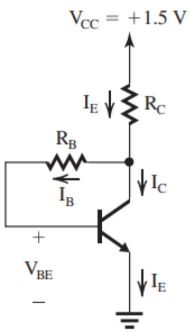
5.1:

Solution:



(a) For the dc analysis, refer to the figure:

$$V_{CC} = I_E R_C + I_B R_B + V_{BE} \rightarrow 1.5 = I_E \times 1 + \frac{I_E}{\beta + 1} \times 47 + 0.7 \rightarrow I_E = \frac{1.5 - 0.7}{1 + \frac{47}{101}} = 0.546 \text{ mA}$$

$$I_C = \alpha I_E = 0.99 \times 0.546 = 0.54 \text{ mA}$$

(b) $g_m = \frac{I_C}{V_T} = \frac{0.54 \text{ mA}}{0.025 \text{ V}} = 21.6 \frac{\text{mA}}{\text{V}}$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{21.6} = 4.63 \text{ k}\Omega$$

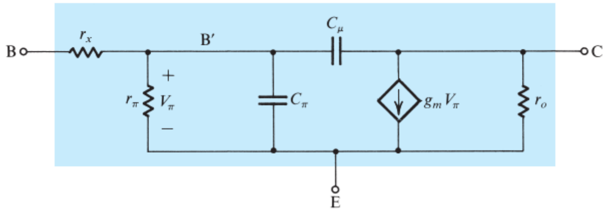
(c) $\frac{V_o}{V_b} = -g_m (R_C || R_L) = -21.6(1||1) = -10.8 \text{ V}$

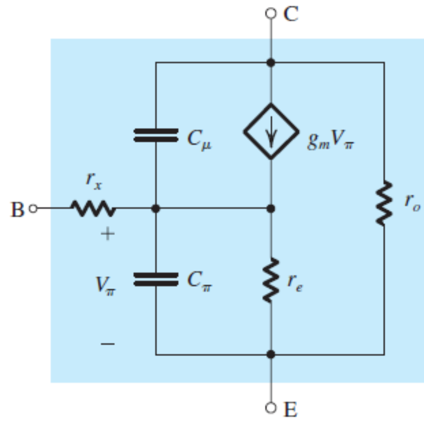
(d) Using Miller's theorem, the component of R_{in} due to R_B can be found as:

$$R_{in}1 = \frac{R_B}{1 - \left(\frac{V_o}{V_b}\right)} = \frac{47 \text{ k}\Omega}{1 - (-10.8)} = 4 \text{ k}\Omega$$

$$R_{in} = R_{in1} || r_\pi = 4 || 4.63 = 2.14 \text{ k}\Omega$$

(e) $G_V = \frac{R_{in}}{R_{in} + R_{sig}} \times \frac{V_o}{V_b} = \frac{2.14}{1 + 2.14} \times -10.8 = -7.4$





(f) $C_{in} = C_\pi + (1 + |\frac{V_o}{V_b}|)C_\mu$

Where:

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{21.6 \times 10^{-3}}{2\pi \times 600 \times 10^{-6}} = 5.73\ \text{pF}$$

$C_\pi = 5.73 - 0.8 = 4.93$ pF

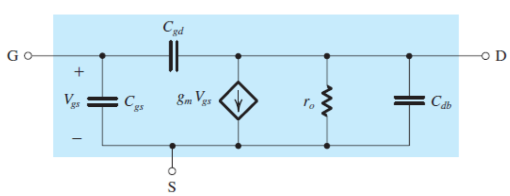
$C_{in} = 4.93 + (1 + 10.8) \times 0.8 = 14.37$ pF

(g) $R'_{sig} = R_{in} \| R_{sig} = 2.14\,\text{k}\Omega \| 1\,\text{k}\Omega = 0.68\,\text{k}\Omega$

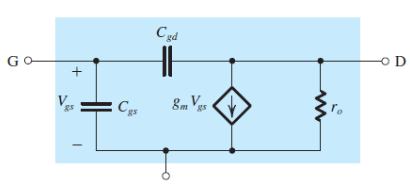
$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi \times 14.37 \times 10^{-12} \times 0.68 \times 10^3} = 16.3\ \text{MHz}$$

5.2:

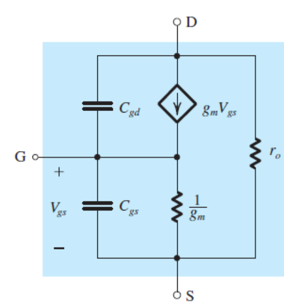
Solution:



(b)



(c)



(d)

Since the total resistance at the drain is r_o, we have:

$A_M = -g_m r_o$
$\tau_{gs} = C_{gs} R_{gs} = C_{gs} R_{sig}$
R_{gd} =R_sig (1+g_m R_L^')+R_L^'= $R_{sig}(1 + g_m r_o) + r_o$
$\tau_{gd} = C_{gd} R_{gd} = C_{gd}[R_{sig}(1 + g_m r_o) + r_o]$
$\tau_{cL} = C_L R_L' = C_L r_o$

Thus,
$\tau_H = \tau_{cL} + \tau_{gd} + \tau_{cs}$= $C_{gs} R_{sig} + C_{gd}[R_{sig}(1 + g_m r_o) + r_o] + C_L r_o$

For the given numerical values:

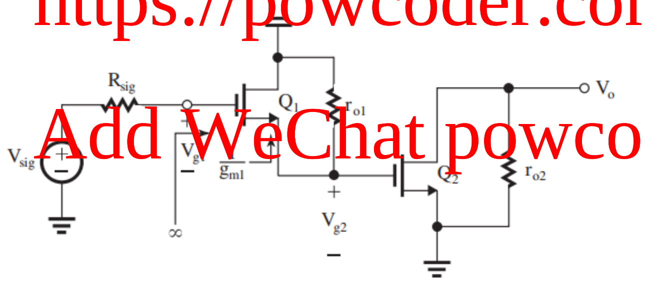
$$A_M = -1 \times 20 = -20$$

$$\tau_H = 20 \times 20 + 5[20(1 + 1 \times 20) + 20] + 10 \times 20 = 400 + 2200 + 200 = 2800\ ps$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 2.8 \times 10^{-9}} = 56.8\ MHz$$

$$GB = 20 \times 56.8 = 1.14\ GHz$$

(b)



We see that:

$$\frac{V_{g1}}{V_{sig}} = 1\ , \qquad \frac{V_{g2}}{V_{g1}} = \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}, \qquad \frac{V_o}{V_{g2}} = -g_{m2} r_{o2}$$

$$A_M = 1 \times \frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}} \times -g_{m2} r_{o2} = -\frac{r_{o1}}{\frac{1}{g_{m1}} + r_{o1}}(g_{m2} r_{o2})$$

Next, we evaluate the open-circuit time constants. Refer to Figure 2:

C_{gd1}: Capacitor C_{gd1} is between G_1 and ground and thus sees the resistance R_{sig},

$$R_{gd1} = R_{sig}$$

$$\tau_{gd1} = C_{gd1}R_{sig}$$

C_{gs1}: To find the resistance R_{gs1} seen by capacitor C_{gs1}, we replace Q_1 with its hybrid-π equivalent circuit with V_{sig} set to zero, $C_{gd1} = 0$ and C_{gs1} replaced by a test voltage V_x. The resulting equivalent circuit is shown in the figure 3:

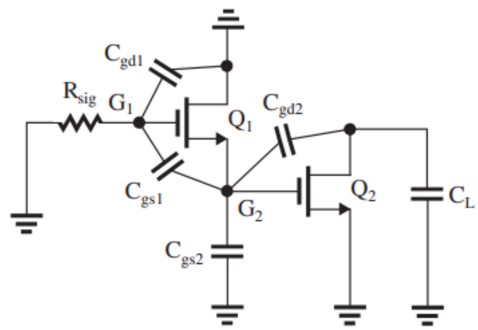


Figure 2

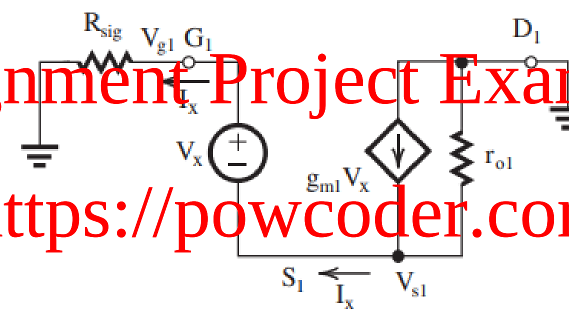


Figure 3

Analysis of the circuit in figure 3 proceeds as follows:

$$V_{g1} = I_X R_{sig}$$

$$V_{s1} = V_{g1} - V_X = I_X R_{sig} - V_X$$

At S_1:

$$I_X = g_{m1}V_X - \frac{V_{S1}}{r_{o1}} = g_{m1}V_X - \frac{I_X R_{sig} - V_X}{r_{o1}} \quad \rightarrow \quad I_X\left(1 + \frac{R_{sig}}{r_{o1}}\right) = V_X\left(g_{m1} + \frac{1}{r_{o1}}\right)$$

$$R_{gs1} = \frac{V_X}{I_X} = \frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

$$\tau_{gs1} = C_{gs1}\frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}}$$

C_{gs2}: Capacitor C_{gs2} sees the resistance between G_2 and ground, which is the output resistance of source follower Q_1

$$R_{gs2} = \frac{1}{g_{m1}}||r_{o1}$$

Thus,

$$\tau_{gs2} = C_{gs2}(\frac{1}{g_{m1}}||r_{o1})$$

C_{gs2}: Transistor Q_2 operates as a CS amplifier with an equivalent signal-source resistance equal to the output resistance of the source follower Q_1, that is $\frac{1}{g_{m1}}||r_{o1}$ and with a gain from gate to drain of $g_{m2}r_{o2}$. Thus, the formula for R_{gd} in a CS amplifier can be adapted as follows:

$$R_{gd}2 = (\frac{1}{g_{m1}}||r_{o1})(1 + g_{m2}r_{o2}) + r_{o2}$$

And thus,

$$\tau_{gd2} = C_{gd2}[(\frac{1}{g_{m1}}||r_{o1})(1 + g_{m2}r_{o2}) + r_{o2}]$$

C_L: Capacitor C_L sees the resistance between D_2 and ground which is r_{o2},

$$\tau_{CL} = C_L r_{o2}$$

Summing τ_{gd1}, τ_{gs1} , τ_{gs2} and τ_{CL} gives τ_H in the problem statement.

For the given numerical values:

$A_M = -\frac{20}{1+20}(1 \times 20) = -19$

$\tau_{gd1} = C_{gd1}R_{sig} = 5 \times 20 = 100\ ps$

$\tau_{gs1} = C_{gs1}\frac{R_{sig} + r_{o1}}{1 + g_{m1}r_{o1}} = 20\frac{20+20}{1 + 1 \times 20} = 38\ ps$

$\tau_{gd2} = C_{gd2}[(\frac{1}{g_{m1}}||r_{o1})(1 + g_{m2}r_{o2}) + r_{o2}]$ = 5[(1||20) (1+20) +20]=200 ps

$\tau_{CL} = C_L r_{o2} = 10 \times 20 = 200\ ps$

$\tau_H = 100 + 38 + 19 + 200 + 200 = 557\ ps$

$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 557 \times 10^{-12}} = 286\ MHz$

GB = $19 \times 286 = 5.43\ GHz$