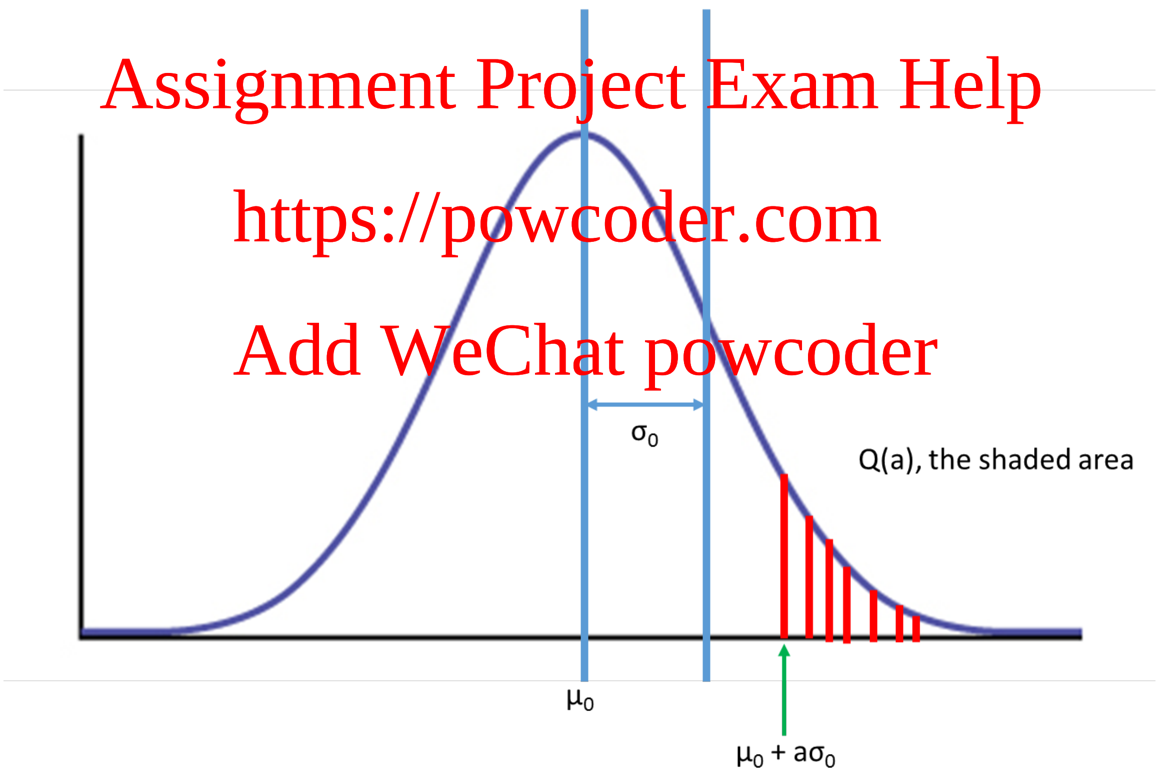
CDMA with Noise Solution

In the class, we have considered CDMA in an ideal system. However, in reality, we have much more complicated scenario: Users will experience noise.

Consider the scenario with one sender and one receiver. The chipping sequence is (-1 -1 -1 +1 -1 +1 +1 +1). Suppose the sender sends bit "1", then the signal sent will be (-1 -1 -1 +1 -1 +1 +1 +1). In the channel, noise is added to the signal so that the received signal will be ($-1+n_1$ $-1+n_2$ $-1+n_3$ $+1+n_4$ $-1+n_5$ $+1+n_6$ $+1+n_7$ $+1+n_8$), where $n_1, \ldots, n_8$ are noise terms. They are independently normally distributed with zero mean and σ^2 variance, in this question, $\sigma^2 = 1$. Formally, $n_i \sim N(0, 1)$. You should know the normal distribution in a prerequisite course.

After the computing the inner product at the receiver, what "value" does the receiver derive? If the value is smaller than 0, it is decoded as -1, otherwise, it is decoded as 1. Use the provided table to find the probability that it is wrongly decoded as -1.

The tail probability (Q function) of a standard normal distribution is given in the attached q_function.pdf.



(1) Inner product/M: $R=1+ (-n_1- n_2 -n_3 -n_4+n_5 +n_5 +n_7 +n_8)/8$

$n_i \sim N(0, 1)$.

$-n_i \sim N(0, 1)$.

$n_i/8 \sim N(0, 1/64)$.

$-n_i/8 \sim N(0, 1/64)$.

This is because if n is a Gaussian random variable with variance σ^2 , a*n is a Gaussian random variable with variance $a^2\sigma^2$

$R \sim N(1,1/8) = (\mu_0, \sigma_0^2)$

This is because the sum of two independent Gaussian random variables is still a Gaussian random variable. See https://en.wikipedia.org/wiki/Sum_of_normally_distributed_random_variables

$$X \sim N(\mu_X, \sigma_X^2)$$
$$Y \sim N(\mu_Y, \sigma_Y^2)$$
$$Z = X + Y,$$

then

$$Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2).$$

$P(R<0)=Q(\frac{|0-\mu 0|}{\sigma 0}) = Q(2.82) \approx 2.4012*10^{-03}$

It follows the definition of Q function, see https://en.wikipedia.org/wiki/Q-function

If Y is a Gaussian random variable with mean μ and variance σ^2, then $X = \frac{Y-\mu}{\sigma}$ is standard normal and

$$P(Y > y) = P(X > x) = Q(x)$$

where $x = \frac{y-\mu}{\sigma}$.