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# Mutation

CSI 3120

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University of Ottawa

# Reasoning about Mutable State is Hard

mutable set

```
insert i s1;  
f x;  
member i s1
```

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immutable set

```
let s1 = insert i s0 in  
f x;  
member i s1
```

Is member i s1 ~~Assignment Project Exam Help~~

- When s1 is mutable, one must look at f to determine if it modifies s1.
- Worse, one must often solve the ~~aliasing problem~~.
- Worse, in a concurrent setting, one must look at *every other function* that *any other thread may be executing* to see if it modifies s1.

## Assignment Project Exam Help Thus far...

We have considered ~~Add WeChat powcoder~~ the (almost) purely functional subset of OCaml.

- We've had a few side effects: printing & raising exceptions.

Two reasons for this emphasis:

- *Reasoning about functional code is easier.*
  - Both formal reasoning
    - equationally, using the substitution model
    - and informal reasoning
  - Data structures are *persistent*.
    - They don't change – we build new ones and let the garbage collector reclaim the unused old ones.
  - *Hence, any invariant you prove about a state*
    - e.g., 3 is a member of set S.
- *To convince you that you don't need side effects for many things where you previously thought you did.*
  - Programming with *basic immutable data like ints, pairs, lists is easy*.
    - types do a lot of testing for you!
    - do not fear recursion!
  - You can implement *expressive, highly reusable functional* data structures like polymorphic 2-3 trees or dictionaries or stacks or queues or sets with reasonable space and time.

## Assignment Project Exam Help But alas...

*Purely functional Add WeChat powcoder*

- Sometimes we really want code to have some effect on the world.
- For example, the OCaml top-level loop prints out your result.
  - Without that printing (a side effect), how would you know that your functions computed the right thing?

*Some algorithms or data structures need mutable state.*

- Hash-tables have (essentially) constant-time access and update.
  - The best functional dictionaries have either:
    - logarithmic access & logarithmic update
    - constant access & linear update
    - constant update & linear access
  - Don't forget that we give up something for this:
    - we can't go back and look at previous versions of the dictionary. We *can* do that in a functional setting.
- Robinson's unification algorithm
  - A critical part of the OCaml type-inference engine.
  - Also used in other kinds of program analyses.
- Depth-first search, more ...

*However, purely mostly functional code is amazingly productive*

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## **OCAML MUTABLE REFERENCES**

## Assignment Project Exam Help References

- New type: `t Add WeChat powcoder`
  - Think of it as a pointer to a *box* that holds a `t` value.
  - The contents of the box can be read or written.

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## Assignment Project Exam Help References

- New type: `t ref` **Add WeChat powcoder**
  - Think of it as a pointer to a *box* that holds a `t` value.
  - The contents of the box can be read or written.
- To create a fresh box: `ref 42`
  - allocates a new box, initializes its contents to 42, and returns a pointer:

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- `ref 42 : int ref`

## Assignment Project Exam Help References

- New type:  $t \text{ ref}$  **Add WeChat powcoder**
  - Think of it as a pointer to a *box* that holds a  $t$  value.
  - The contents of the box can be read or written.
- To create a fresh box:  $\text{ref } 42$ 
  - allocates a new box, initializes its contents to 42, and returns a pointer:

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- $\text{ref } 42 : \text{int ref}$
- To read the contents:  $!r$ 
  - if  $r$  points to a box containing 42, then return 42.
  - if  $r : t \text{ ref}$  then  $!r : t$
- To write the contents:  $r := 5$ 
  - updates the box that  $r$  points to so that it contains 5.
  - if  $r : t \text{ ref}$  then  $r := 5 : \text{unit}$

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```
let c = ref 0 in
```

```
let x = !c in
```

(<sup>\* x will be 0 \*</sup>)

```
c := 42;
```

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```
let y = !c in
```

(<sup>\* y will be 42 \*</sup>)

x will still be 0! \*)

## Assignment Project Exam Help Another Example

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let  $c = \text{ref } 0$   
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let  $\text{next}()$  =

let  $v = !c$  in  
Add WeChat powcoder  
 $(c := v + 1 ; v)$

## Assignment Project Exam Help Another Example

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let https://powcoder.com

let v = !c in  
(c := v+1 ; v)

If e1 : unit  
and e2 : t then  
(e1 ; e2) : t

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You can also write it like this:

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let c = ref 0

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let next() =

let v = !c in

let \_ = c := v + 1 in

v

## Assignment Project Exam Help Another idiom

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Global Mutable Reference

```
let c = ref 0

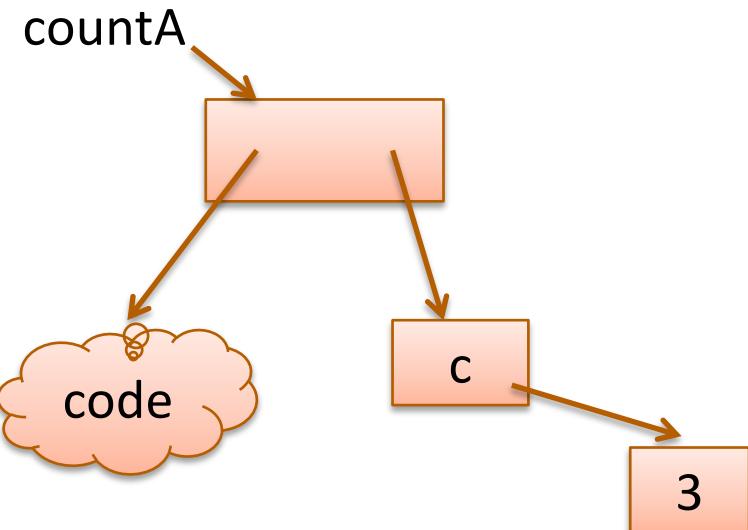
let next () : int =
  let v = !c in
  (c := v+1 ; v)
```

Mutable Reference Captured in Closure

```
let counter () =
  let c = ref 0 in
  fun () ->
    let v = !c in
    (c := v+1 ; v)
```

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```
let countA = counter() in
let countB = counter() in
countA(); (* 0 *)
countA(); (* 1 *)
countB(); (* 0 *)
countB(); (* 1 *)
countA(); (* 2 *)
```

## Assignment Project Exam Help Imperative loops

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```
(* print n .. 0 *)
let count_down (n:int) =
  for i = n downto 0 do
    print_int i;
    print_newline()
done

(* print 0 .. n *)
let count_up (n:int) =
  for i = 0 to n do
    print_int i;
    print_newline()
done
```

```
(* sum of 0 .. n *)

let sum (n:int) =
  let s = ref 0 in
  let current = ref n in
  while current > 0 do
    s := !s + !current;
    current := !current - 1
  done;
  !s
```

# Assignment Project Exam Help? Imperative loops?

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```
(* print n .. 0 *)  
  
let count_down (n:int) =  
  for i = n downto 0 do  
    print_int i;  
    print_newline()  
done
```

```
(* for i=n downto 0 do f i *)  
  
let rec for_down  
  (n : int)  
  (f : int -> unit)  
  : unit =  
  if n >= 0 then  
    (f n; for_down (n-1) f)  
  else
```

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```
let count_down (n:int) =  
  for_down n (fun i =>  
    print_int i;  
    print_newline()  
  )
```

## Assignment Project Exam Help Aliasing

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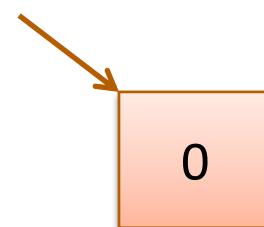
```
let c = ref 0
```

```
let x = Assignment Project Exam Help
```

```
x := 42 ; https://powcoder.com
```

```
!c
```

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## Assignment Project Exam Help Aliasing

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```
let c = ref 0
```

```
let x = Assignment Project Exam Help
```

```
x := 42 ; https://powcoder.com x
```

```
!c
```

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## Assignment Project Exam Help Aliasing

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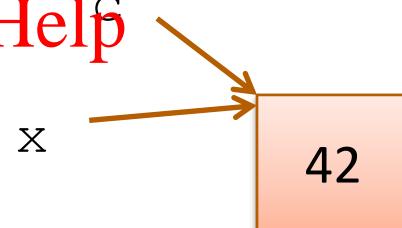
```
let c = ref 0
```

```
let x = Assignment Project Exam Help
```

```
x := 42 ; https://powcoder.com x
```

```
!c
```

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42

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## **REFS AND MODULES**

## Assignment Project Exam Help Types and References

Concrete, first-order type tells you a lot about a data structure:

- int ==> immutable
- int ref ==> mutable
- int \* int ==> immutable
- int \* (int ref) ==> 1<sup>st</sup> component immutable, 2<sup>nd</sup> mutable
- ... etc

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What about higher-order types?

- int -> int ==> the function can't be changed  
==> what happens when we run it?

What about abstract types?

- stack, queue? stack \* queue?

## Assignment Project Exam Help Functional Stacks

```
module type STACK =  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> 'a stack  
  val pop : 'a stack -> 'a stack  
  val top : 'a stack -> 'a option  
  ...  
end
```

## Assignment Project Exam Help Functional Stacks

```
module type STACK =  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> 'a stack  
  val pop : 'a stack -> 'a stack  
  val top : 'a stack -> 'a option  
  ...  
end
```

A functional interface takes in arguments, analyzes them, and produces new results

## Assignment Project Exam Help Imperative Stacks

```
module type IME_STACK =  
  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> unit  
  ...  
end
```

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## Assignment Project Exam Help Imperative Stacks

```
module type IMPL_STACK =  
  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> unit  
  ...  
end
```

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When you see “unit” as the return type, you know the function is being executed for its side effects. (Like void in C/C++/Java.)

## Assignment Project Exam Help Imperative Stacks

```
module type IME_STACK =  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> unit  
  val pop : 'a stack -> 'a option  
end
```

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

## Assignment Project Exam Help Imperative Stacks

```
module type IME_STACK =  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack -> unit  
  val pop : https://powcoder.com 'a option  
end
```

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

## Assignment Project Exam Help Imperative Stacks

```
module type IME_STACK =  
sig  
  type 'a stack  
  val empty : unit -> 'a stack  
  val push : 'a -> 'a stack  
  val pop : https://powcoder.com/option 'a  
end
```

Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

This is a terrific way to use references in ML. Look for these opportunities

Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

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## Assignment Project Exam Help Imperative Stacks

```
module ImpStack : IMP_STACK =  
  struct  
    type 'a stack = ('a list) ref  
  
    let empty() : 'a stack = ref []  
    let push (x:'a) (s:'a stack) : unit =  
      s := x :: (!s)  
    let pop (s:'a stack) : 'a option =  
      match !s with  
        | [] -> None  
        | h :: t -> (s := t ; Some h)  
  end
```

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## Assignment Project Exam Help Imperative Stacks

```
module ImpStack : IMP_STACK =  
  struct  
    type 'a stack = ('a list) ref  
  
    let empty() : 'a stack = ref []  
  
    let push (x:'a) (s:'a stack) : 'a stack =  
      s := x :: (!s)  
  
    let pop (s:'a stack) : 'a option =  
      match !s with  
        | [] -> None  
        | h :: t -> (s := t ; Some h)  
  
  end
```

Note: We don't have to make *everything* mutable.

The list is an immutable data structure stored in a single mutable cell.

# Assignment Project Exam Help Fully Mutable Lists

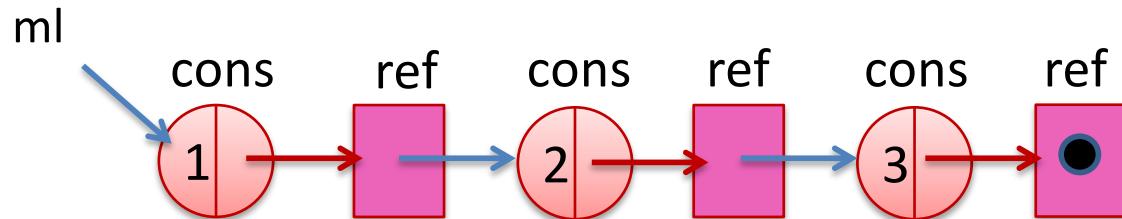
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```
type 'a mlist =  
    Nil | Cons of 'a * ('a mlist ref)
```

```
let ml = Cons(1, ref(Cons(2, ref  
    (Cons(3, ref Nil))))))
```

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# Assignment Project Exam Help Fully Mutable Lists

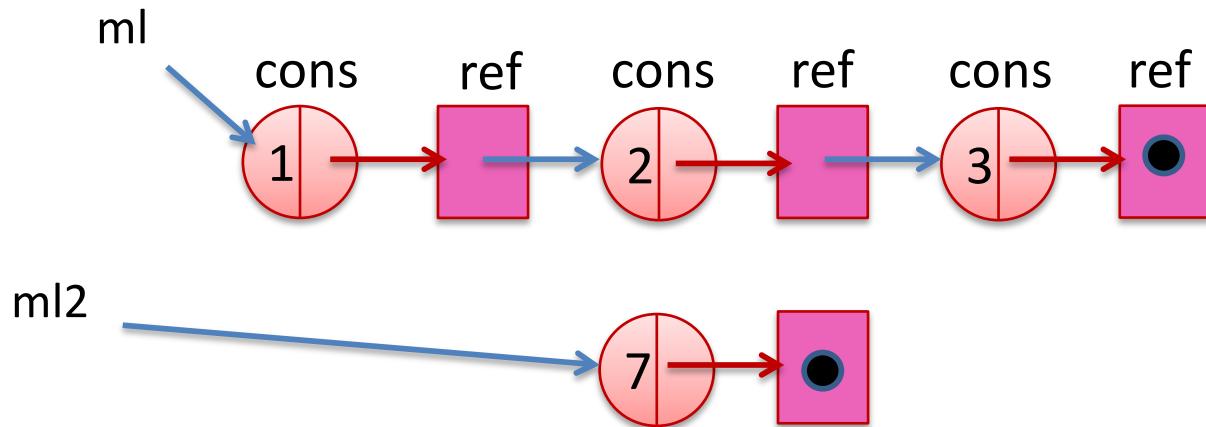
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```
type 'a mlist =  
    Nil | Cons of 'a * ('a mlist ref)
```

```
let ml = Cons(1, ref(Cons(2, ref  
    (Cons(3, ref Nil))))))
```

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let ml2 = Cons(7, ref Nil)

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# Assignment Project Exam Help Fully Mutable Lists

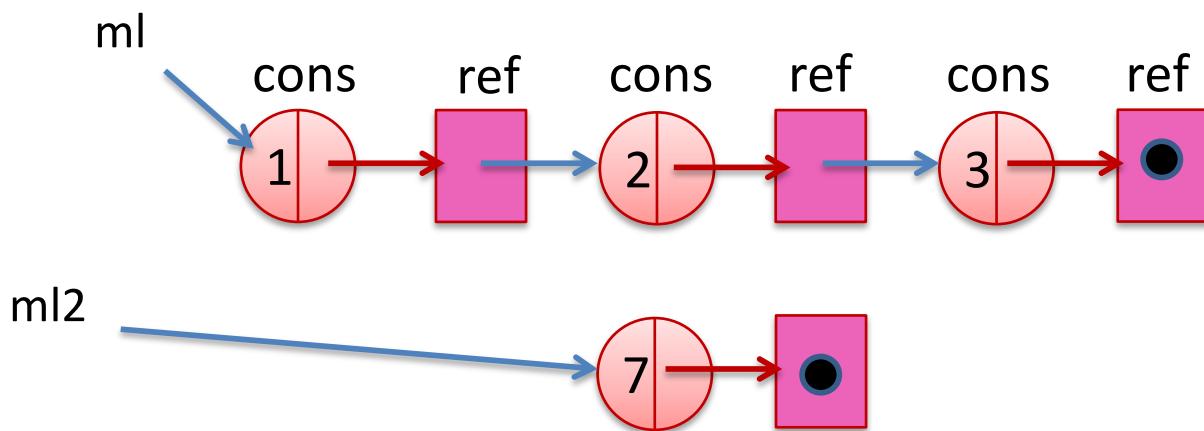
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```
type 'a mlist =  
    Nil | Cons of 'a * ('a mlist ref)
```

```
let rec fudge(l:'a mlist)  
    (m:'a mlist) : unit =  
  match l with  
  | Nil -> ()  
  | Cons(h,t)-> t := m; ()
```

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fudge ml ml2



# Assignment Project Exam Help Fully Mutable Lists

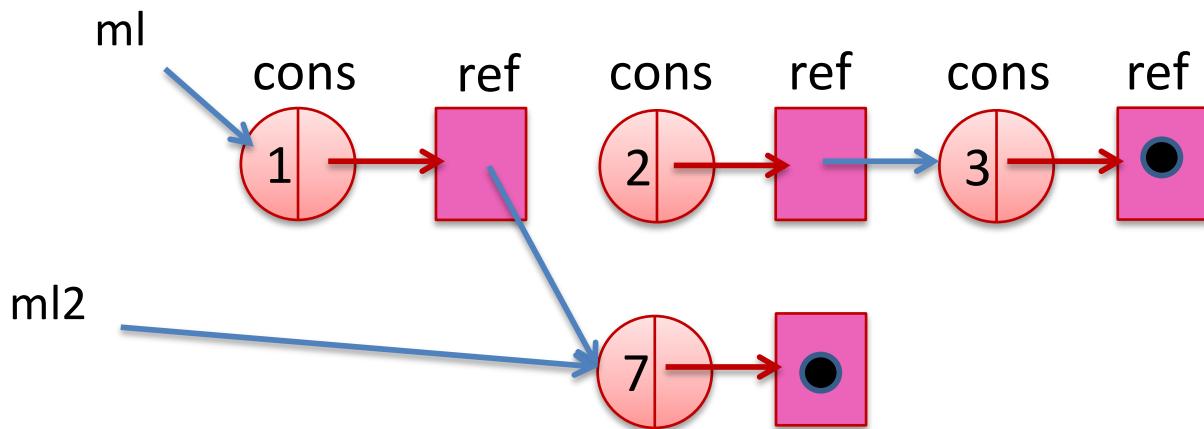
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```
type 'a mlist =  
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let rec fudge(l:'a mlist)  
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```

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fudge ml ml2



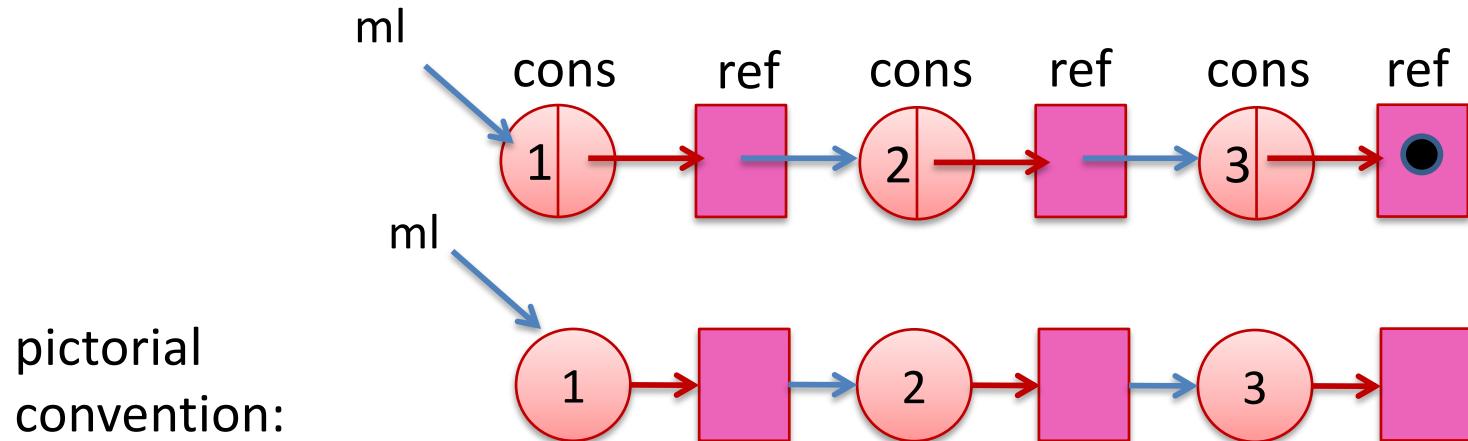
# Assignment Project Exam Helps

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```
type 'a mlist =  
    Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m:'a mlist) : int =  
  match m with  
  | Nil -> 0  
  | Cons(h,t) -> 1 + mlength(!t)
```

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# Assignment Project Exam Help Fraught with Peril

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```
type 'a mlist =  
  Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m: 'a mlist): int =
```

```
  match m with  
  | Nil -> 0  
  | Cons(h,t) -> 1 + length(!t)
```

```
let r = ref Nil ;;
```

```
let m = Cons(3,r) ;;
```

```
r := m ;;
```

```
mlength m ;;
```

# Assignment Project Exam Help Fraught with Peril

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```
type 'a mlist =  
  Nil | Cons of 'a * ('a mlist ref)
```

```
let rec mlength(m : 'a mlist) : int =
```

```
  match m with
```

```
  | Nil -> 0
```

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```
  | Cons(h, t) -> 1 + length(!t)
```

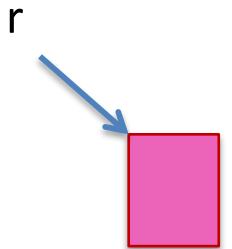
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```
let r = ref Nil in
```

```
let m = Cons(3, r) in
```

```
r := m ;
```

```
mlength m
```



# Assignment Project Exam Help Fraught with Peril

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```
type 'a mlist =  
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```
let rec mlength(m : 'a mlist) : int =
```

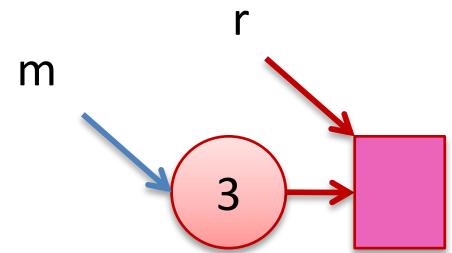
```
  match m with  
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```

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```

```
r := m ;
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```
mlength m
```



# Assignment Project Exam Help Fraught with Peril

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type 'a mlist =  
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let rec mlength(m : 'a mlist) : int =
```

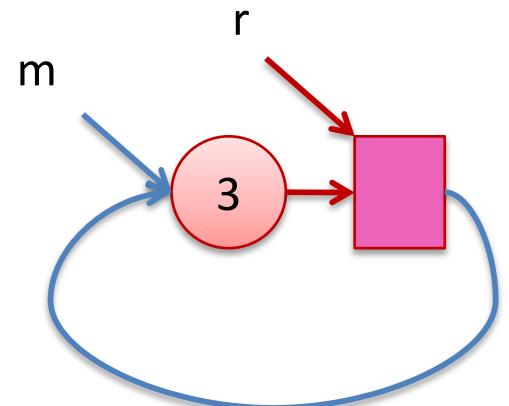
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  | Nil -> 0  
  | Cons(h, t) -> 1 + length(!t)
```

```
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```

```
let m = Cons(3, r) in
```

```
r := m ;
```

```
mlength m
```



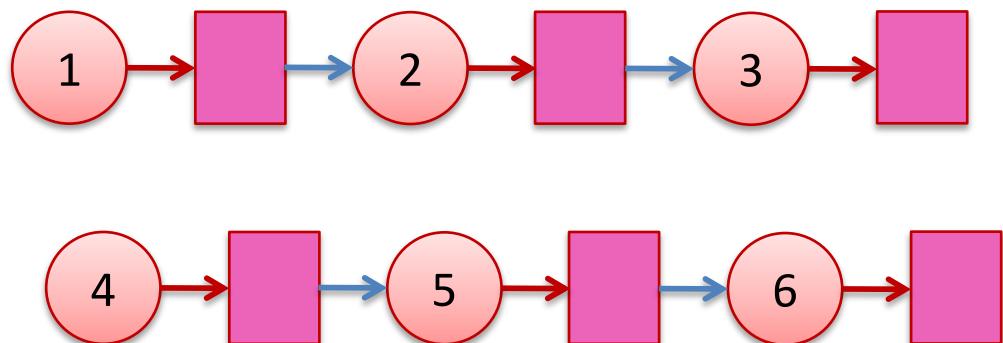
## Assignment Project Exam Help: Another Example:

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```
type 'a mlist =
  Nil | Cons of 'a * ('a mlist ref)
let rec mappend xs ys =
  match xs with
  | Nil -> ys
  | Cons(h, t) ->
    (match !t with
     | Nil -> t := ys
     | Cons(_, _) as m -> mappend m ys)
```

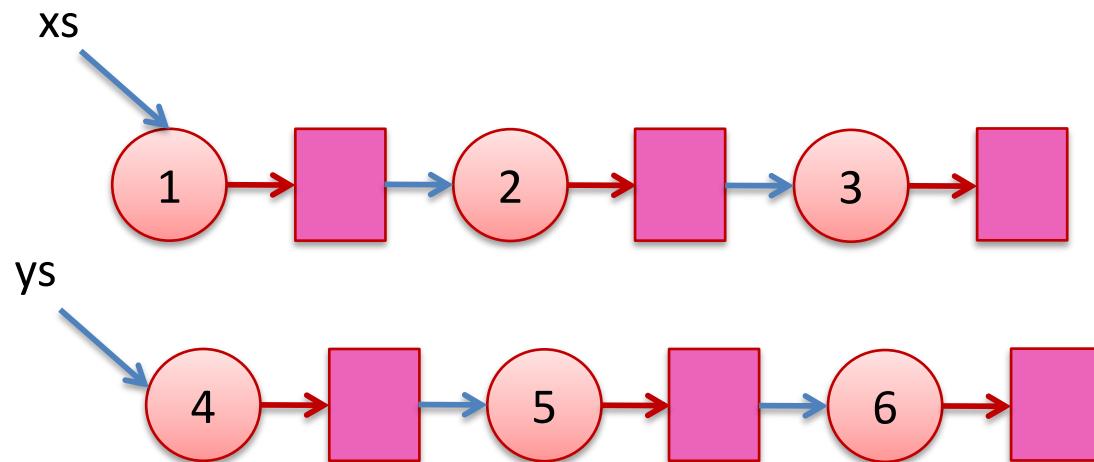
## Assignment Project Exam Help Mutable Append Example:

```
let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;  
mappend x y ;;
```



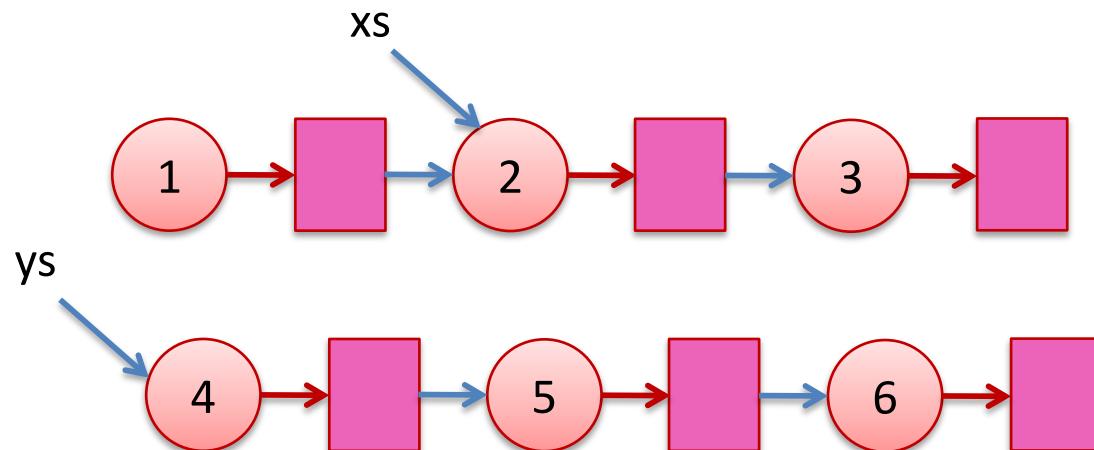
## Assignment Project Exam Help Mutable Append Example:

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mappend x y ;;
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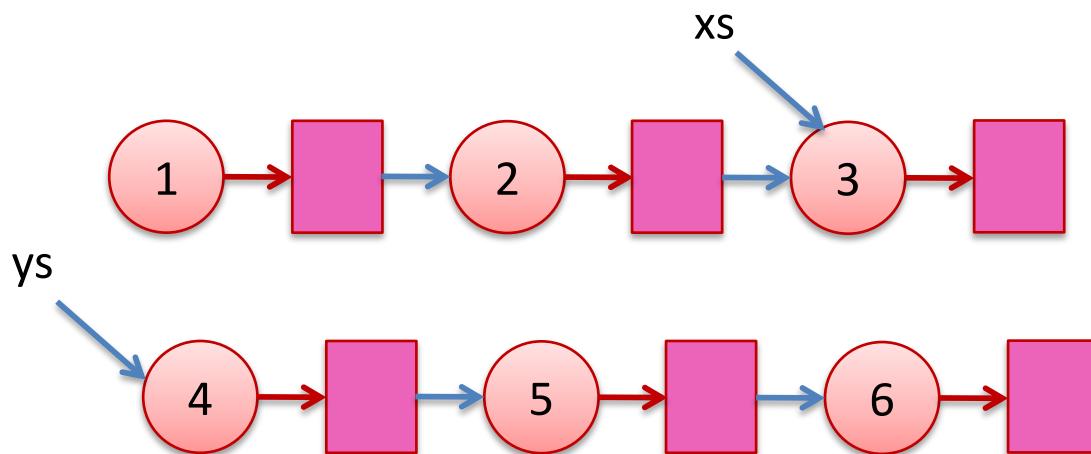
## Assignment Project Exam Help Mutable Append Example:

```
let rec mappend xs ys =  
  match xs with  
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  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;  
mappend x y ;;
```



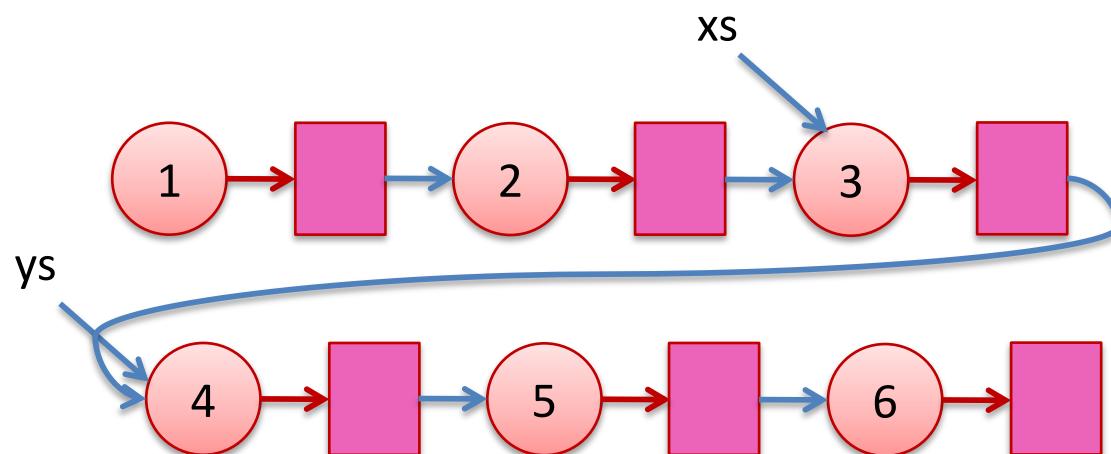
## Assignment Project Exam Help Mutable Append Example:

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    (match !t with  
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mappend x y ;;
```



## Assignment Project Exam Help Mutable Append Example:

```
let rec mappend xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons(h, t) ->  
    (match !t with  
     | Nil -> t := ys;  
     | Cons(_, _) as m -> mappend m ys) ;;  
let x = Cons(1, ref (Cons(2, ref (Cons(3, ref Nil)))))) ;;  
let y = Cons(4, ref (Cons(5, ref (Cons(6, ref Nil)))))) ;;  
mappend x y ;;
```



## Add mutability judiciously

Two types: Add WeChat powcoder

```
type 'a very_mutable_list =  
  Nil  
  | Cons of 'a * ('a very_mutable_list ref)
```

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```
type 'a less_mutable_list = 'a list ref
```

The first makes cyclic lists possible, the second doesn't

- the second preemptively avoids certain kinds of errors.
- often called a *correct-by-construction design*

## Assignment Project Exam Help Is it possible to avoid all state?

Yes! (in single-threaded programs)

- Pass in old values to functions; return new values from functions ...  
but this isn't necessarily the most efficient thing to do

Consider the difference between our functional stacks and our imperative ones:

- fnl\_push : `a -> a stack -> a stack
- imp\_push : `a -> a stack -> unit

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## SUMMARY

## Assignment Project Exam Help Summary: How/when to use state?

- A complicated question!
- In general, try to write the functional version first.
  - e.g., prototype
  - don't have to worry about sharing and updates
  - reasoning is easy (the substitution model is valid!)
- Sometimes you find you can't afford it for efficiency reasons.
  - example: routing tables need to be fast in a switch
  - constant time lookup, update (hash-table)
- When using state, try to *encapsulate* it behind an interface.
  - try to reduce the number of error conditions a client can see
    - correct-by-construction design
  - module implementer must think explicitly about sharing and invariants
  - write these down, write assertions to test them
  - if encapsulated in a module, these tests can be localized
  - *most of your code should still be functional*

## Assignment Project Exam Help Summary

Mutable data structures can lead to efficiency improvements.

- e.g., Hash tables, memoization, depth-first search

But they are *much* harder to get right, so don't jump the gun

- *updating in one place may have an effect on other places.*
- *writing and enforcing invariants becomes more important.*
  - why more important? because the types do less ...
- *cycles in data (other than functions) can't happen until we introduce refs.* Add WeChat powcoder
  - must write operations much more carefully to avoid looping
  - more cases to deal with and the compiler doesn't help you!
- we haven't even gotten to the multi-threaded part.

*So use refs when you must, but try hard to avoid it.*