Integer Numbers

Number Systems

- Unary, or marks:
  - /////// = 7
  - /////// + ////// = ////////////

- Grouping lead to Roman Numerals:
  - VII + V = VVII = XII

- Better: Indo Arabic Numerals:
  - 7 + 5 = 12 = 1·10 + 2

Positional Number System

- The value represented by a digit depends on its *position* in the number.
- Ex: 1832
  - How to decode it?

$$1 * 10^3 + 8 * 10^2 + 3 * 10^1 + 2 * 10^0$$

$$1 * 1000$$
$$8 * 100$$
$$3 * 10$$
$$2 * 1$$

$$1000$$
$$800$$
$$30$$
$$2$$
$$\overline{1832}$$

Positional Number Systems (base b)

- Select a number as the base *b*
- Define an alphabet of *b*–1 symbols plus a symbol for zero to represent all numbers
- Use an *ordered* sequence of 1 or more digits *d* to represent numbers
- The represented number is the sum of all digits, each multiplied by *b* to the power of the digit's position i

$$\text{Number} = \sum_{i=0}^{\text{num digits}} (d_i \cdot b^i)$$

Arabic/Indian Numerals

- Base (or radix): 10 (decimal)
  - The alphabet (digits or symbols) is 0..9
- We use the Arabic symbols for the 10 digits
  - Has ZERO!
- Numerals introduced to Europe by Leonardo Fibonacci in his *Liber Abaci*
  - In 1202
  - So useful!

Arabic/Indian Numerals

- The Italian mathematician Leonardo Fibonacci
- Also known for the Fibonacci sequence
  - 1, 1, 2, 3, 5, 8, 13, 21

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		க	உ	ங	ச	ஞ	ட	ண	அ	கூ

Non-European not on the exam.

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Base Conversion

Three cases:

I. From any base *b* to base 10
II. From base 10 to any base *b*
III. From any base *b* to any other base *c*

From Base *b* to Base 10

- Base (radix): b
- Digits (symbols): 0 … (b – 1)
- $S_{n-1}S_{n-2}\cdots S_2S_1S_0$

$$\text{Value} = \sum_{i=0}^{n-1} (S_i b^i)$$

Use summation to transform any base to decimal

From Base *b* to Base 10

- Example: $1234_5 = ?_{10}$

  $= 1 * 5^3 + 2 * 5^2 + 3 * 5^1 + 4 * 5^0$

  $= 125 + 50 + 15 + 4$

  $= 194$

- Example: $201_5 = ?_{10}$

  $= 2 * 5^2 + 0 * 5^1 + 1 * 5^0$

  $= 50 + 1$

  $= 51$

From Base 10 to Base *b*

- Use successive divisions
- Remember the remainders
- Divide again with the quotients