Ex: 8.1 In the current source of Example 8.1 (Fig. 8.1) we have $I_O = 100\ \mu A$ and we want to reduce the change in output current, ΔI_O, corresponding to a 1-V change in output voltage, ΔV_O, to 1% of I_O.

That is, $\Delta I_O = \dfrac{\Delta V_O}{r_{o2}} = 0.01 I_O \Rightarrow \dfrac{1\text{ V}}{r_{o2}}$

$= 0.01 \times 100\ \mu A$

$$r_{o2} = \frac{1\text{ V}}{1\ \mu A} = 1\text{ M}\Omega$$

$$r_{o2} = \frac{V_A' \times L}{I_O} \Rightarrow 1\text{ M}\Omega = \frac{20 \times L}{100\ \mu A}$$

$$\Rightarrow L = \frac{100\text{ V}}{20\text{ V}/\mu m} = 5\ \mu m$$

To keep V_{OV} of the matched transistors the same as that in Example 8.1, $\dfrac{W}{L}$ of the transistor should remain the same. Therefore,

$$\frac{W}{5\ \mu m} = \frac{10\ \mu m}{1\ \mu m} \Rightarrow W = 50\ \mu m$$

So the dimensions of the matched transistors Q_1 and Q_2 should be changed to

$W = 50\ \mu m$ and $L = 5\ \mu m$

Ex: 8.2 For the circuit of Fig. 8.4 we have

$$I_2 = I_{REF}\frac{(W/L)_2}{(W/L)_1},\ I_3 = I_{REF}\frac{(W/L)_3}{(W/L)_1}$$

and $I_5 = I_4\dfrac{(W/L)_5}{(W/L)_4}$

Since all channel lengths are equal, that is,

$L_1 = L_2 = \cdots = L_5 = 1\ \mu m$

and

$I_{REF} = 10\ \mu A,\ I_2 = 60\ \mu A,\ I_3 = 20\ \mu A,\ I_4 = I_3 = 20\ \mu A,$ and $I_5 = 80\ \mu A,$

we have

$$I_2 = I_{REF}\frac{W_2}{W_1} \Rightarrow \frac{W_2}{W_1} = \frac{I_2}{I_{REF}} = \frac{60}{10} = 6$$

$$I_3 = I_{REF}\frac{W_3}{W_1} \Rightarrow \frac{W_3}{W_1} = \frac{I_3}{I_{REF}} = \frac{20}{10} = 2$$

$$I_5 = I_4\frac{W_5}{W_4} \Rightarrow \frac{W_5}{W_4} = \frac{I_5}{I_4} = \frac{80}{20} = 4$$

To allow the voltage at the drain of Q_2 to go down to within 0.2 V of the negative supply voltage, we need $V_{OV2} = 0.2$ V:

$$I_2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2 = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$60\ \mu A = \frac{1}{2}200\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_2 (0.2)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = \frac{120}{200 \times (0.2)^2} = 15 \Rightarrow W_2$$

$= 15 \times L_2$

$$W_2 = 15\ \mu m,\ \frac{W_2}{W_1} = 6 \Rightarrow W_1 = \frac{W_2}{6} = 2.5\ \mu m$$

$$\frac{W_3}{W_1} = 2 \Rightarrow W_3 = 2 \times W_1 = 5\ \mu m$$

To allow the voltage at the drain of Q_5 to go up to within 0.2 V of positive supply, we need $V_{OV5} = 0.2$ V:

$$I_5 = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_5 V_{OV5}^2$$

$$80\ \mu A = \frac{1}{2}80\frac{\mu A}{V^2}\left(\frac{W}{L}\right)_5 (0.2)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_5 = \frac{2 \times 80}{80 \times (0.2)^2} = 50 \Rightarrow W_5 = 50\ L_5$$

$W_5 = 50\ \mu m$

$$\frac{W_5}{W_4} = 4 \Rightarrow W_4 = \frac{50\ \mu m}{4} = 12.5\ \mu m$$

Thus:

$W_1 = 2.5\ \mu m,\ W_2 = 15\ \mu m,\ W_3 = 5\ \mu m$
$W_4 = 12.5\ \mu m$, and $W_5 = 50\ \mu m$

Ex: 8.3 From Eq. (8.21) we have

$$I_O = I_{REF}\left(\frac{m}{1 + \dfrac{m+1}{\beta}}\right)\left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

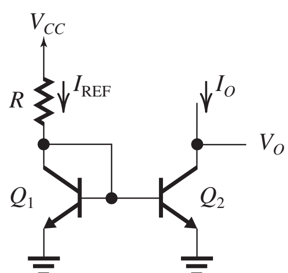
$$I_O = 1\text{ mA}\left(\frac{1}{1 + \dfrac{1+1}{100}}\right)\left(1 + \frac{5 - 0.7}{100}\right)$$

$= 1.02$ mA

$I_O = 1.02$ mA

$$R_o = r_{o2} = \frac{V_A}{I_O} = \frac{100\text{ V}}{1.02\text{ mA}} = 98\text{ k}\Omega \simeq 100\text{ k}\Omega$$

Ex: 8.4



From Eq. (8.23), we have

$$I_O = \frac{I_{REF}}{1 + (2/\beta)}\left(1 + \frac{V_O - V_{BE}}{V_A}\right)$$

where $V_{BE} = V_T \ln\left(\frac{I_O}{I_S}\right)$

$= 0.025 \ln\left(\frac{0.5 \times 10^{-3}}{10^{-15}}\right) = 0.673 \text{ V}$

$0.5 \text{ mA} = \frac{I_{\text{REF}}}{1 + (2/100)} \left(1 + \frac{2 - 0.673}{50}\right) \Rightarrow$

$I_{\text{REF}} = 0.5 \text{ mA} \frac{1.02}{1.026 \text{ mA}} = 0.497 \text{ mA}$

$I_{\text{REF}} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{\text{REF}}}$

$R = \frac{5 - 0.673}{0.497 \text{ mA}} = 8.71 \text{ k}\Omega$

$V_{O\min} = V_{CE\text{sat}} = 0.3 \text{ V}$

For $V_O = 5$ V, From Eq. (8.23) we have

$I_O = \frac{I_{\text{REF}}}{1 + (2/\beta)} \left(1 + \frac{V_O - V_{BE}}{V_A}\right)$

$I_O = \frac{0.497}{1 + (2/100)} \left(1 + \frac{5 - 0.673 \text{ V}}{50}\right) = 0.53 \text{ mA}$

Ex: 8.5 $I_1 = I_2 = \cdots = I_N = I_C|_{Q_{\text{REF}}}$

At the input node,

$I_{\text{REF}} = I_C|_{Q_{\text{REF}}} + I_B|_{Q_{\text{REF}}} + I_{B1} + \cdots + I_{BN}$

$= I_C|_{Q_{\text{REF}}} + (N + 1)\ I_B|_{Q_{\text{REF}}}$

$= I_C|_{Q_{\text{REF}}} + \frac{(N + 1)}{\beta} I_C|_{Q_{\text{REF}}}$

$\Rightarrow I_C|_{Q_{\text{REF}}} = \frac{I_{\text{REF}}}{1 + \frac{N + 1}{\beta}}$

Thus,

$I_1 = I_2 = \cdots = I_N = \frac{I_{\text{REF}}}{1 + \frac{N + 1}{\beta}}$ Q.E.D

For $\beta = 100$, to limit the error to 10%,

$0.1 = \frac{N + 1}{\beta} = \frac{N + 1}{100}$

$\Rightarrow N = 9$

Ex: 8.6

$R_{\text{in}} \simeq \frac{1}{g_{m1}}$

Now, $R_{\text{in}} = 1 \text{ k}\Omega$, thus

$g_{m1} = 1 \text{ mA/V}$

But

$g_{m1} = \sqrt{2(\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 I_{D1}}$

$1 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.1}$

$\Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$

To obtain

$A_{is} = 5$

$5 = A_{is} = \frac{(W/L)_2}{(W/L)_1}$

$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 12.5 = 62.5$

$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{V_{A2}}{5I_{D1}}$

Thus,

$40 \text{ k}\Omega = \frac{V_{A2}}{5 \times 0.1}$

$\Rightarrow V_{A2} = 20 \text{ V}$

But

$V_{A2} = V'_{A2} L_2$

$20 = 20 \times L_2$

$\Rightarrow L_2 = 1 \ \mu\text{m}$

Selecting $L_1 = L_2$, then

$L_1 = L_2 = 1 \ \mu\text{m}$

$W_1 = 12.5 \ \mu\text{m}$

$W_2 = 62.5 \ \mu\text{m}$

Ex: 8.7

Using Eq. (8.42):

$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)} \cdot \sqrt{I_D}$

For $I_D = 10 \ \mu\text{A}$, we have

$g_m = \sqrt{2(387 \ \mu\text{A/V}^2)(10)(10 \ \mu\text{A})}$

$= 0.28 \text{ mA/V}$

Using Eq. (8.46):

$A_0 = V'_A \frac{\sqrt{2\mu_n C_{ox}(W/L)}}{\sqrt{I_D}}$

$= \frac{5 \text{ V}/\mu\text{m} \sqrt{2(387 \ \mu\text{A/V}^2)(10)(0.36)^2}}{\sqrt{10 \ \mu\text{A}}}$

$A_0 = 50 \text{ V/V}$

Since g_m varies with $\sqrt{I_D}$ and A_0 with $\frac{1}{\sqrt{I_D}}$,

for

$I_D = 100 \ \mu\text{A} \Rightarrow g_m = 0.28 \text{ mA/V}\left(\frac{100}{10}\right)^{1/2}$

$= 0.88 \text{ mA/V}$

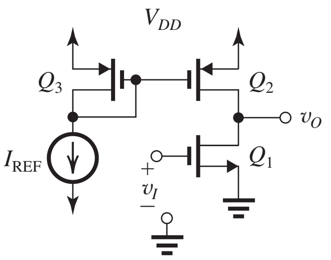
$$A_0 = 50\left(\frac{10}{100}\right)^{1/2} = 15.8\text{ V/V}$$

For $I_D = 1$ mA, we have

$$g_m = 0.28\text{ mA/V}\left(\frac{1}{0.010}\right)^{1/2} = 2.8\text{ mA/V}$$

$$A_0 = 50\left(\frac{0.010}{1}\right)^{1/2} = 5\text{ V/V}$$

Ex: 8.8



Since all transistors have the same

$$\frac{W}{L} = \frac{7.2\ \mu\text{m}}{0.36\ \mu\text{m}},$$

we have

$$I_{REF} = I_{D3} = I_{D2} = I_{D1} = 100\ \mu\text{A}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)}\sqrt{I_{D1}}$$

$$= \sqrt{2(387\ \mu\text{A/V}^2)\left(\frac{7.2}{0.36}\right)(100\ \mu\text{A})}$$

$$= 1.24\text{ mA/V}$$

$$r_{o1} = \frac{V'_{An}L_1}{I_{D1}} = \frac{5\text{ V}/\mu\text{m}\,(0.36\ \mu\text{m})}{0.1\text{ mA}} = 18\text{ k}\Omega$$

$$r_{o2} = \frac{|V'_{Ap}|L_2}{I_{D2}} = \frac{6\text{ V}/\mu\text{m}\,(0.36\ \mu\text{m})}{0.1\text{ mA}} = 21.6\text{ k}\Omega$$

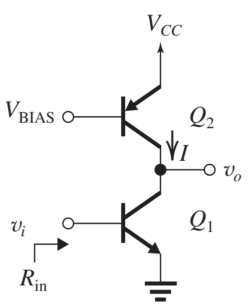
Voltage gain is

$$A_v = -g_{m1}\,(r_{o1}\parallel r_{o2})$$

$$A_v = -(1.24\text{ mA/V})\,(18\text{ k}\Omega \parallel 21.6\text{ k}\Omega)$$

$$= -12.2\text{ V/V}$$

Ex: 8.9



$$I_{C1} = I = 100\ \mu\text{A} = 0.1\text{ mA}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.1\text{ mA}}{25\text{ mV}} = 4\text{ mA/V}$$

$$R_{in} = r_{\pi 1} = \frac{\beta_1}{g_{m1}} = \frac{100}{4\text{ mA/V}} = 25\text{ k}\Omega$$

$$r_{o1} = \frac{V_A}{I} = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$$

$$r_{o2} = \frac{|V_A|}{I} = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$$

$$A_0 = g_{m1}\,r_{o1} = (4\text{ mA/V})\,(500\text{ k}\Omega) = 2000\text{ V/V}$$

$$A_v = -g_{m1}\,(r_{o1}\parallel r_{o2}) = -(4\text{ mA/V})\times$$

$$(500\text{ k}\Omega \parallel 500\text{ k}\Omega) = -1000\text{ V/V}$$

Ex: 8.10 Refer to Fig. 8.18(b),

$$v_o = iR_L$$

$$v_{sig} = i(R_s + R_{in})$$

Thus,

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}} \qquad \text{Q.E.D}$$

Ex: 8.11 Since $g_m r_o \gg 1$, we use Eq. (8.54),

$$R_{in} \simeq \frac{1}{g_m} + \frac{R_L}{g_m r_o}$$

R_L	0	r_o	$(g_m r_o)r_o$	∞
R_{in}	$\frac{1}{g_m}$	$\frac{2}{g_m}$	r_o	∞

Ex: 8.12 For $g_m r_o \gg 1$, we use Eq. (8.58),

$$R_{out} \simeq r_o + (g_m r_o)R_s$$

to obtain

R_s	0	r_o	$(g_m r_o)r_o$	∞
R_{out}	r_o	$(g_m r_o)r_o$	$(g_m r_o)^2 r_o$	∞

Ex: 8.13 A_{vo} remains unchanged at $g_m r_o$. With a load resistance R_L connected,

$$A_v = A_{vo}\frac{R_L}{R_L + R_o}$$

$$= (g_m r_o)\frac{R_L}{R_L + (1 + g_m R_s)r_o}$$

Ex: 8.14 Use Eq. (8.63)

$$R_{in} \simeq r_e\frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}$$

to obtain

R_L	0	r_o	$(\beta+1)r_o$	∞
R_{in}	r_e	$2\ r_e$	$\frac{1}{2}r_\pi$	r_π

Ex: 8.15 Using Eq. (8.68),

$$R_{\text{out}} \simeq r_o + (g_m r_o)(R_e \parallel r_\pi)$$

we obtain

R_e	0	r_e	r_π	r_o	∞
R_{out}	r_o	$2\ r_o$	$\left(\frac{\beta}{2}+1\right)r_o$	$(\beta+1)r_o$	$(\beta+1)r_o$

Ex: 8.16 $R_o = [1 + g_m(R_e \parallel r_\pi)]r_o$

where

$$g_m = 40\text{ mA/V},\ r_\pi = \frac{\beta}{g_m} = 2.5\text{ k}\Omega,$$

$$R_e = 0.5\text{ k}\Omega,\ \text{and } r_o = \frac{V_A}{I_C} = \frac{10}{1} = 10\text{ k}\Omega$$

Thus,

$$R_o = [1 + 40(0.5 \parallel 2.5)] \times 10$$

$$= 177\text{ k}\Omega$$

Without emitter degeneration,

$$R_o = r_o = 10\text{ k}\Omega$$

Ex: 8.17 Since the CG transistor Q_2 increases the output resistance by a factor approximately equal to $g_{m2}r_{o2}$,

$$K \simeq g_{m2}r_{o2}$$

Ex: 8.18 If L is halved $\left(L = \frac{0.55\ \mu\text{m}}{2}\right)$ and $|V_A| = |V_A'| \cdot L$, we obtain

$$|V_A| = 5\text{ V}/\mu\text{m}\left(\frac{0.55\ \mu\text{m}}{2}\right) = 1.375\text{ V}$$

$$R_o = \frac{|V_A|}{|V_{OV}|/2} \cdot \frac{|V_A|}{I_D} = \frac{2\,(1.375\ \text{V})^2}{(0.3\text{ V})\ \ (100\ \mu\text{A})}$$

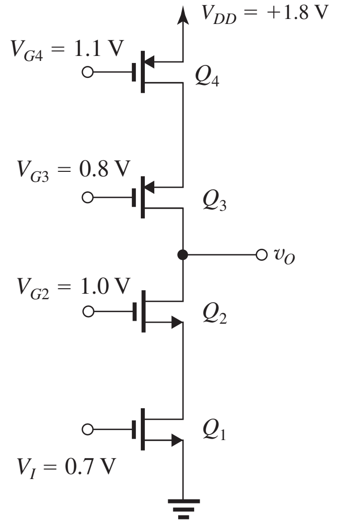
$$= 126\text{ k}\Omega$$

Since $I_D = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)|V_{OV}|^2\left(1 + \frac{V_{SD}}{|V_A|}\right)$

$$\frac{W}{L} = \frac{2\,(100\ \mu\text{A})}{90\ \mu\text{A/V}^2\,(0.3\text{ V})^2\left(1 + \frac{0.3\text{ V}}{1.375\text{ V}}\right)}$$

$$\frac{W}{L} = 20.3$$

Ex: 8.19



If all transistors are matched and are obviously operating at the same I_D, then all $|V_{OV}|$ will be equal and equal to that of Q_1, namely, $|V_{OV}| = 0.7 - 0.5 = 0.2$ V

$$V_{D1} = V_{S2} = V_{G2} - V_{tn} - V_{OV}$$

$$= 1.0 - 0.5 - 0.2 = 0.3\text{ V}$$

The lowest v_{DS2} can go is $|V_{OV}| = 0.2$ V

$$\therefore v_{O\text{min}} = V_{DS1} + V_{DS2} = 0.3 + 0.2 = 0.5\text{ V}$$

Similarly, $V_{SG4} = V_{SG3} = 0.7$ V

$$V_{D4} = V_{S3} = V_{G3} + |V_t| + |V_{OV}|$$

$$= 0.3 + 0.5 - 0.2 = 1.5\text{ V}$$

v_{SD3} can go as low as $|V_{OV}|$, so

$$v_{O\text{max}} = V_{D4} - v_{SD3_{\text{min}}} = 1.5 - 0.2 = 1.3\text{ V}$$

Ex: 8.20 Refer to Fig. 8.33.

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2\ I_D}{|V_{OV}|} = \frac{2 \times 0.2}{0.2}$$

$$= 2\text{ mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A|}{I_D} = \frac{2}{0.2} = 10\text{ k}\Omega$$

$$R_{on} = (g_{m2}r_{o2})r_{o1} = (2 \times 10) \times 10 = 200\text{ k}\Omega$$

$$R_{op} = (g_{m3}r_{o3})r_{o4} = (2 \times 10) \times 10 = 200\text{ k}\Omega$$

$$R_o = R_{on} \parallel R_{op} = 200 \parallel 200 = 100\text{ k}\Omega$$

$$A_v = -g_{m1}R_o = -2 \times 100 = -200\text{ V/V}$$

Ex: 8.21 $g_{m1} = g_{m2} = g_m$

$$= \frac{I_D}{\frac{V_{OV}}{2}} = \frac{0.1\text{ mA}}{(0.2/2)\text{ V}} = 1\text{ mA/V}$$

$r_{o1} = r_{o2} = r_o$

$= \dfrac{V_A}{I_D} = \dfrac{2\text{ V}}{0.1\text{ mA}} = 20\text{ k}\Omega$

so, $g_m r_o = 1\text{ mA/V}\ (20\text{ k}\Omega) = 20$

(a) For $R_L = 20\text{ k}\Omega$,

$$R_{\text{in}2} = \frac{R_L + r_{o2}}{1 + g_{m2} r_{o2}} = \frac{20\text{ k}\Omega + 20\text{ k}\Omega}{1+20} = 1.9\text{ k}\Omega$$

$\therefore A_{v1} = -g_{m1}\,(r_{o1} \parallel R_{\text{in}2})$

$= -1\text{ mA/V}\ (20 \parallel 1.9) = -1.74\text{ V/V}$

or

If we use the approximation of Eq. (8.83),

$$R_{\text{in}2} \approx \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{20\text{ k}\Omega}{20} + \frac{1}{1\text{ mA/V}} = 2\text{ k}\Omega$$

then

$A_{v1} = -1\text{ mA/V}\ (20\text{ k}\Omega \parallel 2\text{ k}\Omega) = -1.82\text{ V/V}$

Continuing, from Eq. (8.80),

$A_v = -g_{m1}\,[(g_{m2} r_{o2} r_{o1}) \parallel R_L]$

$A_v = -1\text{ mA/V}\ \{[(20)\,(20\text{ k}\Omega)] \parallel 20\text{ k}\Omega\}$

$= -19.0\text{ V/V}$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-19.0}{-1.82} = 10.5\text{ V/V}$$

(b) Now, for $R_L = 400\text{ k}\Omega$,

$$R_{\text{in}2} \simeq \frac{R_L}{g_{m2} r_{o2}} + \frac{1}{g_{m2}} = \frac{400\text{ k}\Omega}{20} + \frac{1}{1\text{ mA/V}}$$

$= 21\text{ k}\Omega$

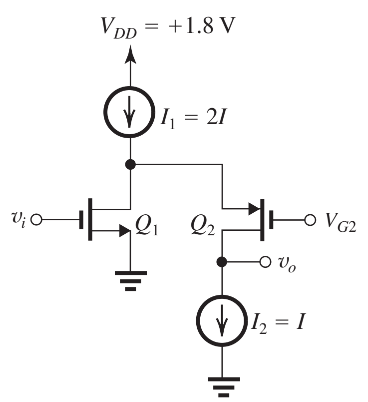
$A_{v1} = -1\text{ mA/V}\ (20\text{ k}\Omega \parallel 21\text{ k}\Omega) = -10.2\text{ V/V}$

$A_v = -1\text{ mA/V}\ [(20)\,(20\text{ k}\Omega)] \parallel 400\text{ k}\Omega$

$= -200\text{ V/V}$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{-200}{-10.2} = 19.6\text{ V/V}$$

Ex: 8.22



(a) $I_{D1} = I$ and $I_{D2} = I$

Since $V_{OV1} = V_{OV2} = 0.2$ V, we have

$$\frac{I_{D2}}{I_{D1}} = \frac{\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2} = \frac{I}{I} = 1$$

Thus,

$$\frac{k_p'\left(\frac{W}{L}\right)_2}{k_n'\left(\frac{W}{L}\right)_1} = 1 \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{k_n'}{k_p'}\left(\frac{W}{L}\right)_1$$

$$= \frac{k_n'}{\frac{k_n'}{4}}\left(\frac{W}{L}\right)_1$$

or $\left(\frac{W}{L}\right)_2 = 4\left(\frac{W}{L}\right)_1$

(b) The minimum voltage required across current source I_1 would be $|V_{OV}| = 0.2$ V, since it is made with a single transistor. If a 0.1-V_{PP} signal swing is to be allowed at the drain of Q_1, the highest dc bias voltage would be

$$V_{DD} - |V_{OV}| - \frac{0.1\ V_{\text{pp}}}{2} = 1.8 - 0.2 - \frac{1}{2}(0.1)$$

$= 1.55$ V

(c) $V_{SG2} = |V_{OV}| + |V_{tp}| = 0.2 + 0.5 = 0.7$ V

V_{G2} can be set at 1.55 – 0.7 = 0.85 V.

(d) Since current source I_2 is implemented with a cascoded current source, the minimum voltage required across it for proper operation is $2V_{OV} = 2\,(0.2\text{ V}) = 0.4$ V.

(e) From parts (c) and (d), the allowable range of signal swing at the output is from 0.4 V to 1.55 V – V_{OV} or 1.35 V.

so, $0.4\text{ V} \le v_O \le 1.35$ V.

Ex: 8.23 Referring to Fig. 8.38,

$R_{op} = (g_{m3} r_{o3})\,(r_{o4} \parallel r_{\pi 3})$ and

$R_{on} = (g_{m2} r_{o2})\,(r_{o1} \parallel r_{\pi 2})$

The maximum values of these resistances are obtained when $r_o \gg r_\pi$ and are given by

$R_{on}\big|_{\max} = (g_{m2} r_{o2})\, r_{\pi 2}$

$R_{op}\big|_{\max} = (g_{m3} r_{o3})\, r_{\pi 3}$

Since $g_m r_\pi = \beta$,

$R_{on}\big|_{\max} = \beta_2 r_{o2}$

$R_{op}\big|_{\max} = \beta_3 r_{o3}$

Since $A_v = -g_{m1}(R_{on} \parallel R_{op})$,

$|A_{v\max}| = g_{m1}(\beta_2 r_{o2} \parallel \beta_3 r_{o3})$

Ex: 8.24 For the *npn* transistors,

$$g_{m1} = g_{m2} = \frac{|I_C|}{|V_T|} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{8 \text{ mA/V}} = 12.5 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{|V_A|}{|I_C|} = \frac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$$

From Fig. 8.38,

$R_{on} = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi 2})$

$= (8 \text{ mA/V})(25 \text{ k}\Omega)(25 \text{ k}\Omega \parallel 12.5 \text{ k}\Omega)$

$R_{on} = 1.67 \text{ M}\Omega$

For the *pnp* transistors,

$$g_{m3} = g_{m4} = \frac{|I_C|}{V_T} = \frac{0.2 \text{ mA}}{25 \text{ mV}} = 8 \text{ mA/V}$$

$$r_{\pi 3} = r_{\pi 4} = \frac{\beta}{g_m} = \frac{50}{8 \text{ mA/V}} = 6.25 \text{ k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_A|}{|I_C|} = \frac{4 \text{ V}}{0.2 \text{ mA}} = 20 \text{ k}\Omega$$

$R_{op} = (g_{m3} r_{o3})(r_{o4} \parallel r_{\pi 3})$

$= (8 \text{ mA/V})(20 \text{ k}\Omega)(20 \text{ k}\Omega \parallel 6.25 \text{ k}\Omega)$

$R_{op} = 762 \text{ k}\Omega$

$A_v = -g_{m1}(R_{on} \parallel R_{op})$

$= -(8 \text{ mA/V})(1.67 \text{ M}\Omega \parallel 762 \text{ k}\Omega)$

$A_v = -4186 \text{ V/V}$

$A_{v\max}$ occurs when r_{o1} and r_{o4} are $\gg r_\pi$.

Then

$R_{on} = (g_{m2} r_{o2})\, r_{\pi 2} = \beta_2 r_{o2}$

$R_{on} = 100\,(25 \text{ k}\Omega) = 2.5 \text{ M}\Omega$

$R_{op} = (g_{m3} r_{o3})\, r_{\pi 3} = \beta_3 r_{o3}$

$R_{op} = 50\,(20 \text{ k}\Omega) = 1 \text{ M}\Omega$

Finally,

$A_{v\max} = -(8 \text{ mA/V})(2.5 \text{ M}\Omega \parallel 1.0 \text{ M}\Omega)$

$A_{v\max} = -5714 \text{ V/V}$

Ex: 8.25 Refer to the circuit in Fig. 8.39. All transistors are operating at $I_D = I_{\text{REF}} = 100\ \mu\text{A}$ and equal V_{OV}, found from

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{3.6}{0.36} \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.227 \text{ V}$

$V_{GS} = 0.227 + 0.5 = 0.727 \text{ V}$

$V_{O\min} = V_{G3} - V_{t3}$

$= V_{GS4} + V_{GS1} - V_{t3}$

Thus,

$V_{O\min} = 2V_{GS} - V_t$

$= V_t + 2\, V_{OV}$

$= 0.5 + 2 \times 0.227 = 0.95 \text{ V}$

$$g_m = \frac{2\, I_D}{V_{OV}} = \frac{2 \times 0.1}{0.227} = 0.88 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$R_o = (g_{m3} r_{o3}) r_{o2} = (0.88 \times 18) \times 18$
$= 285 \text{ k}\Omega$

Ex: 8.26 For the Wilson mirror from Eq. (8.94), we have

$$\frac{I_O}{I_{\text{REF}}} \simeq \frac{1}{1 + \dfrac{2}{\beta^2}} = 0.9998$$

Thus $\dfrac{|I_O - I_{\text{REF}}|}{I_{\text{REF}}} \times 100 = 0.02\%$

whereas for the simple mirror from Eq. (8.18) we have

$$\frac{I_O}{I_{\text{REF}}} = \frac{1}{1 + \dfrac{2}{\beta}} = 0.98$$

Hence $\dfrac{|I_O - I_{\text{REF}}|}{I_{\text{REF}}} \times 100 = 2\%$

For the Wilson current mirror, we have

$$R_o = \frac{\beta r_o}{2} = \frac{100 \times 100 \text{ k}\Omega}{2} = 5 \text{ M}\Omega$$

and for the simple mirror, $R_o = r_o$
$= 100 \text{ k}\Omega$.

Ex: 8.27 For the two current sources designed in Example 8.6, we have

$$g_m = \frac{I_C}{V_T} = \frac{10\ \mu\text{A}}{25 \text{ mV}} = 0.4\ \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{10\ \mu\text{A}} = 10 \text{ M}\Omega,$$

$$r_\pi = \frac{\beta}{g_m} = 250 \text{ k}\Omega$$

For the current source in Fig. 8.43(b), we have

$R_o = r_{o2} = r_o = 10 \text{ M}\Omega$

For the current source in Fig. 8.43, from Eq. (8.102), we have

$R_{out} \simeq [1 + g_m (R_E \parallel r_\pi)]\, r_o$

From Example 8.6, $R_E = R_3 = 11.5$ kΩ;

therefore,

$$R_{out} \simeq \left[1 + 0.4\frac{\text{mA}}{\text{V}}\,(11.5\text{ k}\Omega \parallel 250\text{ k}\Omega)\right] 10\text{ M}\Omega$$

$\therefore R_{out} = 54$ MΩ

Ex: 8.28

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2\text{ mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2 = 0.4\text{ mA/V}$$

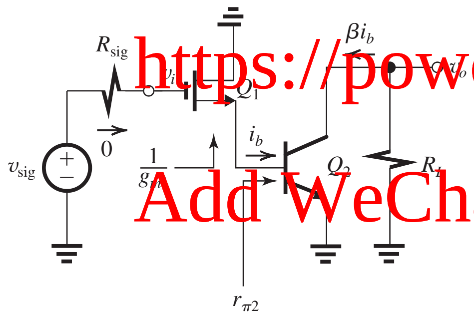
$$r_{o1} = r_{o3} = \frac{V_A}{I_D} = \frac{5}{0.2} = 25\text{ k}\Omega$$

$$R_L = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{mb}} = 25 \parallel 25 \parallel 2.5\text{ k}\Omega$$

$$= 2.083\text{ k}\Omega$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_L + \dfrac{1}{g_m}} = \frac{2.083}{2.083 + \dfrac{1}{2}} = 0.81\text{ V/V}$$

Ex: 8.29



$$g_{m1} = \sqrt{2k_n I_D}$$

$$= \sqrt{2 \times 8 \times 1}$$

$$= 4\text{ mA/V}$$

$$\frac{1}{g_{m1}} = 0.25\text{ k}\Omega$$

$$g_{m2} = 40\text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{40} = 2.5\text{ k}\Omega$$

$$R_{in} = \infty$$

$$i_b = \frac{v_i}{\dfrac{1}{g_{m1}} + r_{\pi 2}} = \frac{v_{sig}}{\dfrac{1}{g_{m1}} + r_{\pi 2}} = \frac{v_{sig}}{0.25 + 2.5}$$

$$= \frac{v_{sig}}{2.75}$$

$$v_o = -\beta i_b R_L = -\frac{100 \times 4}{2.75} v_{sig}$$

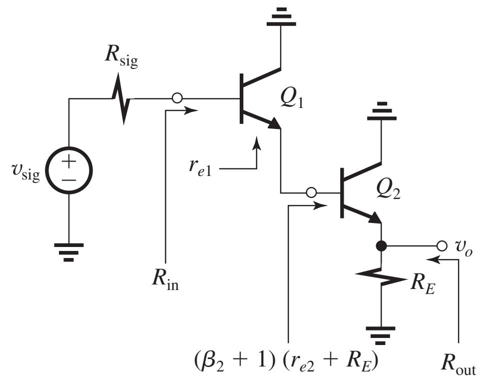
$$G_v \equiv \frac{v_o}{v_{sig}} = -145.5\text{ V/V}$$

These results apply for both $R_{sig} = 4$ kΩ and $R_{sig} = 400$ kΩ. If in the CC–CE amplifier of Example 8.7, $R_{sig} = 400$ kΩ, G_v becomes

$$G_v = \frac{255}{255 + 400} \times 0.99 \times -160$$

$$= -61.7\text{ V/V}$$

Ex: 8.30



From the figure we can write

$$R_{in} = (\beta_1 + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_E)]$$

$$R_{out} = R_E \parallel \left[r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}\right]$$

$$\frac{v_o}{v_{sig}} = \frac{R_E}{R_E + r_{e2} + \dfrac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}}$$

For $I_{E2} = 5$ mA, $\beta_1 = \beta_2 = 100$, $R_E = 1$ kΩ, and $R_{sig} = 100$ kΩ, we obtain

$$r_{e2} = \frac{25\text{ mV}}{5\text{ mA}} = 5\ \Omega$$

$$I_{E1} = \frac{5}{\beta_2 + 1} = \frac{5}{101} \simeq 0.05\text{ mA}$$

$$r_{e1} = \frac{25\text{ mV}}{0.05\text{ mA}} = 500\ \Omega$$

$$R_{in} = 101 \times (0.5 + 101 \times 1.005) = 10.3\text{ M}\Omega$$

$$R_{out} = 1 \parallel \left[0.005 + \frac{0.5 + (100/101)}{101}\right] \simeq 20\ \Omega$$

$$\frac{v_o}{v_{sig}} = \frac{1}{1 + 0.005 + \dfrac{0.5 + (100/101)}{101}}$$

$$= 0.98\text{ V/V}$$

Ex: 8.31 Refer to Fig. 8.49.

$$r_e = 25\ \Omega$$

$$R_{in} = (\beta_1 + 1)(2\, r_e) = 101 \times 0.05 = 5.05\text{ k}\Omega$$

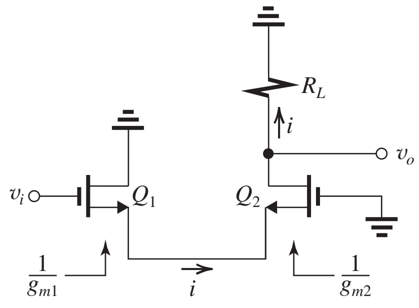
$$\frac{v_o}{v_i} = \frac{\alpha_2 R_L}{2\ r_e} \simeq \frac{5}{0.05} = 100 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \times \frac{v_o}{v_i}$$

$$= \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \times \frac{v_o}{v_i}$$

$$= \frac{5.05}{5.05 + 5} \times 100 = 50 \text{ V/V}$$

Ex: 8.32



(a) From the figure we see that

$$i = \frac{v_i}{2/g_m} = \frac{1}{2} g_m v_i$$

and

$$v_o = iR_L$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} g_m R_L$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{2I}{V_{OV}} R_L = \frac{IR_L}{V_{OV}} \qquad \text{Q.E.D}$$

(b) $I = 0.1$ mA and $R_L = 20$ kΩ, to obtain a gain of 10 V/V,

$$10 = \frac{0.1 \times 20}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

The required W/L can be obtained from

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right) \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

8.1 Referring to Fig. 8.1, $V_{DD} = 1.3$ V,

$I_O = I_{\text{REF}} = 100\ \mu\text{A}$, $L = 0.5\ \mu\text{m}$, $W = 5\ \mu\text{m}$, $V'_A = 5\ \text{V}/\mu\text{m}$, $V_t = 0.4$ V, $k'_n = 500\ \mu\text{A/V}^2$

$$I_O = I_D = \frac{1}{2}k'_n\left(\frac{W}{L}\right)V_{OV}^2$$

$$V_{OV} = \sqrt{\frac{2I_D}{k'_n\left(\frac{W}{L}\right)}}$$

$$= \sqrt{\frac{2\,(100\ \mu\text{A})}{(500\ \mu\text{A/V}^2)\left(\frac{5}{0.5}\right)}} = 0.2\ \text{V}$$

$$V_{DS} = V_{GS} = V_t + V_{OV} = 0.4 + 0.2 = 0.6\ \text{V}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{\text{REF}}} = \frac{1.8 - 0.6}{0.1\ \text{mA}} = 12\ \text{k}\Omega$$

The lowest V_O will be

$$V_{DS2} = V_{OV} = 0.2\ \text{V}$$

$$R_O = r_o = \frac{V'_A L}{I_D} = \frac{5\ \text{V}/\ \mu\text{m} \times 0.5\ \mu\text{m}}{100\ \mu\text{A}} = 25\ \text{k}\Omega$$

$$\Delta I_D \approx \frac{\Delta V_O}{r_o} = \frac{0.5\ \text{V}}{25\ \text{K}} = 20\ \mu\text{A}$$

8.2 Refer to Fig. 8.1.

$$\frac{\Delta I_O}{I_O} = 10\%$$

$$\Delta I_O = 0.1 \times 150 = 15\ \mu\text{A}$$

$$\Delta V_O = 1.8 - 0.3 = 1.5\ \text{V}$$

$$r_o = \frac{\Delta V_O}{\Delta I_O} = \frac{1.5\ \text{V}}{15\ \mu\text{A}} = 100\ \text{k}\Omega$$

But

$$r_o = \frac{V_A}{I_O} = \frac{V'_A L}{I_O}$$

$$100 = \frac{10 \times L}{0.15} \Rightarrow L = 1.5\ \mu\text{m}$$

$$\Rightarrow V_A = 15\ \text{V}$$

$$V_{OV} = V_{DS2\text{min}} = 0.3\ \text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.3 = 0.8\ \text{V}$$

$$I_D = \frac{1}{2}k'_n\left(\frac{W}{L}\right)V_{OV}^2\left(1 + \frac{V_{DS}}{V_A}\right)$$

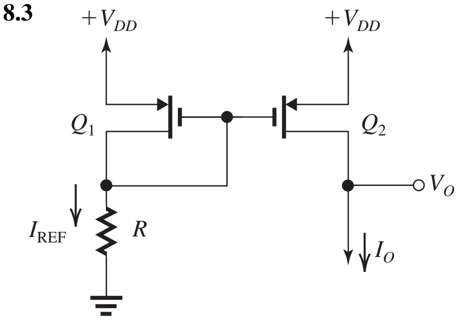
$$150 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.09\left(1 + \frac{0.8}{15}\right)$$

$$\Rightarrow \frac{W}{L} = 7.91$$

$$W = 7.91 \times 1.5 = 11.9\ \mu\text{m}$$

$$R = \frac{V_{DD} - V_{GS}}{I_{\text{REF}}} = \frac{1.8 - 0.8}{0.15} = 6.7\ \text{k}\Omega$$

8.3



Set $|V_{OV}| = V_{DD} - V_{O\text{max}}$

$$= 1.3 - 1.1 = 0.2\ \text{V}$$

$$V_G = V_{DD} - |V_{tp}| - |V_{OV}|$$

$$= 1.3 - 0.4 - 0.2 = 0.7\ \text{V}$$

$$R = \frac{V_G}{I_{D1}} = \frac{0.7\ \text{V}}{80\ \mu\text{A}} = 8.75\ \text{k}\Omega$$

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}|^2$$

thus

$$\frac{W}{L} = \frac{2I_D}{\mu_p C_{ox}|V_{OV}|^2} = \frac{2 \times 80\ \mu\text{A}}{80\ \mu\text{A/V}^2 \times 0.2^2} = 50$$

8.4 Referring to Fig. 8.1, if $W_2 = 5\ W_1$ and we let $L_1 = L_2$, then we obtain

$$I_O = I_{D2} = I_{\text{REF}}\frac{(W/L)_2}{(W/L)_1} = 20\ \mu\text{A} \times 5 = 100\ \mu\text{A}$$

$$V_{O\text{min}} = V_{OV} = 0.2\ \text{V}$$

From Eq. (8.8):

$$I_O = \frac{(W/L)_2}{(W/L)_1} \cdot I_{\text{REF}}\left(1 + \frac{V_O - V_{GS}}{V_{A2}}\right)$$

$$V_{GS} = V_t + V_{OV} = 0.5\ \text{V} + 0.2\ \text{V} = 0.7\ \text{V}$$

Thus, I_D equal $5I_{\text{REF}}$ will be obtained at

$$V_O = V_{GS} = 0.7\ \text{V}$$

For $V_O = V_{GS} + 1 = 1.7$ V

$$I_O = 100\left(1 + \frac{1.7 - 0.7}{20}\right) = 105\ \mu\text{A}$$

The corresponding increase in I_O, ΔI_O is, thus, 5 µA.

8.5 Referring to Fig. P8.5, we obtain

$V_{GS1} = V_{GS2}$ so that $\dfrac{I_{D2}}{I_{D1}} = \dfrac{(W/L)_2}{(W/L)_1}$ and

$$I_{D2} = I_{\text{REF}}\frac{(W/L)_2}{(W/L)_1}$$

$$I_{D3} = I_{D2}$$

$$V_{GS3} = V_{GS4}, \text{ thus } \frac{I_{D4}}{I_{D3}} = \frac{(W/L)_4}{(W/L)_3}$$

$$I_O = I_{D4} = I_{\mathrm{REF}} \frac{(W/L)_2}{(W/L)_1} \cdot \frac{(W/L)_4}{(W/L)_3}$$

8.6 Refer to the circuit of Fig. P8.6. For Q_2 to operate properly (i.e., in the saturation mode) for drain voltages as high as +0.8 V, and provided its width is the minimum possible, we use

$|V_{OV}| = 0.2$ V

Note that all three transistors Q_1, Q_2, and Q_3 will be operated at this value of overdrive voltage. For Q_1,

$I_{D1} = I_{\mathrm{REF}} = 20\ \mu\mathrm{A}$

$$I_{D1} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 |V_{OV}|^2$$

$$20 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_1 \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 10$$

For $L = 0.5\ \mu$m,

$W_1 = 5\ \mu$m

Now, for

$I_2 = 100\ \mu\mathrm{A} = 5 I_{\mathrm{REF}}$, we have

$$\frac{(W/L)_2}{(W/L)_1} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 5 \times 10 = 50$$

$W_2 = 50 \times 0.5 = 25\ \mu$m

For

$I_3 = 40\ \mu\mathrm{A} = 2 I_{\mathrm{REF}}$, we obtain

$$\frac{(W/L)_3}{(W/L)_1} = 2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 20$$

$W_3 = 10\ \mu$m

We next consider Q_4 and Q_5. For Q_5 to operate in saturation with the drain voltage as low as −0.8 V, and for it to have the minimum possible W/L, we operate Q_5 at

$V_{OV} = 0.2$ V

This is the same overdrive voltage at which Q_4 will be operating. Thus, we can write for Q_4,

$I_4 = I_3 = 40\ \mu$A

and using

$$I_{D4} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_4 V_{OV}^2$$

$$40 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_4 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = 5$$

$W_4 = 2.5\ \mu$m

Finally, since

$I_5 = 80\ \mu\mathrm{A} = 2\, I_4$,

$$\left(\frac{W}{L}\right)_5 = 2\left(\frac{W}{L}\right)_4$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 10$$

$W_5 = 5\ \mu$m

To find the value of R, we use

$$|V_{SG1}| = |V_{tp}| + |V_{OV1}|$$

$$= 0.5 + 0.2 = 0.7 \text{ V}$$

$$R = \frac{1 - |V_{SG1}|}{I_{\mathrm{REF}}} = \frac{0.3 \text{ V}}{0.02 \text{ mA}}$$

$$= 15 \text{ k}\Omega$$

The output resistance of the current source Q_2 is

$$r_{o2} = \frac{|V_{A2}|}{I_2} = \frac{|V'_{Ap}| \times L}{I_2}$$

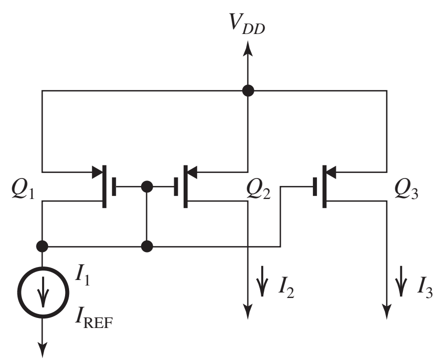
$$= \frac{5 \times 0.5}{0.1 \text{ mA}} = 25 \text{ k}\Omega$$

The output resistance of the current sink Q_5 is

$$r_{o5} = \frac{V_{A5}}{I_5} = \frac{V'_{An} \times L}{I_5}$$

$$= \frac{5 \times 0.5}{80} = 31.25 \text{ k}\Omega$$

8.7 Referring to the figure, suppose that Q_1 has $W = 10\ \mu$m, Q_2 has $W = 20\ \mu$m, and Q_3 has $W = 40\ \mu$m.



(1) With Q_1 diode connected,

$$I_2 = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1} = 100\ \mu\text{A} \left(\frac{20}{10}\right) = 200\ \mu\text{A}$$

$$I_3 = 100\ \mu\text{A} \left(\frac{40}{10}\right) = 400\ \mu\text{A}$$

(2) With Q_2 diode connected, and $W = 20\ \mu\text{m}$,

$$I_1 = 100\ \mu\text{A} \left(\frac{10}{20}\right) = 50\ \mu\text{A}$$

$$I_3 = 100\ \mu\text{A} \left(\frac{40}{20}\right) = 200\ \mu\text{A}$$

(3) If Q_3 with $W = 40\ \mu\text{m}$ is diode connected,

$$I_1 = 100\ \mu\text{A} \left(\frac{10}{40}\right) = 25\ \mu\text{A}$$

$$I_2 = 100\ \mu\text{A} \left(\frac{20}{40}\right) = 50\ \mu\text{A}$$

So, with only one transistor diode connected, we can get $25\ \mu\text{A},\ 50\ \mu\text{A},\ 200\ \mu\text{A},$ and $400\ \mu\text{A}$, or four different currents.

Now, if two transistors are diode connected, the effective width is the sum of the two widths.

(4) If Q_1 and Q_2 are diode connected, then

$W_{\text{eff}} = 20 + 10 = 30\ \mu\text{m}$, so that

$$I_3 = 100\ \mu\text{A} \left(\frac{40}{30}\right) = 133\ \mu\text{A}$$

(5) If Q_2 and Q_3 are diode connected, then

$W_{\text{eff}} = 20 + 40 = 60\ \mu\text{m}$, so that

$$I_1 = 100\ \mu\text{A} \left(\frac{10}{60}\right) = 16.7\ \mu\text{A}$$

(6) If Q_1 and Q_3 are diode connected,

$W_{\text{eff}} = 10 + 40 = 50\ \mu\text{m}$, so that

$$I_2 = 100\ \mu\text{A} \left(\frac{20}{50}\right) = 40\ \mu\text{A}$$

So three different currents are obtained with double-diode connects.

To find V_{SG}, we use the following for the diode-connected transistor(s):

$$I_D = \frac{1}{2}\mu_p C_{ox} \left(\frac{W}{L}\right) (V_{SG} - |V_{tp}|)^2$$

and substitute $I_D = I_{\text{REF}} = 100\ \mu\text{A}$. Thus

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{1\ \mu\text{m}}\right) (V_{SG} - 0.6)^2$$

$$\Rightarrow V_{SG} = 0.6 + \sqrt{\frac{2}{W\,(\mu\text{m})}}$$

For the six cases above we obtain

(1) $W = W_1 = 10\ \mu\text{m} \Rightarrow V_{SG} = 1.05\ \text{V}$

(2) $W = W_2 = 20\ \mu\text{m} \Rightarrow V_{SG} = 0.92\ \text{V}$

(3) $W = W_3 = 40\ \mu\text{m} \Rightarrow V_{SG} = 0.82\ \text{V}$

(4) $W = W_1 + W_2 = 30\ \mu\text{m} \Rightarrow V_{SG} = 0.86\ \text{V}$

(5) $W = W_2 + W_3 = 60\ \mu\text{m} \Rightarrow V_{SG} = 0.78\ \text{V}$

(6) $W = W_1 + W_3 = 50\ \mu\text{m} \Rightarrow V_{SG} = 0.80\ \text{V}$

8.8 (a) If $I_S = 10^{-17}$ A and we ignore base currents, then

$I_{\text{REF}} = I_S\, e^{V_{BE}/V_T}$ so that

$$V_{BE} = V_T \ln\left(\frac{I_{\text{REF}}}{10^{-17}}\right)$$

For $I_{\text{REF}} = 10\ \mu\text{A}$,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-5}}{10^{-17}}\right) = 0.69\ \text{V}$$

For $I_{\text{REF}} = 10\ \text{mA}$,

$$V_{BE} = 0.025 \ln\left(\frac{10^{-2}}{10^{-17}}\right) = 0.863\ \text{V}$$

So for the range of

$10\ \mu\text{A} \le I_{\text{REF}} \le 10\ \text{mA}$,

$0.691\ \text{V} \le V_{BE} \le 0.863\ \text{V}$

(b) Accounting for finite β,

$$I_O = I_{\text{REF}} \cdot \frac{1}{1 + 2/\beta}$$

For $I_{\text{REF}} = 10\ \mu\text{A}$,

$$I_O = \frac{10\ \mu\text{A}}{1 + \dfrac{2}{50}} = 9.62\ \mu\text{A}$$

For $I_{\text{REF}} = 0.1\ \text{mA}$,

$$I_O = \frac{0.1\ \text{mA}}{1 + \dfrac{2}{100}} = 0.098\ \text{mA}$$

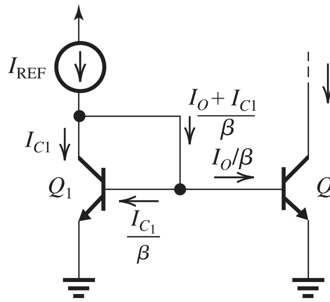
For $I_{\text{REF}} = 1\ \text{mA}$,

$$I_O = \frac{1\ \text{mA}}{1 + \dfrac{2}{100}} = 0.98\ \text{mA}$$

For $I_{\text{REF}} = 10\ \text{mA}$,

$$I_O = \frac{10\ \text{mA}}{1 + \dfrac{2}{50}} = 9.62\ \text{mA}$$

8.9



$I_O = mI_{C1}$

A node equation at the collector of Q_1 yields

$$I_{REF} = I_{C1} + \frac{I_O + I_{C1}}{\beta}$$

Substituting $I_{C1} = I_O/m$ results in

$$\frac{I_O}{I_{REF}} = \frac{m}{1 + \dfrac{m+1}{\beta}} \qquad \text{Q.E.D.}$$

For $\beta = 80$ and the error in the current transfer ratio to be limited to 10%, that is

$$\frac{m}{1 + \dfrac{m+1}{\beta}} \geq 0.9m$$

$$\left(1 + \frac{m+1}{\beta}\right) \leq \frac{1}{0.9}$$

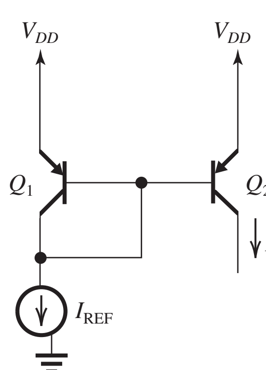
$$\frac{m+1}{\beta} \leq \frac{1}{0.9} - 1$$

$$m \leq \beta\left(\frac{1}{0.9} - 1\right) - 1$$

$$m \leq 80\left(\frac{1}{0.9} - 1\right) - 1 = 7.88$$

Thus, the largest current transfer ratio possible is 7.88.

8.10



For identical transistors, the transfer ratio is

$$\frac{I_O}{I_{REF}} = \frac{1}{1 + 2/\beta} = \frac{1}{1 + \dfrac{2}{50}} = 0.96$$

8.11 Nominally, $I_O = I_{REF} = 1$ mA

$$r_{o2} = \frac{V_{A2}}{I_O} = \frac{90}{1} = 90\ \text{k}\Omega$$

$$r_{o2} = \frac{\Delta V_O}{\Delta I_O} \Rightarrow \frac{10 - 1}{\Delta I_O} = 90 \Rightarrow \Delta I_O = 0.1\ \text{mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.1}{1} = 10\%\ \text{change}$$

8.12 Equation (8.21) gives the current transfer ratio of an *npn* mirror with a nominal ratio of m:

$$I_O = I_{REF}\frac{m}{1 + \dfrac{m+1}{\beta}}\left(1 + \frac{V_O - V_{BE}}{V_{A2}}\right)$$

This equation can be adapted for the *pnp* mirror of Fig. P8.12 by substituting $m = 1$, replacing V_O with the voltage across Q_3, namely $(3 - V_O)$, replacing V_{BE} with V_{EB}, and V_{A2} with $|V_A|$:

$$I_O = I_{REF}\frac{1 + [(3 - V_O - V_{EB})/|V_A|]}{1 + (2/\beta)} \qquad (1)$$

Now, substituting $I_O = 1$ mA, $V_O = 1$ V, $\beta = 50$, $|V_A| = 50$ V, and

$$V_{EB} = V_T \ln\frac{I_O}{I_S} = 0.025 \ln\left(\frac{10^{-3}}{10^{-15}}\right) = 0.691\ \text{V}$$

results in

$$I_{REF} = \frac{1 \times (1 + 0.04)}{1 + \dfrac{3 - 1 - 0.691}{50}} = 1.013\ \text{mA}$$

$$R = \frac{V_{CC} - V_{EB}}{I_{REF}} = \frac{3 - 0.691}{1.013} = 2.28\ \text{k}\Omega$$

Maximum allowed voltage $V_O = 3 - 0.3 = 2.7$ V. For $V_O = 2.7$ V, Eq. (1) yields

$$I_O = 1.013\,\frac{1 + \dfrac{3 - 2.7 - 0.691}{50}}{1.04} = 0.966\ \text{mA}$$

For $V_O = -5$ V, Eq. (1) yields

$$I_O = 1.013\,\frac{1 + \dfrac{3 - (-5) - 0.691}{50}}{1.04} = 1.116\ \text{mA}$$

Thus, the change in I_O is 0.15 mA.

8.13 The solution is given in the circuit diagram. Note that the starting point is calculating the current I in the Q_1–R_1–Q_2 branch. See figure on next page.

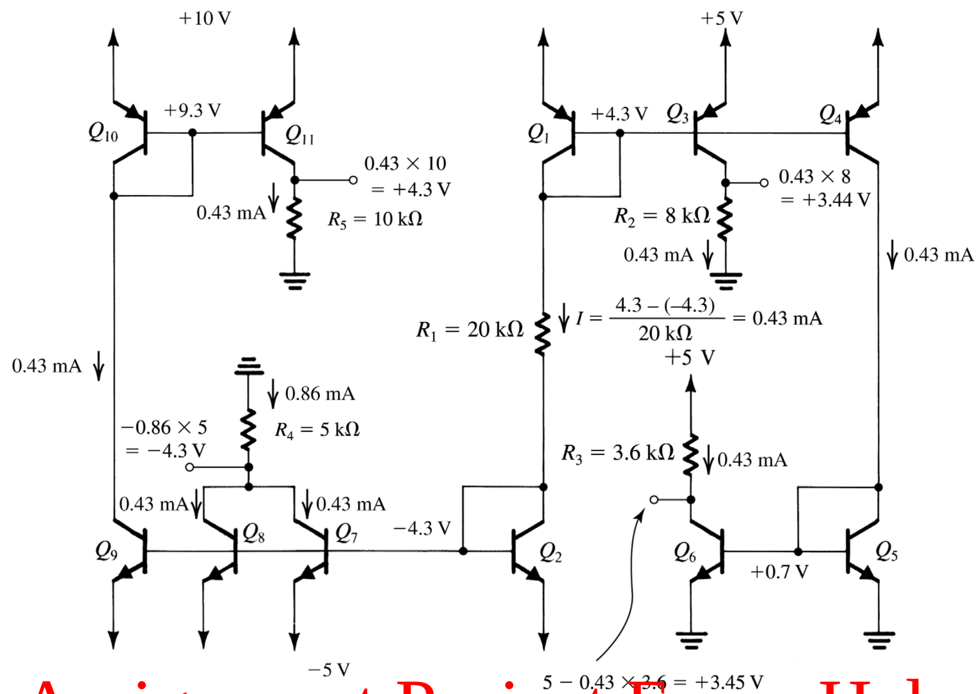
8.14 Refer to the circuit in Fig. P8.14.

$V_2 = 2.7 - V_{EB} = 2.7 - 0.7 = +2$ V

$V_3 = 0 + V_{EB} = +0.7$ V

Thus, Q_3 and Q_4 are operating in the active mode, and each is carrying a collector current of $I/2$. The same current is flowing in Q_2 and Q_1; thus

This figure belongs to Problem **8.13**.



$$V_1 = -2.7 + \frac{I}{2}R$$

But

$$V_1 = -V_{BE1} = -0.7$$

Thus,

$$-0.7 = -2.7 + \frac{1}{2}IR$$

$$\Rightarrow IR = 4 \text{ V}$$

The current I splits equally between Q_5 and Q_6; thus

$$V_4 = -2.7 + \left(\frac{I}{2}\right)R = -2.7 + 2 = -0.7 \text{ V}$$

$$V_5 = -2.7 + \left(\frac{I}{2}\right)\left(\frac{R}{2}\right) = -2.7 + 1 = -1.7 \text{ V}$$

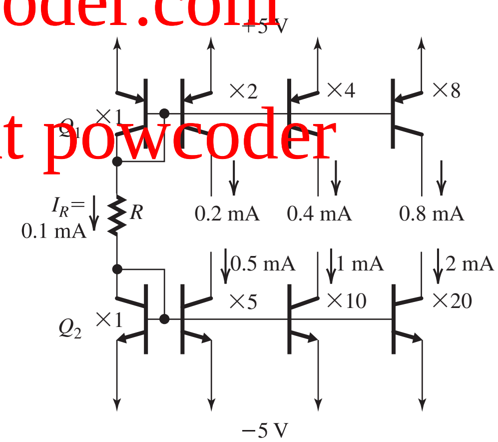
Thus, Q_5 and Q_6 are operating in the active mode as we have implicitly assumed.

Note that the values of V_1, V_2, V_3, V_4, and V_5 do not depend on the value of R. Only I depends on the value of R:

(a) $R = 10\text{ k}\Omega \;\Rightarrow\; I = \frac{4}{10} = 0.4 \text{ mA}$

(b) $R = 100\text{ k}\Omega \;\Rightarrow\; I = \frac{4}{100} = 0.04 \text{ mA}$

8.15 There are various ways this design could be achieved, but the most straightforward is the one shown:



With this scheme,

$$R = \frac{5 - 0.7 - 0.7 - (-5)}{0.1 \text{ mA}} = 86 \text{ k}\Omega$$

and each transistor has EBJ areas proportional to the current required. Multiple, parallel transistors are acceptable.

Note: This large value of R is not desirable in integrated form; other designs may be move suitable.

Even without knowing exact circuitry, we can find the total power dissipation as approximately

$$P_T = P_{CC} + P_{EE}$$

$P_T = 5\text{ V}\,(0.1 + 0.2 + 0.4 + 0.8)\ \text{mA}$

$+5\text{ V}\,(0.1 + 0.5 + 1 + 2)\ \text{mA}$

$P_T = 7.5\text{ mW} + 18\text{ mW} = 25.5\text{ mW}$

8.16 (a)

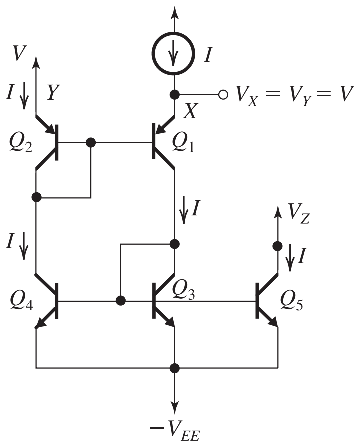


Figure 1

Figure 1 shows the current conveyor circuit with Y connected to a voltage V, X fed with a current source I, and Z connected to a voltage V_Z that keeps Q_5 operating in the active mode. Assuming that all transistors are operating in the active mode and that $\beta \gg 1$, so that we can neglect all base currents, we see that the current I through Q_1 will flow through the two-output mirror Q_3, Q_4, and Q_5. The current I in Q_5 will be drawn from Q_2, which forms a mirror with Q_1. Thus $V_{EB2} = V_{EB1}$ and the voltage that appears at X will be equal to V. The current in Q_5 will be equal to I, thus terminal Z sinks a constant current I.

(b)

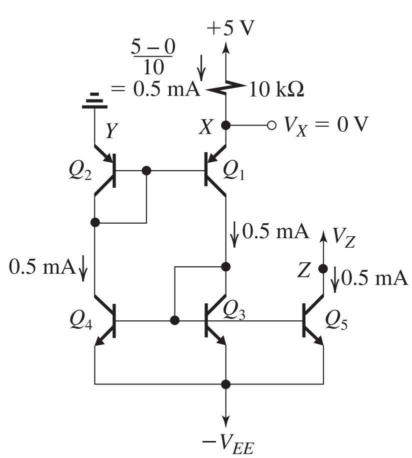


Figure 2

Figure 2 shows the special case of $V = 0$ V. As before, the voltage at X, V_X, will be equal to V. Thus

$V_X = 0$

That is, a virtual ground appears at X, and thus the current I that flows into X can be found from

$$I = \frac{5 - V_X}{10\text{ k}\Omega} = \frac{5 - 0}{10} = 0.5\text{ mA}$$

This is the current that will be mirrored to the output, resulting in $I_Z = 0.5$ mA.

8.17 Using Eq. (8.28),

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}}$$

where

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{V_A' L}{I_{D1}} = \frac{10 \times 0.5}{0.1\text{ mA}} = 50\text{ k}\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2 \times 0.5 \times \left(\frac{10}{0.5}\right) \times 0.1} = 1.414\text{ mA/V}$$

$$\frac{1}{g_{m1}} = 0.71\text{ k}\Omega$$

Thus,

$$R_{\text{in}} = 50 \parallel 0.71 = 0.7\text{ k}\Omega = 700\ \Omega$$

$$A_{is} = \frac{(W/L)_2}{(W/L)_1} = \frac{50/0.5}{10/0.5} = 5\text{ A/A}$$

$$R_O = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V_A' L}{I_{D2}}$$

$$= \frac{10 \times 0.5}{5 \times 0.1} = 10\text{ k}\Omega$$

8.18 $A_{is} = 4 = \dfrac{(W/L)_2}{(W/L)_1}$

Since $L_1 = L_2$, then

$$\frac{W_2}{W_1} = 4$$

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}} \simeq \frac{1}{g_{m1}}$$

For

$$R_{\text{in}} = 500\ \Omega \Rightarrow g_{m1} = 2\text{ mA/V}$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

Thus,

$$2 = \sqrt{2 \times 0.4 \times \left(\frac{W}{L}\right)_1 \times 0.2}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$R_O = r_{o2} = \frac{V_A}{I_{D2}} = \frac{V_A' L}{I_{D2}}$$

Thus,

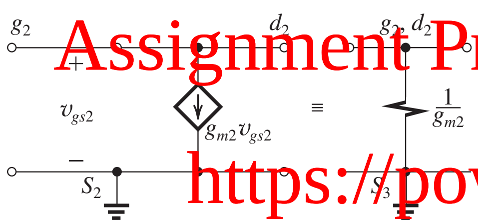
$$20 = \frac{20\,L}{4 \times 0.2}$$

$$\Rightarrow L = 0.8\ \mu\text{m}$$

$$W_1 = 25 \times 0.8 = 20\ \mu\text{m}$$

$$W_2 = 4\ W_1 = 80\ \mu\text{m}$$

8.19 Refer to Fig. P8.19. Consider first the diode-connected transistor Q_2. From the figure we



see that from a small-signal point of view it is equivalent to a resistance $1/g_{m2}$. Thus the voltage gain of Q_1 will be

$$\frac{v_{d1}}{v_i} = -g_{m1} \times \frac{1}{g_{m2}} = -\frac{g_{m1}}{g_{m2}}$$

The signal current in the drain of Q_1, $g_{m1}\ v_i$, will be mirror in the drain of Q_3;

$$i_{d3} = g_{m1}\ v_i \frac{(W/L)_3}{(W/L)_2} = g_{m1} v_i \frac{W_3}{W_2}$$

which flows through R_L and produces the output voltage v_o,

$$v_o = i_{d3} R_L = g_{m1} v_i \frac{W_3}{W_2} R_L$$

Thus, the small-signal voltage gain will be

$$\frac{v_o}{v_i} = g_{m1} R_L\ (W_3/W_2)$$

8.20 Replacing Q_1 and Q_2 with their small-signal hybrid-π models results in the equivalent circuit shown in the figure below. Observe that the controlled source $g_{m1} v_{\pi1}$ appears across its controlling voltage $v_{\pi1}$; thus the controlled source can be replaced with a resistance $(1/g_{m1})$. The input resistance R_{in} can now be obtained by inspection as

$$R_{\text{in}} = r_{o1} \parallel \frac{1}{g_{m1}} \parallel r_{\pi1} \parallel r_{\pi2}$$

Since $r_{o1} \gg r_{\pi1}$,

$$R_{\text{in}} \simeq \frac{1}{g_{m1}} \parallel r_{\pi1} \parallel r_{\pi2} \qquad (1)$$

The short-circuit output current i_o is given by

$$i_o = g_{m2} v_{\pi2}$$

Since $v_{\pi2} = v_{\pi1} = i_i R_{\text{in}}$, then the short-circuit current gain A_{is} is given by

$$A_{is} = \frac{i_o}{i_i} = g_{m2} R_{\text{in}}$$

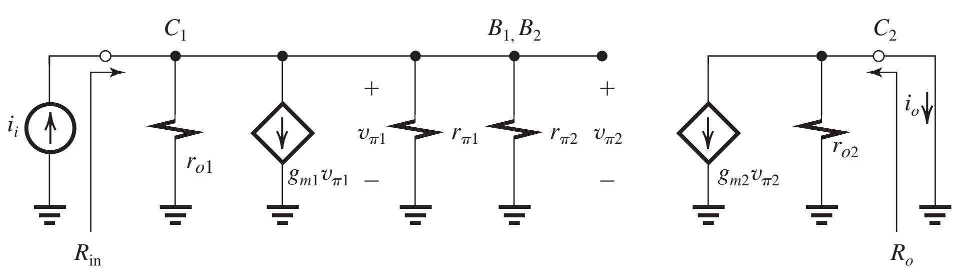
$$= g_{m2}\left(\frac{1}{g_{m1}} \parallel r_{\pi1} \parallel r_{\pi2}\right) \qquad (2)$$

For situations where β_1 and β_2 are large, we can neglect $r_{\pi1}$ and $r_{\pi2}$ in Eqs. (1) and (2) to obtain

$$R_{\text{in}} \simeq 1/g_{m1}$$

$$A_{is} \simeq g_{m1}/g_{m2}$$

This figure belongs to Problem **8.20**.



8.21 (a)

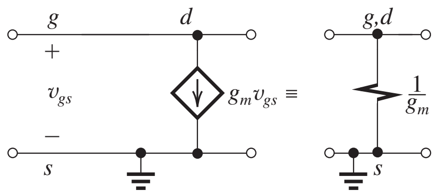


Figure 1

Replacing the MOSFET with its hybrid-π model but neglecting r_o results in the equivalent circuit in Fig. 1. Observing that the controlled-source $g_m v_{gs}$ appears across its control voltage v_{gs}, we can replace it by a resistance $1/g_m$, as indicated. Thus the small-signal resistance of the diode-connected MOS transistor is $1/g_m$. For the given values,

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$$

$$= \sqrt{2 \times 0.2 \times 10 \times 0.1} = 0.632 \text{ mA/V}$$

$$\frac{1}{g_m} = 1.6 \text{ k}\Omega$$

(b) Replacing the BJT with its hybrid-π model results in the equivalent circuit in Fig. 2. Observing that the controlled-source $g_m v_\pi$ appears across its control voltage v_π, we can replace it by a resistance $1/g_m$, as indicated. Next the two parallel resistances $1/g_m$ and r_π can be combined as

$$\frac{\frac{1}{g_m} \times r_\pi}{\frac{1}{g_m} + r_\pi} = \frac{r_\pi}{1 + g_m r_\pi} = \frac{r_\pi}{\beta + 1} = r_e$$

Thus, the diode-connected BJT has a small-signal resistance r_e. For the given data,

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

8.22 Refer to Fig. 8.11.

$$I_{C1} \simeq I_{\text{REF}} = 0.1 \text{ mA}$$

$$V_{BE1} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.1 \text{ mA}}\right)$$

$$= 0.642 \text{ V}$$

$$I_{C3} \simeq I_{B1} + I_{B2} = 2\, I_{B1} = 2 \times \frac{I_{C1}}{\beta}$$

$$= 2 \times \frac{0.1}{100} = 0.002 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1 \text{ mA}}{0.002 \text{ mA}}\right)$$

$$= 0.545 \text{ V}$$

$$V_x = V_{BE3} + V_{BE1} = 1.187 \text{ V}$$

If I_{REF} is increased to 1 mA,

$$V_{BE1} = 0.7$$

$$I_{C3} \simeq 0.02 \text{ mA}$$

$$V_{BE3} = 0.7 - 0.025 \ln\left(\frac{1}{0.02}\right) = 0.6 \text{ V}$$

$$V_x = 1.3 \text{ V}$$

Thus,

$$\Delta V_x = 1.3 - 1.187 = 0.113 \text{ V}$$

When $V_O = V_x$, the Early effect on Q_1 and Q_2 will be the same, and

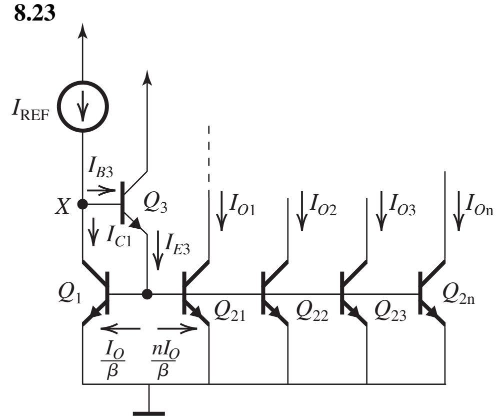
$$I_O = I_{\text{REF}}/(1 + 2/\beta^2)$$

Thus, I_O will be

$I_{\text{REF}} = 100\ \mu\text{A} \Rightarrow I_O = \frac{100}{1 + (2/100^2)} =$ 99.98 μA, for an error of -0.02 μA or -0.02%.

$I_{\text{REF}} = 1 \text{ mA} \Rightarrow I_O = \frac{1}{1 + (2/100^2)} =$ 0.9998 μA, for an error of -0.0002 mA or -0.02%. For proper current-source operation, the minimum required voltage at the output is the value needed to keep Q_2 in the active region, which is approximately 0.3 V.

8.23



This figure belongs to Problem **8.21**, part (b).

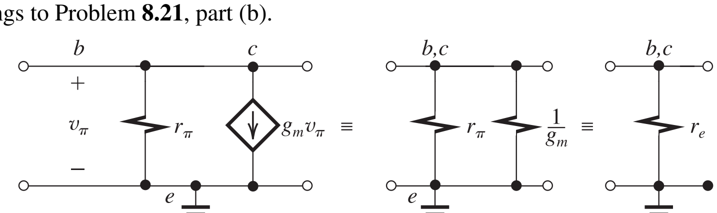


Figure 2

$$I_{O1} = I_{O2} = I_{O3} \cdots = I_{On} = I_O = I_{C1}$$

The emitter of Q_3 supplies the base currents for all transistor, so

$$I_{E3} = \frac{(n+1)\,I_O}{\beta}$$

$$I_{\text{REF}} = I_{B3} + I_O = \frac{(n+1)\,I_O}{\beta\,(\beta+1)} + I_O$$

$$\frac{I_O}{I_{\text{REF}}} = \frac{1}{1 + \dfrac{(n+1)}{\beta\,(\beta+1)}} \simeq \frac{1}{1 + \dfrac{n+1}{\beta^2}}$$

For the deviation from unity to be kept $\leq 0.2\%$

$$\frac{n+1}{\beta^2} \leq 0.002$$

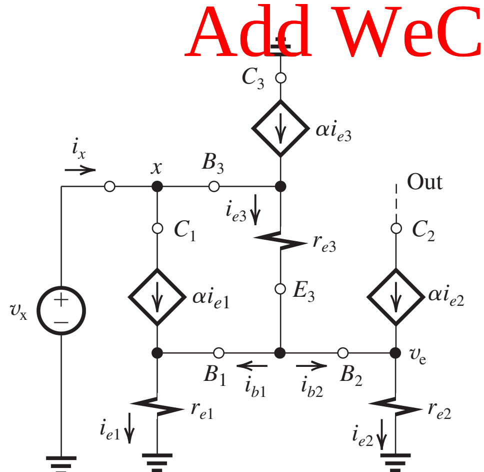
$$\Rightarrow n_{\max} = 0.002 \times 150^2 - 1 = 44$$

8.24 Refer to Fig. 8.11 and observe that $I_{C1} \simeq I_{\text{REF}}$ and $I_{C2} = I_{C1}$; thus each of Q_1 and Q_2 is operating at a collector bias current approximately equal to I_{REF}. Transistor Q_3 is operating at an emitter bias current

$$I_{E3} = I_{B1} + I_{B2} = 2I_B = 2I_C/\beta = \frac{2(1-\alpha)}{\alpha} I_C$$

$$= \frac{2(1-\alpha)}{\alpha} I_{\text{REF}}$$

Replacing each of the three transistors with its T model and applying an input test voltage v_x to determine R_{in}, we obtain the equivalent circuit shown.



In this equivalent circuit,

$$r_{e1} = r_{e2} = r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C} = \frac{\alpha V_T}{I_{\text{REF}}}$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{\alpha V_T}{2(1-\alpha) I_{\text{REF}}}$$

$$i_{e1} = i_{e2} = i_e$$

$$i_{e3} = i_{b1} + i_{b2} = 2(1-\alpha) i_e$$

From the figure we obtain

$$i_x = \alpha i_{e1} + (1-\alpha) i_{e3}$$

$$= \alpha i_e + (1-\alpha) \times 2(1-\alpha) i_e$$

$$= i_e[\alpha + 2(1-\alpha)^2]$$

But $2(1-\alpha)^2 \ll \alpha$. Thus,

$$i_x \simeq \alpha i_e \qquad (1)$$

$$v_x = i_{e3}\, r_{e3} + i_{e1} r_{e1}$$

$$= 2(1-\alpha) i_e \frac{\alpha V_T}{2(1-\alpha) I_{\text{REF}}} + i_e \frac{\alpha V_T}{I_{\text{REF}}}$$

$$v_x = \alpha i_e \left[\frac{V_T}{I_{\text{REF}}} + \frac{V_T}{I_{\text{REF}}} \right]$$

Now, using $i_x = \alpha i_e$ from Eq. (1), we have

$$v_x = i_x \times \frac{2V_T}{I_{\text{REF}}}$$

Thus,

$$R_{\text{in}} \equiv \frac{v_x}{i_x} = \frac{2V_T}{I_{\text{REF}}} \qquad \text{Q.E.D.}$$

For $I_{\text{REF}} = 100\ \mu\text{A} = 0.1$ mA,

$$R_{\text{in}} = \frac{2 \times 25 \text{ mV}}{0.1 \text{ mA}} = 500\ \Omega$$

8.25 For $I = 10\ \mu$A:

$$g_m = \frac{I}{V_T} = \frac{10\ \mu\text{A}}{25 \text{ mV}} = 0.4 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.4 \text{ mA/V}} = 250 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I} = \frac{10 \text{ V}}{10\ \mu\text{A}} = 1 \text{ M}\Omega$$

$$A_0 = g_m r_o = \frac{V_A}{V_I} = \frac{10 \text{ V}}{0.025 \text{ V}} = 400 \text{ V/V}$$

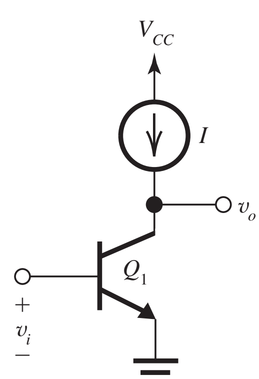
For $I = 100\ \mu$A :

$$g_m = \frac{100\ \mu\text{A}}{25 \text{ mV}} = 4 \text{ mA/V}$$

$$r_\pi = \frac{100}{4 \text{ mA/V}} = 25 \text{ k}\Omega$$

$$r_o = \frac{10 \text{ V}}{100\ \mu\text{A}} = 100 \text{ k}\Omega$$

$$A_0 = 4 \text{ mA/V} \times 100 \text{ k}\Omega = 400 \text{ V/V}$$



For $I = 1$ mA:

$$g_m = \frac{1\text{ mA}}{25\text{ mV}} = 40\text{ mA/V}$$

$$r_\pi = \frac{100}{40\text{ mA/V}} = 2.5\text{ k}\Omega$$

$$r_o = \frac{10\text{ V}}{1\text{ mA}} = 10\text{ k}\Omega$$

$$A_0 = 40\text{ mA/V} \times 10\text{ k}\Omega = 400\text{ V/V}$$

I	g_m	r_π	r_o	A_0
10 μA	0.4 mA/V	250 kΩ	1 MΩ	400 V/V
100 μA	4.0 mA/V	25 kΩ	100 kΩ	400 V/V
1 mA	40 mA/V	2.5 kΩ	10 kΩ	400 V/V

8.26 Refer to Fig. 8.13(b).

$$g_m = \frac{I_C}{V_T} = \frac{I}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20\text{ mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{100\text{ V}}{0.5\text{ mA}} = 200\text{ k}\Omega$$

$$R_{\text{in}} = r_\pi = \frac{\beta}{g_m} = \frac{100}{20\text{ mA/V}} = 5\text{ k}\Omega$$

$$A_{vo} = -A_0 = -g_m r_o = -20 \times 200 = -4000\text{ V/V}$$

$$R_o = r_o = 200\text{ k}\Omega$$

To raise R_{in} by a factor of 5 by changing I, the value of I must be lowered by the same factor to $I = 0.1$ mA.

Now, g_m is reduced by a factor of 5 and r_o is increased by a factor of 5, keeping A_{vo} unchanged at -4000 V/V. However, R_o will be increased to

$$R_o = 5 \times 200\text{ k}\Omega = 1\text{ M}\Omega$$

If the amplifier is fed with a signal source having $R_{\text{sig}} = 5\text{ k}\Omega$ and a 100-kΩ load resistance is connected to the output, the equivalent circuit shown below results.

$$\frac{v_o}{v_{\text{sig}}} = \frac{r_\pi}{r_\pi + R_{\text{sig}}} \times -g_m(r_o \parallel R_L)$$

$$= -\frac{25}{25+5} \times 4\ (1000\text{ k}\Omega \parallel 100\text{ k}\Omega)$$

$$= -303\text{ V/V}$$

8.27

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 10 \times 0.5}{0.2} = 50\text{ V/V}$$

$$g_m = \frac{2I_D}{V_{OV}}$$

$$2 = \frac{2I_D}{0.2} \Rightarrow I_D = 0.2\text{ mA}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 25$$

$$W = 12.5\ \mu\text{m}$$

8.28 From Eq. (8.40) we see that A_0 is inversely proportional to $\sqrt{I_D}$. Thus

$I_D = 100\ \mu$A $A_0 = 50$ V/V

$I_D = 25\ \mu$A $A_0 = 100$ V/V

$I_D = 400\ \mu$A $A_0 = 25$ V/V

From Eq. (8.42), g_m is proportional to $\sqrt{I_D}$. Thus changing I_D from 100 μA to 25 μA reduces g_m by a factor of 2. Changing I_D from 100 μA to 400 μA increases g_m by a factor of 2.

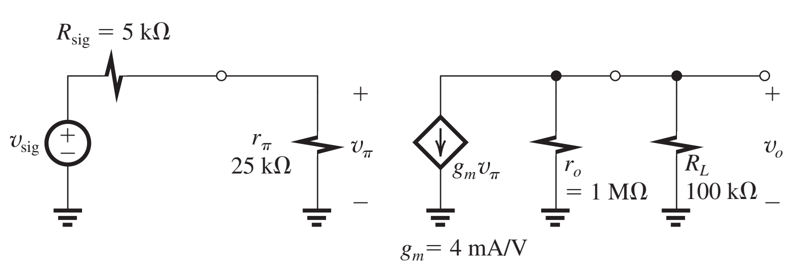
8.29 $A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}}$

$$20 = \frac{2 \times 5 \times L}{0.2}$$

$$\Rightarrow L = 0.4\ \mu\text{m}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

This figure belongs to Problem **8.26**.



$$2 = \frac{2I}{0.2}$$

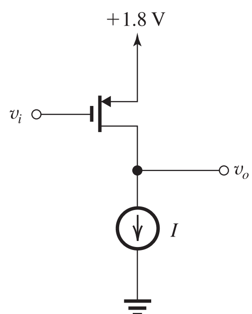
$$\Rightarrow I = 0.2 \text{ mA}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.04$$

$$\Rightarrow \frac{W}{L} = 25$$

8.30



The highest instantaneous voltage allowed at the drain is that which results in a voltage equal to (V_{OV}) across the transistor. Thus

$$v_{O\max} = 1.8 - 0.2 = +1.6 \text{ V}$$

8.31 For the *npn* transistor,

$$g_m = \frac{I_C}{V_T} = \frac{0.1 \text{ mA}}{0.025 \text{ V}} = 4 \text{ mA/V}$$

For the NMOS transistor,

$$g_m = \frac{2\,I_D}{V_{OV}}$$

$$4 = \frac{2\,I_D}{0.25}$$

$$\Rightarrow I_D = 0.5 \text{ mA}$$

8.32 $g_m = \dfrac{2\,I_D}{V_{OV}} = \dfrac{2 \times 0.1}{0.5} = 0.4$ mA/V

From Table J.1 (Appendix J), we find that for the 0.5-μm process $|V_A'| = 20$ V/μm. Thus for our 1-μ-m long transistor, $V_A = 20$ V.

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.1 \text{ mA}} = 200 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.4 \times 200 = 80 \text{ V/V}$$

From Table J.1:

$$\mu_n C_{ox} = 190 \ \mu\text{A/V}^2$$

Now,

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 190 \times \frac{W}{L} \times 0.25$$

$$\Rightarrow \frac{W}{L} = 4.21$$

$$\Rightarrow W = 4.21 \ \mu\text{m}$$

8.33 $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.1}{0.2} = 1$ mA/V

From Table K.1 (Appendix K), for the 0.18-μm process we have

$|V_A'| = 5$ V/μm, $\mu_n C_{ox} = 387$ μA/V^2

Thus, for our NMOS transistor whose $L = 0.3$ μm,

$$V_A = 5 \times 0.3 = 1.5 \text{ V}$$

$$r_o = \frac{V_A}{I_D} = \frac{1.5 \text{ V}}{0.1 \text{ mA}} = 15 \text{ k}\Omega$$

$$A_0 = g_m r_o = 1 \times 15 = 15 \text{ V/V}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

$$100 = \frac{1}{2} \times 387 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 13 \Rightarrow W = 3.9 \ \mu\text{m}$$

8.34 For the BJT cell:

$$g_m = \frac{I_C}{V_T} = \frac{I_C}{0.025 \text{ V}}$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{I_C}$$

$$A_0 = g_m r_o = \frac{V_A}{V_T} = \frac{100 \text{ V}}{0.025 \text{ V}} = 4000 \text{ V/V}$$

$$R_{\text{in}} = r_\pi = \frac{\beta}{g_m} = \frac{100}{g_m}$$

For the MOSFET cell:

$$g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D} = \sqrt{2 \times 0.2 \times 40 \times I_D}$$

$$= \sqrt{16 I_D} = 4\sqrt{I_D} \ \text{ mA/V} \quad (I_D \text{ in mA})$$

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{I_D}$$

$$A_0 = g_m r_o = \frac{40}{\sqrt{I_D}} \text{ V/V} \quad (I_D \text{ in mA})$$

$$R_{\text{in}} = \infty$$

	BJT Cell		**MOSFET Cell**	
Bias current	**$I_C = 0.1$ mA**	**$I_C = 1$ mA**	**$I_D = 0.1$ mA**	**$I_D = 1$ mA**
g_m (mA/V)	4	40	1.26	4
r_o (kΩ)	1000	100	100	10
A_0 (V/V)	4000	4000	126	40
R_{in} (kΩ)	25	2.5	∞	∞

8.35 Using Eq. (8.46),

$$A_0 = \frac{V_A'\sqrt{2(\mu_n C_{ox})\,(WL)}}{\sqrt{I_D}}$$

$$18 = \frac{5\sqrt{2 \times 0.4 \times 8 \times 0.54 \times 0.54}}{\sqrt{I_D}}$$

$$\Rightarrow I_D = 0.144 \text{ mA}$$

8.36 $g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$

$$= \sqrt{2 \times 0.4 \times 8 I_D} = 2.53\sqrt{I_D}$$

$I_D = 25\ \mu\text{A}, \qquad g_m = 2.53\sqrt{0.025} = 0.4 \text{ mA/V}$

$I_D = 250\ \mu\text{A}, \qquad g_m = 2.53\sqrt{0.25} = 1.26 \text{ mA/V}$

$I_D = 2.5 \text{ mA}, \qquad g_m = 2.53\sqrt{2.5} = 4 \text{ mA/V}$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{I_D} = \frac{1.8}{I_D}$$

$$A_0 = g_m r_o$$

$I_D = 25\ \mu\text{A} \quad r_o = \frac{1.8}{0.025} = 72 \text{ k}\Omega$

$A_0 = 0.4 \times 72 = 28.8 \text{ V/V}$

$I_D = 250\ \mu\text{A} \quad r_o = \frac{1.8}{0.25} = 7.2 \text{ k}\Omega$

$A_0 = 1.26 \times 7.2 = 9.1 \text{ V/V}$

$I_D = 2.5 \text{ mA} \quad r_o = \frac{1.8}{2.5} = 0.72 \text{ k}\Omega$

$A_0 = 4 \times 0.72 = 2.88 \text{ V/V}$

8.37

$L = 0.36\ \mu\text{m}, \quad V_{OV} = 0.25 \text{ V}, \quad I_D = 10\ \mu\text{A}$

(a) $g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 10}{0.25} = 80\ \mu\text{A/V}$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D}$$

From Appendix J, Table J.1, $V_A' = 5 \text{ V}/\mu\text{m}$,

$$r_o = \frac{5 \times 0.36}{10} = 0.18 \text{ M}\Omega$$

$$A_0 = g_m r_o = 80 \times 0.18 = 14.4 \text{ V/V}$$

(b) If I_D is increased to 100 µA (i.e., by a factor of 10), V_{OV} increases by a factor of $\sqrt{10} = 3.16$ to

$$V_{OV} = 0.25 \times 3.16 = 0.79 \text{ V}$$

and g_m increases by a factor of $\sqrt{10} = 3.16$ to

$$g_m = 80 \times 3.16 = 253\ \mu\text{A/V} = 0.253 \text{ mA/V}$$

and r_o decreases by a factor of 10 to

$$r_o = \frac{0.18 \text{ M}\Omega}{10} = 18 \text{ k}\Omega$$

Thus, A_0 becomes

$$A_0 = 0.253 \times 18 = 4.55 \text{ V/V}$$

(c) If the device is redesigned with a new value of W so that it operates at

$V_{OV} = 0.25 \text{ V}$ for $I_D = 100\ \mu\text{A}$,

$$g_m = \frac{2I_D}{V_{OV}} = \frac{0.2 \text{ mA}}{0.25 \text{ V}} = 0.8 \text{ mA/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.1} = 18 \text{ k}\Omega$$

$$A_0 = g_m r_o = 0.8 \times 18 = 14.4 \text{ V/V}$$

(d) If the redesigned device in (c) is operated at 10 µA, V_{OV} decreases by a factor equal to $\sqrt{10}$ to 0.08 V, g_m decreases by a factor of $\sqrt{10}$ to 0.253 mA/V, r_o increases by a factor of 10 to 180 kΩ, and A_0 becomes

$$0.253 \times 180 = 45.5 \text{ V/V}$$

which is an increase by a factor of $\sqrt{10}$.

(e) The lowest value of A_0 is obtained with the first design when operated at $I_D = 100\ \mu\text{A}$. The resulting $A_0 = 4.55$ V/V. The highest value of A_0 is obtained with the second design when operated at $I_D = 10\ \mu\text{A}$. The resulting $A_0 = 45.5$ V/V. If in any design W/L is held constant while L is increased by a factor of 10, g_m remains unchanged but r_o increases by a factor of 10, resulting in A_0 increasing by a factor of 10.

8.38

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V_A' L}{V_{OV}} = \frac{2 \times 6 \times 0.5}{0.15} = 40 \text{ V/V}$$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \frac{W}{L} \times 0.15^2$$

$$\Rightarrow \frac{W}{L} = 22.2$$

Thus,

$$W = 22.2 \times 0.5 = 11.1\ \mu\text{m}$$

$$g_m = \frac{2\,I_D}{V_{OV}} = \frac{2 \times 0.1}{0.15} = 1.33 \text{ mA/V}$$

$$r_o = \frac{V_A' L}{I_D} = \frac{6 \times 0.5}{0.1} = 30 \text{ k}\Omega$$

8.39 $A_0 = |A_{vo}| = 100$

$$100 = \frac{2\,V_A}{V_{OV}} = \frac{2\,V_A}{0.2}$$

$$\Rightarrow V_A = 10 \text{ V}$$

Since $V_A' = 20 \text{ V}/\ \mu\text{m}$, we have

$$L = \frac{V_A}{V_A'} = \frac{10}{20} = 0.5\ \mu\text{m}$$

$$I_D = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$50 = \frac{1}{2} \times 200 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 12.5$$

8.40 Refer to Fig. 8.15(a).

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.5 + 0.3 = 0.8 \text{ V}$$

$$V_G = 2.5 - V_{SG2} = 2.5 - 0.8 = 1.7 \text{ V}$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 \times 0.3^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 11.1$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{OV2}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_2 \times 0.3^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 22.2$$

$$A_v = -g_{m1}\ (r_{o1} \parallel r_{o2})$$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.1}{0.3} = 0.67 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{|V_A'|L}{I_D} = \frac{20 \times 0.5}{0.1} = 100 \text{ k}\Omega$$

$$A_v = -0.67 \times (100 \parallel 100) = -33.5 \text{ V/V}$$

8.41 Refer to Fig. 8.15. Since $V_{An}' = |V_{Ap}'|$ and the channel lengths are equal, $V_{An} = |V_{Ap}|$ and $r_{o1} = r_{o2} = r_o$. Thus

$$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -g_{m1}(r_o/2)$$

$$-40 = -\frac{1}{2}g_{m1}r_o$$

$$\Rightarrow g_{m1}r_o = 80$$

$$A_0 = \frac{2V_{An}}{V_{OV}} = \frac{2V_{An}'L}{V_{OV}}$$

$$80 = \frac{2 \times 5 \times L}{0.25}$$

$$\Rightarrow L = 2\ \mu\text{m}$$

$$V_{SG2} = |V_{tp}| + |V_{OV}| = 0.5 + 0.25 = 0.75 \text{ V}$$

$$V_G = V_{DD} - V_{SG2} = 1.8 - 0.75 = 1.05 \text{ V}$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$100 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_1 \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 8$$

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_2 \times 0.25^2$$

$$\left(\frac{W}{L}\right)_2 = 32$$

8.42 Refer to Fig. P8.42. The gain of the first stage is

$$A_{v1} = -g_{m1}(r_{o1}/2)$$

where $(r_{o1}/2)$ is the equivalent resistance at the output of Q_1 and includes r_{o1} in parallel with the output resistance of the current-source load, which is equal to r_{o1}. Similarly, the gain of the second stage is

$$A_{v2} = -g_{m2}(r_{o2}/2)$$

Now because $V_{An} = |V_{Ap}| = |V_A|$ and both Q_1 and Q_2 are operating at equal currents I, we have

$$r_{o1} = r_{o2} = r_o$$

The overall voltage gain A_v will be

$$A_v = A_{v1}A_{v2}$$

$$A_v = \frac{1}{4}\ g_{m1}g_{m2}r_o^2$$

If the two transistors are operated at equal overdrive voltages, $|V_{OV}|$, both will have equal g_m,

$$A_v = \frac{1}{4}(g_m r_o)^2$$

and

$$g_m r_o = \frac{2|V_A|}{|V_{OV}|} = \frac{2 \times 5}{|V_{OV}|} = \frac{10}{|V_{OV}|}$$

$$A_v = 400 = \frac{1}{4} \times \left[\frac{10}{|V_{OV}|}\right]^2$$

$$\Rightarrow |V_{OV}| = 0.25 \text{ V}$$

8.43

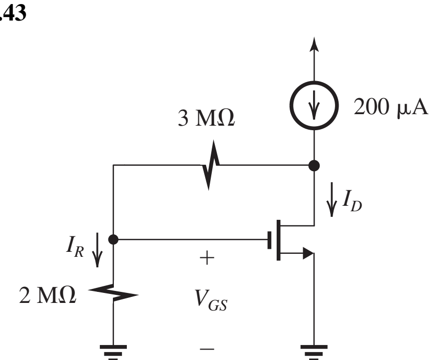


Figure 1

(a) Neglecting the dc current in the feedback network and the Early effect, we see from Fig. 1 that $I_D = 200\ \mu\text{A}$. Now, using

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

we can determine V_{OV}:

$$0.2 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.45\ \text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.5 + 0.45 = 0.95\ \text{V}$$

The current in the feedback network can now be found as

$$I_R = \frac{V_{GS}}{2\ \text{M}\Omega} = \frac{0.95}{2} = 0.475\ \mu\text{A}$$

which indeed is much smaller than the 200 μA delivered by the current source. Thus, we were justified in neglecting I_R above.

(b) Replacing the MOSFET with its hybrid-π model, we obtain the equivalent circuit shown in Fig. 2.

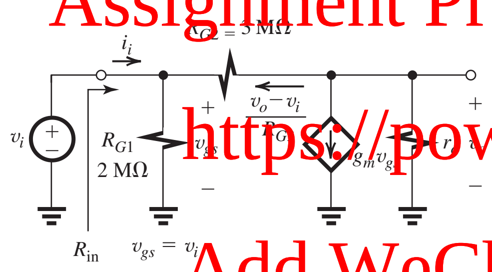


Figure 2

A node equation at the output node yields

$$\frac{v_o}{r_o} + g_m v_{gs} + \frac{v_o - v_i}{R_{G2}} = 0$$

where $v_{gs} = v_i$. Thus,

$$v_o\left(\frac{1}{r_o} + \frac{1}{R_{G2}}\right) = -v_i\left(g_m - \frac{1}{R_{G2}}\right)$$

$$\frac{v_o}{v_i} = -\left(g_m - \frac{1}{R_{G2}}\right)(r_o \parallel R_{G2})$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.45} = 0.894\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100\ \text{k}\Omega$$

$$\frac{v_o}{v_i} = -\left(0.89 - \frac{1}{3000}\right) \times (100 \parallel 3000)$$

$$= -86.5\ \text{V/V}$$

To obtain the maximum allowable negative signal swing at the output, we first determine the dc voltage at the output by referring to Fig. 1,

$$V_{DS} = V_{GS}\left(1 + \frac{R_{G2}}{R_{G1}}\right)$$

$$= 0.95 \times \left(1 + \frac{3}{2}\right) = 2.375\ \text{V}$$

The MOSFET will remain in saturation as long as $V_{DG} \geq -V_t$. Thus at the limit $V_{DG} = -0.5$ V,

$$v_{G\text{max}} = 0.5 + v_{D\text{min}}$$

$$V_{GS} + |\hat{v}_i| = 0.5 + V_{DS} - |\hat{v}_o|$$

$$0.95 + \frac{|\hat{v}_o|}{|A_v|} = 0.5 + 2.375 - |\hat{v}_o|$$

$$|\hat{v}_o| = \frac{0.5 + 2.375 - 0.95}{1 + \dfrac{1}{|A_v|}}$$

Substituting $|A_v| = 86.5$, we obtain

$$|\hat{v}_o| = 1.9\ \text{V}$$

An approximate value of $|\hat{v}_o|$ could have been obtained from

$$v_{O\text{min}} = V_{OV} = 0.45\ \text{V}$$

Thus,

$$V_{DS} - |\hat{v}_o| = V_{OV}$$

$$\Rightarrow |\hat{v}_o| = V_{DS} - V_{OV} = 2.375 - 0.45$$

$$= 1.925\ \text{V}$$

$$|\hat{v}_i| = \frac{|\hat{v}_o|}{86.5} = 22\ \text{mV}$$

(c) To determine R_{in}, refer to Fig. 2,

$$i_i = \frac{v_i}{R_{G1}} - \frac{v_o - v_i}{R_{G2}}$$

$$= \frac{v_i}{R_{G1}} - \frac{A_v v_i - v_i}{R_{G2}}$$

$$= v_i\left[\frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}}\right]$$

$$R_{\text{in}} = \frac{v_i}{i_i} = 1 \Big/ \left[\frac{1}{R_{G1}} + \frac{(1 - A_v)}{R_{G2}}\right]$$

$$= 1 \Big/ \left(\frac{1}{2} + \frac{(1 + 86.5)}{3}\right) = 33.7\ \text{k}\Omega$$

8.44 Refer to Fig. 8.16(a).

$$R_o = 100\ \text{k}\Omega = r_{o1} \parallel r_{o2}$$

But

$$r_{o1} = r_{o2} = \frac{|V_A|}{I_{\text{REF}}} = \frac{5}{I_{\text{REF}}}$$

Thus,

$$100 = \frac{1}{2} \times \frac{5}{I_{\text{REF}}}$$

$$\Rightarrow I_{\text{REF}} = 25\ \mu\text{A}$$

$$A_v = -g_{m1}R_o$$

$-40 = -g_{m1} \times 100$

$\Rightarrow g_{m1} = 0.4$ mA/V

But

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$0.4 = \sqrt{2 \times 0.4\left(\frac{W}{L}\right)_1 \times 0.025}$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 8$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$25 = \frac{1}{2} \times 400 \times 8 \times V_{OV1}^2$$

$\Rightarrow V_{OV1} = 0.125$ V

If Q_2 and Q_3 are operated at $|V_{OV}| = 0.125$ V,

$$I_{D2} = I_{D3} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{2,3} |V_{OV}|^2$$

$$25 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{2,3} \times 0.125^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{2,3} = 32$$

8.45 From the results of Example 8.4, we see that the almost linear region of the transfer characteristic (i.e., region 3) is defined by $V_{IA} = 0.89$ V and $V_{IB} = 0.935$ V. Maximum output signal swing is achieved by biasing Q_1 at the middle of this range; thus

$V_I = 0.913$ V

The peak-to-peak amplitude at the output will be $(V_{OA} - V_{OB}) = 2.47 - 0.335 = 2.135$ V. Thus the peak amplitude will be $\frac{1}{2}(2.135) = 1.07$ V.

8.46 Refer to the solution to Example 8.4.

$V_{OA} = V_{DD} - |V_{OV3}|$

$= 5 - 0.526 = 4.474 \simeq 4.47$ V

The relationship between v_O and v_I in region III of the transfer characteristic can be found as follows:

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 (v_I - V_{tn})^2 \left(1 + \frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{tp}|)^2\left(1 + \frac{V_{DD} - v_O}{|V_{Ap}|}\right)$$

Thus,

$$\frac{k_n'}{k_p'}\frac{1}{|V_{OV3}|^2}\frac{1}{1 + \dfrac{V_{DD}}{|V_{Ap}|}}(v_I - V_{tn})^2$$

$$= \frac{1 - \dfrac{v_O}{V_{DD} + |V_{Ap}|}}{1 + \dfrac{V_O}{V_{An}}}$$

$$\frac{200}{65 \times 0.526^2}\frac{1}{1 + \dfrac{5}{10}}(v_I - 0.6)^2 = \frac{1 - \dfrac{v_O}{5 + 10}}{1 + \dfrac{v_O}{20}}$$

$$7.41(v_I - 0.6)^2 = \frac{1 - 0.07v_O}{1 + 0.05v_O}$$

Substituting $v_O = V_{OA} = 4.47$ V gives

$v_I = V_{IA} = 0.88$ V

To find the coordinates of point B, we note that $V_{OB} = V_{IB} - 0.6$. Thus

$$7.41V_{OB}^2 = \frac{1 - 0.07\ V_{OB}}{1 + 0.05\ V_{OB}}$$

This equation can be solved by trial and error to yield

$V_{OB} = 0.36$ V

and

$V_{IB} = 0.96$ V

Thus at the output the linear region now extends from 0.36 V to 4.47 V as compared to 0.335 V to 2.47 V when the power supply was 3 V: an increase of about the same size as the increase in the power supply.

8.47 Refer to Fig. 8.16(a).

Note that Q_2, Q_3 are not matched:

$I_{D1} = 100$ µA

(a) $I_{D2} = I_{D1} = 100$ µA

$$\frac{I_{D3}}{I_{D2}} = \frac{(W/L)_3}{(W/L)_2} = \frac{W_3}{W_2}$$

(Note that $V_{SG2} = V_{SG3}$)

$$\Rightarrow I_{D3} = 100\ \mu\text{A}\ \frac{10}{40} = 25\ \mu\text{A} \Rightarrow\ I_{\text{REF}} = 25\ \mu\text{A}$$

(b) By referring to Fig. 8.16(d), you notice that in Segment III, both Q_1 and Q_2 are in saturation and the transfer characteristic is quite linear. The output voltage in this segment is limited between V_{OA} and V_{OB}: coordinates of point A:
$v_{OA} = V_{DD} - |V_{OV3}|$

$$|V_{OV3}|^2 = \frac{I_{D3}}{\frac{1}{2}k_p'\left(\frac{W}{L}\right)_3} = \frac{25}{\frac{1}{2}\times 50\times\frac{10}{1}}$$

$\Rightarrow\ |V_{OV3}| = 0.32$ V

$V_{OA} = 3.3 - 0.32 = 2.98$V

At point B: $V_{OB} = V_{IB} - V_{tn}$

Now we find the transfer equation for the linear section: (Refer to Example 8.4)

$i_{D1} = i_{D2} \Rightarrow$ (Note that $|V_{OV2}| = |V_{OV3}|$)

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_1(v_I - V_{tn})^2\left(1 + \frac{v_O}{V_{An}}\right)$$

$$= \frac{1}{2}k_p'\left(\frac{W}{L}\right)_2 V_{OV3}^2\left(1 + \frac{V_{DD} - v_O}{|V_{AP}|}\right)$$

$$\frac{1}{2}\times 100\times\frac{20}{1}(v_I - 0.8)^2\left(1 + \frac{v_O}{100}\right)$$

$$= \frac{1}{2}\times 50\times\frac{40}{1}\times 0.32^2\left(1 + \frac{3.3 - v_O}{50}\right)$$

$$(v_I - 0.8)^2 = 0.32^2\left(1.066 - \frac{v_O}{50}\right)\Big/\left(1 + \frac{v_O}{100}\right)$$

$$(v_I - 0.8)^2 = 0.11\left(\frac{1 - 0.019v_O}{1 + 0.01v_O}\right)$$

$\simeq 0.11(1 - 0.03v_O)$

$(v_I - 0.8)^2 = 0.11(1 - 0.03v_O) \qquad (1)$

Now if we solve for $V_{OB} = V_{IB} - 0.8$

$V_{OB}^2 + 0.0033V_{OB} - 0.11 = 0 \Rightarrow V_{OB} = 0.33$ V

Therefore the extreme values of v_O for which Q_1 and Q_2 are in saturation $0.33\text{ V} \le v_O \le 2.98$ V

(c) From (b) we can find V_{IA} and V_{IB}:

$V_{IB} = V_{OB} + V_t = 0.33 + 0.8 = 1.13$ V

If we solve (1) for $V_{OA} = 2.98$ V, then

$(V_{IA} - 0.8)^2 = 0.11\,(1 - 0.03\times 2.98) \Rightarrow V_{IA}$

$= 1.116$ V

Large-signal voltage gain

$$= \frac{\Delta v_O}{\Delta v_I} = \frac{2.98 - 0.33}{1.13 - 1.116}$$

$$\frac{\Delta v_O}{\Delta v_i} = -189.3\text{ V/V}$$

(d) $v_O = \dfrac{V_{DD}}{2} = \dfrac{3.3}{2} = 1.65$ V

Differentiating both sides of (1) relative to v_I:

$$2\,(v_I - 0.8) = 0.11\times(-0.03)\frac{\partial v_O}{\partial v_I}$$

$$\Rightarrow \frac{\partial v_O}{\partial v_I} = -606.1\,(v_I - 0.8)$$

For $v_O = 1.65$ V, from ① we have

$(v_I - 0.8)^2 = 0.11\,(1 - 0.03\times 1.65) \Rightarrow v_I$

$= 1.123$ V

$$\left.\frac{\partial v_O}{\partial v_I}\right|_{v_I = 1.123} = -195.8\text{ V/V}$$

(e) $R_{out} = r_{o1} \parallel r_{o2}$

$$r_{o1} = \frac{V_{An}}{I_{D1}} = \frac{100\text{ V}}{0.1\text{ mA}} = 1\text{ M}\Omega$$

$$r_{o2} = \frac{V_{Ap}}{I_{D2}} = \frac{50\text{ V}}{0.1\text{ mA}} = 500\text{ k}\Omega$$

$\Rightarrow R_{out} = 500\text{ k}\Omega \parallel 1\text{ M}\Omega$

$R_{out} = 333$ kΩ

$$g_{m1} = \sqrt{2k_n'\left(\frac{W}{L}\right)_1 I_{D1}}$$

$$= \sqrt{2\times 100\times 10^{-6}\times\frac{20}{1}\times 100\times 10^{-6}}$$

$= 0.632$ mA/V

$A_v = -g_{m1}(r_{o1} \parallel r_{o2}) = -210.6$ V/V

Comment: The three estimates of voltage gain obtained in (c), (d) and (e) are all reasonably close; about -200 V/V.

8.48 (a)

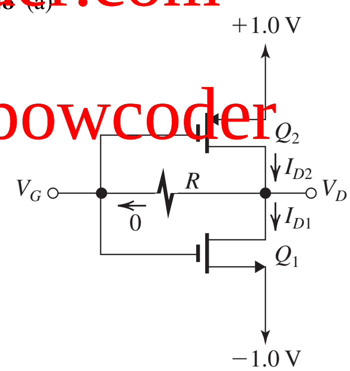


Figure 1

From Fig. 1 we see that since the dc currents into the gates are zero,

$V_D = V_G$

Also, since Q_1 and Q_2 are matched and carry equal drain currents,

$I_{D1} = I_{D2} = I_D$

$V_{SG2} = V_{GS1} = 1$ V

and thus,

$V_G = 0$

Thus,

$$I_D = \frac{1}{2}\times 1\times(1 - 0.5)^2 = 0.125\text{ mA}$$

(b)

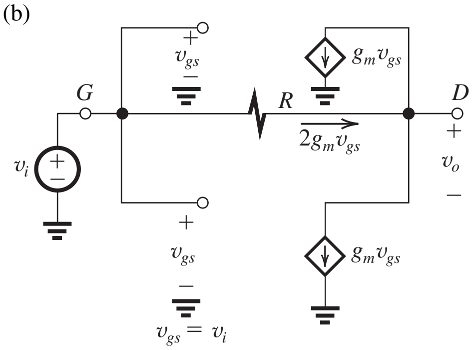


Figure 2

From Fig. 2 we see that

$$v_o = v_i - 2g_m v_{gs} R$$

But

$$v_{gs} = v_i$$

Thus,

$$A_v = \frac{v_o}{v_i} = 1 - 2g_m R$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.125}{1 - 0.5} = 0.5 \text{ mA/V}$$

$$A_v = 1 - 2 \times 0.5 \times 1000 = -999 \text{ V/V}$$

(c)

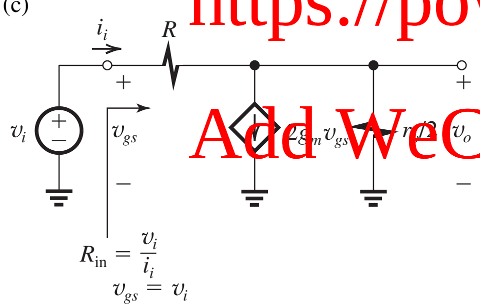


Figure 3

For the circuit in Fig. 3 we can write at the output

$$\frac{v_o}{r_o/2} + 2g_m v_{gs} + \frac{v_o - v_i}{R} = 0$$

Substituting $v_{gs} = v_i$ and rearranging, we obtain

$$\frac{v_o}{v_i} = -2g_m \frac{1 - \dfrac{1}{2g_m R}}{\dfrac{1}{R} + \dfrac{2}{r_o}}$$

But $2g_m R \gg 1$; thus

$$A_v = \frac{v_o}{v_i} \simeq -2g_m \left(R \parallel \frac{r_o}{2}\right)$$

where

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.125} = 160 \text{ k}\Omega$$

$$A_v = -2 \times 0.5(1000 \parallel 80) = -74.1 \text{ V/V}$$

$$R_{in} = \frac{v_i}{i_i} = \frac{v_i}{(v_i - v_o)/R} = R \frac{1}{1 - \dfrac{v_o}{v_i}}$$

$$= \frac{R}{1 - A_v} = \frac{1000}{1 + 74.1} = 13.3 \text{ k}\Omega$$

(d) $\dfrac{v_i}{v_{sig}} = \dfrac{R_{in}}{R_{in} + R_{sig}} = \dfrac{13.3}{20 + 13.3} = 0.4$ V/V

$$G_v = \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} \times \frac{v_o}{v_i}$$

$$= 0.4 \times -74.1 = -29.6 \text{ V/V}$$

(e) Both Q_1 and Q_2 remain in saturation for output voltages that ensure that the minimum voltage across each transistor is equal to $|V_{OV}| = 0.5$ V. Thus, the output voltage can range from -0.5 V to $+0.5$ V.

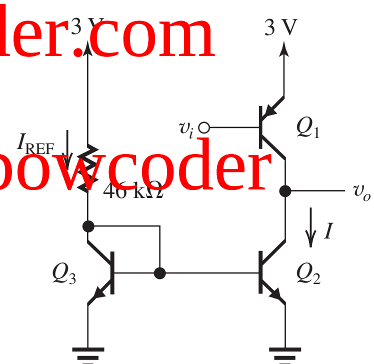
8.49 (a) $I_{REF} = I_{C3} = \dfrac{3 - V_{BE3}}{46 \text{ k}\Omega}$

$$I_{REF} = \frac{3 - 0.7}{46}$$

$$= 0.05 \text{ mA}$$

$$\Rightarrow I_{C2} = 5 I_{C3}$$

$$I_{C2} = I = 0.25 \text{ mA} \Rightarrow I = 0.25 \text{ mA}$$



(b) $|V_A| = 50$ V $\Rightarrow r_{o1} = \dfrac{|V_A|}{I} = \dfrac{30}{0.25}$
$= 120$ kΩ

$$r_{o2} = \frac{30}{0.25} = 120 \text{ k}\Omega$$

Total resistance at the collector of Q_1 is equal to $r_{o1} \parallel r_{o2}$, thus

$$r_{tot} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

(c) $g_{m1} = \dfrac{I_{C1}}{V_T} = \dfrac{0.25}{0.025} = 10$ mA/V

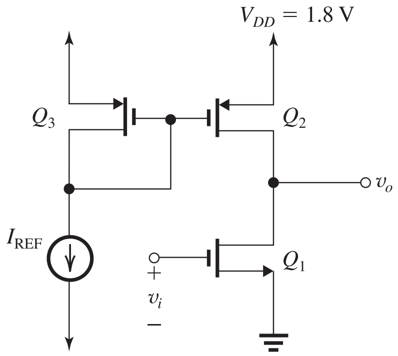
$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{50}{10} = 5 \text{ k}\Omega$$

(d) $R_{in} = r_{\pi 1} = 5$ kΩ

$$R_o = r_{o1} \parallel r_{o2} = 120 \text{ k}\Omega \parallel 120 \text{ k}\Omega = 60 \text{ k}\Omega$$

$$A_v = -g_{m1} R_o = -10 \times 60 = -600 \text{ V/V}$$

8.50



For an output of 1.6 V,

$V_{SD2\min} = |V_{OV2}| = 1.8 - 1.6 = 0.2$ V,

For an output of 0.2 V,

$V_{DS1\min} = 0.2$ V,

thus

$V_{OV1} = 0.2$ V

Since $I_{D2} = I_{D3} = I_{D1} = 50\ \mu$A

and $I_D = \frac{1}{2}(\mu_p C_{ox})(W/L)V_{OV}^2$, we have

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_3 = \frac{2I_{D2}}{(\mu_p C_{ox})(V_{OV})^2}$$

$$= \frac{2\,(50\ \mu\text{A})}{(86\ \mu\text{A/V}^2)\,(0.2\ \text{V})^2} = 29.1$$

For Q_1,

$$\left(\frac{W}{L}\right)_1 = \frac{2\,(50\ \mu\text{A})}{(387\ \mu\text{A/V}^2)\,(0.2\ \text{V})^2} = 6.46$$

A_v must be at least -10 V/V

and $A_v = -g_m\,(r_{o1} \parallel r_{o2})$

$$g_{m1} = \frac{2I_D}{V_{OV1}} = \frac{2 \times 0.05}{0.2} = 0.5\ \text{mA/V}$$

$$r_{o1} \parallel r_{o2} = \frac{10}{0.5} = 20\ \text{k}\Omega$$

But

$$r_{o1} = \frac{V_{A1}}{I_{D1}} = \frac{V'_{An}L}{I_{D1}} = \frac{5L}{0.05} = 100L$$

$$r_{o2} = \frac{|V_{A2}|}{I_{D2}} = \frac{|V'_{Ap}|L}{I_{D2}} = \frac{6L}{0.05} = 120L$$

Thus,

$100L \parallel 120L = 20\ \text{k}\Omega$

$\Rightarrow L = 0.367\ \mu$m

If L is to be an integer multiple of 0.18 μm, then

$L = 0.54\ \mu$m

To raise the gain to 20 V/V, $r_{o1} \parallel r_{o2}$ has to be raised to 40 kΩ, which requires

$L = 2 \times 0.367 = 0.734$

Again, to use a multiple of 0.18 μm we select $L = 0.9\ \mu$m. This represents an increase in L by a factor of $\frac{0.90}{0.54} = \frac{5}{3}$. Ws will have to increase by the same factor. Thus, the area of each transistor will increase by a factor of $\left(\frac{5}{3}\right)^2$ and the total area will increase as follows:

Initial total area =
Area of Q_1 + Area of Q_2 + Area of Q_3

$= 6.46 \times 0.54^2 + 29.1 \times 0.54^2 + 29.1 \times 0.54^2$

$= 18.85\ \mu\text{m}^2$

New total area =
$6.46 \times 0.9^2 + 29.1 \times 0.9^2 + 29.1 \times 0.9^2$

$= 52.37\ \mu\text{m}^2$

Thus, the increase is by a factor of 2.78.

8.51 Refer to Fig. 8.18.

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{20 + 20}{1 + 2 \times 20} = 980\ \Omega$$

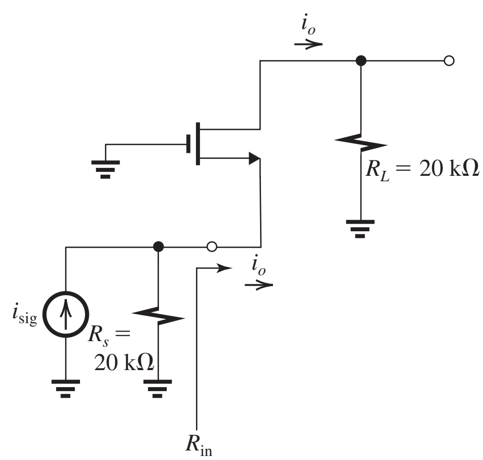
$R_{out} = r_o + R_s + g_m r_o R_s$

$= 20 + 1 + 2 \times 20 \times 1 = 61\ \text{k}\Omega$

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}}$$

$$= \frac{20}{1 + 0.98} = 10.1\ \text{V/V}$$

8.52



$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{20 + 20}{1 + 2 \times 20} = 980\ \Omega$$

Since $i_s = i_o$,

$$\frac{i_o}{i_{sig}} = \frac{R_s}{R_s + R_{in}} = \frac{20}{20 + 0.98} = 0.95\ \text{A/A}$$

If R_L increases by a factor of 10, R_{in} becomes

$$R_{\text{in}} = \frac{20 + 200}{1 + 2 \times 20} = 5.37 \text{ k}\Omega$$

and the current gain becomes

$$\frac{i_o}{i_{\text{sig}}} = \frac{20}{20 + 5.37} = 0.79 \text{ A/A}$$

Thus an increase in R_L by a factor of 10 resulted in a decrease in the current gain from 0.95 A/A to 0.79 A/A, a change of only -17%. This indicates that the CG amplifier functions as an effective current buffer.

8.53 Refer to Fig. P8.53.

$I_D = 0.2$ mA $\quad V_{OV} = 0.2$ V

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20}{0.2} = 100 \text{ k}\Omega$$

$$R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$500 = 100 + R_s(1 + 2 \times 100)$$

$$\Rightarrow R_s = \frac{400}{201} \simeq 2 \text{ k}\Omega$$

$$V_{\text{BIAS}} = I_D R_S + V_{GS}$$

$$= I_D R_S + V_t + V_{OV}$$

$$= 0.2 \times 2 + 0.5 + 0.2$$

$$= 1.1 \text{ V}$$

8.54 Refer to Fig. P8.54. To obtain maximum output resistance, we use the largest possible R_s consistent with $I_D R_s \leq 0.3$ V. Thus

$$R_s = \frac{0.3 \text{ V}}{0.1 \text{ mA}} = 3 \text{ k}\Omega$$

Now, for Q_2 we have

$g_m = 1$ mA/V $\qquad$ and $\qquad V_A = 10$ V

Thus,

$$r_o = \frac{V_A}{I_D} = \frac{10 \text{ V}}{0.1 \text{ mA}} = 100 \text{ k}\Omega$$

$$R_{\text{out}} = r_o + R_s + g_m r_o R_s$$

$$= 100 + 3 + 1 \times 100 \times 3$$

$$= 403 \text{ k}\Omega$$

8.55 Refer to Fig. P8.55.

(a) $I_{D1} = I_{D2} = I_{D3} = 100$ μA

Using $I_{D1} = \frac{1}{2} k_n'(W/L)_1 V_{OV1}^2$, we obtain

$$0.1 = \frac{1}{2} \times 4 \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.224 \text{ V}$$

$$V_{GS1} = V_t + V_{OV1} = 0.8 + 0.224 = 1.024 \text{ V}$$

$$V_{\text{BIAS}} = V_{GS} + I_{D1} R_s$$

$$= 1.024 + 0.1 \times 0.05 = 1.03 \text{ V}$$

(b) $g_{m1} = \dfrac{2\, I_{D1}}{V_{OV1}} = \dfrac{2 \times 0.1}{0.224} = 0.9$ mA/V

All transistors are operating at $I_D = 0.1$ mA and have $|V_A| = 20$ V. Thus all have equal values for r_o:

$$r_o = \frac{|V_A|}{I_D} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

(c) For Q_2, $R_L = r_{o2} = 200$ kΩ,

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o}$$

$$= \frac{200 + 200}{1 + 0.9 \times 200} = 2.2 \text{ k}\Omega$$

(d) $R_{\text{out}} = r_o + R_s + g_m r_o R_s$

$$= 200 + 0.05 + 0.9 \times 200 \times 0.05$$

$$= 209 \text{ k}\Omega$$

(e) $\dfrac{v_i}{v_{\text{sig}}} = \dfrac{R_{\text{in}}}{R_{\text{in}} + R_s} = \dfrac{2.2}{2.2 + 0.05} = 0.98$ V/V

$$\frac{v_o}{v_i} = \frac{R_L}{R_{\text{in}}} = \frac{200}{2.2} = 90.9 \text{ V/V}$$

$$\frac{v_o}{v_{\text{sig}}} = 90.9 \times 0.98 = 89 \text{ V/V}$$

(f) The value of v_o can range from $V_{\text{BIAS}} - V_t = 1.03 - 0.8 = 0.23$ V to $(V_{DD} - V_{OV2})$. Since $I_{D2} = I_{D1}$ and $k_n = k_p$, then $V_{OV2} = V_{OV1}$. Thus the maximum value of v_o is $3.3 - 0.224 = 3.076$ V. Thus the peak-to-peak value of v_o is $3.076 - 0.23 = 2.85$ V. Correspondingly, the peak-to-peak value of v_{sig} will be

$$v_{\text{sig}} \text{ (peak to peak)} = \frac{2.85}{89} = 32 \text{ mV}$$

8.56 Given Eq. (8.63):

$$R_{\text{in}} \simeq r_e \frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}$$

We can write

$$\frac{R_{\text{in}}}{r_e} = \frac{1 + (R_L/r_o)}{1 + [R_L/(\beta + 1) r_o]} = \frac{1 + (R_L/r_o)}{1 + (R_L/101 r_o)}$$

R_L/r_o	0	1	10	100	1000	∞
R_{in}/r_e	1	2	10	50.8	91.8	101

Observe that the range of R_{in} is r_e to $(\beta + 1) r_e$.

8.57 $R_{in} \simeq r_e \dfrac{r_o + R_L}{r_o + R_L/(\beta + 1)}$

$R_{in} \simeq 2r_e$ is obtained when

$$\frac{r_o + R_L}{r_o + R_L/(\beta + 1)} = 2$$

$$\Rightarrow R_L \simeq r_o$$

8.58 Equation (8.66):

$$R_{out} = r_o + (R_e \parallel r_\pi) + (R_e \parallel r_\pi) g_m r_o$$

$$= r_o + (r_e \parallel r_\pi)(1 + g_m r_o)$$

For $g_m r_o \gg 1$,

$$R_{out} \simeq r_o + g_m r_o (R_e \parallel r_\pi)$$

$$\frac{R_{out}}{r_o} = 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e}$$

$$= 1 + \frac{\beta R_e}{(\beta + 1) r_e + R_e}$$

Thus,

$$\frac{R_{out}}{r_o} = 1 + \frac{\beta (R_e/r_e)}{\beta + 1 + (R_e/r_e)}$$

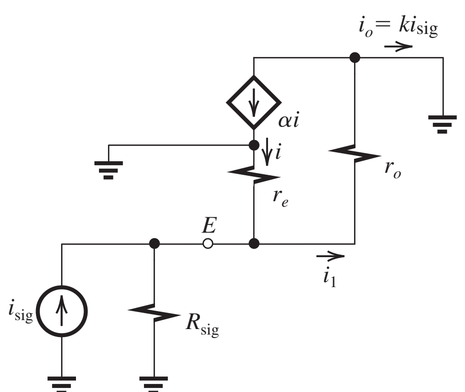
For $\beta = 100$,

$$\frac{R_{out}}{r_o} = 1 + \frac{100(R_e/r_e)}{101 + (R_e/r_e)}$$

R_e/r_e	0	1	2	10	$\beta/2$	β	1000
R_{out}/r_o	1	2	2.9	10	34	51	92

Observe that R_{out} ranges from r_o to $(\beta + 1) r_o$, with the maximum value obtained for $R_e = \infty$.

8.59 Refer to Fig. P8.59. To obtain the short-circuit current gain k, we replace the BJT with its T model and short circuit the collector to ground, resulting in the circuit shown in the figure.



At the emitter node we see that there are three parallel resistances to ground: r_e, r_o, and R_{sig}. Thus,

$$i = -i_{sig} \frac{1/r_e}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}}$$

and

$$i_1 = i_{sig} \frac{1/r_o}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}}$$

At the collector node, we can write

$$i_o \equiv k i_{sig} = i_1 - \alpha i$$

Thus,

$$k i_{sig} = i_{sig} \frac{1/r_o + (\alpha/r_e)}{\dfrac{1}{r_e} + \dfrac{1}{r_o} + \dfrac{1}{R_{sig}}} \qquad (1)$$

Now $r_o \gg r_e$ and for the case $R_{sig} \gg r_e$, we obtain

$$k \simeq \frac{\alpha/r_e}{1/r_e} = \alpha$$

For our case,

$$\alpha = \frac{\beta}{\beta + 1}$$

$$k = \alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

The output resistance R_{out} is given by

$$R_{out} = r_o + (R_{sig} \parallel r_\pi)(1 + g_m r_o)$$

where

$$r_o = \frac{V_A}{I_C} = \frac{50\ \text{V}}{0.1\ \text{mA}} = 500\ \text{k}\Omega$$

$$g_m = \frac{I_C}{V_T} = \frac{0.1\ \text{mA}}{0.025\ \text{V}} = 4\ \text{mA/V}$$

$$g_m r_o = 4 \times 500 = 2000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{4} = 25\ \text{k}\Omega$$

Thus,

$$R_{out} = 500 + (10 \parallel 25) \times 2001$$

$$= 14.8\ \text{M}\Omega$$

Thus the CB amplifier has a current gain of nearly unity and a very high output resistance: a near-ideal current buffer!

A more exact value of k can be obtained using Eq. (1); $k = 0.975$.

8.60 Refer to Fig. P8.60.

$$I = I_C = \alpha I_E = 0.99 \times \frac{5 - 0.7}{4.3} \simeq 1\ \text{mA}$$

$$r_o = \frac{V_A}{I_C} = \frac{100\ \text{V}}{1\ \text{mA}} = 100\ \text{k}\Omega$$

$$R_{out} = r_o + (R_E \parallel r_\pi)(1 + g_m r_o)$$

where

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{0.025\text{ V}} = 40\text{ mA/V}$$

$$g_m r_o = 40 \times 100 = 4000$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40} = 2.5\text{ k}\Omega$$

$$R_E = 4.3\text{ k}\Omega$$

Thus,

$$R_{\text{out}} = 100 + (4.3 \parallel 2.5) \times 4001 = 6.4\text{ M}\Omega$$

For

$$\triangle V_C = 10\text{ V}$$

$$\triangle I = \frac{10\text{ V}}{6.4\text{ M}\Omega} = 1.6\ \mu\text{A}$$

A very small change indeed!

8.61 Refer to Fig. 8.27.

$$R_{\text{out}} = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

$$\simeq r_o + (R_e \parallel r_\pi)(g_m r_o)$$

$$\frac{R_{\text{out}}}{r_o} = 1 + g_m(R_e \parallel r_\pi)$$

$$= 1 + \frac{g_m r_\pi R_e}{r_\pi + R_e}$$

$$= 1 + \frac{\beta R_e}{(\beta/g_m) + R_e}$$

For our case $\beta = 100$,

$$g_m = \frac{I_C}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20\text{ mA/V, thus}$$

$$\frac{R_{\text{out}}}{r_o} = 1 + \frac{100\ R_e}{5 + R_e} \qquad (1)$$

where R_e is in kilohms.

(a) For $R_{\text{out}} = 5\ r_o$, Eq. (1) gives $R_e = 0.208\text{ k}\Omega = 208\ \Omega$

(b) For $R_{\text{out}} = 10\ r_o$, Eq. (1) gives $R_e = 0.495\text{ k}\Omega \simeq 500\ \Omega$.

(c) For $R_{\text{out}} = 50\ r_o$, Eq. (1) gives $R_e = 4.8\text{ k}\Omega$. From Eq. (1) we see that the maximum value of R_{out}/r_o is obtained with $R_e = \infty$ and its value is 101, which is $(\beta + 1)$.

8.62 $50 = g_{m2} r_{o2}$

$$= A_{02} = \frac{2V_A}{V_{OV}}$$

$$V_A = 50 \times V_{OV}/2$$

$$= 25 \times 0.2 = 5\text{ V}$$

$$V_A = V_A' L$$

$$5 = 5 \times L \Rightarrow L = 1\ \mu\text{m}$$

8.63 Refer to Fig. 8.32

$$R_o = g_{m3} r_{o3} r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{V_{OV}} \qquad \text{Q.E.D.}$$

(a) $I = 0.1$ mA

$$0.1 \times R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 1.6\text{ M}\Omega$$

To obtain the W/L values,

$$I = I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 50$$

(b) $I = 0.5$ mA

$$0.5 R_o = \frac{2 \times 4^2}{0.2} = 160$$

$$R_o = 320\text{ k}\Omega$$

$$I = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$500 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 250$$

8.64 Refer to Fig. 8.32.

$$R_o = (g_{m3} r_{o3}) r_{o4}$$

For identical transistors,

$$R_o = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \times \frac{|V_A|}{I}$$

Thus,

$$IR_o = \frac{2|V_A|^2}{|V_{OV}|}$$

Substituting

$$|V_A| = |V_A'|\ L$$

$$IR_o = \frac{2|V_A'|^2}{|V_{OV}|} L^2 \qquad \text{Q.E.D.}$$

Now, for

$$L = 0.18\ \mu\text{m}, \qquad IR_o = \frac{2 \times 5^2}{0.2} \times 0.18^2 = 8.1\ \text{V}$$

$$L = 0.36\ \mu\text{m}, \qquad IR_o = \frac{2 \times 5^2}{0.2} \times 0.36^2 = 32.4\ \text{V}$$

$$L = 0.54\ \mu\text{m}, \qquad IR_o = \frac{2 \times 5^2}{0.2} \times 0.54^2 = 72.9\ \text{V}$$

To fill out the table we use

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2I}{|V_{OV}|} = \frac{2I}{0.2} = 10I$$

$$A_v = g_m(R_o/2)$$

(a) The price paid is the increase in circuit area.

(b) As I is increased, g_m increases and hence the current-driving capability of the amplifier, and as we will see later, its bandwidth.

(c) The circuit with the largest area ($58n$) as compared to the circuit with the smallest area ($0.065n$): A_v is $364.5/40.5 = 9$ times larger; g_m is 100 times larger, but R_o is 11.1 times lower.

8.65 Refer to Fig. 8.33(a).

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}}$$

$$2 = \frac{2I}{0.25}$$

$$\Rightarrow I = 0.25\ \text{mA}$$

For identical transistors,

$$R_o = (g_m r_o) r_o = \frac{2V_A}{V_{OV}} \frac{V_A}{I} = \frac{2V_A^2}{V_{OV}\ I}$$

$$200 = \frac{2V_A^2}{0.25 \times 0.25}$$

$$\Rightarrow V_A = 2.5\ \text{V}$$

$$V_A = V_A' L$$

$$L = \frac{V_A}{V_A'} = \frac{2.5}{5} = 0.5\ \mu\text{m}$$

To obtain W/L, we use

$$I_D = I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$250 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right) \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 20$$

To obtain maximum negative signal swing at the output, we select V_G so that the voltage at the drain of Q_1 is the minimum permitted, which is equal to V_{OV} (i.e., 0.25 V). Thus

$$V_G = 0.25 + V_{GS2}$$

$$= 0.25 + V_{OV2} + V_t$$

$$= 0.25 + 0.25 + 0.5 = 1.0\ \text{V}$$

The minimum permitted output voltage is

$$V_G - V_t = 1 - 0.5 = 0.5\ \text{V or } 2V_{OV}.$$

8.66 Refer to Fig. 8.33.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2I}{V_{OV}} = \frac{2 \times 0.2}{0.2} = 2\ \text{mA/V}$$

Since all transistors are operating at the same I_D and $|V_{OV}|$, all have equal values of g_m. Also because all have equal $|V_A| = 4$ V, all r_o's will be equal.

$$r_o = \frac{|V_A|}{I_D} = \frac{|V_A|}{I} = \frac{4}{0.2} = 20\ \text{k}\Omega$$

$$R_{on} = (g_m r_o) r_o = (2 \times 20) \times 20 = 800\ \text{k}\Omega$$

$$R_{op} = (g_m r_o) r_o = (2 \times 20) \times 20 = 800\ \text{k}\Omega$$

$$R_o = R_{on} \parallel R_{op} = 400\ \text{k}\Omega$$

$$A_v = -g_{m1} R_o = -2 \times 400 = -800\ \text{V/V}$$

8.67 Refer to Fig. 8.33.

$$A_v = -g_{m1} R_o$$

$$-280 = -1 \times R_o \Rightarrow R_o = 280\ \text{k}\Omega$$

This table belongs to Problem **8.64**.

	$L = L_{min} = 0.18\ \mu m$, $IR_o = 8.1$ V				$L = 2L_{min} = 0.36\ \mu m$, $IR_o = 32.4$ V				$L = 3L_{min} = 0.54\ \mu m$, $IR_o = 72.9$ V			
	g_m (mA/V)	R_o (kΩ)	A_v (V/V)	$2WL$ (μm^2)	g_m (mA/V)	R_o (kΩ)	A_v (V/V)	$2WL$ (μm^2)	g_m (mA/V)	R_o (kΩ)	A_{vo} (V/V)	$2WL$ (μm^2)
$I = 0.01$ mA, $W/L = n$	0.1	810	−40.5	$0.065\ n$	0.1	3,240	−162	$0.26n$	0.1	7,290	−364.5	$0.58n$
$I = 0.1$ mA, $W/L = 10\ n$	1.0	81	−40.5	$0.65n$	1.0	324	−162	$2.6n$	1.0	729	−364.5	$5.8n$
$I = 1.0$ mA, $W/L = 100\ n$	10.0	8.1	−40.5	$6.5n$	10.0	32.4	−162	$26n$	10.0	72.9	−364.5	$58n$

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2I}{V_{OV}} \Rightarrow I = \frac{1}{2} g_{m1} V_{OV}$$

$$= \frac{1}{2} \times 1 \times 0.25 = 0.125 \text{ mA}$$

All four transistors are operated at the same value of I_D and the same value of $|V_{OV}|$. Also all have the same channel length and $|V_A'|$; thus all r_o values are equal. Thus

$$R_{on} = R_{op} = 2R_o = 2 \times 280 = 560 \text{ k}\Omega$$

$$560 = (g_m r_o) r_o$$

$$= \frac{2|V_A|}{|V_{OV}|} \frac{|V_A|}{I}$$

$$= \frac{2|V_A|^2}{0.25 \times 0.125}$$

$$\Rightarrow V_A = 2.96 \text{ V}$$

$$L = \frac{V_A}{V_A'} = \frac{2.96}{5} = 0.6 \text{ µm}$$

For each of the NMOS devices,

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$125 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 10$$

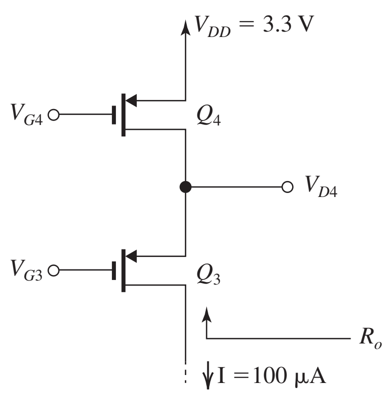
For each of the PMOS transistors,

$$I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$125 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 40$$

8.68



$$V_{SG4} = |V_{tp}| + |V_{OV}|$$

$$= 0.8 + 0.2 = 1 \text{ V}$$

Thus,

$$V_{G4} = V_{DD} - V_{SG4} = 3.3 - 1 = 2.3 \text{ V}$$

To obtain the largest possible signal swing at the output, we maximize the allowable positive signal swing by setting V_{D4} at its highest possible value of $V_{DD} - |V_{OV}| = 3.3 - 0.2 = 3.1$ V. This will be obtained by selecting V_{GS} as follows:

$$V_{G3} = V_{D4} - V_{SG3}$$

Since

$$V_{SG3} = V_{SG4} = 1 \text{ V}$$

$$V_{G3} = 3.1 - 1 = 2.1 \text{ V}$$

the highest allowable voltage at the output will be

$$v_{D3\max} = V_{G3} + |V_{tp}|$$

$$= 2.1 + 0.8 = 2.9 \text{ V}$$

Since both Q_3 and Q_4 carry the same current $I = 100$ µA and are operated at the same overdrive voltage, $|V_{OV}| = 0.2$ V, their W/L ratios will be the same and can be found from

$$I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 60 \times \left(\frac{W}{L}\right)_{3,4} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 83.3$$

To obtain R_o, we first find g_m and r_o of both devices,

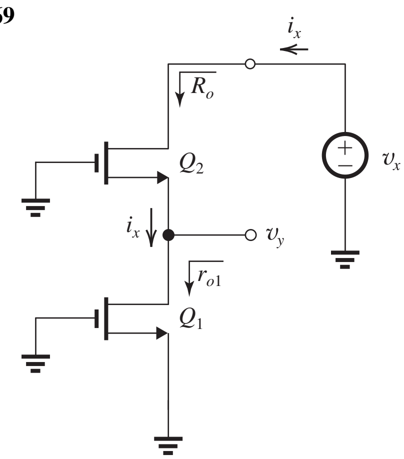
$$g_{m3,4} = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3,4} = \frac{|V_A|}{I_D} = \frac{5}{0.1} = 50 \text{ k}\Omega$$

$$R_o = (g_{m3} r_{o3}) r_{o4}$$

$$= 1 \times 50 \times 50 = 2.5 \text{ M}\Omega$$

8.69



While v_x appears across R_o, v_y appears across r_{o1}, Thus,

$$\frac{v_y}{v_x} = \frac{r_{o1}}{R_o}$$

$$= \frac{r_{o1}}{r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1}}$$

For $g_{m2} r_{o2} \gg 1$ and $g_{m2} r_{o1} \gg 1$,

$$\frac{v_y}{v_x} \simeq \frac{1}{g_{m2} r_{o2}}$$

8.70 Refer to Fig. P8.70.

(a) For the circuit in (a),

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) V_{OVa}^2 \qquad (1)$$

For the circuit in (b),

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{4L} \right) V_{OVb}^2 \qquad (2)$$

Comparing Eqs. (1) and (2) we see that

$V_{OVb} = 2V_{OVa}$ Q.E.D.

Now,

$$g_m = \frac{2I_D}{V_{OV}}$$

Thus for the circuit in (a),

$$g_{ma} = \frac{2I}{V_{OVa}}$$

and for the circuit in (b),

$$g_{mb} = \frac{2I}{V_{OVb}} = \frac{2I}{2\ V_{OVa}} = \frac{I}{V_{OVa}}$$

Thus,

$$g_{mb} = \frac{1}{2}\ g_{ma} \qquad \text{Q.E.D.}$$

Since the channel length in (b) is four times that in (a),

$$V_{Ab} = 4V_{Aa}$$

and

$$r_{ob} = 4r_{oa}$$

Thus

$$A_{va} = -g_{ma} r_{oa}$$

and

$$A_{vb} = -g_{mb} r_{ob}$$

$$= -\frac{1}{2} g_{ma} \times 4\ r_{oa}$$

$$= 2A_{va} \qquad \text{Q.E.D.}$$

(b) For the cascode circuit in (c) to have the same minimum voltage requirement at the drain as that for circuit (b), which is equal to $V_{OVb} = 2\ V_{OVa}$, we must operate each of the two transistors in the cascode amplifier at $V_{OV} = V_{OVa}$. Thus each of the two transistors in the cascode circuit will have $g_m = g_{ma}$. Also, each will have $r_o = r_{oa}$. Thus

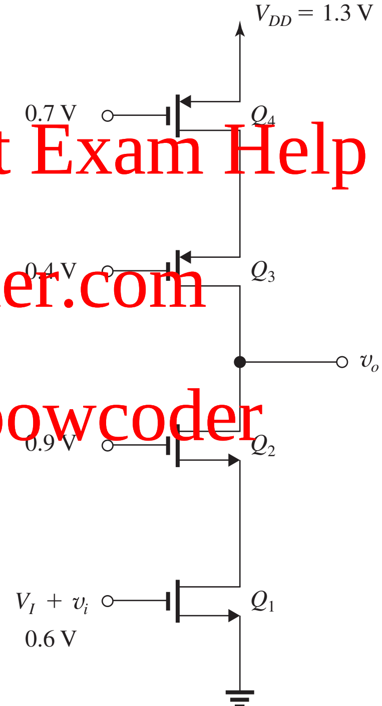
$$A_{vc} = -g_m R_o$$

$$\simeq -g_m[(g_m r_o) r_o]$$

$$= -A_{va}^2$$

Obviously, the cascode delivers a much greater gain than that achieved by quadrupling the channel length of the CS amplifier.

8.71



Since all four transistors have equal transconductance parameters, k, and all four have the same bias current, their overdrive voltages will be equal. We can obtain $|V_{OV}|$ by considering either Q_1 or Q_4. For Q_1,

$$V_{GS} = V_I = 0.6\text{ V} = V_t + V_{OV}$$

Thus,

$$V_{OV} = 0.6 - 0.4 = 0.2\text{ V}$$

Similarly, for Q_4,

$$V_{SG} = V_{DD} - V_{G4} = 1.3 - 0.7 = 0.6\text{ V}$$

Thus,

$|V_{OV}| = V_{SG} - |V_t| = 0.6 - 0.4 = 0.2$ V

The maximum allowable voltage at the output is

$v_{O\max} = V_{GS} + |V_{t3}|$

$= 0.4 + 0.4 = 0.8$ V

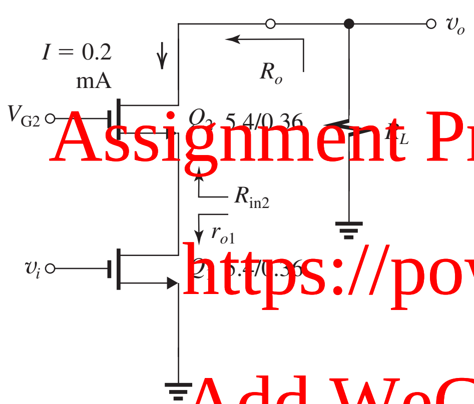
The minimum allowable voltage at the output is

$v_{O\min} = V_{G2} - |V_{t2}|$

$= 0.9 - 0.4 = 0.5$ V

Thus the output voltage can range from 0.5 V to 0.8 V.

8.72



$$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}$$

$$= \sqrt{2 \times 0.4 \times \frac{5.4}{0.36} \times 0.2} = 1.55 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5 \times 0.36}{0.2} = 9 \text{ k}\Omega$$

$$R_o = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} = 9 + 9 + 1.55 \times 9 \times 9$$

$$= 143.6 \text{ k}\Omega$$

$$A_v = -g_{m1}(R_o \parallel R_L)$$

$$-100 = -1.55(R_o \parallel R_L)$$

$$\Rightarrow R_o \parallel R_L = 64.5 \text{ k}\Omega$$

$$\frac{1}{R_o} + \frac{1}{R_L} = \frac{1}{64.5}$$

$$\frac{1}{R_L} = \frac{1}{64.5} - \frac{1}{143.6} = \frac{1}{117}$$

$$\Rightarrow R_L = 117 \text{ k}\Omega$$

$$R_{in2} = \frac{r_{o2} + R_L}{1 + g_{m2} r_{o2}}$$

$$= \frac{9 + 143.6}{1 + 1.55 \times 9} = 10.2 \text{ k}\Omega$$

$$R_{d1} = r_{o1} \parallel R_{in2} = 9 \parallel 10.2 = 4.8 \text{ k}\Omega$$

$$A_1 = -g_{m1} R_{d1} = -1.55 \times 4.8 = -7.41 \text{ V/V}$$

8.73 Refer to Fig. P8.73.

(a) $R_1 = r_{o1} = r_o$

$R_2 \simeq (g_m r_o) r_o$

$$R_3 = \frac{R_2 + r_o}{g_m r_o} = \frac{g_m r_o^2 + r_o}{g_m r_o} \simeq r_o$$

(b) $i_1 = g_m v_i$

$$i_2 = i_1 \frac{R_3}{R_3 + r_o} = g_m v_i \frac{r_o}{r_o + r_o} = \frac{1}{2} g_m v_i$$

$$i_3 = i_1 - i_2 = \frac{1}{2} g_m v_i$$

$$i_4 = i_3 = \frac{1}{2} g_m v_i$$

$$i_5 = i_4 = \frac{1}{2} g_m v_i$$

$i_6 = 0$ (because $v_{sg4} = 0$)

$$i_7 = i_5 = \frac{1}{2} g_m v_i$$

(c) $v_1 = -i_2 r_o = -\frac{1}{2}(g_m r_o) v_i$

$$v_2 = -i_4 R_2 = -\frac{1}{2} g_m (g_m r_o) r_o v_i$$

$$= -\frac{1}{2}(g_m r_o)^2 v_i$$

$$v_3 = -i_5 R_1 = -\frac{1}{2} g_m v_i r_o = -\frac{1}{2}(g_m r_o) v_i$$

(d) v_i is a 5-mV peak sine wave.

$$\hat{v}_1 = -\frac{1}{2} \times 20 \times v_i = -10 \times 5 = -50 \text{ mV}$$

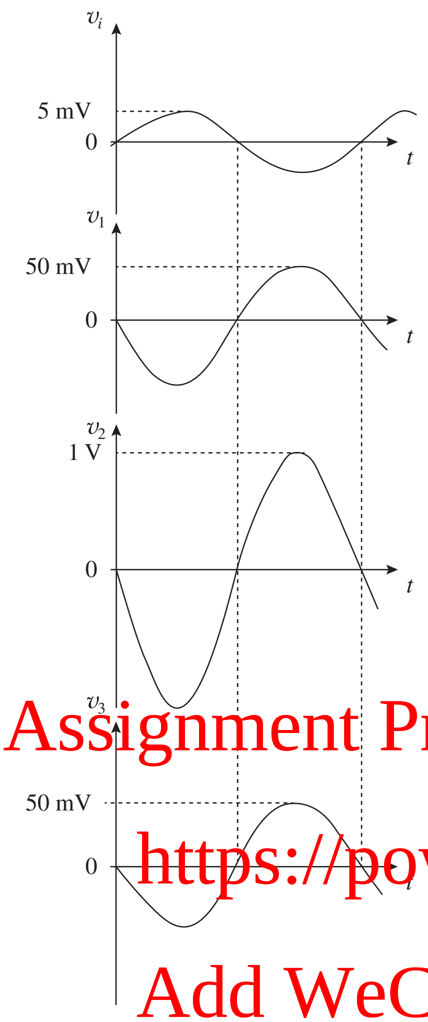
Thus, v_1 is a 50-mV peak sine wave that is 180° out of phase with v_i.

$$\hat{v}_2 = -\frac{1}{2} \times 20^2 \times 5 = -1 \text{ V}$$

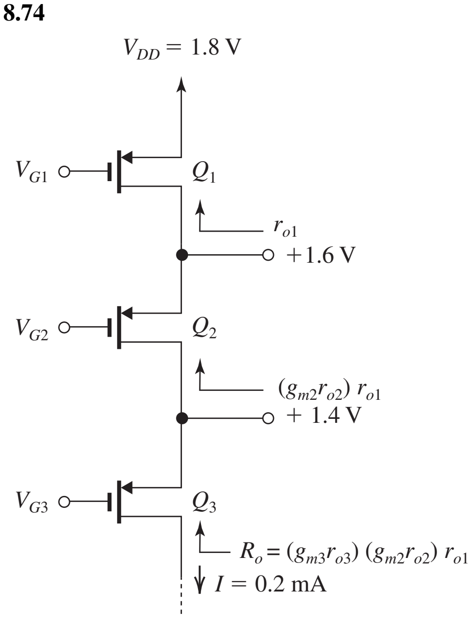
Thus, v_2 is a 1-V peak sine wave, 180° out of phase relative to v_i.

$$\hat{v}_3 = -\frac{1}{2} \times 20 \times 5 = -50 \text{ mV}$$

Thus, v_3 is a 50-mV peak sine wave, 180° out of phase relative to v_i.



8.74



We design for a minimum voltage of $|V_{OV}|$ across each of Q_1 and Q_2.

$$V_{G1} = V_{DD} - V_{SG1} = V_{DD} - |V_{tp}| - |V_{OV}|$$
$$= 1.8 - 0.4 - 0.2 = 1.2 \text{ V}$$

$$V_{G2} = V_{S2} - V_{SG2}$$
$$= 1.6 - 0.4 - 0.2 = 1.0 \text{ V}$$

$$V_{G3} = V_{S3} - V_{SG3}$$
$$= 1.4 - 0.4 - 0.2 = 0.8 \text{ V}$$

All transistors carry the same $I_D = 0.2$ mA and operate at the same value of $|V_{OV}| = 0.2$ V. Thus, their W/L ratios will be equal,

$$0.2 = \frac{1}{2} \times 0.1 \times \frac{W}{L} \times 0.2^2$$

$$\Rightarrow \frac{W}{L} = 100$$

$$R_o = (g_m r_o)^2 r_o$$

where

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_o = \frac{|V_A'|L}{I_D} = \frac{6 \times 0.4}{0.2} = 12 \text{ k}\Omega$$

$$R_o = (2 \times 12)^2 \times 12 = 6.91 \text{ M}\Omega$$

8.75 Refer to Fig. P8.75.

(a) $R_{o1} = r_o$

$R_{o2} = r_o$

$R_{o5} = r_o$

$R_{o4} = (g_m r_o) r_o$

$$R_{o3} = r_{o3} + (g_{m3} r_{o3})(R_{o1} \parallel R_{o2})$$

$$= r_o + g_m r_o \times \frac{1}{2} r_o$$

$$\simeq r_o(1 + \frac{1}{2} g_m r_o) \simeq \frac{1}{2}(g_m r_o) r_o$$

$$R_{\text{in}3} = \frac{r_{o3} + R_{o4}}{1 + g_{m3} r_{o3}} \simeq \frac{r_o + g_m r_o r_o}{g_m r_o}$$

$$= \frac{1}{g_m} + r_o \simeq r_o$$

(b) $R_o = R_{o3} \parallel R_{o4}$

$$= \frac{1}{2}(g_m r_o) r_o \parallel (g_m r_o) r_o$$

$$= \frac{1}{3}(g_m r_o) r_o$$

(c) When v_o is short-circuited to ground, $R_{\text{in}2}$ becomes equal to $1/g_{m3}$. This resistance will be much smaller than the two other resistances between the drain of Q_1 and ground, namely,

$R_{o1} = r_o$ and $R_{o2} = r_o$. Thus the signal current in the drain of Q_1, $g_{m1}v_i$ will mostly flow into $1/g_{m3}$, that is, into the source of Q_3 and out of the drain of Q_3 to ground. Thus, the output short-circuit current will be equal to $g_{m1}v_i$; thus the short-circuit transconductance G_m will be

$$G_m = g_{m1} \qquad \text{Q.E.D.}$$

(d) $\dfrac{v_o}{v_i} = -g_{m1}R_o$

$$= -g_m \times \frac{1}{3}(g_m r_o)r_o$$

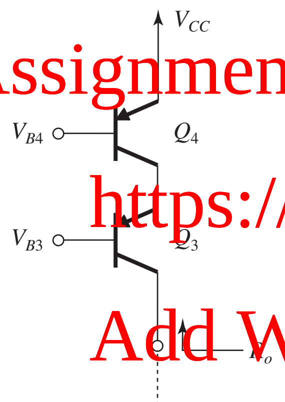
$$= -\frac{1}{3}(g_m r_o)^2$$

For

$$g_m = 2 \text{ mA/V} \qquad \text{and} \qquad A_0 = 30$$

$$\frac{v_o}{v_i} = -\frac{1}{3}(30)^2 = -300 \text{ V/V}$$

8.76



$$R_o = (g_{m3}r_{o3})(r_{o4} \parallel r_{\pi 3})$$

$$I = 0.2 \text{ mA}$$

$$g_{m3} = \frac{I}{V_T} = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_{o3} = r_{o4} = \frac{V_A}{I} = \frac{5}{0.2} = 25 \text{ k}\Omega$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{50}{8} = 6.25 \text{ k}\Omega$$

$$R_o = (8 \times 25)(25 \parallel 6.25)$$

$$= 1 \text{ M}\Omega$$

8.77 When Eq. (8.88) is applied to the case of identical *pnp* transistors, it becomes

$$R_o = (g_m r_o)(r_o \parallel r_\pi)$$

Now,

$$g_m = \frac{I}{V_T} \qquad r_o = \frac{|V_A|}{I}$$

$$g_m r_o = |V_A|/V_T$$

$$r_\pi = \frac{\beta}{g_m}$$

Thus,

$$IR_o = \frac{|V_A|}{V_T}\frac{Ir_o\, r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|\, r_\pi}{r_o + r_\pi}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|}{1 + \dfrac{r_o}{r_\pi}}$$

$$= \frac{|V_A|}{V_T}\frac{|V_A|}{1 + \dfrac{1}{\beta}\, g_m r_o}$$

$$= \frac{|V_A|}{V_T}\frac{1}{\dfrac{1}{|V_A|} + \dfrac{1}{\beta}\dfrac{1}{V_T}}$$

$$= \frac{|V_A|}{(V_T/|V_A|) + (1/\beta)} \qquad \text{Q.E.D.}$$

For $|V_A| = 5$ V and $\beta = 50$ we obtain

$$IR_o = \frac{5}{(0.025/5) + (1/50)} = 200 \text{ V}$$

I (mA)	0.1	0.5	1
R_o (kΩ)	2000	400	200

8.78 Refer to Fig. 8.38. When all transistors have equal β and r_o, and, since they conduct equal currents, they have equal g_m, then

$$R_{on} = R_{op} = g_m r_o(r_o \parallel r_\pi)$$

$$R_o = R_{on} \parallel R_{op} = \frac{1}{2}(g_m r_o)(r_o \parallel r_\pi)$$

$$A_v = -g_m R_o$$

$$= -\frac{1}{2}(g_m r_o)g_m(r_o \parallel r_\pi)$$

$$= -\frac{1}{2}(g_m r_o)\frac{g_m r_o r_\pi}{r_\pi + r_o}$$

$$= -\frac{1}{2}(g_m r_o)\frac{1}{\dfrac{1}{g_m r_o} + \dfrac{1}{g_m r_\pi}}$$

Substituting $g_m r_o = \dfrac{|V_A|}{V_T}$ and $g_m r_\pi = \beta$,

$$A_v = -\frac{1}{2}\frac{|V_A|/V_T}{(V_T/|V_A|) + (1/\beta)}$$

For $|V_A| = 5$ V and $\beta = 50$ we obtain

$$A_v = -\frac{1}{2}\frac{5/0.025}{(0.025/5) + (1/50)} = -4000 \text{ V/V}$$

8.79 The output resistance of the cascode amplifier (excluding the load) is

$R_o = g_m r_o (r_o \parallel r_\pi)$

Thus,

$A_v = -g_m(R_o \parallel R_L)$

$= -g_m(R_o \parallel \beta r_o)$

For $|V_A| = 100$ V, $\beta = 50$, and $I = 0.2$ mA we obtain

$g_m = \dfrac{I}{V_T} = \dfrac{0.2}{0.025} = 8$ mA/V

$r_\pi = \dfrac{\beta}{g_m} = \dfrac{50}{8} = 6.25$ kΩ

$r_o = \dfrac{|V_A|}{I} = \dfrac{100}{0.2} = 500$ kΩ

$R_o = 8 \times 500 \times (500 \parallel 6.25)$

$= 24{,}691$ kΩ

$A_v = -8(24.7 \parallel 25) \times 10^3$

$= -99.4 \times 10^3 \simeq -10^5$ V/V

8.80 (a) Refer to circuit in Fig. P8.80(a).

$g_{m1} = \dfrac{I}{V_T} = \dfrac{0.1}{0.025} = 4$ mA/V

$g_{m2} = g_{m1} = 4$ mA/V

$r_{\pi 1} = r_{\pi 2} = \dfrac{\beta}{g_m} = \dfrac{100}{4} = 25$ kΩ

$r_{o1} = r_{o2} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$R_{\text{in}} = r_{\pi 1} = 25$ kΩ

$R_o = g_{m2} r_{o2} (r_{o1} \parallel r_{\pi 2})$

$= (4 \times 50)(50 \parallel 25) = 3.33$ MΩ

$A_{vo} = -g_m R_o$

$= -4 \times 3.33 \times 10^3 = -13{,}320$ V/V

(b) Refer to the circuit in Fig. P8.80(b).

$g_{m1} = \dfrac{I}{V_T} = \dfrac{0.1}{0.025} = 4$ mA/V

$g_{m2} = \dfrac{2I_{D2}}{V_{OV}} = \dfrac{2I}{V_{OV}} = \dfrac{2 \times 0.1}{0.2} = 1$ mA/V

$r_{\pi 1} = \dfrac{\beta}{g_{m1}} = \dfrac{100}{4} = 25$ kΩ

$r_{o1} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$r_{o2} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$R_{\text{in}} = r_{\pi 1} = 25$ kΩ

$R_o = g_{m2} r_{o2} r_{o1}$

$= 1 \times 50 \times 50 = 2.5$ MΩ

$A_{vo} = -g_{m1} R_o$

$= -4 \times 2.5 \times 10^3 = -10{,}000$ V/V

(c) Refer to the circuit in Fig. P8.80(c).

$g_{m1} = g_{m2} = \dfrac{2I_D}{|V_{OV}|} = \dfrac{2I}{|V_{OV}|} = \dfrac{2 \times 0.1}{0.2} =$ 1 mA/V

$r_{o1} = r_{o2} = \dfrac{|V_A|}{I_D} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$R_{\text{in}} = \infty$

$R_o = g_{m2} r_{o2} r_{o1}$

$= 1 \times 50 \times 50 = 2.5$ MΩ

$A_{vo} = -g_{m1} R_o = -1 \times 2.5 \times 18^3 = -2500$ V/V

(d) Refer to the circuit in Fig. P8.80(c).

$g_{m1} = \dfrac{2I_D}{|V_{OV}|} = \dfrac{2I}{|V_{OV}|} = \dfrac{2 \times 0.1}{0.2} = 1$ mA/V

$g_{m2} = \dfrac{I}{V_T} = \dfrac{0.1}{0.025} = 4$ mA/V

$r_{o1} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$r_{o2} = \dfrac{|V_A|}{I} = \dfrac{5}{0.1} = 50$ kΩ

$r_{\pi 2} = \dfrac{\beta}{g_{m2}} = \dfrac{100}{4} = 25$ kΩ

$R_{\text{in}} = \infty$

$R_o = (g_{m2} r_{o2})(r_{o1} \parallel r_{\pi 2})$

$= 4 \times 50(50 \parallel 25)$

$= 3.33$ MΩ

$A_{vo} = -g_{m1} R_o$

$= -1 \times 3.33 \times 10^6 = -3330$ V/V

Comment: The highest voltage gain (13,320 V/V) is obtained in circuit (a). However, the input resistance is only 25 kΩ. Of the two circuits with infinite input resistance (c and d), the circuit in (d) has the higher voltage gain. Observe that combining MOSFETs with BJTs results in circuits superior to those with exclusively MOSFETs or BJTs.

8.81 (a) Refer to the circuit in Fig. P8.81(a).

$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\dfrac{W}{L}\right) I_D}$

$= \sqrt{2 \times 0.4 \times 25 \times 0.1}$

$= 1.41$ mA/V

$$r_{o1} = \frac{V_A}{I_D} = \frac{1.8}{0.1} = 18\ \text{k}\Omega$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1}{0.025} = 4\ \text{mA/V}$$

$$r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18\ \text{k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{125}{4} = 31.25\ \text{k}\Omega$$

$$G_m = g_{m1} = 1.41\ \text{mA/V}$$

$$R_o = g_{m2} r_{o2}(r_{o1} \parallel r_{\pi 2})$$

$$= 4 \times 18 \times (18 \parallel 31.25) = 822.3\ \text{k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 822.3 = -1159\ \text{V/V}$$

(b) Refer to circuit in Fig. P8.81(b).

$$g_{m1} = g_{m2} = \sqrt{2 \times 0.4 \times 25 \times 0.1}$$

$$= 1.41\ \text{mA/V}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I} = \frac{1.8}{0.1} = 18\ \text{k}\Omega$$

$$G_m = g_{m1} = 1.41\ \text{mA/V}$$

$$R_o = g_{m2} r_{o2} r_{o1}$$

$$= 1.41 \times 18 \times 18 = 457\ \text{k}\Omega$$

$$A_{vo} = -G_m R_o = -1.41 \times 457 = -644\ \text{V/V}$$

We observe that the circuit with a cascode transistor provides higher gain.

8.82 Refer to Fig. 8.39.

$$I_O = I_{\text{REF}} \frac{(W/L)_2}{(W/L)_1}$$

$$= 20 \frac{40/1}{4/1} = 200\ \mu\text{A}$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$20 = \frac{1}{2} \times 160 \times \frac{4}{1} \times V_{OV1}^2$$

$$\Rightarrow V_{OV1} = 0.25\ \text{V}$$

$$V_{G2} = V_{GS1} = V_t + V_{OV1} = 0.6 + 0.25 = 0.85\ \text{V}$$

$$V_{OV4} = V_{OV1}$$

Thus,

$$V_{GS4} = V_{GS1} = 0.85\ \text{V}$$

$$V_{G3} = 0.85 + 0.85 = 1.7\ \text{V}$$

The lowest voltage at the output while Q_3 remains in saturation is

$$V_{O\min} = V_{G3} - V_{t3}$$

$$= 1.7 - 0.6 = 1.1\ \text{V}$$

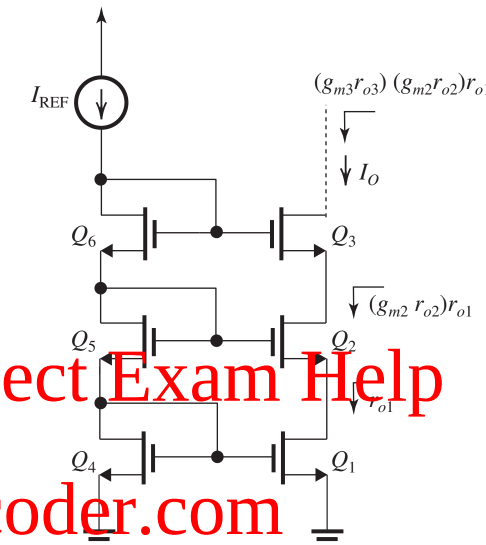
$$g_{m2} = g_{m3} = \frac{2 I_{D2,3}}{V_{OV2,3}} = \frac{2 \times 0.2}{0.25} = 1.6\ \text{mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_D} = \frac{10}{0.2} = 50\ \text{k}\Omega$$

$$R_o = g_{m3} r_{o3} r_{o2}$$

$$= 1.6 \times 50 \times 50 = 4\ \text{M}\Omega$$

8.83



From the figure we see that

$$R_o = (g_{m3} r_{o3})(g_{m2} r_{o2}) r_{o1}$$

8.84 Refer to Eq. (8.95),

$$R_o = \beta_3 r_{o3}/2$$

where

$$r_{o3} = \frac{V_A}{I} = \frac{100\ \text{V}}{1\ \text{mA}} = 100\ \text{k}\Omega$$

Thus,

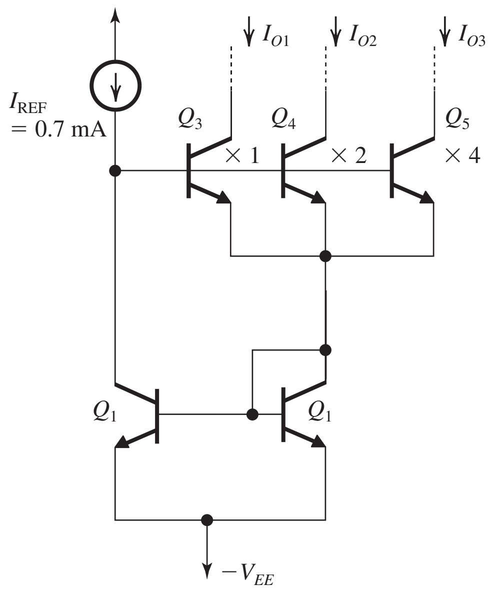
$$R_o = \frac{100 \times 100}{2} = 5\ \text{M}\Omega$$

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{10\ \text{V}}{5\ \text{M}\Omega} = 2\ \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = \frac{2\ \mu\text{A}}{1\ \text{mA}} = 0.002 \qquad \text{or } 0.2\%$$

8.85 (a) $I_{O1} = I_{O2} = \dfrac{1}{2} \dfrac{I_{\text{REF}}}{1 + \dfrac{2}{\beta^2}}$

(b)



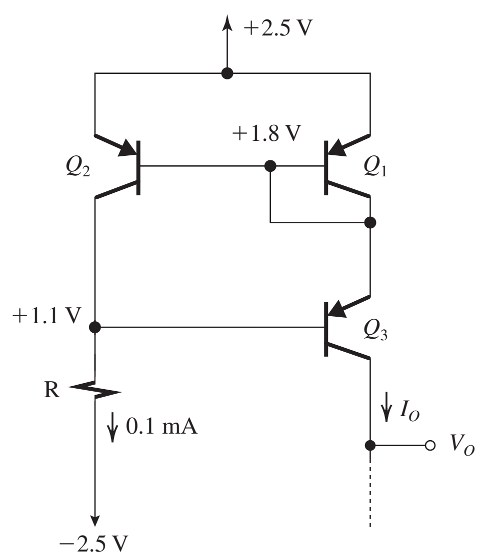
The figure shows the required circuit. Observe that the output transistor is split into three transistors having base–emitter junctions with area ratio 1:2:4. Thus

$$I_{O1} = \frac{0.1}{1 + \frac{2}{\beta^2}} = \frac{0.1}{1 + \frac{2}{50^2}} = 0.0999 \text{ mA}$$

$$I_{O2} = \frac{0.2}{1 + \frac{2}{50^2}} = 0.1998 \text{ mA}$$

$$I_{O4} = \frac{0.4}{1 + \frac{2}{50^2}} = 0.3997 \text{ mA}$$

8.86



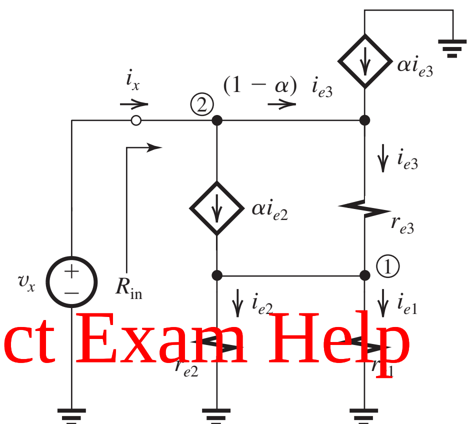
$$R = \frac{1.1 + 2.5}{0.1} = 36 \text{ k}\Omega$$

$V_{O\max}$ is limited by Q_3 saturating. Thus

$$V_{O\max} = V_{E3} - V_{EC\text{sat}}$$

$$= 1.8 - 0.3 = 1.5 \text{ V}$$

8.87 Replacing each of the transistors in the Wilson mirror of Fig. 8.40 with its T model while neglecting r_o results in the circuit shown below.



Note that the diode-connected transistor Q_1 reduces to a resistance r_{e1}. To determine R_{in}, we have applied a test voltage v_x. In the following we analyze the circuit to find i_x and hence R_{in}, as

$$R_{in} = \frac{v_x}{i_x}$$

Note that all three transistors are operating at equal emitter currents, approximately equal to I_{REF}. Thus

$$r_{e1} = r_{e2} = r_{e3} = \frac{V_T}{I_{REF}}$$

Analysis of the circuit proceeds as follows. Since $r_{e1} = r_{e2}$, we obtain

$$i_{e2} = i_{e1} \qquad (1)$$

Node equation at node 1:

$$i_{e3} + \alpha i_{e2} = i_{e1} + i_{e2}$$

Using Eq. (1) yields

$$i_{e3} = (2 - \alpha) i_{e1} \qquad (2)$$

Node equation at node 2:

$$i_x = \alpha i_{e2} + (1 - \alpha) i_{e3}$$

Using Eqs. (1) and (2) yields

$$i_x = i_{e1}[\alpha + (1 - \alpha)(2 - \alpha)]$$

$$i_x = i_{e1}[2 - 2\alpha + \alpha^2] \qquad (3)$$

Finally, v_x can be expressed as the sum of the voltages across r_{e3} and r_{e1},

$$v_x = i_{e3} r_e + i_{e1} r_e$$

Using Eq. (2) yields

$$v_x = i_{e1} r_e (3 - \alpha) \qquad (4)$$

Dividing Eq. (4) by Eq. (3) yields

$$R_{\text{in}} = \frac{v_x}{i_x} = r_e \frac{3 - \alpha}{2 - 2\alpha + \alpha^2}$$

For $\alpha \simeq 1$,

$$R_{\text{in}} = 2r_e = 2\frac{V_T}{I_{\text{REF}}} \qquad \text{Q.E.D.}$$

Thus, for $I_{\text{REF}} = 0.2$ mA,

$$R_{\text{in}} = 250\,\Omega$$

8.88 Refer to circuit in Fig. 8.41(a).

(a) Each of the three transistors is operating at $I_D = I_{\text{REF}}$. Thus

$$I_{\text{REF}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$180 = \frac{1}{2} \times 400 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.3 \text{ V}$$

$$V_{G3} = V_{tn} + V_{OV} = 0.5 + 0.3 = 0.8 \text{ V}$$

(b) Q_1 is operating at $V_{DS} = V_{GS} = 0.8$ V

Q_2 is operating at $V_{DS} = 2V_{GS} = 1.6$ V

Thus,

$$I_{\text{REF}} - I_O = \frac{\Delta V_{DS}}{r_o}$$

where

$$r_o = \frac{V_A}{I_{\text{REF}}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

$$I_{\text{REF}} - I_O = \frac{0.8}{100} = 0.008 \text{ mA} = 8\ \mu\text{A}$$

$$I_O = 180 - 8 = 172\ \mu\text{A}$$

(c) Refer to Fig. 8.41(c). Since Q_1 and Q_2 are now operating at equal V_{DS}, we estimate $I_O = I_{\text{REF}} = 180\ \mu$A.

(d) The minimum allowable V_O is the value at which Q_3 leaves the saturation region:

$$V_{O\text{min}} = V_{G3} - V_t$$

$$= V_{GS3} + V_{GS1} - V_t$$

$$= 0.8 + 0.8 - 0.5 = 1.1 \text{ V}$$

(e) Diode-connected transistor Q_4 has an incremental resistance $1/g_{m4}$. Reference to Fig. 8.41(b) indicates that the incremental resistance of Q_4 would appear in series with the gate of Q_3 and thus carries zero current. Thus including Q_4 has no effect on the value of R_o, which can be found from Eq. (8.96):

$$R_o = g_{m3} r_{o3} r_{o2}$$

where

$$g_{m3} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.18}{0.3} = 1.2 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{V_A}{I_{\text{REF}}} = \frac{18}{0.18} = 100 \text{ k}\Omega$$

Thus,

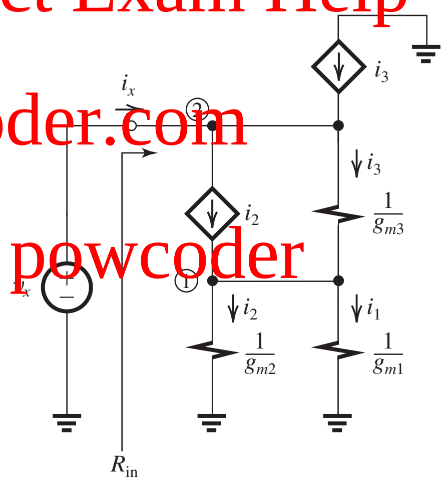
$$R_o = 1.2 \times 100 \times 100 = 12 \text{ M}\Omega$$

(f) For $\Delta V_O = 1$ V, we obtain

$$\Delta I_O = \frac{\Delta V_O}{R_o} = \frac{1 \text{ V}}{12 \text{ M}\Omega} = 0.08\ \mu\text{A}$$

$$\frac{\Delta I_O}{I_O} = 0.04\%$$

8.89 Replacing each of the three transistors in the Wilson current mirror in Fig. 8.41(a) with its T model results in the circuit in the figure.



Here, we have applied a test voltage v_x to determine R_{in},

$$R_{\text{in}} \equiv \frac{v_x}{i_x}$$

Since all three transistors are identical and are operating at the same I_D,

$$g_{m1} = g_{m2} = g_{m3}$$

Now from the figure we see that

$$i_1 = i_2$$

and

$$i_2 + i_3 = i_2 + i_1$$

Thus

$$i_3 = i_1 = i_2$$

A node equation at node 2 gives

$i_x + i_3 = i_2 + i_3$

Thus

$i_x = i_2$

The voltage v_x can be expressed as the sum of the voltages across $1/g_{m3}$ and $1/g_{m1}$:

$v_x = (i_3/g_{m3}) + (i_1/g_{m1})$

Substituting $i_3 = i_2$ and $i_1 = i_2$, $g_{m1} = g_{m3} = g_m$, and

$v_x = 2\ i_2/g_m$

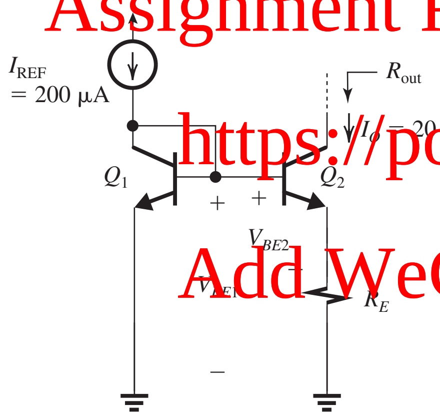
But $i_2 = i_x$; thus

$v_x = 2\ i_x/g_m$

and thus

$R_{\text{in}} = \dfrac{2}{g_m}$ Q.E.D.

8.90



(a) Assuming β is high so that we can neglect base currents,

$$I_O R_E = V_T \ln\left(\frac{I_{\text{REF}}}{I_O}\right)$$

Substituting $I_O = 20\ \mu\text{A}$ and $I_{\text{REF}} = 200\ \mu\text{A}$ results in

$$0.02\ R_E = 0.025\ \ln\left(\frac{200}{20}\right)$$

$\Rightarrow R_E = 2.88\ \text{k}\Omega$

(b) $R_{\text{out}} = (R_E \parallel r_{\pi 2}) + r_{o2} + g_{m2} r_{o2} (R_E \parallel r_{\pi 2})$

where

$$g_{m2} = \frac{0.02}{0.025} = 0.8\ \text{mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{50}{0.02} = 2500\ \text{k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_m} = \frac{200}{0.8} = 250\ \text{k}\Omega$$

$R_{\text{out}} = (2.9 \parallel 250) + 2500 + 0.8 \times 2500 \times (2.9 \parallel 250)$

$= 8.2\ \text{M}\Omega$

A 5-V change in V_O gives rise to

$$\Delta I_O = \frac{5}{7.1} = 0.7\ \mu\text{A}$$

8.91 Refer to Fig. 8.42.

(a) To obtain a current transfer ratio of 0.8 (i.e., $I_O/I_{\text{REF}} = 0.8$ and $I_O = 80\ \mu\text{A}$), we write

$$I_O R_E = V_T \ln\left(\frac{I_{\text{REF}}}{I_O}\right)$$

$$0.08 R_E = 0.025\ \ln\left(\frac{100}{80}\right)$$

$\Rightarrow R_E = 69.7\ \Omega$

$$g_{m2} = \frac{0.08}{0.025} = 3.2\ \text{mA}$$

$$r_{o2} = \frac{50}{0.08} = 625\ \text{k}\Omega$$

$r_{\pi 2} = \infty$ (because $\beta = \infty$)

$R_{\text{out}} = R_E + r_{o2} + g_{m2} r_{o2} R_E$

$= 0.069 + 625 + 3.2 \times 625 \times 0.0697$

$= 764.5\ \text{k}\Omega$

Relative to the value of r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = 1.22$$

(b) To obtain $I_O/I_{\text{REF}} = 0.1$, that is, $I_O = 10\ \mu\text{A}$, we write

$$0.01\ R_E = V_T\ \ln\left(\frac{100}{10}\right)$$

$\Rightarrow R_E = 5.76\ \text{k}\Omega$

$$g_{m2} = \frac{0.01}{0.025} = 0.4\ \text{mA/V}$$

$$r_{o2} = \frac{50}{0.01} = 5000\ \text{k}\Omega$$

$r_{\pi 2} = \infty$

$R_{\text{out}} = R_E + r_{o2} + g_{m2} r_{o2} R_E$

$= 5.76 + 5000 + 0.4 \times 5000 \times 5.76$

$= 16.5\ \text{M}\Omega$

Compared to r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = \frac{16.5}{5} = 3.3$$

(c) To obtain $I_O/I_{\text{REF}} = 0.01$, that is, $I_O = 1\ \mu\text{A}$, we write

$$0.001\ R_E = 0.025\ \ln\left(\frac{100}{1}\right)$$

$$\Rightarrow R_E = 115\ \text{k}\Omega$$

$$g_{m2} = \frac{0.001}{0.025} = 0.04\ \text{mA/V}$$

$$r_{o2} = \frac{50}{0.001} = 50 \times 10^3\ \text{k}\Omega$$

$$R_{\text{out}} = 115 + 50 \times 10^3 + 0.04 \times 50 \times 10^3 \times 115$$

$$= 280\ \text{M}\Omega$$

Relative to the value of r_{o2},

$$\frac{R_{\text{out}}}{r_{o2}} = \frac{280}{50} = 5.6$$

8.92 (a) Refer to the circuit in Fig. P8.92. Neglecting the base currents, we see that all three transistors are operating at $I_C = 10\ \mu\text{A}$, and thus

$$V_{BE1} = V_{BE2} = V_{BE3} = 0.7 - 0.025\ \ln\left(\frac{1\ \text{mA}}{10\ \mu\text{A}}\right)$$

$$= 0.585\ \text{V}$$

From the circuit we see that the voltage across R is $V_{BE} = 0.585$ V, thus

$$I_O R = V_{BE}$$

$$R = \frac{0.585}{0.01} = 58.5\ \text{k}\Omega$$

(b) $g_{m3} = \dfrac{0.01}{0.025} = 0.4\ \text{mA/V}$

$$r_{o3} = \frac{40}{0.01} = 4000\ \text{k}\Omega$$

$$r_{\pi 3} = \frac{\beta}{g_{m3}} = \frac{100}{0.4} = 250\ \text{k}\Omega$$

$$R_{\text{out}} = (R \parallel r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (R \parallel r_{\pi 3})$$

$$= (58.5 \parallel 250) + 4000 + 0.4 \times 4000 \times (58.5 \parallel 250)$$

$$= 79.9\ \text{M}\Omega$$

8.93 Refer to the circuit in Fig. P8.93. Since Q_1 and Q_2 are matched and conducting equal currents I, their V_{GS} values will be equal. Thus from the loop Q_1, Q_6, R, and Q_2, we see that

$$IR = V_{EB6}$$

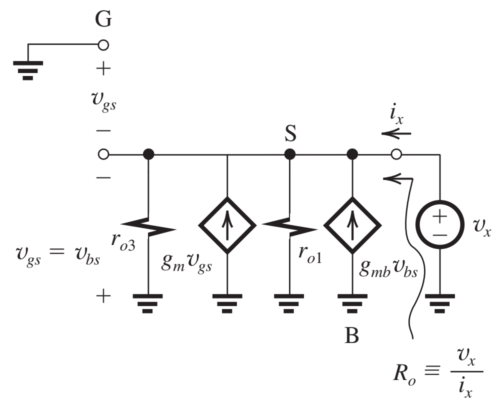
$$= V_T\ \ln\left(\frac{I}{I_S}\right) \qquad \text{Q.E.D.}$$

Now to obtain $I = 0.2$ mA, we write

$$0.2R = 0.7 - 0.025\ \ln\left(\frac{1\ \text{mA}}{0.2\ \text{mA}}\right)$$

$$\Rightarrow R = 3.3\ \text{k}\Omega$$

8.94



The figure shows the equivalent circuit of the source follower prepared for finding R_o. Observe that we have set $v_i = 0$ and applied a test voltage v_x. We note that

$$v_{gs} = v_{bs} = -v_x \qquad (1)$$

and

$$i_x = -g_{mb} v_{bs} + \frac{v_x}{r_{o1}} - g_m v_{gs} + \frac{v_x}{r_{o3}}$$

Thus,

$$i_x = g_{mb} v_x + \frac{v_x}{r_{o1}} + g_m v_x + \frac{v_x}{r_{o3}}$$

from which we obtain

$$R_o \equiv \frac{v_x}{i_x} = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_m + g_{mb}} \qquad \text{Q.E.D.}$$

8.95 The dc level shift provided by a source follower is equal to its V_{GS}. Thus, to obtain a dc level shift of 0.9 V, we write

$$V_{GS} = 0.9\ \text{V} = V_t + V_{OV}$$

$$\Rightarrow V_{OV} = 0.9 - 0.6 = 0.3\ \text{V}$$

To obtain the required bias current, we use

$$I = I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$= \frac{1}{2} \times 0.2 \times \frac{20}{0.5} \times 0.3^2$$

$$I = 0.36\ \text{mA} = 360\ \mu\text{A}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.36}{0.3} = 2.4\ \text{mA/V}$$

$$g_{mb} = \chi g_m = 0.2 \times 2.4 = 0.48\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{20 \times 0.5}{0.36} = 27.8\ \text{k}\Omega$$

To determine A_{vo}, we note [refer to Fig. 8.45(b)] that the total effective resistance between the MOSFET source terminal and ground is

$r_{o1} \parallel r_{o3} \parallel \dfrac{1}{g_{mb}}$. Denoting this resistance R, we have

$$R = r_o \parallel r_o \parallel \frac{1}{g_{mb}}$$

$$= 27.8 \parallel 27.8 \parallel \frac{1}{0.48}$$

$$= 1.81 \text{ k}\Omega$$

Thus, the open-circuit voltage gain is

$$A_{vo} = \frac{R}{R + \dfrac{1}{g_m}}$$

$$= \frac{1.81}{1.81 + \dfrac{1}{2.4}} = 0.81 \text{ V/V}$$

$$R_o = R // \frac{1}{g_m}$$

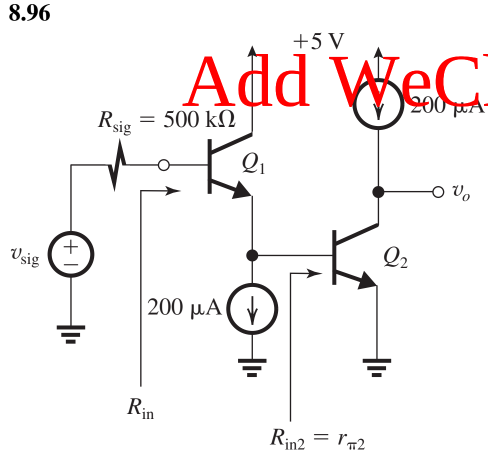
$$= 1.81 \text{ k}\Omega // \frac{1}{2.4 \text{ mA/V}}$$

$$= 0.339 \text{ k}\Omega$$

When a load resistance of 2 kΩ is connected to the output, the total resistance between the output node and ground become $R \parallel R_L = 1.81 \parallel 2 =$ 0.95 kΩ. Thus, the voltage gain becomes

$$A_v = \frac{0.95}{0.95 + \dfrac{1}{2.4}} = 0.7 \text{ V/V}$$

8.96



Each of Q_1 and Q_2 is operating at an I_C approximately equal to 200 μA. Thus for both devices,

$$g_m = \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$r_e \simeq \frac{1}{g_m} = 0.125 \text{ k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{8} = 12.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.2} = 250 \text{ k}\Omega$$

(a) $R_{in2} = r_{\pi 2} = 12.5 \text{ k}\Omega$

$$R_{in} = (\beta_1 + 1)[r_{e1} + (r_{\pi 2} \parallel r_{o1})]$$

$$= 101[0.125 + (12.5 \parallel 250)]$$

$$= 1.215 \text{ M}\Omega$$

$$\frac{v_{b1}}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{1.215}{1.215 + 0.5} = 0.71 \text{ V/V}$$

$$\frac{v_{e1}}{v_{b1}} = \frac{r_{\pi 2} \parallel r_{o1}}{(r_{\pi 2} \parallel r_{o1}) + r_{e1}} = 0.99 \text{ V/V}$$

$$\frac{v_o}{v_{b1}} = -g_{m2} r_{o2} = -8 \times 250 = -2000 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{sig}} = 0.71 \times 0.99 \times -2000 = -1405 \text{ V/V}$$

(b) Increasing the bias current by a factor of 10 (i.e., to 2 mA) results in

$$g_m = 80 \text{ mA/V}$$

$$r_e = 0.0125 \text{ k}\Omega$$

$$r_\pi = 1.25 \text{ k}\Omega$$

$$r_o = 25 \text{ k}\Omega$$

$$R_{in2} = r_{\pi 2} = 1.25 \text{ k}\Omega$$

$$R_{in} = 101[0.0125 + (1.25 \parallel 25)] = 121.5 \text{ k}\Omega$$

Thus, R_{in} has been reduced by a factor of 10.

$$\frac{v_{b1}}{v_{sig}} = \frac{121.5}{121.5 + 500}$$

$= 0.2$ V/V (considerably reduced)

$$\frac{v_{e1}}{v_{b1}} = \frac{(1.25 \parallel 25)}{(1.25 \parallel 25) + 0.0125}$$

$= 0.99$ V/V (unchanged)

$$\frac{v_o}{v_{b1}} = -g_{m2} r_o = -80 \times 25$$

$= -2000$ V/V (unchanged)

$$G_v = \frac{v_o}{v_{sig}} = 0.2 \times 0.99 \times -2000 = -396 \text{ V/V}$$

which has been reduced by a factor of 3.5! All this reduction in gain is a result of the reduction in R_{in}.

8.97

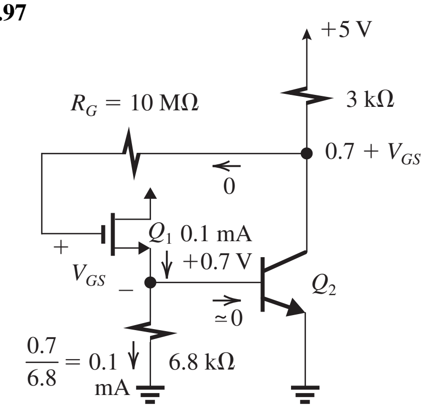


Figure 1

(a) From Fig. 1 we see that

$I_{D1} \simeq 0.1$ mA/V

But

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 2 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.316 \text{ V}$$

$$V_{GS} = V_t + V_{OV} = 1.316 \text{ V}$$

Thus,

$$V_{C2} = V_{G2} = 0.7 + V_{GS} = 2.016 \text{ V}$$

$$I_{C2} = \frac{V_{CC} - V_{C2}}{3 \text{ k}\Omega} = \frac{5 - 2.016}{3} \simeq 1 \text{ mA}$$

(b) $g_{m1} = \dfrac{2I_{D1}}{V_{OV}} = \dfrac{2 \times 0.1}{0.316} = 0.632$ mA/V

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1 \text{ mA}}{0.025} = 40 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{200}{40} = 5 \text{ k}\Omega$$

(c) Neglecting R_G, we can write

$$\frac{v_{b2}}{v_i} = \frac{r_{\pi 2} \parallel 6.8 \text{ k}\Omega}{(r_{\pi 2} \parallel 6.8 \text{ k}\Omega) + \dfrac{1}{g_{m1}}}$$

$= 0.65$ V/V

$$\frac{v_o}{v_{b2}} = -g_{m2}(3 \parallel 1)$$

$$= -40 \times \frac{3}{4} = -30 \text{ V/V}$$

$$\frac{v_o}{v_i} = 0.65 \times -30 = -19.5 \text{ V/V}$$

(d)

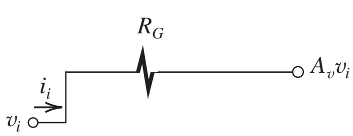


Figure 2

From Fig. 2 we can find i_i as

$$i_i = \frac{v_i - A_v v_i}{R_G}$$

$$= \frac{v_i + 19.4\, v_i}{R_G}$$

Thus,

$$R_{in} \equiv \frac{v_i}{i_i} = \frac{R_G}{20.5} = \frac{10 \text{ M}\Omega}{20.5} = 487 \text{ k}\Omega$$

Thus the overall voltage gain becomes

$$\frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} \times A_v$$

$$\frac{v_o}{v_{sig}} = \frac{487}{487 + 500} \times -19.5$$

$= -9.6$ V/V

(e) The suggested configuration, shown partially in Fig. 3, will have

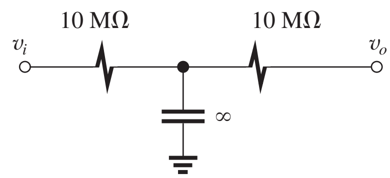


Figure 3

no effect on the dc bias of each transistor. However, it will have a profound effect on R_{in}, as R_{in} now is 10 MΩ, and

$$\frac{v_o}{v_{sig}} = \frac{10}{10 + 0.5} \times -19.5 = -18.6 \text{ V/V}$$

This is nearly double the value we had before!

8.98 From Fig. P8.98 we see that

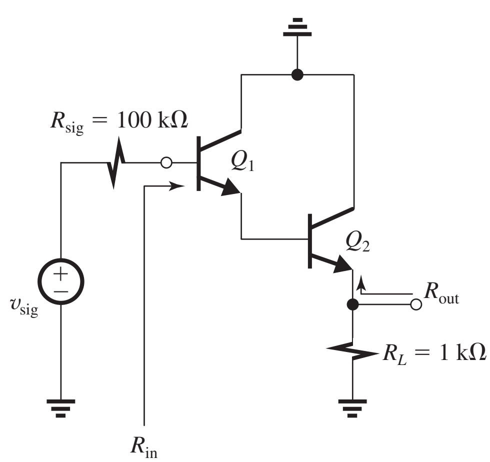
$I_{E2} = 10$ mA

$$I_{E1} = \frac{I_{E2}}{\beta_2 + 1} \simeq \frac{10}{100} = 0.1 \text{ mA}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{10 \text{ mA}} = 2.5\ \Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

The Darlington follower circuit prepared for small-signal analysis is shown in the figure.



$$R_{in} = (\beta + 1)[r_{e1} + (\beta_2 + 1)(r_{e2} + R_L)]$$

$$= 101[0.25 + (101)(0.0025 + 1)]$$

$$= 10.25 \text{ M}\Omega$$

$$R_{out} = r_{e2} + \frac{r_{e1} + R_{sig}/(\beta_1 + 1)}{\beta_2 + 1}$$

$$= 2.5 + \frac{250 + \dfrac{100 \times 10^3}{101}}{101} = 14.8\ \Omega$$

With R_L removed,

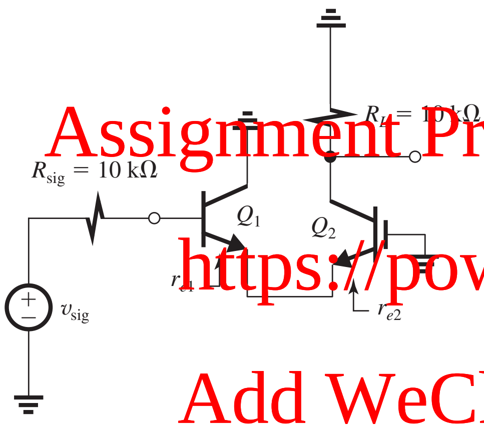
$$G_{vo} = \frac{v_o}{v_{sig}} = 1$$

With R_L connected,

$$G_v = \frac{v_o}{v_{sig}} = G_{vo}\frac{R_L}{R_L + R_{out}}$$

$$= 1 \times \frac{1}{1 + 0.0148} = 0.985$$

8.99



The figure shows the circuit prepared for signal analysis.

$$G_v = \frac{v_o}{v_{sig}} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha R_L}{\dfrac{R_{sig}}{\beta + 1} + r_{e1} + r_{e2}}$$

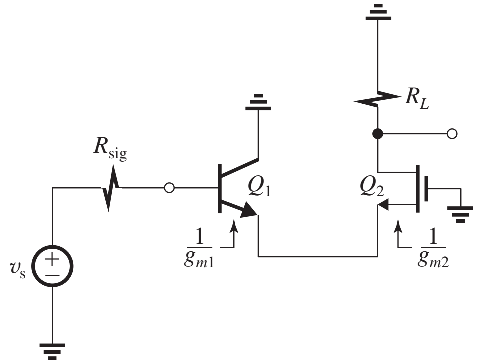
where

$$\alpha \simeq 1$$

$$r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$$G_v = \frac{10}{\dfrac{10}{101} + 0.05 + 0.05} = 50.2\ \text{V/V}$$

8.100



From the figure we can determine the overall voltage gain as

$$G_v = \frac{v_o}{v_{sig}} = \frac{\text{Total resistance in the drain}}{\text{Total resistance in the sources}}$$

$$= \frac{R_L}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}} = \frac{1}{2}g_m R_L$$

where

$$g_m = g_{m1} = g_{m2} = 5\ \text{mA/V}$$

$$G_v = \frac{1}{2} \times 5 \times 10 = 25\ \text{V/V}$$

8.101 Refer to Fig. P8.101. All transistors are operating at $I_E = 0.5$ mA. Thus,

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

(a) Refer to Fig. P8.101(a).

$$\frac{v_o}{v_{sig}} = -\frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitter}}$$

$$= \frac{-\alpha \times 10\ \text{k}\Omega}{\dfrac{10\ \text{k}\Omega}{\beta + 1} + r_e}$$

For

$$\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 0.99$$

$$G_v = \frac{-0.99 \times 10}{\dfrac{10}{101} + 0.05} = -66.4\ \text{V/V}$$

(b) Refer to Fig. P8.101(b).

$$i_{b1} = \frac{v_{sig}}{10 + (\beta + 1)r_{e1}} = \frac{v_{sig}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta i_{b1} = \frac{100\ v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$i_{c2} = \alpha i_{c1} = \frac{0.99 \times 100\ v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$v_o = -i_{c2} \times 10$$

$$G_v \equiv \frac{v_o}{v_{\text{sig}}} = -\frac{10 \times 0.99 \times 100}{10 + 101 \times 0.05} = -65.8 \text{ V/V}$$

(c) Refer to Fig. P8.101(c).

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{\alpha \times \text{Total resistance in collector}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 10}{\dfrac{10}{\beta + 1} + 2\, r_e} = \frac{0.99 \times 10}{\dfrac{10}{101} + 2 \times 0.05}$$

$$= 49.7 \text{ V/V}$$

(d) Refer to Fig. P8.101(d).

R_{in} (at the base of Q_1) $= (\beta_1 + 1)[r_{e1} + r_{\pi 2}]$

where

$$r_{e1} = 50\ \Omega$$

$$r_{\pi 2} = (\beta + 1) r_{e2} = 101 \times 50 = 5.05 \text{ k}\Omega$$

Thus,

$$R_{\text{in}} = 101(0.05 + 5.05) = 515 \text{ k}\Omega$$

$$\frac{v_{b1}}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{515}{515 + 10} = 0.98 \text{ V/V}$$

$$\frac{v_{b2}}{v_{b1}} = \frac{r_{\pi 2}}{r_{\pi 2} + r_{e1}} = \frac{5.05}{5.05 + 0.05} = 0.98 \text{ V/V}$$

$$\frac{v_o}{v_{b2}} = -g_{m2} \times 10 \text{ k}\Omega$$

$$= -20 \times 10 = -200 \text{ V/V}$$

$$G_v = \frac{v_o}{v_{\text{sig}}} = 0.98 \times 0.98 \times -200 = -194 \text{ V/V}$$

(e) Refer to Fig. P8.101(e).

$$i_{b1} = \frac{v_{\text{sig}}}{10 + (\beta + 1) r_{e1}} = \frac{v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$i_{c1} = \beta_1 i_{b1} = \frac{100\ v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$i_{e2} = i_{c1}$$

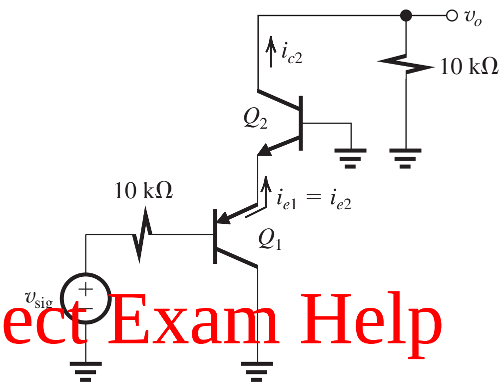
$$i_{c2} = \alpha i_{e2} = \alpha i_{c1} = \frac{0.99 \times 100\ v_{\text{sig}}}{10 + 101 \times 0.05}$$

$$v_o = i_{c2} \times 10 = \frac{0.99 \times 100 \times 10\ v_{\text{sig}}}{10 + 101 \times 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{0.99 \times 100 \times 10}{10 + 101 \times 0.05} = 65.8 \text{ V/V}$$

(f)



$$i_{e1} = i_{e2} = \frac{v_{\text{sig}}}{\dfrac{10}{\beta_1 + 1} + r_{e1} + r_{e2}}$$

$$= \frac{v_{\text{sig}}}{\dfrac{10}{101} + 0.05 + 0.05}$$

$$i_{c2} = \alpha i_{e2} = \frac{0.99\ v_{\text{sig}}}{\dfrac{10}{101} + 0.05 + 0.05}$$

$$v_o = i_{c2} \times 10 \text{ k}\Omega = \frac{0.99 \times 10\ v_{\text{sig}}}{\dfrac{10}{101} + 0.05 + 0.05}$$

Thus,

$$G_v = \frac{v_o}{v_{\text{sig}}} = 49.7 \text{ V/V}$$