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Mutation

CSI 3120

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Reasoning about Mutable State is Hard

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mutable set immutable set

```
insert i s1  
f x;  
member i s1
```

```
let s1 = insert i s0 in  
f x;  
member i s1
```

Is member i s1 Assignment Project Exam Help

- When s1 is mutable, one must look at f to determine if it modifies s1.
- Worse, one must often solve the *aliasing problem*.
- Worse, in a concurrent setting, one must look at *every other function* that *any other thread may be executing* to see if it modifies s1.

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Thus far...

We have considered the (almost) purely functional subset of OCaml.

- We've had a few side effects: printing & raising exceptions.

Two reasons for this emphasis:

- *Reasoning about functional code is easier.*
 - Both formal reasoning
 - equationally, using the substitution model
 - and informal reasoning
 - Data structures are *persistent*.
 - They don't change – we build new ones and let the garbage collector reclaim the unused old ones.
 - *Hence, any invariant you prove about a state*
 - e.g., 3 is a member of set S.
- *To convince you that you don't need side effects for many things where you previously thought you did.*
 - Programming with *basic immutable data like ints, pairs, lists is easy*.
 - types do a lot of testing for you!
 - do not fear recursion!
 - You can implement *expressive, highly reusable functional* data structures like polymorphic 2-3 trees or dictionaries or stacks or queues or sets with reasonable space and time.

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But alas...

Purely functional Assignment Project Exam Help

- Sometimes we really want code to have some effect on the world.
- For example, the OCaml top-level loop prints out your result.
 - Without that printing (a side effect), how would you know that your functions computed the right thing?

Some algorithms or data structures need mutable state.

- Hash-tables have (essentially) constant-time access and update.
 - The best functional dictionaries have either:
 - logarithmic access & logarithmic update
 - constant access & linear update
 - constant update & linear access
 - Don't forget that we give up something for this:
 - we can't go back and look at previous versions of the dictionary. We *can* do that in a functional setting.
- Robinson's unification algorithm
 - A critical part of the OCaml type-inference engine.
 - Also used in other kinds of program analyses.
- Depth-first search, more ...

However, *purely mostly functional code is amazingly productive*

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OCAML MUTABLE REFERENCES

- New type: **Assignment Project Exam Help**
 - Think of it as a pointer to a *box* that holds a t value.
 - The contents of the box can be read or written.

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- New type: **Assignment Project Exam Help**
 - Think of it as a pointer to a *box* that holds a *t* value.
 - The contents of the box can be read or written.
- To create a fresh box: `ref 42`
 - allocates a new box, initializes its contents to 42, and returns a pointer:


The diagram illustrates a pointer variable named `ref 42`. It consists of two parts: a variable name and a value. An arrow points from the variable name to a small orange square representing memory. Inside the square is the number `42`, representing the value stored at that memory location.

- New type: **ref**
 - Think of it as a pointer to a *box* that holds a *t* value.
 - The contents of the box can be read or written.
- To create a fresh box: **ref 42**
 - allocates a new box, initializes its contents to 42, and returns a pointer:


The diagram shows a pointer variable `r` pointing to a box containing the value `42`. The pointer variable `r` is shown as a red box with the text `42` inside. An arrow points from the variable `r` to the box.
- To read the contents: **!r**
 - if `r` points to a box containing 42, then return 42.
 - if `r : t ref` then `!r : t`
- To write the contents: **r := 5**
 - updates the box that `r` points to so that it contains 5.
 - if `r : t ref` then `r := 5 : unit`

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```
let c = ref 0 in
```

```
let x = !c in
```

(^{* x will be 0 *}) Assignment Project Exam Help

```
(c := 42; https://powcoder.com
```

```
let y = !c in
```

(^{* y will be 42!} Add WeChat powcoder

```
x will still be 0! *)
```

```
...)
```

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Another Example

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let $c = \text{ref } 0$
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let $\text{next}()$ =

let $v = !c$ in
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 $(c := v + 1 ; v)$

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let $c = \text{ref } 0$
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let $\text{next}()$ =

let $v = !c$ in
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 $(c := v + 1 ; v)$

If $e1 : \text{unit}$
and $e2 : t$ then
 $(e1 ; e2) : t$

<https://powcoder.com> You can also write it like this:

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let c = ref 0

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let next() =

let v = !c in

let _ = c := v + 1 in

v

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Global Mutable Reference

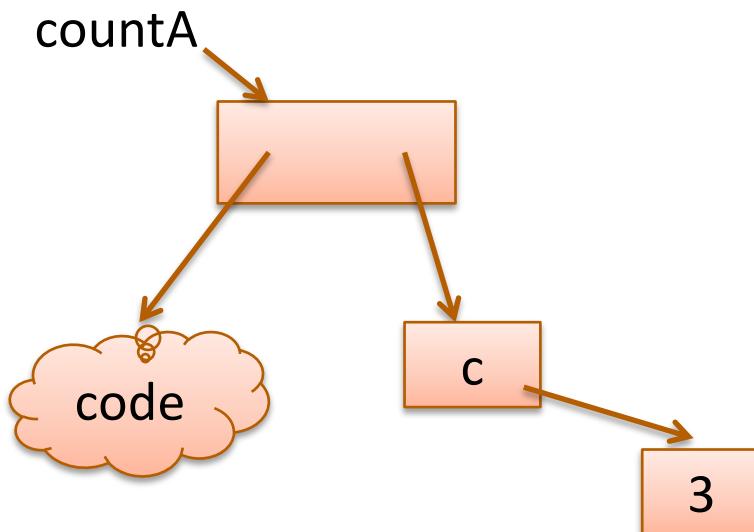
```
let c = ref 0 Add WeChat powcoder  
  
let next () : int =  
  let v = !c in Assignment Project Exam Help  
  (c := v+1 ; v)
```

Mutable Reference Captured in Closure

```
let counter () =  
  let c = ref 0 in  
  fun () ->  
    let v = !c in Assignment Project Exam Help  
    (c := v+1 ; v)
```

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```
let countA = counter() in  
let countB = counter() in  
countA(); (* 0 *)  
countA(); (* 1 *)  
countB(); (* 0 *)  
countB(); (* 1 *)  
countA(); (* 2 *)
```



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```
(* print n .. 0 *)  
let count_down (n:int) =  
  for i = n downto 0 do  
    print_int i;  
    print_newline()  
done  
  
(* print 0 .. n *)  
let count_up (n:int) =  
  for i = 0 to n do  
    print_int i;  
    print_newline()  
done
```

```
(* sum of 0 .. n *)  
  
let sum (n:int) =  
  let s = ref 0 in  
  let current = ref n in  
  while current > 0 do  
    s := !s + !current;  
    current := !current - 1  
  done;  
  !s
```

<https://powcoder.com> Imperative loops?

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```
(* print n ..0 *)  
Add WeChat powcoder  
  
let count_down (n:int) =  
  for i = n downto 0 do  
    print_int i;  
    print_newline()  
done
```

```
(* for i=n downto 0 do f i *)  
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let rec for_down  
  (n : int)  
  (f : int -> unit)  
  : unit =  
  if n >= 0 then  
    (f n; for_down (n-1) f)  
  else
```

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```
let count_down (n:int) =  
  for_down n (fun i ->  
    print_int i;  
    print_newline()  
  )
```

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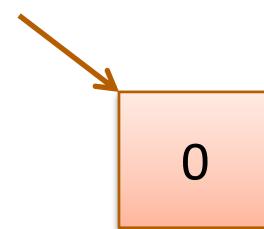
```
let c = ref 0;;
```

```
let x = Assignment Project Exam Help
```

```
x := 42;; https://powcoder.com
```

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```
!c;;
```



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```
let c = ref 0;;
```

```
let x = Assignment Project Exam Help
```

```
x := 42;; https://powcoder.com x
```

```
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```

```
!c;;
```



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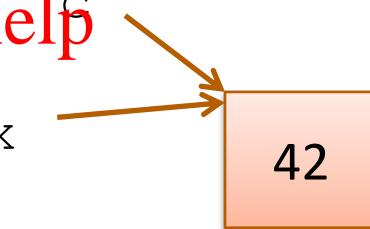
```
let c = ref 0;;
```

```
let x = Assignment Project Exam Help
```

```
x := 42;; https://powcoder.com x
```

```
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```

```
!c;;
```



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REFS AND MODULES

Concrete, first order type tells you a lot about a data structure:

- int Add WeChat powcoder
 ==> immutable
- int ref ==> mutable
- int * int ==> immutable
- int * (int ref) Add WeChat powcoder
 ==> 1st component immutable, 2nd mutable
- ... etc <https://powcoder.com>

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What about higher-order types?

- int -> int ==> the function can't be changed
 ==> what happens when we run it?

What about abstract types?

- stack, queue? stack * queue?

module Assignment Project Exam Help

```
sig      Add WeChat powcoder
  type 'a stack
  val empty : unit -> 'a stack
  val push : 'a -> 'a stack -> 'a stack
  val pop  : 'a stack -> 'a stack
  val top  : 'a stack -> 'a option
  ...
end
```

module Assignment Project Exam Help

sig Add WeChat powcoder

type 'a stack

val empty : unit -> 'a stack

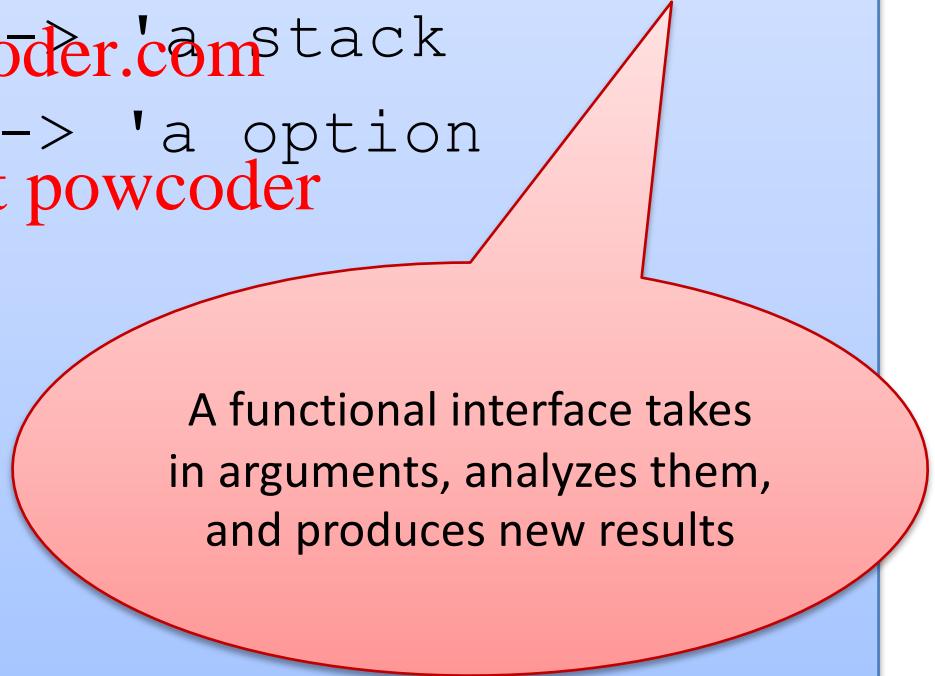
val push : 'a -> 'a stack -> 'a stack

val pop : 'a stack -> 'a stack

val top : 'a stack -> 'a option

...
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end



A functional interface takes in arguments, analyzes them, and produces new results

```
module Assignment Project Exam Help
```

```
sig      Add WeChat powcoder
```

```
type 'a stack
```

```
val empty : unit -> 'a stack
```

```
val push : 'a -> 'a stack -> unit
```

```
...
```

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```
end
```

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```
module Assignment_Project_Exam_Help
```

```
sig      Add WeChat powcoder
```

```
type 'a stack
```

```
val empty : unit -> 'a stack
```

```
val push : 'a -> 'a stack -> unit
```

```
...
```

```
end
```

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<https://powcoder.com>

When you see “unit” as the return type, you know the function is being executed for its side effects. (Like void in C/C++/Java.)

```
module Assignment Project Exam Help
```

```
sig      Add WeChat powcoder
```

```
type 'a stack
```

```
val empty : unit -> 'a stack
```

```
val push : 'a -> 'a stack -> unit
```

```
val pop : 'a stack -> 'a option
```

```
end
```

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

```
module Assignment_Project_Exam_Help
```

```
sig      Add WeChat powcoder
```

```
type 'a stack
```

```
val empty : unit -> 'a stack
```

```
val push : 'a -> 'a stack -> unit
```

```
val pop : 'a stack -> 'a option
```

```
end
```

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

```
module Assignment_Project_Exam_Help
```

```
sig      Add WeChat powcoder
```

```
type 'a stack
```

```
val empty : unit -> 'a stack
```

```
val push : 'a -> 'a stack -> 'a stack
```

```
val pop : https://powcoder.com/stack -> 'a option
```

```
end
```

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

This is a terrific way to use references in ML. Look for these opportunities

Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

```
module IMPERATIVE_STACKS

structure Stack : IMPERATIVE_STACKS = {
  type 'a stack = ('a list) ref

  let empty() : 'a stack = ref []
  let push (x:'a) (s:'a stack) : unit =
    s := x :: (!s)
  let pop (s:'a stack) : 'a option =
    match !s with
    | [] -> None
    | h :: t -> (s := t ; Some h)
}

end
```

```
module Assignment Project Exam Help
```

```
struct
```

```
type 'a stack = ('a list) ref
```

```
let empty() : 'a stack = ref []
```

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```
let push (x:'a) (s:'a stack) : 'a stack =
  s := x :: (!s)
```

Note: We don't have to
make *everything* mutable.

The list is an immutable
data structure stored in a
single mutable cell.

```
let pop (s:'a stack) : 'a option
```

```
match !s with
```

```
| [] -> None
```

```
| h :: t -> (s := t ; Some h)
```

```
end
```

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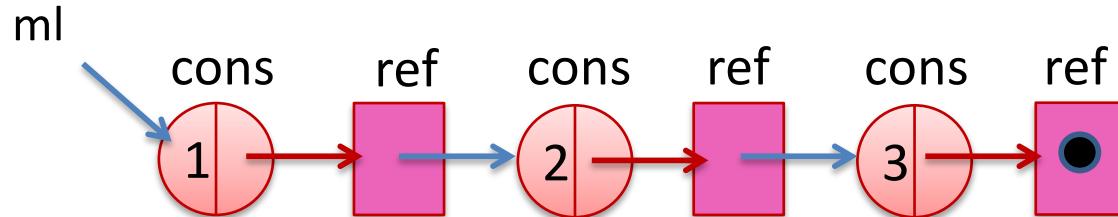
type 'a mlist =

Nil | Cons of 'a * ('a mlist ref)

let ml = Cons(1, ref(Cons(2, ref(Cons(3, ref Nil))))))

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```
type 'a mlist =
```

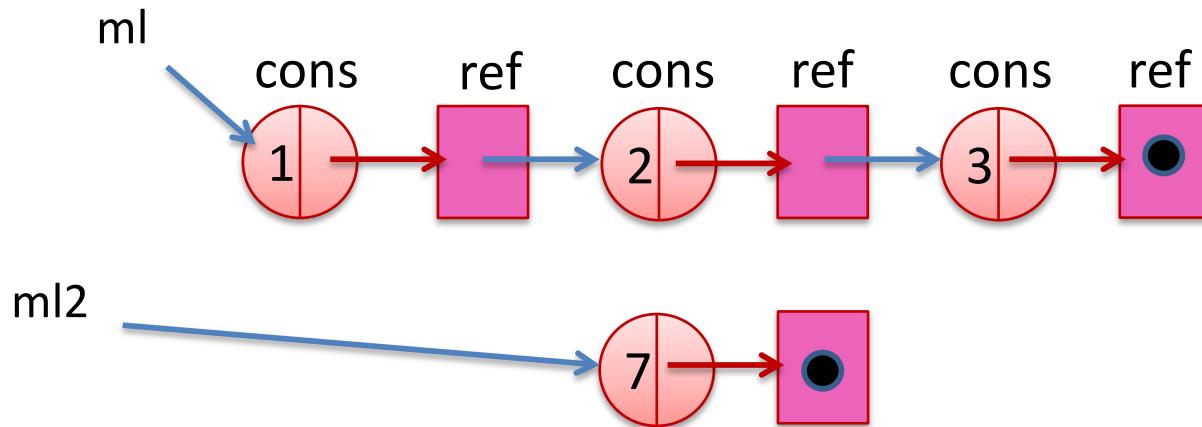
```
Nil | Cons of 'a * ('a mlist ref)
```

```
let ml = Cons(1, ref(Cons(2, ref  
(Cons(3, ref Nil))))))
```

<https://powcoder.com>

```
let ml2 = Cons(7, ref Nil)
```

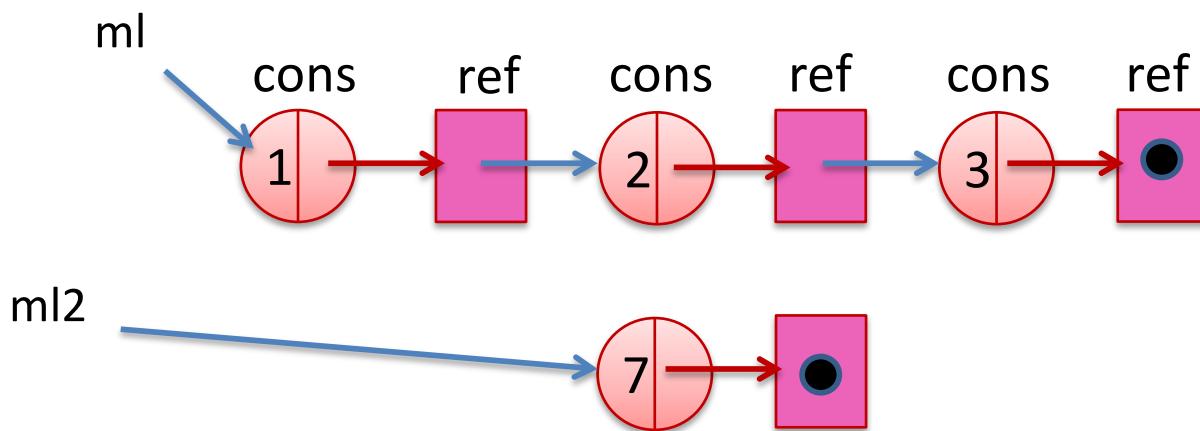
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Assignment Project Exam Help

```
type 'a mlist =  
  Nil | Cons of 'a * ('a mlist ref)  
  
let rec fudge(l:'a mlist)  
  (m:'a mlist) : unit =  
  match l with  
  | Nil -> ()  
  | Cons(h,t)-> t := m; ()
```

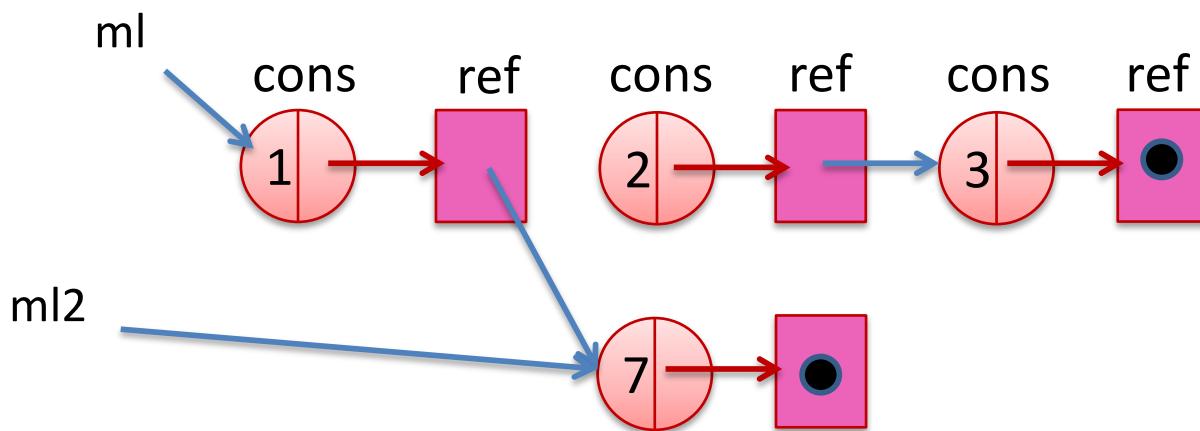
Assignment Project Exam Help
fudge ml ml2



Assignment Project Exam Help

```
type 'a mlist =  
  Nil | Cons of 'a * ('a mlist ref)  
  
let rec fudge(l:'a mlist)  
  (m:'a mlist) : unit =  
  match l with  
  | Nil -> ()  
  | Cons(h,t)-> t := m; ()
```

Assignment Project Exam Help
fudge ml ml2



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type 'a mlist =

Nil | Cons of 'a * ('a mlist ref)

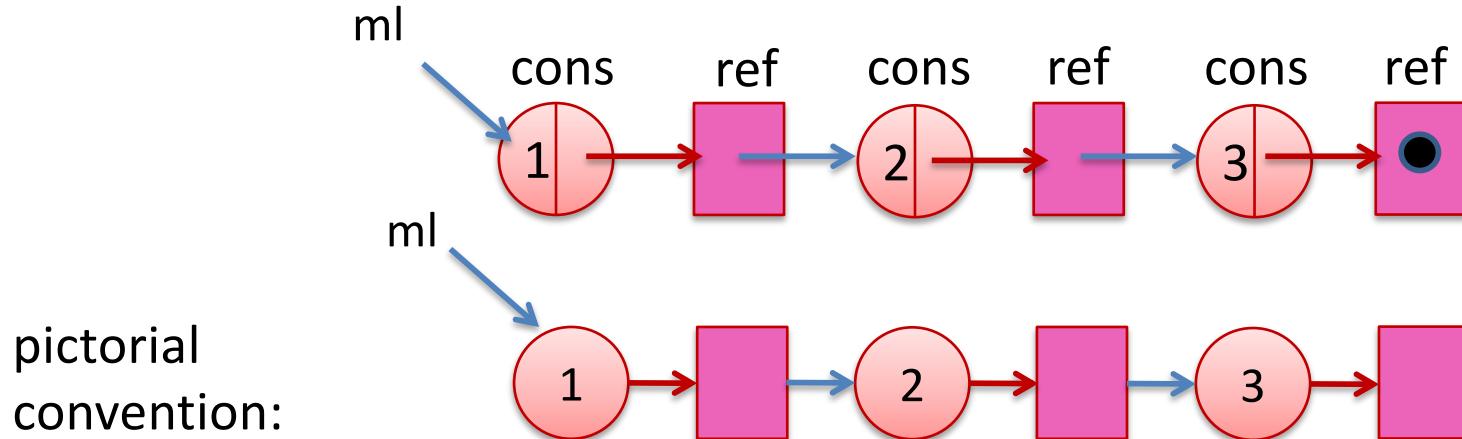
let rec mlength(m:'a mlist) : int =

match m with

| Nil -> 0

| Cons(h,t) -> 1 + mlength(!t)

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Assignment Project Exam Help

```
type 'a mlist = Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m:'a mlist):int =
```

```
match m with
```

```
| Nil -> 0
```

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```
| Cons(h,t) -> 1 + length(!t)
```

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```
let r = ref Nil in
```

```
let m = Cons(3,r) in
```

```
(r := m ;
```

```
mlength m)
```

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```
type 'a mlist = Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m:'a mlist):int =
```

```
match m with
```

```
| Nil -> 0
```

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```
| Cons(h,t) -> 1 + length(!t)
```

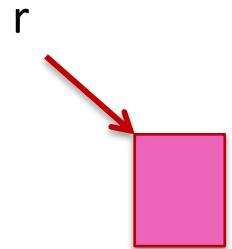
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```
let r = ref Nil in
```

```
let m = Cons(3,r) in
```

```
(r := m ;
```

```
mlength m)
```



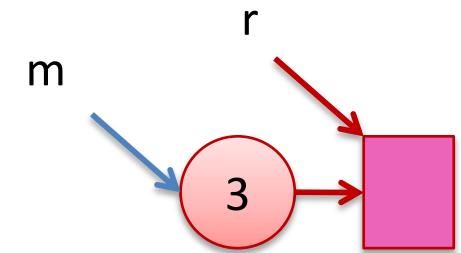
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type 'a mlist = Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m:'a mlist):int =
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match m with  
| Nil -> 0  
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```

```
let r = ref Nil in  
let m = Cons(3,r) in  
(r := m ;  
mlength m)
```



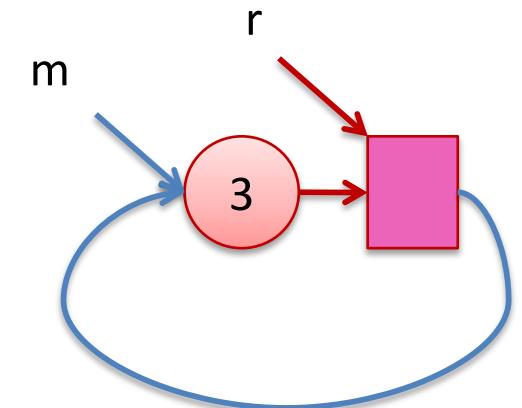
Assignment Project Exam Help

```
type 'a mlist = Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m:'a mlist):int =
```

```
match m with  
| Nil -> 0  
| Cons(h,t) -> 1 + length(!t)
```

```
let r = ref Nil in  
let m = Cons(3,r) in  
(r := m ;  
mlength m)
```



<https://powcoder.com> Another Example:

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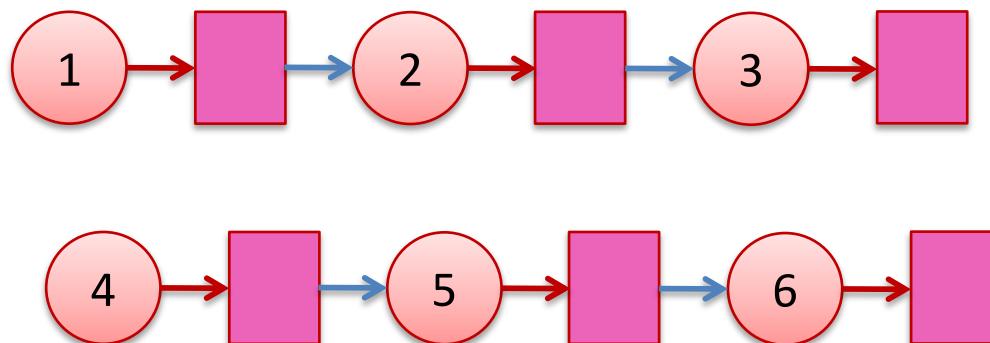
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```
type 'a mlist =
  Nil | Cons of 'a * ('a mlist ref)

let rec mappend xs ys =
  match xs with
  | Nil -> ys
  | Cons(h, t) ->
    (match !t with
     | Nil -> t := ys
     | Cons(_, _) as m -> mappend m ys)
```

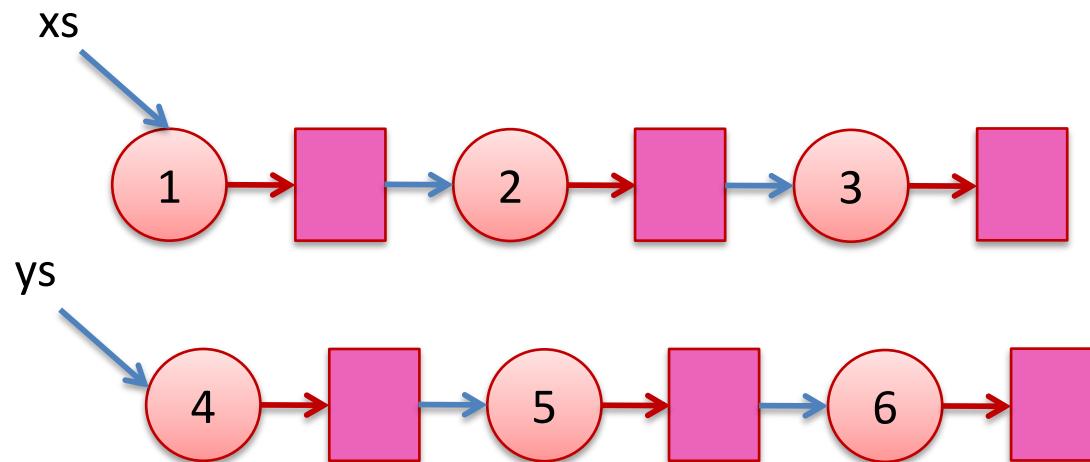
<https://powcoder.com> Example:

```
let rec mappend xs ys = Assignment Project Exam Help
  match xs with Add WeChat powcoder
  | Nil -> ()
  | Cons(h, t) ->
    (match !t with
     | Nil -> t := ys; Assignment Project Exam Help
     | Cons(_, _) as m -> mappend m ys) ;;
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;
mappend x y ;; Add WeChat powcoder
```



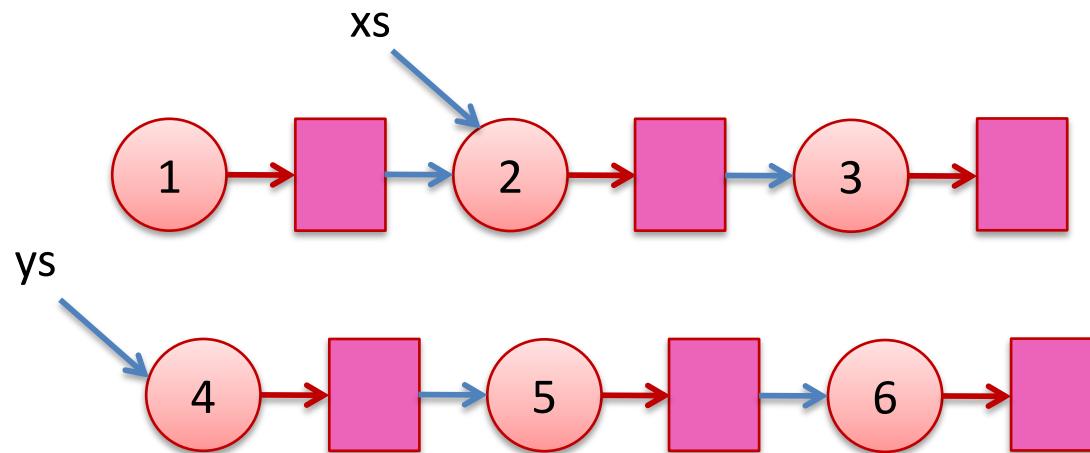
<https://powcoder.com> Example:

```
let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
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mappend x y ;;
```



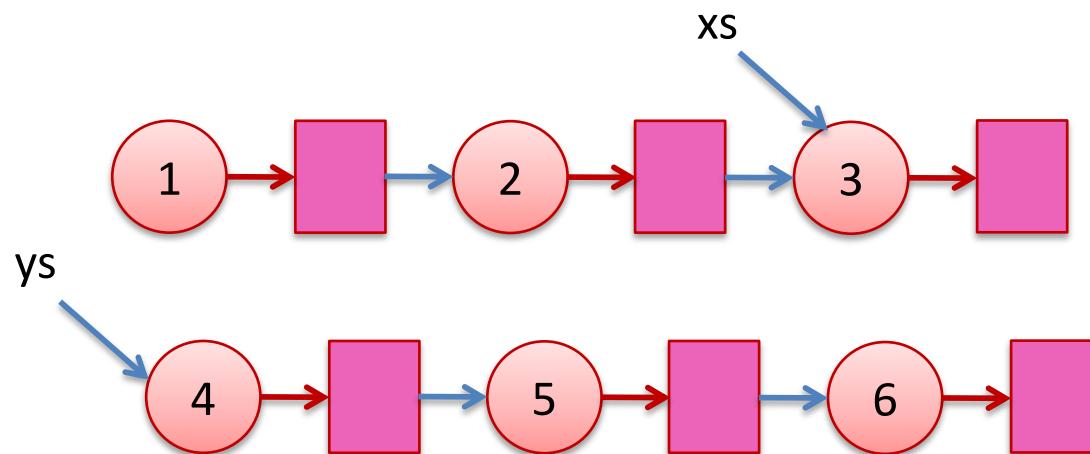
<https://powcoder.com> Example:

```
let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
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let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
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mappend x y ;;
```



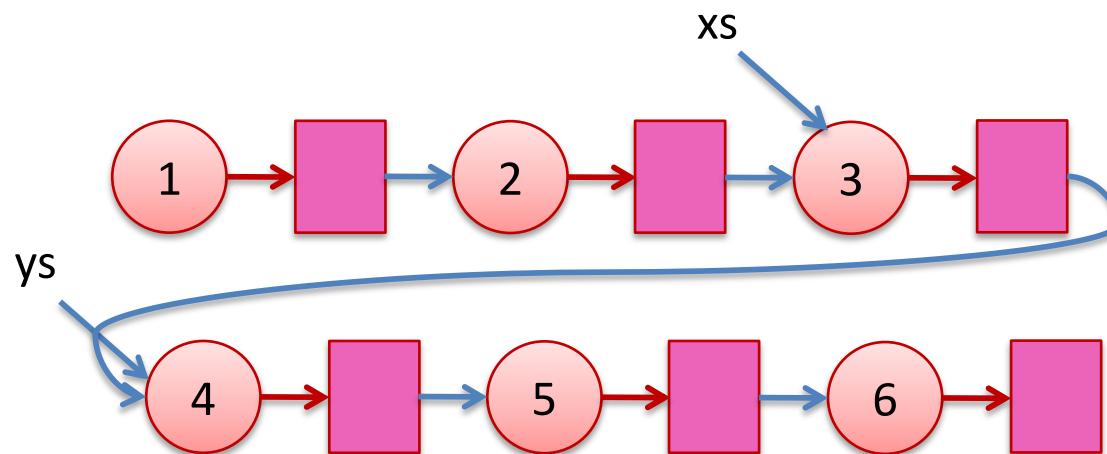
<https://powcoder.com> Example:

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let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;  
mappend x y ;;
```



<https://powcoder.com> Example:

```
let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;  
mappend x y ;;
```



<https://powcoder.com> Add mutability judiciously

Two types Assignment Project Exam Help

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```
type 'a very_mutable_list =  
  Nil  
 | Cons of 'a * ('a very_mutable_list ref)
```

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```
type 'a less_mutable_list = 'a list ref
```

The first makes cyclic lists possible, the second doesn't

- the second preemptively avoids certain kinds of errors.
- often called a *correct-by-construction design*

Is it possible to avoid all state?

Yes! (in single-threaded programs)

- Pass in old values to functions; return new values from functions ...
but this isn't necessarily the most efficient thing to do

Consider the difference between our functional stacks and our imperative ones:

- fnl_push : ' $a \rightarrow !a\ stack \rightarrow !a\ stack$ '
- imp_push : ' $a \rightarrow !a\ stack \rightarrow unit$ '

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SUMMARY

Summary: How/when to use state?

- A complicated question!
- In general, try to write the functional version first.
 - e.g., prototype
 - don't have to worry about sharing and updates
 - reasoning is easy (the substitution model is valid!)
- Sometimes you find you can't afford it for efficiency reasons.
 - example: routing tables need to be fast in a switch
 - constant time lookup, update (hash-table)
- When using state, try to *encapsulate* it behind an interface.
 - try to reduce the number of error conditions a client can see
 - correct-by-construction design
 - module implementer must think explicitly about sharing and invariants
 - write these down, write assertions to test them
 - if encapsulated in a module, these tests can be localized
 - *most of your code should still be functional*

Mutable ~~Assignment Project Exam Help~~ can lead to efficiency improvements.

- e.g., Hash tables, memoization, depth-first search
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But they are *much* harder to get right, so don't jump the gun

- *updating in one place may have an effect on other places.*
- *writing and enforcing invariants becomes more important.*
 - why more important? because the types do less ...
- *cycles in data (other than functions) can't happen until we introduce refs.* [Add WeChat powcoder](#)
 - must write operations much more carefully to avoid looping
 - more cases to deal with and the compiler doesn't help you!
- we haven't even gotten to the multi-threaded part.

So use refs when you must, but try hard to avoid it.