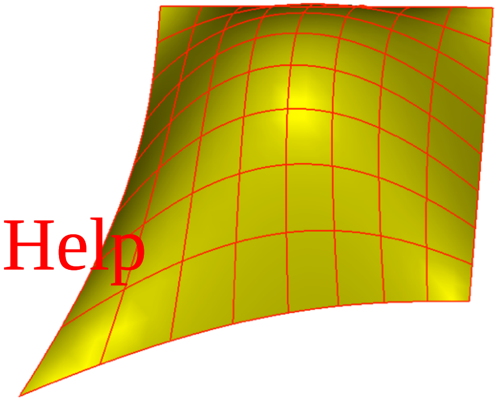
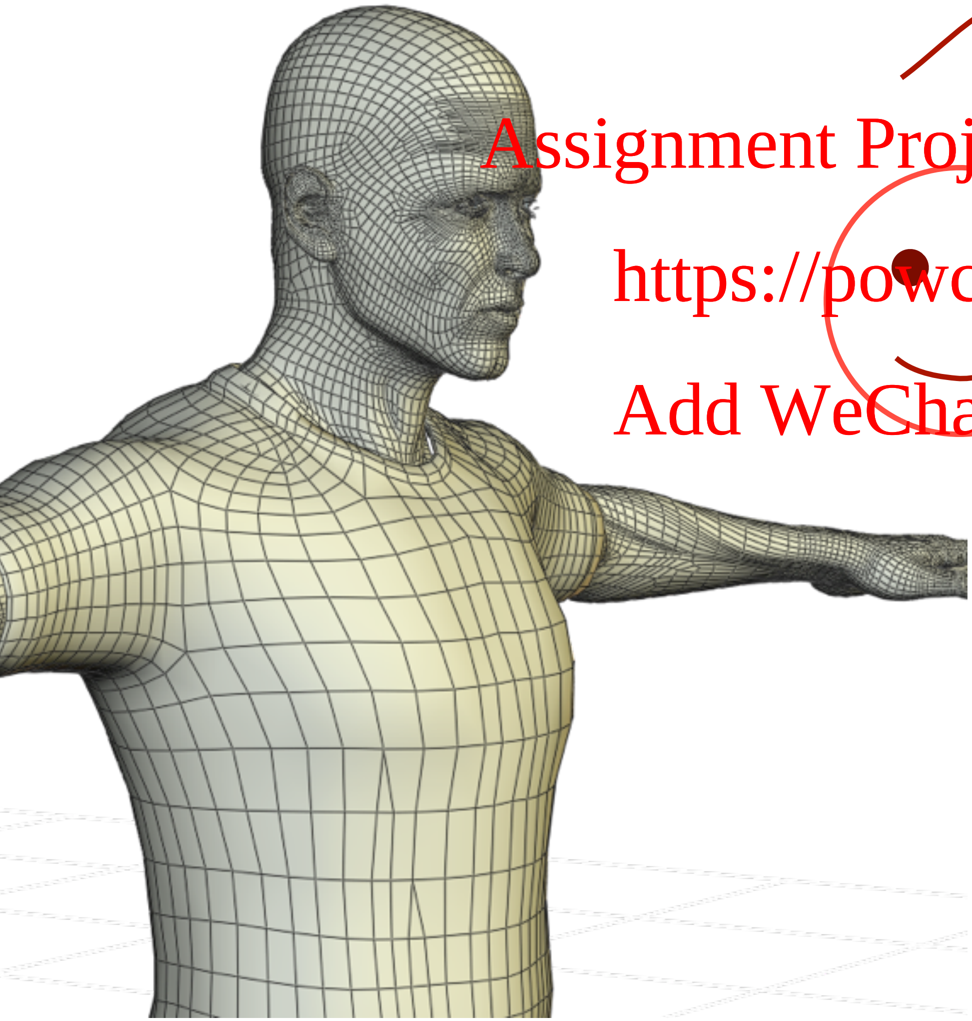
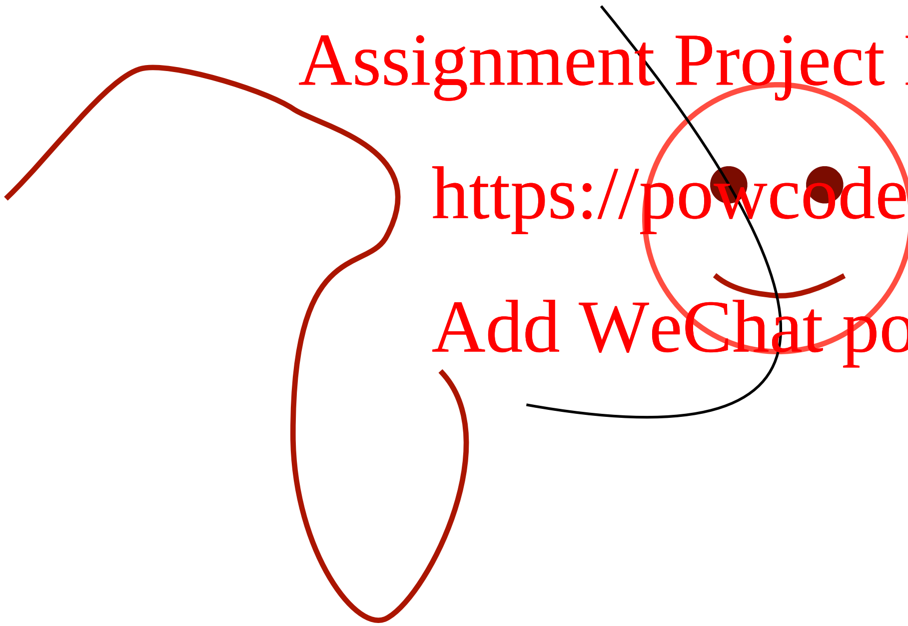
Curves and Surfaces





Curve Design



Representing curves

Explicit: $y = f(x),\ z = g(x)$

- Cannot get multiple values for single x, infinite slopes
- E.g. cannot represent a circle

Implicit (2D): $f(x,y) = 0$

- Cannot easily compare tangent vectors at joints
- Above/below test, normals from gradient

Parametric: $x = f_x(t),\ y = f_y(t),\ z = f_z(t)$

- Often the most convenient formulation
- Tangent of curve $(f(t), g(t), h(t))$ is $(f'(t), g'(t), h'(t))$ where ' indicates derivative wrt to t

Describing curves by means of polynomials

Reminder:

Lth degre polynomial

$$p(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_L t^L$$

$a_0, \ldots a_L$ are the coefficients

L : is the degree

$L+1$ is the order

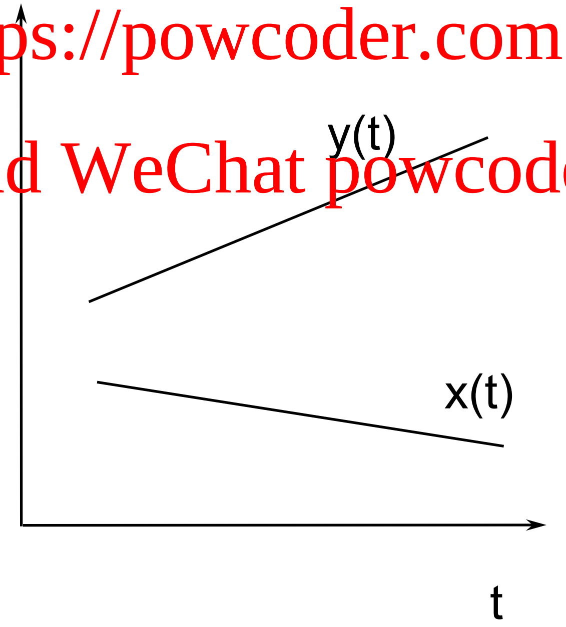
Polynomial Curves of Degree 1

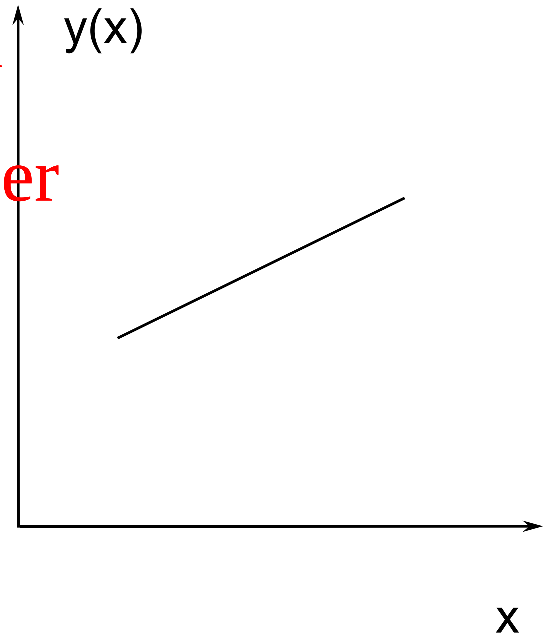
Parametric and implicit forms are linear

$$x(t) = at + b$$

$$y(t) = ct + d$$

$$F(x,y) = kx + ly + m$$





Polynomial Curves of Degree 2

Parametric

$x(t) = at^2+2bt+c$

$y(t) = dt^2+2et+f$

t *in* $\mathbf{R}$

For any choice of constants

$a,d,c,d,e,f \rightarrow$ parabola

Implicit

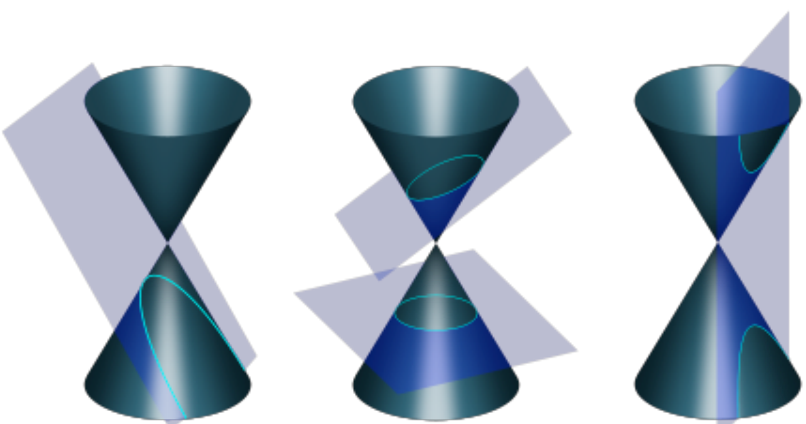
$F(x,y) = Ax^2+2Byx+Cy^2+Dx+Ey+G$

Let $d = AC-B^2$

$d > 0 \rightarrow F(x,y) = 0$ is an ellipse

$d = 0 \rightarrow F(x,y) = 0$ is a parabola

$d < 0 \rightarrow F(x,y) = 0$ is a hyperbola



1 2 3

1. Parabola
2. Circle and Ellipse
3. Hyperbola

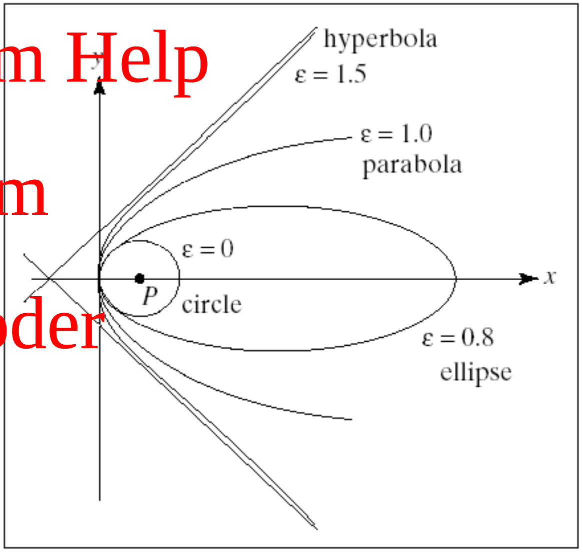
Courtesy of Pbroks13, Wikipedia

Polynomial curves of degree 2

Common Vertex form:

$$y^2 = 2px - (1 - \varepsilon^2)x^2$$

FIGURE 11.5 The common-vertex equations of the conics.



Rational Quadratic Parametric Curves

$$P(t) = \frac{P_0(1-t)^2 + 2wP_1t(1-t) + P_2t^2}{(1-t)^2 + 2wt(1-t) + t^2}$$

w < 1 ellipse

w = 1 parabola

w > 1 hyperbola

Other kinds of curves

Sinusoidal

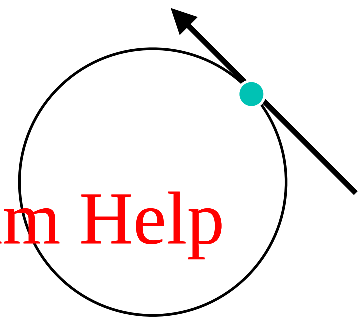
Exponential

Complex

Fractals

Differential Geometry Concepts

Tangent vector



- Given a parameterized curve $p(t)$: $\tau(t) = p'(t) = dp(t)/dt$ defines the forward orientation of the curve and the parametric velocity

- Notice that it is a function of t and that it is a **vector**
- It defines locally (instantaneously) the shape of the curve at each point and can be used for interactive design tools
- Unit tangent: $\mathbf{T}(t) = \tau(t) / \|\tau(t)\|$

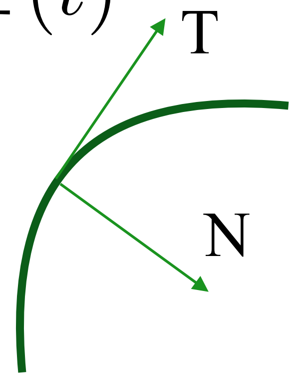
Differential Geometry Concepts

Normal vector (AKA curvature vector)

- Given the unit tangent vector $\mathbf{T}(t)$:

$$e_2(t) = p''(t) - (p''(t) \cdot \mathbf{T}(t))\,\mathbf{T}(t)$$

$$\mathbf{N}(t) = \frac{e_2(t)}{||e_2(t)||}$$



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- Indicates how far from a straight line the curve is (Line has 0 curvature)
- Normal to the tangent vector $\mathbf{T}$

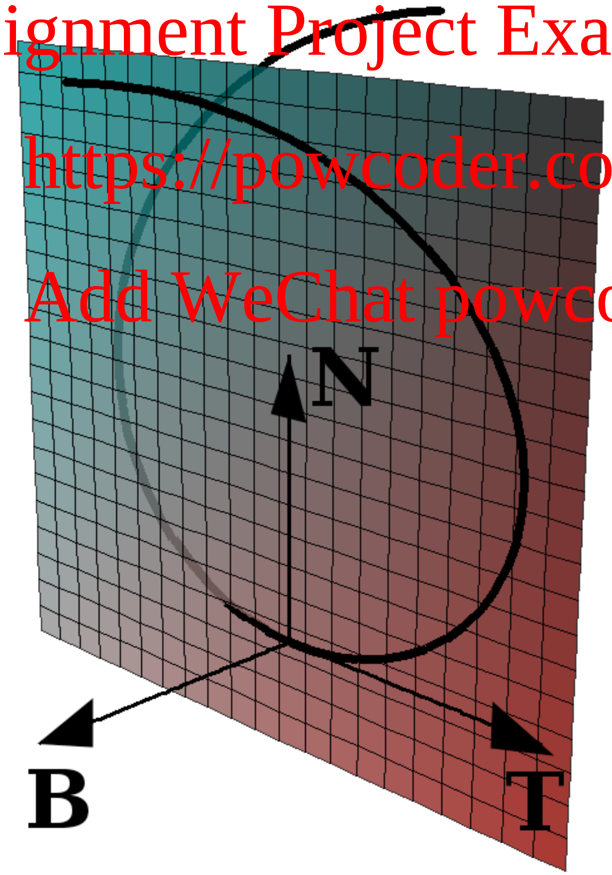
Differential Geometry Concepts

Binormal

- Normal to both tangent and normal vectors
- In 3D:
  B = **T** ⊗ **N**

Frenet frame

The three vectors together form the Frenet frame



Osculating plane defined by **N**,**T**

Curve design in graphics and animation

- In graphics we often want to interpolate or approximate a set of values in an efficient and predictable way

- We use parametric polynomials and constrain them to create desired types of curves

Parametric polynomial curve design in graphics

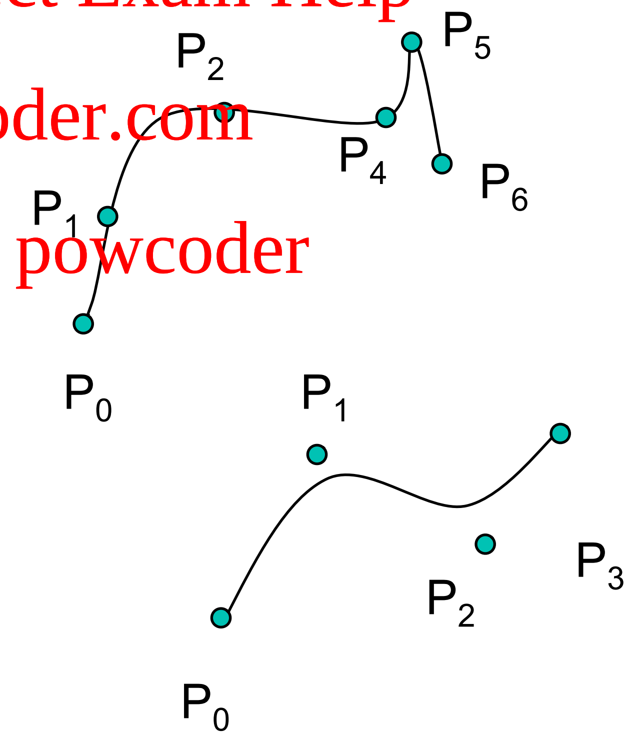
Geometric approach

Any t

$P_0, \ldots, P_L \rightarrow$ Curve generation $\rightarrow P(t)$

Constraints *Polynomial* *Curve*

Interpolation vs Approximation



Parametric polynomial curve design in graphics

Geometric approach

Any t

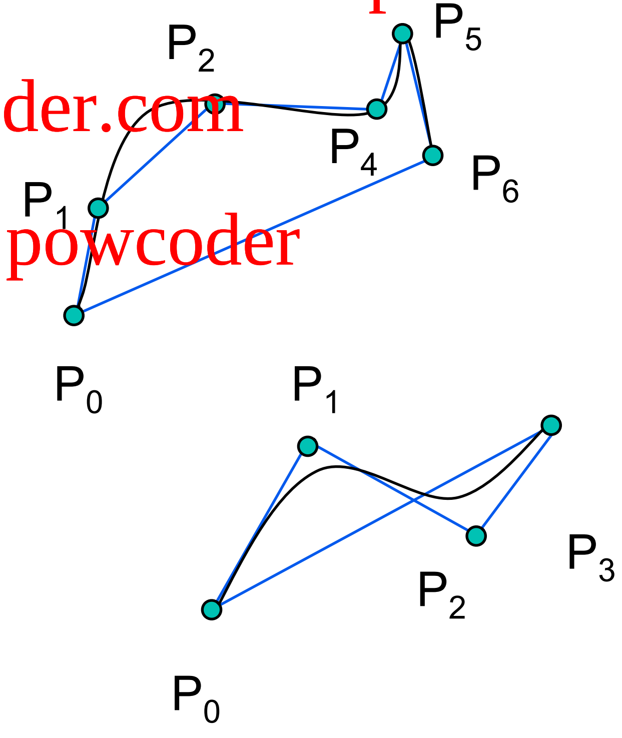
$P_0, \ldots, P_L \rightarrow$ Curve generation $\rightarrow P(t)$

Constraints *Polynomial* *Curve*

P_i control points

$P_0 \ldots P_L$ control polygon

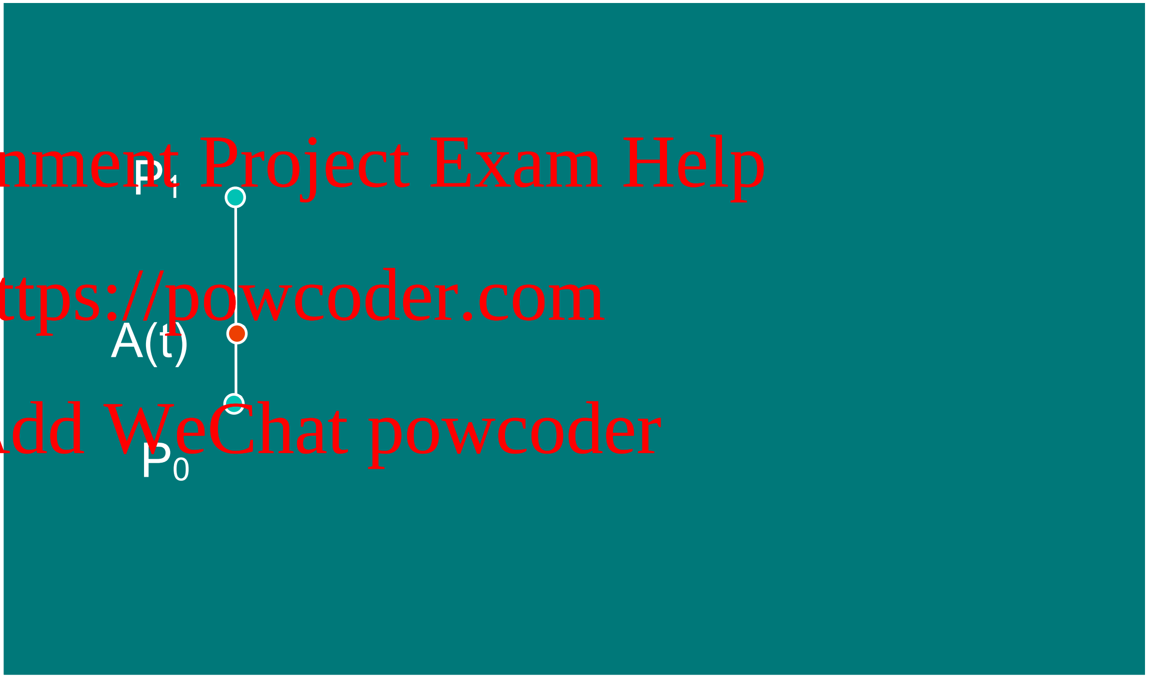
Interpolation vs Approximation



De Casteljau Algorithm

Tweening

Two points=line

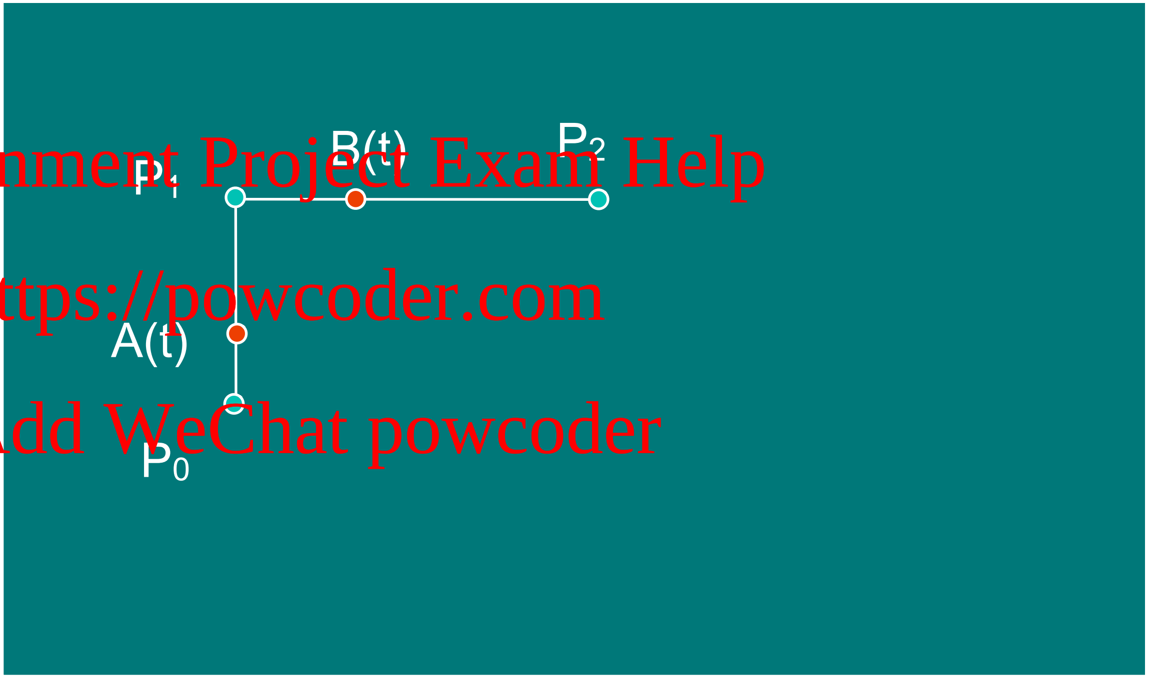


$A(t) = (1-t)P_0 + tP_1$

De Casteljau Algorithm

Tweening

Three points



$A(t) = (1\text{-}t)P_0 + tP_1$

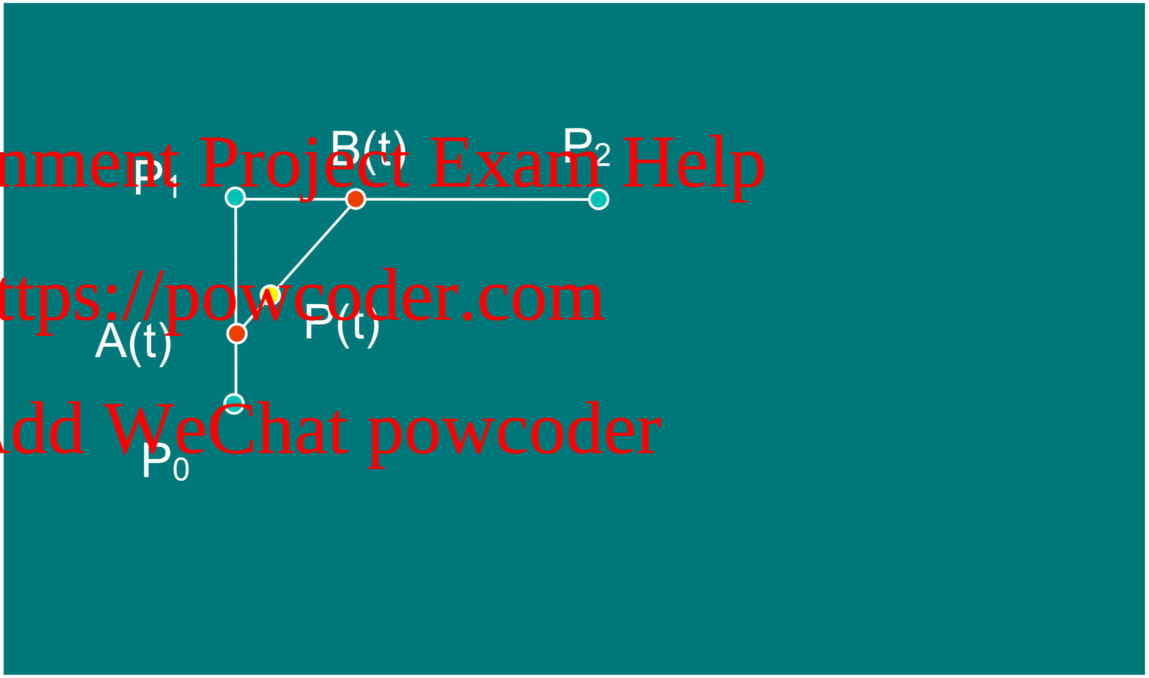
$B(t) = (1\text{-}t)P_1 + tP_2$

De Casteljau Algorithm

Tweening

Three points

(parabola)



$A(t) = (1-t)P_0 + tP_1$

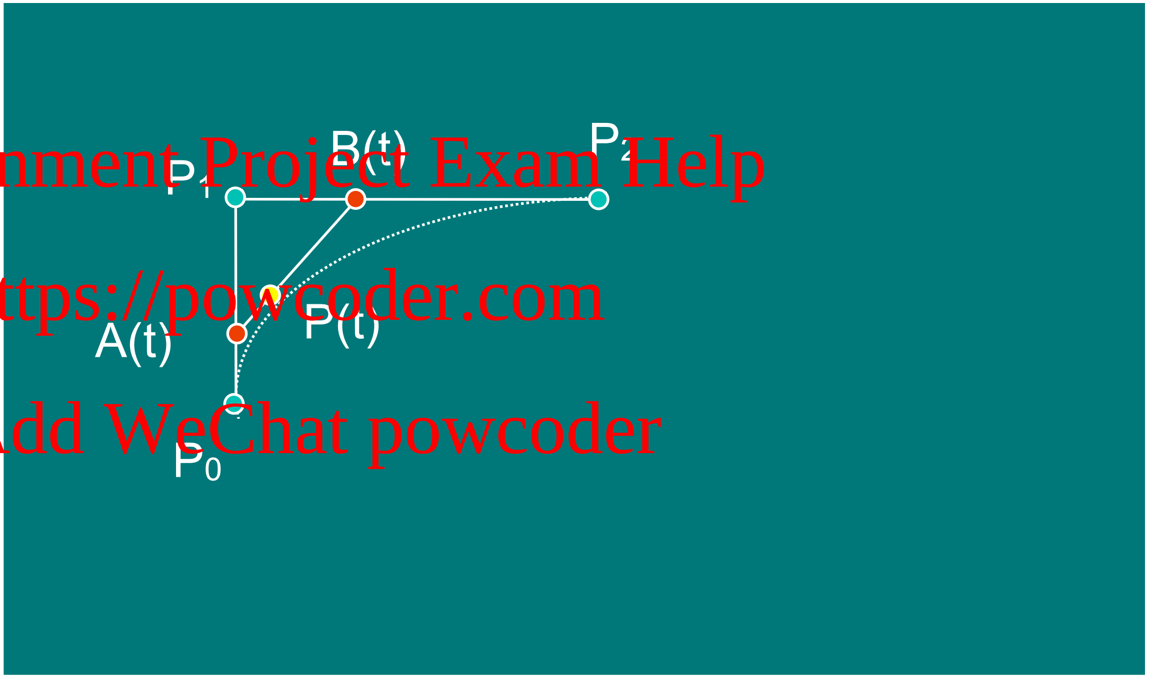
$B(t) = (1-t)P_1 + tP_2$

$P(t) = (1-t)\,A(t) + tB(t) = (1-t)^2P_0 + 2t(1-t)P_1 + t^2P_2$

De Casteljau Algorithm

Tweening

Three points
(parabola)



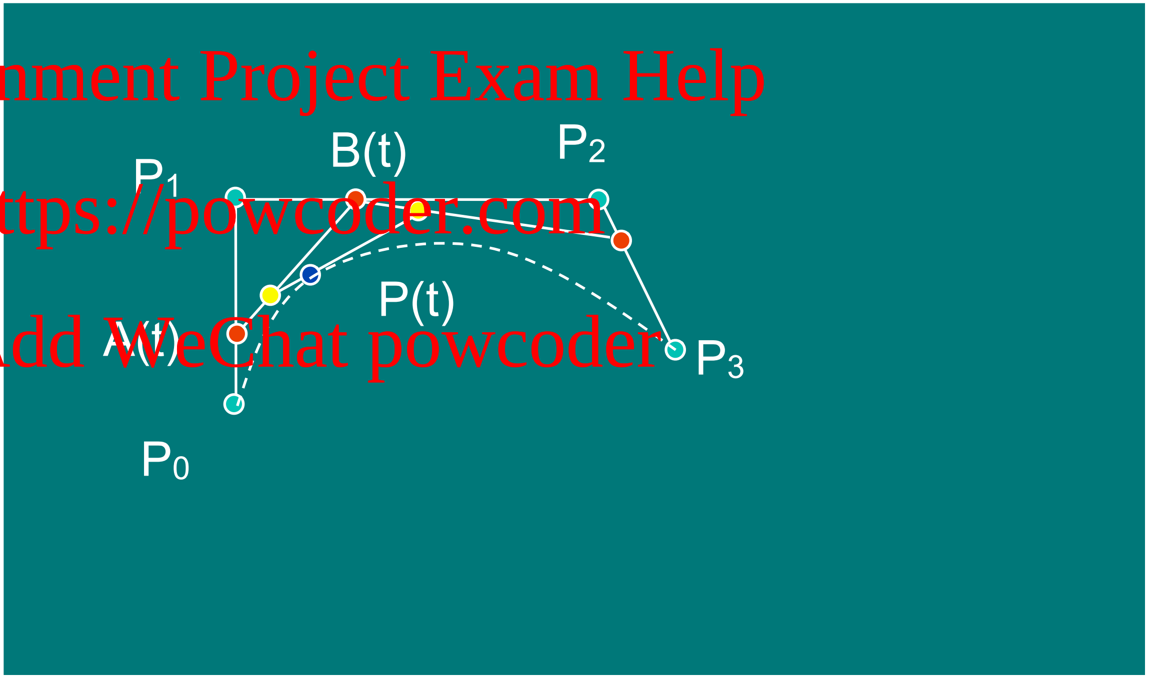
$A(t) = (1-t)P_0 + tP_1$

$B(t) = (1-t)P_1 + tP_2$

$P(t) = (1-t)\,A(t) + tB(t) = (1-t)^2P_0 + 2t(1-t)P_1 + t^2P_2$

De Calsteljau (cont)

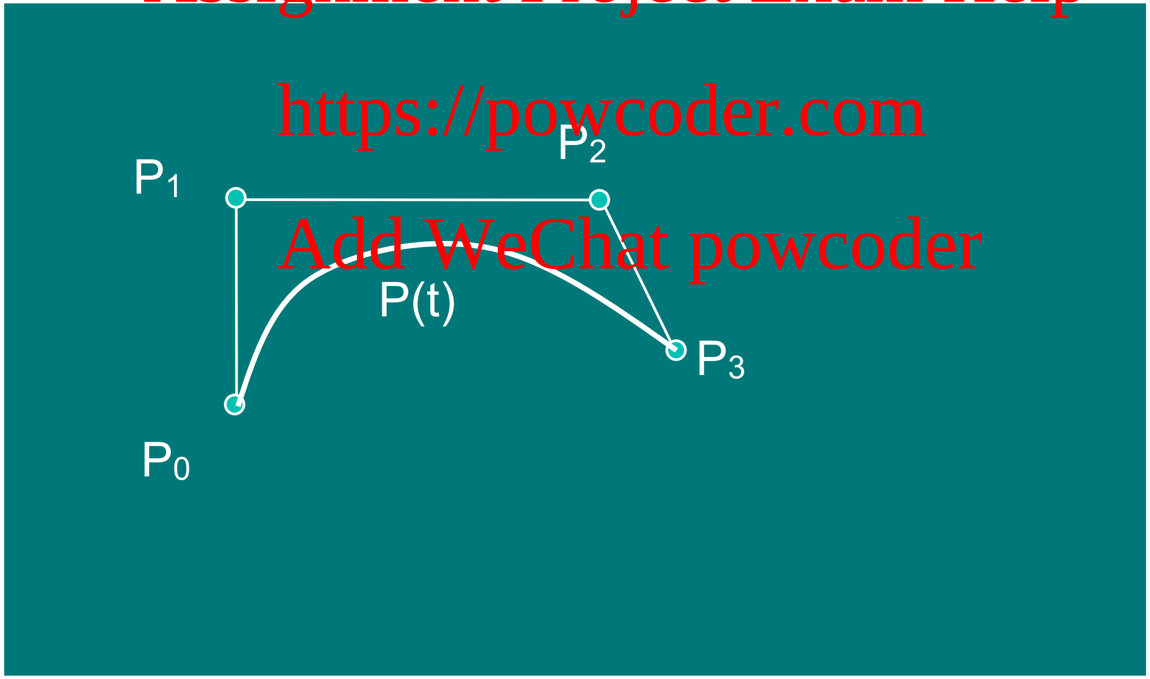
Tweening with four points



$$P(t) = (1-t)^3P_0+3(1-t)^2tP_1+3(1-t)t^2P_2+t^3P_3$$

Bezier Curves

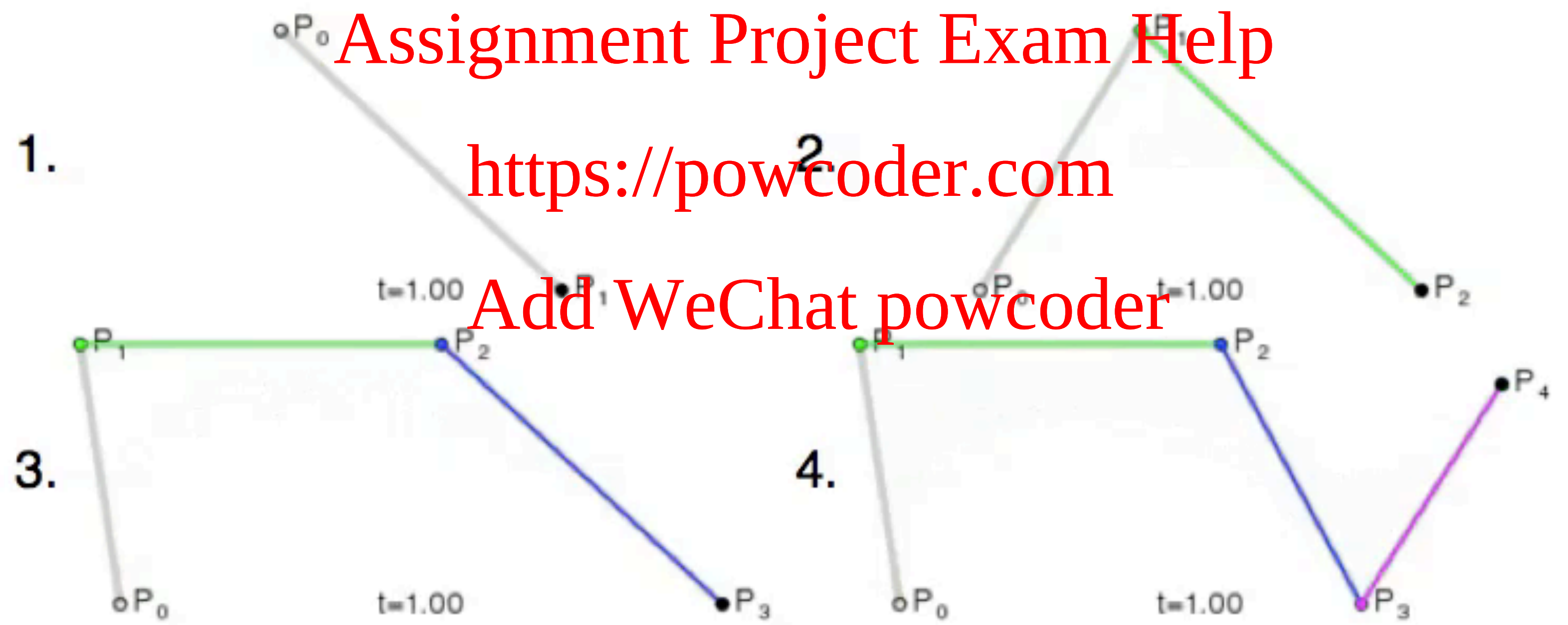
One of the most fundamental concepts in curve design



$$P(t) = (1-t)^3P_0+3(1-t)^2tP_1+3(1-t)t^2P_2+t^3P_3$$

Visualization

- Courtesy of Phil Tregoning, Wikipedia



Bézier curves: 1. linear; 2. quadratic; 3. cubic; 4. quartic.

Coefficients of Bezier Curves: Berstein polynomials

$$P(t) = (1\text{-}t)^3P_0+3(1\text{-}t)^2tP_1+3(1\text{-}t)t^2P_2+t^3P_3$$

$B^3_0\ (t) = (1-\ t)^3$

$B^3_1\ (t) = 3(1-\ t)^2t$

$B^3_2\ (t) = 3(1\text{-}t)t^2$

$B^3_3\ (t) = t^3$

Expansion of $[(1-\ t) + t]^3 = (1-\ t)^3 +3(1-\ t)^2t + 3(1-\ t)t^2 +t^3 \rightarrow$

$\Sigma\ B^3_k\ (t) = 1,\ k = 0,1,2,3$

Affine combination of points

Berstein Polynomials of L degree

L + 1 control points, P_k

$$P(t) = \sum_{k=0}^{L} B_k^L(t) P_k, \quad \text{where}$$

$$B_k^L(t) = \begin{pmatrix} L \\ K \end{pmatrix} (1-t)^{L-k} t^k$$

$$\begin{pmatrix} L \\ K \end{pmatrix} = \frac{L!}{k!(L-k)!}, \quad \text{for} \;\; L \geq k$$

$$\text{Affine combination:} \sum_{k=0}^{L} B_k^L(t) = 1, \quad \text{for all} \;\; t$$

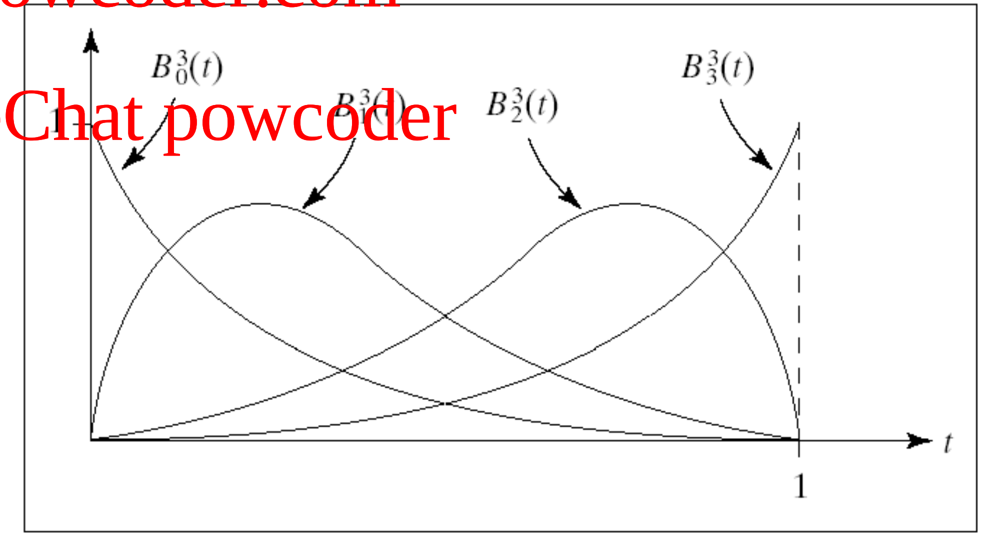
Expansion of $[(1-t) + t]^L$

Other Berstein Polynomials Properties

Allways positive

Zero only at t =0 or 1

Degree 3



Properties of Bezier curves

- End point interpolation
- Affine Invariance: $$T(P(t)) = \sum_{k=0}^{L} B_k^L(t)T(P)_k$$
- Invariance under affine transformation of the parameter
- Convex Hull property for t in [0,1] $$P = \sum_{k=0}^{L} a_k P_k, \text{ where } \sum_{k=0}^{L} a_k = 1 \text{ and } a_k > 0$$
- Linear precision by collapsing convex hull
- Variation Diminishing property: No straight line cuts the curve more times than it cuts the control polygon

Derivatives of Bezier curves

It can be shown that:

Any derivative also a Bezier curve of lower degree

$$P'(t) = L\sum_{k=0}^{L-1} B_k^{L-1}(t)\Delta P_k \text{ where } \Delta P_k = P_{k+1} - P_k$$

$$P''(t) = L(L-1)\sum_{k=0}^{L-2} B_k^{L-2}(t)\Delta^2 P_k \text{ where } \Delta^2 P_k = \Delta P_{k+1} - \Delta P_k$$

Which degree is best?

Cubic curves

- Lower order not enough flexibility
- Higher order too many wiggles and computationally expensive
- Cubic curves are lowest degree polynomial curves that are not planar in 3D

More complex curves

- Piecewise cubics

The general case for Cubic Parametric Curves (piece)

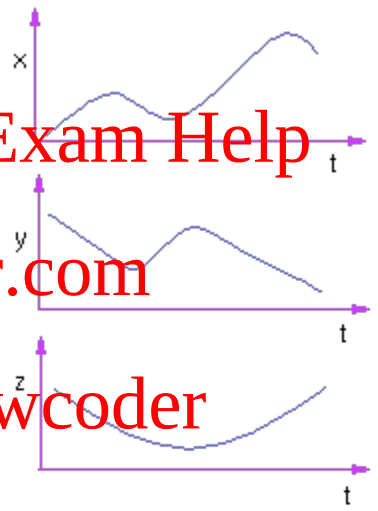
- Bezier Cubic Curves are only one possible family of cubic curves.

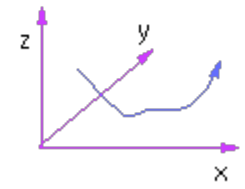
- We want more control of the constraints on the curves
  - *location*
  - *shape*

-

General form of Cubic parametric curves

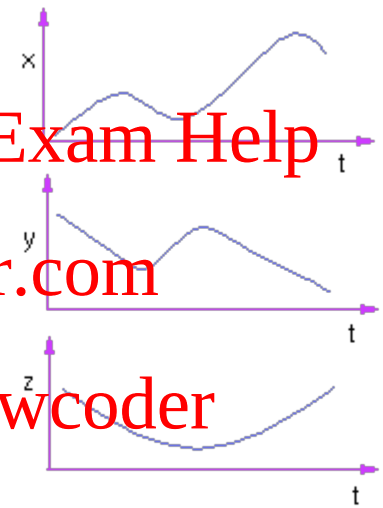
$$x(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$
$$y(t) = b_3 t^3 + b_2 t^2 + b_1 t + b_0$$
$$z(t) = c_3 t^3 + c_2 t^2 + c_1 t + c_0$$
$$t \in [0, 1]$$

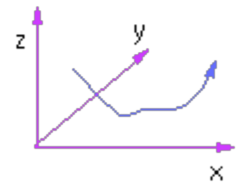




Matrix Form

$$x(t) = a_3t^3 + a_2t^2 + a_1t + a_0$$
$$y(t) = b_3t^3 + b_2t^2 + b_1t + b_0$$
$$z(t) = c_3t^3 + c_2t^2 + c_1t + c_0$$
$$t \in [0, 1]$$





$$x(t) = \begin{bmatrix} t^3 & t^2 & t^1 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$x(t) = TA$$
$$y(t) = TB$$
$$z(t) = TC$$

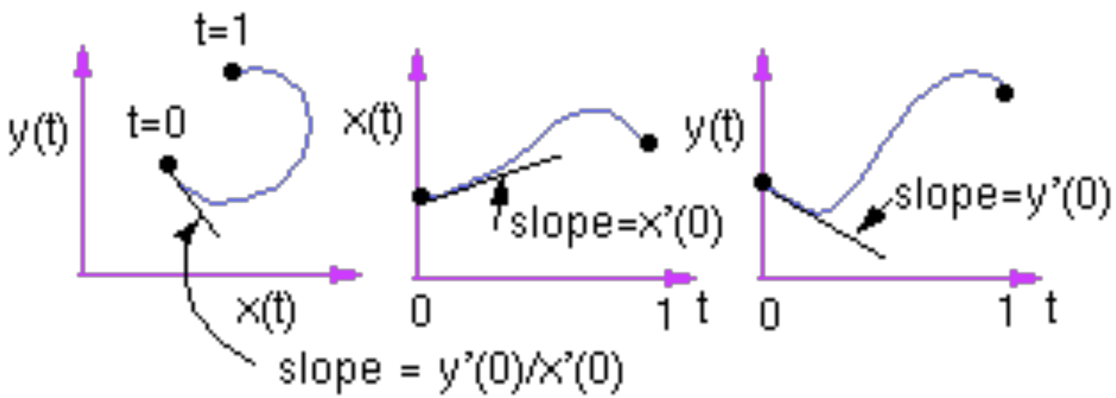
Derivative of Cubic Parametric Curves - tangent vector

Where the curve goes for some t

$$x(t) = \begin{bmatrix} t^3 & t^2 & t^1 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

How the curve is shaped for some t

$$x'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$



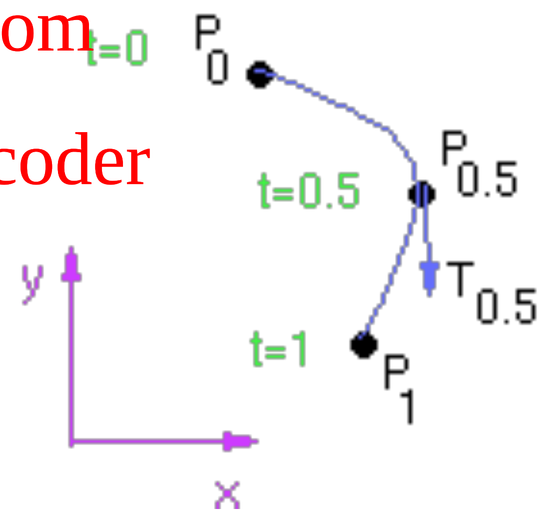
For a given set of constraints we get a family of curves

Constraints

Endpoints and a tangent at midpoint

$$x(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$x'(t) = \frac{dx}{dt} = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$



Setting up the curve

Constraints

$$x(t) = \begin{bmatrix} t^3 & t^2 & t^1 & 1 \end{bmatrix} A$$

$$x'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} A$$

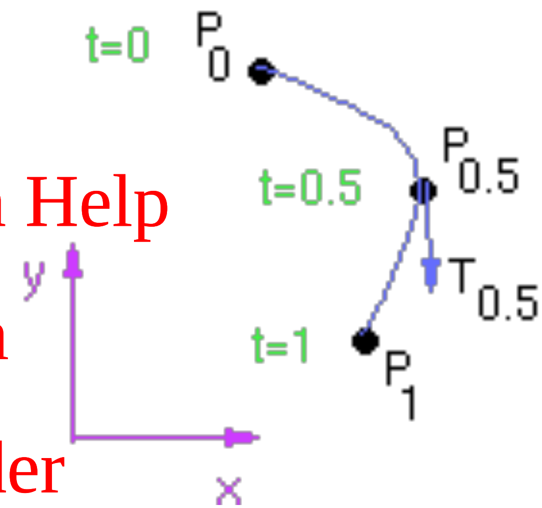
$$x(0) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} A$$

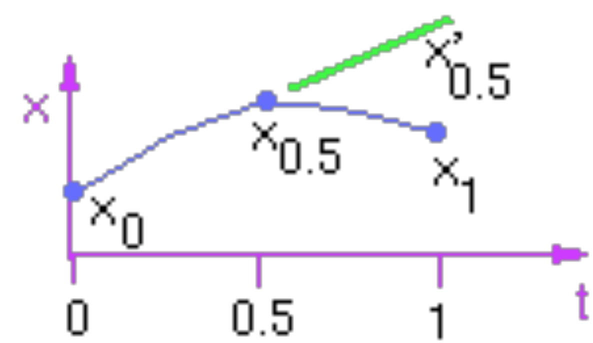
$$x(0.5) = \begin{bmatrix} 0.5^3 & 0.5^2 & 0.5 & 1 \end{bmatrix} A$$

$$x'(0.5) = \begin{bmatrix} 3(0.5)^2 & 2(0.5) & 1 & 0 \end{bmatrix} A$$

$$x(1) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} A$$

$$G_x = BA$$



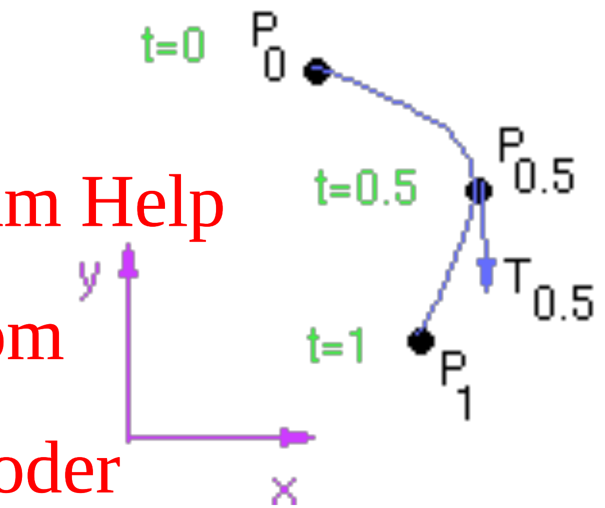


Solving for A

Constraints

$$x(t) = \begin{bmatrix} t^3 & t^2 & t^1 & 1 \end{bmatrix} A = TA$$

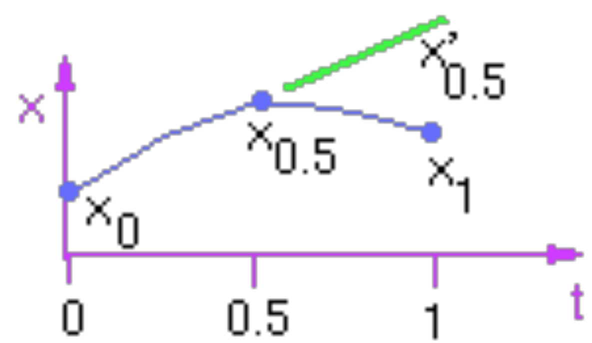
$$x'(t) = \begin{bmatrix} 3t^2 & 2t & 1 & 0 \end{bmatrix} A = T'A$$



$$\begin{bmatrix} x_0 \\ x_{0.5} \\ x'_{0.5} \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.5^3 & 0.5^2 & 0.5 & 1 \\ 3(0.5)^2 & 2(0.5) & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} A$$

$$G_x = BA \Rightarrow A = B^{-1}G_x$$

$$x(t) = TA \Rightarrow \boxed{x(t) = TB^{-1}G_x}$$



Final form

Basis matrix

$$x(t) = TB^{-1}G_x$$

$$Set\ \ M = B^{-1}$$

$$x(t) = TMG_x$$

$$y(t) = TMG_y$$

$$z(t) = TMG_z$$

$$P(t) = TMG$$

For the example

$$M = \begin{bmatrix} -4 & 0 & -4 & 4 \\ 8 & -4 & 6 & -4 \\ -5 & 5 & -2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Blending functions

T*M

$$x(t) = TMG_x \Rightarrow$$
$$x(t) = \left[\begin{array}{cccc} f_1(t) & f_2(t) & f_3(t) & f_4(t) \end{array}\right] G_x$$

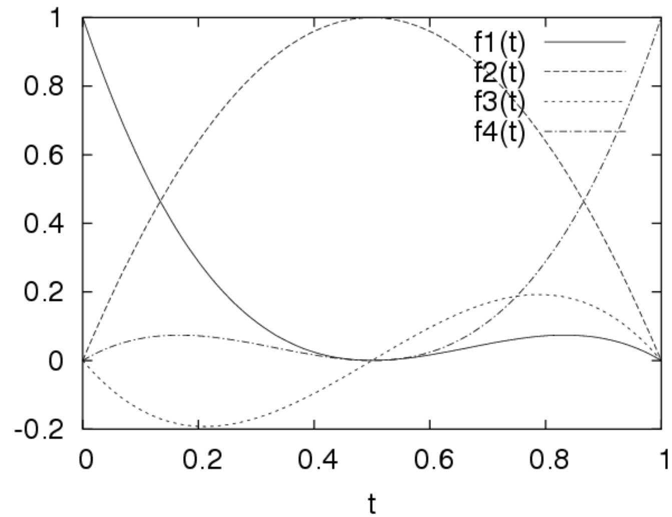
For the example

$$f_1(t) = -4t^3 + 8t^2 - 5t + 1$$

$$f_2(t) = -4t^2 + 4t$$
$$f_3(t) = -4t^3 + 6t^2 - 2t$$

$$f_4(t) = 4t^3 - 4t^2 + t$$

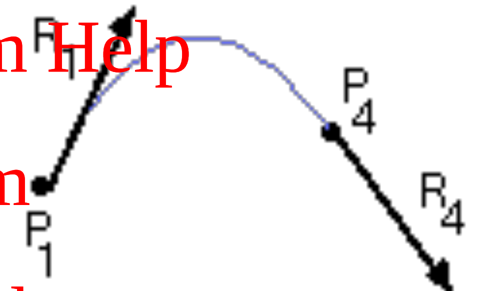
Each blending function weights the contribution of one of the constraints



Hermite Curves

Constraints

Two points and two tangents

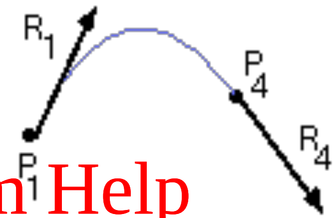


$$G_h = \left[\begin{array}{cccc} P_1 & P_4 & R_1 & R_4 \end{array} \right]$$
$$x(t) = TA_h = TM_hG_h$$

$$x(0) = P_1 = \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array} \right] A_h$$
$$x(1) = P_4 = \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \right] A_h$$
$$x'(0) = R_1 = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array} \right] A_h$$
$$x'(1) = R_4 = \left[\begin{array}{cccc} 3 & 2 & 1 & 0 \end{array} \right] A_h$$
$$G_h = B_hA_h$$
$$A_h = B_h^{-1}G_h$$
$$x(t) = TA_h$$

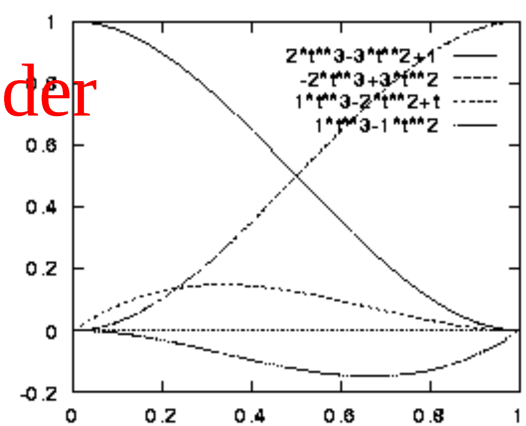
Hermite Curves

Blending functions



$$M_h = B_h^{-1} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

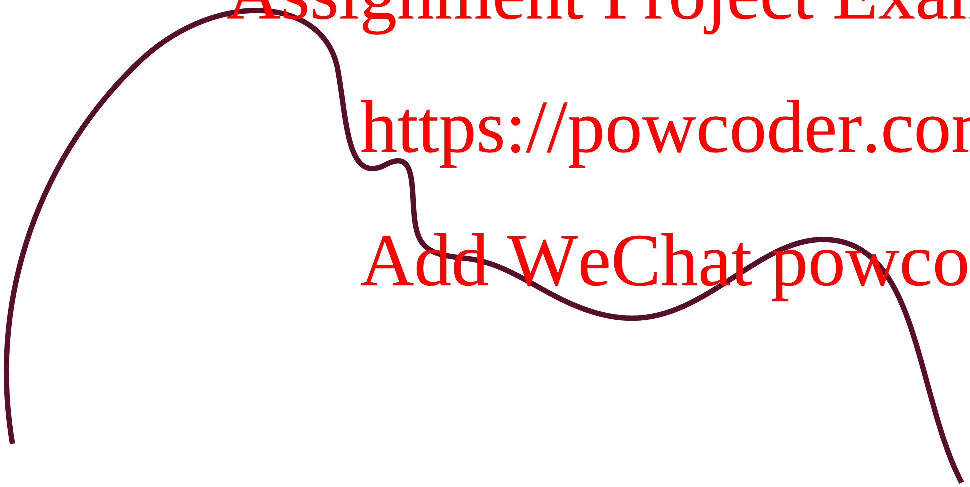
$$x(t) = T M_h G_h \Rightarrow$$
$$x(t) = \begin{bmatrix} f_1(t) & f_2(t) & f_3(t) & f_4(t) \end{bmatrix} G_h$$



$$f_1(t) = 2t^3 - 3t^2 + 1$$
$$f_2(t) = -2t^3 + 3t^2$$
$$f_3(t) = t^3 - 2t^2 + t$$
$$f_4(t) = t^3 - t^2$$

What does the magnitude of the tangent do?

Interactive demo



Bezier Curves

Special case of Hermite curves

$$P_{1,h} = P_1$$
$$P_{4,h} = P_4$$
$$R_{1,h} = 3(P_2 - P_1)$$
$$R_{4,h} = 3(P_4 - P_3)$$

Bezier Curves

Special case of Hermite curves

$$P_{1,h} = P_1$$

$$P_{4,h} = P_4$$

$$R_{1,h} = 3(P_2 - P_1)$$

$$R_{4,h} = 3(P_4 - P_3)$$

$$\begin{bmatrix} P_{1,h} \\ P_{4,h} \\ R_{1,h} \\ R_{4,h} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

Bezier Curves

Special case of Hermite curves

$$\begin{bmatrix} P_{1,h} \\ P_{4,h} \\ R_{1,h} \\ R_{4,h} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$G_h = M_{bh} G_b$$

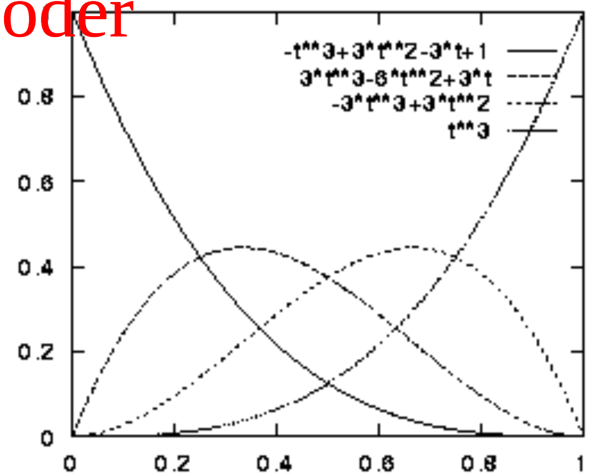
$$P(t) = T M_h G_h \Rightarrow P(t) = T M_h M_{bh} G_b \Rightarrow$$
$$P(t) = T M_b G_b$$

Bezier Curves

Special case of Hermite curves

We can verify that TM_b are the bernstein polynomials

$$f_1(t) = (1-t)^3$$
$$f_2(t) = 3t(1-t)^2$$
$$f_3(t) = 3t^2(1-t)$$
$$f_4(t) = t^3$$

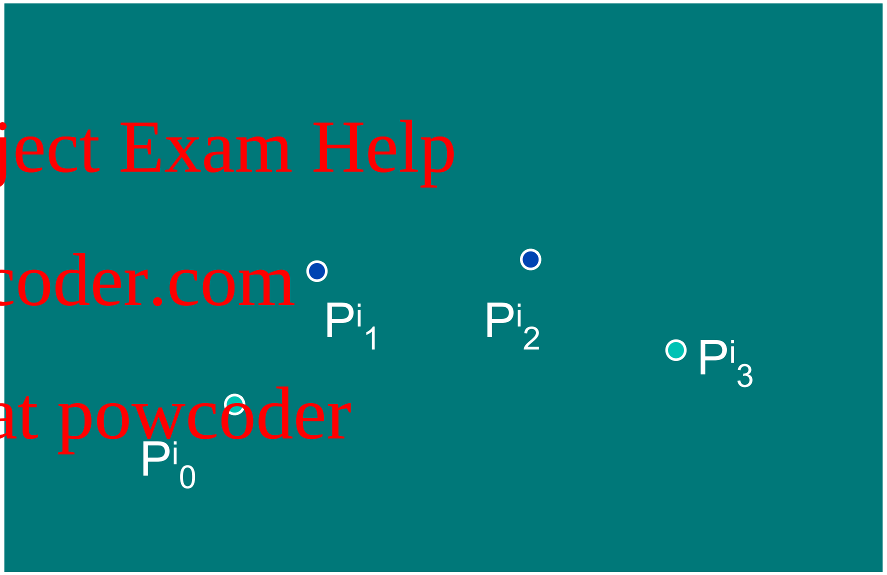


Transforming between representations

Just like Bezier and Hermite curves can be transformed into each other with a matrix multiplication, other families of curves can do so as well

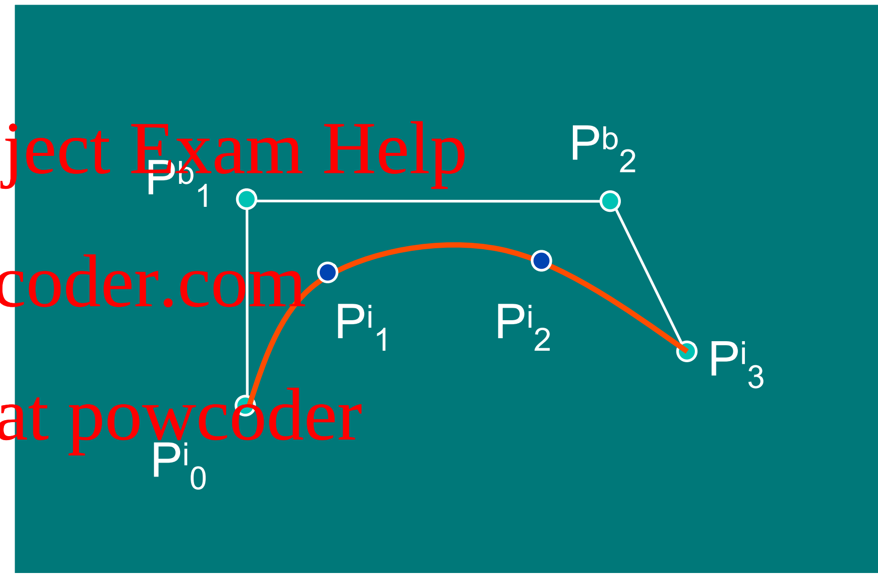
Bezier to Interpolating curves

- Curve must interpolate $P^i_0, P^i_1, P^i_2, P^i_3$



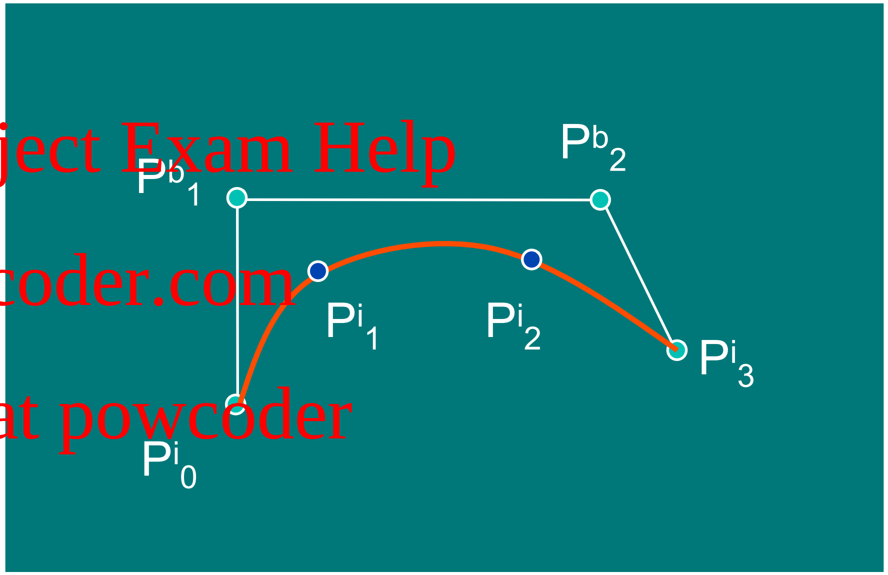
Bezier to Interpolating curves

- Curve must interpolate $P^i_0, P^i_1, P^i_2, P^i_3$
- How can we find the Bezier points P^b from the P^is so that the resulting Bezier curve interpolates the Pi points?



Bezier to Interpolating curves

For the next three slides points are row vectors!!



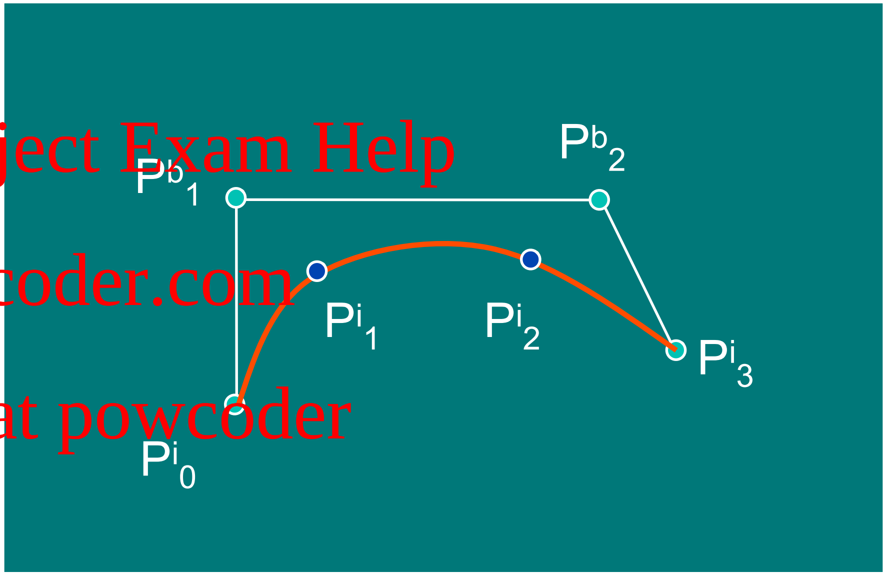
$$P_j^i = [\ P_j^i,x \quad P_j^i,y \quad P_j^i,z\ ]$$

$$G^b = \begin{pmatrix} P_0^b \\ P_1^b \\ P_2^b \\ P_3^b \end{pmatrix}$$

The t-values are chosen arbitrarily

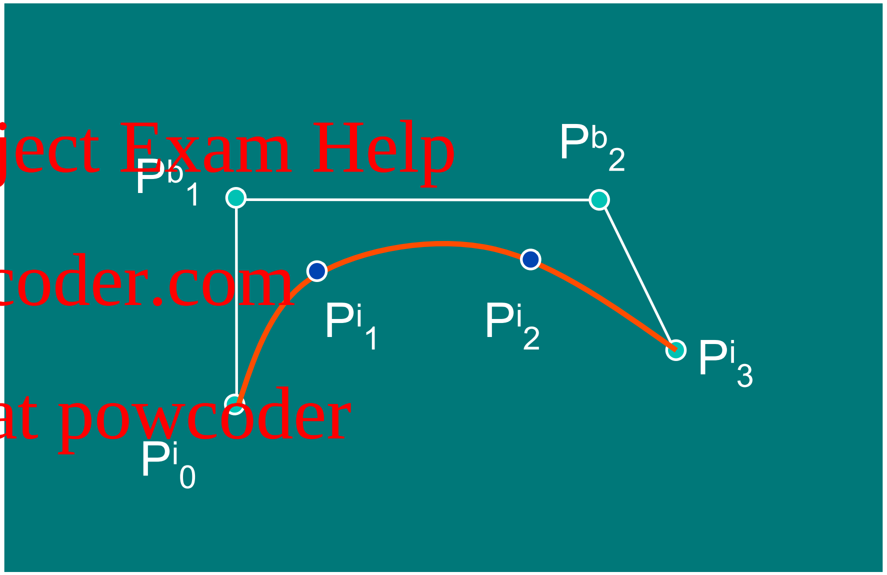
$$\begin{array}{l} P_0^i = T(0)M_bG^b \\ P_1^i = T(\frac{1}{3})M_bG^b \\ P_2^i = T(\frac{2}{3})M_bG^b \\ P_3^i = T(1)M_bGx^b \end{array} \rightarrow \begin{pmatrix} P_0^i \\ P_1^i \\ P_2^i \\ P_3^i \end{pmatrix} = \begin{pmatrix} T(0) \\ T(\frac{1}{3}) \\ T(\frac{2}{3}) \\ T(1) \end{pmatrix} M_b \begin{pmatrix} P_0^b \\ P_1^b \\ P_2^b \\ P_3^b \end{pmatrix}$$

Bezier to Interpolating curves



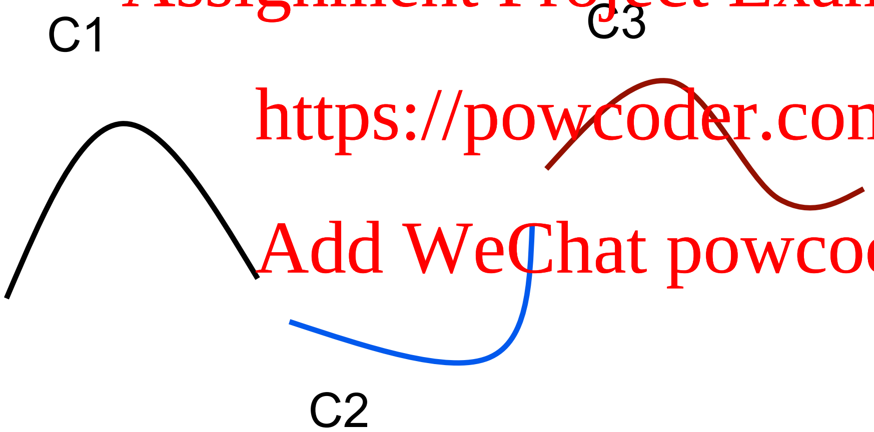
$$\begin{pmatrix} P_0^i \\ P_1^i \\ P_2^i \\ P_3^i \end{pmatrix} = \begin{pmatrix} T(0) \\ T(\frac{1}{3}) \\ T(\frac{2}{3}) \\ T(1) \end{pmatrix} M_b \begin{pmatrix} P_0^b \\ P_1^b \\ P_2^b \\ P_3^b \end{pmatrix} \Rightarrow \mathbf{P}^i = T M_b \mathbf{P}^b$$

Bezier to Interpolating curves



$$\mathbf{P}^i = TM_b\mathbf{P}^b \Leftrightarrow \mathbf{P}^b = (TM_b)^{-1}\mathbf{P}^i$$

Piecewise cubic curves



Connection?

Continuity

Geometric G^k-continuity

$P^{(i)}(t-) = c_i P^{(i)}(t+)$ $\forall t$ in [a,b]

for $i = 0,\ldots,k$ and

for some c_i positive constants

Parametric C^k-continuity

$P^{(i)}$ exists and is continuous $\forall t$ in [a,b], for $i = 0,\ldots,k$

Terminology:

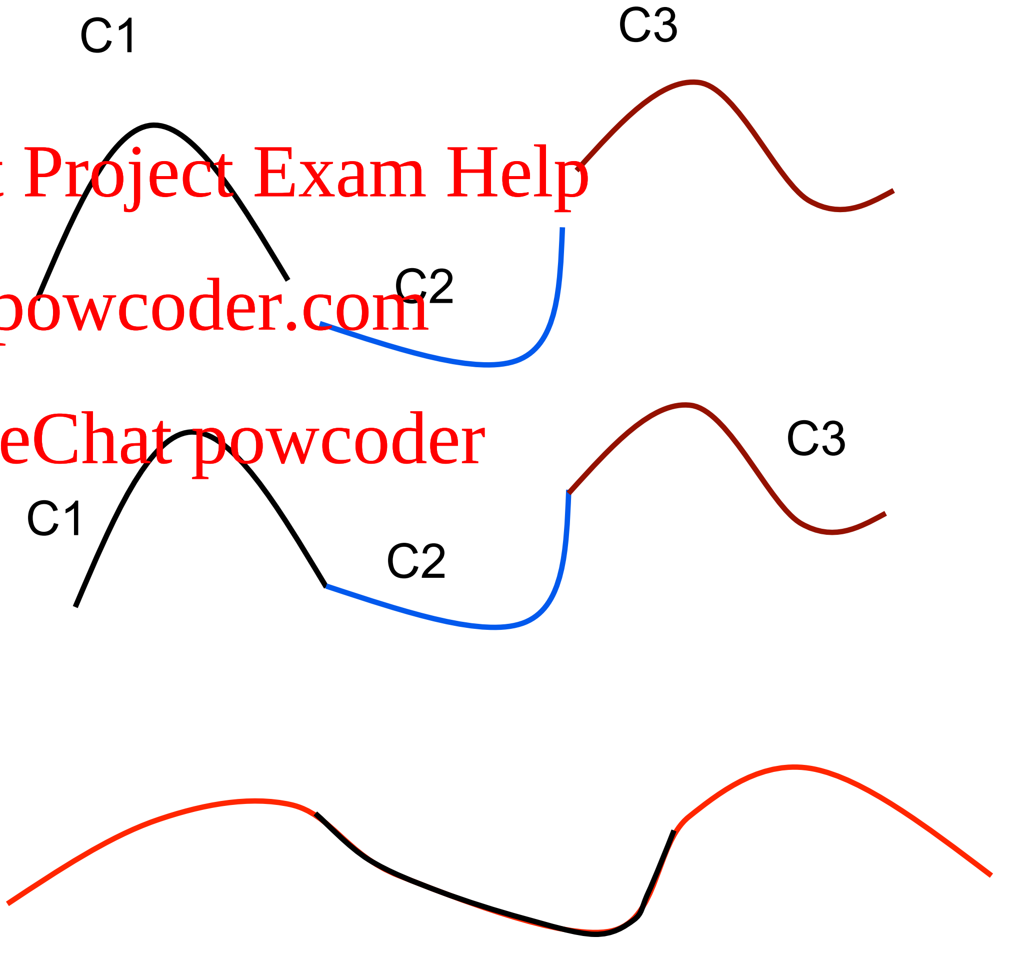
P is k-smooth

P has kth-order continuity

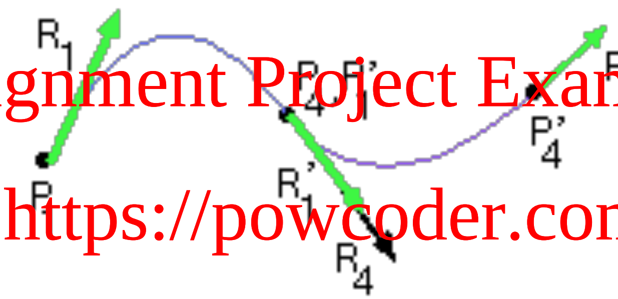
($P^{(i)}$ is the i-th derivative)

Is a C^k-continuous function G^K continuous as well?

Examples

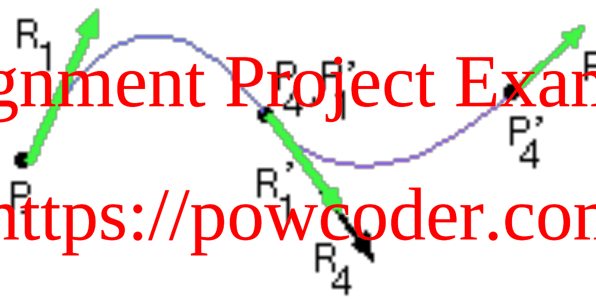


Piecewise Cubic Hermite Curves



What are the conditions for G1 continuity?

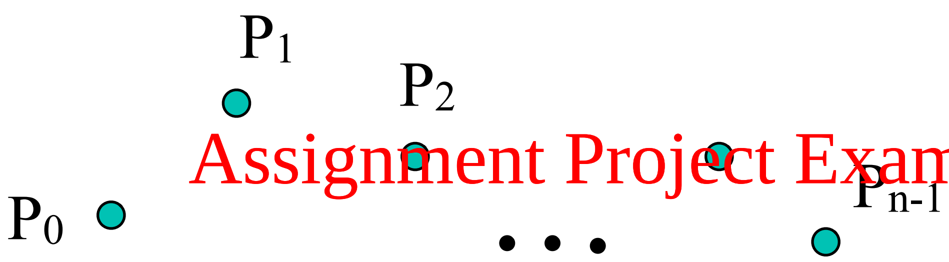
Piecewise Cubic Hermite Curves



R’1 = kR4

P1’= P4

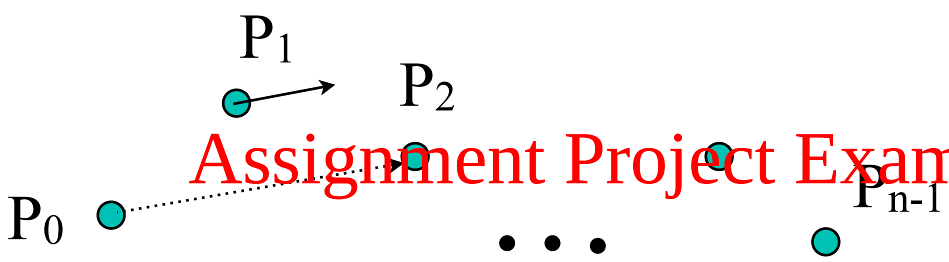
Catmull-Rom splines



Fit a piece wise cubic curve to the points

- P_i, $i = 0,1,\ldots,n-1$

Catmull-Rom splines



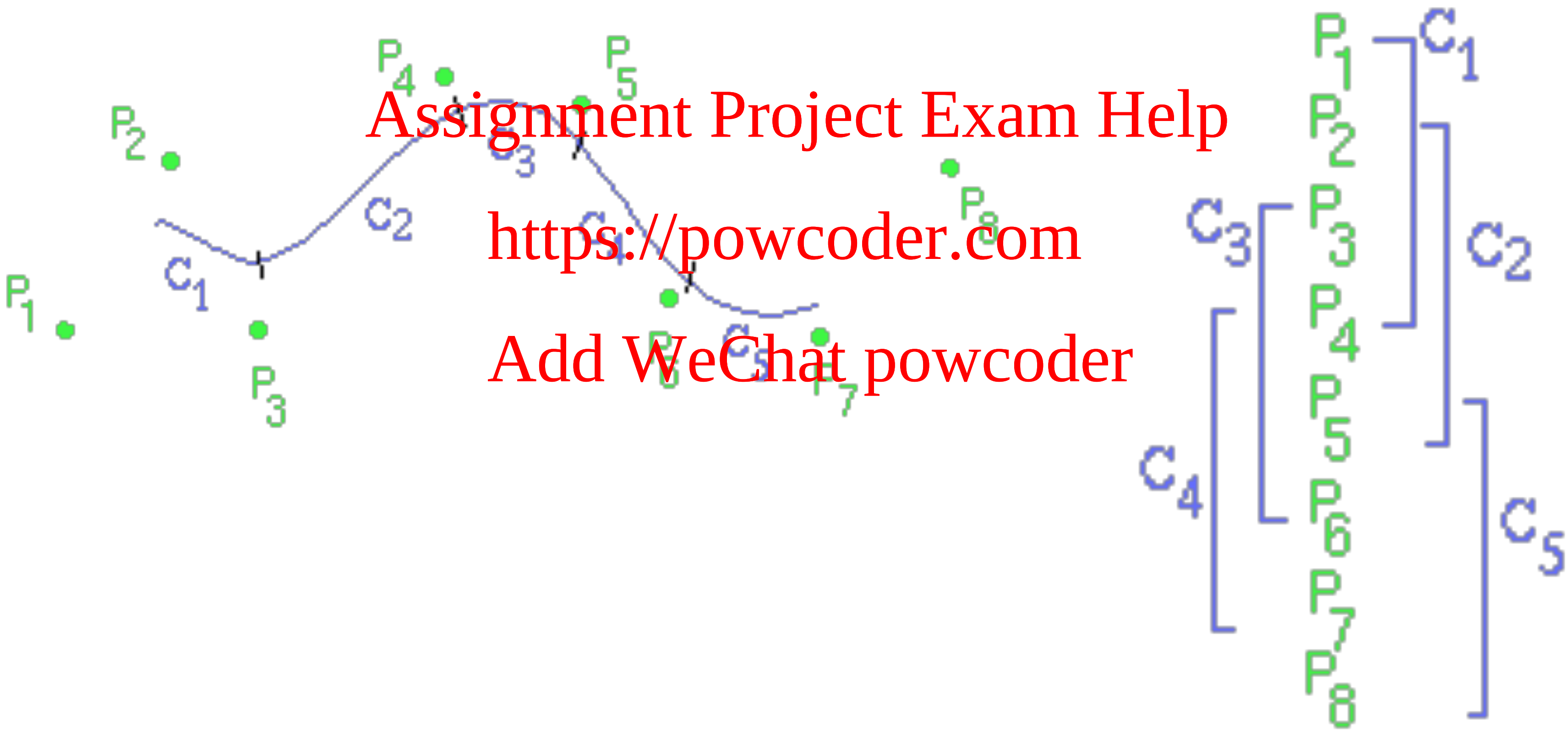
Fit a piece wise hermite curve to the points

- P_i, $i = 0,1,\ldots,n-1$

Second order accurate tangents

- $T_i = (P_{i+1} - P_{i-1})/2.0$ for $i = 1,\ldots,n-2$
- $T_0 = 2(P_1 - P_0) - (P_2-P_0)/2$, similarly for T_{n-1}

Uniform BSplines



Matrix form

For a bspline curve with:

- m+1 control points $P_0, \ldots, P_m$
- m-2 segments $Q_3, \ldots, Q_m$
- t in [3,…,m]

$$Q_i(t) = \begin{bmatrix} (t-t_i)^3 & (t-t_i)^2 & (t-t_i) & 1 \end{bmatrix} \mathrm{M}_{bspline} \begin{bmatrix} P_{i-3} \\ P_{i-2} \\ P_{i-1} \\ P_i \end{bmatrix}$$

Properties

C2 continuous

Convex hull property

NO invariace under perspective projection!

NURBS: Nonuniform Rational B-splines

$X(t) = X(t) / W(t)$

$Y(t) = Y(t) / W(t)$

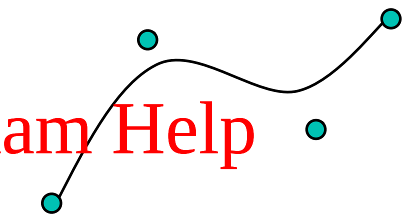
$Z(t) = Z(t) / W(t)$

- Exact conic sections

- Invariance under perspective projection

Summary: General problem

$P_0, \ldots, P_L \rightarrow$ Curve generation $\rightarrow P(t)$



$$P(t) = \sum_{i=0}^{3} B_i(t) P_i, \quad t \in [a, b], \quad a, b \in \Re$$

where

$B_i(t)$: Blending functions

$P_i, i = 1, \cdots, L$: Control Points

Blending functions

Weight the influence of each constraint (e.g. control point) on the curve created.

Wish list for blending functions

- Easy to compute and stable
- Sum to unity for every t in [a,b]
- Support over portion of [a,b]
- Interpolate certain control points
- Sufficient smoothness

Example: Bezier curves

- Sum up to unity
- Smooth
- Interpolate first and last
- Expensive to compute for large L
- No local control

$$P(t) = \sum_{k=0}^{L} B_k^L(t) P_k \quad \text{where}$$

$$B_k^L(t) = \begin{pmatrix} L \\ K \end{pmatrix} (1-t)^{L-k} t^k$$

$$\begin{pmatrix} L \\ K \end{pmatrix} = \frac{L!}{k!(L-k)!}, \quad \text{for} \quad L \geq k$$

$$\text{Affine combination:} \sum_{k=0}^{L} B_k^L(t) = 1, \quad \text{for all} \quad t$$

Rendering parametric curves

Transform into primitives we know how to handle

- Line segments

Single cubic segment

C(t): t in [0,1]

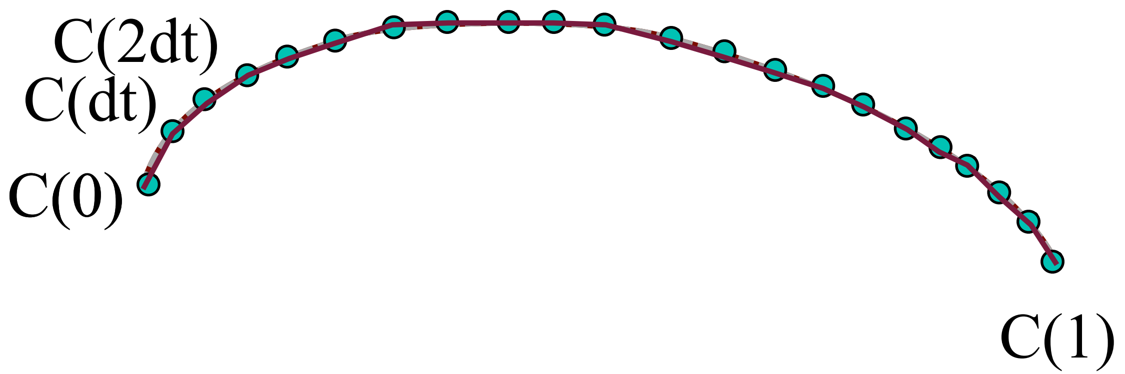
Converting to Lines

Straightforward Uniform subdivision

Evaluate C(t) at n points spaced by dt=1/(n-1)

C(0), C(dt), C(2dt),…,C((n-1)*dt=1)

Draw as lines



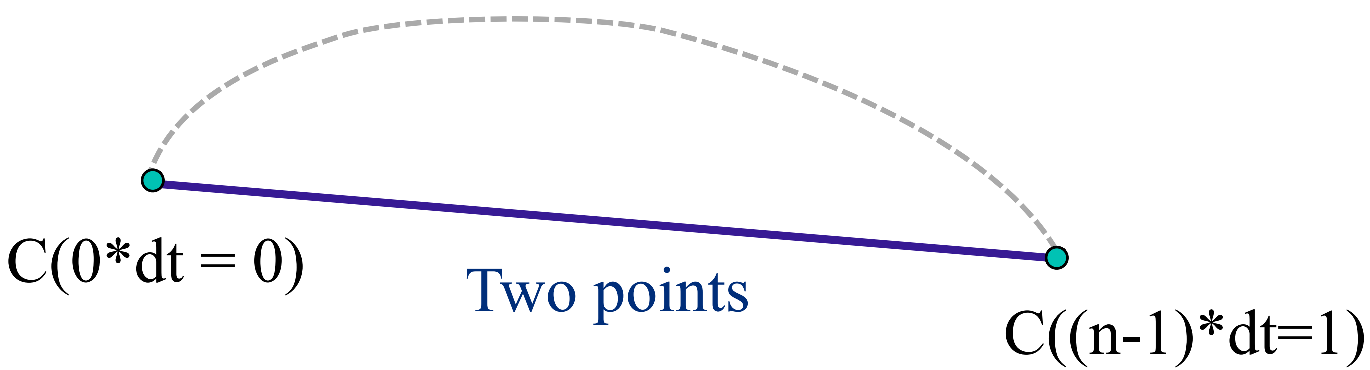
Converting to Lines

Straightforward Uniform subdivision

Evaluate C(t) at n points spaced by dt=1/(n-1)

C(0), C(dt), C(2dt),…,C((n-1)*dt=1)

Here n = 2 and dt = 1/(n-1) = 1



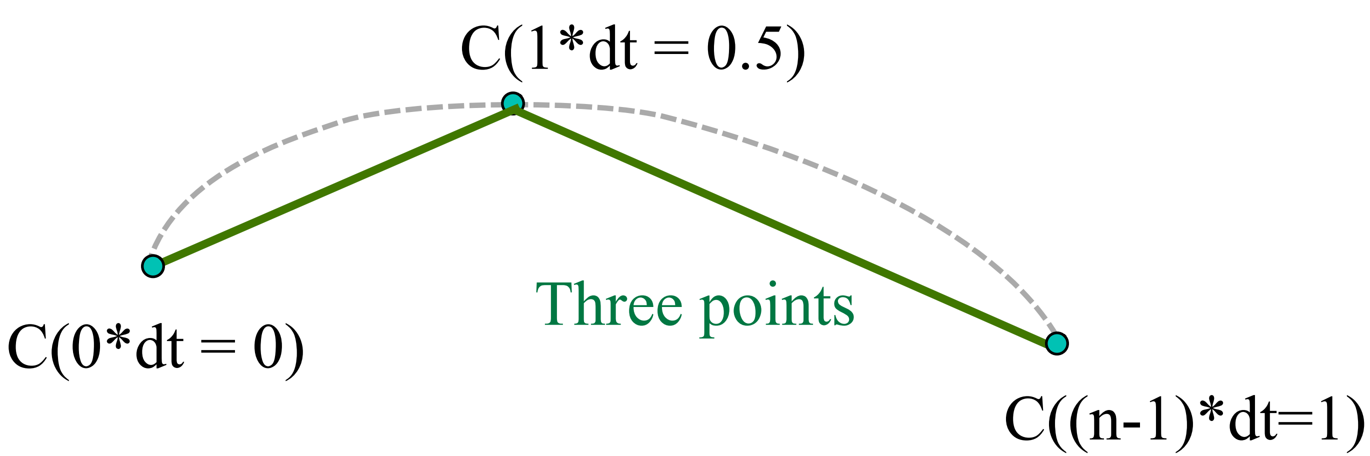
Converting to Lines

Straightforward Uniform subdivision

Evaluate C(t) at n points spaced by dt=1/(n-1)

C(0), C(dt), C(2dt),…,C((n-1)*dt=1)

Here n = 3 and dt = 1/(n-1)= 0.5



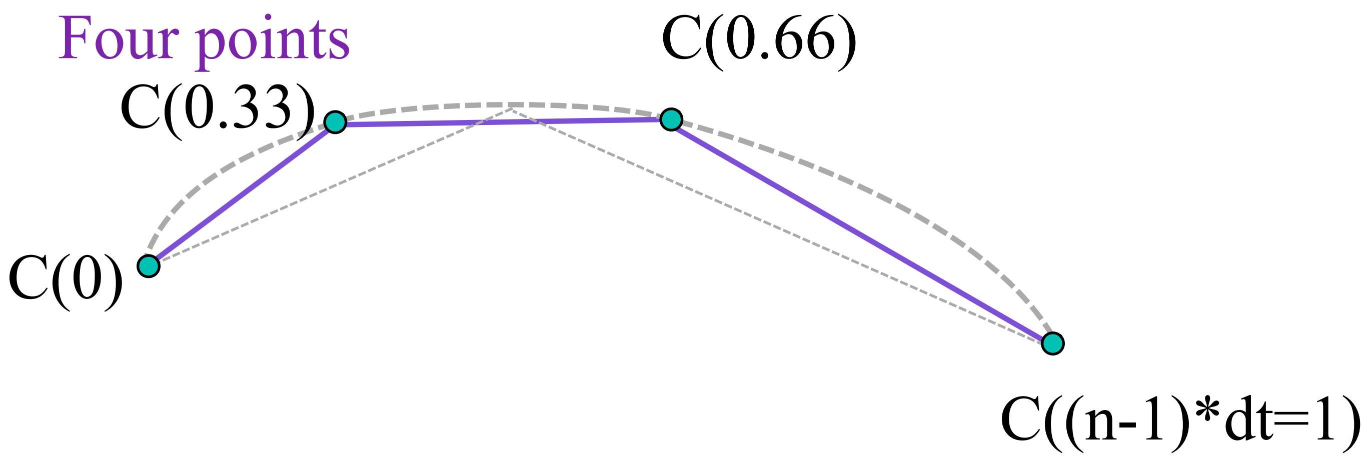
Converting to Lines

Straightforward Uniform subdivision

Evaluate C(t) at n points spaced by dt=1/(n-1)

C(0), C(dt), C(2dt),…,C((n-1)*dt=1)

Here n = 4 and dt = 1/(n-1) = 0.33



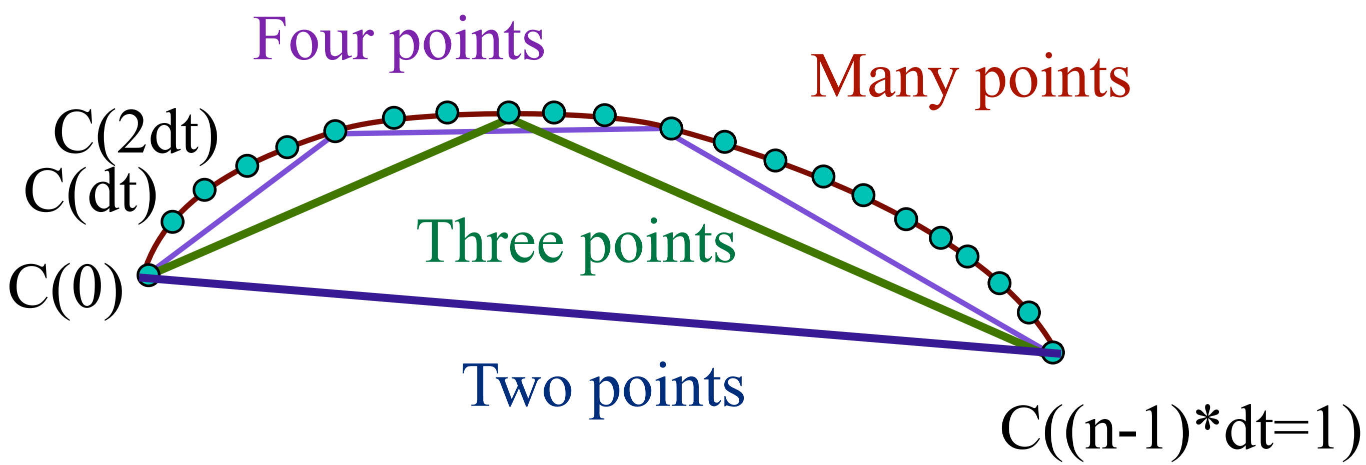
Converting to Lines

Straightforward Uniform subdivision

Evaluate C(t) at n points spaced by dt=1/(n-1)

C(0), C(dt), C(2dt),…,C((n-1)*dt=1)

All together for comparison

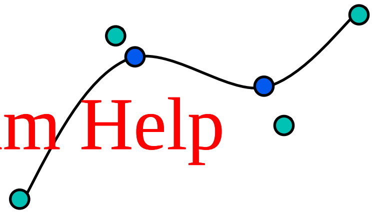


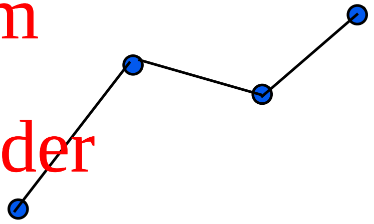
How many evaluation points are enough for Bezier curves?

Not too few

Not too many

Ok, how many?





Subdivision schemes

Uniform

Non-uniform

- Adaptive

Error metrics that define how much information we are losing

An example...

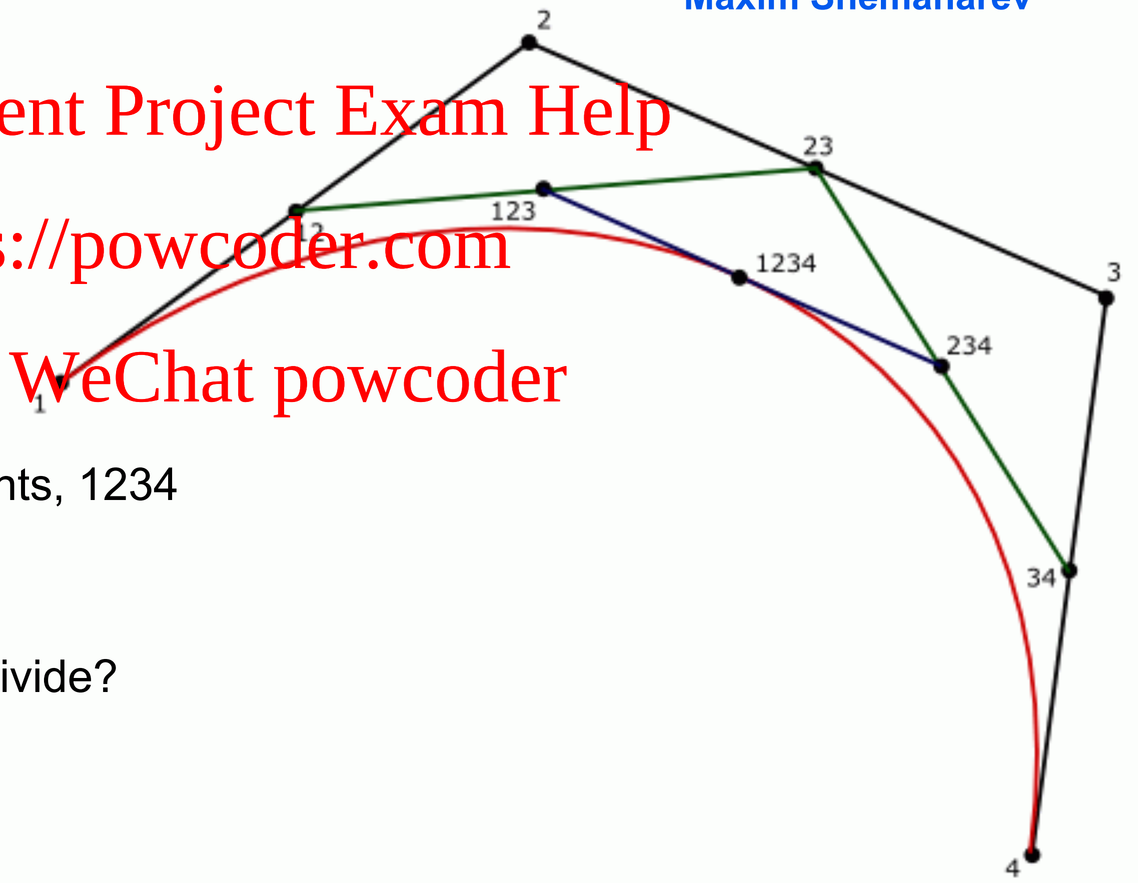
Adaptive Subdivision of Bezier Curves

de Casteljau subdivision

One Bezier curve becomes 2 flatter curves

Images courtesy of **Maxim Shemanarev**

Original points 1,2,3,4 →
Midpoints 12, 23, 34
Midpoints of midpoints: 123, 234
Midpoints of midpoints of midpoints, 1234
Remember: tweening for t = 0.5
Can chose any t we want
Ok, how many times do we subdivide?

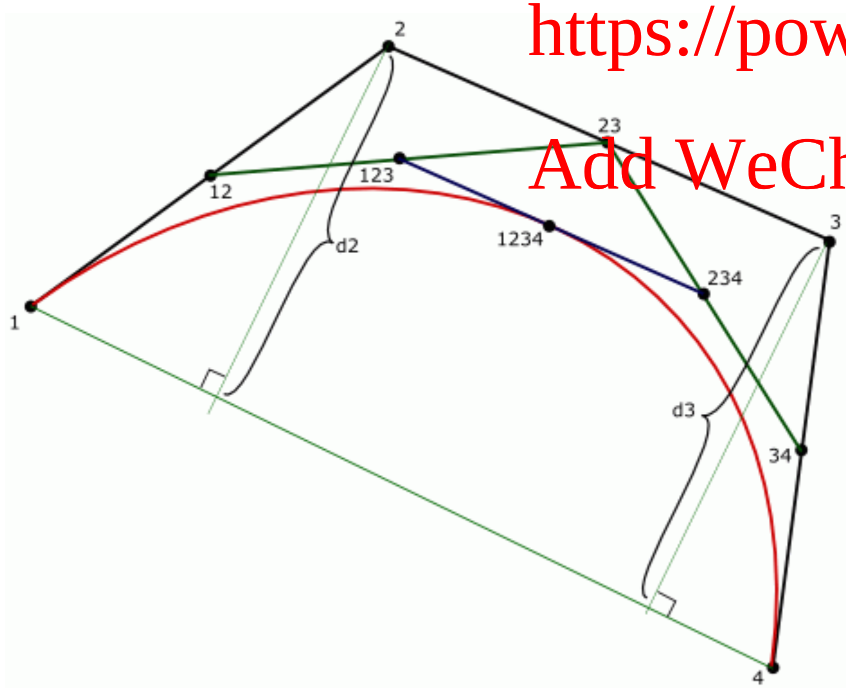


Error metrics

Images courtesy of
Maxim Shemanarev

Examples:

Point distance



Tangent distance