Static Program Analysis

Part 9 – pointer analysis

http://cs.au.dk/~amoeller/spa/

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Agenda

- **Introduction to points-to analysis**
- Andersen's analysis
- Steensgaards's analysis
- Interprocedural points-to analysis
- Null pointer analysis
- Flow-sensitive points-to analysis

Analyzing programs with pointers

How do we perform e.g. constant propagation analysis when the programming language has pointers?
(or object references?)

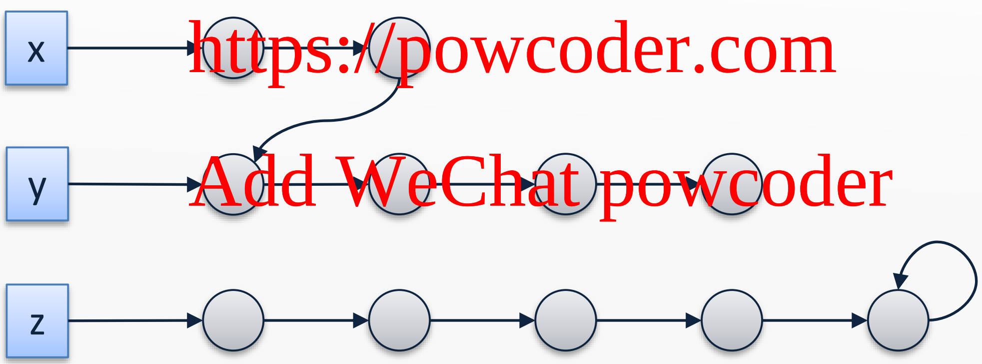
$E \rightarrow$ `&`X
 | `alloc` E
 | `*`E
 | `null`
 | …

$S \rightarrow$ `*`X `=` E`;`
 | …

```
...
*x = 42;
*y = -87;
z = *x;
// is z 42 or -87?
```

Heap pointers

- For simplicity, we ignore records
  - `alloc` then only allocates a single cell
  - only linear structures can be built in the heap



- Let’s at first also ignore functions as values
- We still have many interesting analysis challenges...

Pointer targets

- The fundamental question about pointers:
  What cells can they point to?
- We need a suitable abstraction
- The set of (abstract) cells, *Cells*, contains
  - `alloc-`*i* for each allocation site with index *i*
  - *X* for each program variable named *X*
- This is called ***allocation site abstraction***
- Each abstract cell may correspond to many concrete memory cells at runtime

Points-to analysis

- Determine for each pointer variable *X* the set *pt*(*X*) of the cells *X* may point to

```
...
*x = 42;
*y = -87;
z = *x;
// is z 42 or -87?
```


- A *conservative* ("may points-to") analysis:

  – the set may be too large
  – can show absence of aliasing: $pt(X) \cap pt(Y) = \varnothing$

- We'll focus on *flow-insensitive* analyses:
  – take place on the AST
  – before or together with the control-flow analysis

Obtaining points-to information

- An almost-trivial analysis (called *address-taken*):
  - include all `alloc-`*i* cells
  - include the `X` cell if the expression `&X` occurs in the program

- Improvement for a typed language:
  - eliminate those cells whose types do not match

- This is sometimes good enough
  - and clearly very fast to compute

Pointer normalization

- Assume that all pointer usage is normalized:
  - $X = $ `alloc` P where P is `null` or an integer constant
  - $X = \&Y$
  - $X = Y$
  - $X = *Y$
  - $*X = Y$
  - $X = $ `null`
- Simply introduce lots of temporary variables…
- All sub-expressions are now named
- We choose to ignore the fact that the cells created at variable declarations are uninitialized

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Andersen's analysis (1/2)

- For every cell c, introduce a constraint variable $\llbracket c \rrbracket$ ranging over sets of cells, i.e. $\llbracket \cdot \rrbracket : \mathit{Cells} \rightarrow 2^{\mathit{Cells}}$

- Generate constraints:

  - $X = \texttt{alloc}\ P$: $\quad \texttt{alloc-}i \in \llbracket X \rrbracket$
  - $X = \&Y$: $\quad Y \in \llbracket X \rrbracket$

  - $X = Y$: $\quad \llbracket Y \rrbracket \subseteq \llbracket X \rrbracket$
  - $X = {}^*Y$: $\quad \mathrm{c} \in \llbracket Y \rrbracket \Rightarrow \llbracket \mathrm{c} \rrbracket \subseteq \llbracket X \rrbracket$ for each $\mathrm{c} \in \mathit{Cells}$
  - ${}^*X = Y$: $\quad \mathrm{c} \in \llbracket X \rrbracket \Rightarrow \llbracket Y \rrbracket \subseteq \llbracket \mathrm{c} \rrbracket$ for each $\mathrm{c} \in \mathit{Cells}$
  - $X = \texttt{null}$: $\quad$ (no constraints)

Andersen's analysis (2/2)

- The points-to map is defined as:

$$pt(X) = [\![X]\!]$$

- The constraints fit into the cubic framework ☺

- Unique minimal solution in time $O(n^3)$

- In practice, for Java: $O(n^2)$

- The analysis is flow-insensitive but *directional*
  - models the direction of the flow of values in assignments

Example program

```
var p,q,x,y,z;
p = alloc null;
x = y;
x = z;
*p = z;
p = q;
q = &y;
x = *p;
p = &z;
```


Cells = {p, q, x, y, z, `alloc-1`}

Applying Andersen

- Generated constraints:

$\texttt{alloc-1} \in \llbracket \mathrm{p} \rrbracket$

$\llbracket \mathrm{y} \rrbracket \subseteq \llbracket \mathrm{x} \rrbracket$

$\llbracket \mathrm{z} \rrbracket \subseteq \llbracket \mathrm{x} \rrbracket$

$\mathrm{c} \in \llbracket \mathrm{p} \rrbracket \Rightarrow \llbracket \mathrm{z} \rrbracket \subseteq \llbracket \alpha \rrbracket$ for each $\mathrm{c} \in \mathit{Cells}$

$\llbracket \mathrm{q} \rrbracket \subseteq \llbracket \mathrm{p} \rrbracket$

$\mathrm{y} \in \llbracket \mathrm{q} \rrbracket$

$\mathrm{c} \in \llbracket \mathrm{p} \rrbracket \Rightarrow \llbracket \alpha \rrbracket \subseteq \llbracket \mathrm{x} \rrbracket$ for each $\mathrm{c} \in \mathit{Cells}$

$\mathrm{z} \in \llbracket \mathrm{p} \rrbracket$

- Smallest solution:

$\mathit{pt}(\mathrm{p}) = \{\, \texttt{alloc-1}, \mathrm{y}, \mathrm{z} \,\}$

$\mathit{pt}(\mathrm{q}) = \{\, \mathrm{y} \,\}$

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Steensgaard's analysis

- View assignments as being bidirectional
- Generate constraints:
  - $X = \texttt{alloc}\ P$: $\texttt{alloc-}i \in \llbracket X \rrbracket$
  - $X = \&Y$: $Y \in \llbracket X \rrbracket$
  - $X = Y$: $\llbracket X \rrbracket = \llbracket Y \rrbracket$
  - $X = {*}Y$: $c \in \llbracket Y \rrbracket \Rightarrow \llbracket c \rrbracket = \llbracket X \rrbracket$ for each $c \in \mathit{Cells}$
  - ${*}X = Y$: $c \in \llbracket X \rrbracket \Rightarrow \llbracket Y \rrbracket = \llbracket c \rrbracket$ for each $c \in \mathit{Cells}$

- Extra constraints:

$$c_1, c_2 \in \llbracket c \rrbracket \Rightarrow \llbracket c_1 \rrbracket = \llbracket c_2 \rrbracket \quad \text{and} \quad \llbracket c_1 \rrbracket \cap \llbracket c_2 \rrbracket \neq \emptyset \Rightarrow \llbracket c_1 \rrbracket = \llbracket c_2 \rrbracket$$

(whenever a cell may point to two cells, they are essentially merged into one)

- Steensgaard's original formulation uses conditional unification for $X = Y$: $c \in \llbracket Y \rrbracket \Rightarrow \llbracket X \rrbracket = \llbracket Y \rrbracket$ for each $c \in \mathit{Cells}$ (avoids unifying if Y is never a pointer)

Steensgaard's analysis

- Reformulate as term unification
- Generate constraints:
  - $X = \texttt{alloc}\ P$: $\quad \llbracket X \rrbracket = \&\llbracket \texttt{alloc-}i \rrbracket$
  - $X = \&Y$: $\quad \llbracket X \rrbracket = \&\llbracket Y \rrbracket$
  - $X = Y$: $\quad \llbracket X \rrbracket = \llbracket Y \rrbracket$
  - $X = {*}Y$: $\quad \llbracket Y \rrbracket = \&\alpha \ \wedge\ \llbracket X \rrbracket = \alpha$ where α is fresh
  - ${*}X = Y$: $\quad \llbracket X \rrbracket = \&\alpha \ \wedge\ \llbracket Y \rrbracket = \alpha$ where α is fresh

- Terms:
  - term variables, e.g. $\llbracket X \rrbracket$, $\llbracket \texttt{alloc-}i \rrbracket$, α (each representing the possible values of a cell)
  - a single (unary) term constructor $\&t$ (representing pointers)
  - $\llbracket X \rrbracket$ is now a term variable, not a constraint variable holding a set of cells
- Fits with our unification solver! (union-find…)
- The points-to map is defined as $\mathrm{pt}(X) = \{\, c \in \mathit{Cells} \mid \llbracket X \rrbracket = \&\llbracket c \rrbracket \,\}$
- Note that there is only one kind of term constructor, so unification never fails

Applying Steensgaard

- Generated constraints:

$\texttt{alloc-1} \in [\![p]\!]$

$[\![y]\!] = [\![x]\!]$

$[\![z]\!] = [\![x]\!]$

$\alpha \in [\![p]\!] \Rightarrow [\![z]\!] = [\![\alpha]\!]$

$[\![q]\!] = [\![p]\!]$

$y \in [\![q]\!]$

$\alpha \in [\![p]\!] \Rightarrow [\![\alpha]\!] = [\![x]\!]$

$z \in [\![p]\!]$

\+ the extra constraints

- Smallest solution:

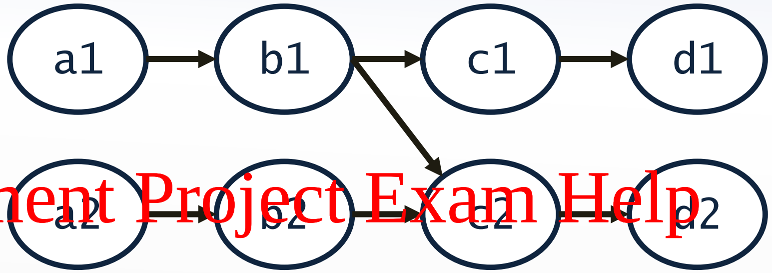
$pt(p) = \{\ \texttt{alloc-1}, y, z\ \}$

$pt(q) = \{\ \texttt{alloc-1}, y, z\ \}$

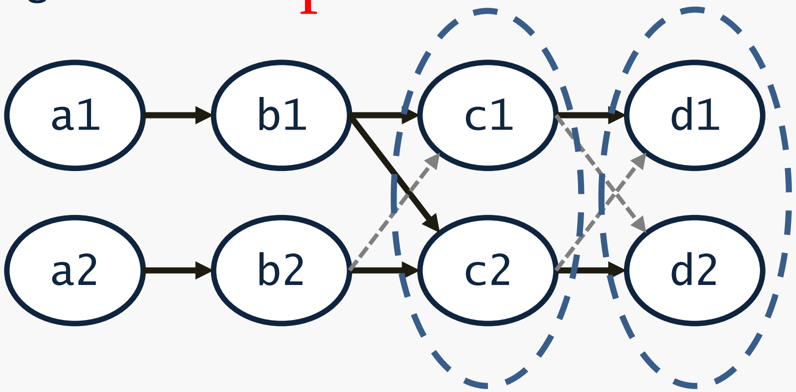
Another example

```
a1 = &b1;
b1 = &c1;
c1 = &d1;
a2 = &b2;
b2 = &c2;
c2 = &d2;
b1 = &c2;
```

Andersen:



Steensgaard:



Recall our type analysis…

- Focusing on pointers…
- Constraints:
  - $X = \texttt{alloc}\ P$: $[\![X]\!] = \&[\![P]\!]$
  - $X = \&Y$: $[\![X]\!] = \&[\![Y]\!]$
  - $X = Y$: $[\![X]\!] = [\![Y]\!]$
  - $X = {*}Y$: $\&[\![X]\!] = [\![Y]\!]$
  - ${*}X = Y$: $[\![X]\!] = \&[\![Y]\!]$
- Implicit extra constraint for term equality:

$$\&t_1 = \&t_2 \Rightarrow t_1 = t_2$$

- Assuming the program type checks, is the solution for pointers the same as for Steensgaard's analysis?

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Interprocedural points-to analysis

- In TIP, function values and pointers may be mixed together:

```
(***x)(1,2,3)
```


- In this case the CFA and the points-to analysis must happen *simultaneously*!

- The idea: Treat function values as a kind of pointers

Function call normalization

- Assume that all function calls are of the form

  $x = y(a_1, \ldots, a_n)$

- y may be a variable whose value is a function pointer

- Assume that all return statements are of the form

  `return` z;

- As usual, simply introduce lots of temporary variables…

- Include all function names in *Cells*

CFA with Andersen

- For the function call

  $x = y(a_1, \ldots, a_n)$

  and every occurrence of

  $f(x_1, \ldots, x_n)$ { … return z; }

  add these constraints:

  $f \in [\![f]\!]$

  $f \in [\![y]\!] \Rightarrow ([\![a_i]\!] \subseteq [\![x_i]\!]$ for i=1,…,$n \land [\![z]\!] \subseteq [\![x]\!])$

Andersen's analysis is already closely connected to control-flow analysis!

- (Similarly for simple function calls)
- Fits directly into the cubic framework!

CFA with Steensgaard

- For the function call

  $$x = y(a_1, \ldots, a_n)$$

  and every occurrence of

  $$f(x_1, \ldots, x_n)\ \{ \ldots \texttt{return}\ z;\ \}$$

  add these constraints:

  $$f \in [\![f]\!]$$

  $$f \in [\![y]\!] \Rightarrow ([\![a_i]\!] = [\![x_i]\!] \text{ for i=1,...,} n \wedge [\![z]\!] = [\![x]\!])$$

- (Similarly for simple function calls)
- Fits into the unification framework, but requires a generalization of the ordinary union-find solver

Context-sensitive pointer analysis

- Generalize the abstract domain $\mathit{Cells} \rightarrow 2^{\mathit{Cells}}$ to

$$\mathit{Contexts} \rightarrow \mathit{Cells} \rightarrow 2^{\mathit{Cells}}$$

  (or equivalently: $\mathit{Cells} \times \mathit{Contexts} \rightarrow 2^{\mathit{Cells}}$)
  where *Contexts* is a (finite) set of call contexts

- As usual, many possible choices of *Contexts*
  - recall the call string approach and the functional approach
- We can also track the set of reachable contexts for each function (like the use of lifted lattices earlier):

$$\mathit{Contexts} \rightarrow \mathrm{lift}(\mathit{Cells} \rightarrow 2^{\mathit{Cells}})$$

- Does this still fit into the cubic solver?

Context-sensitive pointer analysis

```
foo(a) {
  return *a;
}

bar() {
  ...
  x = alloc null; // alloc-1
  y = alloc null; // alloc-2
  *x = alloc null; // alloc-3
  *y = alloc null; // alloc-4
  ...
  q = foo(x);
  w = foo(y);
  ...
}
```


Are q and w aliases?

Context-sensitive pointer analysis

```
mk() {
  return alloc null; // alloc-1
}

baz() {
  var x,y;
  x = mk();
  y = mk();
  ...
}
```


Are x and y aliases?

Context-sensitive pointer analysis

- We can go one step further and introduce *context-sensitive heap* (a.k.a. *heap cloning*)
- Let each abstract cell be a pair of

  - `alloc-`*i* (the `alloc` with index *i*) or *X* (a program variable)

  - a heap context from a (finite) set *HeapContexts*

- This allows abstract cells to be named by the source code allocation site ***and (information from) the current context***
- One choice:
  - set *HeapContexts* = *Contexts*
  - at `alloc`, use the entire current call context as heap context

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Null pointer analysis

- Decide for every dereference `*p`,
  is `p` different from `null`?

- (Why not just treat null as a special cell in an Andersen or Steensgaard-style analysis?)

- Use the monotone framework
  - assuming that a points-to map *pt* has been computed

- Let us consider an intraprocedural analysis
  (i.e. we ignore function calls)

A lattice for null analysis

- Define the simple lattice *Null*:

$$\begin{array}{c} ? \\ | \\ \text{NN} \end{array}$$

where NN represents "definitely **n**ot **n**ull"
and ? represents "maybe null"

- Use for every program point the map lattice:

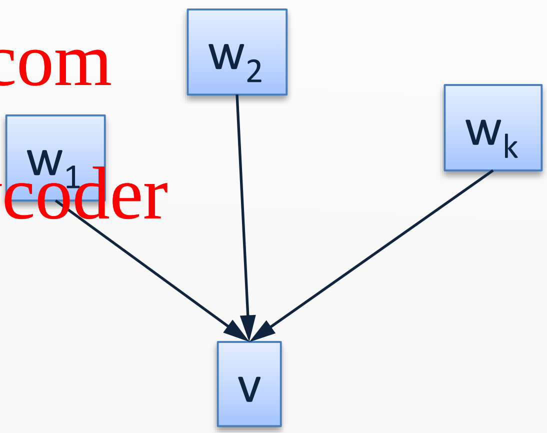
$$\mathit{Cells} \rightarrow \mathit{Null}$$

Setting up

- For every CFG node, v, we have a variable $[\![v]\!]$:
  - a map giving abstract values for all cells at the program point *after* v

- Auxiliary definition:

$$\mathit{JOIN}(v) = \bigsqcup_{w \in \mathit{pred}(v)} [\![w]\!]$$



(i.e. we make a *forward* analysis)

Null analysis constraints

- For operations involving pointers:
  - X = `alloc` P: $\llbracket v \rrbracket$ = ??? where P is null or an integer constant
  - $X = \&Y$: $\llbracket v \rrbracket$ = ???
  - $X = Y$: $\llbracket v \rrbracket$ = ???
  - $X = *Y$: $\llbracket v \rrbracket$ = ???
  - $*X = Y$: $\llbracket v \rrbracket$ = ???
  - X = `null`: $\llbracket v \rrbracket$ = ???
- For all other CFG nodes:
  - $\llbracket v \rrbracket = JOIN(v)$

Null analysis constraints

- For a heap store operation $*X = Y$ we need to model the change of whatever X points to
- That may be *multiple* abstract cells (i.e. the cells $pt(X)$)
- With the present abstraction, each abstract heap cell `alloc-`i may describe *multiple* concrete cells
- So we settle for **weak** update:

$$*X = Y: \qquad [\![v]\!] = store(JOIN(v), X, Y)$$

$$\text{where } store(\sigma, X, Y) = \sigma[\alpha \underset{\alpha \in pt(X)}{\mapsto} \sigma(\alpha) \sqcup \sigma(Y)]$$

Null analysis constraints

- For a heap load operation $X = {*}Y$ we need to model the change of the program variable X
- Our abstraction has a *single* abstract cell for X

- That abstract cell represents a *single* concrete cell

- So we can use **strong** update:

$$X = {*}Y: \qquad [\![v]\!] = load(JOIN(v), X, Y)$$

$$\text{where } \; load(\sigma, X, Y) = \sigma[X \mapsto \bigsqcup_{\alpha \in pt(Y)} \sigma(\alpha)]$$

Strong and weak updates

```
mk() {
  return alloc null; // alloc-1
}

...
a = mk();
b = mk();
*a = alloc null; // alloc-2
n = null;
*b = n; // strong update here would be unsound!
c = *a;
```


is c null here?

The abstract cell `alloc-1` corresponds to *multiple concrete cells*

Strong and weak updates

```
a = alloc null; // alloc-1
b = alloc null; // alloc-2
*a = alloc null; // alloc-3
*b = alloc null; // alloc-4
if (...) {
  x = a;
} else {
  x = b;
}
n = null;
*x = n; // strong update here would be unsound!
c = *x;
```


is c null here?

The points-to set for x contains *multiple abstract cells*

Null analysis constraints

- $X = \mathtt{alloc}\ P$: $\llbracket v \rrbracket = \mathit{JOIN}(v)[X \mapsto \mathrm{NN}, \mathtt{alloc}\text{-}\mathtt{i} \mapsto ?]$
- $X = \&Y$: $\llbracket v \rrbracket = \mathit{JOIN}(v)[X \mapsto \mathrm{NN}]$
- $X = Y$: $\llbracket v \rrbracket = \mathit{JOIN}(v)[X \mapsto \mathit{JOIN}(v)(Y)]$
- $X = \mathtt{null}$: $\llbracket v \rrbracket = \mathit{JOIN}(v)[X \mapsto ?]$

could be improved...

- In each case, the assignment modifies a program variable
- So we can use strong updates, as for heap load operations

Strong and weak updates, revisited

- Strong update: $\sigma[c \mapsto \textit{new-value}]$
  - possible if *c* is known to refer to a single concrete cell
  - works for assignments to local variables (as long as TIP doesn't have e.g. nested functions)

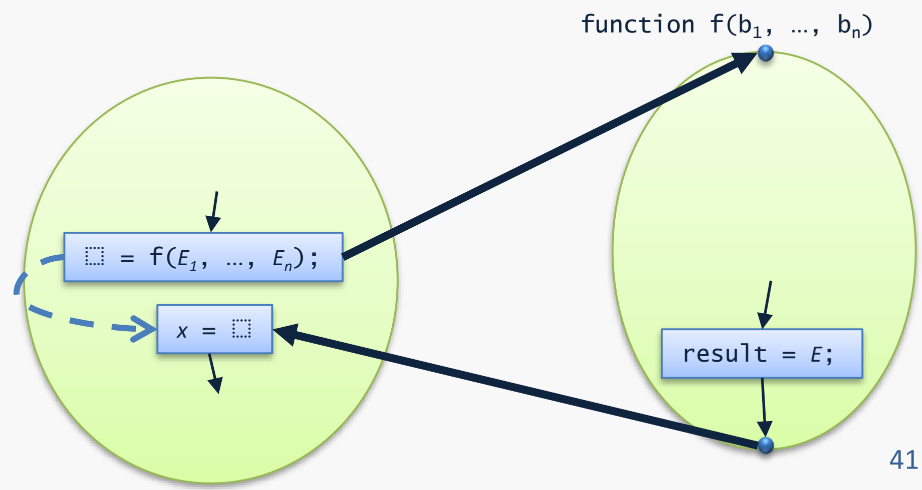
- Weak update: $\sigma[c \mapsto \sigma(c) \sqcup \textit{new-value}]$
  - necessary if *c* may refer to multiple concrete cells
  - bad for precision, we lose some of the power of flow-sensitivity
  - required for assignments to heap cells (unless we extend the analysis abstraction!)

Interprocedural null analysis

- Context insensitive or context sensitive, as usual…
  – at the after-call node, use the heap from the callee
- But be careful!

  Pointers to local variables may escape to the callee

  – the abstract state at the after-call node cannot simply copy the abstract values for local variables from the abstract state at the call node



Using the null analysis

- The pointer dereference *p is "safe" at entry of v if

  JOIN(v)(p) = NN

- The quality of the null analysis depends on the quality of the underlying points-to analysis

Example program

```
p = alloc null;
q = &p;
n = null;
*q = n;
*p = n;
```


Andersen generates:

pt(p) = {`alloc-1`}

pt(q) = {p}

pt(n) = Ø

Generated constraints

$\llbracket \texttt{p=alloc null} \rrbracket = \bot[\texttt{p} \mapsto \text{NN}, \texttt{alloc-1} \mapsto ?]$

$\llbracket \texttt{q=\&p} \rrbracket = \llbracket \texttt{p=alloc null} \rrbracket[\texttt{q} \mapsto \text{NN}]$

$\llbracket \texttt{n=null} \rrbracket = \llbracket \texttt{q=\&p} \rrbracket[\texttt{n} \mapsto ?]$

$\llbracket \texttt{*q=n} \rrbracket = \llbracket \texttt{n=null} \rrbracket[\texttt{p} \mapsto \llbracket \texttt{n=null} \rrbracket(\texttt{p}) \sqcup \llbracket \texttt{n=null} \rrbracket(\texttt{n})]$

$\llbracket \texttt{*p=n} \rrbracket = \llbracket \texttt{*q=n} \rrbracket[\texttt{alloc-1} \mapsto \llbracket \texttt{*q=n} \rrbracket(\texttt{alloc-1}) \sqcup \llbracket \texttt{*q=n} \rrbracket(\texttt{n})]$

Solution

$[\![\texttt{p=alloc null}]\!] = [p \mapsto NN, q \mapsto NN, n \mapsto NN, \texttt{alloc-1} \mapsto ?]$

$[\![\texttt{q=\&p}]\!] = [p \mapsto NN, q \mapsto NN, n \mapsto NN, \texttt{alloc-1} \mapsto ?]$

$[\![\texttt{n=null}]\!] = [p \mapsto NN, q \mapsto NN, n \mapsto ?, \texttt{alloc-1} \mapsto ?]$

$[\![\texttt{*q=n}]\!] = [p \mapsto ?, q \mapsto NN, n \mapsto ?, \texttt{alloc-1} \mapsto ?]$

$[\![\texttt{*p=n}]\!] = [p \mapsto ?, q \mapsto NN, n \mapsto ?, \texttt{alloc-1} \mapsto ?]$

- At the program point before the statement `*q=n` the analysis now knows that q is definitely non-null
- … and before `*p=n`, the pointer p is maybe null
- Due to the weak updates for all heap store operations, precision is bad for `alloc-i` cells

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Points-to graphs

- Graphs that describe possible heaps:
  - nodes are abstract cells
  - edges are possible pointers between the cells

- The lattice of points-to graphs is $2^{Cells \times Cells}$ ordered under subset inclusion
  (or alternatively, $Cells \rightarrow 2^{Cells}$)

- For every CFG node, v, we introduce a constraint variable ⟦v⟧ describing the state *after* v

- Intraprocedural analysis (i.e. ignore function calls)

Constraints

- For pointer operations:
  - $X = \texttt{alloc}\ P$: $[\![v]\!] = JOIN(v){\downarrow}X \cup \{\, (X, \texttt{alloc-}i) \,\}$
  - $X = \&Y$: $[\![v]\!] = JOIN(v){\downarrow}X \cup \{\, (X, Y) \,\}$
  - $X = Y$: $[\![v]\!] = JOIN(v){\downarrow}X \cup \{\, (X, t) \mid (Y, t) \in JOIN(v) \}$
  - $X = {*}Y$: $[\![v]\!] = JOIN(v){\downarrow}X \cup \{\, (X, t) \mid (Y, s) \in \sigma, (s, t) \in JOIN(v) \}$
  - ${*}X = Y$: $[\![v]\!] = JOIN(v) \cup \{\, (s, t) \mid (X, s) \in JOIN(v), (Y, t) \in JOIN(v) \}$
  - $X = \texttt{null}$: $[\![v]\!] = JOIN(v){\downarrow}X$

note: weak update!

where $\sigma{\downarrow}X = \{\, (s,t) \in \sigma \mid s \neq X \}$

$$JOIN(v) = \bigcup_{w \in pred(v)} [\![w]\!]$$

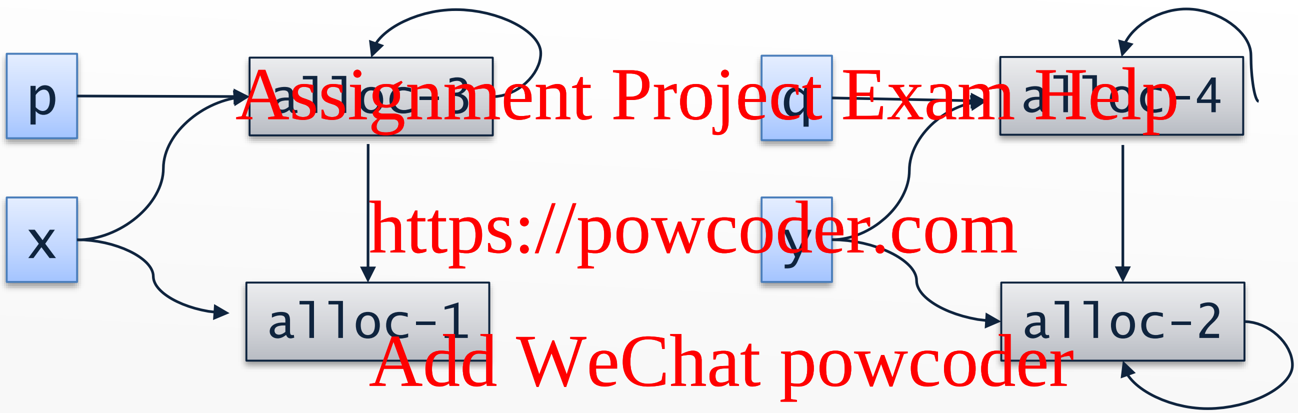
- For all other CFG nodes:
  - $[\![v]\!] = JOIN(v)$

Example program

```
var x,y,n,p,q;
x = alloc null; y = alloc null;
*x = null; *y = y;
n = input;
while (n>0) {
  p = alloc null; q = alloc null;
  *p = x; *q = y;
  x = p; y = q;
  n = n-1;
}
```

Result of analysis

- After the loop we have this points-to graph:



- We conclude that x and y will always be disjoint

Points-to maps from points-to graphs

- A points-to map for each program point v:

$$pt(X) = \{ \, t \mid (X,t) \in [\![v]\!] \, \}$$

- More expensive, but more precise:
  - Andersen: $pt(\mathrm{x}) = \{ \mathrm{y}, \mathrm{z} \}$
  - flow-sensitive: $pt(\mathrm{x}) = \{ \mathrm{z} \}$


```
x = &y;
x = &z;
```

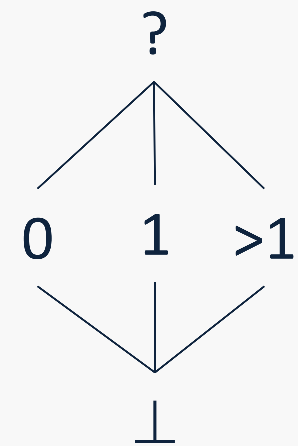
Improving precision with abstract counting

- The points-to graph is missing information:
  - `alloc-2` nodes always form a self-loop in the example

- We need a more detailed lattice:

$$2^{Cell \times Cell} \times (Cell \rightarrow Count)$$

  where we for each cell keep track of how many concrete cells that abstract cell describes

$Count =$ 

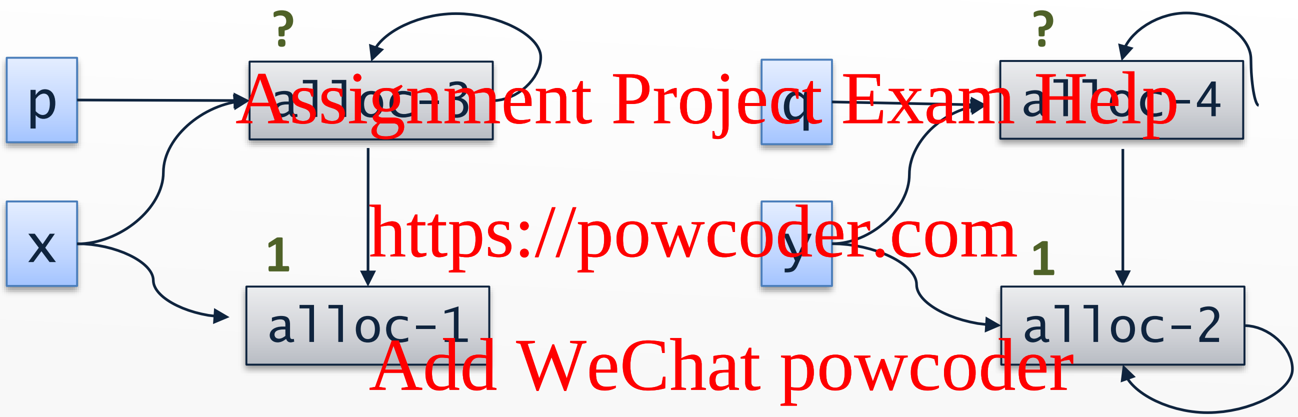
- This permits **strong updates** on those that describe precisely 1 concrete cell

Constraints

- X= `alloc` P: ...
- $*X = Y$: ...
- ...

Better results

- After the loop we have this extended points-to graph:



- Thus, `alloc-2` nodes form a self-loop

Escape analysis

- Perform a points-to analysis
- Look at return expression
- Check reachability in the points-to graph to arguments or variables defined in the function itself

- None of those

  ⇓

  no escaping stack cells

```
baz()  {
  var x;
  return &x;
}

main() {
  var p;
  p=baz();
  *p=1;
  return *p;
}
```