COMP 5416 Assignment 1

Due: 5pm, Friday, 25/AUG/2017

(20)

Question 1 (Review of Probability A). X and Y are two independent random variables. X is a uniformly distributed in $[0,3]$, and Y is uniformly distributed in $[3,6]$. Find the probability density function (PDF) of $X+Y$.

Let $pdf_X(\cdot)$, $pdf_Y(\cdot)$ denote pdfs of X and Y.

$$cdf_{X+Y}(z) = P(X+Y \le z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} pdf_X(x)\, pdf_Y(y)\, \mathbb{1}(x+y \le z)\, dy\, dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{z-x} pdf_X(x)\, pdf_Y(y)\, dy\, dx$$

(3) ~~Let~~ $pdf_{X+Y}(z) = \dfrac{\partial\, cdf_{X+Y}(z)}{\partial z}$

$$= \int_{-\infty}^{\infty} pdf_X(x)\, pdf_Y(z-x)\, dx$$

(1) $pdf_X(x) = \begin{cases} \frac{1}{3} & x \in [0,3] \\ 0 & \text{otherwise} \end{cases}$

(2) $pdf_Y(x) = \begin{cases} \frac{1}{3} & x \in [3,6] \\ 0 & \text{otherwise} \end{cases}$

(1), (2) $\Rightarrow$ (3).

$$pdf_{X+Y}(z) = \begin{cases} 0 & z \in (-\infty, 3) \\ \frac{1}{9}(z-3) & z \in (3, 6] \\ \frac{1}{9}(9-z) & z \in (6, 9] \\ 0 & z \in (9, +\infty) \end{cases}$$

(10)

Question 2 (Review of Probability B). T is a random variable that follows exponential distribution. The probability density function of T is

$$f(t) = \begin{cases} 0, & \text{if } t < 0, \\ \lambda e^{-\lambda t}, & \text{otherwise.} \end{cases} \tag{1}$$

Prove that $\mathbb{P}(T > a + b | T > a) = \mathbb{P}(T > b)$.

$$P(T>a+b \mid T>a)$$
$$= \frac{P(T>a+b,\ T>a)}{P(T>a)}$$
$$= \frac{P(T>a+b)}{P(T>a)}$$
$$= \frac{e^{-\lambda(a+b)}}{e^{-\lambda a}}$$
$$= e^{-\lambda b}$$
$$= P(T>b).$$

$$P(T>a) = 1 - CDF_T(a)$$
$$= e^{-\lambda a}$$
$$P(T>a+b) = e^{-\lambda(a+b)}$$
$$P(T>b) = e^{-\lambda b}$$