Ex: 10.1 $A_M = -\dfrac{R_G}{R_G + R_{\text{sig}}} g_m (R_D \parallel R_L)$

$= -\dfrac{10}{10 + 0.1} \times 2(10 \parallel 10)$

$= -9.9$ V/V

$f_{P1} = \dfrac{1}{2\pi C_{C1}(R_{\text{sig}} + R_G)}$

$= \dfrac{1}{2\pi \times 1 \times 10^{-6}(0.1 + 10) \times 10^6}$

$= 0.016$ Hz

$f_{P2} = \dfrac{g_m + 1/R_S}{2\pi C_S}$

$= \dfrac{(2 + 0.1) \times 10^{-3}}{2\pi \times 1 \times 10^{-6}} = 334.2$ Hz

$f_{P3} = \dfrac{1}{2\pi C_{C2}(R_D + R_L)}$

$= \dfrac{1}{2\pi \times 1 \times 10^{-6}(10 + 10) \times 10^3}$

$= 8$ Hz

$f_Z = \dfrac{1}{2\pi C_S R_S}$

$= \dfrac{1}{2\pi \times 1 \times 10^{-6} \times 10 \times 10^3}$

$= 15.9$ Hz

Since the highest-frequency pole is $f_{P2} = 334.2$ Hz and the next highest-frequency singularity is f_Z at 15.9 Hz, the lower 3-dB frequency f_L will be

$f_L \simeq f_{P2} = 334.2$ Hz

Ex: 10.2 Refer to Fig. 10.10.

$\tau_{C1} = C_{C1}[R_{\text{sig}} + (R_B \parallel r_\pi)]$

$= 1 \times 10^{-6}[5 + (100 \parallel 2.5)] \times 10^3$

$= 7.44$ ms

$\tau_{CE} = C_E \left[R_E \parallel \left(r_e + \dfrac{R_B \parallel R_{\text{sig}}}{\beta + 1} \right) \right]$

$\beta = g_m r_\pi = 40 \times 2.5 = 100$

$r_e \simeq 1/g_m = 25\ \Omega$

$\tau_{CE} = 1 \times 10^{-6} \left[5 \parallel \left(0.025 + \dfrac{100 \parallel 5}{101} \right) \right] \times 10^3$

$\tau_{CE} = 0.071$ ms

$\tau_{C2} = C_{C2}(R_C + R_L)$

$= 1 \times 10^{-6}(8 + 5) \times 10^3$

$= 13$ ms

$f_L = \dfrac{1}{2\pi} \left[\dfrac{1}{\tau_{C1}} + \dfrac{1}{\tau_{CE}} + \dfrac{1}{\tau_{C2}} \right]$

$= \dfrac{1}{2\pi} \left[\dfrac{1}{7.44} + \dfrac{1}{0.071} + \dfrac{1}{13} \right] \times 10^3$

$= 2.28$ kHz

$f_Z = \dfrac{1}{2\pi C_E R_E}$

$= \dfrac{1}{2\pi \times 1 \times 10^{-6} \times 5 \times 10^3} = 31.8$ Hz

Since f_Z is much lower than f_L it will have a negligible effect on f_L.

Ex: 10.3 $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}} = \dfrac{3.45 \times 10^{-11}\ \text{F/m}}{10 \times 10^{-9}\ \text{m}}$

$= 3.45 \times 10^{-3}$ F/m^2

$= 3.45$ fF/µm^2

$C_{ov} = W L_{ov} C_{ox}$

$= 10 \times 0.05 \times 3.45 = 1.72$ fF

$C_{gs} = \dfrac{2}{3} W L C_{ox} + C_{ov}$

$= \dfrac{2}{3} \times 10 \times 1 \times 3.45 + 1.72$

$= 24.72$ fF

$C_{gd} = C_{ov} = 1.72$ fF

$C_{sb} = \dfrac{C_{sb0}}{\sqrt{1 + \dfrac{V_{SB}}{V_0}}} = \dfrac{10}{\sqrt{1 + \dfrac{1}{0.6}}} = 6.1$ fF

$C_{db} = \dfrac{C_{db0}}{\sqrt{1 + \dfrac{V_{DB}}{V_0}}} = \dfrac{10}{\sqrt{1 + \dfrac{2+1}{0.6}}} = 4.1$ fF

Ex: 10.4 $g_m = \sqrt{2k_n'(W/L)I_D}$

$= \sqrt{2 \times 0.16 \times (10/1) \times 0.1} = 0.566$ mA/V

$f_T = \dfrac{g_m}{2\pi(C_{gs} + C_{gd})}$

$= \dfrac{0.566 \times 10^{-3}}{2\pi(24.72 + 1.72) \times 10^{-15}}$

$f_T = 3.4$ GHz

Ex: 10.5 $C_{de} = \tau_F g_m$

where

$\tau_F = 20$ ps

$g_m = \dfrac{I_C}{V_T} = \dfrac{1\ \text{mA}}{0.025\ \text{V}} = 40$ mA/V

Thus,

$C_{de} = 20 \times 10^{-12} \times 40 \times 10^{-3} = 0.8 \text{ pF}$

$C_{je} \simeq 2C_{je0}$

$= 2 \times 20 = 40 \text{ fF}$

$C_\pi = C_{de} + C_{je}$

$= 0.8 + 0.04 = 0.84 \text{ pF}$

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

$$= \frac{20}{\left(1 + \frac{2}{0.5}\right)^{0.33}} = 12 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(0.84 + 0.012) \times 10^{-12}} = 7.47 \text{ GHz}$$

Ex: 10.6 $|h_{fe}| = 10$ at $f = 50$ MHz

Thus,

$f_T = 10 \times 50 = 500$ MHz

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$= 12.7$ pF

$C_\pi = 12.7 - 2 = 10.7$ pF

Ex: 10.7 $C_\pi = C_{de} + C_{je}$

$10.7 = C_{de} + 2$

$\Rightarrow C_{de} = 8.7$ pF

Since C_{de} is proportional to g_m and hence I_C, at $I_C = 0.1$ mA,

$C_{de} = 0.87$ pF

and

$C_\pi = 0.87 + 2 = 2.87$ pF

$$f_T = \frac{4 \times 10^{-3}}{2\pi(2.87 + 2) \times 10^{-12}} = 130.7 \text{ MHz}$$

Ex: 10.8 $A_M = -\dfrac{R_G}{R_G + R_{\text{sig}}} g_m R'_L$

$$= -\frac{4.7}{4.7 + 0.01} \times 7.14 = -7.12 \text{ V/V}$$

$$f_H = \frac{1}{2\pi C_{\text{in}}(R_{\text{sig}} \parallel R_G)}$$

$$= \frac{1}{2\pi \times 4.26 \times 10^{-12}(0.01 \parallel 4.7) \times 10^6}$$

$= 3.7$ MHz

Ex: 10.9 $f_H = \dfrac{1}{2\pi C_{\text{in}}(R_{\text{sig}} \parallel R_G)}$

$$1 \times 10^6 = \frac{1}{2\pi C_{\text{in}}(0.1 \parallel 4.7) \times 10^6}$$

$\Rightarrow C_{\text{in}} = 1.625$ pF

But,

$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$1.625 = 1 + C_{gd}(1 + 7.14)$

$\Rightarrow C_{gd} = 0.08$ pF

Ex: 10.10 To reduce the midband gain to half the value found, we reduce R'_L by the same factor, thus

$R'_L = 1.5$ kΩ

But,

$R'_L = r_o \parallel R_C \parallel R_L$

$1.5 = 100 \parallel 8 \parallel R_L$

$\Rightarrow R_L = 1.9$ kΩ

$C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 7 + 1(1 + 40 \times 1.5)$

$= 68$ pF

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 68 \times 10^{-12} \times 1.65 \times 10^3}$$

$= 1.42$ MHz

Thus, by accepting a reduction in gain by a factor of 2, the bandwidth is increased by a factor of $1.42/0.754 = 1.9$, approximately the same factor as the reduction in gain.

Ex: 10.11 $f_t = |A_M| f_H$

$$2 \times 10^9 = \frac{12.5}{2\pi(C_L + C_{gd}) \times 10 \times 10^3}$$

$\Rightarrow C_L + C_{gd} = 99.5$ fF

$C_L = 99.5 - 5 = 94.5$ fF

Ex: 10.12 $T(s) = \dfrac{1000}{1 + \dfrac{s}{2\pi \times 10^5}}$

$\text{GB} = 1000 \times 100 \times 10^3 = 10^8$ Hz

Ex: 10.13 $T(j\omega) = \dfrac{A_M}{\left(1 + j\dfrac{\omega}{\omega_{P1}}\right)\left(1 + j\dfrac{\omega}{\omega_{P2}}\right)}$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega}{\omega_{P2}}\right)^2\right]}}$$

$$|T| = \frac{|A_M|}{\sqrt{\left[1 + \left(\frac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega}{k\omega_{P1}}\right)^2\right]}}$$

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{k\omega_{P1}}\right)^2\right]$$

For $\omega_H = 0.9\omega_{P1}$,

$$2 = (1 + 0.81)\left(1 + \frac{0.81}{k^2}\right)$$

$\Rightarrow k = 2.78$

For $\omega_H = 0.99\omega_{P1}$,

$$2 = (1 + 0.99^2)\left(1 + \frac{0.99^2}{k^2}\right)$$

$\Rightarrow k = 9.88$

Ex: 10.14 $2 = \left[1 + \left(\dfrac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega_H}{k\omega_{P1}}\right)^2\right]$

For $k = 1$,

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]$$

$$= \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]^2$$

$\Rightarrow \omega_H = 0.64\omega_{P1}$

The approximate value using Eq. (10.77) is

$$\omega_H \simeq 1\bigg/\sqrt{\frac{2}{\omega_{P1}^2}} = \frac{\omega_{P1}}{\sqrt{2}}$$

$= 0.71\omega_{P1}$

For $k = 2$, the exact value of ω_H is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{2\omega_{P1}}\right)^2\right]$$

$$= 1 + \frac{5}{4}\left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \frac{1}{4}\left(\frac{\omega_H}{\omega_{P1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{P1}}\right)^4 + 5\left(\frac{\omega_H}{\omega_{P1}}\right)^2 - 4 = 0$$

$\Rightarrow \omega_H = 0.84\omega_{P1}$

The approximate value of ω_H is obtained from Eq. (10.77),

$$\omega_H \simeq 1\bigg/\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{4\omega_{P1}^2}}$$

$\Rightarrow \omega_H = 0.89\omega_{P1}$

For $k = 4$, the exact value of ω_H is obtained from

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{4\omega_{P1}}\right)^2\right]$$

$$= 1 + \frac{17}{16}\left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \frac{1}{16}\left(\frac{\omega_H}{\omega_{P1}}\right)^4$$

$$\left(\frac{\omega_H}{\omega_{P1}}\right)^4 + 17\left(\frac{\omega_H}{\omega_{P1}}\right)^2 - 16 = 0$$

$\Rightarrow \omega_H = 0.95\omega_{P1}$

The approximate value of ω_H is found from Eq. (10.77):

$$\omega_H \simeq 1\bigg/\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{16\omega_{P1}^2}}$$

$= 0.97\omega_{P1}$

Ex: 10.15 $C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 20 + 5(1 + 1.25 \times 10)$

$= 87.5$ fF

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 87.5 \times 10^{-15} \times 10 \times 10^3}$$

$= 181.9$ MHz

This is greater than the value obtained in Example 10.8, $f_H = 135.5$ MHz, by 34%. The value obtained in Example 10.8 is a better estimate of f_H as it takes into account the effect of C_L.

Ex: 10.16
$|A_M| = g_m R'_L = 1.25 \times 10 = 12.5$ V/V

$\text{GB} = |A_M| f_H$

$= 12.5 \times 135.5 = 1.69$ GHz

Ex: 10.17 $|A_M| = \dfrac{1}{2} \times 12.5 = 6.25$ V/V

$R_{gs} = 10$ kΩ

$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 10(1 + 6.25) + 5 = 77.5\ \text{k}\Omega$

$R_{C_L} = R'_L = 5\ \text{k}\Omega$

$\tau_{gs} = C_{gs}R_{gs} = 20 \times 10^{-15} \times 10 \times 10^3 = 200\ \text{ps}$

$\tau_{gd} = C_{gd}R_{gd} = 5 \times 10^{-15} \times 77.5 \times 10^3 = 387.5\ \text{ps}$

$\tau_{C_L} = C_L R_{C_L} = 25 \times 10^{-15} \times 5 \times 10^3 = 125\ \text{ps}$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 200 + 387.5 + 125$

$= 712.5\ \text{ps}$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 712.5 \times 10^{-12}} = 223.4\ \text{MHz}$$

$\text{GB} = 6.25 \times 223.4 = 1.4\ \text{GHz}$

Ex: 10.18 $g_m = \sqrt{2\mu_n C_{ox}\dfrac{W}{L}I_D}$

Since I_D is increased by a factor of 4, g_m doubles:

$g_m = 2 \times 1.25 = 2.5\ \text{mA/V}$

Since R'_L is $r_o/2$, increasing I_D by a factor of four results in r_o and hence R'_L decreasing by a factor of 4, thus

$$R'_L = \frac{1}{4} \times 10 = 2.5\ \text{k}\Omega$$

$|A_M| = g_m R'_L = 2.5 \times 2.5 = 6.25\ \text{V/V}$

$R_{gs} = R'_{\text{sig}} = 10\ \text{k}\Omega$

$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 10(1 + 6.25) + 2.5$

$= 75\ \text{k}\Omega$

$R_{C_L} = R'_L = 2.5\ \text{k}\Omega$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= C_{gs}R'_{\text{sig}} + C_{gd}R_{gd} + C_L R_{C_L}$

$= 20 \times 10^{-15} \times 10 \times 10^3 + 5 \times 10^{-15} \times 75$

$\quad \times 10^3 + 25 \times 10^{-15} \times 2.5 \times 10^3$

$= 200 + 375 + 62.5$

$= 637.5\ \text{ps}$

$$f_H = \frac{1}{2\pi \times 637.5 \times 10^{-12}} = 250\ \text{MHz}$$

$\text{GB} = |A_M| f_H$

$= 6.25 \times 250 = 1.56\ \text{GHz}$

Ex: 10.19 (a) $g_m = 40\ \text{mA/V}$

$$r_\pi = \frac{200}{40} = 5\ \text{k}\Omega$$

$$r_{on} = \frac{V_{An}}{I} = \frac{130}{1} = 130\ \text{k}\Omega$$

$$r_{op} = \frac{|V_{Ap}|}{I} = \frac{50}{1} = 50\ \text{k}\Omega$$

$R'_L = r_{on} \parallel r_{op} = 130 \parallel 50 = 36.1\ \text{k}\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}} g_m R'_L$$

$$= -\frac{5}{5 + 0.2 + 36} \times 40 \times 36.1$$

$= -175\ \text{V/V}$

(b) $C_{\text{in}} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 16 + 0.3(1 + 40 \times 36.1)$

$= 450\ \text{pF}$

$R'_{\text{sig}} = r_\pi \parallel (r_x + R_{\text{sig}})$

$= 5 \parallel (0.2 + 36) = 4.39\ \text{k}\Omega$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 450 \times 10^{-12} \times 4.39 \times 10^3}$$

$= 80.6\ \text{kHz}$

(c) $R_\pi = R'_{\text{sig}} = 4.39\ \text{k}\Omega$

$R_\mu = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 4.39(1 + 40 \times 36.1) + 36.1$

$= 6.38\ \text{M}\Omega$

$R_{CL} = R'_L = 36.1\ \text{k}\Omega$

$\tau_H = C_\pi + C_\mu R_\mu + C_L R_{CL}$

$= 16 \times 4.39 + 0.3 \times 6.38 \times 10^3 + 5 \times 36.1$

$= 70.2 + 1914 + 180.5$

$= 2164.7\ \text{ns}$

$$f_H = \frac{1}{2\pi \times 2164.7 \times 10^{-9}}$$

$= 73.5\ \text{kHz}$

(d) $f_Z = \dfrac{g_m}{2\pi C_\mu}$

$$= \frac{40 \times 10^{-3}}{2\pi \times 0.3 \times 10^{-12}} = 21.2\ \text{GHz}$$

(e) $\text{GB} = 175 \times 73.5 = 12.9\ \text{MHz}$

Ex: 10.20 $R_{\text{in}} = \dfrac{R_L + r_o}{1 + g_m r_o}$

$= \dfrac{500 + 20}{1 + 25} = 20\ \text{k}\Omega$

$G_v = \dfrac{R_L}{R_{\text{sig}} + R_{\text{in}}} = \dfrac{500}{10 + 20} = 16.7\ \text{V/V}$

$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}} = 10 \parallel 20 = 6.7\ \text{k}\Omega$

$R_{gd} = R_L \parallel R_o$

$= 500 \parallel 280 = 179.5\ \text{k}\Omega$

$\tau_H = C_{gs}R_{gs} + (C_{gd} + C_L)R_{gd}$

$= 20 \times 10^{-15} \times 6.7 \times 10^3 + (5 + 25) \times 10^{-15}$

$\times 179.5 \times 10^3$

$= 134 + 5385$

$= 5519\ \text{ps}$

$f_H = \dfrac{1}{2\pi \times 5519 \times 10^{-12}} = 28.8\ \text{MHz}$

Thus, while the midband gain has been increased substantially (by a factor of 9.7), the bandwidth has been substantially lowered (by a factor of 9.4). Thus, the high-frequency advantage of the CG amplifier is completely lost!

Ex: 10.21 (a) $A_{\text{CS}} = -g_m(R_L \parallel r_o)$

$= -g_m(r_o \parallel r_o) = -\dfrac{1}{2}g_m r_o$

$= -\dfrac{1}{2} \times 40 = -20\ \text{V/V}$

$A_{\text{cascode}} = -g_m(R_L \parallel R_o)$

$= -g_m(r_o \parallel g_m r_o r_o)$

$\simeq -g_m r_o = -40\ \text{V/V}$

Thus,

$\dfrac{A_{\text{cascode}}}{A_{\text{CS}}} = 2$

(b) For the CS amplifier,

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd}$

where

$R_{gs} = R_{\text{sig}}$

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$\simeq R_{\text{sig}}(1 + g_m R'_L)$

$= R_{\text{sig}}\left(1 + \dfrac{1}{2}g_m r_o\right)$

$= R_{\text{sig}}\left(1 + \dfrac{1}{2} \times 40\right) = 21R_{\text{sig}}$

$\tau_H = C_{gs}R_{\text{sig}} + C_{gd} \times 21R_{\text{sig}}$

$= C_{gs}R_{\text{sig}} + 0.25C_{gs} \times 21R_{\text{sig}}$

$= 6.25C_{gs}R_{\text{sig}}$

$f_H = \dfrac{1}{2\pi \times 6.25C_{gs}R_{\text{sig}}}$

For the cascode amplifier,

$\tau_H \simeq R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1}R_{d1})]$

where

$R_{d1} = r_{o1} \parallel R_{\text{in}2} = r_o \parallel \dfrac{r_o + r_o}{g_m r_o}$

$= r_o \parallel \dfrac{2}{g_m} = \dfrac{\dfrac{2}{g_m}r_o}{\dfrac{2}{g_m} + r_o}$

$= \dfrac{2r_o}{2 + g_m r_o} = \dfrac{2r_o}{2 + 40} = \dfrac{r_o}{21}$

$\tau_H = C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \dfrac{g_m r_o}{21}\right)\right]$

$= C_{gs}R_{\text{sig}}\left[1 + 0.25\left(1 + \dfrac{40}{21}\right)\right]$

$= 1.73C_{gs}R_{\text{sig}}$

$f_H = \dfrac{1}{2\pi \times 1.73\ C_{gs}R_{\text{sig}}}$

Thus,

$\dfrac{f_H(\text{cascode})}{f_H(\text{CS})} = \dfrac{6.25}{1.73} = 3.6$

(c) $\dfrac{f_t(\text{cascode})}{f_t(\text{CS})} = 2 \times 3.6 = 7.2$

Ex: 10.22 $g_m = 40\ \text{mA/V}$

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{200}{40} = 5\ \text{k}\Omega$

$R_{\text{in}} = r_\pi + r_x = 5 + 0.2 = 5.2\ \text{k}\Omega$

$A_0 = g_m r_o$

$= 40 \times 130 = 5200$

$R_{o1} = r_{o1} = 130\ \text{k}\Omega$

$R_{\text{in}2} = r_{e2}\dfrac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)}$

$= 25\dfrac{130 + 50}{130 + \dfrac{50}{201}}$

$= 35\ \Omega$

$R_o \simeq \beta_2 r_{o2} = 200 \times 130 = 26\ \text{M}\Omega$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m (R_o \parallel R_L)$$

$$= -\frac{5}{5 + 0.2 + 36} 40(26000 \parallel 50)$$

$$A_M = -242 \text{ V/V}$$

$$R'_{sig} = r_{\pi 1} \parallel (r_{x1} + R_{sig})$$

$$= 5 \parallel (0.2 + 36) = 4.39 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{sig} = 4.39 \text{ k}\Omega$$

$$R_{c1} = r_{o1} \parallel R_{in2}$$

$$= 130 \text{ k}\Omega \parallel 35\ \Omega \simeq 35\ \Omega$$

$$R_{\mu 1} = R'_{sig}(1 + g_{m1} R_{c1}) + R_{c1}$$

$$= 4.39(1 + 40 \times 0.035) + 0.035$$

$$= 10.6 \text{ k}\Omega$$

$$\tau_H = C_{\pi 1} R_{\pi 1} + C_{\mu 1} R_{\mu 1} + C_{\pi 2} R_{c1} + (C_L + C_{\mu 2})(R_L \parallel R_o)$$

$$= 16 \times 4.39 + 0.3 \times 10.6 + 16 \times 0.035 + (5 + 0.3)(50 \parallel 26{,}000)$$

$$= 70.24 + 3.18 + 0.56 + 264.5$$

$$= 338.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 338.5 \times 10^{-9}} = 470 \text{ kHz}$$

$$f_t = |A_M| f_H = 242 \times 470 = 113.8 \text{ MHz}$$

Thus, in comparison to the CE amplifier of Exercise 10.19, we see that $|A_M|$ has increased from 175 V/V to 242 V/V, f_H has increased from 73.5 kHz to 470 kHz, and f_t has increased from 12.9 MHz to 113.8 MHz.

To have f_H equal to 1 MHz,

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 1 \times 10^6} = 159.2 \text{ ns}$$

Thus,

$$159.2 = 70.24 + 3.18 + 0.56 + (C_L + C_\mu)(50 \parallel 26000)$$

$$\Rightarrow C_L + C_\mu = 1.71 \text{ pF}$$

Thus, C_L must be reduced to 1.41 pF.

Ex: 10.23 From Eq. (10.120), we obtain

$$R_{gs} = \frac{R_{sig}}{g_m R'_L + 1} + \frac{R'_L}{g_m R'_L + 1} = \frac{R_{sig} + R'_L}{g_m R'_L + 1}$$

$$R_{gd} = R_{sig}$$

$$R_{C_L} = \frac{R'_L}{g_m R'_L + 1}$$

Ex: 10.24 From Example 10.11, we get

$$\tau_H = b_1 = 104 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 104 \times 10^{-12}} = 1.53 \text{ GHz}$$

This is lower than the exact value found in Example 10.11 (i.e., 1.86 GHz) by about 18%, still not a bad estimate!

Ex: 10.25 $g_m = 40$ mA/V

$$r_e = 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$R'_{sig} = R_{sig} + r_x = R_{sig} = 1 \text{ k}\Omega$$

$$R'_L = R_L \parallel r_o = 1 \parallel 100 = 0.99 \text{ k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \dfrac{R'_{sig}}{\beta + 1}}$$

$$= \frac{0.99}{0.99 + 0.025 + (1/101)} = 0.97 \text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$$= 15.9 \text{ pF}$$

$$C_\mu = 2 \text{ pF}$$

$$C_\pi = 13.9 \text{ pF}$$

$$f_Z = \frac{1}{2\pi C_\pi r_e} = \frac{1}{2\pi \times 13.9 \times 10^{-12} \times 25}$$

$$= 458 \text{ MHz}$$

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \dfrac{R'_L}{r_e}\right)\right] R'_{sig} + \left[C_\pi + C_L\left(1 + \dfrac{R'_{sig}}{r_\pi}\right)\right] R'_L}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{sig}}{r_\pi}}$$

$$= \frac{\left[13.9 + 2\left(1 + \dfrac{0.99}{0.025}\right)\right] \times 1 + (13.9 + 0)0.99}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$$= 2.66 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{C_\pi C_\mu R'_L R'_{sig}}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{sig}}{r_\pi}} = \frac{13.9 \times 2 \times 0.99 \times 1}{1 + \dfrac{0.99}{0.025} + \dfrac{1}{2.5}}$$

$$= 0.671 \times 10^{-18}$$

ω_{P1} and ω_{P2} are the roots of the equation

$1 + b_1 s + b_2 s^2 = 0$

Solving we obtain,

$f_{P1} = 67$ MHz

$f_{P2} = 563$ MHz

Since $f_{P1} \ll f_{P2}$,

$f_H \simeq f_{P1} = 67$ MHz

Ex: 10.26 (a) $I_{D1,2} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)_{1,2} V_{OV}^2$

$0.4 = \dfrac{1}{2} \times 0.2 \times 100 V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.4}{0.2} = 4$ mA/V

(b) $A_d = g_m(R_D \parallel r_o)$

where

$r_o = \dfrac{V_A}{I_D} = \dfrac{20}{0.4} = 50\ \text{k}\Omega$

$A_d = 4(5 \parallel 50) = 4 \times 4.545$

$= 18.2$ V/V

(c) $f_H = \dfrac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$= \dfrac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 4.545 \times 10^3}$

$= 292$ MHz

(d) $\tau_{gs} = C_{gs}R_{\text{sig}} = 50 \times 10 = 500$ ps

$\tau_{gd} = C_{gd}R_{gd} = C_{gd}[R_{\text{sig}}(1 + g_m R_L') + R_L']$

$= 10[10(1 + 18.2) + 4.545]$

$= 1965.5$ ps

$\tau_{C_L} = (C_L + C_{db})R_L' = 110 \times 4.545 = 500$ ps

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 500 + 1965.5 + 500 = 2965.5$ ps

$f_H = \dfrac{1}{2\pi \times 2965.5 \times 10^{-12}}$

$= 53.7$ MHz

Ex: 10.27 $f_Z = \dfrac{1}{2\pi R_{SS} C_{SS}}$

$= \dfrac{1}{2\pi \times 75 \times 10^3 \times 0.4 \times 10^{-12}}$

$= 5.3$ MHz

Thus, the 3-dB frequency of the CMRR is 5.3 MHz.

Ex: 10.28 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$g_{m1,2} = \dfrac{0.5}{0.025} = 20$ mA/V

$r_{o2} = r_{o4} = \dfrac{100}{0.5} = 200\ \text{k}\Omega$

$A_d = 20(200 \parallel 200) = 2000$ V/V

The dominant high-frequency pole is that introduced at the output node,

$f_H = \dfrac{1}{2\pi C_L(r_{o2} \parallel r_{o4})}$

$= \dfrac{1}{2\pi \times 2 \times 10^{-12} \times 100 \times 10^3}$

$= 0.8$ MHz

Ex: 10.29 (a) $A_M = -g_m R_L'$

where

$R_L' = R_L \parallel r_o = 20 \parallel 20 = 10\ \text{k}\Omega$

$A_M = -2 \times 10 = -20$ V/V

$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_L'$

$= C_{gs}R_{\text{sig}} + C_{gd}[R_{\text{sig}}(1 + g_m R_L') + R_L'] + C_L R_L'$

$= 20 \times 20 + 5[20(1 + 20) + 10] + 5 \times 10$

$= 400 + 2150 + 50$

$= 2600$ ps

$f_H = \dfrac{1}{2\pi\tau_H}$

$= \dfrac{1}{2\pi \times 2600 \times 10^{-12}}$

$= 61.2$ MHz

$\text{GB} = |A_M| f_H$

$= 20 \times 61.2$

$= 1.22$ GHz

(b) $G_m = \dfrac{g_m}{1 + g_m R_s} = \dfrac{2}{1 + 2} = 0.67$ mA/V

$R_o \simeq r_o(1 + g_m R_s)$

$= 20 \times 3 = 60\ \text{k}\Omega$

$R_L' = R_L \parallel R_o = 20 \parallel 60 = 15\ \text{k}\Omega$

$A_M = -G_m R_L'$

$= -0.67 \times 15 = -10$ V/V

$R_{gd} = R_{\text{sig}}(1 + G_m R_L') + R_L'$

$= 20(1 + 10) + 15$

$= 235\ \text{k}\Omega$

$$R_{C_L} = R'_L = 15\ \text{k}\Omega$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}}R_s/(r_o + R_L)}{1 + g_m R_s\left(\dfrac{r_o}{r_o + R_L}\right)}$$

where

$$R_s = \frac{2}{g_m} = 1\ \text{k}\Omega$$

$$R_{gs} = \frac{20 + 1 + \dfrac{20 \times 1}{20 + 20}}{1 + 2 \times \dfrac{20}{20 + 20}}$$

$$= 10.75\ \text{k}\Omega$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$$

$$= 20 \times 10.75 + 5 \times 235 + 5 \times 15$$

$$= 215 + 1175 + 75 = 1465\ \text{ps}$$

$$f_H = \frac{1}{2\pi \times 1465 \times 10^{-12}} = 109\ \text{MHz}$$

$$\text{GB} = 10 \times 109 = 1.1\ \text{GHz}$$

Ex: 10.30 Refer to Fig. 10.42(b).

$$A_M = \frac{2r_\pi}{2r_\pi + R_{\text{sig}}} \times \frac{1}{2} \times g_m R_L$$

where

$$g_m = 20\ \text{mA/V}$$

$$r_\pi = \frac{100}{20} = 5\ \text{k}\Omega$$

$$A_M = \frac{10}{10 + 10} \times \frac{1}{2} \times 20 \times 10 = 50\ \text{V/V}$$

$$f_{P1} = \frac{1}{2\pi\left(\dfrac{C_\pi}{2} + C_\mu\right)(2r_\pi \parallel R_{\text{sig}})}$$

$$= \frac{1}{2\pi\left(\dfrac{6}{2} + 2\right) \times 10^{-12}(10 \parallel 10) \times 10^3}$$

$$= 6.4\ \text{MHz}$$

$$f_{P2} = \frac{1}{2\pi C_\mu R_L}$$

$$= \frac{1}{2\pi \times 2 \times 10^{-12} \times 10 \times 10^3}$$

$$= 8\ \text{MHz}$$

$$T(s) = \frac{50}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

$$|T(j\omega)| = \frac{50}{\sqrt{\left[1 + \left(\dfrac{\omega}{\omega_{P1}}\right)^2\right]\left[1 + \left(\dfrac{\omega}{\omega_{P2}}\right)^2\right]}}$$

At $\omega = \omega_H$, $|T| = 50/\sqrt{2}$, thus

$$2 = \left[1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\right]\left[1 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2\right]$$

$$= 1 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2 + \left(\frac{\omega_H}{\omega_{P2}}\right)^2 + \left(\frac{\omega_H}{\omega_{P1}}\right)^2\left(\frac{\omega_H}{\omega_{P2}}\right)^2$$

$$\frac{\omega_H^4}{\omega_{P1}^2\omega_{P2}^2} + \omega_H^2\left(\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{f_{P1}^2 f_{P2}^2} + f_H^2\left(\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}\right) - 1 = 0$$

$$\frac{f_H^4}{6.4^2 \times 8^2} + f_H^2\left(\frac{1}{6.4^2} + \frac{1}{8^2}\right) - 1 = 0$$

$\Rightarrow f_H = 4.6$ MHz (Exact value)

Using Eq. (10.164), an approximate value for f_H can be obtained:

$$f_H \simeq 1\bigg/\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1\bigg/\sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5\ \text{MHz}$$

10.1 Refer to Fig. 10.3(b).

$$\frac{V_g}{V_{sig}} = \frac{R_G}{R_G + R_{sig} + \dfrac{1}{sC_{C1}}}$$

where

$R_G = R_{G1} \parallel R_{G2} = 2\ \text{M}\Omega \parallel 1\ \text{M}\Omega = 667\ \text{k}\Omega$

and

$R_{sig} = 200\ \text{k}\Omega$

$$\frac{V_g}{V_{sig}} = \frac{R_G}{R_G + R_{sig}} \frac{s}{s + \dfrac{1}{C_{C1}(R_G + R_{sig})}}$$

Thus,

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{sig})}$$

We required

$f_{P1} \le 10$ Hz

thus we select C_{C1} so that

$$\frac{1}{2\pi C_{C1}(R_G + R_{sig})} \le 10$$

$$C_{C1} \ge \frac{1}{2\pi \times 10 \times (667 + 200) \times 10^3} = 18.4\ \text{nF}$$

$\Rightarrow C_{C1} = 20$ nF

10.2 Refer to Fig. 10.3(b).

$$V_o = -I_d \frac{R_D}{R_D + \dfrac{1}{sC_{C2}} + R_L} \times R_L$$

$$\frac{V_o}{I_d} = -\frac{R_D R_L}{R_D + R_L} \frac{s}{s + \dfrac{1}{C_{C2}(R_D + R_L)}}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

where

$R_D = 10\ \text{k}\Omega$ and $R_L = 10\ \text{k}\Omega$

To make $f_{P3} \le 10$ Hz,

$$\frac{1}{2\pi C_{C2}(R_D + R_L)} \le 10$$

$$\Rightarrow C_{C2} \ge \frac{1}{2\pi \times 10 \times (10 + 10) \times 10^3} = 0.8\ \mu\text{F}$$

Select, $C_{C2} = 0.8\ \mu$F.

10.3 Refer to Fig. 10.3(b).

$$I_s = \frac{V_g}{\dfrac{1}{g_m} + Z_S}$$

$$I_s = \frac{g_m V_g Y_S}{Y_S + g_m}$$

$$\frac{I_s}{V_g} = \frac{g_m\left(\dfrac{1}{R_S} + sC_S\right)}{g_m + \dfrac{1}{R_S} + sC_S}$$

$$= g_m \frac{s + 1/C_S R_S}{s + \dfrac{g_m + 1/R_S}{C_S}}$$

Thus,

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

where

$g_m = 5$ mA/V and $R_S = 1.8\ \text{k}\Omega$

To make $f_{P2} \le 100$ Hz,

$$\frac{g_m + 1/R_S}{2\pi C_S} \le 100$$

$$\Rightarrow C_S \ge \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 100} = 8.8\ \mu\text{F}$$

Select $C_S = 10\ \mu$F.

Thus,

$$f_{P2} = \frac{5 \times 10^{-3} + (1/1.8 \times 10^3)}{2\pi \times 10 \times 10^{-6}} = 88.4\ \text{Hz}$$

and

$$f_Z = \frac{1}{2\pi \times 10 \times 10^{-6} \times 1.8 \times 10^3} = 8.84\ \text{Hz}$$

10.4 Refer to Fig. 10.3.

$$A_M = -\frac{R_G}{R_G + R_{sig}} \times g_m(R_D \parallel R_L)$$

where

$R_G = R_{G1} \parallel R_{G2} = 47\ \text{M}\Omega \parallel 10\ \text{M}\Omega$

$= 8.246\ \text{M}\Omega$

$R_{sig} = 100\ \text{k}\Omega,\ g_m = 5\ \text{mA/V},\ R_D = 4.7\ \text{k}\Omega$ and $R_L = 10\ \text{k}\Omega$.

Thus,

$$A_M = -\frac{8.426}{8.426 + 0.1} \times 5(4.7 \parallel 10)$$

$= -15.8$ V/V

$$f_{P1} = \frac{1}{2\pi C_{C1}(R_G + R_{sig})}$$

$$= \frac{1}{2\pi \times 0.01 \times 10^{-6}(8.426 + 0.1) \times 10^6}$$

$= 1.9$ Hz

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$= \frac{5\times10^{-3} + 0.5\times10^{-3}}{2\pi \times 10\times10^{-6}} = 87.5 \text{ Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S}$$

$$= \frac{1}{2\pi\times10\times10^{-6}\times2\times10^3} = 8 \text{ Hz}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

$$= \frac{1}{2\pi\times1\times10^{-6}(4.7+10)\times10^3} = 10.8 \text{ Hz}$$

Since

$f_{P2} \gg f_{P1}, f_{P3}, f_Z$,

$f_L \simeq f_{P2} = 87.5$ Hz

10.5

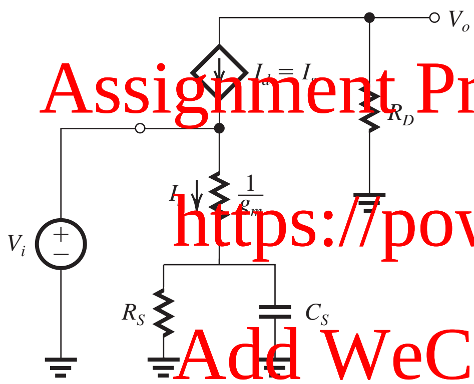


Figure 1

Replacing the MOSFET with its T model results in the circuit shown in Fig. 1.

(a) $A_M \equiv \dfrac{V_o}{V_i} = -g_m R_D$

$-20 = -2 \times R_D$

$\Rightarrow R_D = 10$ kΩ

(b) $f_P = \dfrac{g_m + 1/R_S}{2\pi C_S}$

$$100 = \frac{2\times10^{-3} + (1/4.5\times10^3)}{2\pi C_S}$$

$\Rightarrow C_S = 3.53$ μF

(c) $f_Z = \dfrac{1}{2\pi C_S R_S} =$

$$\frac{1}{2\pi\times3.53\times10^{-6}\times4.5\times10^3} = 10 \text{ Hz}$$

(d) Since $f_P \gg f_Z$,

$f_L \simeq f_P = 100$ Hz

(e) The Bode plot for the gain is shown in Fig. 2.

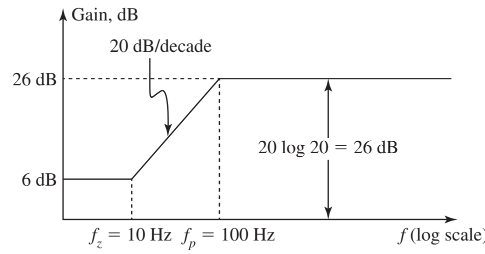


Figure 2

Observe that the dc gain is 6 dB, i.e. 2 V/V. This makes perfect sense since from Fig. 1 we see that at dc, capacitor C_S behaves as open circuit and the gain becomes

$$\text{DC gain} = -\frac{R_D}{\dfrac{1}{g_m} + R_S} = -\frac{10 \text{ k}\Omega}{\left(\dfrac{1}{2} + 4.5\right)}$$

$= -2$ V/V

10.6 See figure on next page. Replacing the MOSFET with its T model results in the circuit shown in the figure.

$$A_M = -\frac{R_G}{R_G + R_{sig}} \times g_m(R_D \parallel R_L)$$

$$= -\frac{2}{2+0.5} \times 3(20 \parallel 10)$$

$= -16$ V/V

To minimize the total capacitance we select C_S so as to place f_{P2} (usually the highest-frequency low-frequency pole) at 100 Hz. Thus,

$$100 = \frac{g_m}{2\pi C_S}$$

$$= \frac{3\times10^{-3}}{2\pi C_S}$$

$\Rightarrow C_S = 4.8$ μF

Select $C_S = 5$ μF.

Of the two remaining poles, the one caused by C_{C2} has associated relatively low-valued resistances (R_D and R_L are much lower than R_G), thus to minimize the total capacitance we place f_{P3} at 10 Hz and f_{P1} at 1 Hz. Thus,

$$10 = \frac{1}{2\pi C_{C2}(R_D + R_L)}$$

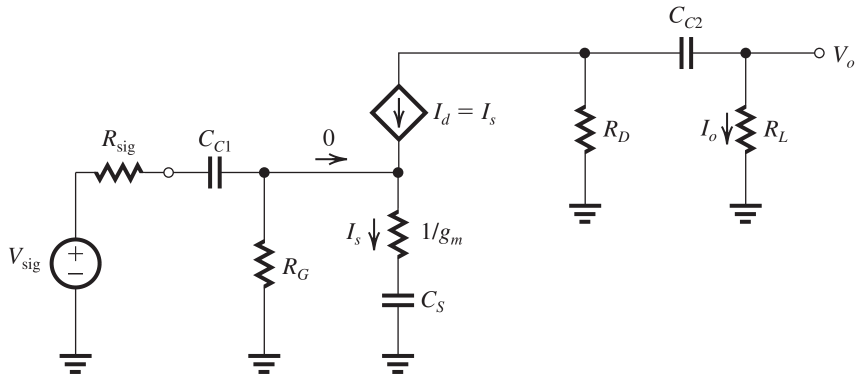
$$\equiv \frac{1}{2\pi C_{C2}(20+10)\times10^3}$$

$\Rightarrow C_{C2} = 0.53$ μF

Select $C_{C2} = 1$ μF.

$$1 = \frac{1}{2\pi C_{C1}(R_G + R_{sig})}$$

This figure belongs to Problem 10.6.



$$1 = \frac{1}{2\pi C_{C1}(2 + 0.5) \times 10^6}$$

$$\Rightarrow C_{C1} = 63.7 \text{ nF}$$

Select $C_{C1} = 100$ nF $= 0.1$ μF.

With the selected capacitor values, we obtain

$$f_{P1} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 2.5 \times 10^6} = 0.64 \text{ Hz}$$

$$f_{P2} = \frac{3 \times 10^{-3}}{2\pi \times 5 \times 10^{-6}} = 95.5 \text{ Hz}$$

$$f_Z = 0 \text{ (dc)}$$

$$f_{P3} = \frac{1}{2\pi \times 1 \times 10^{-6}(20 + 10) \times 10^3} = 5.3 \text{ Hz}$$

Since $f_{P2} \gg f_{P1}$ and f_{P3} we have

$$f_L \simeq f_{P2} = 95.5 \text{ Hz}$$

10.7 The amplifier in Fig. P10.7 will have the equivalent circuit in Fig. 10.9 except with $R_E = \infty$ (i.e. omitted). Also, the equivalent circuits in Fig. 10.10 can be used to determine the three short-circuit time constants, again with $R_E = \infty$. Since the amplifier is operating at $I_C \simeq I_E = 100$ μA $= 0.1$ mA and $\beta = 100$,

$$r_e = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250 \ \Omega$$

$$g_m = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25 \text{ k}\Omega$$

Using the equivalent circuit in Fig. 10.10(b), we get

$$\tau_{CE} = C_E\left(r_e + \frac{R_B \parallel R_{sig}}{\beta + 1}\right)$$

To make C_E responsible for 80% of f_L, we use

$$\frac{1}{\tau_{CE}} = 0.8\,\omega_L = 0.8 \times 2\pi f_L$$

$$\Rightarrow \tau_{CE} = \frac{1}{0.8 \times 2\pi \times 100} \simeq 2 \text{ ms}$$

Thus,

$$C_E\left[250 + \frac{(200 \parallel 20) \times 10^3}{101}\right] = 2 \times 10^{-3}$$

$$\Rightarrow C_E = 4.65 \ \mu\text{F}$$

Select $C_E = 5$ μF.

Using the information in Fig. 10.10(a), we determine τ_{C1} as

$$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{sig}]$$

To make the contribution of C_{C1} to the determination of f_L equal to 10%, we use

$$\frac{1}{\tau_{C1}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C1} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$$C_{C1}[(200 \parallel 25) \times 10^3 + 20 \times 10^3] = 15.92 \times 10^{-3}$$

$$\Rightarrow C_{C1} = 0.38 \ \mu\text{F}$$

Select $C_{C1} = 0.5$ μF.

For C_{C2} we use the information in Fig. 10.10(c) to determine τ_{C2}:

$$\tau_{C2} = C_{C2}(R_C + R_L)$$

To make the contribution of C_{C2} to the determination of f_L equal to 10%, we use

$$\frac{1}{\tau_{C2}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$$\Rightarrow \tau_{C2} = \frac{1}{0.1 \times 2\pi \times 100} = 15.92 \text{ ms}$$

Thus,

$C_{C2}(20 + 10) \times 10^3 = 15.92 \times 10^{-3}$

$\Rightarrow C_{C2} = 0.53\ \mu\text{F}$

Although, to be conservative we should select $C_{C2} = 1\ \mu\text{F}$; in this case we can select

$C_{C2} = 0.5\ \mu\text{F}$

because the required value is very close to 0.5 μF and because we have selected C_{C1} and C_E larger than the required values. The resulting f_L will be

$$f_L = \frac{1}{2\pi}\left[\frac{1}{\tau_{CE}} + \frac{1}{\tau_{C1}} + \frac{1}{\tau_{C2}}\right]$$

$$\tau_{CE} = 5 \times 10^{-6} \times \left[250 + \frac{(200 \parallel 20) \times 10^3}{101}\right]$$

$= 2.15$ ms

$\tau_{C1} = 0.5 \times 10^{-6}[(200 \parallel 25) \times 10^3 + 20 \times 10^3]$

$= 21.1$ ms

$\tau_{C2} = 0.5 \times 10^{-6}(20 + 10) \times 10^3 = 15$ ms

$$f_L = \frac{1}{2\pi}\left[\frac{10^3}{2.15} + \frac{10^3}{21.1} + \frac{10^3}{15}\right]$$

$= 92.2$ Hz

which is lower (hence more conservative) than the required value of 100 Hz.

$C_{\text{total}} = 5 + 0.5 + 0.5 = 6\ \mu\text{F}$

10.8 Refer to Fig. 10.9.

In the midband,

$R_{\text{in}} = R_{B1} \parallel R_{B2} \parallel r_\pi$

where

$R_{B1} = 33\ \text{k}\Omega,\ R_{B2} = 22\ \text{k}\Omega$

$$g_m = \frac{I_C}{V_T} = \frac{0.3\ \text{mA}}{0.025\ \text{V}} = 12\ \text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.3\ \text{mA}} = 83.3\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10\ \text{k}\Omega$$

Thus,

$R_{\text{in}} = 33 \parallel 22 \parallel 10 = 5.7\ \text{k}\Omega$

$$A_M = -\frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m (R_C \parallel R_L)$$

where

$R_{\text{sig}} = 5\ \text{k}\Omega,\ R_C = 4.7\ \text{k}\Omega,\ \text{and}\ R_L = 5.6\ \text{k}\Omega$

Thus,

$$A_M = -\frac{5.7}{5.7 + 5} \times 12(4.7 \parallel 5.6)$$

$= -16.3$ V/V

Using the method of short-circuit time constants and the information in Fig. 10.10, we obtain

$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$

$= C_{C1}(R_{\text{in}} + R_{\text{sig}})$

$= 1 \times 10^{-6}(5.7 + 5) \times 10^3 = 10.7$ ms

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

$$= 20 \times 10^{-6}\left[3.9 \times 10^3 \parallel \left(83.3 + \frac{(33 \parallel 22 \parallel 5) \times 10^3}{121}\right)\right]$$

$= 2.2$ ms

$\tau_{C2} = C_{C2}(R_C + R_L)$

$= 1 \times 10^{-6}(4.7 + 5.6) \times 10^3 = 10$ ms

$$f_L \simeq \frac{1}{2\pi}\left(\frac{1}{\tau_{C1}} + \frac{1}{\tau_{CE}} + \frac{1}{\tau_{C2}}\right)$$

$$= \frac{1}{2\pi}\left(\frac{1}{10.7 \times 10^{-3}} + \frac{1}{2.2 \times 10^{-3}} + \frac{1}{10.3 \times 10^{-3}}\right)$$

$= 102.7$ Hz

10.9 Refer to the data given in the statement for Problem 10.8.

$R_B = R_{B1} \parallel R_{B2} = 33\ \text{k}\Omega \parallel 22\ \text{k}\Omega = 13.2\ \text{k}\Omega$

$I_C \simeq I_E \simeq 0.3$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.3\ \text{mA}}{0.025\ \text{V}} = 12\ \text{mA/V}$$

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.3\ \text{mA}} = 83.3\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{12} = 10\ \text{k}\Omega$$

From Fig. 10.10, we have

$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$

For C_{C1} to contribute 10% of f_L, we use

$$\frac{1}{\tau_{C1}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$= 0.1 \times 2\pi \times 50$

$\Rightarrow \tau_{C1} = 31.8$ ms

Thus,

$C_{C1}[(13.2 \parallel 10) + 5] \times 10^3 = 31.8 \times 10^{-3}$

$\Rightarrow C_{C1} = 3\ \mu\text{F}$

$\tau_{C2} = C_{C2}(R_C + R_L)$

For C_{C2} to contribute 10% of f_L, we use

$$\frac{1}{\tau_{C2}} = 0.1\omega_L = 0.1 \times 2\pi f_L$$

$= 0.1 \times 2\pi \times 50$

$\Rightarrow \tau_{C2} = 31.8$ ms

Thus,

$C_{C2}(4.7 + 5.6) \times 10^3 = 31.8 \times 10^{-3}$

$\Rightarrow C_{C2} = 3.09\ \mu\text{F} \simeq 3\ \mu\text{F}$

Finally,

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

For C_E to contribute 80% of f_L, we use

$$\frac{1}{\tau_{CE}} = 0.8\omega_L = 0.8 \times 2\pi f_L$$

$= 0.8 \times 2\pi \times 50$

$\Rightarrow \tau_{CE} = 3.98\ \text{ms}$

Thus,

$$C_E\left[3900 \parallel \left(83.3 + \frac{(13.2 \parallel 5) \times 1000}{121}\right)\right]$$

$= 3.98 \times 10^{-3}$

$\Rightarrow C_E = 36.2\ \mu\text{F}$

10.10 Using the information in Fig. 10.10, we get

$\tau_{C1} = C_{C1}[(R_B \parallel r_\pi) + R_{\text{sig}}]$

$= C_{C1}[(10 \parallel 1) + 5] \times 10^3$

$= C_{C1} \times 5.91 \times 10^3$

$$\tau_{CE} = C_E\left[R_E \parallel \left(r_e + \frac{R_B \parallel R_{\text{sig}}}{\beta + 1}\right)\right]$$

where

$$r_e = \frac{r_\pi}{\beta + 1} = \frac{1000}{101} \simeq 10\ \Omega$$

$\tau_{CE} =$

$$C_E\left[1.5 \times 10^3 \parallel \left(10 + \frac{(10 \parallel 5) \times 1000}{101}\right)\right]$$

$= C_E \times 41.8$

For C_{C1} and C_E to contribute equally to f_L,

$\tau_{C1} = \tau_{CE}$

Thus,

$C_{C1} \times 5.91 \times 10^3 = C_E \times 41.8$

$$\Rightarrow \frac{C_E}{C_{C1}} = 141.4$$

10.11 Replacing the BJT with its T model results in the equivalent circuit shown in Fig. 1 below.

(a) At midband, C_E and C_C act as short circuits. Thus

$R_{\text{in}} = (\beta + 1)r_e$

$$\frac{V_o}{V_{\text{sig}}} = -\frac{(\beta + 1)r_e}{(\beta + 1)r_e + R_{\text{sig}}}\, g_m(R_C \parallel R_L)$$

$$= -\frac{\beta(R_C \parallel R_L)}{(\beta + 1)r_e + R_{\text{sig}}}$$

(b) Because the controlled current source αI_e is ideal, it effectively separates the input circuit from the output circuit. The result is that the poles caused by C_E and C_C do not interact. The pole due to C_E will have frequency ω_{PE}:

$$\omega_{PE} = \frac{1}{C_E\left[r_e + \dfrac{R_{\text{sig}}}{\beta + 1}\right]}$$

and the pole due to C_C will have a frequency ω_{PC}

$$\omega_{PC} = \frac{1}{C_C(R_C + R_L)}$$

(c) The overall voltage transfer function can be expressed as

$$\frac{V_o}{V_{\text{sig}}} = A_M \frac{s}{s + \omega_{PE}}\, \frac{s}{s + \omega_{PC}}$$

This figure belongs to Problem 10.11, part (a).

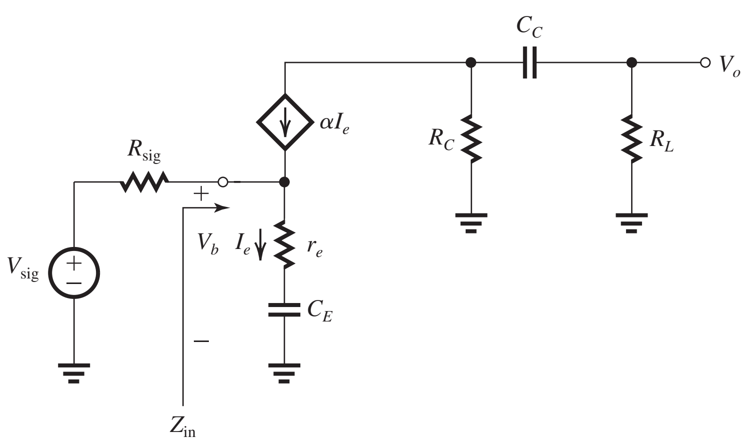


Figure 1

(d) $r_e = \dfrac{V_T}{I_E} = \dfrac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$

$$A_M = -\frac{100(10\text{ k}\Omega \parallel 10\text{ k}\Omega)}{101 \times 25 \times 10^{-3}\text{ k}\Omega + 10\text{ k}\Omega}$$

$= -40$ V/V

(e) To minimize the total capacitance we choose to make the pole caused by C_E the dominant one and make its frequency equal to $f_L = 100$ Hz,

$$2\pi \times 100 = \frac{1}{C_E\left[25 + \dfrac{10,000}{101}\right]}$$

$\Rightarrow C_E = 12.83\ \mu\text{F}$

Placing the pole due to C_C at 10 Hz, we obtain

$$2\pi \times 10 = \frac{1}{C_C(10 + 10) \times 10^3}$$

$\Rightarrow C_C = 0.8\ \mu\text{F}$

(f) A Bode plot for the gain magnitude is shown in Fig. 2.

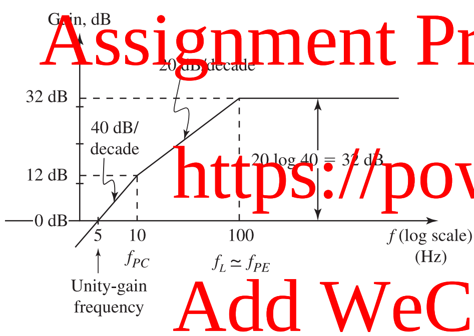


Figure 2

The gain at f_{P2} (10 Hz) is 12 dB. Since the gain decreases by 40 dB/decade or equivalently 12 dB/octave, it reaches 0 dB (unity magnitude) at $f = f_{PC}/2 = 5$ Hz.

10.12

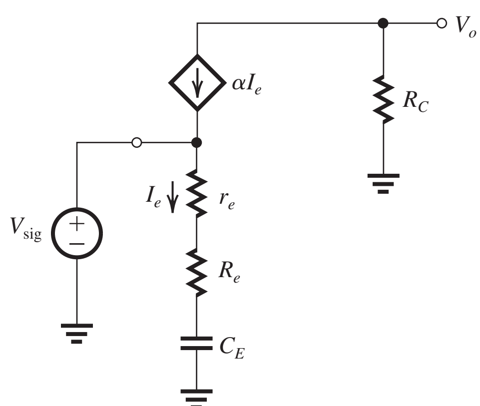


Figure 1

Replacing the BJT with its T model results in the circuit shown in Fig. 1.

(a) $I_e = \dfrac{V_{sig}}{r_e + R_e + \dfrac{1}{sC_E}}$

$V_o = -\alpha I_e R_C$

Thus,

$$\frac{V_o}{V_{sig}} = -\frac{\alpha R_C}{r_e + R_e}\,\frac{s}{s + \dfrac{1}{C_E(r_e + R_e)}} \quad (1)$$

From this expression we obtain

$$A_M = -\frac{\alpha R_C}{r_e + R_e} \quad (2)$$

and

$$f_L = f_P = \frac{1}{2\pi C_E(r_e + R_e)} \quad (3)$$

(b) From Eq. (2) we see that

$$|A_M| = \frac{\alpha R_C}{r_e}\,\frac{1}{1 + \dfrac{R_e}{r_e}}$$

Thus, including R_e reduces the gain magnitude by the factor $\left(1 + \dfrac{R_e}{r_e}\right)$.

(c) From Eq. (3) we obtain

$$f_L = \frac{1}{2\pi C_E r_e}\,\frac{1}{1 + \dfrac{R_e}{r_e}}$$

Thus, including R_e reduces f_L by the factor $\left(1 + \dfrac{R_e}{r_e}\right)$. This is the same factor by which the magnitude of the gain is reduced. Thus, R_e can be used to tradeoff gain for decreasing f_L (that is, increasing the amplifier bandwidth).

(d) $I = 0.25$ mA, $\quad R_C = 10\text{ k}\Omega, \quad C_E = 10\ \mu\text{F}$

$$r_e = \frac{V_T}{I} = \frac{25\text{ mV}}{0.25\text{ mA}} = 100\ \Omega$$

For $R_e = 0$:

$$|A_M| = \frac{\alpha R_C}{r_e} \simeq \frac{10\text{ k}\Omega}{100\ \Omega} = 100\text{ V/V}$$

$$f_L = \frac{1}{2\pi \times 10 \times 10^{-6} \times 100} = 159.2\text{ Hz}$$

To lower f_L by a factor of 10, we use

$$1 + \frac{R_e}{r_e} = 10$$

$\Rightarrow R_e = 900\ \Omega$

The gain now becomes

$$|A_M| = \frac{100}{1 + \dfrac{R_e}{r_e}} = \frac{100}{10} = 10\text{ V/V}$$

See Fig. 2 for the Bode plot.

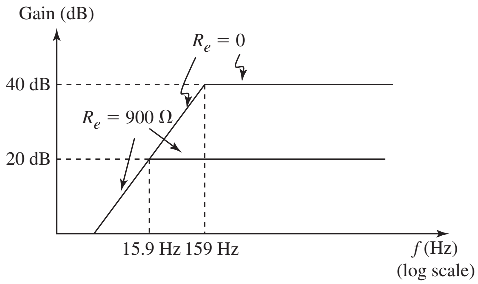


Figure 2

10.13 $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}}$

$$= \frac{3.45 \times 10^{-11}\ \text{F/m}}{8 \times 10^{-9}\ \text{m}} = 0.43 \times 10^{-2}\ \text{F/m}^2$$

$$= 0.43 \times 10^{-2} \times 10^{-12}\ \text{F}/\mu\text{m}^2$$

$$= 4.3\ \text{fF}/\mu\text{m}^2$$

$$k_n' = \mu_n C_{ox}$$

$$= 450 \times 10^8\ (\mu\text{m}^2/\text{V}\cdot\text{s}) \times 4.3 \times 10^{-15}\ \text{F}/\mu\text{m}^2$$

$$= 193.5\ \mu\text{A/V}^2$$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2 (1 + \lambda V_{DS})$$

$$200 = \frac{1}{2} \times 193.5 \times 20 \times V_{OV}^2 (1 + 0.05 \times 1.5)$$

$$\Rightarrow V_{OV} = 0.31\ \text{V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.31} = 1.3\ \text{mA/V}$$

$$\chi = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

$$= \frac{0.5}{2\sqrt{0.65 + 1}} = 0.19$$

$$g_{mb} = \chi g_m = 0.25\ \text{mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{1}{|\lambda| I_D} = \frac{1}{0.05 \times 0.2} = 100\ \text{k}\Omega$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$= \frac{2}{3} \times 20 \times 1 \times 4.3 + 20 \times 0.05 \times 4.3$$

$$= 57.3 + 4.3 = 61.6\ \text{fF}$$

$$C_{gd} = WL_{ov}C_{ox} = 20 \times 0.05 \times 4.3$$

$$= 4.3\ \text{fF}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \dfrac{|V_{SB}|}{V_0}}}$$

$$= \frac{20}{\sqrt{1 + \dfrac{1}{0.7}}} = 12.8\ \text{fF}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \dfrac{|V_{DB}|}{V_0}}}$$

$$= \frac{20}{\sqrt{1 + \dfrac{2.5}{0.7}}} = 9.4\ \text{fF}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1.3 \times 10^{-3}}{2\pi(61.6 + 4.3) \times 10^{-15}} = 3.1\ \text{GHz}$$

10.14 $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.2}{0.3} = 1.33\ \text{mA/V}$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$= \frac{1.33 \times 10^{-3}}{2\pi \times (25 + 5) \times 10^{-15}} = 7.1\ \text{GHz}$$

10.15 $f_T = \dfrac{g_m}{2\pi(C_{gs} + C_{gd})}$

For $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \quad (1)$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component $(WL_{ov}C_{ox})$ is small, we get

$$C_{gs} \simeq \frac{2}{3} WLC_{ox} \quad (2)$$

The transconductance g_m is given by

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \quad (3)$$

Substituting from (2) and (3) into (1), we get

$$f_T = \frac{\sqrt{2\mu_n C_{ox}\left(\dfrac{W}{L}\right) I_D}}{2\pi \times \dfrac{2}{3} WLC_{ox}}$$

$$= \frac{1.5}{\pi L} \sqrt{\frac{\mu_n I_D}{2C_{ox}WL}} \qquad \text{Q.E.D.}$$

We observe that for a given device, f_T is proportional to $\sqrt{I_D}$; thus to obtain faster operation the MOSFET is operated at a higher I_D.

Also, we observe that f_T is inversely proportional to $L\sqrt{WL}$; thus faster operation is obtained from smaller devices.

10.16 $f_T = \dfrac{g_m}{2\pi(C_{gs} + C_{gd})}$

For $C_{gs} \gg C_{gd}$

$$f_T \simeq \frac{g_m}{2\pi C_{gs}} \qquad (1)$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

If the overlap component is small, we get

$$C_{gs} \simeq \frac{2}{3}WLC_{ox} \qquad (2)$$

The transconductance g_m can be expressed as

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV} \qquad (3)$$

Substituting from (2) and (3) into (1), we obtain

$$f_T = \frac{\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}}{2\pi \times \dfrac{2}{3}WLC_{ox}}$$

$$= \frac{3\mu_n V_{OV}}{4\pi L^2}$$

We note that for a given channel length, f_T can be increased by operating the MOSFET at a higher V_{OV}.

For $L = 0.5\ \mu$m and $\mu_n = 450$ cm^2/V·s,

we have

$$V_{OV} = 0.2 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.2}{4\pi \times 0.5^2}$$

$$= 5.73 \text{ GHz}$$

$$V_{OV} = 0.4 \text{ V} \Rightarrow f_T = \frac{3 \times 450 \times 10^8 \times 0.4}{4\pi \times 0.5^2}$$

$$= 11.46 \text{ GHz}$$

10.17 $A_0 = \dfrac{2V_A}{V_{OV}} = \dfrac{2V_A'L}{V_{OV}}$

$$A_0 = \frac{2 \times 5 \times L}{0.2} = 50L,\ \text{V/V}\ (L \text{ in } \mu\text{m})$$

$$f_T \simeq \frac{3\mu_n V_{OV}}{4\pi L^2} = \frac{3 \times 400 \times 10^8 \times 0.2}{4\pi L^2}$$

$$f_T = \frac{1.91}{L^2},\ \text{GHz}\ (L \text{ in } \mu\text{m})$$

The expressions for A_0 and f_T can be used to obtain their values for different values of L. The results are given in the following table.

L	L_{min} 0.13 μm	$2L_{min}$ 0.26 μm	$3L_{min}$ 0.39 μm	$4L_{min}$ 0.52 μm	$5L_{min}$ 0.65 μm
A_0 (V/V)	6.5	13	19.5	26	32.5
f_T (GHz)	113	28.3	12.6	7.1	4.5

10.18 $f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

where

$$g_m = \frac{I_C}{V_T} = \frac{0.5 \text{ mA}}{0.025 \text{ V}} = 20 \text{ mA/V}$$

$$C_\pi = 8 \text{ pF}$$

$$C_\mu = 1 \text{ pF}$$

Thus,

$$f_T = \frac{20 \times 10^{-3}}{2\pi \times (8+1) \times 10^{-12}} = 353.7 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta} = \frac{353.7}{100} = 3.54 \text{ MHz}$$

10.19 See figure on next page. $C_\pi = C_{de} + C_{je}$

where C_{de} is proportional to I_C.

At $I_C = 0.5$ mA,

$8 = C_{de} + 2 \Rightarrow C_{de} = 6$ pF

At $I_C = 0.25$ mA, $C_{de} = \dfrac{1}{2} \times 6 = 3$ pF, and $C_\pi = 3 + 2 = 5$ pF.

Also, at $I_C = 0.25$ mA, $g_m = 10$ mA/V. Thus f_T at $I_C = 0.25$ mA is

$$f_T = \frac{10 \times 10^{-3}}{2\pi(5+1) \times 10^{-12}} = 265.3 \text{ MHz}$$

10.20 $r_x = 100\ \Omega$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{1} = 50 \text{ k}\Omega$$

$$C_{de} = \tau_F g_m = 30 \times 10^{-12} \times 40 \times 10^{-3} = 1.2 \text{ pF}$$

$$C_{je0} = 20 \text{ pF}$$

$$C_\pi = C_{de} + 2C_{je0} = 1.2 + 2 \times 0.02 = 1.24 \text{ pF}$$

$$C_\mu = \frac{C_{je0}}{\left(1 + \dfrac{V_{CB}}{V_{0c}}\right)^m}$$

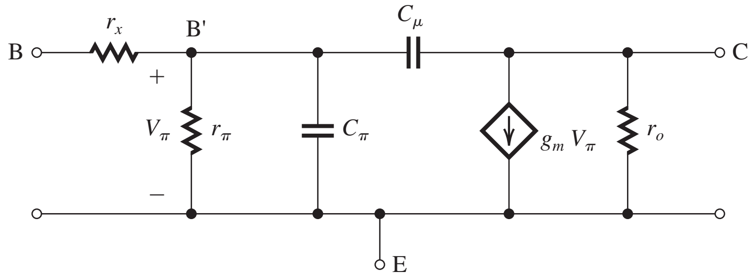
$$C_\mu = \frac{20}{\left(1 + \dfrac{2}{0.75}\right)^{0.5}} = 10.4 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(1.24 + 0.01) \times 10^{-12}}$$

$$= 5.1 \text{ GHz}$$

This figure belongs to Problem 10.20.



10.21 For $f \gg f_\beta$,

$$|h_{fe}| = \frac{f_T}{f}$$

At $f = 50$ MHz and $I_C = 0.2$ mA,

$$|h_{fe}| = 10 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 500 \text{ MHz}$$

At $f = 50$ MHz and $I_C = 1.0$ mA,

$$|h_{fe}| = 12 = \frac{f_T}{50}$$

$$\Rightarrow f_T = 600 \text{ MHz}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

where

$$C_\pi = C_{de} + C_{je}$$

$$= \tau_F g_m + C_{je}$$

$$C_\mu = 0.1 \text{ pF}$$

At $I_C = 0.2$ mA, $g_m = \dfrac{0.2}{0.025} = 8$ mA/V, thus

$$500 \times 10^6 = \frac{8 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 2.45 \text{ pF}$$

$$\tau_F \times 8 \times 10^{-3} + C_{je} = 2.45 \times 10^{-12} \qquad (1)$$

At $I_C = 1$ mA, $g_m = \dfrac{1}{0.025} = 40$ mA/V, thus

$$600 \times 10^6 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 10.51 \text{ pF}$$

$$\tau_F \times 40 \times 10^{-3} + C_{je} = 10.51 \times 10^{-12} \qquad (2)$$

Solving (1) together with (2) yields

$$\tau_F = 252 \text{ ps}$$

$$C_{je} = 0.43 \text{ pF}$$

10.22 $f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$10 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + 0.1) \times 10^{-12}}$$

$$\Rightarrow C_\pi = 0.54 \text{ pF}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3 \text{ k}\Omega$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{10 \times 10^9}{120} = 83.3 \text{ MHz}$$

10.23 For $f \gg f_\beta$,

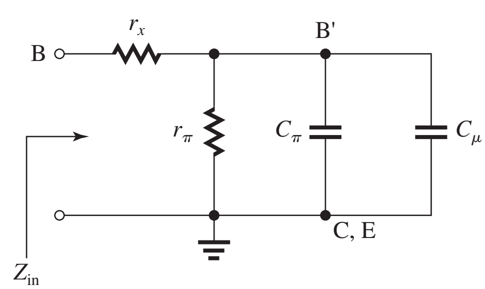
$$|h_{fe}| \simeq \frac{f_T}{f}$$

$$40 = \frac{2000 \text{ MHz}}{f}$$

$$\Rightarrow f = 50 \text{ MHz}$$

$$f_\beta = \frac{f_T}{\beta_0} = \frac{2000 \text{ MHz}}{200} = 10 \text{ MHz}$$

10.24



With the emitter and the collector grounded, the equivalent circuit takes the form shown in the figure, and the input impedance becomes

$$Z_{in} = r_x + \frac{1}{\dfrac{1}{r_\pi} + j\omega(C_\pi + C_\mu)}$$

$$= r_x + \frac{r_\pi}{1 + j\omega(C_\pi + C_\mu)r_\pi}$$

Since $\omega_\beta = \dfrac{1}{(C_\pi + C_\mu)r_\pi}$, then

$$Z_{\text{in}} = r_x + \frac{r_\pi}{1 + j\left(\dfrac{\omega}{\omega_\beta}\right)}$$

$$= r_x + r_\pi \frac{1 - j\left(\dfrac{\omega}{\omega_\beta}\right)}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2}$$

$$R_e(Z_{\text{in}}) = r_x + \frac{r_\pi}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2}$$

For the real part to be an estimate of r_x accurate to within 10%, we require

$$\frac{r_\pi}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \le 0.1 r_x$$

$$\frac{1}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \le 0.1\left(\frac{r_x}{r_\pi}\right)$$

But $r_x \le \dfrac{r_\pi}{10}$, thus $\dfrac{r_x}{r_\pi} \le 0.1$;

$$\frac{1}{1 + \left(\dfrac{\omega}{\omega_\beta}\right)^2} \le 0.1 \times 0.1$$

or, equivalently,

$$1 + \left(\frac{\omega}{\omega_\beta}\right)^2 \ge 100$$

$$\Rightarrow \omega \ge 10\omega_\beta$$

10.25 To complete the table we use the following relationships:

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{I_E \text{ (mA)}}$$

$$g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} \simeq \frac{I_E}{V_T} = \frac{I_E \text{ (mA)}}{0.025 \text{ V}}$$

$$r_\pi = \frac{\beta_0}{g_m \text{ (mA/V)}}, \text{ k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$f_\beta = \frac{1}{2\pi(C_\pi + C_\mu)r_\pi}$$

$$f_\beta = \frac{f_T}{\beta_0}$$

10.26 $C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 1 + 0.1(1 + 39)$

$= 5$ pF

$$f_{sdB} = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 5 \times 10^{-12} R_{\text{sig}}}$$

For $f_{sdB} = 1$ MHz,

$$R_{\text{sig}} < \frac{1}{2\pi \times 5 \times 10^{-12} \times 1 \times 10^6} = 31.8 \text{ k}\Omega$$

10.27 (a) $V_o = -AV_i$

If the current flowing through R_{sig} is denoted I_i, we obtain

$$Y_{\text{in}} = \frac{I_i}{V_i} = \frac{sC(V_i - V_o)}{V_i}$$

$$= sC\left(1 - \frac{V_o}{V_i}\right)$$

$= sC(1 + A)$

Thus,

$C_{\text{in}} = C(1 + A)$

This table belongs to Problem 10.25.

Transistor	I_E (mA)	r_e (Ω)	g_m (mA/V)	r_π (kΩ)	β_0	f_T (MHz)	C_μ (pF)	C_π (pF)	f_β (MHz)
(a)	2	12.5	80	12.5	100	500	2	23.5	5
(b)	1	25	40	3.13	125	500	2	10.7	4
(c)	1	25	40	2.5	100	500	2	10.7	5
(d)	10	2.5	400	0.25	100	500	2	125.3	5
(e)	0.1	250	4	25	100	150	2	2.2	1.5
(f)	1	25	40	0.25	10	500	2	10.7	50
(g)	1.25	20	50	0.2	10	800	1	9	80

(b) $\dfrac{V_i(s)}{V_{sig}(s)} = \dfrac{1/sC_{in}}{R_{sig} + \dfrac{1}{sC_{in}}}$

$= \dfrac{1}{1 + sC_{in}R_{sig}}$

$\dfrac{V_o(s)}{V_{sig}(s)} = -\dfrac{A}{1 + sC_{in}R_{sig}}$

(c) DC gain = 40 dB = 100 V/V,

$\Rightarrow A = 100$ V/V

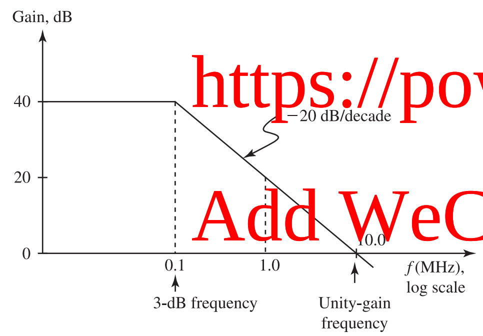
$f_{3dB} = \dfrac{1}{2\pi C_{in} R_{sig}}$

$100 \times 10^3 = \dfrac{1}{2\pi C_{in} \times 1 \times 10^3}$

$\Rightarrow C_{in} = 1591.5$ pF

$C = \dfrac{C_{in}}{A+1} = \dfrac{1591.5}{101} = 15.8$ pF

(d) The Bode plot is shown in the figure below. From the figure we see that the gain reduces to unity two decades higher than f_{3dB}, that is at 10 MHz.



10.28 $C_{in} = 0.2(1 + 1000)$

$= 200.2$ pF

$\dfrac{V_o(s)}{V_{sig}(s)} = -\dfrac{1000}{1 + sC_{in}R_{sig}}$

$f_{3dB} = \dfrac{1}{2\pi C_{in} R_{sig}}$

$= \dfrac{1}{2\pi \times 200.2 \times 10^{-12} \times 1 \times 10^3}$

$= 795$ kHz

The gain falls off at the rate of 20 dB/decade. For the gain to reach 0 dB (unity), the gain has to fall by 60 dB. This requires three decades or a factor of 1000, thus

$f_{\text{unity gain}} = 795 \times 1000 = 795$ MHz

10.29 $f_H = \dfrac{1}{2\pi C_{in} R_{sig}}$

For $f_H \geq 6$ MHz

$C_{in} \leq \dfrac{1}{2\pi f_H R_{sig}} = \dfrac{1}{2\pi \times 6 \times 10^6 \times 1 \times 10^3}$

$C_{in} \leq 26.5$ pF

But,

$C_{in} = C_{gs} + (1 + g_m R'_L) C_{gd}$

$= 5 + (1 + g_m R'_L) \times 1,$ pF

$= 6 + g_m R'_L,$ pF

For $C_{in} \leq 26.5$ pF we have

$g_m R'_L \leq 20.5$

$R'_L \leq \dfrac{20.5}{5} = 4.1$ kΩ

Corresponding to $R'_L = 4.1$ kΩ, we have

$|A_M| = g_m R'_L = 20.5$ V/V

$GB = |A_M| f_H$

$= 20.5 \times 6 = 123$ MHz

If $f_H = 2$ MHz, we obtain

$C_{in} = 26.5 \times 3 = 79.5$ pF

$g_m R'_L = 79.5 - 6 = 73.5$

Thus,

$|A_M| = 73.5$ V/V

$GB = 73.5 \times 2 = 147$ MHz

10.30 Refer to Example 10.3. If the transistor is replaced with another whose W is half that of the original transistor, we obtain

$W_2 = \dfrac{1}{2} W_1$

Since

$C_{gs} = \dfrac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$

then

$C_{gs2} = \dfrac{1}{2} C_{gs1} = 0.5$ pF

Also,

$C_{gd} = WL_{ov}C_{ox}$

thus,

$C_{gd2} = \dfrac{1}{2} C_{gd1} = 0.2$ pF

Since

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

then

$$g_{m2} = \frac{1}{\sqrt{2}} g_{m1} = 0.71 \text{ mA/V}$$

Since

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

then

$$V_{OV2} = \sqrt{2} V_{OV1}$$

Finally,

$$r_{o2} = r_{o1} = 150 \text{ k}\Omega$$

Thus,

$$R'_{L2} = R'_{L1} = 7.14 \text{ k}\Omega$$

Thus,

$$C_{\text{in}2} = C_{gs2} + (g_{m2}R'_{L2} + 1)C_{gd2}$$

$$= 0.5 + (0.71 \times 7.14 + 1) \times 0.2$$

$$= 1.71 \text{ pF}$$

This should be compared to $C_{\text{in}1} = 4.26$ pF. Thus,

$$f_{H2} = \frac{1}{2\pi C_{\text{in}2}(R_{\text{sig}} \parallel R_G)}$$

$$= \frac{1}{2\pi \times 1.71 \times 10^{-12}(0.1 \parallel 4.7) \times 10^6}$$

$$= 952 \text{ MHz}$$

in comparison to $f_{H1} = 398$ MHz

$$|A_{M2}| = \frac{4.7}{4.7 + 0.1} \times g_{m2}R'_{L2}$$

$$= \frac{4.7}{4.8} \times 0.71 \times 7.14$$

$$= 5 \text{ V/V}$$

in comparison to $|A_{M1}| = 7$ V/V.

$$GB_2 = 5 \times 952 = 4.73 \text{ GHz}$$

in comparison to $GB_1 = 7 \times 398 = 2.79$ GHz.

10.31 $A_M = -\dfrac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m R'_L$

where

$$R'_L = R_D \parallel R_L \parallel r_o$$

$$= 8 \text{ k}\Omega \parallel 10 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$= 4.1 \text{ k}\Omega$$

$$A_M = -\frac{100}{100 + 100} \times 3 \times 4.1$$

$$= -6.1 \text{ V/V}$$

$$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R'_L) \quad (1)$$

$$= 1 + 0.2(1 + 3 \times 4.1)$$

$$= 3.66 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}} \quad (2)$$

where

$$R'_{\text{sig}} = R_{\text{sig}} \parallel R_{\text{in}}$$

$$= 100 \text{ k}\Omega \parallel 100 \text{ k}\Omega = 50 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi \times 3.66 \times 10^{-12} \times 50 \times 10^3}$$

$$= 870 \text{ kHz}$$

To double f_H by changing R_{in}, Eq. (2) indicates that R'_{sig} must be halved:

$$R'_{\text{sig}} = 25 \text{ k}\Omega$$

which requires R_{in} to be changed to $R_{\text{in}2}$,

$$25 \text{ k}\Omega = 100 \parallel R_{\text{in}2}$$

$$\Rightarrow R_{\text{in}2} = 33.3 \text{ k}\Omega$$

This change will cause $|A_M|$ to become

$$|A_{M2}| = \frac{33.3}{33.3 + 100} \times 3 \times 4.1$$

$$= 3.1 \text{ V/V}$$

which is about half the original value.

To double f_H by changing R_L, Eq. (2) indicates that C_{in} must be halved:

$$C_{\text{in}2} = \frac{1}{2} \times 3.66 = 1.83 \text{ pF}$$

Using Eq. (1), we obtain

$$1.83 = 1 + 0.2(1 + g_m R'_{L2})$$

$$\Rightarrow g_m R'_{L2} = 3.15$$

Thus,

$$R'_{L2} = 1.05 \text{ k}\Omega$$

and R_{L2} can be found from

$$1.05 = R_L \parallel 8 \text{ k}\Omega \parallel 50 \text{ k}\Omega$$

$$\Rightarrow R_L = 1.24 \text{ k}\Omega$$

and the midband gain becomes

$$|A_{M2}| = \frac{100}{100 + 100} \times 3.15 = 1.6 \text{ V/V}$$

which is about a quarter of the original gain. Clearly, changing R_{in} is the preferred course of action!

10.32 (a) $A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$

where

$R'_L = R_D \parallel R_L \parallel r_o$

$= 20 \text{ k}\Omega \parallel 20 \text{ k}\Omega \parallel 100 \text{ k}\Omega$

$= 9.1 \text{ k}\Omega$

$A_M = -\frac{2 \text{ M}\Omega}{2 \text{ M}\Omega + 0.5 \text{ M}\Omega} \times 5 \times 9.1$

$= -36.4$ V/V

(b) $f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 3 + 0.5(1 + 5 \times 9.1)$

$= 26.25$ pF

and

$R'_{sig} = R_{sig} \parallel R_G$

$= 500 \text{ k}\Omega \parallel 2000 \text{ k}\Omega$

$= 400 \text{ k}\Omega$

Thus,

$f_H = \frac{1}{2\pi \times 26.25 \times 10^{-12} \times 400 \times 10^3}$

$= 15.2$ kHz

(c) $f_Z = \frac{g_m}{2\pi C_{gd}}$

$= \frac{5 \times 10^{-3}}{2\pi \times 0.5 \times 10^{-12}}$

$= 1.6$ GHz

10.33 $R_G = R_{G1} \parallel R_{G2} = 47 \text{ M}\Omega \parallel 10 \text{ M}\Omega$

$= 8.25 \text{ M}\Omega$

$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$

where

$R'_L = R_L \parallel R_D \parallel r_o$

$= 10 \text{ k}\Omega \parallel 4.7 \text{ k}\Omega \parallel 100 \text{ k}\Omega$

$= 3.1 \text{ k}\Omega$

$A_M = -\frac{8.25}{8.25 + 0.1} \times 3 \times 3.1$

$= -9.2$ V/V

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_m R'_L)$

$= 1 + 0.2(1 + 3 \times 3.1)$

$= 3.06$ pF

and

$R'_{sig} = R_{sig} \parallel R_G$

$= 100 \text{ k}\Omega \parallel 8.25 \text{ M}\Omega$

$= 99 \text{ k}\Omega$

Thus,

$f_H = \frac{1}{2\pi \times 3.06 \times 10^{-12} \times 99 \times 10^3}$

$= 525$ kHz

10.34 $g_m = \sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}$

$= \sqrt{2 \times 0.09 \times 100 \times 0.1}$

$= 1.34$ mA/V

$r_{o1} = \frac{|V_{A1}|}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$

$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$

The total resistance at the output node, R'_L, is given by

$R'_L = r_{o1} \parallel r_{o2} = 128 \text{ k}\Omega \parallel 192 \text{ k}\Omega$

$= 76.8 \text{ k}\Omega$

$A_M = -g_m R'_L$

$= -1.34 \times 76.8 = -103$ V/V

$f_H = \frac{1}{2\pi C_{in} R_{sig}}$

where

$C_{in} = C_{gs} + C_{gd}(1 + g_{m1} R'_L)$

$= 0.2 + 0.015(1 + 103)$

$= 1.76$ pF

Thus,

$f_H = \frac{1}{2\pi \times 1.76 \times 10^{-12} \times 200 \times 10^3}$

$= 452$ kHz

$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.34 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}}$

$= 14.2$ GHz

10.35 $g_m R'_L = 50$

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 10 + 1(1 + 50)$

$= 61$ pF

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 61 \times 10^{-12} \times 5 \times 10^3}$$

$= 522$ kHz

10.36

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

where

$R'_L = r_o \parallel R_C \parallel R_L$

$= 100$ kΩ $\parallel$ 10 kΩ $\parallel$ 10 kΩ

$= 4.76$ kΩ

and

$r_\pi = \beta / g_m = 100/40 = 2.5$ kΩ

$$A_M = -\frac{100}{100 + 10} \times \frac{2.5}{2.5 + 0.1 + (10 \parallel 100)} \times 40 \times 4.76$$

$= -37$ V/V

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 10 + 1 \times (1 + 40 \times 4.76)$

$= 201.4$ pF

and

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 2.5 \parallel [0.1 + (100 \parallel 10)]$

$= 2$ kΩ

$$f_H = \frac{1}{2\pi \times 201.4 \times 10^{-12} \times 2 \times 10^3}$$

$= 395$ kHz

10.37 Refer to Example 10.4. Since I_E is doubled to 2 mA, we have

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{25 \text{ mV}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{80 \text{ mA/V}} = 1.25 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{2 \text{ mA}} = 50 \text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{80 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 16 \text{ pF}$$

$C_\mu = 1$ pF

$C_\pi = 15$ pF

$r_x = 50\ \Omega$

Also, now

$R_B = 50$ kΩ

$R_C = 4$ kΩ

The new value of A_M is

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_B \parallel R_{sig})} (g_m R'_L)$$

where

$R'_L = r_o \parallel R_C \parallel R_L$

$= 50 \parallel 4 \parallel 5 = 2.13$ kΩ

Thus,

$g_m R'_L = 80 \times 2.13 = 170$ V/V

and

$$A_M = -\frac{50}{50 + 5} \times \frac{1.25}{1.25 + 0.05 + (50 \parallel 5)} \times 170$$

$= -33$ V/V

and

$20 \log |A_M| = 30.4$ dB

This should be compared to the previous value of 39 V/V (32 dB). To determine f_H, we first find C_{in},

$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$

$= 15 + 1(1 + 170)$

$= 186$ pF

and the effective source resistance R'_{sig},

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$= 1.25 \parallel [0.05 + (50 \parallel 5)]$

$= 0.98$ kΩ

Thus

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 186 \times 10^{-12} \times 0.98 \times 10^3}$$

$= 873$ kHz

This should be compared to the previous value of 754 kHz. The gain-bandwidth product becomes

$\text{GB} = |A_M| f_H = 33 \times 873 = 28.8$ MHz

This should be compared to the previous value of $39 \times 754 = 29.4$ MHz. Thus, increasing the bias current by a factor of 2 results in an increase in f_H by a factor of 1.16—that is, by about 16%. However, because of the attendant reduction in input resistance, the overall gain decreased by about the same factor and GB remained nearly constant. The price paid for the slight increase in f_H is an increase in power dissipation by a factor of about two.

10.38 (a) $A_M =$

$$-\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_{sig} \parallel R_B)} g_m R'_L$$

For $R_B \gg R_{sig}$, $r_x \ll R_{sig}$, $R_{sig} \gg r_\pi$,

$$A_M \simeq -\frac{r_\pi}{R_{sig}} g_m R'_L = -\beta R'_L / R_{sig} \qquad \text{Q.E.D.}$$

(b) $C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$

For $g_m R'_L \gg 1$ and $g_m R'_L C_\mu \gg C_\pi$,

$C_{in} \simeq g_m R'_L C_\mu$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

where

$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})]$

$\simeq r_\pi \parallel R_{sig} \simeq r_\pi$

Thus,

$$f_H \simeq \frac{1}{2\pi g_m R'_L C_\mu r_\pi}$$

$$f_H = \frac{1}{2\pi C_\mu \beta R'_L} \qquad \text{Q.E.D.}$$

(c) $\text{GB} = |A_M| f_H$

$$= \beta \frac{R'_L}{R_{sig}} \frac{1}{2\pi C_\mu \beta R'_L} = \frac{1}{2\pi C_\mu R_{sig}} \qquad \text{Q.E.D.}$$

For $R_{sig} = 25$ kΩ and $C_\mu = 1$ pF,

$$\text{GB} = \frac{1}{2\pi \times 1 \times 10^{-12} \times 25 \times 10^3} = 6.37 \text{ MHz}$$

For $I_C = 1$ mA and $\beta = 100$,

(i) $R'_L = 25$ kΩ:

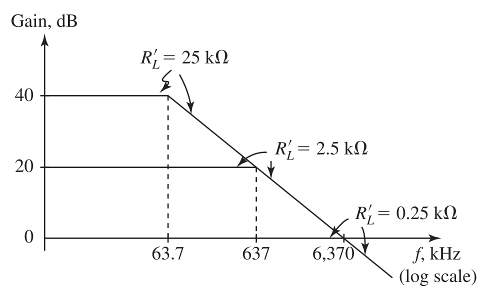
$$A_M = -100 \times \frac{25}{25} = -100 \text{ V/V}$$

$$f_H = \frac{\text{GB}}{|A_M|} = \frac{6.37 \text{ MHz}}{100 \text{ V/V}} = 63.7 \text{ kHz}$$

(ii) $R'_L = 2.5$ kΩ:

$$A_M = -100 \times \frac{2.5}{25} = -10 \text{ V/V}$$

$$f_H = \frac{\text{GB}}{|A_M|} = \frac{6.37 \text{ MHz}}{10 \text{ V/V}} = 637 \text{ kHz}$$



The Bode plots are shown in the figure.

If the midband gain is unity,

$f_H = \text{GB} = 6.37$ MHz

This is obtained when R'_L is

$$1 = 100 \times \frac{R'_L}{25}$$

$\Rightarrow R'_L = 0.25 \text{ k}\Omega = 250\ \Omega$

10.39 $R_B = R_{B1} \parallel R_{B2} = 68 \text{ k}\Omega \parallel 27 \text{ k}\Omega$

$= 19.3$ kΩ

$$g_m = \frac{I_C}{V_T} = \frac{0.8 \text{ mA}}{0.025 \text{ V}} = 32 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{32} = 6.25 \text{ k}\Omega$$

$R'_{sig} = r_\pi \parallel R_B \parallel R_{sig}$

$= 6.25 \text{ k}\Omega \parallel 19.3 \text{ k}\Omega \parallel 10 \text{ k}\Omega$

$= 3.2$ kΩ

$R'_L = R_C \parallel R_L = 4.7 \text{ k}\Omega \parallel 10 \text{ k}\Omega$

$= 3.2$ kΩ

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + (R_{sig} \parallel R_B)} g_m R'_L$$

$$= -\frac{19.3}{19.3 + 10} \frac{6.25}{6.25 + (10 \parallel 19.3)} \times 32 \times 3.2$$

$= -32.8$ V/V

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)}$$

$$1 \times 10^9 = \frac{32 \times 10^{-3}}{2\pi (C_\pi + C_\mu)}$$

$\Rightarrow C_\pi + C_\mu = 5.1$ pF

$C_\pi = 5.1 - 0.8 = 4.3$ pF

$C_{in} = C_\pi + (g_m R'_L + 1) C_\mu$

$= 4.3 + (32 \times 3.2 + 1) \times 0.8$

$= 87$ pF

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 87 \times 10^{-12} \times 3.2 \times 10^3}$$

$= 572$ kHz

10.40 $R_{in} = \dfrac{R}{1 - K}$

$$= \frac{100 \text{ k}\Omega}{1 - 0.9} = 1000 \text{ k}\Omega = 1 \text{ M}\Omega$$

10.41 Using Miller's theorem, we obtain

$$Z_{in} = \frac{Z}{1-A}, \quad Z_{out} = \frac{Z}{1-\frac{1}{A}}$$

For

$$Z = \frac{1}{j\omega C}$$

$$Z_{in} = \frac{1}{j\omega C(1-A)} \Rightarrow C_{in} = C(1-A)$$

$$Z_{out} = \frac{1}{j\omega C\left(1-\frac{1}{A}\right)} \Rightarrow C_{out} = C\left(1-\frac{1}{A}\right)$$

(a) $A = -1000$ V/V, $\quad C = 1$ pF

$$C_{in} = 1(1+1000) = 1001 \text{ pF}$$

$$C_{out} = 1\left(1+\frac{1}{1000}\right) = 1.001 \text{ pF}$$

(b) $A = -10$ V/V, $\quad C = 10$ pF

$$C_{in} = 10(1+10) = 110 \text{ pF}$$

$$C_{out} = 10\left(1+\frac{1}{10}\right) = 11 \text{ pF}$$

(c) $A = -1$ V/V, $\quad C = 10$ pF

$$C_{in} = 10(1+1) = 20 \text{ pF}$$

$$C_{out} = 10(1+1) = 20 \text{ pF}$$

(d) $A = +1$ V/V, $\quad C = 10$ pF

$$C_{in} = C(1-1) = 0$$

$$C_{out} = C(1-1) = 0$$

(e) $A = +10$ V/V, $\quad C = 10$ pF

$$C_{in} = 10(1-10) = -90 \text{ pF}$$

$$C_{out} = 10\left(1-\frac{1}{10}\right) = 9 \text{ pF}$$

The -90 pF input capacitance can be used to cancel an equal (+90 pF) capacitance between the input node and ground.

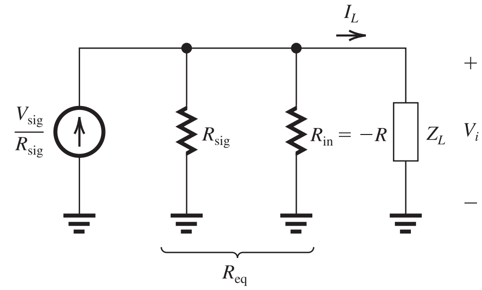


Figure 2

(a) Refer to Fig. 1.

$$I_i = \frac{V_i - V_o}{R} = \frac{V_i - 2V_i}{R} = -\frac{V_i}{R}$$

Thus,

$$R_{in} \equiv \frac{V_i}{I_i} = -R$$

(b) Replacing the signal source with its equivalent Norton's form results in the circuit in Fig. 2. Observe that $R_{eq} = \infty$ when $R_{sig} = R$. In this case,

$$I_L = \frac{V_{sig}}{R_{sig}} = \frac{V_{sig}}{R}$$

(c) If $Z_L = \frac{1}{sC}$,

$$V_i = I_L Z_L$$

$$= \frac{V_{sig}}{R} \times \frac{1}{sC}$$

$$= \frac{1}{sCR} V_{sig}$$

and

$$V_o = 2V_i = \frac{2}{sCR} V_{sig}$$

Thus,

$$\frac{V_o}{V_{sig}} = \frac{2}{sCR}$$

which is the transfer function of an ideal noninverting integrator.

10.42

This figure belongs to Problem 10.42.

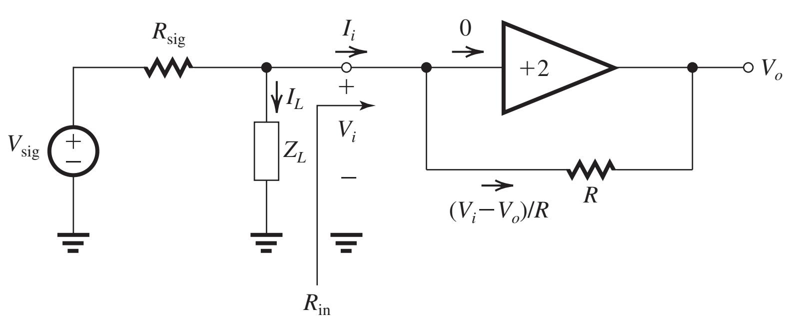
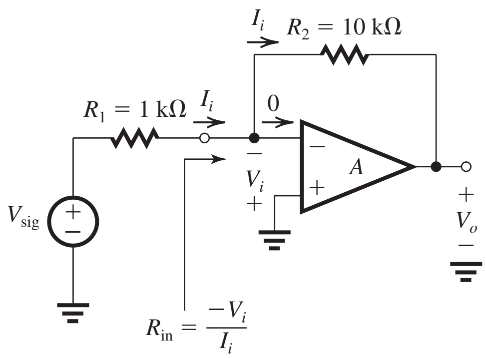


Figure 1

10.43



From the figure we see that

$$V_o = AV_i \qquad (1)$$

From Miller's theorem, we have

$$R_{in} = \frac{R_2}{1 - \left(\frac{V_o}{-V_i}\right)} = \frac{R_2}{1 + \frac{V_o}{V_i}} = \frac{R_2}{1 + A} \qquad (2)$$

Using the voltage divider rule at the input, we get

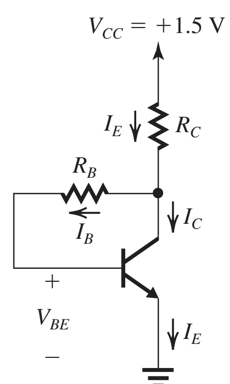
$$-V_i = V_{sig}\frac{R_{in}}{R_{in} + R_1}$$

$$\Rightarrow V_i = -V_{sig}\frac{R_{in}}{R_{in} + R_1} \qquad (3)$$

For each value of A we use Eq. (2) to determine R_{in}, Eq. (3) to determine V_i (for $V_{sig} = 1$ V), Eq. (1) to determine V_o, and finally we calculate the value of V_o/V_{sig}. The results are given in the table below.

A (V/V)	**R_{in} (kΩ)**	**V_i (V)**	**V_o (V)**	**V_o/V_{sig} (V/V)**
10	9.091×10^{-1}	-0.476	-4.76	-4.76
100	9.901×10^{-2}	-0.0901	-9.01	-9.01
1000	9.990×10^{-3}	-9.89×10^{-3}	-9.89	-9.89
10,000	9.999×10^{-4}	-9.989×10^{-4}	-9.99	-9.99

10.44



(a) For the dc analysis, refer to the figure.

$$V_{CC} = I_E R_C + I_B R_B + V_{BE}$$

$$1.5 = I_E \times 1 + \frac{I_E}{\beta + 1} \times 47 + 0.7$$

$$\Rightarrow I_E = \frac{1.5 - 0.7}{1 + \frac{47}{101}} = 0.546 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.546 = 0.54 \text{ mA}$$

(b) $g_m = \frac{I_C}{V_T} = \frac{0.54 \text{ mA}}{0.025 \text{ V}} = 21.6$ mA/V

$$r_\pi = \frac{\beta}{g_m} = \frac{200}{21.6} = 4.63\ \text{k}\Omega$$

(c) $\frac{V_o}{V_b} = -g_m(R_C \parallel R_L)$

$$= -21.6(1 \parallel 1) = -10.8 \text{ V/V}$$

(d) Using Miller's theorem, the component of R_{in} due to R_B can be found as

$$R_{in1} = \frac{R_B}{1 - (V_o/V_b)}$$

$$= \frac{47\ \text{k}\Omega}{1 - (-10.8)} = 4\ \text{k}\Omega$$

$$R_{in} = R_{in1} \parallel r_\pi$$

$$= 4 \parallel 4.63 = 2.14\ \text{k}\Omega$$

(e) $G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{V_o}{V_b}$

$$= \frac{2.14}{2.14 + 1} \times -10.8 = -7.4 \text{ V/V}$$

(f) $C_{in} = C_\pi + \left(1 + \left|\frac{V_o}{V_b}\right|\right) C_\mu$

where

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \times 600 \times 10^6}$$

$$C_\pi + C_\mu = 5.73 \text{ pF}$$

$$C_\pi = 5.73 - 0.8 = 4.93 \text{ pF}$$

$$C_{in} = 4.93 + (1 + 10.8) \times 0.8$$

$$= 14.37 \text{ pF}$$

(g) $R'_{sig} = R_{in} \parallel R_{sig}$

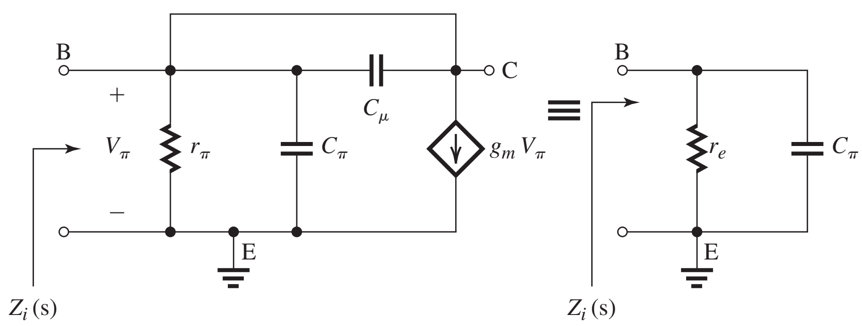
$$= 2.14\ \text{k}\Omega \parallel 1\ \text{k}\Omega = 0.68\ \text{k}\Omega$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.68 \times 10^3}$$

$$= 16.3 \text{ MHz}$$

10.45



From the figure we see that the controlled current-source $g_m V_\pi$ appears across its control voltage V_π, thus we can replace the current source with a resistance $1/g_m$. Now, the parallel equivalent of r_π and $1/g_m$ is

$$\frac{r_\pi(1/g_m)}{r_\pi + \frac{1}{g_m}} = \frac{r_\pi}{g_m r_\pi + 1} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the equivalent circuit simplifies to that of r_e in parallel with C_π,

$$Z_i(s) = \frac{1}{\frac{1}{r_e} + sC_\pi} = \frac{r_e}{1 + sC_\pi r_e}$$

$$Z_i(j\omega) = \frac{r_e}{1 + j\omega C_\pi r_e}$$

$Z_i(j\omega)$ will have a 45° phase at

$$\omega_{45} C_\pi r_e = 1$$

$$\Rightarrow \omega_{45} = \frac{1}{C_\pi r_e}$$

Now,

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

At high bias currents,

$$C_\pi \gg C_\mu$$

and

$$f_T \simeq \frac{g_m}{2\pi C_\pi}$$

Since $g_m \simeq 1/r_e$, we have

$$f_T \simeq \frac{1}{2\pi C_\pi r_e}$$

Thus,

$$f_{45^\circ} \simeq f_T = 400 \text{ MHz}$$

If the bias current is reduced to the value that results in $C_\pi \simeq C_\mu$,

$$f_T = \frac{g_m}{2\pi \times 2C_\pi} = \frac{g_m}{4\pi C_\pi}$$

Again, $g_m \simeq \frac{1}{r_e}$, thus

$$f_T \simeq \frac{1}{4\pi C_\pi r_e}$$

It follows that in this case,

$$f_{45^\circ} = \frac{1}{2} f_T = 200 \text{ MHz}$$

10.46 $A_M = -g_m R'_L$

$= -4 \times 20 = -80$ V/V

$$f_{3dB} = f_H = \frac{1}{2\pi(C_L + C_{gd})R'_L}$$

$$= \frac{1}{2\pi(2 + 0.1) \times 10^{-12} \times 20 \times 10^3}$$

$= 3.8$ MHz

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{4 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 6.4 \text{ GHz}$$

$f_t = |A_M| f_H$

$= 80 \times 3.8 = 304$ MHz

10.47 $f_t = \dfrac{g_m}{2\pi(C_L + C_{gd})}$

$$C_L + C_{gd} = \frac{g_m}{2\pi f_t}$$

$$= \frac{2 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 0.159 \text{ pF}$$

To reduce f_t to 1 GHz, an additional capacitance of 0.159 pF must be connected to the output node. (Doubling the effective capacitance at the output node reduces f_t by a factor of 2.)

10.48 Refer to Fig. P10.48. To determine g_{m1} we use

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.090 \times \frac{100}{1.6} \times 0.1}$$

$= 1.06$ mA/V

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{12.8}{0.1} = 128 \text{ k}\Omega$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{19.2}{0.1} = 192 \text{ k}\Omega$$

$$R'_L = r_{o1} \parallel r_{o2} = 128 \parallel 192 = 76.8 \text{ k}\Omega$$

$$A_M = -g_{m1} R'_L$$

$$= -1.06 \times 76.8 = -81.4 \text{ V/V}$$

$$C_L = C_{db1} + C_{db2} + C_{gd2}$$

$$= 20 + 36 + 15 = 71 \text{ fF}$$

$$f_H = \frac{1}{2\pi (C_L + C_{gd1}) R'_L}$$

$$f_H = \frac{1}{2\pi (71 + 15) \times 10^{-15} \times 76.8 \times 10^3}$$

$$= 24.1 \text{ MHz}$$

$$f_Z = \frac{g_{m1}}{2\pi C_{gd1}} = \frac{1.06 \times 10^{-3}}{2\pi \times 0.015 \times 10^{-12}}$$

$$= 11.2 \text{ GHz}$$

10.49 Figure 1 shows the amplifier high-frequency equivalent circuit. A node equation at the output provides

$$\left(\frac{1}{r_o} + sC_L\right) V_o + g_m V_\pi + sC_\mu (V_o - V_\pi) = 0$$

Replacing V_π by V_i and collecting terms results in

$$V_o \left[\frac{1}{r_o} + s(C_L + C_\mu)\right] = -V_i (g_m - sC_\mu)$$

$$\Rightarrow \frac{V_o}{V_i} = -g_m r_o \frac{1 - s(C_\mu / g_m)}{1 + s(C_L + C_\mu) r_o} \qquad \text{Q.E.D.}$$

For $I_C = 200\ \mu\text{A} = 0.2$ mA and $V_A = 100$ V,

$$g_m = \frac{I_C}{V_T} = \frac{0.2 \text{ mA}}{0.025 \text{ V}} = 8 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.2} = 500 \text{ k}\Omega$$

$$\text{DC gain} = -g_m r_o = 8 \times 500 = -4000 \text{ V/V}$$

$$f_{3dB} = \frac{1}{2\pi (C_L + C_\mu) r_o}$$

$$= \frac{1}{2\pi (1 + 0.2) \times 10^{-12} \times 500 \times 10^3}$$

$$= 265.3 \text{ kHz}$$

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{8 \times 10^{-3}}{2\pi \times 0.2 \times 10^{-12}} = 6.37 \text{ GHz}$$

$$f_t = |A_{dc}| f_{3dB}$$

$$= 4000 \times 265.3 = 1.06 \text{ GHz}$$

The Bode plot is shown in Figure 2.

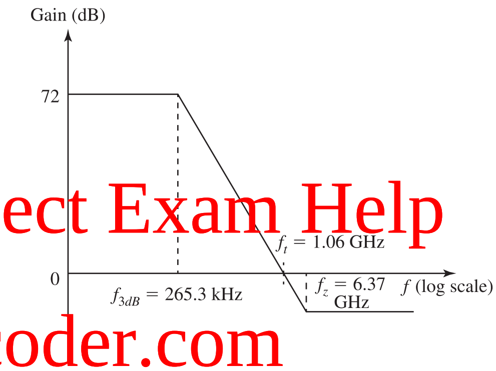


Figure 2

10.50 The equivalent circuit is shown in the figure.

$$g_m = \frac{I_C}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{80} = 1.5 \text{ k}\Omega$$

(a) $$A_M = -\frac{r_\pi}{r_\pi + r_x} g_m R'_L$$

$$-10 = -\frac{1.5}{1.5 + 0.1} \times g_m R'_L$$

$$\Rightarrow g_m R'_L = 10.7 \text{ V/V}$$

$$R'_L = 0.133 \text{ k}\Omega$$

This figure belongs to Problem 10.49, part (a).

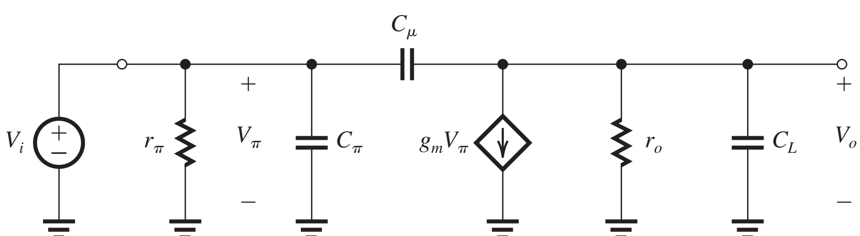
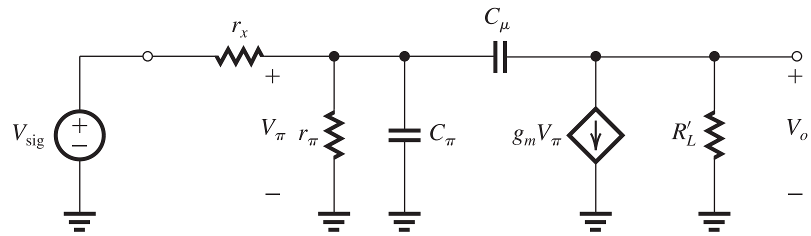


Figure 1

This figure belongs to Problem 10.50.



$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T}$

$= \dfrac{80 \times 10^{-3}}{2\pi \times 2 \times 10^9} = 6.37$ pF

$C_\pi = 6.37 - 1 = 5.37$ pF

$C_{\text{in}} = C_\pi + (g_m R'_L + 1)C_\mu$

$C_{\text{in}} = 5.37 + (10.7 + 1) \times 1$

$= 17.07$ pF

$R'_{\text{sig}} = r_\pi \parallel r_x = 1.5\ \text{k}\Omega \parallel 0.1\ \text{k}\Omega$

$= 0.094\ \text{k}\Omega$

$f_H = \dfrac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$

$= \dfrac{1}{2\pi \times 17.07 \times 10^{-12} \times 0.094 \times 10^3}$

$= 99.2$ MHz

(b) If $|A_M|$ is reduced to 1, we obtain

$1 = \dfrac{1.5}{1.6} \times g_m R'_L$

$\Rightarrow g_m R'_L = 1.07$

$C_{\text{in}} = C_\pi + (g_m R'_L + 1)C_\mu$

$= 5.37 + (1.07 + 1) \times 1$

$= 7.44$ pF

$f_H = \dfrac{1}{2\pi \times 7.44 \times 10^{-12} \times 0.094 \times 10^3}$

$= 227.6$ MHz

10.51 $A_M = 40$ dB $\Rightarrow 100$ V/V

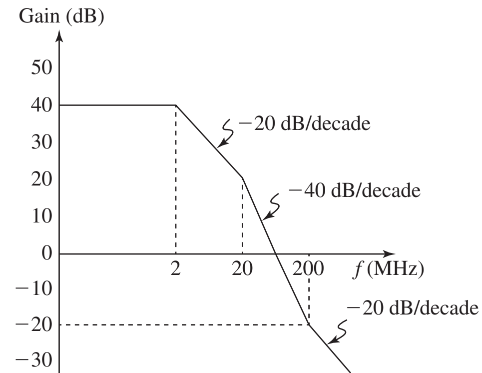
$A(s) =$

$$100 \frac{1 + s/(2\pi \times 200 \times 10^6)}{\left(1 + \dfrac{s}{2\pi \times 2 \times 10^6}\right)\left(1 + \dfrac{s}{2\pi \times 20 \times 10^6}\right)}$$

Since $f_{P1} \ll f_{P2} \ll f_Z$, we have

$f_{3dB} \simeq f_{P1} = 2$ MHz

The Bode plot is shown in the figure.



10.52 (a) $A(s) = 1000 \dfrac{1}{1 + s/(2\pi \times 10^5)}$

(b)

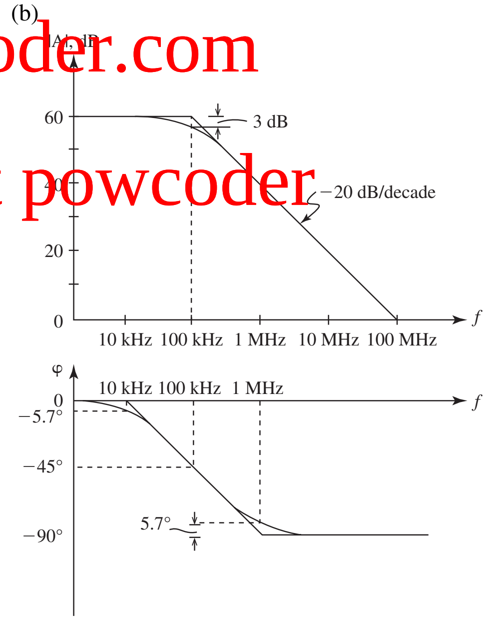


Figure 1

Figure 1 shows the Bode plot for the gain magnitude and phase.

(c) GB = 1000 × 100 kHz = 100 MHz

(d) The unity-gain frequency f_t is

$f_t = 100$ MHz

(e)

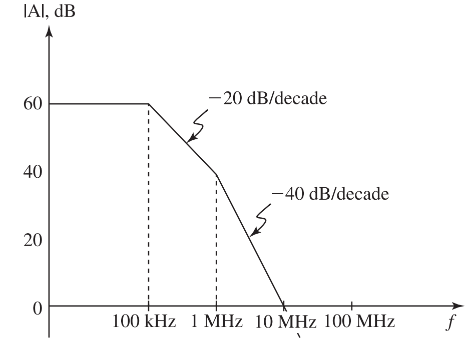


Figure 2

Figure 2 shows the magnitude response when a second pole at 1 MHz appears in the transfer function. The unity-gain frequency f_t now is

$f_t = 10$ MHz

which is different from the gain-bandwidth product,

GB = 100 MHz

10.53 Using the dominant-pole approximation,

$\omega_H \simeq \omega_{P1}$

Using the root-sum-of-squares formula, we get

$$\omega_H \simeq \frac{1}{\sqrt{\dfrac{1}{\omega_{P1}^2} + \dfrac{1}{\omega_{P2}^2}}} = \frac{\omega_{P1}}{\sqrt{1 + \left(\dfrac{\omega_{P1}}{\omega_{P2}}\right)^2}}$$

(a) For a difference of 10%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\dfrac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.9\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 4.26$$

(b) For a difference of 1%,

$$\frac{\omega_{P1}}{\sqrt{1 + \left(\dfrac{\omega_{P1}}{\omega_{P2}}\right)^2}} = 0.99\omega_{P1}$$

$$\Rightarrow \frac{\omega_{P2}}{\omega_{P1}} = 49.3$$

10.54 $A(s) = -10^3 \dfrac{1 + \dfrac{s}{10^4}}{\left(1 + \dfrac{s}{10^3}\right)\left(1 + \dfrac{s}{10^5}\right)}$

(a) $\omega_H \simeq 10^3$ rad/s

(b) $\omega_H = 1 \Bigg/ \sqrt{\left(\dfrac{1}{10^6} + \dfrac{1}{10^{10}}\right) - \dfrac{2}{10^8}}$

= 1010 Hz

If the frequency of the finite zero is lowered to 10^3 rad/s the zero will cancel the pole at 10^3 rad/s and the transfer function becomes

$$A(s) = -10^3 \frac{1}{1 + \dfrac{s}{10^5}}$$

The 3-dB frequency now becomes

$\omega_{3dB} = 10^5$ rad/s

10.55 If at $\omega = 10^7$ rad/s the excess phase due to the 3 coincident poles (at frequency ω_P) is 30°, then each pole is contributing 10°. Thus,

$$\tan^{-1}\frac{10^7}{\omega_P} = 10°$$

$$\omega_P = \frac{10^7}{\tan 10°} = 5.67 \times 10^7 \text{ rad/s}$$

10.56 $\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L}$

where

$C_{gs} = 30$ fF

$R_{gs} = R'_{sig} = 10$ kΩ

$C_{gd} = 5$ fF

$R_{gd} = R'_{sig}(1 + g_m R'_L) + R'_L$

$= 10(1 + 2 \times 20) + 20$

$= 430$ kΩ

$C_L = 30$ fF

$R_{CL} = R'_L = 20$ kΩ

Thus,

$\tau_H = 30 \times 10 + 5 \times 430 + 30 \times 20$

= 3050 ps

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 3050 \times 10^{-12}}$$

= 52.2 MHz

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{2 \times 10^{-3}}{2\pi \times 5 \times 10^{-15}} = 63.7 \text{ GHz}$$

10.57 $A_M = -\dfrac{R_G}{R_G + R_{sig}} g_m R'_L$

$$= -\frac{0.65}{0.65 + 0.15} \times 5 \times 10$$

$= -40.6$ V/V

$\tau_{gs} = C_{gs}R_{gs}$

$= C_{gs}R'_{\text{sig}}$

$= C_{gs}(R_{\text{sig}} \parallel R_G)$

$= 2 \times 10^{-12} \times (150 \parallel 650) \times 10^3$

$= 243.8 \text{ ns}$

$\tau_{gd} = C_{gd}R_{gd}$

$= C_{gd}[R'_{\text{sig}}(1 + g_m R'_L) + R'_L]$

$= 0.5 \times 10^{-12}[(150 \parallel 650)(1+5 \times 10)+10] \times 10^3$

$= 3112.8 \text{ ns}$

$\tau_{CL} = C_L R'_L$

$= 30 \times 10^{-12} \times 10 \times 10^3$

$= 300 \text{ ns}$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 243.8 + 3112.8 + 300$

$= 3656.6 \text{ ns}$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 3656.6 \times 10^{-9}}$$

$= 43.5 \text{ MHz}$

10.58 The figure shows the equivalent circuit of the two-stage amplifier where we have modeled each stage as a transconductance amplifier. At node 1:

$R_{\text{eq1}} = R_{\text{sig}} \parallel R_{\text{in1}}$

$= 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 5 \text{ k}\Omega$

$C_{\text{eq1}} = C_{\text{in1}} + C_{\text{sig}} = 10 + 10 = 20 \text{ pF}$

Thus,

$$f_{P1} = \frac{1}{2\pi C_{\text{eq1}} R_{\text{eq1}}}$$

$$f_{P1} = \frac{1}{2\pi \times 20 \times 10^{-12} \times 5 \times 10^3}$$

$= 1.59 \text{ MHz}$

At node 2:

$R_{\text{eq2}} = R_{o1} \parallel R_{\text{in2}} = 2 \text{ k}\Omega \parallel 10 \text{ k}\Omega$

$= 1.67 \text{ k}\Omega$

$C_{\text{eq2}} = C_{o1} + C_{\text{in2}}$

$= 2 + 10 = 12 \text{ pF}$

$$f_{P2} = \frac{1}{2\pi C_{\text{eq2}} R_{\text{eq2}}}$$

$$= \frac{1}{2\pi \times 12 \times 10^{-12} \times 1.67 \times 10^3}$$

$= 7.94 \text{ MHz}$

At node 3:

$R_{\text{eq3}} = R_{o2} \parallel R_L = 2 \text{ k}\Omega \parallel 1 \text{ k}\Omega = 0.67 \text{ k}\Omega$

$C_{\text{eq3}} = C_{o2} + C_L = 2 + 7 = 9 \text{ pF}$

$$f_{P3} = \frac{1}{2\pi C_{\text{eq3}} R_{\text{eq3}}}$$

$$f_{P3} = \frac{1}{2\pi \times 9 \times 10^{-12} \times 0.67 \times 10^3}$$

$= 26.4 \text{ MHz}$

Thus, the three poles have frequencies 1.59 MHz, 7.94 MHz, and 26.4 MHz. Since the frequency of the second pole is more than two octaves higher than that of the first pole, the 3-dB frequency will be mostly determined by f_{P1},

$f_{3dB} \simeq f_{P1} = 1.59 \text{ MHz}$

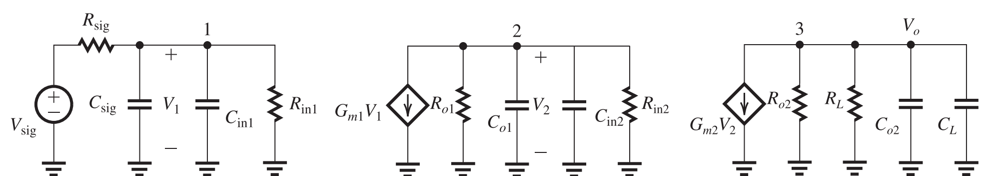
A slightly better estimate of f_{3dB} can be determined using the root-sum-of-squares formula,

$$f_{3dB} = 1/\sqrt{\left(\frac{1}{f_{P1}}\right)^2 + \left(\frac{1}{f_{P2}}\right)^2 + \left(\frac{1}{f_{P3}}\right)^2}$$

$$= 1/\sqrt{\frac{1}{1.59^2} + \frac{1}{7.94^2} + \frac{1}{26.4^2}}$$

$= 1.56 \text{ MHz}$

This figure belongs to Problem 10.58.



10.59 $A_M = -g_m R'_L$

$= -4 \times 20 = -80$ V/V

$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$

$= 2 + 0.1(80 + 1)$

$= 10.1$ pF

Using the Miller approximation, we obtain

$$f_H \simeq \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 10.1 \times 10^{-12} \times 20 \times 10^3}$$

$= 788$ kHz

Using the open-circuit time constants, we get

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R'_{\text{sig}}$

$= 2 \times 20 = 40$ ns

$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 80) + 20 = 1640$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1640 = 164$ ns

$\tau_{CL} = C_L R'_L$

$= 2 \times 20 = 40$ ns

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL} \qquad (1)$$

$= 40 + 164 + 40 = 244$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 244 \times 10^{-9}} = 652 \text{ kHz}$$

The estimate obtained using the open-circuit time constants is more appropriate as it takes into account the effect of C_L. We note from Eq. (1) that τ_{CL} is 16.4% of τ_H, thus C_L has a significant effect on the determination of f_H.

10.60 $\tau_{gs} = C_{gs} R_{gs} = C_{gs} R'_{\text{sig}}$

$= 5 \times 10 = 50$ ns

$R_{gd} = R'_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 10(1 + 5 \times 10) + 10$

$= 520$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 1 \times 520 = 520$ ns

$R_{C_L} = C_L R_L$

$= 5 \times 10 = 50$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 50 + 520 + 50 = 620$ ns

$$f_H = \frac{1}{2\pi \tau_H} \qquad (1)$$

$$= \frac{1}{2\pi \times 620 \times 10^{-9}} = 257 \text{ kHz}$$

The interaction of R'_{sig} with the input capacitance contributes all of τ_{gs} (50 ns) and a significant part of τ_{gd}, namely

$C_{gd}[R'_{\text{sig}}(1 + g_m R'_L)] = 1 \times 10(1 + 50) = 510$ ns

Thus, the total contribution of R'_{sig} is

$50 + 510 = 560$ ns

which is $\frac{560}{620} = 90.3\%$ of τ_H. To double f_H, we must reduce τ_H to half of its value:

$\tau_H = \frac{1}{2} \times 620 = 310$ ns

Now,

$$\tau_H = R'_{\text{sig}}[C_{gs} + C_{gd}(1 + g_m R'_L)] + C_{gd} R'_L + C_L R'_L$$

$310 = R'_{\text{sig}}[5 + 1(1 + 50)] + 1 \times 10 + 5 \times 10$

$\Rightarrow R'_{\text{sig}} = 4.46$ kΩ

10.61 To lower f_H from 135.5 MHz (see Example 10.8) to 100 MHz, τ_H must be increased to

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 100 \times 10^6}$$

$= 1591.5$ ps

Now,

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$1591.5 = 200 + 725 + \tau_{CL}$

$\Rightarrow \tau_{CL} = 666.5$

But,

$\tau_{CL} = C'_L R'_L$

$665.5 = C'_L \times 10$

$\Rightarrow C'_L = 66.6$ fF

Thus, the original C_L of 25 fF must be increased by

$66.6 - 25 = 41.6$ fF

10.62 We will assume that the value given in the problem statement is for R_{sig} (not R'_{sig}):

$R_{\text{sig}} = 5$ kΩ

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$R'_{sig} = r_\pi \parallel R_{sig} = 5\ \text{k}\Omega \parallel 5\ \text{k}\Omega$

$= 2.5\ \text{k}\Omega$

$\tau_H = C_\pi R_\pi = C_\pi R'_{sig}$

$= 10 \times 2.5 = 25\ \text{ns}$

$\tau_\mu = C_\mu R_\mu$

$= C_\mu [R'_{sig}(1 + g_m R'_L) + R'_L]$

$= 1 \times [2.5(1 + 20 \times 5) + 5]$

$= 257.5\ \text{ns}$

$\tau_{CL} = C_L R'_L$

$= 10 \times 5 = 50\ \text{ns}$

$\tau_H = \tau_\pi + \tau_\mu + \tau_{CL}$

$= 25 + 257.5 + 50 = 332.5\ \text{ns}$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 332.5 \times 10^{-9}} = 479\ \text{kHz}$$

$$A_M = -\frac{r_\pi}{r_\pi + R_{sig}} g_m R'_L$$

$$= -\frac{5}{5+5} \times 20 \times 5$$

$= -50\ \text{V/V}$

10.63 Refer to Fig. 10.11(a). Since R_B is not specified, we assume that its value is very large.

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R'_L$$

where

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\ \text{k}\Omega$$

Thus,

$$A_M = -\frac{2.5}{2.5 + 0.1 + 1} \times 40 \times 5$$

$= -138.9\ \text{V/V}$

Using the Miller approximation, we obtain

$C_{in} = C_\pi + C_\mu(1 + g_m R'_L)$

$= 10 + 0.3(1 + 40 \times 5)$

$= 70.3\ \text{pF}$

$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$

$= 2.5 \parallel (0.1 + 1) = 0.76\ \text{k}\Omega$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 70.3 \times 10^{-12} \times 0.76 \times 10^3}$$

$= 2.98\ \text{MHz}$

Using the open-circuit time constants, we get

$\tau_\pi = C_\pi R_\pi = C_\pi R'_{sig}$

$= 10 \times 0.76 = 7.6\ \text{ns}$

$\tau_\mu = C_\mu [R'_{sig}(g_m R'_L + 1) + R'_L]$

$= 0.3[0.76(40 \times 5 + 1) + 5]$

$= 47.3\ \text{ns}$

$\tau_{C_L} = C_L R'_L = 3 \times 5 = 15\ \text{ns}$

$\tau_H = \tau_\pi + \tau_\mu + \tau_{C_L}$

$= 7.6 + 47.3 + 15 = 69.9\ \text{ns}$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 69.9 \times 10^{-9}} = 2.28\ \text{MHz}$$

This is a more realistic estimate of f_H as it takes into account the effect of C_L.

10.64 CS amplifier with:

$R'_{sig} = r_o/2$

$R'_L = r_o/2$

$C_{gs} = C_{gd} = 0.1\ \text{pF}$

Using the Miller approximation, we get

$C_{in} = C_{gs} + C_{gd}(g_m R'_L + 1)$

$$= 0.1 + 0.1\left(\frac{1}{2}\, g_m r_o + 1\right)$$

$$= 0.1\left(\frac{1}{2}\, g_m r_o + 2\right)$$

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$$

$$= \frac{1}{2\pi \times 0.1\left(\frac{1}{2}\, g_m r_o + 2\right)(r_o/2) \times 10^{-9}}$$

$$= \frac{1}{0.1\pi\left(\frac{1}{2}\, g_m r_o + 2\right) r_o \times 10^{-9}}$$

where r_o is in kΩ.

For initial design,

$g_m = 2\ \text{mA/V}, \quad r_o = 20\ \text{k}\Omega$

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2} \times 40 + 2\right) \times 20 \times 10^{-9}}$$

$= 7.23\ \text{MHz}$

(i) For the case, I is reduced by a factor of 4:
Since

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

reducing I_D by a factor of 4 reduces g_m by a factor of 2,

$$g_m = 1 \text{ mA/V}$$

Since

$$r_o = \frac{V_A}{I_D},$$

reducing I_D by a factor of 4, increases r_o by a factor of 4,

$$r_o = 80 \text{ k}\Omega$$

Thus,

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2} \times 80 + 2\right) \times 80 \times 10^{-9}}$$

$$= 0.95 \text{ MHz}$$

(ii) For the case, I is increased by a factor of 4, g_m increases by a factor of 2,

$$g_m = 4 \text{ mA/V}$$

and r_o decreases by a factor of 4,

$$r_o = 5 \text{ k}\Omega$$

Thus,

$$f_H = \frac{1}{0.1\pi\left(\frac{1}{2} \times 20 + 2\right) \times 5 \times 10^{-9}}$$

$$= 53.1 \text{ MHz}$$

10.65 $R'_L = r_o \parallel R_L = 20 \text{ k}\Omega \parallel 12 \text{ k}\Omega$

$$= 7.5 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs}R_{gs} = C_{gs}R'_{\text{sig}}$$

$$= 0.2 \times 100 = 20 \text{ ns}$$

$$\tau_{gd} = C_{gd}[R'_{\text{sig}}(g_m R'_L + 1) + R'_L]$$

$$= 0.2[100(1.5 \times 7.5 + 1) + 7.5]$$

$$= 246.5 \text{ ns}$$

(a) $C_L = 0$

$$\tau_{C_L} = 0$$

$$\tau_H = \tau_{gs} + \tau_{gd} = 20 + 246.5 = 266.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 266.5 \times 10^{-9}} = 597 \text{ kHz}$$

(b) $C_L = 10$ pF

$$\tau_{C_L} = C_L R'_L = 10 \times 7.5 = 75 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= 20 + 246.5 + 75 = 341.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 341.5 \times 10^{-9}} = 466 \text{ kHz}$$

(c) $C_L = 50$ pF

$$\tau_{C_L} = C_L R'_L = 50 \times 7.5 = 375 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 20 + 246.5 + 375$$

$$= 641.5 \text{ ns}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 641.5 \times 10^{-9}} = 248 \text{ kHz}$$

Using the Miller approximation, since C_L is not taken into account, then for all three cases we obtain

$$C_{\text{in}} = C_{gs} + C_{gd}(g_m R'_L + 1)$$

$$= 0.2 + 0.2(1.5 \times 7.5 + 1)$$

$$= 2.65 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R'_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 2.65 \times 10^{-12} \times 100 \times 10^3}$$

$$= 600 \text{ kHz}$$

which is very close to the estimate obtained using the method of open-circuit time constants for the case $C_L = 0$. However, as C_L is increased, the estimate obtained using the Miller approximation becomes less and less realistic, which is due to the fact that it does not take C_L into account.

10.66 Refer to Fig. 10.26(c).

$$\frac{V_o}{V_{\text{sig}}} = \frac{1/g_m}{\frac{1}{g_m} + R_{\text{sig}}} g_m R_L$$

$$= \frac{1/5}{\frac{1}{5} + 1} \times 5 \times 10$$

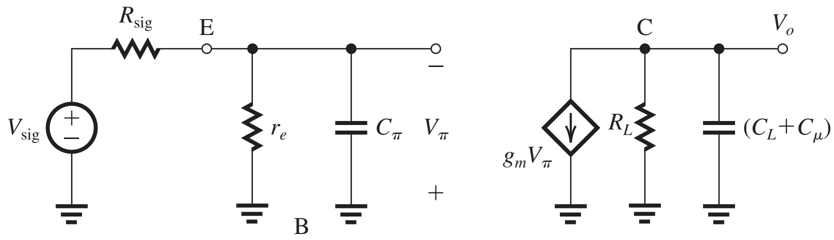
$$= 8.3 \text{ V/V}$$

$$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{\text{sig}} \parallel \frac{1}{g_m}\right)}$$

$$f_{P1} = \frac{1}{2\pi \times 4 \times 10^{-12}\left(1 \parallel \frac{1}{5}\right) \times 10^3}$$

$$= 239 \text{ MHz}$$

This figure belongs to Problem 10.67.



$$f_{P2} = \frac{1}{2\pi (C_L + C_{gd}) R_L}$$

$$= \frac{1}{2\pi (2 + 0.2) \times 10^{-12} \times 10 \times 10^3}$$

$= 7.23$ MHz

Since $f_{P1} \gg f_{P2}$, f_{P2} will be dominant and

$f_H \simeq f_{P2} = 7.23$ MHz

10.67 See figure above. Replacing the BJT with its high-frequency T model while neglecting r_o and r_x results in the equivalent circuit shown in the figure.

(a) There are two separate poles, one at the input given by

$$f_{P1} = \frac{1}{2\pi C_\pi (R_{\text{sig}} \parallel r_e)}$$

and the other at the output, given by

$$f_{P2} = \frac{1}{2\pi (C_L + C_\mu) R_L} \quad \text{Q.E.D.}$$

(b) $I_C = 1$ mA,

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 25\ \Omega$$

$$f_{P1} = \frac{1}{2\pi \times 10 \times 10^{-12} (1 \parallel 0.025) \times 10^3}$$

$= 652.5$ MHz

$$f_{P2} = \frac{1}{2\pi (1 + 1) \times 10^{-12} \times 10 \times 10^3}$$

$= 7.96$ MHz

Since $f_{P2} \ll f_{P1}$, f_{P2} will be dominant and

$f_H \simeq f_{P2} = 7.96$ MHz

10.68 Refer to Fig. 10.26 with

$R_L = r_o$

$R_{\text{sig}} = r_o/2$

$C_L = C_{gs}$

Now,

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{r_o + r_o}{1 + g_m r_o}$$

$$\simeq \frac{2r_o}{g_m r_o} = \frac{2}{g_m}$$

$$R_{gs} = R_{\text{sig}} \parallel R_{\text{in}}$$

$$= \frac{r_o}{2} \parallel \frac{2}{g_m}$$

$$= \frac{\frac{r_o}{2} \times \frac{2}{g_m}}{\frac{r_o}{2} + \frac{2}{g_m}} = \frac{r_o}{\frac{1}{2} g_m r_o + 2}$$

$$\simeq \frac{r_o}{\frac{1}{2} g_m r_o} = \frac{2}{g_m}$$

$$\tau_{gs} = C_{gs} R_{gs} \simeq \frac{2C_{gs}}{g_m}$$

$$R_o = r_o + R_{\text{sig}} + g_m r_o R_{\text{sig}}$$

$$R_o = r_o + \frac{1}{2} r_o + \frac{1}{2} g_m r_o r_o$$

$$\simeq \frac{1}{2} g_m r_o^2$$

$$R_{gd} = R_L \parallel R_o$$

$$= r_o \parallel \frac{1}{2} g_m r_o^2$$

$$\simeq r_o$$

$$\tau_{gd} = (C_L + C_{gd}) R_{gd}$$

$$= (C_{gs} + C_{gd}) r_o$$

$$\tau_H = \tau_{gs} + \tau_{gd}$$

$$= \frac{2C_{gs}}{g_m} + (C_{gs} + C_{gd}) r_o$$

Since $g_m r_o \gg 1$, we obtain

$$\tau_H \simeq (C_{gs} + C_{gd}) r_o$$

and

$$f_H = \frac{1}{2\pi \tau_H}$$

$$f_H = \frac{1}{2\pi (C_{gs} + C_{gd}) r_o}$$

Since

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

then

$$f_H = \frac{f_T}{g_m r_o} \qquad \text{Q.E.D.}$$

10.69 Refer to Example 10.9. To reduce f_H to 200 MHz, τ_H must become

$$\tau_H = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 200 \times 10^6}$$

$$= 795.8 \text{ ps}$$

Since τ_{gs} remains constant at 26.6 ps, τ_{CL} must be increased to

$$\tau_{CL} = 795.8 - 26.6 = 769.2 \text{ ps}$$

But,

$$\tau_{CL} = (C_{gd} + C_L)R_{gd}$$

thus,

$$769.2 = (5 + C_L) \times 18.7$$

$$\Rightarrow C_L + 5 = 41.1 \text{ fF}$$

$$C_L = 36.1 \text{ fF}$$

Thus, the amount of additional capacitance to be connected at the output is

$$36.1 - 25 = 11.1 \text{ fF}$$

10.70 $r_o = \dfrac{V_A}{I_D} = \dfrac{10}{0.1} = 100 \text{ k}\Omega$

$$R_{\text{sig}} = r_o/2 = 50 \text{ k}\Omega$$

$$R_L = r_o = 100 \text{ k}\Omega$$

$$g_m r_o = 1.5 \times 100 = 150$$

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{100 + 100}{1 + 150} = 1.32 \text{ k}\Omega$$

$$R_o = R_{\text{sig}} + r_o + g_m r_o R_{\text{sig}}$$

$$= 50 + 100 + 150 \times 50 = 7650 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs}(R_{\text{in}} \parallel R_{\text{sig}})$$

$$\tau_{gs} = 0.2(1.32 \parallel 50)$$

$$= 0.26 \text{ ns}$$

$$R_{gd} = R_L \parallel R_o$$

$$= 100 \parallel 7650 = 98.7 \text{ k}\Omega$$

$$\tau_{gd} = (C_{gd} + C_L + C_{db})R_{gd}$$

$$= (0.015 + 0.03 + 0.02) \times 98.7$$

$$= 6.42 \text{ ns}$$

$$\tau_H = \tau_{gs} + \tau_{gd} = 0.26 + 6.42$$

$$= 6.68 \text{ ns}$$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 6.68 \times 10^{-9}} = 23.8 \text{ MHz}$$

10.71 $R_o = r_{o2} + r_{o1} + (g_{m2} r_{o2}) r_{o1}$

$$= 2r_o + g_m r_o^2$$

$$= 2 \times 20 + 2 \times 20 \times 20 = 840 \text{ k}\Omega$$

$$A_v = -g_{m1}(R_o \parallel R_L)$$

$$= -2(840 \parallel 1000)$$

$$= -913 \text{ V/V}$$

Using Eq. (10.109), we obtain

$$\tau_H = R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})]$$
$$+ R_{d1}(C_{gd1} + C_{db1} + C_{gs2})$$
$$+ (R_L \parallel R_o)(C_L + C_{gd2})$$

where

$$R_{d1} = r_{o1} \parallel R_{\text{in2}}$$

$$R_{\text{in2}} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{20 + 1000}{1 + 2 \times 20} = 24.9 \text{ k}\Omega$$

$$R_{d1} = 20 \parallel 24.9 = 11.1 \text{ k}\Omega$$

Thus,

$$\tau_H = 100[20 + 5(1 + 2 \times 11.1)]$$
$$+ 11.1(5 + 5 + 20)$$
$$+ (1000 \parallel 840)(20 + 5)$$

$$= 13587 + 333 + 11413$$

$$= 25,333 \text{ ps} = 25.33 \text{ ns}$$

$$f_H = \frac{1}{2\pi \times 25.33 \times 10^{-9}} = 6.28 \text{ MHz}$$

10.72 (a) $A_M = -g_m R_L'$

where

$$R_L' = R_L \parallel r_o = 20 \parallel 20 = 10 \text{ k}\Omega$$

Thus,

$$A_M = -4 \times 10 = -40 \text{ V/V}$$

$$\tau_{gs} = C_{gs} R_{gs}$$

$$= C_{gs} R_{\text{sig}} = 2 \times 20 = 40 \text{ ns}$$

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 4 \times 10) + 10$

$= 830\ \text{k}\Omega$

$\tau_{gd} = C_{gd} R_{gd} = 0.2 \times 830 = 166\ \text{ns}$

$\tau_{C_L} = C_L R'_L = 1 \times 10 = 10\ \text{ns}$

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= 40 + 166 + 10 = 216\ \text{ns}$

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 216 \times 10^{-9}} = 737\ \text{kHz}$$

$\text{GB} \equiv |A_M| f_H = 40 \times 737 = 29.5\ \text{MHz}$

(b) $A_M = -g_{m1}(R_o \parallel R_L)$

where

$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1}$

$= 2r_o + g_m r_o^2$

$= 2 \times 20 + 4 \times 20 \times 20 = 1640\ \text{k}\Omega$

$A_M = -4(1640 \parallel 20) = -79\ \text{V/V}$

$$R_{\text{in}2} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{20 + 20}{1 + 4 \times 20} \simeq 0.49\ \text{k}\Omega$$

$R_{d1} = r_{o1} \parallel R_{\text{in}2} = 20 \parallel 0.49 = 0.48\ \text{k}\Omega$

Using Eq. (10.109), we obtain

$\tau_H = R_{\text{sig}}[C_{gs1} + C_{gd1}(1 + g_{m1} R_{d1})]$

$\quad + R_{d1}(C_{gd1} + C_{db1} + C_{gs2})$

$\quad + (R_L \parallel R_o)(C_L + C_{gd2})$

$= 20[2 + 0.2(1 + 4 \times 0.48)]$

$\quad + 0.48(0.2 + 0.2 + 2)$

$\quad + (20 \parallel 1640)(1 + 0.2)$

$\tau_H = 51.7 + 1.15 + 23.7 = 76.6\ \text{ns}$

$$f_H = \frac{1}{2\pi \times 76.6 \times 10^{-9}} = 2.08\ \text{MHz}$$

$\text{GB} \equiv |A_M| f_H = 79 \times 2.08 = 164\ \text{MHz}$

Note the increase in bandwidth and in GB.

10.73 $20 \log |A_M| = 74\ \text{dB}$

$\Rightarrow |A_M| = 5000$

$R_o \simeq (g_m r_o) r_o$

$R_L = R_o$

$|A_M| = g_m(R_L \parallel R_o)$

$$5000 = \frac{1}{2} g_m R_o$$

$$= \frac{1}{2}(g_m r_o)^2$$

$\Rightarrow g_m r_o = 100$

$$\frac{2V_A}{V_{OV}} = 100$$

$$\Rightarrow V_{OV} = \frac{2 \times 10}{100} = 0.2\ \text{V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times 50 \times 0.2^2$$

$= 0.2\ \text{mA}$

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2\ \text{mA/V}$$

$$f_t = \frac{2 \times 10^{-3}}{2\pi(1 + 0.1) \times 10^{-12}}$$

$= 289.4\ \text{MHz}$

$$f_{3dB} = \frac{f_t}{|A_M|} = \frac{289.4}{5000} = 57.9\ \text{kHz}$$

If the cascode transistor is removed,

$A_M = -g_m(r_o \parallel R_L)$

$\simeq -g_m r_o = -100\ \text{V/V}$

10.74 (a) For the CS amplifier,

$$|A_M| = g_m(r_o \parallel r_o) = \frac{1}{2} g_m r_o$$

$$f_H = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

$$= \frac{1}{2\pi\left[C_{gs} + C_{gd}\left(\frac{1}{2} g_m r_o + 1\right)\right] R_{\text{sig}}} \quad (1)$$

For the cascode amplifier,

$|A_M| = g_m(R_o \parallel r_o)$

$= g_m[(g_m r_o) r_o \parallel r_o]$

$\simeq g_m r_o$

Thus, the gain increases by a factor of 2.

$$f_H = \frac{1}{2\pi C_{\text{in}} R_{\text{sig}}}$$

where

$$C_{\text{in}} = C_{gs} + C_{gd}(1 + g_m R_{d1})$$

$$R_{d1} = r_o \parallel R_{\text{in2}}$$

$$= r_o \parallel \frac{R_L + r_o}{g_m r_o}$$

$$R_{d1} = r_o \parallel \frac{r_o + r_o}{g_m r_o}$$

$$= r_o \parallel \frac{2}{g_m} \simeq \frac{2}{g_m}$$

$$C_{\text{in}} = C_{gs} + C_{gd}\left(1 + g_m \times \frac{2}{g_m}\right)$$

$$= C_{gs} + 3C_{gd}$$

$$f_H = \frac{1}{2\pi(C_{gs} + 3C_{gd})R_{\text{sig}}} \qquad (2)$$

From (1) and (2), the ratio N of f_H of the cascode amplifier to f_H of the CS amplifier is

$$N = \frac{C_{gs} + C_{gd}\left(\frac{1}{2}g_m r_o + 1\right)}{C_{gs} + 3C_{gd}}$$

Thus,

$$N \simeq \frac{C_{gs} + \frac{1}{2}(g_m r_o)C_{gd}}{C_{gs} + 3C_{gd}} \qquad \text{Q.E.D}$$

(b) $50 = \frac{1}{2}g_m r_o$

$$\Rightarrow g_m r_o = 100$$

$$N = \frac{C_{gs} + \frac{1}{2} \times 100 \times 0.1C_{gs}}{C_{gs} + 3 \times 0.1C_{gs}}$$

$$= \frac{1 + 5}{1 + 0.3} = 4.6$$

(c) $g_m r_o = \dfrac{2V_A}{V_{OV}}$

$$100 = \frac{2 \times 10}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 0.4 \times 10 \times 0.2^2$$

$$= 0.08 \text{ mA} = 80\ \mu\text{A}$$

10.75 (a) From Fig. 10.29, we have

$$f_t = \frac{g_m}{2\pi(C_L + C_{gd})}$$

$$= \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{2\pi(C_L + C_{gd})}\sqrt{I_D} \qquad \text{Q.E.D.} \qquad (1)$$

(b) $g_m = \sqrt{2\mu_n C_{ox}(W/L)}\,\sqrt{I_D}$ (2)

$$V_{OV} = \sqrt{\frac{I_D}{\frac{1}{2}\mu_n C_{ox}(W/L)}} \qquad (3)$$

$$r_o = \frac{V_A}{I_D} \qquad (4)$$

$$R_o = (g_m r_o)r_o \qquad (5)$$

$$A_M = -g_m(R_o \parallel R_L)$$

$$= -g_m(R_o \parallel R_o)$$

$$= -\frac{1}{2}g_m R_o \qquad (6)$$

Substituting

$\mu_n C_{ox} = 0.4$ mA/V^2, $W/L = 20$, $C_L = 20$ fF,

$C_{gd} = 5$ fF and $V_A = 10$ V in Eqs. (1)–(6), we obtain the following results in the table below.

10.76 Refer to Fig. 10.30.

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5 \text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{\text{sig}}}g_m(\beta r_o \parallel R_L)$$

$$= -\frac{2.5}{2.5 + 0.05 + 5}40(100 \times 100 \parallel 2)$$

$$\simeq -26.5 \text{ (V/V)}$$

This table belongs to Problem 10.75, part (b).

I_D (mA)	f_t (GHz)	V_{OV} (V)	g_m (mA/V)	r_o (kΩ)	R_o (MΩ)	A_M (V/V)	f_H (MHz)
0.1	8	0.16	1.26	100	12.6	−7938	1
0.2	11.5	0.22	1.80	50	4.5	−4050	2.8
0.5	18	0.35	2.83	20	1.13	−1600	11.3

$R'_{sig} = r_\pi \parallel (r_x + R_{sig})$

$= 2.5 \parallel (0.05 + 5) = 1.67\text{ k}\Omega$

$R_{\pi 1} = R'_{sig} = 1.67\text{ k}\Omega$

$$R_{c1} = r_{o1} \parallel \left[r_{e2} \frac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)} \right]$$

$$= 100 \parallel \left[0.025 \frac{100 + 2}{100 + \dfrac{2}{101}} \right]$$

$= 25.5\ \Omega$

$R_{\mu 1} = R'_{sig}(1 + g_{m1}R_{c1}) + R_{c1}$

$= 1.67(1 + 40 \times 0.0255) + 0.0255$

$= 3.4\text{ k}\Omega$

$R_o = \beta_2 r_{o2} = 100 \times 100 = 10{,}000\text{ k}\Omega$

$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + (C_{cs1} + C_{\pi 2})R_{c1}$

$\quad + (C_L + C_{cs2} + C_{\mu 2})(R_L \parallel R_o)$

$= 10 \times 1.67 + 2 \times 3.4 + (0 + 10) \times 0.0255$

$\quad + (0 + 0 + 2)(2 \parallel 10{,}000)$

$= 16.7 + 6.8 + 0.255 + 4 = 27.8\text{ ns}$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 27.8 \times 10^{-9}}$$

$= 5.7\text{ MHz}$

10.77 (a) Gain from base to collector of $Q_1 = -1$. Thus,

$C_{in} = C_{\pi 1} + C_{\mu 1}(1 + 1)$

$= C_{\pi 1} + 2C_{\mu 1}$

$$f_{P1} = \frac{1}{2\pi R'_{sig} C_{in}}$$

$$= \frac{1}{2\pi R'_{sig}(C_{\pi 1} + 2C_\mu)} \qquad \text{Q.E.D.}$$

At the output node, the total capacitance is $(C_L + C_{c2} + C_{\mu 2})$ and since r_o is large, R_o will be very large, thus the total resistance will be R_L. Thus the pole introduced at the output node will have a frequency f_{P2},

$$f_{P2} = \frac{1}{2\pi(C_L + C_{c2} + C_{\mu 2})R_L} \qquad \text{Q.E.D.}$$

(b) $I = 1$ mA

$$g_m = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\text{ k}\Omega$$

(i) $R_{sig} = 1\text{ k}\Omega$

$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 1 = 0.71\text{ k}\Omega$

$$f_{P1} = \frac{1}{2\pi \times 0.71 \times 10^3(10 + 2 \times 2) \times 10^{-12}}$$

$= 16$ MHz

$$f_{P2} = \frac{1}{2\pi(0 + 0 + 2) \times 10^{-12} \times 2 \times 10^3}$$

$\simeq 40$ MHz

$$f_H = 1 \Bigg/ \sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2}}$$

$$= 1 \Bigg/ \sqrt{\frac{1}{16^2} + \frac{1}{40^2}} = 14.9\text{ MHz}$$

(ii) $R_{sig} = 10\text{ k}\Omega$

$R'_{sig} = r_\pi \parallel R_{sig} = 2.5 \parallel 10 = 2\text{ k}\Omega$

$$f_{P1} = \frac{1}{2\pi \times 2 \times 10^3(10 + 4) \times 10^{-12}}$$

$= 5.7$ MHz

$f_{P2} = 40$ MHz

$$f_H = 1 \Bigg/ \sqrt{\frac{1}{5.7^2} + \frac{1}{40^2}} = 5.6\text{ MHz}$$

10.78 Refer to Fig. 10.30.

$I_C = 0.1$ mA

$$g_m = \frac{0.1\text{ mA}}{0.025\text{ V}} = 4\text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_D} = \frac{100\text{ V}}{0.1\text{ mA}} = 1000\text{ k}\Omega$$

$$r_e \simeq \frac{1}{g_m} = 0.25\text{ k}\Omega$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m(\beta r_o \parallel R_L)$$

$$= -\frac{r_\pi}{r_\pi + r_\pi} g_m(\beta r_o \parallel \beta r_o)$$

$$= -\frac{1}{4}\beta g_m r_o$$

$$= -\frac{1}{4} \times 100 \times 4 \times 1000$$

$= -100{,}000$ V/V

$$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}} = r_\pi \parallel r_\pi = \frac{1}{2}r_\pi = 12.5 \text{ k}\Omega$$

$$R_{\pi 1} = R'_{\text{sig}} = 12.5 \text{ k}\Omega$$

$$R_{c1} = r_o \parallel r_e\left(\frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}\right)$$

$$= 1000 \parallel 0.25\left(\frac{r_o + \beta r_o}{r_o + \dfrac{\beta}{\beta+1}r_o}\right)$$

$$\simeq 1000 \parallel 0.25 \times \frac{\beta + 1}{2}$$

$$= 1000 \parallel 12.5 = 12.3 \text{ k}\Omega$$

$$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1}R_{c1}) + R_{c1}$$

$$= 12.5(1 + 4 \times 12.3) + 12.3 = 639.8 \text{ k}\Omega$$

$$R_o = \beta r_o = 100 \times 1000 = 100 \text{ M}\Omega$$

$$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + (C_{cs1} + C_{\pi 2})R_{c1}$$
$$+ (C_L + C_{c2} + C_{\mu 2})(R_L \parallel R_o)$$

To determine C_π, we use

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$1 \times 10^9 = \frac{4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 0.64 \text{ pF}$$

$$C_\pi = 0.64 - 0.1 = 0.54 \text{ pF}$$

$$\tau_H = 0.54 \times 12.5 + 0.1 \times 639.8 + 0.54 \times 12.3$$
$$+ 0.1 \times \frac{1}{2} \times 100 \times 1000$$

$$= 6.8 + 64 + 6.6 + 5000 \text{ ns}$$

Obviously the last term, which is due to the pole at the output node, is dominant. The frequency of the output pole is

$$f_P = \frac{1}{2\pi \times 5000 \times 10^{-9}} = 31.8 \text{ kHz}$$

$$f_H \simeq f_P = 31.8 \text{ MHz}$$

Because the other poles are at much higher frequencies, an estimate of the unity-gain frequency can be found as

$$f_t = |A_M|f_P = 10^5 \times 31.8 \times 10^3 = 3.18 \text{ GHz}$$

This estimate of f_t is not very good (too high!). The other three poles have frequencies much lower than 3.18 GHz and will cause the gain to decrease faster, reaching the 0 dB value at a frequency lower than 3.18 GHz. Also note that f_T of the BJTs is 1 GHz and the models we use for the BJT do not hold at frequencies approaching f_T.

10.79 $A_M = \dfrac{R'_L}{R'_L + \dfrac{1}{g_m}}$

where

$$R'_L = R_L \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 2 \parallel 20 = 1.82 \text{ k}\Omega$$

$$A_M = \frac{1.82}{1.82 + \dfrac{1}{5}} = 0.91 \text{ V/V}$$

$$R_o = r_o \parallel \frac{1}{g_m} = 20 \parallel \frac{1}{5} \simeq 0.2 \text{ k}\Omega = 200 \ \Omega$$

$$f_Z = \frac{g_m}{2\pi C_{gs}}$$

$$= \frac{5 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 398 \text{ MHz}$$

Next, we evaluate b_1 and b_2:

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R'_L + 1}\right)R_{\text{sig}} + \left(\frac{C_{gs} + C_L}{g_m R'_L + 1}\right)R'_L$$

$$= \left(0.1 + \frac{2}{5 \times 1.82 + 1}\right)20 + \left(\frac{2+1}{5 \times 1.82 + 1}\right) \times 1.82$$

$$= 5.96 + 0.54 = 6.50 \times 10^{-9} \text{ s}$$

$$b_2 = \frac{(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}}{g_m R'_L + 1}R_{\text{sig}}R'_L$$

$$= \frac{(2 + 0.1) \times 1 + 2 \times 0.1}{5 \times 1.82 + 1} \times 20 \times 1.82$$

$$= 8.3 \times 10^{-18}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.3}}{6.5} = 0.44$$

Thus, the poles are real and their frequencies can be obtained by finding the roots of the polynomial $(1 + b_1 s + b_2 s^2)$

$$= 1 + 6.5 \times 10^{-9}s + 8.3 \times 10^{-18}s^2$$

which are

$$\omega_{P1} = 0.21 \times 10^9 \text{ rad/s}$$

and

$\omega_{P2} = 0.57 \times 10^9$ rad/s

Thus,

$$f_{P1} = \frac{\omega_{P1}}{2\pi} = 33.4 \text{ MHz}$$

$$f_{P2} = \frac{\omega_{P2}}{2\pi} = 90.7 \text{ MHz}$$

Since the two poles are relatively close to each other, an estimate of f_H can be obtained using

$$f_H = 1 \Big/ \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$= 31.6$ MHz

10.80 $f_H \simeq f_{P1} \simeq \dfrac{1}{2\pi b_1}$

where

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R'_L + 1}\right) R_{\text{sig}} + \left(\frac{C_{gs} + C_L}{g_m R'_L + 1}\right) R'_L$$

For $C_L = 0$,

$$b_1 = C_{gd} R_{\text{sig}} + \frac{C_{gs}}{g_m R'_L + 1}(R_{\text{sig}} + R'_L)$$

For $R_{\text{sig}} \gg R'_L$,

$$b_1 \simeq C_{gd} R_{\text{sig}} + \frac{C_{gs}}{g_m R'_L + 1} R_{\text{sig}}$$

$$= \left(C_{gd} + \frac{C_{gs}}{g_m R'_L + 1}\right) R_{\text{sig}}$$

and

$$f_H = \frac{1}{2\pi R_{\text{sig}}\left(C_{gd} + \dfrac{C_{gs}}{g_m R'_L + 1}\right)} \qquad \text{Q.E.D.}$$

For the given numerical values,

$$f_H = \frac{1}{2\pi \times 100 \times 10^3 \left[10 + \dfrac{2}{5 \times (2 \parallel 20) + 1}\right] \times 10^{-12}}$$

$= 156$ kHz

(Note: An error was made in the first printing of the book and the values of C_{gs} and C_{gd} were exchanged. The above value of f_H corresponds to the numbers in the first printing.)

For $C_{gs} = 10$ pF and $C_{gd} = 2$ pF,

$$f_H = \frac{1}{2\pi \times 100 \times 10^3 \left[2 + \dfrac{10}{5 \times (2 \parallel 20) + 1}\right] \times 10^{-12}}$$

$= 532$ kHz

10.81 Refer to Fig. 10.31(c). Replacing C_{gs} with an input capacitance between G and ground, we get

$$C_{\text{eq}} = C_{gs}(1 - K)$$

where

$$K = \frac{g_m R'_L}{1 + g_m R'_L}$$

then

$$C_{\text{eq}} = C_{gs}/(1 + g_m R'_L)$$

and the total input capacitance becomes

$$C_{\text{in}} = C_{gd} + C_{\text{eq}}$$

$$= C_{gd} + \frac{C_{gs}}{1 + g_m R'_L}$$

The frequency of the input pole is

$$f_{P1} = \frac{1}{2\pi R_{\text{sig}}\left(C_{gd} + \dfrac{C_{gs}}{1 + g_m R'_L}\right)}$$

$$f_H \simeq f_{P1}$$

10.82 For a maximally flat response we have

$$Q = \frac{1}{\sqrt{2}}$$

$\omega_0 = \omega_{3dB} = 2\pi \times 10^6$ rad/s

Thus, the transfer function will be

$$\frac{V_o(s)}{V_i(s)} = \frac{\text{dc gain}}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$= \frac{0.8}{s^2 + s\sqrt{2} \times 2\pi \times 10^6 + (2\pi \times 10^6)^2}$$

$$= \frac{0.8}{s^2 + s\ 8.886 \times 10^6 s + 39.48 \times 10^{12}}$$

10.83 With $g_{mb} = 0$ and r_o large, we obtain

$$R'_L \simeq R_L$$

and

$$A_M = \frac{g_m R_L}{g_m R_L + 1}$$

For $A_M = 0.9$,

$$0.9 = \frac{g_m R_L}{g_m R_L + 1}$$

$$\Rightarrow g_m R_L = 9$$

Now, for a maximally-flat response, $Q = 1/\sqrt{2}$.
Using the expression for Q in Eq. (10.129), we get

$$Q = \frac{\sqrt{g_m R_L + 1}\sqrt{[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]R_{\text{sig}}R_L}}{[C_{gs} + C_{gd}(g_m R_L + 1)]R_{\text{sig}} + (C_{gs} + C_L)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{9+1}\sqrt{[(10+1)10 + 10 \times 1] \times 100 \times R_L}}{[10 + 1(9+1)] \times 100 + (10+10)R_L}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}\sqrt{R_L}}{10\left(1 + \dfrac{R_L}{100}\right)}$$

$$\Rightarrow \left(\frac{R_L}{100}\right)^2 - 4\left(\frac{R_L}{100}\right) + 1 = 0$$

This equation results in two solutions,

$R_L = 27\ \text{k}\Omega$ and $R_L = 373\ \text{k}\Omega$

The second answer is not very practical as it implies the transistor is operating at $g_m = 9/373 = 0.024$ mA/V, a very small transconductance! We will pursue only the first answer. Thus

$$R_L = 27\ \text{k}\Omega$$

$$g_m = 0.33\ \text{mA/V}$$

and the 3-dB frequency is found using Eq (10.127):

$$f_{3dB} = f_0 = \frac{1}{2\pi\sqrt{b_2}}$$

$$\omega_{3dB} = \sqrt{\frac{g_m R_L + 1}{R_{\text{sig}}R_L[(C_{gs} + C_{gd})C_L + C_{gs}C_{gd}]}}$$

$$= \sqrt{\frac{9+1}{100 \times 27[(10+1) \times 10 + 10 \times 1] \times 10^6 \times 10^{-24}}}$$

$$= 5.55\ \text{Mrad/s}$$

$$f_{3dB} = 884\ \text{kHz}$$

10.84 Refer to Fig. 10.33.

$$I_C = 1\ \text{mA}$$

$$g_m = 40\ \text{mA/V}, \quad r_e = 25\ \Omega$$

$$r_\pi = \frac{100}{40} = 2.5\ \text{k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{20}{1} = 20\ \text{k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$2 \times 10^9 = \frac{40 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$\Rightarrow C_\pi + C_\mu = 3.2\ \text{pF}$$

$$C_\pi = 3.2 - 0.1 = 3.1\ \text{pF}$$

$$R'_L = R_L \parallel r_o = 1 \parallel 20 = 0.95\ \text{k}\Omega$$

$$R'_{\text{sig}} = R_{\text{sig}} + r_x = 1 + 0.1 = 1.1\ \text{k}\Omega$$

$$A_M = \frac{R'_L}{R'_L + r_e + \dfrac{R'_{\text{sig}}}{\beta + 1}}$$

$$A_M = \frac{0.95}{0.95 + 0.025 + \dfrac{1.1}{101}}$$

$$= 0.96\ \text{V/V}$$

$$f_Z = \frac{1}{2\pi C_\pi r_e}$$

$$= \frac{1}{2\pi \times 3.1 \times 10^{-12} \times 25}$$

$$\simeq 2\ \text{GHz}$$

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \dfrac{R'_L}{r_e}\right)\right]R'_{\text{sig}} + \left[C_\mu + C_L\left(1 + \dfrac{R'_{\text{sig}}}{r_\pi}\right)\right]R'_L}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{\text{sig}}}{r_\pi}}$$

$$= \frac{\left[3.1 + 0.1\left(1 + \dfrac{0.95}{0.025}\right)\right] \times 1.1 + (3.1 + 0) \times 0.95}{1 + \dfrac{0.95}{0.025} + \dfrac{1.1}{2.5}}$$

$$= 0.27 \times 10^{-9}$$

$$b_2 = \frac{[(C_\pi + C_\mu)C_L + C_\pi C_\mu]R'_L R'_{\text{sig}}}{1 + \dfrac{R'_L}{r_e} + \dfrac{R'_{\text{sig}}}{r_\pi}}$$

$$= \frac{(0 + 3.1 \times 0.1) \times 0.95 \times 1.1}{1 + \dfrac{0.95}{0.025} + \dfrac{1.1}{2.5}}$$

$$= 8.2 \times 10^{-21}$$

$$Q = \frac{\sqrt{b_2}}{b_1} = \frac{\sqrt{8.2 \times 10^{-21}}}{0.27 \times 10^{-9}} = 0.335$$

Thus, the poles are real and their frequencies can be found as the roots of the polynomial $(1 + b_1 s + b_2 s^e)$

$$= 1 + 0.27 \times 10^{-9}s + 8.2 \times 10^{-21}s^2$$

$$= \left(1 + \frac{s}{\omega_{P1}}\right)\left(1 + \frac{s}{\omega_{P2}}\right)$$

$\Rightarrow \omega_{P1} = 4.25 \times 10^9$ rad/s

$\omega_{P2} = 28.6 \times 10^9$ rad/s

Thus,

$f_{P1} = 676$ MHz

$f_{P2} = 4.6$ GHz

Thus,

$f_{3dB} \simeq f_{P1} = 676$ MHz

10.85 $I = 0.4$ mA

(a) $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$0.2 = \frac{1}{2} \times 0.4 \times 16V_{OV}^2$

$\Rightarrow V_{OV} = 0.25$ V

$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.25}$

$= 1.6$ mA/V

(b) $r_o = \frac{V_A}{I_D} = \frac{20}{0.2}$

$= 100$ kΩ

$R_D \parallel r_o = 10 \parallel 100$

$= 9.1$ kΩ

$A_d = g_m(R_D \parallel r_o)$

$= 1.6 \times 9.1 = 14.5$ V/V

(c) R_{sig} small and the frequency response is determined by the output pole:

$$f_{P2} = \frac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$$

$$= \frac{1}{2\pi(100 + 5 + 5) \times 10^{-15} \times 9.1 \times 10^3}$$

$= 159$ MHz

$f_H \simeq 159$ MHz

(d) $R_{\text{sig}} = 40$ kΩ

$\tau_{gs} = C_{gs}R_{gs}$

$= C_{gs}R_{\text{sig}}$

$= 40 \times 10^{-15} \times 40 \times 10^3$

$= 1.6$ ns

$R_{gd} = R_{\text{sig}}(g_m R'_L + 1) + R'_L$

$= 40(1.6 \times 9.1 + 1) + 9.1$

$= 631.5$ kΩ

$\tau_{gd} = C_{gd}R_{gd} = 5 \times 631.5 = 3.16$ ns

$\tau_{CL} = (C_L + C_{db})R'_L$

$= (100 + 5) \times 9.1$

$= 955.5$ ps $= 0.96$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 1.6 + 3.16 + 0.96 = 5.72$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 5.72 \times 10^{-9}}$$

$= 27.8$ MHz

10.86 The common-mode gain will have a zero at

$$f_Z = \frac{1}{2\pi R_{SS}C_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \times 1 \times 10^{-12}}$$

$= 1.59$ MHz

Thus, the CMRR will have two poles, one at f_Z, i.e. at 1.59 MHz, and the other at the dominant pole of A_d, 20 MHz. Thus, the 3-dB frequency of CMRR will be approximately equal to f_Z,

$f_{3dB} = 1.59$ MHz

10.87 At low frequencies,

$A_d = 100$ V/V

$A_{cm} = 0.1$ V/V

$\text{CMRR} = \frac{A_d}{A_{cm}} = 1000$ or 60 dB

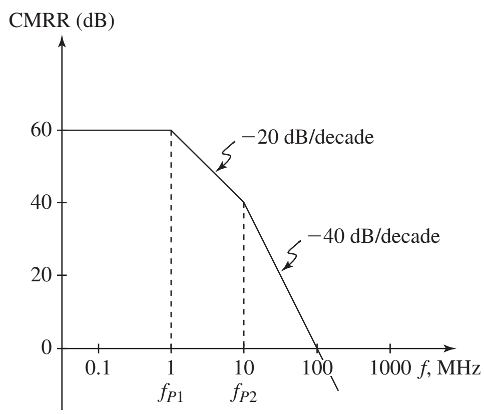
The first pole of CMRR is coincident with the zero of the common-mode gain,

$f_{P1} = 1$ MHz

The second pole is coincident with the dominant pole of the differential gain,

$f_{P2} = 10$ MHz

A sketch for the Bode plot for the gain magnitude is shown in the figure.



10.88 $R_{SS} = \dfrac{V_A}{I} = \dfrac{40}{0.1}$

$= 400$ kΩ

$C_{SS} = 100$ fF

$$f_Z = \frac{1}{2\pi C_{SS} R_{SS}}$$

$$= \frac{1}{2\pi \times 100 \times 10^{-15} \times 400 \times 10^3} = 4 \text{ MHz}$$

If V_{OV} of the current source is reduced by a factor of 2 while I remains unchanged, (W/L) must be increased by a factor of 4. Assume L remains unchanged, W must be increased by a factor of 4. Since C_{SS} is proportional to W, its value will be quadrupled:

$C_{SS} = 400$ fF

The output resistance R_{SS} will remain unchanged. Thus, f_Z will decrease by a factor of 4 to become

$f_Z = 1$ MHz

10.89 $I = 80$ µA

$\dfrac{W}{L} = 100, \quad k_n' = 0.2 \text{ mA/V}^2, \quad V_A = 20 \text{ V},$

$C_{gs} = 50 \text{ fF}, \quad C_{gd} = 10 \text{ fF}, \quad C_{db} = 10 \text{ fF},$

$R_D = 20$ kΩ,

$C_L = 100$ fF

(a) $I_D = \dfrac{1}{2} k_n' \left(\dfrac{W}{L}\right) V_{OV}^2$

$0.040 = \dfrac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.063$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.04}{0.063} = 1.27 \text{ mA/V}$$

(b) $r_o = \dfrac{V_A}{I_D} = \dfrac{20}{0.04} = 500$ kΩ

$A_d = g_m(R_D \parallel r_o)$

$= 1.27(20 \parallel 500) = 1.27 \times 19.2 = 24.4$ V/V

(c) $f_H \simeq f_{P2} = \dfrac{1}{2\pi(C_L + C_{gd} + C_{db})(R_D \parallel r_o)}$

$$= \frac{1}{2\pi(100 + 10 + 10) \times 10^{-15} \times 19.2 \times 10^3}$$

$= 69.1$ MHz

(d) $R_{sig} = 100$ kΩ

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}} = 50 \times 100 = 5$ ns

$R_{gd} = R_{\text{sig}}(1 + g_m R_L') + R_L'$

$= 100(1 + 1.27 \times 19.2) + 19.2$

$= 2559$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 10 \times 2559$

$= 25.6$ ns

$\tau_{CL} = (C_L + C_{db}) R_L'$

$= (100 + 10) \times 19.2 = 2.1$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 5 + 25.6 + 2.1 = 32.7$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 32.7 \times 10^{-9}}$$

$= 4.9$ MHz

10.90 $g_m = \frac{I_C}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{10} = 10 \text{ k}\Omega$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

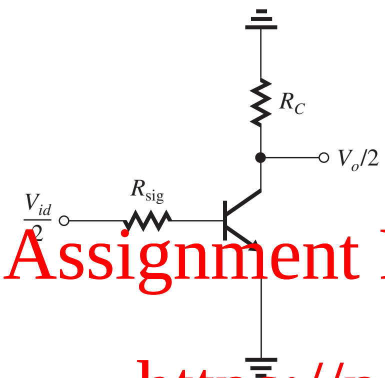
$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$

$= \frac{10 \times 10^{-3}}{2\pi \times 500 \times 10^6}$

$= 3.2 \text{ pF}$

$C_\pi = 3.2 - 0.5 = 2.7 \text{ pF}$

(a)



The figure shows the differential half-circuit and its high-frequency equivalent circuit.

(b) $A_d \equiv \frac{V_o}{V_{id}} = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m R_C$

$= -\frac{10}{10 + 0.1 + 10} \times 10 \times 10$

$= -49.8 \text{ V/V}$

(c) $C_{in} = C_\pi + C_\mu(1 + g_m R_C)$

$= 2.7 + 0.5(1 + 10 \times 10)$

$= 53.2 \text{ pF}$

$f_H = \frac{1}{2\pi C_{in} R'_{sig}}$

where

$R'_{sig} = r_\pi \parallel (R_{sig} + r_x)$

$= 10 \parallel 10.1 \simeq 5 \text{ k}\Omega$

$f_H = \frac{1}{2\pi \times 53.2 \times 10^{-12} \times 5 \times 10^3} = 598 \text{ kHz}$

$\text{GB} = |A_d| f_H = 49.8 \times 598 = 29.8 \text{ MHz}$

10.91 The common-mode gain will have a zero at

$f_Z = \frac{1}{2\pi \times 1 \times 10^6 \times 1 \times 10^{-12}}$

$= 159 \text{ kHz}$

Thus, the CMRR will have two poles: The first will be coincident with the zero of A_{cm},

$f_{P1} = 159 \text{ kHz}$

and the second will be coincident with the pole of A_d,

$f_{P2} = 2 \text{ MHz}$

10.92 $g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}} = \frac{I}{V_{OV1,2}}$

$= \frac{0.2 \text{ mA}}{0.2 \text{ V}} = 1 \text{ mA/V}$

$r_{o2} = r_{o4} = \frac{|V_A|}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$

$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$= 1(100 \parallel 100) = 50 \text{ V/V}$

$f_{P1} = \frac{1}{2\pi C_L R_o}$

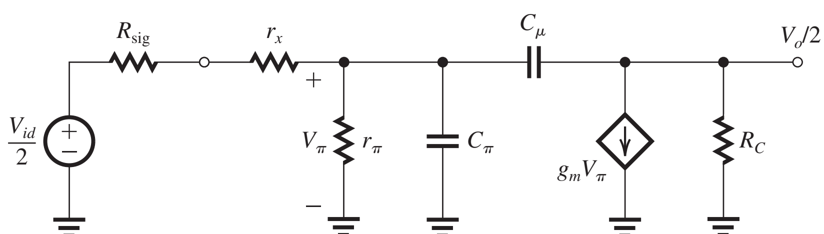
$= \frac{1}{2\pi C_L(r_{o2} \parallel r_{o4})}$

$= \frac{1}{2\pi \times 0.2 \times 10^{-12}(100 \parallel 100) \times 10^3}$

$= \frac{1}{2\pi \times 0.2 \times 50 \times 10^{-9}} = 15.9 \text{ MHz}$

$f_{P2} = \frac{g_{m3}}{2\pi C_m}$

This figure belongs to Problem 10.90.



where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|} = 1 \text{ mA/V}$$

$$f_{P2} = \frac{1 \times 10^{-3}}{2\pi \times 0.1 \times 10^{-12}} = 1.59 \text{ GHz}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = 2 \times 1.59 = 3.18 \text{ GHz}$$

10.93 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$$g_{m1,2} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

$$A_d = \frac{I}{|V_{OV}|}\left(\frac{2|V_A|}{I} \parallel \frac{2|V_A|}{I}\right)$$

$$A_d = \frac{|V_A|}{|V_{OV}|}$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

where

$$R_o = r_{o2} \parallel r_{o4} = \frac{|V_A|}{I}$$

$$f_{P1} = \frac{I}{2\pi C_L |V_A|} \quad (1)$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m}$$

where

$$g_{m3} = \frac{2I_D}{|V_{OV}|} = \frac{I}{|V_{OV}|}$$

$$C_m = \frac{C_L}{4}$$

$$f_{P2} = \frac{4I}{2\pi C_L |V_{OV}|} \quad (2)$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} = 2f_{P2} = \frac{8I}{2\pi C_L |V_{OV}|} \quad (3)$$

Dividing (2) by (1), we obtain

$$\frac{f_{P2}}{f_{P1}} = 4\frac{|V_A|}{|V_{OV}|} = 4\,A_d \qquad \text{Q.E.D.}$$

Since $f_{P2} = 4\,A_d\,f_{P1}$, the unity-gain frequency f_t is equal to GB, thus

$$f_t = A_d\,f_{P1}$$

$$= \frac{|V_A|}{|V_{OV}|}\,\frac{I}{2\pi C_L |V_A|}$$

$$f_t = \frac{I/|V_{OV}|}{2\pi C_L}$$

$$= \frac{g_m}{2\pi C_L} \qquad \text{Q.E.D.}$$

For the numerical values given, we have

$$A_d = \frac{20}{0.2} = 100 \text{ V/V}$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.2}{0.2} = 1 \text{ mA/V}$$

$$f_{P1} = \frac{I}{2\pi C_L |V_A|}$$

$$= \frac{0.2 \times 10^{-3}}{2\pi \times 100 \times 10^{-15} \times 20}$$

$= 15.9$ MHz

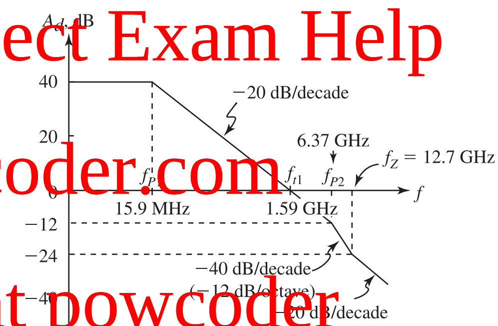
$f_t = 15.9 \times 100 = 1.59$ GHz

$f_{P2} = 4A_d f_{P1} = 4 \times 100 \times 15.9$

$= 6.37$ GHz

$f_Z = 2f_{P2} = 12.7$ GHz

A sketch of the Bode plot for $|A_d|$ is shown in the figure.



10.94 See figure on next page. The mirror high-frequency equivalent circuit is shown in the figure. Note that we have neglected r_x and r_o. The model of the diode-connected transistor Q_1 reduces to r_{e1} in parallel with $C_{\pi 1}$.

To obtain the current-transfer function $I_o(s)/I_i(s)$, we first determine V_π in terms of I_i. Observe that the short-circuit at the output causes $C_{\mu 2}$ to appear in parallel with $C_{\pi 1}$ and $C_{\pi 2}$. Thus,

$$V_\pi = I_i(s)\frac{1}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})} \quad (1)$$

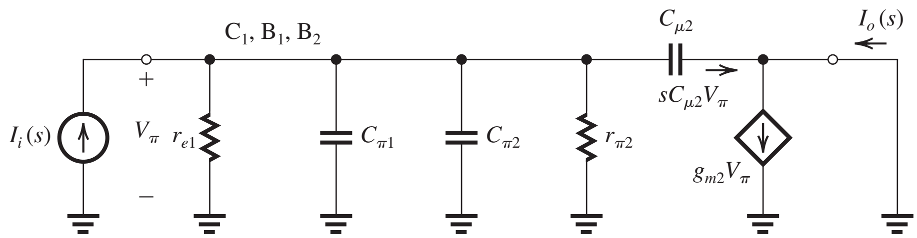
At the output node we have

$$I_o(s) = g_{m2}V_\pi - sC_{\mu 2}V_\pi \quad (2)$$

Combining Eqs. (1) and (2) gives

$$\frac{I_o(s)}{I_i(s)} = \frac{g_{m2} - sC_{\mu 2}}{\left(\frac{1}{r_{e1}} + \frac{1}{r_{\pi 2}}\right) + s(C_{\pi 1} + C_{\pi 2} + C_{\mu 2})}$$

This figure belongs to Problem 10.94.



Since the two transistors are operating at approximately equal dc bias currents, their small-signal parameters will be equal, thus

$$\frac{I_o(s)}{I_i(s)} = \frac{g_m - sC_\mu}{\frac{1}{r_e}\left(1 + \frac{1}{\beta+1}\right) + s(2C_\pi + C_\mu)}$$

$$= \frac{g_m r_e}{1 + \frac{1}{\beta+1}} \; \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

$$= \frac{\alpha}{1 + \frac{1}{\beta+1}} \; \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

$$= \frac{1}{1 + 2/\beta} \; \frac{1 - s(C_\mu/g_m)}{1 + s\left[(2C_\pi + C_\mu)r_e/\left(1 + \frac{1}{\beta+1}\right)\right]}$$

Thus we see that the low-frequency transmission is

$$\frac{I_o}{I_i}(0) = \frac{1}{1 + \frac{2}{\beta}}$$

as expected. The pole is at f_P,

$$f_P \simeq \frac{1}{2\pi\left[(2C_\pi + C_\mu)r_e/\left(1 + \frac{1}{\beta}\right)\right]}$$

and the zero is at

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

For the numerical values given,

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$r_e \simeq 25\ \Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6}$$

$$= 12.7\text{ pF}$$

$$C_\pi = 12.7 - 2 = 10.7\text{ pF}$$

$$f_P = \frac{1}{2\pi[(2 \times 10.7 + 2) \times 10^{-12} \times 25/1.01]}$$

$$= \frac{1.01 \times 10^{12}}{2\pi \times 23.4 \times 25}$$

$$= 274.8\text{ MHz}$$

$$f_Z = \frac{g_m}{2\pi C_\mu}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 2 \times 10^{-12}} = 3.18\text{ GHz}$$

10.95 Refer to Eqs. (10.146)–(10.153). For our case,

$$G_m = \frac{g_m}{1 + g_m R_s} \quad (1)$$

R_o = very large

$$R'_L = R_L \parallel R_o = R_L$$

$$A_m = -G_m R_L = \frac{-g_m R_L}{1 + g_m R_s} \quad (2)$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R_L) + R_L \quad (3)$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s}{1 + g_m R_s} \quad (4)$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{gd}$$

$$f_H = \frac{1}{2\pi\tau_H}$$

For the numerical values given:

(a) $R_s = 0$

$$G_m = g_m = 5\text{ mA/V}$$

$$A_M = -g_m R_L = -5 \times 5 = -25\text{ V/V}$$

$$R_{gd} = 100(1 + 5 \times 5) + 5 = 2605\text{ k}\Omega$$

$$\tau_H = 10 \times 100 + 2 \times 2605 = 6.21\text{ ns}$$

$$f_H = \frac{1}{2\pi \times 6.21 \times 10^{-9}} = 26.6\text{ MHz}$$

$$\text{GB} = 26.6 \times 25 = 641\text{ MHz}$$

(b) $R_s = 100\ \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.1} = 3.33\text{ mA/V}$$

$A_M = -3.33 \times 5 = -16.7$ V/V

$R_{gd} = 100(1 + 3.33 \times 5) + 5$

$= 1771.7$ kΩ

$$R_{gs} = \frac{100 + 0.1}{1 + 5 \times 0.1} = 66.7 \text{ k}\Omega$$

$\tau_H = 10 \times 66.7 + 2 \times 1771.7 = 4.21$ ns

$$f_H = \frac{1}{2\pi \times 4.21 \times 10^{-9}} = 37.8 \text{ MHz}$$

GB $= 631$ MHz

(c) $R_s = 200\ \Omega$

$$G_m = \frac{5}{1 + 5 \times 0.2} = 2.5 \text{ mA/V}$$

$A_M = -2.5 \times 5 = -12.5$ V/V

$R_{gd} = 100(1 + 2.5 \times 5) + 5 = 1355$ kΩ

$$R_{gs} = \frac{100 + 0.2}{1 + 5 \times 0.2} = 50.1 \text{ k}\Omega$$

$\tau_H = 10 \times 50.1 + 2 \times 1355$ ns

$$f_H = \frac{1}{2\pi \times 3.21 \times 10^{-9}} = 49.6 \text{ MHz}$$

GB $= 49.6 \times 12.5 = 620$ MHz

A summary of the results is provided in the following table:

	$R_s = 0$	$R_s = 100\ \Omega$	$R_s = 200\ \Omega$
$\|A_M\|$ (V/V)	25	16.7	12.5
$f_H\|$ (MHz)	26.6	37.8	49.6
GB (MHz)	641	631	620

Observe that increasing R_s trades off gain for bandwidth while GB remains approximately constant.

10.96 (a) $A_M = -g_m R'_L$

where

$R'_L = R_L \parallel r_o$

$= 40 \parallel 40 = 20$ kΩ

$A_M = -5 \times 20 = -100$ V/V

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}}$

$= 2 \times 20 = 40$ ns

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= 20(1 + 5 \times 20) + 20$

$= 2040$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 2040$

$= 204$ ns

$\tau_{CL} = C_L R'_L$

$= 1 \times 20 = 20$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 40 + 204 + 20 = 264$ ns

$$f_H = \frac{1}{2\pi \times 264 \times 10^{-9}} = 603 \text{ kHz}$$

GB $= 100 \times 603 = 60.3$ MHz

(b) With $R_s = 400\ \Omega$,

$$G_m = \frac{g_m}{1 + g_m R_s}$$

$$= \frac{5}{1 + 5 \times 0.4} = 1.67 \text{ mA/V}$$

$R_o = r_o(1 + g_m R_s)$

$= 40(1 + 5 \times 0.4) = 120$ kΩ

$R'_L = R_L \parallel R_o = 40 \parallel 120 = 30$ kΩ

$A_M = -G_m R'_L$

$= -1.67 \times 30 = -50$ V/V

$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R'_L$

$= 20(1 + 1.67 \times 30) + 30$

$= 1050$ kΩ

$\tau_{gd} = C_{gd} R_{gd} = 0.1 \times 1050 = 105$ ns

$\tau_{CL} = C_L R_{CL}$

$= C_L R'_L$

$= 1 \times 30 = 30$ ns

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s/(r_o + R_L)}{1 + g_m R_s \left(\dfrac{r_o}{r_o + R_L}\right)}$$

$$= \frac{20 + 0.4 + 20 \times 0.4/(40 + 40)}{1 + 5 \times 0.4\left(\dfrac{40}{40 + 40}\right)}$$

$= 10.25$ kΩ

$\tau_{gs} = C_{gs} R_{gs} = 2 \times 10.25 = 20.5$ ns

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$

$= 20.5 + 105 + 30 = 155.5$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 155.5 \times 10^{-9}} = 1.02 \text{ MHz}$$

GB $= 51.2$ MHz

10.97 (a) $\text{GB} = |A_M| f_H$

$$= \frac{1}{2\pi C_{gd} R_{\text{sig}}}$$

$$= \frac{1}{2\pi \times 0.2 \times 10^{-12} \times 100 \times 10^3}$$

$$= 7.96 \text{ MHz}$$

(b) $|A_M| = 20$ V/V

$$f_H = \frac{7.96}{20} = 398 \text{ kHz}$$

(c) $A_0 = g_m r_o$

$$100 = 5 \times r_o$$

$$\Rightarrow r_o = 20 \text{ k}\Omega$$

$$G_m = \frac{g_m}{1 + g_m R_s} = \frac{5}{1 + g_m R_s}$$

$$R_o = r_o(1 + g_m R_s) = 20(1 + g_m R_s)$$

$$R'_L = R_L \parallel R_o = 20 \parallel 20(1 + g_m R_s)$$

$$A_M = -G_m R'_L$$

$$20 = \frac{5}{1 + g_m R_s}[20 \parallel 20(1 + g_m R_s)]$$

$$4(1 + g_m R_s) = \frac{20 \times 20(1 + g_m R_s)}{20 + 20(1 + g_m R_s)}$$

$$\Rightarrow 1 + g_m R_s = 4$$

$$\Rightarrow R_s = \frac{3}{g_m} = 0.6 \text{ k}\Omega = 600 \ \Omega$$

10.98 $G_m = \dfrac{g_m}{1 + g_m R_s} = \dfrac{g_m}{1 + k}$

$$R'_L = R_L \parallel R_o$$

$$= r_o \parallel r_o(1 + g_m R_s)$$

$$= r_o \parallel r_o(1 + k)$$

$$= \frac{r_o \times r_o(1 + k)}{r_o + r_o(1 + k)}$$

$$= r_o \frac{1 + k}{2 + k}$$

$$A_M = -G_m R'_L = -\frac{g_m r_o}{2 + k}$$

Thus,

$$A_M = \frac{-A_0}{2 + k} \qquad \text{Q.E.D.}$$

$$R_{gs} = \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s/(r_o + R_L)}{1 + g_m R_s \left(\dfrac{r_o}{r_o + R_L} \right)}$$

$$= \frac{R_{\text{sig}} + R_s + R_{\text{sig}} R_s / 2r_o}{1 + \dfrac{1}{2} g_m R_s}$$

For $R_{\text{sig}} \gg R_s$,

$$R_{gs} \simeq \frac{R_{\text{sig}}(1 + R_s/2r_o)}{1 + (k/2)}$$

For $r_o \gg R_s$,

$$R_{gs} \simeq \frac{R_{\text{sig}}}{1 + (k/2)}$$

$$\tau_{gs} = C_{gs} R_{gs} = \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)}$$

$$R_{gd} = R_{\text{sig}}(1 + G_m R'_L) + R_L$$

Utilizing the expressions for R'_L and $G_m R'_L$ derived earlier, we obtain

$$R_{gd} = R_{\text{sig}} \left[1 + \frac{A_0}{2 + k} \right] + r_o \left(\frac{1 + k}{2 + k} \right)$$

$$\tau_{gd} = C_{gd} R_{gd} =$$

$$C_{gd} R_{\text{sig}} \left(1 + \frac{A_0}{2 + k} \right) + C_{gd} r_o \left(\frac{1 + k}{2 + k} \right)$$

$$\tau_{C_L} = C_L R'_L$$

$$= C_L r_o \frac{1 + k}{2 + k}$$

Thus,

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$$

$$= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}} \left(1 + \frac{A_0}{2 + k} \right)$$

$$+ C_{gd} r_o \left(\frac{1 + k}{2 + k} \right) + C_L r_o \left(\frac{1 + k}{2 + k} \right)$$

$$= \frac{C_{gs} R_{\text{sig}}}{1 + (k/2)} + C_{gd} R_{\text{sig}} \left(1 + \frac{A_0}{2 + k} \right)$$

$$+ (C_L + C_{gd}) r_o \left(\frac{1 + k}{2 + k} \right) \qquad \text{Q.E.D.}$$

10.99 Substituting the given numerical values in the expressions for A_M and τ_H given in the statement for Problem 10.98 and noting that $A_0 = g_m r_o = 5 \times 40 = 200$, we obtain

$$f_H = \frac{1}{2\pi \tau_H}$$

and

$$\text{GB} = |A_M| f_H$$

we obtain the results in the following table.

k	$\lvert A_M \rvert$, V/V	τ_H ns	f_H (MHz)	GB (MHz)
0	100	264	0.603	60.3
1	66.7	191.3	0.832	55.6
2	50	155	1.03	51.5
3	40	133.2	1.19	47.6
4	33.3	118.7	1.34	44.6
5	28.6	108.3	1.47	42.0
6	25	100.5	1.58	39.5
7	22.2	94.4	1.69	37.5
8	20	89.6	1.78	35.6
9	18.2	85.7	1.86	33.9
10	16.7	82.3	1.93	32.2
11	15.4	79.6	2.00	30.8
12	14.3	77.2	2.06	29.5
13	13.3	75.1	2.12	28.2
14	12.5	73.3	2.17	27.1
15	11.8	71.6	2.22	26.2

To obtain $f_H = 2$ MHz, we see from the table that

$k = 11$

Thus,

$1 + g_m R_s = 11$

$$\Rightarrow R_s = \frac{10}{2} = 5\ \text{k}\Omega$$

The gain achieved is

$|A_M| = 15.4$ V/V

10.100 (a) Refer to Fig. P10.100(a). Since the total resistance at the drain is r_o, we have

$A_M = -g_m r_o$ Q.E.D.

$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{\text{sig}}$

$R_{gd} = R_{\text{sig}}(1 + g_m R'_L) + R'_L$

$= R_{\text{sig}}(1 + g_m r_o) + r_o$

$\tau_{gd} = C_{gd} R_{gd} = C_{gd}[R_{\text{sig}}(1 + g_m r_o) + r_o]$

$\tau_{C_L} = C_L R'_L = C_L r_o$

Thus,

$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L}$

$= C_{gs} R_{\text{sig}} + C_{gd}[R_{\text{sig}}(1 + g_m r_o) + r_o]$

$+ C_L r_o$ Q.E.D.

For the given numerical values,

$A_M = -1 \times 20 = -20$ V/V

$\tau_H = 20 \times 20 + 5[20(1 + 1 \times 20) + 20] + 10 \times 20$

$= 400 + 2200 + 200 = 2800$ ps $= 2.8$ ns

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 2.8 \times 10^{-9}}$$

$= 56.8$ MHz

GB $= 20 \times 56.8 = 1.14$ GHz

(b) From Fig. 1 we see that

$$\frac{V_{g1}}{V_{\text{sig}}} = 1$$

$$\frac{V_{g2}}{V_{g1}} = \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}$$

$$\frac{V_o}{V_{g2}} = -g_{m2} r_{o2}$$

Thus,

$$A_M = 1 \times \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}} \times -g_{m2} r_{o2}$$

$$= -\frac{r_{o1}}{1/g_{m1} + r_{o1}}(g_{m2} r_{o2}) \qquad \text{Q.E.D.}$$

This figure belongs to Problem 10.100, part (b).

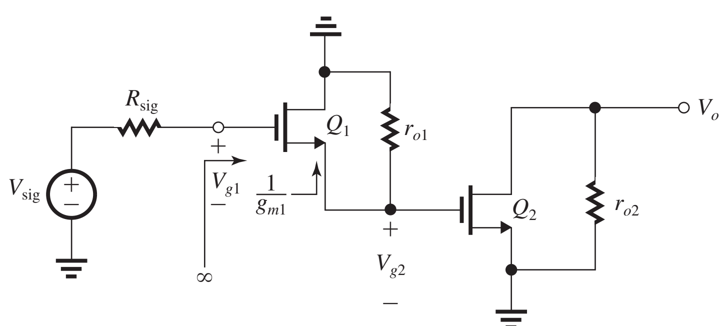


Figure 1

Next we evaluate the open-circuit time constants. Refer to Fig. 2.

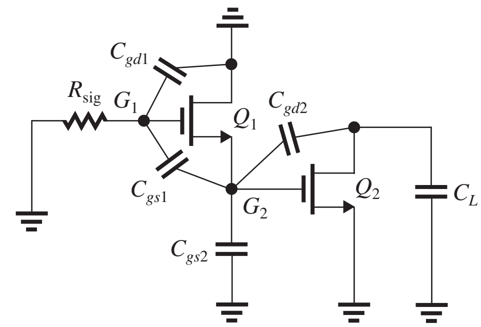


Figure 2

C_{gd1}: Capacitor C_{gd1} is between G_1 and ground and thus sees the resistance R_{sig},

$$R_{gd1} = R_{sig}$$

$$\tau_{gd1} = C_{gd1} R_{sig}$$

C_{gs1}: To find the resistance R_{gs1} seen by capacitor C_{gs1}, we replace Q_1 with its hybrid-π equivalent circuit with V_{sig} set to zero, $C_{gd1} = 0$, and C_{gs1} replaced by a test voltage V_x. The resulting equivalent circuit is shown in Fig. 3.

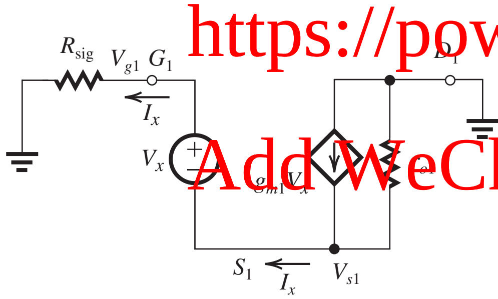


Figure 3

Analysis of the circuit in Fig. 3 proceeds as follows:

$$V_{g1} = I_x R_{sig}$$

$$V_{s1} = V_{g1} - V_x = I_x R_{sig} - V_x$$

Node equation at S_1,

$$I_x = g_{m1} V_x - \frac{V_{s1}}{r_{o1}}$$

$$= g_{m1} V_x - \frac{I_x R_{sig} - V_x}{r_{o1}}$$

$$I_x\left(1 + \frac{R_{sig}}{r_{o1}}\right) = V_x\left(g_{m1} + \frac{1}{r_{o1}}\right)$$

Thus,

$$R_{gs1} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + r_{o1}}{1 + g_{m1} r_{o1}}$$

$$\tau_{gs1} = C_{gs1} R_{gs1}$$

$$= C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_{m1} r_{o1}}$$

C_{gs2}: Capacitor C_{gs2} sees the resistance between G_2 and ground, which is the output resistance of source follower Q_1,

$$R_{gs2} = \frac{1}{g_{m1}} \parallel r_{o1}$$

Thus,

$$\tau_{gs2} = C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$$

C_{gd2}: Transistor Q_2 operates as a CS amplifier with an equivalent signal-source resistance equal to the output resistance of the source follower Q_2, that is, $\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$ and with a gain from gate to drain of $g_{m2} r_{o2}$. Thus, the formula for R_{gd} in a CS amplifier can be adapted as follows:

$$R_{gd2} = \left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_{m2} r_{o2}) + r_{o2}$$

and thus,

$$\tau_{gd2} = C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_{m2} r_{o2}) + r_{o2}\right]$$

C_L: Capacitor C_L sees the resistance between D_2, and ground which is r_{o2},

$$\tau_{C_L} = C_L r_{o2}$$

Summing τ_{gd1}, τ_{gs1}, τ_{gs2}, τ_{gd2} and τ_{C_L} gives τ_H in the problem statement. Q.E.D.

For the given numerical values:

$$A_M = -\frac{20}{1 + 20}(1 \times 20)$$

$$= -19 \text{ V/V}$$

$$\tau_{gd1} = C_{gd1} R_{sig} = 5 \times 20 = 100 \text{ ps}$$

$$\tau_{gs1} = C_{gs1} \frac{R_{sig} + r_{o1}}{1 + g_m r_{o1}}$$

$$= 20\,\frac{20 + 20}{1 + 1 \times 20} = 38 \text{ ps}$$

$$\tau_{gs2} = C_{gs2}\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)$$

$$= 20 \times (1 \parallel 20) = 19 \text{ ps}$$

$$\tau_{gd2} = C_{gd2}\left[\left(\frac{1}{g_{m1}} \parallel r_{o1}\right)(1 + g_m r_{o2}) + r_{o2}\right]$$

$$= 5[(1 \parallel 20)(1 + 20) + 20]$$

$$= 200 \text{ ps}$$

$\tau_{C_L} = C_L r_{o2} = 10 \times 20 = 200$ ps

$\tau_H = 100 + 38 + 19 + 200 + 200 = 557$ ps

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 557 \times 10^{-12}}$$

$= 286$ MHz

$\text{GB} = 19 \times 286 = 5.43$ GHz

Thus, while the dc gain remained approximately the same both f_H and GB increased by a factor of about 5!

10.101 At an emitter bias current of 0.1 mA, Q_1 and Q_2 have

$g_m = 4$ mA/V

$r_e = 250\ \Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.1} = 1000\text{ k}\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{4 \times 10^{-3}}{2\pi \times 200 \times 10^6} = 3.2\text{ pF}$$

$C_\mu = 0.2$ pF

$C_\pi = 3$ pF

To determine R_{in} and the voltage gain A_M, refer to the circuit in Fig. 10.40(a). Here, however, R_L is r_{o2}.

$R_{\text{in}2} = r_{\pi 2} = 25\text{ k}\Omega$

$R_{\text{in}} = (\beta_1 + 1)[r_{e1} + (r_{o1} \parallel R_{\text{in}2})]$

$= 101[0.25 + (1000 \parallel 25)]$

$\simeq 2.5\text{ M}\Omega$

$$\frac{V_{b1}}{V_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{2.5\text{ M}\Omega}{2.5\text{ M}\Omega + 10\text{ k}\Omega} \simeq 1\text{ V/V}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{(R_{\text{in}2} \parallel r_{o1})}{(R_{\text{in}2} \parallel r_{o1}) + r_{e1}}$$

$$= \frac{25 \parallel 1000}{(25 \parallel 1000) + 0.25} = 0.99 \simeq 1\text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2} r_{o2} = -4 \times 1000 = -4000\text{ V/V}$$

Thus,

$$A_M = \frac{V_o}{V_{\text{sig}}} = -4000\text{ V/V}$$

To determine f_H we use the method of open-circuit time constants. Figure 10.40(b) shows the circuit with $V_{\text{sig}} = 0$ and the four capacitances indicated. Again, recall that here $R_L = r_{o2}$. Also, in our present circuit there is a capacitance C_L at the output.

Capacitance $C_{\mu 1}$ sees a resistance $R_{\mu 1}$,

$R_{\mu 1} = R_{\text{sig}} \parallel R_{\text{in}}$

$= 10\text{ k}\Omega \parallel 2.5\text{ M}\Omega \simeq 10\text{ k}\Omega$

To find the resistance $R_{\pi 1}$ we refer to the circuit in Fig. 10.40(c) where $R_{\text{in}2}$ is considered to include r_{o2},

$R_{\text{in}2} = 25\text{ k}\Omega \parallel 1000\text{ k}\Omega = 24.4\text{ k}\Omega$

We use the formula for $R_{\pi 1}$ given in Example 10.13:

$$R_{\pi 1} = \frac{R_{\text{sig}} + R_{\text{in}2}}{1 + \dfrac{R_{\text{sig}}}{r_{\pi 1}} + \dfrac{R_{\text{in}2}}{r_{e1}}}$$

$$R_{\pi 1} = \frac{10 + 24.4}{1 + \dfrac{10}{25} + \dfrac{24.4}{0.25}} = 347\ \Omega$$

Capacitance $C_{\pi 2}$ sees a resistance $R_{\pi 2}$:

$R_{\pi 2} = R_{\text{in}2} \parallel R_{\text{out}1}$

$$= r_{\pi 2} \parallel r_{o1} \parallel \left[r_{e1} + \frac{R_{\text{sig}}}{\beta_1 + 1}\right]$$

$$= 25 \parallel 1000 \parallel \left[0.25 + \frac{10}{101}\right]$$

$= 344\ \Omega$

Capacitance $C_{\mu 2}$ sees a resistance $R_{\mu 2}$:

$R_{\mu 2} = (1 + g_{m2} r_{o2})(R_{\text{in}2} \parallel R_{\text{out}1}) + r_{o2}$

$= (1 + 4 \times 1000) \times 0.344 + 1000$

$= 2376\text{ k}\Omega$

We can determine τ_H from

$$\tau_H = C_{\mu 1} R_{\mu 1} + C_{\pi 1} R_{\pi 1} + C_{\mu 2} R_{\mu 2} + C_{\pi 2} R_{\pi 2} + C_L r_o$$

$$= 0.2 \times 10 + 3 \times 0.347 + 0.2 \times 2376 + 3 \times 0.344 + 1 \times 1000$$

$\tau_H = 2 + 1 + 475.2 + 1 + 1000$

$= 1479.2$ ns

Observe that there are two dominant capacitances: the most significant is C_L and the second most significant is $C_{\mu 2}$.

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 1479.2 \times 10^{-9}} = 107.6\text{ kHz}$$

10.102 $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2(I/2)}{V_{OV}}$

$$= \frac{I}{V_{OV}} = \frac{0.2\text{ mA}}{0.2\text{ V}} = 1\text{ mA/V}$$

$$\frac{V_o}{V_{sig}} = \frac{R_D}{2/g_m}$$

$$= \frac{1}{2}g_m R_D = \frac{1}{2} \times 1 \times 50 = 25\text{ V/V}$$

The high-frequency analysis can be performed in an analogous manner to that used in the text for the bipolar circuit. Refer to Fig. 10.42(b) and adapt the circuit for the MOS case. Thus,

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\dfrac{C_{gs}}{2} + C_{gd}\right)}$$

$$= \frac{1}{2\pi \times 100 \times 10^3\left(\dfrac{4}{2} + 0.5\right) \times 10^{-12}}$$

$$= 637\text{ kHz}$$

and

$$f_{P2} = \frac{1}{2\pi R_D C_\mu}$$

$$= \frac{1}{2\pi \times 50 \times 10^3 \times 0.5 \times 10^{-12}}$$

$$= 6.37\text{ MHz}$$

Since $f_{P2} \simeq 10 f_{P1}$, the pole at f_{P1} will dominate and

$$f_H \simeq f_{P1} = 637\text{ kHz}$$

10.103 $g_m = \dfrac{I_C}{V_T} \simeq \dfrac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$

$$r_e \simeq 25\ \Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{120}{40} = 3\text{ k}\Omega$$

$$R_{in} = 2r_\pi = 6\text{ k}\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \frac{\alpha R_L}{2r_e}$$

$$\simeq \frac{6}{6 + 12} \times \frac{10}{2 \times 0.025} = 66.7\text{ V/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 500 \times 10^6} = 12.7\text{ pF}$$

$$C_\mu = 0.5\text{ pF}$$

$$C_\pi = 12.2\text{ pF}$$

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\dfrac{C_\pi}{2} + C_\mu\right)}$$

$$= \frac{1}{2\pi \times 12 \times 10^3\left(\dfrac{12.2}{2} + 0.5\right) \times 10^{-12}}$$

$$= 2\text{ MHz}$$

$$f_{P2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 0.5 \times 10^{-12}} = 31.8\text{ MHz}$$

Thus, f_{P1} is the dominant pole and

$$f_H \simeq f_{P1} = 2\text{ MHz}$$

10.104 Using an approach analogous to that utilized for the BJT circuit (Fig. 10.42), we see that there is a pole at the input with frequency f_{P1}:

$$f_{P1} = \frac{1}{2\pi R_{sig}\left(\dfrac{C_{gs}}{2} + C_{gd}\right)}$$

$$f_{P1} = \frac{1}{2\pi \times 20 \times 10^3\left(\dfrac{2}{2} + 0.1\right) \times 10^{-12}}$$

$$= 7.2\text{ MHz},$$

and a pole at the output with frequency f_{P2},

$$f_{P2} = \frac{1}{2\pi(C_{gd} + C_L)R_L}$$

$$= \frac{1}{2\pi \times (0.1 + 1) \times 10^{-12} \times 20 \times 10^3}$$

$$= 7.2\text{ MHz}$$

Thus,

$$f_{P1} = f_{P2} = 7.2\text{ MHz}$$

The midband gain A_M is obtained as

$$A_M = \frac{R_L}{2/g_m} = \frac{1}{2}g_m R_L$$

$$= \frac{1}{2} \times 5 \times 20 = 50\text{ V/V}$$

Thus, the amplifier transfer function is

$$\frac{V_o(s)}{V_{sig}(s)} = \frac{50}{\left(1 + \dfrac{s}{2\pi \times 7.2 \times 10^6}\right)^2}$$

$$\left|\frac{V_o}{V_{sig}}\right| = \frac{50}{1 + \left(\dfrac{\omega}{2\pi \times 7.2 \times 10^6}\right)^2}$$

At $\omega = \omega_{3dB}$, $\left|\frac{V_o}{V_i}\right| = \frac{50}{\sqrt{2}}$, thus

$$\sqrt{2} = 1 + \left(\frac{\omega_{3dB}}{2\pi \times 7.2 \times 10^6}\right)^2$$

$$f_{3dB} = \sqrt{\sqrt{2} - 1} \times 7.2 \text{ MHz}$$

$$= 4.6 \text{ MHz}$$

10.105 (a) A CS amplifier for which the gain is low so that the Miller effect is negligible and for which $C_{gd} \ll C_{gs}$ has an input capacitance that is approximately given by

$$C_{\text{in}} \simeq C_{gs}$$

Now,

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \simeq \frac{g_m}{C_{gs}}$$

thus,

$$C_{gs} \simeq \frac{g_m}{\omega_T}$$

and

$$C_{\text{in}} \simeq \frac{g_m}{\omega_T}$$

If r_o can be neglected, the hybrid-π equivalent circuit reduces to that shown in Fig. P10.105(a).

(b) Replacing Q_1 by its equivalent circuit and replacing the diode-connected Q_2 by its equivalent circuit which consists of the input capacitance (g_{m2}/ω_T) and the resistance $1/g_{m2}$ and including the input capacitance of the subsequent stage (g_{m1}/ω_T) gives the equivalent circuit shown in the figure above. The gain can easily be determined as

$$\frac{V_o}{V_i} = -\frac{g_{m1}}{g_{m2} + s\dfrac{g_{m1} + g_{m2}}{\omega_T}}$$

$$= -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + s\dfrac{1 + g_{m1}/g_{m2}}{\omega_T}}$$

Denoting

$$\frac{g_{m1}}{g_{m2}} = G_0,$$

then,

$$\frac{V_o}{V_i} = -\frac{G_0}{1 + \dfrac{s}{\omega_T/(G_0 + 1)}}$$

The dc gain G_0 can be related to W_1 and W_2 as follows: The bias current I divides between Q_1 and Q_2 as,

$$I_1 = \frac{1}{2}k_n'\left(\frac{W_1}{L}\right)V_{OV}^2$$

$$I_2 = \frac{1}{2}k_n'\left(\frac{W_2}{L}\right)V_{OV}^2$$

where we have utilized the fact that Q_1 and Q_2 are operating at the same value of V_{OV}. Thus,

$$\frac{I_1}{I_2} = \frac{W_1}{W_2}$$

Now,

$$g_{m1} = \frac{2I_1}{V_{OV}}$$

and

$$g_{m2} = \frac{2I_2}{V_{OV}}$$

Thus,

$$\frac{g_{m1}}{g_{m2}} = \frac{I_1}{I_2} = \frac{W_1}{W_2}$$

and

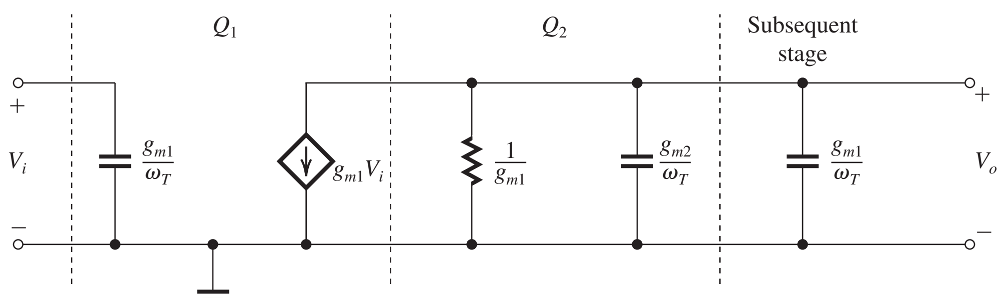
$$G_0 = \frac{g_{m1}}{g_{m2}} = \frac{W_1}{W_2}$$

(c) $G_0 = 3$

$$\Rightarrow \frac{W_1}{W_2} = 3$$

$$W_1 = 3 \times 25 = 75 \text{ μm}$$

This figure belongs to Problem 10.105, part (b).



$$I_1 = \frac{1}{2}k_n'\frac{W_1}{L}V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{75}{0.5} \times 0.3^2 = 1.35 \text{ mA}$$

$$I_2 = \frac{1}{2} \times 0.2 \times \frac{25}{0.5} \times 0.3^2 = 0.45 \text{ mA}$$

$$I = I_1 + I_2 = 1.35 + 0.45 = 1.8 \text{ mA}$$

$$f_{3dB} = \frac{f_T}{G_0 + 1} = \frac{12}{3+1} = 3 \text{ GHz}$$

10.106 (a) For each of Q_1 and Q_2,

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_m r_o = 100$$

$$\frac{V_o}{V_{sig}} = (g_m r_o)^2 = 10{,}000 \text{ V/V}$$

(b) $\tau_{gs1} = C_{gs}R_{sig}$

$= 20 \times 10 = 200$ ps

$R_{gd1} = R_{sig}(1 + g_{m1}r_{o1}) + r_{o1}$

$= 10(1 + 100) + 100$

$= 1110 \text{ k}\Omega$

$\tau_{gd1} = C_{gd1}R_{gd1} = 5 \times 1110 = 5550$ ps

At the drain of Q_1 we have $(C_{db1} + C_{gs2})$ and the resistance seen is r_o:

$\tau_{d1} = (C_{db1} + C_{gs2})r_o$

$= (5 + 20) \times 100 = 2500$ ps

$R_{gd2} = r_{o1}(1 + g_{m2}r_{o2}) + r_{o2}$

$= 100(1 + 100) + 100 = 1110 \text{ k}\Omega$

$\tau_{gd2} = C_{gd2}R_{gd2} = 5 \times 1110 = 5550$ ps

$\tau_{d2} = C_{db2}r_{o2}$

$= 5 \times 100 = 500$ ps

$\tau_H = \tau_{gs1} + \tau_{gd1} + \tau_{d1} + \tau_{gd2} + \tau_{d2}$

$= 200 + 5550 + 2500 + 5550 + 500$

$= 14{,}300$ ps $= 14.3$ ns

$$f_H = \frac{1}{2\pi\tau_H}$$

$$= \frac{1}{2\pi \times 14.3 \times 10^{-9}} = 11.1 \text{ MHz}$$

10.107 (a)

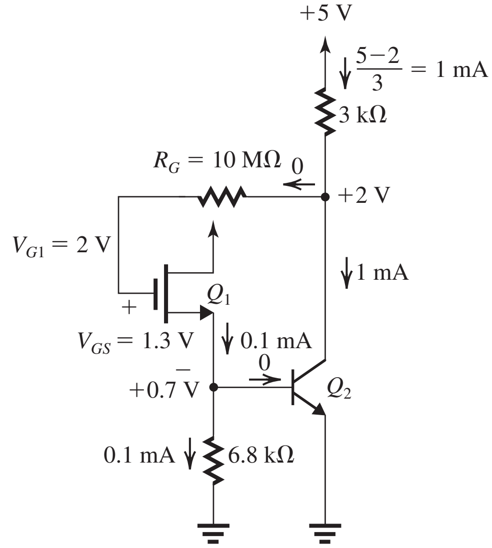


Figure 1

The dc analysis is shown in Fig. 1. It is based on $V_{S1} = V_{BE2} = 0.7$ V. Neglecting I_{B2}, we obtain

$$I_{D1} = \frac{0.7 \text{ V}}{6.8 \text{ k}\Omega} \simeq 0.1 \text{ mA} \qquad \text{Q.E.D.}$$

$$I_{D1} = \frac{1}{2}k_n'(W/L)V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$\Rightarrow V_{OV} \simeq 0.3$ V

$V_{GS} = V_t + V_{OV} = 1 + 0.3 = 1.3$ V

$V_{G1} = 0.7 + 1.3 = 2$ V

$V_{C1} = V_{G1} = 2$ V

$$I_{C2} = \frac{5-2}{3} = 1 \text{ mA} \qquad \text{Q.E.D.}$$

(b) $g_{m1} = \dfrac{2I_{D1}}{V_{OV}} = \dfrac{2 \times 0.1}{0.3} = 0.67$ mA/V

$C_{gs} = C_{gd} = 1$ pF

$$g_{m2} = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

$$C_{\pi 2} + C_{\mu 2} = \frac{g_{m2}}{2\pi f_{T2}}$$

$$= \frac{40 \times 10^{-3}}{2\pi \times 600 \times 10^6} = 10.6 \text{ pF}$$

$C_{\mu 2} = 0.8$ pF

$C_{\pi 2} = 9.8$ pF

(c) Q_1 acts as a source follower, thus

$$\frac{V_{b2}}{V_i} = \frac{6.8\text{ k}\Omega \parallel r_{\pi 2}}{\dfrac{1}{g_{m1}} + (6.8\text{ k}\Omega \parallel r_{\pi 2})}$$

$$= \frac{(6.8 \parallel 5)}{1.5 + (6.8 \parallel 5)} = 0.66\text{ V/V}$$

Neglecting R_G, we obtain

$$\frac{V_o}{V_{b2}} = -g_{m2}(3\text{ k}\Omega \parallel 1\text{ k}\Omega)$$

$$= -40(3 \parallel 1) = -30\text{ V/V}$$

Thus,

$$\frac{V_o}{V_i} = 0.66 \times -30$$

$$\simeq -20\text{ V/V}$$

Using Miller's theorem, the input resistance R_{in} is found as

$$R_{\text{in}} = \frac{R_G}{1 - \dfrac{V_o}{V_i}} = \frac{10\text{ M}\Omega}{1-(-20)}$$

$$= 476\text{ k}\Omega$$

$$\frac{V_i}{V_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

$$= \frac{476}{476 + 100} = 0.83\text{ V/V}$$

$$\frac{V_o}{V_{\text{sig}}} = 0.83 \times 20 = 16.5\text{ V/V}$$

(c) The pole due to C_1 has a frequency f_1:

$$f_1 = \frac{1}{2\pi C_1(R_{\text{sig}} + R_{\text{in}})}$$

$$= \frac{1}{2\pi \times 0.1 \times 10^{-6}(100+476) \times 10^3}$$

$$= 2.8\text{ Hz}$$

The pole due to C_2 has a frequency f_2:

$$f_2 = \frac{1}{2\pi C_2(3+1) \times 10^3}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-6} \times 4 \times 10^3} = 40\text{ Hz}$$

Since $f_2 \gg f_1$, the lower 3-dB frequency f_L will be

$$f_L \simeq f_2 = 40\text{ Hz}$$

(d) $\tau_{gd1} = C_{gd1}(R_{\text{in}} \parallel R_{\text{sig}})$

$$= 1 \times 10^{-12}(476 \parallel 100) \times 10^3$$

$$= 82.6\text{ ns}$$

To determine the resistance R_{gs} seen by C_{gs}, refer to Fig. 2.

We can show that

$$R_{gs} \equiv \frac{V_x}{I_x} = \frac{R_{\text{sig}} + R_s}{1 + g_{m1}R_s}$$

where

$$R_s = 6.8\text{ k}\Omega \parallel r_{\pi 2}$$

$$= (6.8 \parallel 5) = 2.88\text{ k}\Omega$$

$$R_{gs} = \frac{100 + 2.88}{1 + 0.67 \times 2.88} = 35.1\text{ k}\Omega$$

$$\tau_{gs} = C_{gs}R_{gs} = 1 \times 10^{-12} \times 35.1 \times 10^3 = 35.1\text{ ns}$$

$$\tau_{\pi 2} = C_{\pi 2}(r_{\pi 1} \parallel 6.8\text{ k}\Omega)$$

$$= 9.8 \times 10^{-12} \times 2.88 \times 10^3$$

$$= 28.2\text{ ns}$$

$$R_{\mu 2} = \left(\frac{1}{g_{m1}} \parallel 6.8\text{ k}\Omega\right)[1 + g_{m2}(3 \parallel 1)] + (3 \parallel 1)$$

$$= (1.5 \parallel 6.8)\left(1 + 40 \times \frac{3}{4}\right) + 0.75$$

$$= 38.8\text{ k}\Omega$$

This figure belongs to Problem 10.107, part (d).

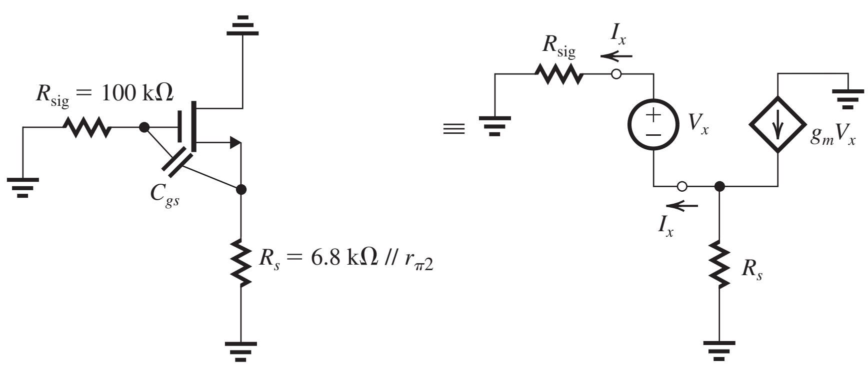


Figure 2

$\tau_{\mu 2} = C_{\mu 2} R_{\mu 2} = 0.8 \times 38.8 = 31.1$ ns

$\tau_H = \tau_{gd} + \tau_{gs} + \tau_{\pi 2} + \tau_{\mu 2}$

$= 82.6 + 35.1 + 38.8 + 31.1$

$= 187.6$ ns

$$f_H = \frac{1}{2\pi \tau_H}$$

$$= \frac{1}{2\pi \times 187.6 \times 10^{-9}} = 848 \text{ kHz}$$

10.108 All transistors are operating at $I_E = 0.5$ mA. Thus,

$g_m \simeq 20$ mA/V

$r_e \simeq 50\ \Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{20} = 5 \text{ k}\Omega$$

$r_o =$ very high (neglect)

$r_x =$ very small (neglect)

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{20 \times 10^{-3}}{2\pi \times 400 \times 10^6}$$

$= 8$ pF

$C_\mu = 2$ pF

$C_\pi = 6$ pF

(a) CE amplifier

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R_L$$

$$= -\frac{5}{5 + 10} \times 20 \times 10 = -66.7 \text{ V/V}$$

Since $C_L = 0$, we can obtain a good estimate of f_H using the Miller approximation:

$C_{\text{in}} = C_\pi + C_\mu (g_m R_L + 1)$

$= 6 + 2(20 \times 10 + 1)$

$= 408$ pF

$$f_H = \frac{1}{2\pi R_{\text{sig}} C_{\text{in}}}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 408 \times 10^{-12}}$$

$= 39$ kHz

(b) This is a cascode amplifier. Refer to Fig. 10.30 for the analysis equations.

$$A_M = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m (\beta r_o \parallel R_L)$$

$$\simeq -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m R_L$$

$= -66.7$ V(V) (same as the CE in (a))

$R'_{\text{sig}} = r_\pi \parallel R_{\text{sig}} = 5 \parallel 10 = 3.33$ kΩ

$R_{\pi 1} = R'_{\text{sig}}$

$\tau_{\pi 1} = C_{\pi 1} R_{\pi 1} = 6 \times 3.33 = 20$ ns

$R_{c1} = r_{e2} = 50\ \Omega$

$R_{\mu 1} = R'_{\text{sig}}(1 + g_{m1} R_{c1}) + R_{c1}$

$= 3.33(1 + 40 \times 0.05) + 0.05$

$= 3.33(1 + 2) + 0.05 = 10.05$ kΩ

$\tau_{\mu 1} = C_{\mu 1} R_{\mu 1} = 2 \times 10.05 = 20.1$ ns

$\tau_{c1} = C_{\pi 2} R_{c1} = 6 \times 0.05 = 0.3$ ns

$\tau_{\mu 2} = C_{\mu 2} R_L = 2 \times 10 = 20$ ns

$\tau_H = 20 + 20.1 + 0.3 + 20 = 60.4$ ns

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 60.4 \times 10^{-9}}$$

$= 2.6$ MHz

(c) This is a CC-CB cascade similar to the circuit analyzed in Fig. 10.42. There are two poles: one at the input,

$$f_{P1} = \frac{1}{2\pi (R_{\text{sig}} \parallel 2r_\pi)\left(\dfrac{C_\pi}{2} + C_\mu\right)}$$

$$f_{P1} = \frac{1}{2\pi (10 \parallel 10) \times 10^3 (3 + 2) \times 10^{-12}}$$

$$= \frac{1}{2\pi \times 5 \times 5 \times 10^{-9}}$$

$= 6.4$ MHz

and one at the output,

$$f_{P2} = \frac{1}{2\pi R_L C_\mu}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 2 \times 10^{-12}}$$

$= 8$ MHz

Since the two poles are relatively close to each other, we use the root-sum-of-the-squares formula to obtain an estimate for f_H:

$$f_H = 1 \Big/ \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

$$= 1 \Big/ \sqrt{\frac{1}{6.4^2} + \frac{1}{8^2}} = 5 \text{ MHz}$$

$$A_M = \frac{R_L}{2r_e + \dfrac{R_{\text{sig}}}{\beta + 1}}$$

$$= \frac{10}{2 \times 0.05 + \dfrac{10}{101}} \simeq 50 \text{ V/V}$$

(d) This is a CC-CE cascade similar to the circuit analyzed in Example 10.13.

$R_{in} = (\beta_1 + 1)(r_{e1} + r_{\pi 2})$

$= 101(0.05 + 5) = 510\ \text{k}\Omega$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{510}{510 + 10}$$

$= 0.98$ V/V

$$\frac{V_{b2}}{V_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5}{5 + 0.05} = 0.99 \text{ V/V}$$

$$\frac{V_o}{V_{b2}} = -g_{m2}R_L = -20 \times 10 = -200 \text{ V/V}$$

$$A_M = \frac{V_o}{V_{sig}} = -0.98 \times 0.99 \times 200 = -194 \text{ V/V}$$

$R_{\mu 1} = R_{sig} \parallel R_{in}$

$= 10 \parallel 510 = 9.81\ \text{k}\Omega$

$\tau_{\mu 1} = C_{\mu 1}R_{\mu 1} = 2 \times 9.81 = 19.6$ ns

$$R_{\pi 1} = \frac{R_{sig} + R_{in2}}{1 + \dfrac{R_{sig}}{r_{\pi 1}} + \dfrac{R_{in2}}{r_{e1}}}$$

$$= \frac{10 + 5}{1 + \dfrac{10}{5} + \dfrac{5}{0.05}} = 0.15\ \text{k}\Omega$$

$\tau_{\pi 1} = C_{\pi 1}R_{\pi 1} = 6 \times 0.15 = 0.9$ ns

$R_{\pi 2} = R_{in2} \parallel R_{out1}$

$$= r_{\pi 2} \parallel \left(\frac{R_{sig}}{\beta_1 + 1} + r_{e1}\right)$$

$$= 5 \parallel \left(\frac{10}{101} + 0.05\right) = 0.15\ \text{k}\Omega$$

$\tau_{\pi 2} = C_{\pi 2}R_{\pi 2} = 6 \times 0.15 = 0.9$ ns

$R_{\mu 2} = (1 + g_{m2}R_L)(R_{in2} \parallel R_{out1}) + R_L$

$$= (1 + 20 \times 10)\left[5 \parallel \left(\frac{10}{101} + 0.05\right)\right] + 10$$

$= 39.1\ \text{k}\Omega$

$\tau_{\mu 2} = C_{\mu 2}R_{\mu 2} = 2 \times 39.1 = 78.2$ ns

$\tau_H = 19.6 + 0.9 + 0.9 + 78.2 = 99.6$ ns

$$f_H = \frac{1}{2\pi \times 99.6 \times 10^{-9}} = 1.6 \text{ MHz}$$

(e) This is a folded cascode amplifier. The analysis is identified to that for (b) above.

$A_M = -66.7$ V/V

$f_H = 2.6$ MHz

(f) This is a CE-CB cascade. The analysis is identical to that for case (c) above.

$A_M = 50$ V/V

$f_H = 5$ MHz

Summary of Results

Case	Configuration	A_M (V/V)	f_H (MHz)
a	CE	−66.7	0.039
b	Cascode	−66.7	2.6
c	CC-CB	50	5
d	CC-CE	−194	1.6
e	Folded Cascode	−66.7	2.6
f	CC-CB	50	5