

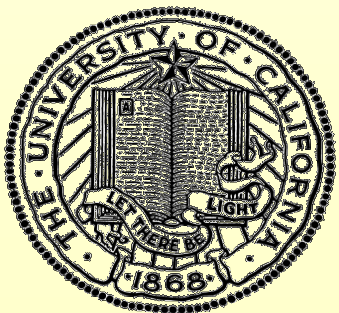
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Data Representation

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Using binary numbers to represent
information



Data Representation

- Goal: Store numbers, characters, sets, database records in the computer.
- What we got: Circuit that stores 2 voltages, one for logic 0 (0 volts) and one for logic 1 (ex: 3.3 volts).

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Storing Information

Value Representation		Value Representation		Value Representation	
H	0	False	0	1e-4	0
T	1	True	1	5	1

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- Use more bits for more items
- Three bits can represent 8 things: 000,001,...,111
- N bits can represent 2^N things

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N bits	Can represent	Which is approximately
8	256	256
16	65,536	65 thousand (64K where K=1024)
32	4,294,967,296	4 billion
64	1.8446×10^{19}	20 billion billion



Storing Information

Byte is a unit of information. Remember 1
byte = 8 bits

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Most computers today use:

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Type	# of bits
Character	8-16
Integers	32-64
Addresses	32-64



Integer Representation

Usual answers:

1. Represent 0 and consecutive positive integers
 - Unsigned integers
2. Represent positive and negative integers
 - Signed magnitude
 - One's complement
 - Two's complement

Unsigned and two's complement the most common



Unsigned Integers

- Integer represented is binary value of bits:

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0000 -> 0, 0001 -> 1, 0010 -> 2, ...

- Encodes only positive values and zero
- Range: 0 to $2^n - 1$, for n bits



Unsigned Integers

If we have 4 bit numbers:

To find range make $n = 4$. Thus $2^4 - 1$ is 15

Thus the values possible are 0 to 15

[0:15] = 16 different numbers

7 would be 0111

17 not represent-able

-3 not represent-able

For 32 bits:

Range is 0 to $2^{32} - 1 = [0: 4,294,967,295]$

Which is 4,294,967,296 different numbers



Signed Magnitude Integers

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- A human readable way of getting both positive and negative integers.

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Signed Magnitude Integers

Representation:

- Use 1 bit of integer to represent the sign of the integer.
 - ◆ Sign bit is msb: 0 is “+”, 1 is “-”
- Rest of the integer is a magnitude, with same encoding as unsigned integers.
- To get the additive inverse of a number, just flip (invert, complement) the sign bit.
- Range: $-(2^{n-1} - 1)$ to $2^{n-1} - 1$

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Signed Magnitude - Example

If 4 bits then range is:

$$-2^3 + 1 \text{ to } 2^3 - 1$$

which is -7 to +7

Given only 4 bits to represent signed magnitude, what integer the following binary numbers represent:

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- 0101 is $0\ 101 = +5$
- -3 is ? $1\ 011 = 1011$
- +12 is ? • Not possible range is -7 +7
- $[-7, \dots, -1, 0, +1, \dots, +7] = 7 + 1 + 7 = 15 < 16 =$

- What problems does this cause?



Signed Magnitude - Example

If 4 bits then range is:

$$-2^3 + 1 \text{ to } 2^3 - 1$$

which is -7 to +7

Questions: Assignment Project Exam Help

- 0101 is 0 101 = + 5
- -3 is ? is 1 011 = -3
- +12 is ? Not possible range is -7 to +7
- $[-7, \dots, -1, 0, +1, \dots, +7] = 7 + 1 + 7 = 15 < 16 =$

- What problems does this cause?

0 = 1 000 (negative zero)

0 = 0 000 (positive zero)

You are wasting two unique binary numbers (1000 and 0000) in representing the same integer value!



One's Complement

- Historically important (in other words, not used today!!!)
- Early computers built by Semour Cray (while at CDC) were based on 1's complement integers.
- Positive integers use the same representation as unsigned.
 - 0000 is 0
 - 0111 is 7, etc
- Negation is done by taking a bitwise complement of the positive representation.
 - Complement = Invert = Not = Flip = {0 -> 1, 1 -> 0}
 - A logical operation done on a single bit
- Top bit is sign bit

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One's Complement Representation

To get 1's complement of -1

- Take $+1$: 0001
- Complement each bit: 1110
- Don't add or take away any bits.

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Another example:

- 1100 \rightarrow flip 0011 $\rightarrow +3$
- This must be a negative number. To find out which, find the inverse.
- 0011 is $+3$
- 1100 in 1's Complement must be?

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Properties of 1's complement:

- Any negative number will have a 1 in the MSB
- What is 0000? 0
- What is 1111? -0



Two's Complement

- Variation on 1's complement that does not have 2 representations for 0.
- This makes the hardware that does arithmetic simpler and faster than the other representations.
- The negative values are all “slid” by one, eliminating the -0 case.
- How to get 2's complement *representation*:
 - Positive: just as if unsigned binary
 - Negative:
 - Take the positive value
 - Take the 1's complement of it
 - Add 1

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Two's Complement

Example, what is -5 in 2SC?

1. What is 5? 0101
2. Invert all the bits: 1010 (basically find the 1SC)
3. Add one: $1010 + 1 = 1011$ which is -5 in 2SC

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To get the additive inverse of a 2's complement integer

1. Take the 1's complement
2. Add 1



What value is my negative number?

- Assume 4-bit number...
 - ◆ 1100 is negative, but what number is it?
 - ◆ Take 2SC again using same method!
 - ★ Invert all bits
 - ★ Add 1

1100 > 0011
+ 0001
0100

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Visualizing Signed Numbers

- Signed Magnitude
- One's Complement
- And... Two's Complement

	unsigned	signed magnitude	1's Complement	2's Complement
000	0	0	0	0
001	1	1	1	1
010	2	2	2	2
011	3	3	3	3
100	4	-0	-3	-4
101	5	-1	-2	-3
110	6	-2	-1	-2
111	7	-3	-0	-1



Two's Complement

Number of integers representable is -2^{n-1} to $2^{n-1}-1$

So if 4 bits: **Assignment Project Exam Help**

$[-8, \dots, -1, 0, +1, \dots, +7] = 8 + 1 + 7 = 16 = 2^4$ numbers

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Interesting observation about 2SC representation

- Assume you use n bits to represent your 2SC number
- The integer 0 is always represented as $0\dots 0$ (n times)
- The integer -1 is always represented as $11\dots 111$ (n times)

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Sign Extension

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How to change a number with a smaller number of bits into the same number (same representation) with a larger number of bits?

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This must be done frequently by arithmetic units

0010 = 2 (4 bits)

0000 0010 = 2 (8 bits)



Sign Extension - unsigned

Unsigned representation:

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Copy the original integer into the LSBs, and put 0's elsewhere.

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Thus for 5 bits to 8 bits:

xxxxx -> 000xxxxx



Sign Extension – signed magnitude

Signed magnitude:

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Copy the original integer's magnitude into the LSBs
& put the original sign into the MSB, put 0's
elsewhere.

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Thus for 6 bits to 8 bits

sxxxxx -> s00xxxxx



Sign Extension – 1SC and 2SC

1's and 2's complement:

1. Copy the original $n-1$ bits into the LSBs
2. Take the MSB of the original and copy it elsewhere

Thus for 6 bits to 8 bits:

$sxxxxx \rightarrow sssxxxxx$



Sign Extension

What is -12 in 8-bit 2's complement form
12

0000 1100 -> Binary

1111 0011 -> flip

1111 0100 -> +1

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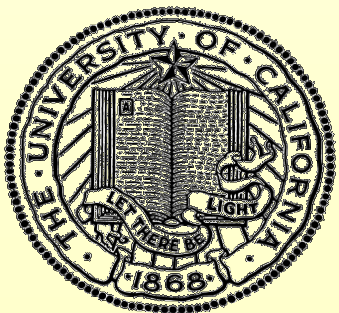


Arithmetic and Logical Operations

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Logical Operations

- Operate on raw bits with 1 = true and 0 = false

		AND	OR	NAND	NOR	XOR	XNOR
In1	In2	$\&$	$ $	$\sim(\&)$	$\sim()$	$\hat{}$	$\sim(\hat{})$
0	0	0	0	1	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	1	0	0	0	1

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Logical Operations

- “bit-wise” logical operations are done in parallel for corresponding bits

- ◆ Example & (AND):

- ★ $X = 0011$

- ★ $Y = 1010$

- ★ $X \text{ AND } Y = ?$

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Logical Operations

- “bit-wise” logical operations are done in parallel for corresponding bits

- ◆ Example & (AND):

- ★ $X = 0011$

- ★ $Y = 1010$

- ★ $X \text{ AND } Y = X \& Y = 0010$

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