Polymorphic Higher-Order Programming

CSI 3120

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Some Design & Coding Rules

- *Laziness* can be a really good force in design.
- Never write the same code twice.
  - factor out the common bits into a reusable procedure.
  - better, use someone else's (well-tested, well-documented, and well-maintained) procedure.
- Why is this a good idea?
  - why don't we just cut-and-paste snippets of code using the editor instead of creating new functions?

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- Why is this a good idea?
  - why don't we just cut-and-paste snippets of code using the editor instead of creating new functions?
  - find and fix a bug in one copy, have to fix in all of them.
  - decide to change the functionality, have to track down all of the places where it gets used.

Factoring Code in OCaml

Consider these definitions:

```
let rec inc_all (xs:int list) : int list =
  match xs with
  | [] -> []
  | hd::tl -> (hd+1)::(inc_all tl)
```

```
let rec square_all (xs:int list) : int list =
  match xs with
  | [] -> []
  | hd::tl -> (hd*hd)::(square_all tl)
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  match xs with
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```

The code is almost identical – factor it out!

Factoring Code in OCaml

A *higher-order* function captures the recursion pattern:

```
let rec map (f:int->int) (xs:int list) : int list =
  match xs with
  | [] -> []
  | hd::tl -> (f hd)::(map f tl)
```

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Uses of the function:

```
let inc x = x+1
let inc_all xs = map inc xs
```

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Uses of the function:

```
let inc x = x+1
let inc_all xs = map inc xs

let square y = y*y
let square_all xs = map square xs
```

Writing little functions like inc just so we can call map is a pain.

Factoring Code in OCaml

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let rec map (f:int->int) (xs:int list) : int list =
  match xs with
  | [] -> []
  | hd::tl -> (f hd) :: (map f tl)
```

We can use an *anonymous* function instead.

Uses of the function:

```
let inc_all xs = map (fun x -> x + 1) x

let square_all xs = map (fun y -> y * y) xs
```

Originally, Church wrote this function using λ instead of **fun**: (λx. x+1) or (λy. y*y)

Here's an annoying thing

```
let rec map (f:int->int) (xs:int list) : int list =
  match xs with
  | [] -> []
  | hd::tl -> (f hd)::(map f tl);;
```

What if I want to increment a list of floats?

Alas, I can't just call this map. It works on ints!

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What if I want to increment a list of floats?

Alas, I can't just call this map. It works on ints!

```
let rec mapfloat (f:float->float) (xs:float list) :
            float list =
  match xs with
  | [] -> []
  | hd::tl -> (f hd)::(mapfloat f tl);;
```

Turns out

```
let rec map f xs =
  match xs with
  | [] -> []
  | hd::tl -> (f hd)::(map f tl)

let ints = map (fun x -> x + 1) [1; 2; 3; 4]

let floats = map (fun x -> x +. 2.0) [3.1415; 2.718]

let strings = map String.uppercase_ascii [“sarah”; “joe”]
```

Type of the undecorated map?

```
let rec map f xs =
  match xs with
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map : ('a -> 'b) -> 'a list -> 'b list
```

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We often use greek letters like α or β to represent type variables.

Read as:

- for any types 'a and 'b,
- if you give map a function from 'a to 'b,
- it will return a function
  - which when given a list of 'a values
  - returns a list of 'b values.

We can say this explicitly

```
let rec map (f:'a -> 'b) (xs:'a list) : 'b list =
  match xs with
   | [] -> []
   | hd::tl -> (f hd)::(map f tl)

map : ('a -> 'b) -> 'a list -> 'b list
```

The OCaml compiler is smart enough to figure out that this is the *most general* type that you can assign to the code.

We say map is *polymorphic* in the types 'a and 'b – just a fancy way to say map can be used on any types 'a and 'b.

Java generics derived from ML-style polymorphism (but added after the fact and more complicated due to subtyping)

Summary

- Map is a *higher-order function* that captures a very common *recursion pattern*

- We can write clear, terse, reusable code by exploiting:
  - higher-order functions
  - anonymous functions
  - first-class functions
  - polymorphism