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## Assignment Project Exam Help

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For a short humorous talk on  
languages without strong typing:

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<https://powcoder.com>

<https://www.destroyallsoftware.com/talks/wat>

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[Broader point: No one (few people) knows what  
their programs do in untyped languages.]

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Assignment Project Exam Help

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Assignment Project Exam Help  
Type Checking Basics

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Amy Felty

University of Ottawa

## Assignment Project Exam Help

Last Time

Functional programming history

- Church & the lambda calculus
- Scheme
- ML (OCaml)
- Modern times: F#, Clojure, Scala, Map Reduce, ...

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OCaml

- Functional language, gets most work done by analyzing old data and producing *new, immutable* data
- Simple, typed programming language based on the lambda calculus
- Immutable data is the default; mutable data is possible (imperative, object-oriented)

## Assignment Project Exam Help Type Checking

- Every value has a type and so does every expression
- This is a concept that is familiar from Java but it becomes more important when programming in a functional language
- The type of an expression is determined by the type of its subexpressions
- We write  $(e : t)$  to say that expression  $e$  has type  $t$ . eg:

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$2 : \text{int}$

$\text{"hello"} : \text{string}$

$2 + 2 : \text{int}$

$\text{"I say " ^ "hello"} : \text{string}$

## Assignment Project Exam Help Type Checking Rules

- There are a set of simple rules that govern type checking
  - programs that do not follow the rules will not type check and OCaml will refuse to compile them for you (the nerve!)
  - at first you may find this to be a pain ...

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- But types are a great thing:
  - they *help us think* about *how to construct* our programs
  - they help us *find stupid programming errors*
  - they help us track down compatibility errors quickly when we edit and *maintain our code*
  - they allow us to *enforce powerful invariants* about our data structures

## Assignment Project Exam Help Type Checking Rules

- Example rules:
  - (1) `0 : int` (and similarly for any other integer constant `n`)
  - (2) `"abc" : string` (and similarly for any other string constant `"..."`)

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## Assignment Project Exam Help Type Checking Rules

- Example rules. [Add WeChat powcoder](#)

(1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )

(2)  $"abc" : \text{string}$  (and similarly for any other string constant "...")

## Assignment Project Exam Help

(3) if  $e1 : \text{int}$  and  $e2 : \text{int}$  then  $e1 + e2 : \text{int}$  (4) if  $e1 : \text{int}$  and  $e2 : \text{int}$  then  $e1 * e2 : \text{int}$

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## Assignment Project Exam Help Type Checking Rules

- Example rules.

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- Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)

## Assignment Project Exam Help Type Checking Rules

- Example rules.

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$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)

Therefore,  $(2 + 3) : \text{int}$  (By rule 3)

## Assignment Project Exam Help Type Checking Rules

- Example rules.

(1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )

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### Assignment Project Exam Help

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- Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)

Therefore,  $(2 + 3) : \text{int}$  (By rule 3)

$5 : \text{int}$  (By rule 1)

## Assignment Project Exam Help Type Checking Rules

- Example rules. [Add WeChat powcoder](#)

(1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )

(2)  $"\text{abc}" : \text{string}$  (and similarly for any other string constant  $s$ )

(3) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 + e2 : \text{int}$  [https://powcoder.com](#) **Fri: This is a *formal proof* that the expression is well-typed!**

(5) if  $e1 : \text{string}$  and  $e2 : \text{string}$   
then  $e1 \wedge e2 : \text{string}$  [Add WeChat powcoder](#)

- Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)

Therefore,  $(2 + 3) : \text{int}$  (By rule 3)

$5 : \text{int}$  (By rule 1)

Therefore,  $(2 + 3) * 5 : \text{int}$  (By rule 4 and our previous work)

## Assignment Project Exam Help Type Checking Rules

- Example rules.

(1) `0 : int` (and similarly for any other integer constant `n`)

(2) `"abc" : string` (and similarly for any other string constant `"..."`)

## Assignment Project Exam Help

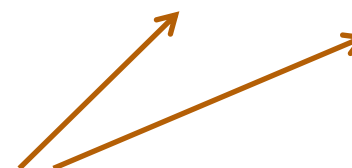
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(5) if `e1 : string` and `e2 : string` then `e1 ^ e2 : string` (6) if `e : int` then `string_of_int e : string`

- Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type `int` in place of the `????`

`???? * ???? : int`



## Assignment Project Exam Help Type Checking Rules

- Example rules. Add WeChat powcoder

(1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )

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### Assignment Project Exam Help

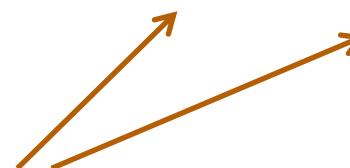
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- Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type  $\text{int}$  in place of the  $????$

$7 \quad * \quad ???? \quad : \text{int}$



## Assignment Project Exam Help Type Checking Rules

- Example rules. Add WeChat powcoder

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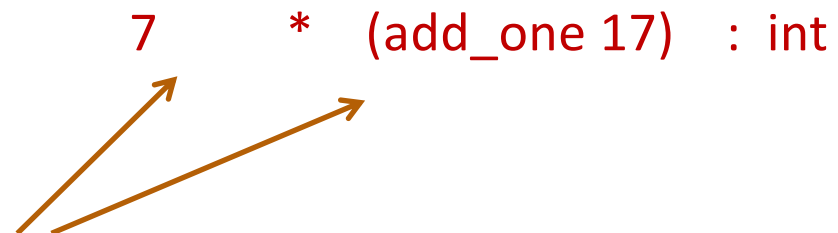
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- Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type  $\text{int}$  in place of the ????

$7 * (\text{add\_one } 17) : \text{int}$



## Assignment Project Exam Help Type Checking Rules

- You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
```

```
OCaml Version 4.07.0
```

```
#
```

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## Assignment Project Exam Help Type Checking Rules

- You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
OCaml Version 4.07.0
# 3 + 1;
```

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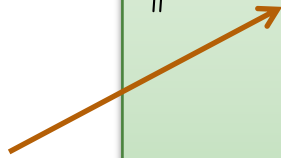
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- You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
OCaml Version 4.07.0
# 3 + 1;
- : int = 4
#
```

press  
return  
and you  
find out  
the type  
and the  
value

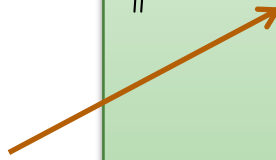


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- You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
OCaml Version 4.07.0
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
#
```

press  
return  
and you  
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value



## Assignment Project Exam Help Type Checking Rules

- You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
OCaml Version 4.07.0
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
# #quit;;
$
```

## Assignment Project Exam Help Type Checking Rules

- Example rules.

(1) `0 : int` (and similarly for any other integer constant `n`)

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- Violating the rules:

`"hello" : string`

(By rule 2)

`1 : int`

(By rule 1)

`1 + "hello" : ??`

(NO TYPE! Rule 3 does not apply!)

## Assignment Project Exam Help Type Checking Rules

- Violating the rules:

```
# "hello" + 1;;
```

Error: This expression has type string but an expression was expected of type int

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- The type error message tells you the type that was **expected** and the type that it **inferred** for your subexpression
- By the way, this was one of the nonsensical expressions that did not evaluate to a value
- It is a **good thing** that this expression does not type check!

*“Well typed programs do not go wrong”*

*Robin Milner, 1978*

## Assignment Project Exam Help Type Checking Rules

- Violating the rules:

```
# "hello" + 1;;
```

Error: This expression has type string but an expression was expected of type int

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- A possible fix:

```
# "hello" ^ (string_of_int 1);;  
- : string = "hello1"
```

- One of the keys to becoming a good ML programmer is to understand type error messages.*

## Assignment Project Exam Help Example Type-checking Rules

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if  $e1 : \text{bool}$   
and  $e2 : t$  and  $e3 : t$  (the same type  $t$ , for some type  $t$ )  
then if  $e1$  then  $e2$  else  $e3 : t$  (that same type  $t$ )

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## Assignment Project Exam Help Type Checking Rules

- Type errors for if statements can be confusing sometimes.  
Example: We create a string from s, concatenating it n times:

```
let rec concatn s n =  
  if n <= 0 then  
    ""  
  else  
    s ^ (concatn s (n-1))
```

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```

OCaml says:

**Error: This expression has type int but an  
expression was expected of type string**

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???

OCaml says:

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**Error: This expression has type int but an  
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```

they don't  
agree!



???

OCaml says:

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**Error: This expression has type int but an  
expression was expected of type string**

## Assignment Project Exam Help Type Checking Rules

- Type errors for if statements can be confusing sometimes.  
Example. We create a string from s, concatenating it n times:

they don't  
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```
let rec concatn s n =  
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```

???

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The type checker points to the correct branch as the cause of an error because it does not AGREE with the type of an earlier branch.  
Really, the error is in the earlier branch.

Moral: *Sometimes you need to look in an earlier branch for the error*  
even though the type checker points to a later branch.  
The type checker doesn't know what the user wants.

## Assignment Project Exam Help A Tactic: Add Typing Annotations

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```
let rec concatn (s:string) (n:int) : string =  
  if n <= 0 then  
    0  
  else  
    s ^ (concatn s (n-1))
```

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**Error: This expression has type int but an  
expression was expected of type string**

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**EXCEPTIONS:**

**DO THEY CAUSE PROGRAMS TO**

**"GO WRONG"?**

## Assignment Project Exam Help Type Checking Rules

- What about this expression:  
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```
# 3 / 0 ;;  
Exception: Division_by_zero.
```

- Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?  
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## Assignment Project Exam Help Type Checking Rules

- What about this expression:

```
# 3 / 0 ;;  
Exception: Division_by_zero.
```

- Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?
  - In general, detecting a divide-by-zero error requires we know that the divisor evaluates to 0.
  - In general, deciding whether the divisor evaluates to 0 requires solving the halting problem:

```
# 3 / (if turing_machine_halts m then 0 else 1) ;;
```

- There are type systems that will rule out divide-by-zero errors, but they require programmers to supply proofs to the type checker

## Assignment Project Exam Help Isn't that cheating?

*"Well typed programs do not go wrong"*

*Robin Milner, 1978*

(3 / 0) is well typed. Does it "go wrong?" Answer: No.

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*"Go wrong"* is a technical term meaning "have no defined semantics." Raising an exception is perfectly well defined semantics, which we can reason about, which we can handle in ML with an exception handler.

So, it's not cheating.

## Assignment Project Exam Help Type Soundness

*“Well typed programs do not go wrong”*

Programming languages with this property have *sound* type systems. They are called *safe* languages.

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Safe languages are generally immune to buffer overrun vulnerabilities, uninitialized pointer vulnerabilities, etc. (but not immune to all bugs!)

Safe languages: ML, Java, Python, ...

Unsafe languages: C, C++, Pascal

# Well typed programs do not go wrong

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Turing Award, 1991



Robin Milner

“For three distinct and complete achievements:

1. **LCF**, the mechanization of Scott's Logic of Computable Functions, probably the first theoretically based yet practical tool for machine assisted proof construction;
2. **ML**, the first language to include polymorphic type inference together with a type-safe exception-handling mechanism;
3. **CCS**, a general theory of concurrency.

In addition, he formulated and strongly advanced full abstraction, the study of the relationship between operational and denotational semantics.”

*“Well typed programs do not go wrong”*

*Robin Milner, 1978*

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**SUMMARY**

## Assignment Project Exam Help

OCaml is a *functional* programming language

- Java gets most work done by *modifying* data
- OCaml gets most work done by producing *new, immutable* data

## Assignment Project Exam Help

OCaml is a *typed* programming language

- the *type* of an expression *correctly predicts* the kind of *value* the expression will generate when it is executed
- there are systematic rules defining when any expression (or program) type checks
  - these rules actually form a formal logic ... it is not a coincidence that languages like ML are used inside theorem provers
- the type system is *sound*; the language is *safe*