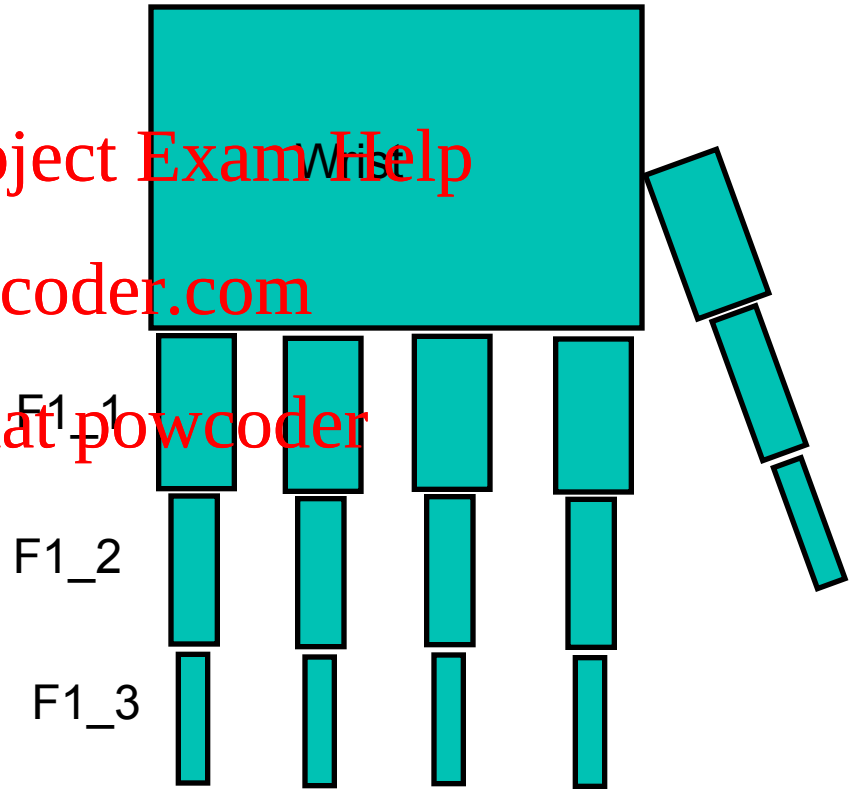
Hierarchical structures

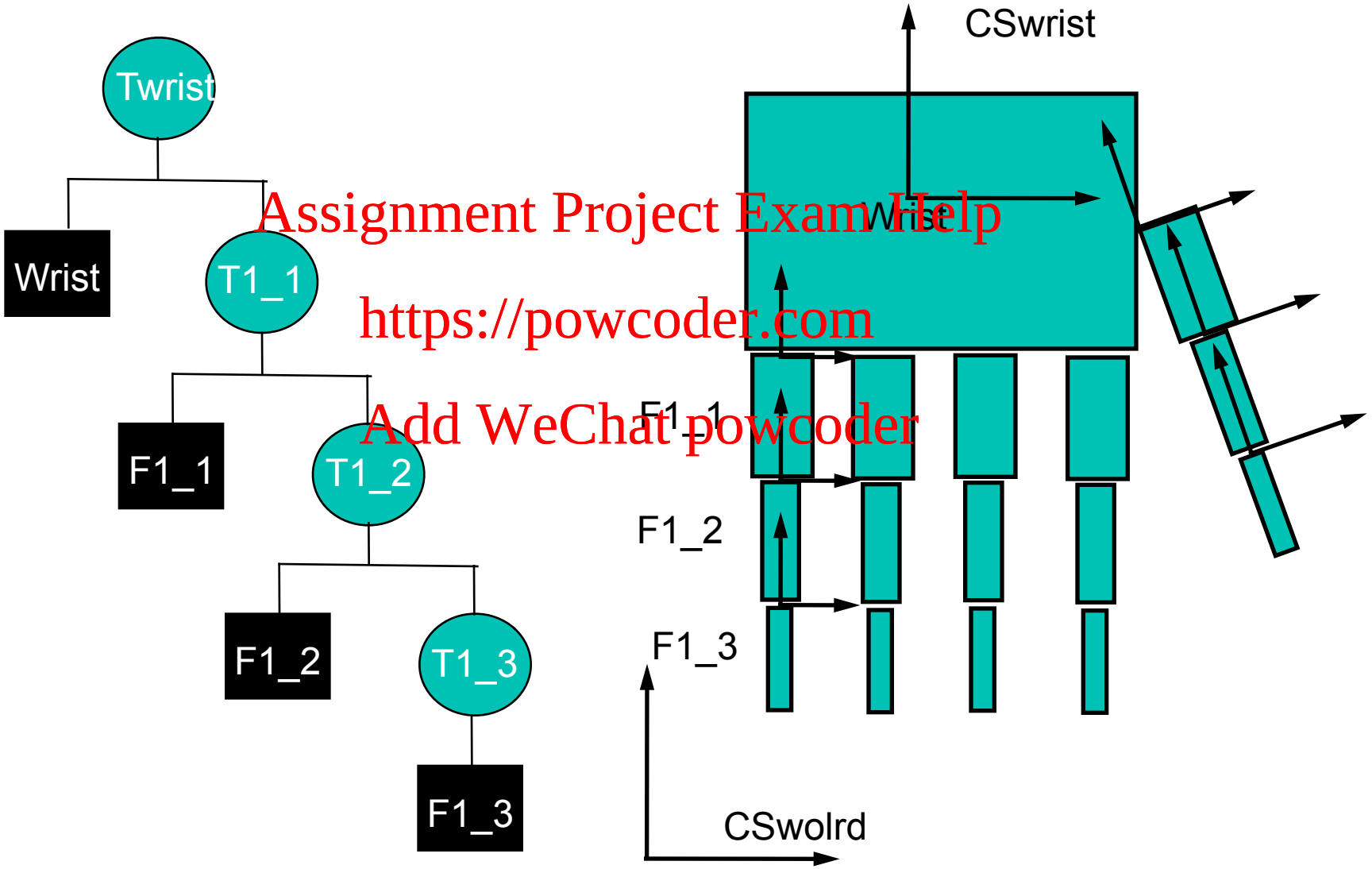
Wrist and 5 fingers

We want the fingers to stay attached to the wrist as the wrist moves.

Each segment is abstracted by a coordinate system and has its own transformation matrix with respect to its parent's system thus modelling relative motion.



Hierarchy of systems

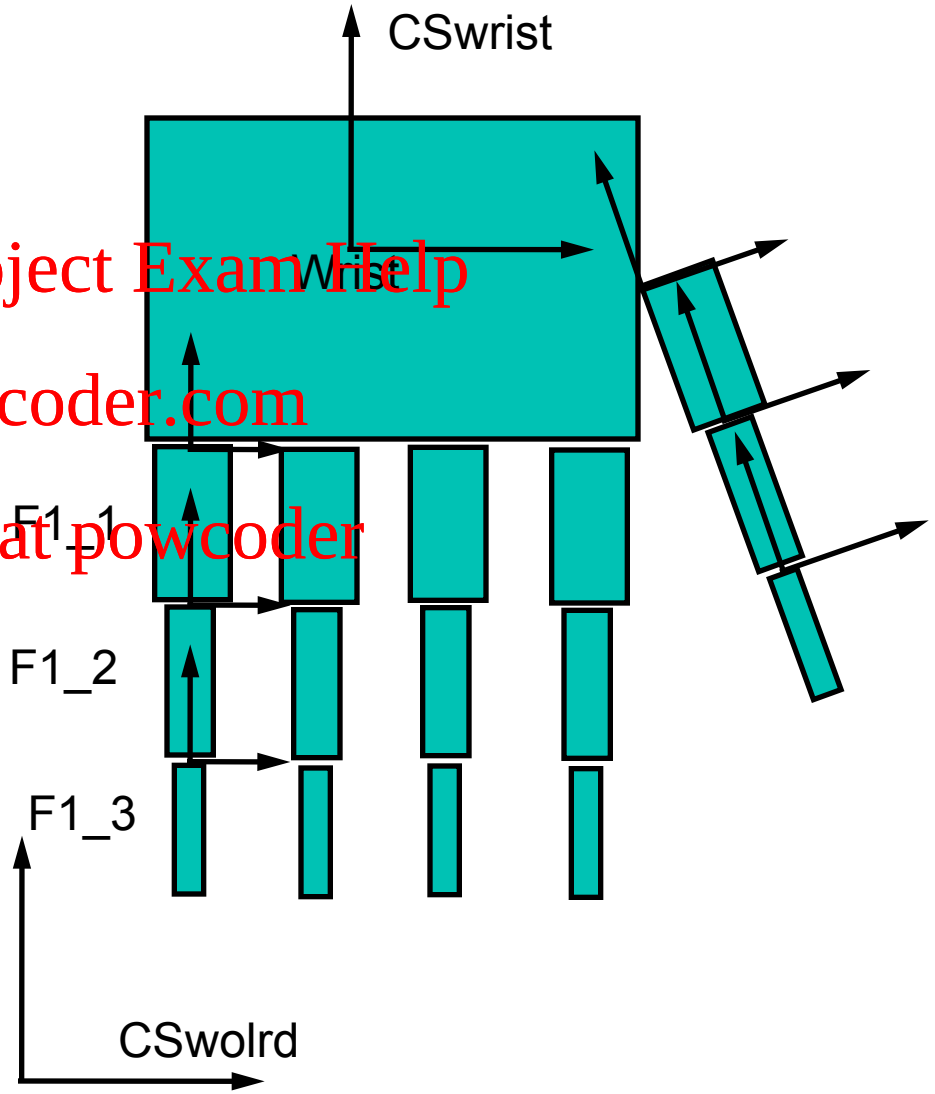


Hierarchy of systems

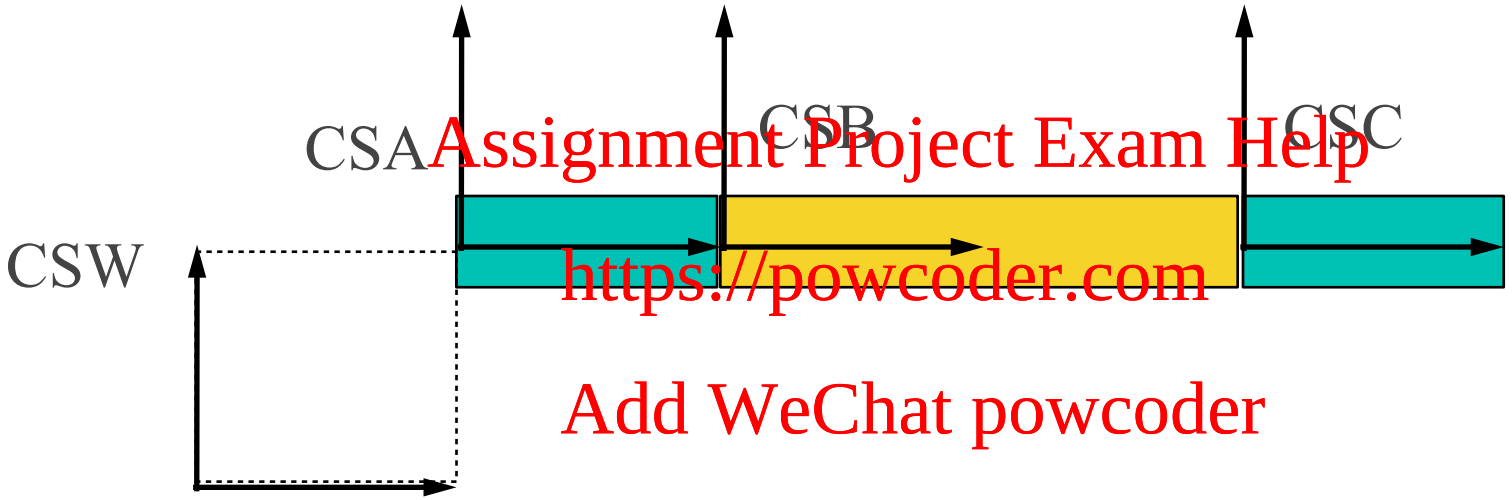
CSF1_1 = T1_1(CSwrist)

CSF1_2 = T1_2(CSF1_1)

CSF1_3 = T1_3(CSF1_2)



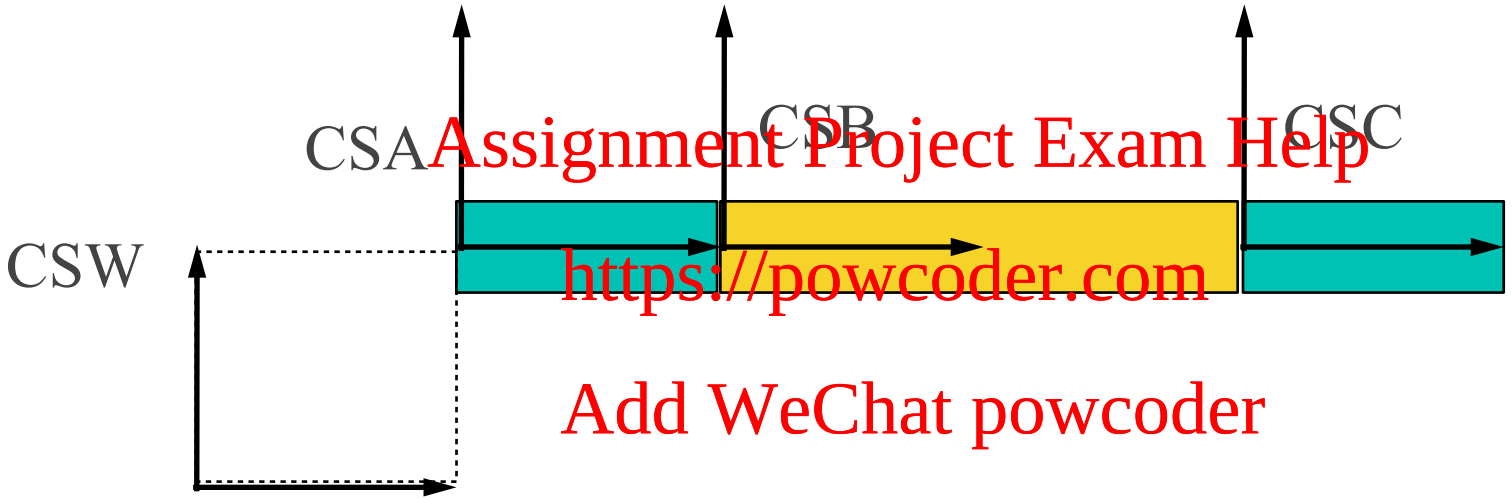
Example: Kinematic Chain



$$P_w = {}_wM_{AA}M_{BB}M_CP_C$$

What do these matrices look like?

Example: Kinematic Chain

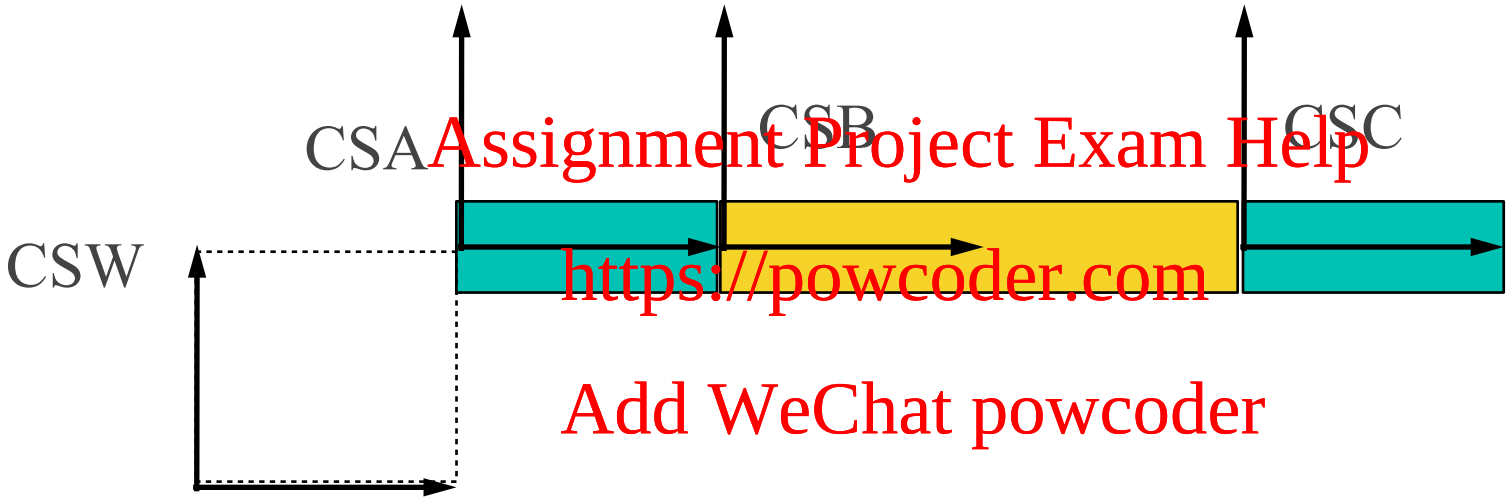


$$P_w = {}_wM_{AA}M_{BB}M_CP_C$$

What do these matrices look like?

Depends on the structure and the degrees of freedom we want

Example: Kinematic Chain

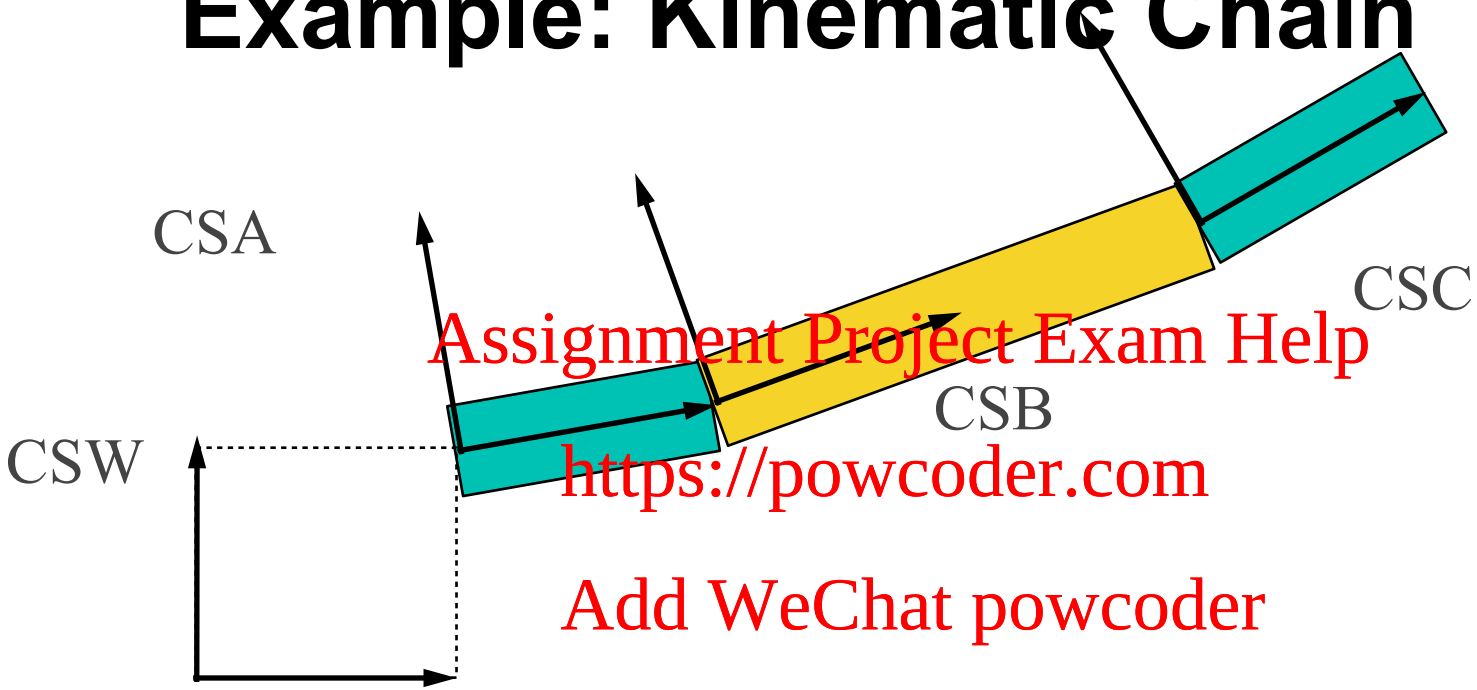


$P_w = {}_wM_{AA}M_{BB}M_CP_C$

One possibility *(three degrees of freedom)*

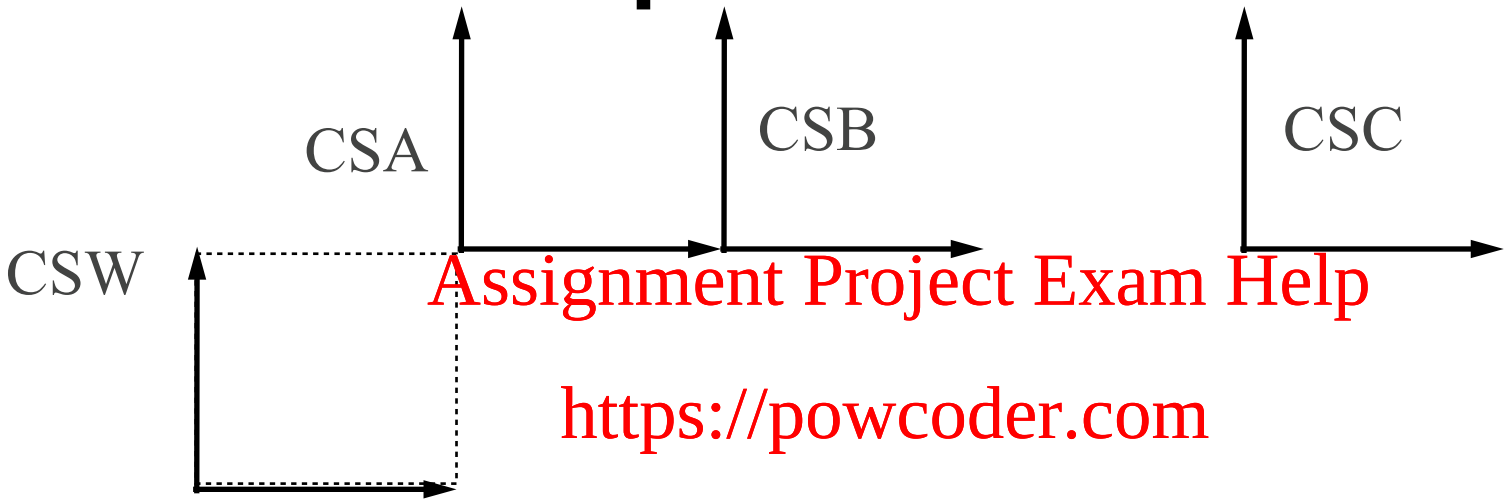
$$P_w(\theta_1, \theta_2, \theta_3) =$$
$$(T(1,1)R(z,\theta_1))(T(1,0)R(z,\theta_2))(T(2,0)R(z,\theta_3))P_c$$

Example: Kinematic Chain



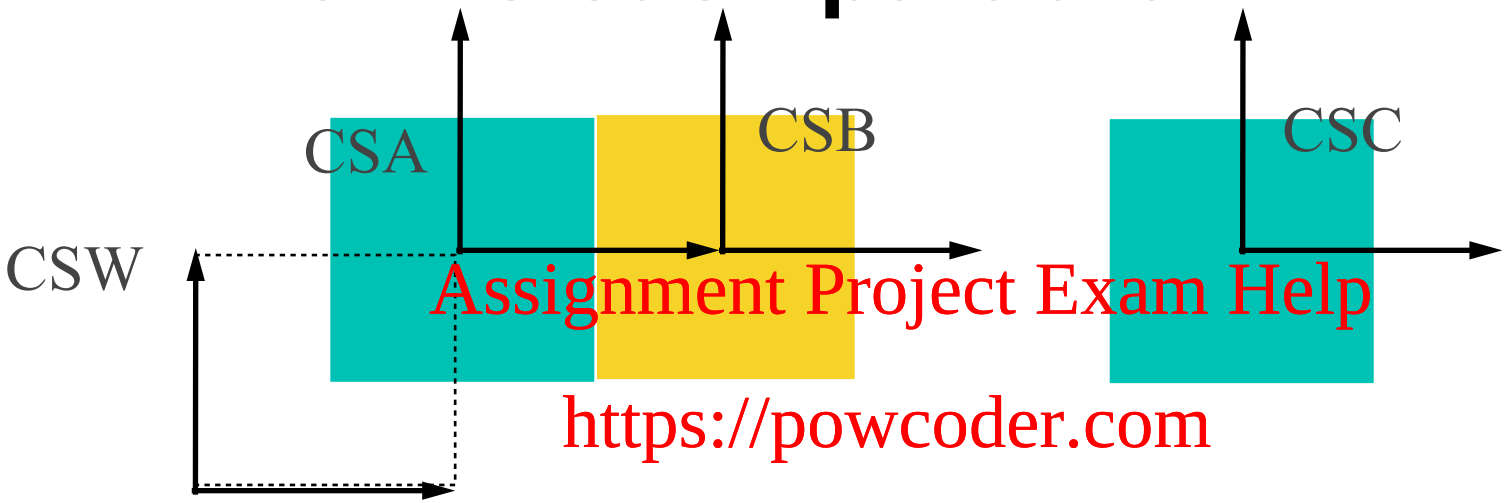
$$P_w(10,10,10) =$$
$$(T(1,1)R(z,10))(T(1,0)R(z,10))(T(2,0)R(z,10))P_c$$

How is each part drawn in code?



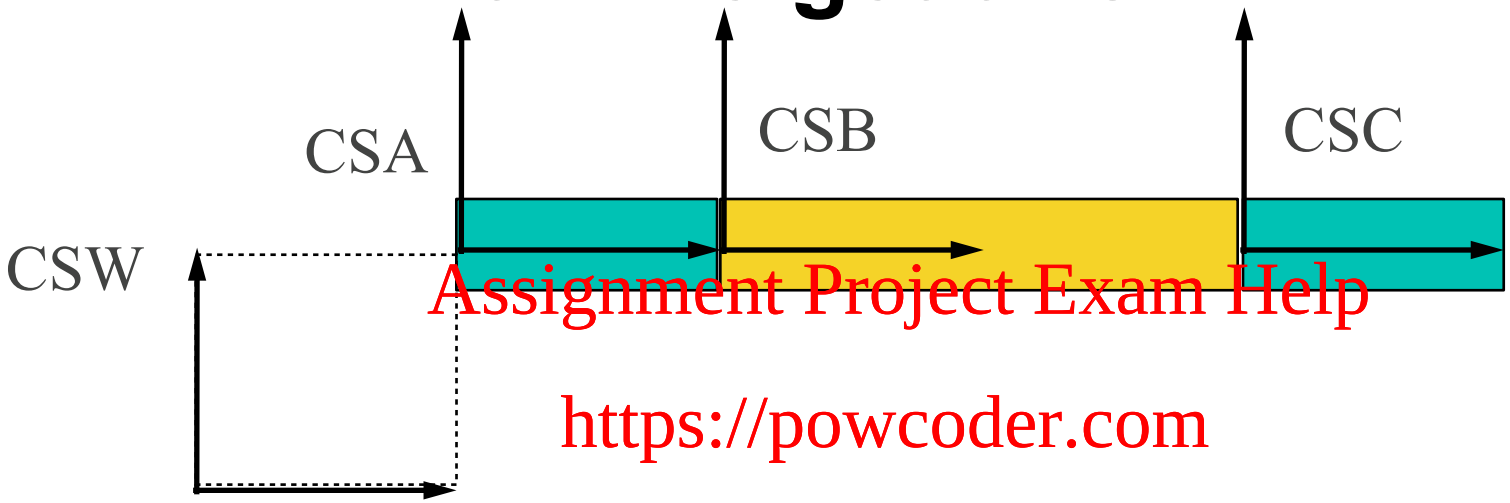
$T(1,1)$	$ModelMat = T(1,1)$
$R(z,\theta_1)$	$ModelMat = T(1,1)R(z,\theta_1)$
$drawCube()$	Cube size 1 centered at the origin
$T(1,0)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)$
$R(z,\theta_2)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)$
$drawCube()$	
$T(2,0)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)T(2,0)$
$R(z,\theta_3)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)T(2,0)Rz(\theta_3)$
$dawCube()$	

How is each part drawn?

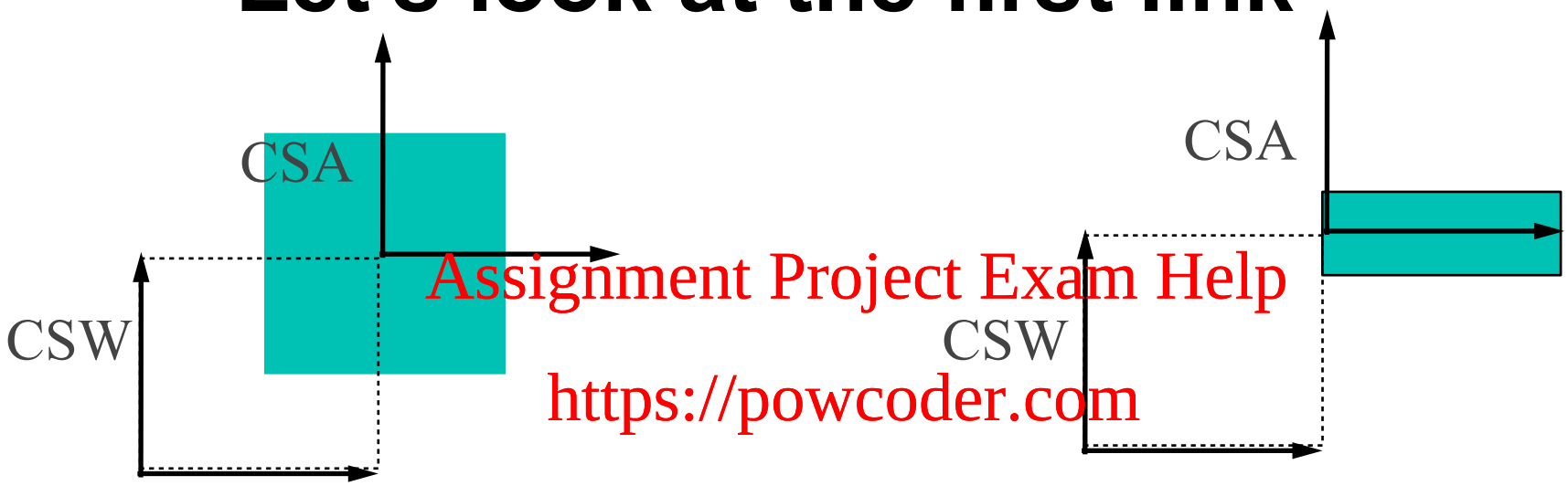


$T(1,1)$	$ModelMat = T(1,1)$
$R(z,\theta_1)$	$ModelMat = T(1,1)R(z,\theta_1)$
$drawCube()$	Cube size 1 centered at the origin
$T(1,0)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)$
$R(z,\theta_2)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)$
$drawCube()$	
$T(2,0)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)T(2,0)$
$R(z,\theta_3)$	$ModelMat = T(1,1)R(z,\theta_1)T(1,0)R(z,\theta_2)T(2,0)Rz(\theta_3)$
$dawCube()$	

How we get this?



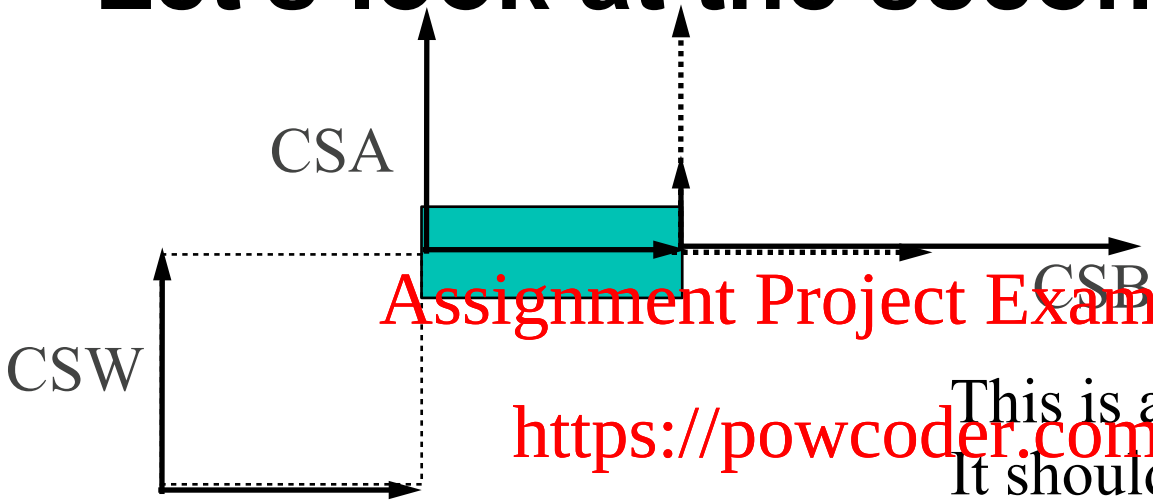
Let's look at the first link



$T(1,1)$	$M = T(1,1)$
$R(z,\theta_1)$	$M = T(1,1)R(z,\theta_1)$
$drawCube()$	Cube size 1 centered at the origin

$T(1,1)$	$M = T(1,1)$
$R(z,\theta_1)$	$M = T(1,1)R(z,\theta_1)$
$T(0.5,0)$	$M = T(1,1)R(z,\theta_1)T(0.5,0)$
$S(1,0.3)$	$M = T(1,1)R(z,\theta_1)T(0.5,0)S(1,0.3)$
$drawCube()$	Cube size 1 centered at the origin

Let's look at the second link



This is a problem now!
It should only apply to the first cube,
not the second system. I could undo the transformations.
However,

$T(1,1)$	$M = T(1,1)$
$R(z,\theta_1)$	$M = T(1,1)R(z,\theta_1)$
$T(0.5,0)$	$M = T(1,1)R(z,\theta_1)T(0.5,0)$
$S(1,0.3)$	$M = T(1,1)R(z,\theta_1)T(0.5,0)S(1,0.3)$
$drawCube()$	Cube size 1 centered at the origin

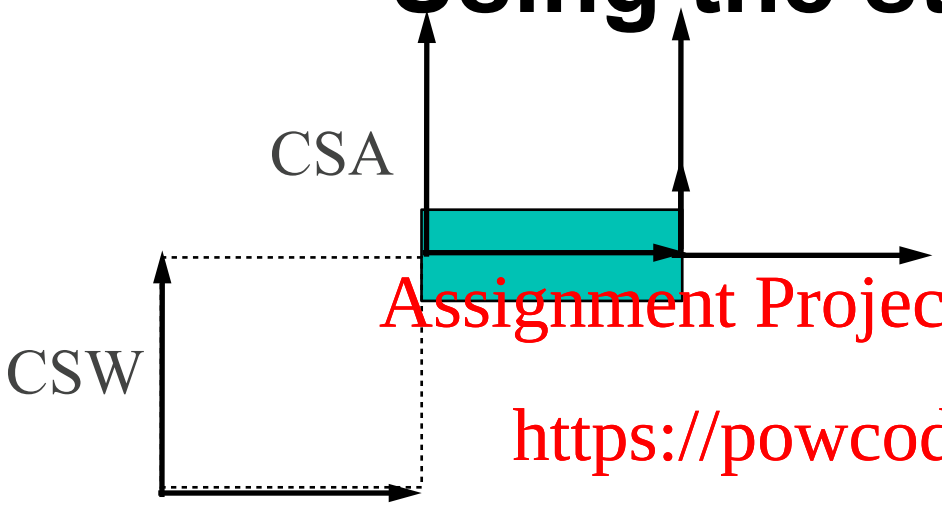
Matrix Stack

When we have more than one branch or multiple objects, it is often convenient to use a matrix stack to load and unload matrices

- Using the push(*) and pop() operations of a stack

- stack.push(M); // in the code gPush()
- draw();
- stack.pop(M); // in the code gPop()

Using the stack



$stack.init()$	$[\,]$
$T(1,1)$	$[M = T(1,1)]$
$R(z,\theta_1)$	$[M = T(1,1)R(z,\theta_1)]$
$stack.push()$	$[M, M]$
$T(0.5,0)$	$[M = T(1,1)R(z,\theta_1), M' = MT(0.5,0)]$
$S(1,0.3)$	$[M = T(1,1)R(z,\theta_1), M' = MT(0.5,0)S(1,0.3)]$
$drawCube()$	Cube size 1 centered at the origin
$stack.pop()$	$[M = T(1,1)R(z,\theta_1)]$

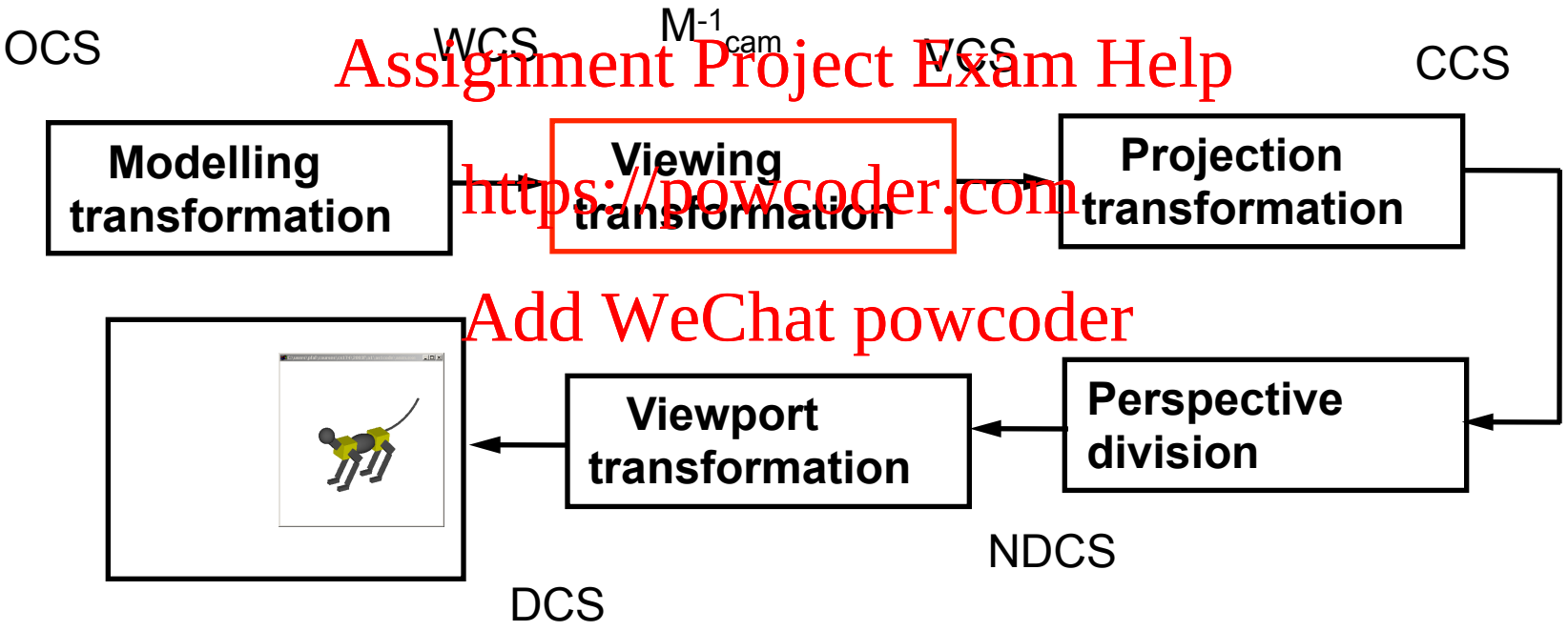
Hybrid way of thinking

Use TOP to BOTTOM to position a coordinate system

Then use BOTTOM to TOP to position the objects within that system

Often it is easier to do it in the opposite order

Graphics Pipeline



Taking a snapshot of a 3D Scene

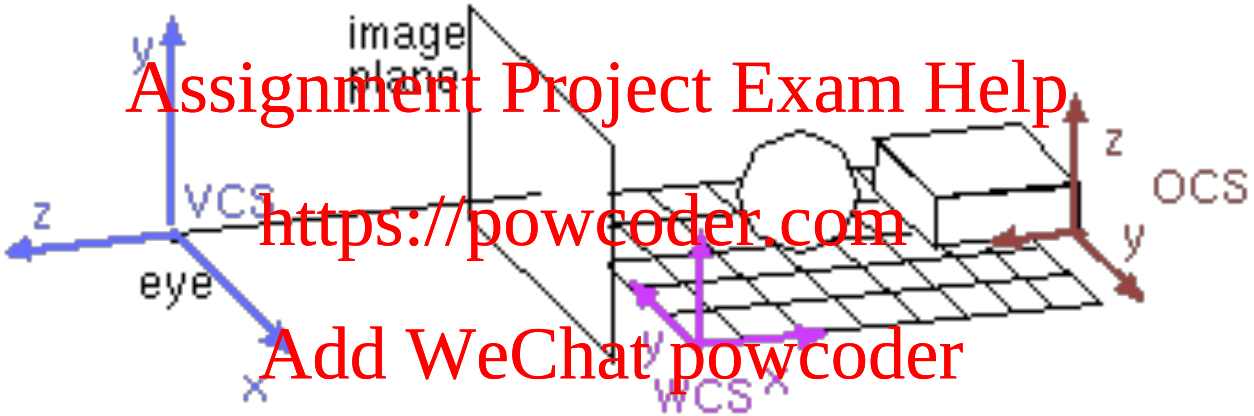
Image copyright E. Angel

OpenGL Assumption

The camera system is:

Camera transformation

Transforms objects to camera coordinates



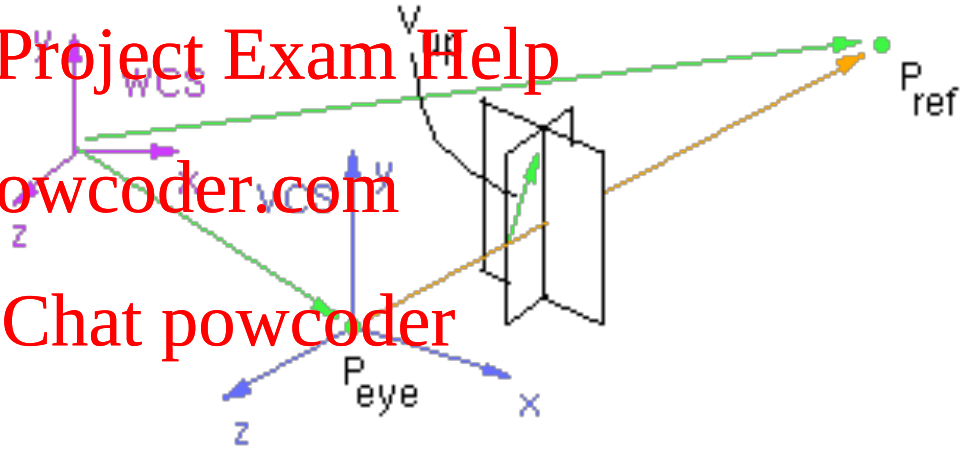
$$\left.\begin{aligned} P_{wcs} = M_{cam} P_{vcs} \rightarrow P_{vcs} = M_{cam}^{-1} P_{wcs} \\ P_{wcs} = M_{mod} P_{obj} \end{aligned}\right\} \rightarrow$$

$$P_{vcs} = M_{cam}^{-1} M_{mod} P_{obj}$$

Defining Mcam

Common way

Eye point
Reference point
Upvector



To build Mcam we need to define a camera coordinate system (origin, **i**, **j**, **k**)

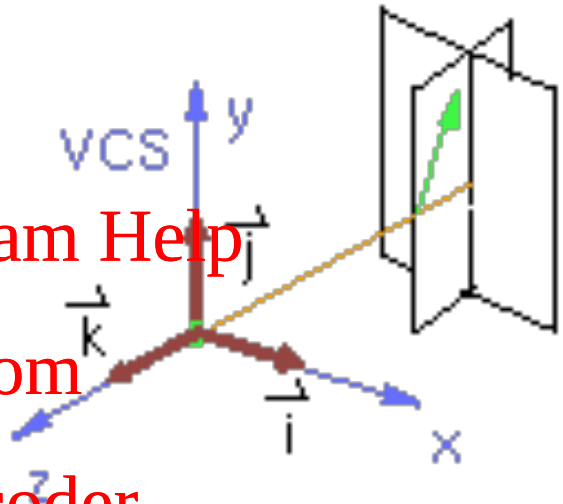
Camera Coordinate system

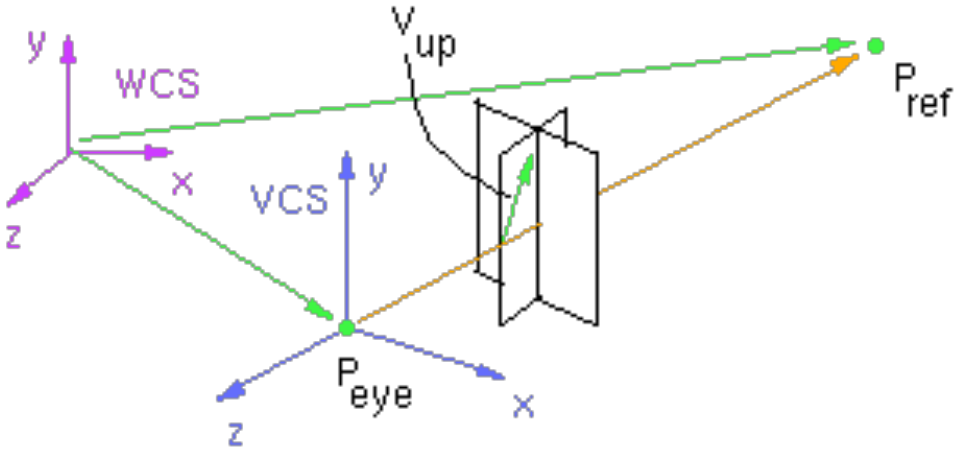
$$\mathbf{k} = \frac{P_{eye} - P_{ref}}{|P_{eye} - P_{ref}|}$$

$$\mathbf{I} = \mathbf{v}_{up} \times \mathbf{k}$$

$$\mathbf{i} = \frac{\mathbf{I}}{|\mathbf{I}|}$$

$$\mathbf{j} = \mathbf{k} \times \mathbf{i}$$

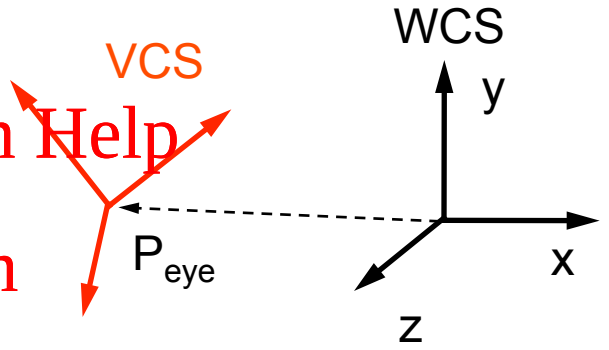




Building Mcam

Change of basis

Our reference system is WCS, we know the camera parameters with respect to the world



Align WCS with VCS

$$M_{cam} = \begin{bmatrix} 1 & 0 & 0 & P_{eye,x} \\ 0 & 1 & 0 & P_{eye,y} \\ 0 & 0 & 1 & P_{eye,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{wcs} = M_{cam} P_{vcs}$$

Building Mcam inverse

Invert smart

$$M_{cam}^{-1} = \left(\begin{bmatrix} 1 & 0 & 0 & P_{eye,x} \\ 0 & 1 & 0 & P_{eye,y} \\ 0 & 0 & 1 & P_{eye,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

$$= \left(\begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 0 & 0 & P_{eye,x} \\ 0 & 1 & 0 & P_{eye,y} \\ 0 & 0 & 1 & P_{eye,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

Building Mcam inverse

Invert smart

$$M_{cam}^{-1} = \left(\begin{bmatrix} i_x & j_x & k_x & 0 \\ i_y & j_y & k_y & 0 \\ i_z & j_z & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} 1 & 0 & 0 & P_{eye,x} \\ 0 & 1 & 0 & P_{eye,y} \\ 0 & 0 & 1 & P_{eye,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1}$$

Transpose

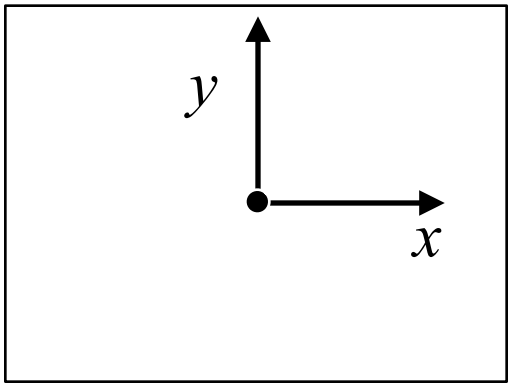
$$= \begin{bmatrix} i_x & i_y & i_z & 0 \\ j_x & j_y & j_z & 0 \\ k_x & k_y & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -P_{eye,x} \\ 0 & 1 & 0 & -P_{eye,y} \\ 0 & 0 & 1 & -P_{eye,z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{vcs} = M_{cam}^{-1} P_{wcs}$$

Camera Transform

Summary

- The camera transformation is really another affine transformation
- It transforms the scene so that the camera is at zero looking down the -z axis



End of Modelling transformations

1. ***Preservation of affine combinations of points.***
2. ***Preservation of lines and planes.***
3. ***Preservation of parallelism of lines and planes.***
4. ***Relative ratios on a line are preserved***
5. ***Affine transformations are composed of elementary ones.***

Camera transformation as a change of basis.

Graphics Pipeline