

# Assignment Project Exam Help

# Thinking Inductively

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CSI 3120

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# Options

A value **v** has type **t option** if it is either:

- the value **None**, or
- a value **Some v'**, and **v'** has type **t**

Options can signal ~~Assignment Project Exam Help~~ there is no useful result to the computation

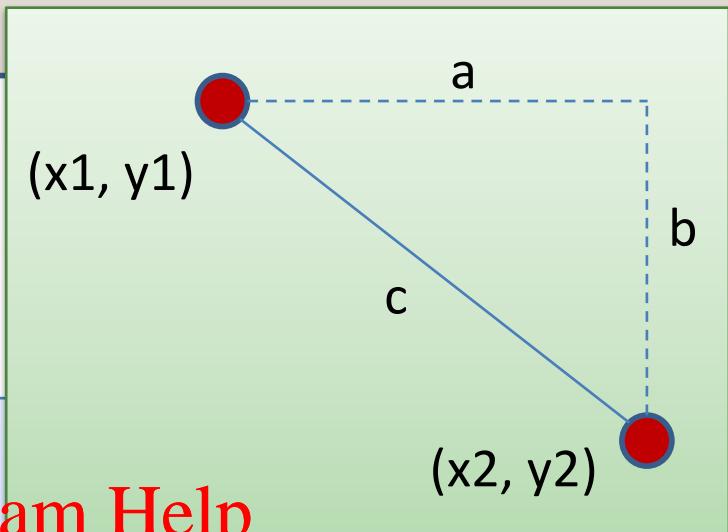
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Example: we look up a value in a hash table using a key.

- If the key is present, return **Some v** where **v** is the associated value
- If the key is not present, we return **None**

# Slope between two points

```
type point = float * float  
let slope (p1:point) (p2:point) : float =  
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```



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# Slope between two points

```
type point = float * float
```

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```
let slope (p1:point) (p2:point) : float =  
    let (x1,y1) = p1 in
```

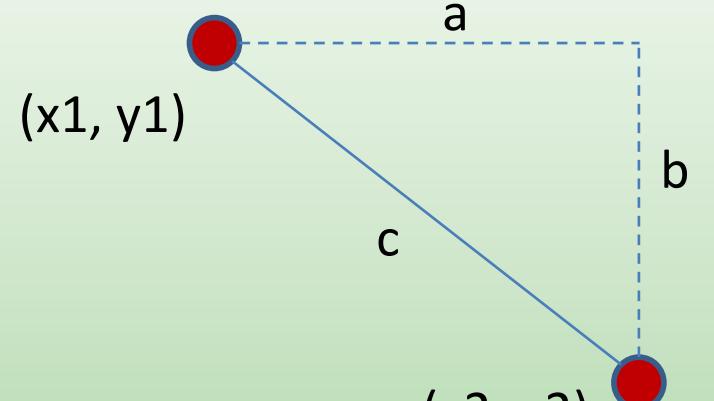
let (x2,y2) = p2 in

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deconstruct tuple

# Slope between two points



```
type point = float * float
```

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```
let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    ???
```

← avoid divide by zero

← what can we return?

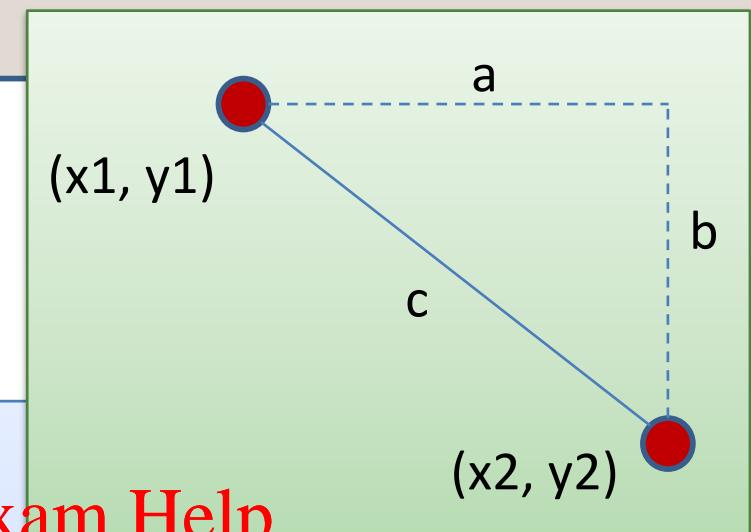
# Slope between two points

```
type point = float * float
```

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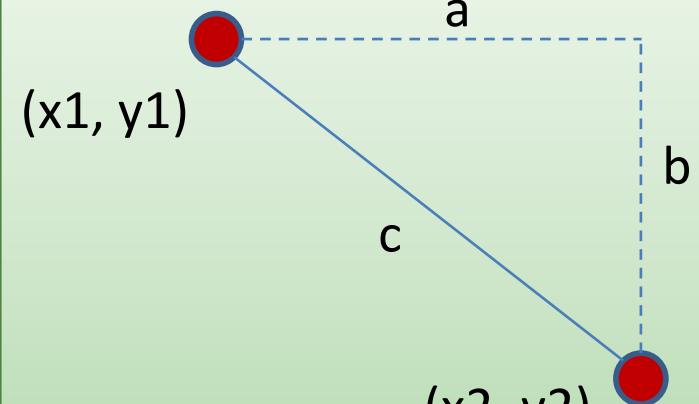
```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    ???
  else
    ???
```

we need an option type as the result type

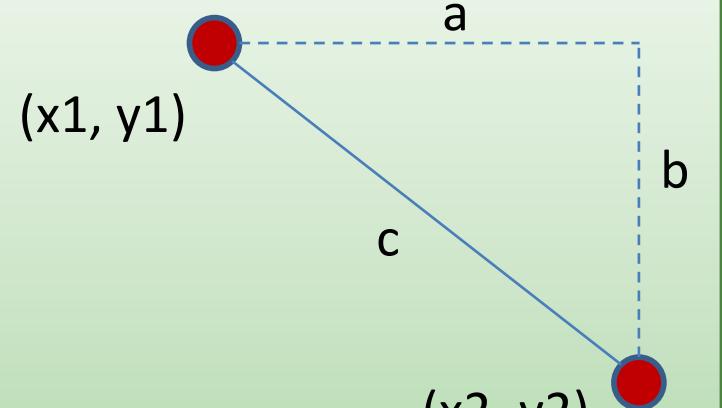


# Slope between two points

```
type point = float * float  
Assignment Project Exam Help  
let slope (p1:point) (p2:point) : float option =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in  
  let xd = x2 -. x1 in  
  if xd != 0.0 then  
    Some ((y2 -. y1) /. xd)  
  else  
    None
```



# Slope between two points



```
type point = float * float
```

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```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
```

$\frac{(y2 - y1)}{xd}$

else

None

Has type float

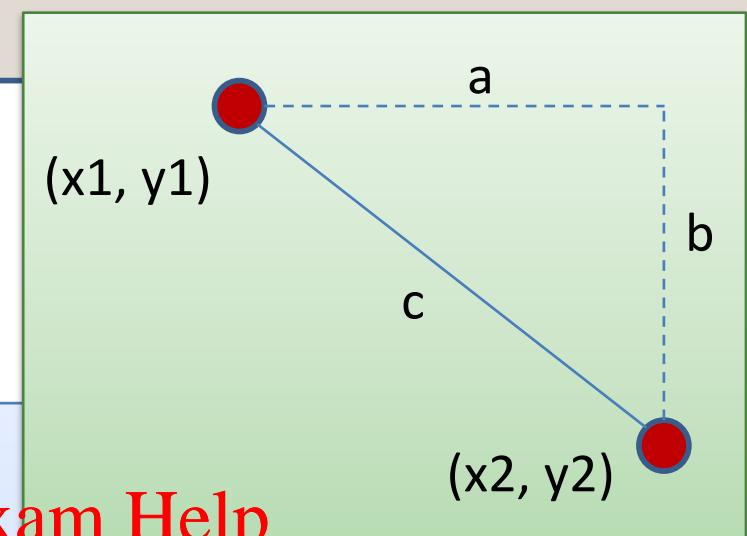
Can have type float option

# Slope between two points

```
type point = float * float
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
     $(y2 - y1) /. xd$ 
  else
    None
    
```

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Has type float  
 Can have type float option  
 WRONG: Type mismatch



# Slope between two points

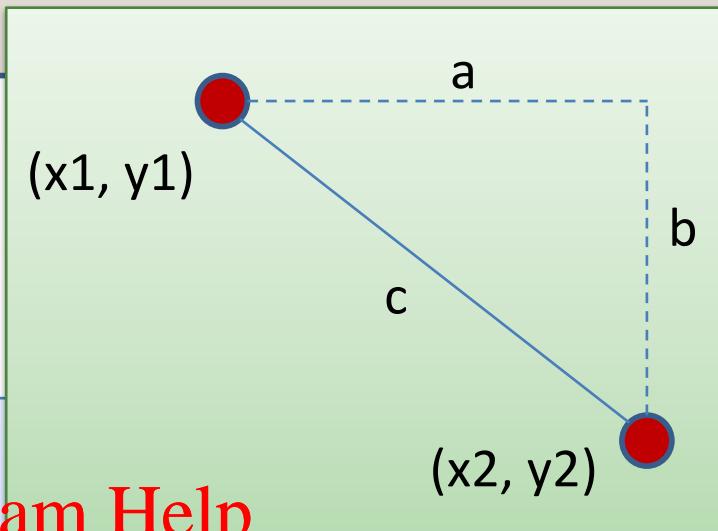
```
type point = float * float
```

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```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    None
```

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doubly WRONG:  
result does not  
match declared result

Has type **float**

# Remember the typing rule for if

```
if e1 : bool  
and e2 : t and e3 : t (for some type t)  
then if e1 then e2 else e3 : t
```

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Returning <https://powcoder.com> an optional value from an if statement:

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```
if ... then  
  
    None      : t option  
  
else  
  
    Some ( ... )  : t option
```

# How do we use an option?

slope : point -> point -> float option

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returns a float option

# How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
```

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# How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
    slope p1 p2
```

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returns a float option;  
to print we must discover if it is  
None or Some

# How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with
```

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# How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with
```

```
    Some s ->  
    | None ->
```

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There are two possibilities

Vertical bar separates possibilities

# How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
```

```
  match slope p1 p2 with
```

```
    Some s ->
```

```
    | None ->
```

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The "Some s" pattern includes the variable s

The object between | and -> is called a pattern

# How do we use an option?

```
slope : point -> point -> float option
```

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```
let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
      print_string ("slope:" ^ string_of_float s)
    | None ->
      print_string "Vertical line.\n"
```

# Writing Functions Over Typed Data

- Steps to writing functions over typed data:
  1. Write down the function and argument names
  2. Write down argument and result types
  3. Write down some examples (in a comment)
  4. Deconstruct input data structures
  5. Build new output values
  6. Clean up by identifying repeated patterns
- For option types:  
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when the **input** has type **t option**,  
deconstruct with:

```
match ... with
| None -> ...
| Some s -> ...
```

when the **output** has type **t option**,  
construct with:

Some (...)

None

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## MORE PATTERN MATCHING

# Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1, y1) = p1 in
  let (x2, y2) = p2 in
  sqrt (square (x2 -. x1) + square (y2 -. y1))
```

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# Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    let (x1,y1) = p1 in
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) + square (y2 -. y1))
;;
```

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( $x_2, y_2$ ) is an example of a pattern – a pattern for tuples.

So let declarations can contain patterns just like match statements

The difference is that a match allows you to consider multiple different data shapes

# Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1, y1) -> https://powcoder.com
    let (x2, y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



There is only 1 possibility when matching a pair

# Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1, y1) -> https://powcoder.com
    match p2 with
    | (x2, y2) ->
      sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



We can nest one match expression inside another.

(We can nest any expression inside any other, if the expressions have the right types)

# Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1, y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;

```

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Pattern for a pair of pairs: **((variable, variable), (variable, variable))**  
All the variable names in the pattern must be different.

# Complex Patterns

```
type point = float * float

let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    match (p1, p2) with
    | ((x1, y1), (x2, y2)) ->
        sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

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A pattern must be **consistent with** the type of the expression  
in between **match ... with**

For example,  $x_1 : \text{float}$ ,  $x_2 : \text{float}$ ,  $p_1 : \text{float} * \text{float}$

# Pattern-matching in function parameters

```
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) + square (y2 -. y1))
;;
```

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Function parameters are patterns too!

# What's the best style?

```
let distance (p1:point) (p2:point) : float =  
    let square x = x *. x in  
    let (x1,y1) = p1 in  
    let (x2,y2) = p2 in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

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```
let distance ((x1,y1):point) ((x2,y2):point) : float =  
    let square x = x *. x in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.

Code with unnecessary nested matches/lets is particularly ugly to read.

# What's the best style?

```
let distance (x1,y1) (x2,y2) =  
    let square x = x *. x in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

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Note that the types for tuples plus the tuple patterns are a little ugly/verbose, and they can be inferred from the operators used such as `+.' ... but for now in class, use the explicit type annotations.

# Other Patterns

```
type point = float * float

(* returns a nearby point in the graph if one exists *)
nearby : graph -> point -> point option

let printer (g:graph) (p:point) : unit =
  match nearby g p with
  | None -> print_string "could not find one\n"
  | Some (x,y) ->
    print_float x;
    print_string ", ";
    print_float y;
    print_newline();
;;
```

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# Other Patterns

- Constant values can be used as patterns

```
let small_prime (n:int) : bool =  
  match n with  
  | 2 -> true  
  | 3 -> true  
  | 5 -> true  
  | _ -> false  
;;
```

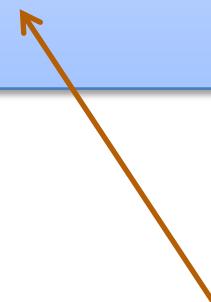
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```
let iffy (b:bool) : int =  
  match b with  
  | true -> 0  
  | false -> 1  
;;
```

the underscore pattern  
matches anything  
it is the "don't care" pattern



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**INDUCTIVE THINKING**

# Inductive Programming and Proving

An *inductive data type T* is a data type defined by:

- a collection of base cases
  - that don't refer to T
- a collection of inductive cases that build new values of type T from pre-existing data of type T
  - the pre-existing data is guaranteed to be *smaller* than the new values

Programming principle: <https://powcoder.com>

- solve programming problem for base cases
- solve programming problem for inductive cases by calling function recursively (inductively) on *smaller* data value

Proving principle:

- prove program satisfies property P for base cases
- prove inductive cases satisfy property P assuming inductive calls on *smaller* data values satisfy property P

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## LISTS: AN INDUCTIVE DATA TYPE

# Lists are Recursive Data

- In OCaml, a list value is:
  - [] (the empty list)
  - $v :: vs$  (a value  $v$  followed by a shorter list of values  $vs$ )



# Lists are Inductive Data

- In OCaml, a list value is:
  - [] (the empty list)
  - v :: vs (a value v followed by a shorter list of values vs)
- An example:[Assignment Project Exam Help](https://powcoder.com)
  - 2 :: 3 :: 5 :: [] has type int list  
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  - is the same as: 2 :: (3 :: (5 :: []))
  - "::" is called "cons"
- An alternative syntax ("syntactic sugar" for lists):
  - [2; 3; 5]
  - But this is just a shorthand for 2 :: 3 :: 5 :: []. If you ever get confused fall back on the 2 basic *constructors*: :: and []

# Typing Lists

- Typing rules for lists:

(1) [ ] may have any list type **t list**

(2) if  $e_1 : t$  and  $e_2 : t \text{ list}$   
then  $(e_1 :: e_2) : t \text{ list}$

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# Typing Lists

- Typing rules for lists:

(1) [ ] may have any list type  $t$  list

(2) if  $e_1 : t$  and  $e_2 : t$  list  
then  $(e_1 :: e_2) : t$  list

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- More examples:

$(1 + 2) :: (3 + 4) :: [] : ??$  Add WeChat powcoder

$(2 :: []) :: (5 :: 6 :: []) :: [] : ??$

$[ [2]; [5; 6] ] : ??$

# Typing Lists

- Typing rules for lists:

(1) [ ] may have any list type **t list**

(2) if  $e_1 : t$  and  $e_2 : t \text{ list}$   
then  $(e_1 :: e_2) : t \text{ list}$

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- More examples:

$(1 + 2) :: (3 + 4) :: [] : \text{int list}$

$(2 :: []) :: (5 :: 6 :: []) :: [] : (\text{int list}) \text{ list}$  or just **int list list**

$[ [2]; [5; 6] ] : \text{int list list}$

(Remember that the 3<sup>rd</sup> example is an abbreviation for the 2<sup>nd</sup>)

## Another Example

- What type does this have?

[ 2 ] :: [ 3 ]

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# Another Example

- What type does this have?

[ 2 ] :: [ 3 ]  
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int list <https://powcoder.com> int list

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```
# [2] :: [3];;  
Error: This expression has type int but an  
expression was expected of type  
int list
```

#

# Another Example

- What type does this have?

[ 2 ] :: [ 3 ]  
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int list <https://powcoder.com> int list

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- What is a simple fix that makes the expression type check?

# Another Example

- What type does this have?

[ 2 ] :: [ 3 ]  
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int list <https://powcoder.com> int list

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- What is a simple fix that makes the expression type check?

Either:      2 :: [ 3 ]      : int list

Or:      [ 2 ] :: [ [ 3 ] ]      : int list list

# Analyzing Lists

- Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

(\* return Some v, if v is the first list element;  
return None, if the list is empty \*)

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```
let head (xs : int list) : int option =
```

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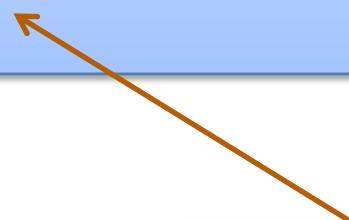
# Analyzing Lists

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(\* return Some v, if v is the first list element;  
return None, if the list is empty \*)

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```
let head (xs : int list) : int option =  
  match xs with  
  | [] ->  
  | hd :: _ ->
```



we don't care about the contents of the tail of the list so we use the underscore

# Analyzing Lists

- Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

(\* return Some v, if v is the first list element;  
return None, if the list is empty \*)

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```
let head (xs : int list) : int option =  
  match xs with  
  | [] -> None  
  | hd :: _ -> Some hd
```

- This function isn't recursive -- we only extracted a small , fixed amount of information from the list -- the first element

## A more interesting example

```
(* Given a list of pairs of integers,  
 produce the list of products of the pairs  
  
    prods [ (2,3), (4,7), (5,2) ] == [6, 28; 10]  
*)
```

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## A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs
```

```
    prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
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```

## A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs  
  
   prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] ->  
    | (x,y) :: tl ->
```

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## A more interesting example

(\* Given a list of pairs of integers,  
produce the list of products of the pairs

```
    prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] -> []  
    | (x,y) :: tl ->
```

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## A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs  
  
   prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] -> []  
    | (x,y) :: tl -> ?? :: ??
```

the result type is int list, so we can speculate  
that we should create a list

## A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs  
  
   prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] -> []  
    | (x,y) :: tl -> (x * y) :: ??
```

the first element is the product



## A more interesting example

```
(* Given a list of pairs of integers,  
   produce the list of products of the pairs  
  
   prods [(2,3), (4,7), (5,2)] == [6, 28; 10]  
*)
```

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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] -> []  
    | (x,y) :: tl -> (x * y) :: ??
```



to complete the job, we must compute  
the products for the rest of the list

## A more interesting example

(\* Given a list of pairs of integers,  
produce the list of products of the pairs

```
    prods [(2,3); (4,7); (5,2)] = [6; 28; 10]  
*)
```

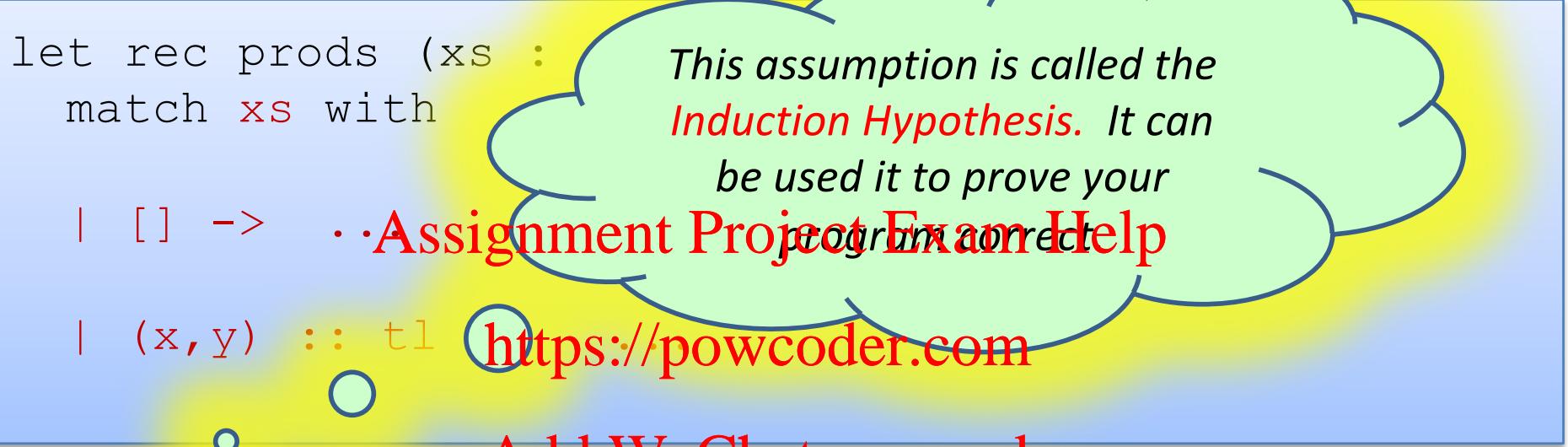
<https://powcoder.com>

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
    | [] -> []  
    | (x,y) :: tl -> (x * y) :: prods tl
```

# Three Parts to Constructing a Function

(1) Think about how to *break down* the input into cases:

```
let rec prods (xs :  
  match xs with  
  | [] -> ...Assignment Project Exam Help  
  | (x, y) :: tl -> ...  
    ...
```



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(2) *Assume* the recursive call on smaller data is correct.

(3) Use the result of the recursive call to *build* correct answer.

```
let rec prods (xs : (int*int) list) : int list =  
  ...  
  | (x, y) :: tl -> ... prods tl ...
```

## Another example: zip

(\* Given two lists of integers,  
return None if the lists are different lengths  
otherwise ~~stick the lists together to create~~  
Some of a list of pairs

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```
zip [2; 3] [4; 5] == Some [(2,4); (3,5)]  
zip [5; 3] [A] == None  
zip [4; 5; 6] [8; 9; 10; 11; 12] == None  
*)
```

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)  
  : (int * int) list option =
```

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## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

match (xs, ys) with

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## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

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```
match (xs, ys) with
| ([], []) -> https://powcoder.com
| ([], y :: ys') ->
| (x :: xs', []) -> Add WeChat powcoder
| (x :: xs', y :: ys') ->
```

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

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```
match (xs, ys) with
| ([], []) -> Some []
| ([], y :: ys') ->
| (x :: xs', []) -> Add WeChat powcoder
| (x :: xs', y :: ys') ->
```

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

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```
match (xs, ys) with
| ([], []) -> Some []
| ([], y :: ys') -> None
| (x :: xs', []) -> None
| (x :: xs', y :: ys') ->
```

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None

None

None

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') -> (x, y) :: zip xs' ys'
```

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is this ok?

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') -> (x, y) :: zip xs' ys'
```

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No! zip returns a list option, not a list!

We need to match it and decide if it is Some or None.



## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     None -> None
     | Some zs -> (x, y) :: zs)
```



Is this ok?

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     None -> None
     | Some zs -> Some ((x, y) :: zs))
```

## Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     None -> None
     | Some zs -> Some ((x, y) :: zs))
  | (_, _) -> None
```

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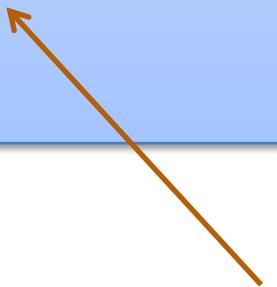
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None -> None

| Some zs -> Some ((x, y) :: zs))

| (\_, \_) -> None



Clean up.

Reorganize the cases.

Pattern matching proceeds in order.

# A bad list example

```
let rec sum (xs : int list) : int =  
  match xs with
```

```
  | hd :: tl -> hd + sum tl
```

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# A bad list example

```
let rec sum (xs : int list) : int =  
  match xs with
```

```
  | hd :: tl -> hd + sum tl
```

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```
# Characters 39-78:
```

```
..match xs with  
  hd :: tl -> hd + sum tl..
```

Warning 8: this pattern-matching is not exhaustive.

Here is an example of a value that is not matched: []

```
val sum : int list -> int = <fun>
```

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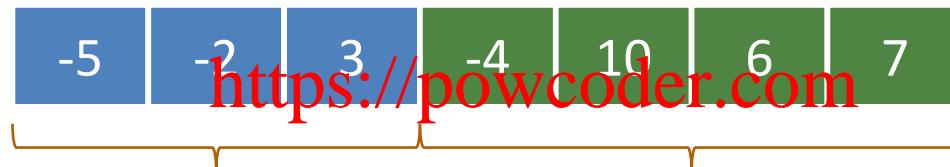
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**INSERTION SORT**

# Recall Insertion Sort

- At any point during the insertion sort:
  - some initial segment of the array will be sorted
  - the rest of the array will be in the same (unsorted) order as it was originally

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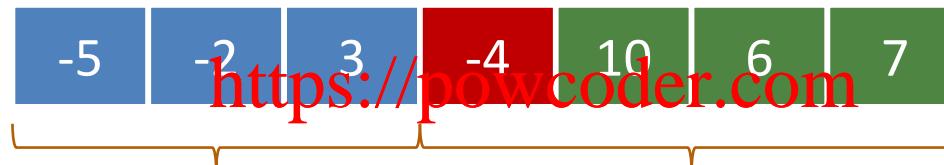


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# Recall Insertion Sort

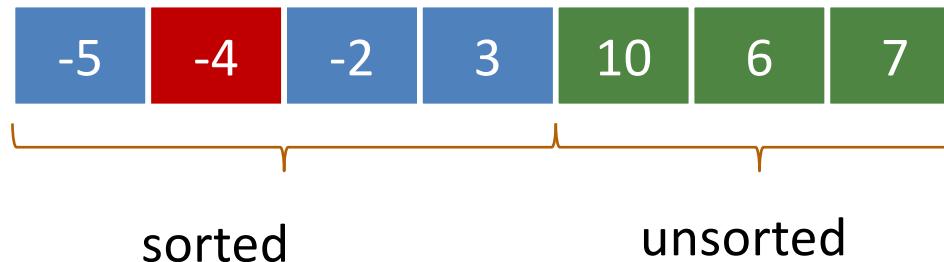
- At any point during the insertion sort:
  - some initial segment of the array will be sorted
  - the rest of the array will be in the same (unsorted) order as it was originally

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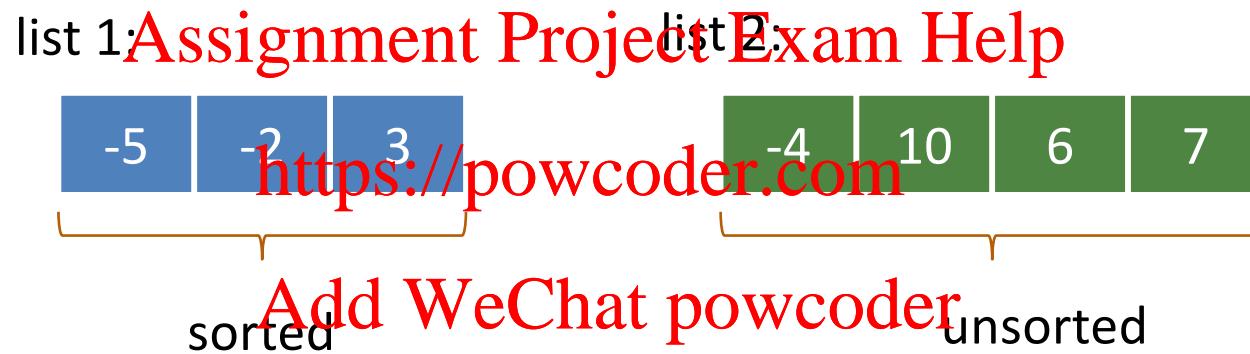
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- At each step, take the next item in the array and insert it in order into the sorted portion of the list



# Insertion Sort With Lists

- The algorithm is similar, except instead of *one array*, we will maintain *two lists*, a sorted list and an unsorted list



- We'll factor the algorithm:
  - a function to insert into a sorted list
  - a sorting function that repeatedly inserts

# Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =
```

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# Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
    | [] ->  
    | hd :: tl -> https://powcoder.com
```

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a familiar pattern:  
analyze the list by cases

# Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
    | [] -> [x] ← https://powcoder.com insert x into the empty list  
    | hd :: tl -> https://powcoder.com
```

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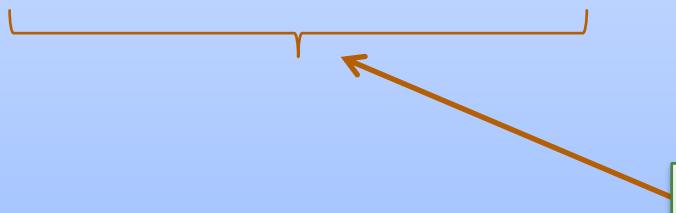
# Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
    | [] -> [x]  
    | hd :: tl -> if hd < x then  
        hd :: insert x tl  
      else tl
```

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- 
- build a new list with:
- hd at the beginning
  - the result of inserting x in to the tail of the list afterwards

# Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
    | [] -> [x]  
    | hd :: tl -> if hd < x then  
        hd :: insert x tl  
      else  
        x :: xs
```



put x on the front of the list,  
the rest of the list follows

# Insertion Sort

```
type il = int list  
  
insert : int -> il -> il
```

(\* insertion sort \*)

let rec insert\_sort =

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# Insertion Sort

```
type il = int list  
  
insert : int -> il -> il
```

(\* insertion sort \*)

```
let rec insert_sort (xs : il) : il
```

```
let rec aux (sorted : il) (x : int) : il =
```

in

# Insertion Sort

```
type il = int list  
  
insert : int -> il -> il
```

(\* insertion sort \*)

```
let rec insert_sort (xs : il) : il
```

```
let rec aux (sorted : il)(unsorted : il) : il =
```

in

aux [] xs

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# Insertion Sort

```
type il = int list  
  
insert : int -> il -> il
```

(\* insertion sort \*)

```
let rec insert_sort (xs : il) : il
```

```
let rec aux (sorted : il)(unsorted : il) : il =  
  match unsorted with  
  | [] ->  
  | hd :: tl ->  
    in  
    aux [] xs
```

# Insertion Sort

```
type il = int list  
  
insert : int -> il -> il
```

(\* insertion sort \*)

let rec insert\_sort (xs : il) : il

```
let rec aux (sorted : il)(unsorted : il) : il =  
  match unsorted with  
  | [] -> sorted  
  | hd :: tl -> aux (insert hd sorted) tl  
in  
aux [] xs
```

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## A SHORT JAVA RANT

# Definition and Use of Java Pairs

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a,int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
        return p2;  
    }  
}
```

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What could go wrong?

# A Paucity of Types

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a,int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
        return p2;  
    }  
}
```

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The input **p1** to swap may be **null** and we forgot to check.

Java has no way to define a pair data structure that is *just a pair*.

*How many students in the class have seen an accidental null pointer exception thrown in their Java code?*

# From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

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# From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

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And if you write code like this:

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```
let swap_java_pair (p:java_pair) : java_pair =  
  let (x,y) = p in  
    (y,x)
```

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# From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

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And if you write code like this:

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```
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
```

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You get a *helpful* error message like this:

```
# ... Characters 91-92:
```

```
let (x,y) = p in (y,x);;
```

```
Error: This expression has type java_pair = (int * int) option
      but an expression was expected of type 'a * 'b
```

# From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up late trying to finish your assignment and you accidentally wrote the following statement?

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```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
    | Some (x,y) -> Some (y,x)
```

# From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
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And what if you were up late trying to finish your assignment and you accidentally wrote the following statement?

<https://powcoder.com>

```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
    | Some (x,y) -> Some (y,x)
```

*OCaml to the rescue!*

```
..match p with
  | Some (x,y) -> Some (y,x)
```

Warning 8: this pattern-matching is not exhaustive.

Here is an example of a value that is not matched:

None

# From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up late trying to finish your assignment and you accidentally wrote the following statement?

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```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
    | Some (x,y) -> Some (y,x)
```



```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
    | None -> None
    | Some (x,y) -> Some (y,x)
```

## From Java Pairs to OCaml Pairs

*Moreover, your pairs are probably almost never null!*

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Defensive programming & always checking for null can be  
annoying.

# From Java Pairs to OCaml Pairs

There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer

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In OCaml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk"

```
type pair = int * int
```

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Once you know OCaml, it is *hard* to write swap incorrectly

Your *bullet-proof* code is much simpler than in Java.

```
let swap (p:pair) : pair =
  let (x,y) = p in (y,x)
```

# Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe just the triples
- There is no type to describe the pairs of pairs
- There is no type ...

OCaml has many more types <https://powcoder.com>

- use option when things may be null
- do not use option when things are not null
- OCaml types describe data structures more precisely
  - programmers have fewer cases to worry about
  - entire classes of errors just go away
  - type checking and pattern analysis help prevent programmers from ever forgetting about a case

# Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe the tuple
- There is no type to describe the list
- There is no type to describe the map

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OCaml

- use
- do

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**SCORE: OCAML 1, JAVA 0**  
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• type checker can analyze code without ever looking about a case help prevent programmers from

# C, C++ Rant

Java has a paucity of types

- but at least when you forget something,  
it *throws an exception* instead of *silently going off the rails!*
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If you forget to check <https://powcoder.com> program,

- no type-check error at compile time
- no exception at run time
- it might crash right away (that would be best), or
- it might permit a buffer-overrun (or similar) vulnerability

# Summary of C, C++ rant

Java has a paucity of types

- but at least when you forget something it **throws an exception** instead of crashing off a trolley!

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If you

- no type safety
- it's not checked
- it's not checked
- so the hardware

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SCORE:

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OCAML 1, JAVA 0, C -1

1, or similar, vulnerability