Answers to Quiz 1

Qn 1

a)

$$\frac{X^2}{a^2} + \frac{Y^2}{a^2} = sec^2\alpha$$

$X = \sec\alpha \cos\beta$
$Y = \sec\alpha \sin\beta$
$Z = b \tan\alpha$

$\beta \in |0, 2\pi)$
$\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

Paper 1: $a = 2,\ b = 4$
Paper 2: $a = 3,\ b = 6$

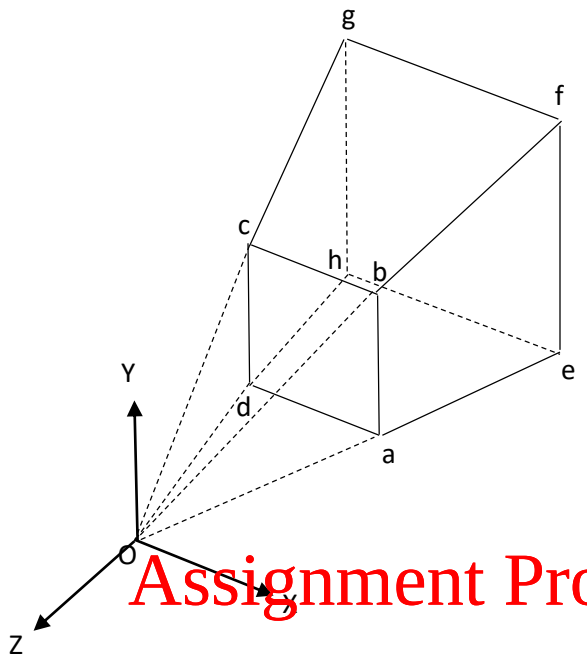
b) Any reasonable answer, for example,

$X = a\ sec^{s_1}\alpha\ cos^{s_2}\beta$
$Y = a\ sec^{s_1}\alpha\ sin^{s_2}\beta$
$Z = b\ tan^{s_1}\alpha$

or

$X = a\ cos^{-s_1}\alpha\ cos^{s_2}\beta$
$Y = a\ cos^{-s_1}\alpha\ sin^{s_2}\beta$
$Z = b\ tan^{s_1}\alpha$

Qn 2



$O = (0, 0, 0)$

H height W Width

$$\tan 30^o = \frac{H}{2\sqrt{3}} \Longrightarrow H = 2 = W$$

$a = (1, -1, -\sqrt{3})$ $b = (1, 1, -\sqrt{3})$

$$\overrightarrow{Oa} \times \overrightarrow{Ob} = \begin{vmatrix} i & j & k \\ 1 & -1 & -\sqrt{3} \\ 1 & 1 & -\sqrt{3} \end{vmatrix} = (2\sqrt{3}, 0, 2)$$

The set of inequalities

$\sqrt{3}X + Z < 0$
$-\sqrt{3}X + Z < 0$
$\sqrt{3}Y + Z < 0$
$-\sqrt{3}Y + Z < 0$
$-100 < Z < -\sqrt{3}$

Qn 3

$$M_{P\leftarrow CT} = [T(10,10,10)R_z(-135^o)R_x(100^o)R_z(30^o)]^{-1}$$
$$= R_z(-30^o)R_x(-100^o)R_z(135^o)T(-a,-a,-a)$$

glRotatef (-30, 0, 0, 1);
glRotatef (-100, 1, 0, 0);
glRotatef (135, 0, 0, 1);
glTranslatef (-a, -a, -a);

Paper 1: $a = 10$
Paper 2: $a = 20$

Qn 4

VRP = (0, 30, 30) VPN = $(0,30,30)$ VUP = (0, 1, 0)

$Z_{VC} = |\text{VPN}| = (0, 1/\sqrt{2}, 1/\sqrt{2})$

$$\text{VUP} \times \text{VPN} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{vmatrix} = (1/\sqrt{2}, 0, 0)$$

$X_{VC} = |\text{VUP} \times \text{VPN}| = (1,0,0)$

$$Y_{VC} = Z_{VC} \times X_{VC} = \begin{vmatrix} i & j & k \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{vmatrix} = (0, 1/\sqrt{2}, -1/\sqrt{2})$$

$$M_{WC\leftarrow CC} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 30 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 30 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 30 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 30 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 30 \\ 30 - \sqrt{2} \\ 1 \end{pmatrix}$$

The world coordinates are $(0, 30, 30 - \sqrt{2})$