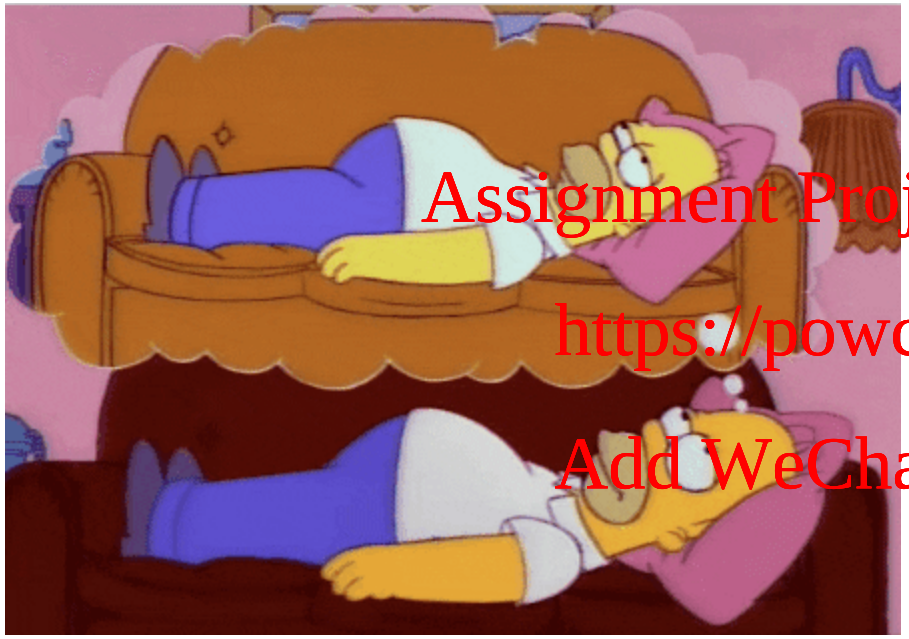
- Keep dividing decimal value by 2 until the value is 0;
- Example: 61_{10}



$61_{10} = 111101_2$

Knowing The Powers Of Two

- Know them in your sleep



2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1024

Yes it's on the exam.

☺

Binary to Octal Conversion

- Group into 3 starting at *least significant bit*
  - Why 3?
  - Add leading 0 as needed
    - Why not trailing 0's?
- Write one octal digit for each group

Binary to Octal Conversion: Examples

- `100 010 111` (binary)

  `4 2 7` (octal)

- `10 101 110` (binary)

  (octal)

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Binary to Octal Conversion: Examples

- `100 010 111` (binary)

  `4 2 7` (octal)

- `010 101 110` (binary)

  `2 5 6` (octal)

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal to Binary Conversion

- Write down the 3-bit binary code for each octal digit
- Example:
  - 047

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal to Binary Conversion

- Write down the 3-bit binary code for each octal digit
- Example:
  - 047

000 100 111

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Binary to Hex Conversion

- Group into 4 starting at least significant bit
  - Why 4?
  - Add leading 0 if needed
- Write one hex digit for each group

Binary to Hex Conversion: Examples

```
1001 1110 0111 0000  (binary)

 9    E    7    0    (hex)
```

```
 1 1111 1010 0011  (binary)

                   (hex)
```


Hex	Bin	Hex	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Binary to Hex Conversion: Examples

```
1001 1110 0111 0000  (binary)

  9    E    7    0   (hex)

0001 1111 1010 0011  (binary)

  1    F    A    3   (hex)
```


Hex	Bin	Hex	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Hex to Binary Conversion

- Write down the 4-bit binary code for each hex digit
- Example:
  - 0x 3 9 C 8

0011 1001 1100 1000

Know this in your sleep

Hex	Bin	Hex	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Hex to Binary Conversion

- Write down the 4-bit binary code for each hex digit
- Example:
  - 0x 3 9 C 8

0011 1001 1100 1000

Know this in your sleep

Hex	Bin	Hex	Bin
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Conversion Table

Decimal	Hexadecimal	Octal	Binary
0	0	0	0000
1	1	1	0001
2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	8	10	1000
9	9	11	1001
10	A	12	1010
11	B	13	1011
12	C	14	1100
13	D	15	1101
14	E	16	1110
15	F	17	1111

A practical example: HTML Color Codes

RGB

2 digit Hex value for Red, 2 digit Hex value for Green, 2 digit Hex value for Blue

FFFFFF	000000	333333	666666	999999	CCCCCC	CCCC99	9999CC	666699
660000	663300	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	660066
990000	993300	CC9900	006600	336666	0033FF	000099	660099	990066
CC0000	CC3300	FFCC00	[illegible]	[illegible]	[illegible]	[illegible]	663399	CC0099
FF0000	FF3300	FFFF00	00CC00	009999	0099FF	0000FF	9900CC	FF0099
CC3333	FF6600	FFFF33	[illegible]	[illegible]	[illegible]	[illegible]	9933FF	FF00FF
FF6666	FF6633	FFFF66	66FF66	66CCCC	00FFFF	3399FF	9966FF	FF66FF
FF9999	FF9966	FFFF99	99FF99	66FFCC	99FFFF	66CCFF	9999FF	FF99FF
FFCCCC	FFCC99	FFFFCC	CCFFCC	99FFCC	CCFFFF	99CCFF	CCCCFF	FFCCFF

Remember Base Conversion

Three cases:

I. From any base *b* to base 10
II. From base 10 to any base *b*
III. From any base *b* to any other base *c*

More Conversions

- Hex → Octal
  - Do it in 2 steps
  - Hex → binary → octal
- Decimal → Hex
  - Do it in 2 steps
  - Decimal → binary → hex
- So why use hex and octal and not just binary and decimal?

Largest Number

- What is the largest number that we can represent in *n* digits…
  - In base 10? 10^n -1 e.g. n = 2, 100-1 = 99
  - In base 2? 2^n -1
  - In octal? 8^n -1
  - In hex? 16^n -1
  - In base 7? 7^n -1
  - In base *b*? b^n -1
- How many different numbers can we represent with *n* digits in base *b*?

  b^n (remember 0 is a number 2, inclusion)

How many bits are necessary?

- How many bits are necessary?
  - log2(444) = 8.8…
  - ceil(8.8) = 9
- Where is log2 on my calculator?

$\log_y(x) = \log(x)/\log(y)$

$\log_2(x) = \log(x)/\log(2)$