Ex: 9.1 Referring to Fig. 9.3,

If R_D is doubled to 5 kΩ,

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2} R_D$$

$$= 1.5 - \frac{0.4 \text{ mA}}{2} (5 \text{ k}\Omega) = 0.5 \text{ V}$$

$$V_{CM_{\max}} = V_t + V_D = 0.5 + 0.5 = +1.0 \text{ V}$$

Since the currents I_{D1}, and I_{D2} are still 0.2 mA each,

$$V_{GS} = 0.82 \text{ V}$$

So, $V_{CM_{\min}} = V_{SS} + V_{CS} + V_{GS}$

$$= -1.5 \text{ V} + 0.4 \text{ V} + 0.82 \text{ V} = -0.28 \text{ V}$$

So, the common-mode range is

-0.28 V to $+1.0$ V

Ex: 9.2 (a) The value of v_{id} that causes Q_1 to conduct the entire current is $\sqrt{2}\, V_{OV}$

$$\rightarrow \sqrt{2} \times 0.316 = 0.45 \text{ V}$$

then, $V_{D1} = V_{DD} - I \times R_D$

$$= 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

$$V_{D2} = V_{DD} = +1.5 \text{ V}$$

(b) For Q_2 to conduct the entire current:

$$v_{id} = -\sqrt{2}\, V_{OV} = -0.45 \text{ V}$$

then,

$$V_{D1} = V_{DD} = +1.5 \text{ V}$$

$$V_{D2} = 1.5 - 0.4 \times 2.5 = 0.5 \text{ V}$$

(c) Thus the differential output range is

$V_{D2} - V_{D1}$: from $1.5 - 0.5 = \;+1$ V

to $0.5 - 1.5 = -1$ V

Ex: 9.3 Refer to answer table for Exercise 9.3 where values were obtained in the following way:

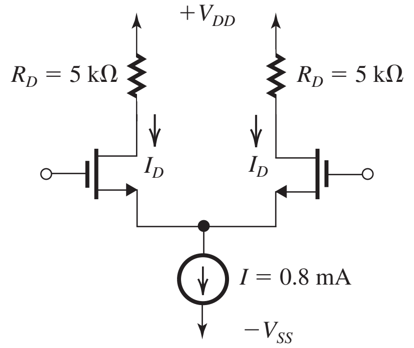
$$V_{OV} = \sqrt{I/k_n W/L} \Rightarrow \frac{W}{L} = \frac{I}{k_n V_{OV}^2}$$

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.1 \rightarrow v_{id} = 2\, V_{OV}\sqrt{0.1}$$

Ex: 9.4 $I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4 \text{ mA}$

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{OV})^2$$



Thus,

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}} = \sqrt{\frac{2\,(0.4 \text{ mA})}{0.2\,(\text{mA/V}^2)\,(100)}}$$

$$= 0.2 \text{ V}$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{0.4 \text{ mA} \times 2}{0.2 \text{ V}} = 4 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{20 \text{ V}}{0.4 \text{ mA}} = 50 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$A_d = (4 \text{ mA/V})(5 \text{ k}\Omega \parallel 50 \text{ k}\Omega) = 18.2 \text{ V/V}$$

Ex: 9.5 With $I = 200$ µA, for all transistors,

$$I_D = \frac{I}{2} = \frac{200 \text{ µA}}{2} = 100 \text{ µA}$$

$$L = 2(0.18 \text{ µm}) = 0.36 \text{ µm}$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V_A'|\,L}{I_D}$$

$$= \frac{(10 \text{ V/ µm})\,(0.36 \text{ µm})}{0.1 \text{ mA}} = 36 \text{ k}\Omega$$

Since $I_{D1} = I_{D2} = \dfrac{1}{2}\,\mu_n C_{ox}\left(\dfrac{W}{L}\right) V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2}$$

$$\frac{2(100 \text{ µA})}{(400 \text{ µA/V}^2)\,(0.2 \text{ V})^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2}$$

$$\frac{2(100 \text{ µA})}{(100 \text{ µA/V}^2)\,(0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100 \text{ µA})\,(2)}{0.2 \text{ V}} = 1 \text{ mA/V},$$

so

$$A_d = g_{m1}(r_{o1} \parallel r_{o3}) = 1(\text{mA/V})\,(36 \text{ k}\Omega \parallel 36 \text{ k}\Omega)$$

$$= 18 \text{ V/V}$$

Ex: 9.6 $L = 2\,(0.18\ \mu\text{m}) = 0.36\ \mu\text{m}$

All $r_o = \dfrac{|V_A'| \cdot L}{I_D}$

The drain current for all transistors is

$I_D = \dfrac{I}{2} = \dfrac{200\ \mu\text{A}}{2} = 100\ \mu\text{A}$

$r_o = \dfrac{(10\ \text{V}/\ \mu\text{m})\,(0.36\ \mu\text{m})}{0.1\ \text{mA}} = 36\ \text{k}\Omega$

Refering to Fig. 9.13(a),

Since $I_D = \dfrac{1}{2}\,\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}^2$ for all NMOS transistors,

$\left(\dfrac{W}{L}\right)_1 = \left(\dfrac{W}{L}\right)_2 = \left(\dfrac{W}{L}\right)_3 = \left(\dfrac{W}{L}\right)_4$

$= \dfrac{2I_D}{\mu_n C_{ox} V_{OV}^2} = \dfrac{2(100\ \mu\text{A})}{400\ \mu\text{A/V}^2(0.2\ \text{V})^2} = 12.5$

$\left(\dfrac{W}{L}\right)_5 = \left(\dfrac{W}{L}\right)_6 = \left(\dfrac{W}{L}\right)_7 = \left(\dfrac{W}{L}\right)_8$

$= \dfrac{2I_D}{\mu_p C_{ox} V_{OV}^2} = \dfrac{2(100\ \mu\text{A})}{100\ \mu\text{A/V}^2(0.2\ \text{V})^2} = 50$

For all transistors,

$g_m = \dfrac{|I_D|}{|V_{OV}|/2} = \dfrac{(0.1\ \text{mA})\,(2)}{(0.2\ \text{V})} = 1\ \text{mA/V}$

From Fig. 9.13(b),

$R_{on} = (g_{m3} r_{o3})\, r_{o1} = (1 \times 36) \times 36$

$= 1.296\ \text{M}\Omega$

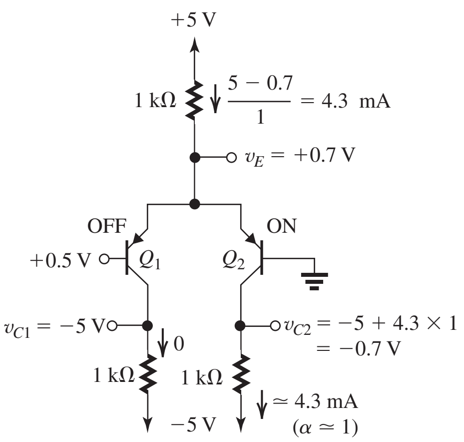
$R_{op} = (g_{m5} r_{o5})\, r_{o7} = (1 \times 36) \times 36$

$= 1.296\ \text{M}\Omega$

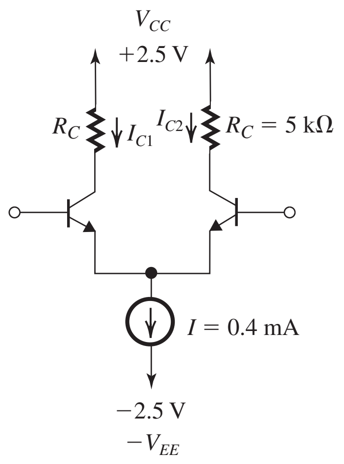
$A_d = g_{m1}\left(R_{on} \parallel R_{op}\right)$

$= (1\ \text{mA/V})\,(1.296\ \text{M}\Omega \parallel 1.296\ \text{M}\Omega)$
$= 648\ \text{V/V}$

Ex: 9.7



Ex: 9.8



$I_{C1} = I_{C2} \simeq I_{E1} = I_{E2} = \dfrac{I}{2} = \dfrac{0.4\ \text{mA}}{2}$

$= 0.2\ \text{mA}$

$V_{CM\,\max} \simeq V_C + 0.4\ \text{V}$

$= V_{CC} - I_C R_C + 0.4\ \text{V}$

$= 2.5 - 0.2\ \text{mA}\,(5\ \text{k}\Omega) + 0.4\ \text{V} = +1.9\ \text{V}$

$V_{CM\,\min} = -V_{EE} + V_{CS} + V_{BE}$

$V_{CM\,\min} = -2.5\ \text{V} + 0.3\ \text{V} + 0.7\ \text{V} = -1.5\ \text{V}$

Input common-mode range is -1.5 V to $+1.9$ V

Ex: 9.9 Substituting $i_{E1} + i_{E2} = I$ in Eqn. (9.45) yields

$i_{E1} = \dfrac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$

$0.99\,I = \dfrac{I}{1 + e^{(v_{B2} - v_{B1})/V_T}}$

$v_{B1} - v_{B2} = -V_T \ln\left(\dfrac{1}{0.99} - 1\right)$

$= -25 \ln (1/99)$

$= 25 \ln (99) = 115\ \text{mV}$

Ex: 9.10 (a) The DC current in each transister is 0.5 mA. Thus V_{BE} for each will be

$V_{BE} = 0.7 + 0.025 \ln\left(\dfrac{0.5}{1}\right)$

$= 0.683\ \text{V}$

$\Rightarrow v_E = 5 - 0.683 = +4.317\ \text{V}$

(b) $g_m = \dfrac{I_C}{V_T} = \dfrac{0.5}{0.025} = 20\ \dfrac{\text{mA}}{\text{V}}$

(c) $i_{C1} = 0.5 + g_{m1}\,\Delta v_{BE1}$

$= 0.5 + 20 \times 0.005 \sin (2\pi \times 1000t)$

$= 0.5 + 0.1 \sin (2\pi \times 1000t)$, mA

$i_{C2} = 0.5 - 0.1 \sin(2\pi \times 1000t)$, mA

(d) $v_{C1} = (V_{CC} - I_C R_C) - 0.1 \times R_C \sin(2\pi \times 1000t)$

$= (15 - 0.5 \times 10) - 0.1 \times 10 \sin(2\pi \times 1000t)$

$= 10 - 1 \sin(2\pi \times 1000t)$, V

$v_{C2} = 10 + 1 \sin(2\pi \times 1000t)$, V

(e) $v_{C2} - v_{C1} = 2 \cdot \sin(2\pi \times 1000t)$, V

(f) Voltage gain $\equiv \dfrac{v_{C2} - v_{C1}}{v_{B1} - v_{B2}}$

$= \dfrac{2 \text{ V peak}}{0.01 \text{ V peak}} = 200$ V/V

Ex: 9.11 The transconductance for each transistor is

$g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$

$I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4$ mA

Thus,

$g_m = \sqrt{2 \times 0.2 \times 100 \times 0.4} = 4$ mA/V

The differential gain for matched R_D values is $A_d = \dfrac{v_{O2} - v_{O1}}{v_{id}} = g_m R_D$

If we ignore the 1% here, then we obtain

$A_d = g_m R_D = (4 \text{ mA/V})(5 \text{ k}\Omega) = 20$ V/V

$|A_{cm}| = \left(\dfrac{R_D}{2R_{SS}}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

$= \left(\dfrac{5}{2 \times 25}\right)(0.01) = 0.001$ V/V

CMRR (dB) $= 20 \log \dfrac{|A_d|}{|A_{CM}|} = 20 \log\left(\dfrac{20}{0.001}\right)$

$= 86$ dB

Ex: 9.12 From Exercise 9.11,

$W/L = 100$, $\mu_n C_{ox} = 0.2$ mA/V^2,

$I_D = \dfrac{I}{2} = \dfrac{0.8 \text{ mA}}{2} = 0.4$ mA

$g_m = \sqrt{2\mu_n C_{ox} \left(\dfrac{W}{L}\right) I_D}$

$= \sqrt{2\,(0.2 \text{ mA/V}^2)\,(100)\,(0.4 \text{ mA})}$

$g_m = 4$ mA/V

Using Eq. (9.88) and the fact that $R_{SS} = 25$ kΩ, we obtain

$\text{CMRR} = \dfrac{(2\, g_m R_{SS})}{\left(\dfrac{\Delta g_m}{g_m}\right)} = \dfrac{2(4 \text{ mA/V})\,(25 \text{ k}\Omega)}{0.01}$

$= 20{,}000$

CMRR (dB) $= 20 \log_{10} (20{,}000) = 86$ dB

Ex: 9.13 If the output of a MOS differential amplifier is taken single-endedly, then

$|A_d| = \dfrac{1}{2} g_m R_D$

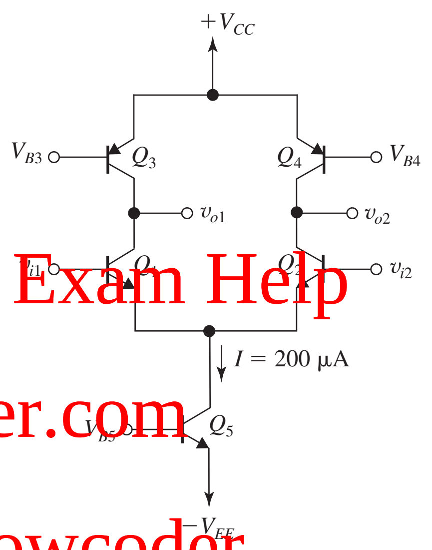
(that is, half the gain obtained with the output taken differentially), and from Fig. 9.25(d) we have

$|A_{cm}| \simeq \dfrac{R_D}{2R_{SS}}$

Thus,

$\text{CMRR} \equiv \dfrac{|A_d|}{|A_{cm}|} = g_m R_{SS}$ Q.E.D.

Ex: 9.14



$I = 200\ \mu$A

Since $\beta \gg 1$,

$I_{C1} \approx I_{C2} \approx \dfrac{I}{2} = \dfrac{200\ \mu\text{A}}{2} = 100\ \mu$A

$g_{m1} = g_{m2} = g_m = \dfrac{I_C}{V_T} = \dfrac{100\ \mu\text{A}}{25 \text{ mV}} = 4$ mA/V

$R_{C1} = R_{C2} = R_C = r_o = \dfrac{|V_A|}{I_C}$

$= \dfrac{10 \text{ V}}{100\ \mu\text{A}} = 100$ kΩ

$r_{o1} = r_{o2} = \dfrac{V_A}{I/2} = \dfrac{10}{0.1} = 100$ kΩ

$r_{e1} = r_{e2} = r_e = \dfrac{V_T}{I_E} = \dfrac{25 \text{ mV}}{0.1 \text{ mA}} = 0.25$ kΩ

$|A_d| = \dfrac{R_C \parallel r_o}{r_e} = \dfrac{100 \text{ k}\Omega \parallel 100 \text{ k}\Omega}{0.25 \text{ k}\Omega}$

$= 200$ V/V

$R_{id} = 2r_\pi$, $\quad r_\pi = \dfrac{\beta}{g_m} = \dfrac{100}{4 \text{ mA/V}} = 25$ kΩ

$R_{id} = 2(25 \text{ k}\Omega) = 50$ kΩ

$$R_{EE} = \frac{V_A}{I} = \frac{10\text{ V}}{200\ \mu\text{A}} = 50\text{ k}\Omega$$

If the total load resistance is assumed to be mismatched by 1%, then we have

$$|A_{cm}| = \frac{R_C}{2R_{EE}}\frac{\Delta R_C}{R_C}$$

$$= \frac{100}{2 \times 50} \times 0.01 = 0.01\text{ V/V}$$

$$\text{CMRR (dB)} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| = 20\log_{10}\left|\frac{200}{0.01}\right|$$

$$= 86\text{ dB}$$

Using Eq. (9.96), we obtain

$$R_{icm} = \beta R_{EE} \cdot \frac{1 + \dfrac{R_C}{\beta r_o}}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 50 \times \frac{1 + \dfrac{100}{100 \times 100}}{1 + \dfrac{100 + 2 \times 50}{100}}$$

$$R_{icm} \simeq 1.68\text{ M}\Omega$$

Ex: 9.15 From Exercise 9.4:

$$V_{OV} = 0.2\text{ V}$$

Using Eq. (9.101) we obtain V_{OS} due to $\Delta R_D/R_D$ as:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right) \cdot \left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{0.2}{2} \times 0.02 = 0.002\text{ V} \quad \text{i.e. } 2\text{ mV}$$

To obtain V_{OS} due to $\dfrac{\Delta(W/L)}{W/L}$,

use Eq. (9.106),

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta(W/L)}{W/L}\right)$$

$$\Rightarrow V_{OS} = \left(\frac{0.2}{2}\right) \times 0.02 = 0.002$$

$$\Rightarrow 2\text{ mV}$$

The offset voltage arising from ΔV_t is obtained from Eq. (9.109):

$$V_{OS} = \Delta V_t = 2\text{ mV}$$

Finally, from Eq. (9.110) the total input offset is

$$V_{OS} =$$

$$\left[\left(\frac{V_{OV}}{2}\frac{\Delta R_D}{R_D}\right)^2 + \left(\frac{V_{OV}}{2}\frac{\Delta(W/L)}{W/L}\right)^2 + (\Delta V_t)^2\right]^{1/2}$$

$$= \sqrt{(2 \times 10^{-3})^2 + (2 \times 10^{-3})^2 + (2 \times 10^{-3})^2}$$

$$= \sqrt{3 \times (2 \times 10^{-3})^2}$$

$$= 3.46\text{ mV}$$

Ex: 9.16 From Eq. (9.120), we get

$$V_{OS} = V_T\sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_S}{I_S}\right)^2}$$

$$= 25\sqrt{(0.02)^2 + (0.1)^2}$$

$$= 2.55\text{ mV}$$

$$I_B = \frac{100}{2(\beta + 1)} = \frac{100}{2 \times 101} \cong 0.5\ \mu\text{A}$$

$$I_{OS} = I_B\left(\frac{\Delta\beta}{\beta}\right)$$

$$= 0.5 \times 0.1\ \mu\text{A} = 50\text{ nA}$$

Ex: 9.17 $I_D = \dfrac{1}{2}I = 0.4\text{ mA}$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2\text{ V}$$

$$g_{m1,2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4\text{ mA/V}$$

$$G_m = g_{m1,2} = 4\text{ mA/V}$$

$$r_{o2} = \frac{V_{An}}{I_D} = \frac{20}{0.4} = 50\text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{20}{0.4} = 50\text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25\text{ k}\Omega$$

$$A_d = G_m R_o = 4 \times 25 = 100\text{ V/V}$$

Ex: 9.18 $G_m = g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{0.4\text{ mA}}{0.025\text{ V}}$

$$= 16\text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{V_A}{I_C} = \frac{V_A}{I/2} = \frac{100}{0.4} = 250\text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 250 \parallel 250 = 125\text{ k}\Omega$$

$$A_d = G_m R_o = 16 \times 125 = 2000\text{ V/V}$$

$$R_{id} = 2r_\pi = 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{160}{16} = 20\text{ k}\Omega$$

Ex: 9.19

$$G_m = g_{m1} = g_{m2} \simeq \frac{I/2}{V_T} = \frac{0.5\text{ mA}}{0.025\text{ V}} = 20\text{ mA/V}$$

$$r_{o4} = \frac{V_A}{I/2} = \frac{100\text{ V}}{0.5\text{ mA}} = 200\text{ k}\Omega$$

$R_{o4} \simeq \beta_4 r_{o4} = 50 \times 200 = 10{,}000 \text{ k}\Omega = 10 \text{ M}\Omega$

$R_{o5} = \frac{1}{2}\beta_5 r_{o5}$

where

$r_{o5} = \frac{V_A}{I/2} = 200 \text{ k}\Omega$

Thus,

$R_{o5} = \frac{1}{2} \times 100 \times 200 = 10 \text{ M}\Omega$

$R_o = R_{o4} \parallel R_{o5} = 10 \parallel 10 = 5 \text{ M}\Omega$

$A_d = G_m R_o$

$= 20 \text{ mA/V} \times 5000 \text{ k}\Omega = 10^5 \text{ V/V or } 100 \text{ dB}$

Ex: 9.20 From Exercise 9.17, we get

$I_D = 0.4 \text{ mA}$

$V_{OV} = 0.2 \text{ V} \quad g_{m1,2} = 4 \text{ mA/V}$

$G_m = 4 \text{ mA/V} \quad A_d = 100 \text{ V/V}$

Now,

$R_{SS} = 25 \text{ k}\Omega$

$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_p I_D}$

$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4 \text{ mA/V}$

$|A_{cm}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005 \text{ V/V}$

$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005}$

$= 20{,}000 \text{ or } 20 \log 20{,}000 = 86 \text{ dB}$

Ex: 9.21 From Exercise 9.18, we get

$I = 0.8 \text{ mA}, \; I_C \simeq 0.4 \text{ mA}, \; V_A = 100 \text{ V}$

$g_{m1,2} = 16 \text{ mA/V}, \; G_m = 16 \text{ mA/V}$

$r_{o2} = r_{o4} = 250 \text{ k}\Omega, \; A_d = 2000 \text{ V/V}$

Now,

$R_{EE} = \frac{100 \text{ V}}{0.8 \text{ mA}} = 125 \text{ k}\Omega$

Using Eq. (9.165),

$|A_{cm}| = \frac{r_{o4}}{\beta_3 R_{EE}} = \frac{250}{160 \times 125} = 0.0125 \text{ V/V}$

$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{0.0125} = 160{,}000 \text{ V/V}$

$20 \log \text{CMRR} = 104 \text{ dB}$

Ex: 9.22 Refer to Fig. (9.40).

(a) Using Eq. (9.170), we obtain

$I_6 = \frac{(W/L)_6}{(W/L)_4}(I/2)$

$\Rightarrow 100 = \frac{(W/L)_6}{100} \times 50$

thus, $(W/L)_6 = 200$

Using Eq. (9.171), we get

$I_7 = \frac{(W/L)_7}{(W/L)_5} I$

$\Rightarrow 100 = \frac{(W/L)_7}{200} \times 100$

thus, $(W/L)_7 = 200$

(b) For Q_1,

$\frac{I}{2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 V_{OV_1}^2$

$\Rightarrow V_{OV1} = \sqrt{\frac{50}{\frac{1}{2} \times 30 \times 200}} = 0.129 \text{ V}$

Similarly for Q_2, $V_{OV2} = 0.129 \text{ V}$

For Q_6,

$100 = \frac{1}{2} \times 90 \times 200 V_{OV6}^2$

$\Rightarrow V_{OV6} = 0.105 \text{ V}$

(c) $g_m = \frac{2I_D}{V_{OV}}$

	I_D	V_{OV}	g_m
Q_1	50 µA	0.129 V	0.775 mA/V
Q_2	50 µA	0.129 V	0.775 mA/V
Q_6	100 µA	0.105 V	1.90 mA/V

(d) $r_{o2} = 10/0.05 = 200 \text{ k}\Omega$

$r_{o4} = 10/0.05 = 200 \text{ k}\Omega$

$r_{o6} = 10/0.1 = 100 \text{ k}\Omega$

$r_{o7} = 10/0.1 = 100 \text{ k}\Omega$

(e) Eq. (9.168):

$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$

$= -0.775\,(200 \parallel 200) = -77.5 \frac{\text{V}}{\text{V}}$

Eq. (9.169):

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

$= -95 \text{ V/V}$

Overall voltage gain is

$A_1 \times A_2 = -77.5 \times -95 = 7363$ V/V

Ex: 9.23 $R_{id} = 20.2\ \mathrm{k\Omega}$

$A_{vo} = 8513$ V/V

$R_o = 152\ \Omega$

With $R_S = 10\ \mathrm{k\Omega}$ and $R_L = 1\ \mathrm{k\Omega}$,

$$G_v = \frac{20.2}{20.2 + 10} \times 8513 \times \frac{1}{(1 + 0.152)}$$

$= 4943$ V/V

Ex: 9.24 $\dfrac{i_{e8}}{i_{b8}} = \beta_8 + 1 = 101$

$$\frac{i_{b8}}{i_{c7}} = \frac{R_5}{R_5 + R_{i4}} = \frac{15.7}{15.7 + 303.5} = 0.0492$$

$$\frac{i_{c7}}{i_{b7}} = \beta_7 = 100$$

$$\frac{i_{b7}}{i_{c5}} = \frac{R_3}{R_3 + R_{i3}} = \frac{3}{3 + 234.8} = 0.0126$$

$$\frac{i_{c5}}{i_{b5}} = \beta_5 = 100$$

$$\frac{i_{b5}}{i_{c2}} = \frac{R_1 + R_2}{R_1 + R_2 + R_{i2}} = \frac{40}{40 + 5.05} = 0.888$$

$$\frac{i_{c2}}{i_1} = \beta_2 = 100$$

Thus the overall current gain is

$$\frac{i_{e8}}{i_1} = 101 \times 0.0492 \times 100 \times 0.0126 \times 100$$

$\times\ 0.888 \times 100$

$= 55{,}599$ A/A

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = \frac{R_6}{R_{i1}} \cdot \frac{i_{e8}}{i_1}$$

$$= \frac{3}{20.2} \times 55599 = 8257 \text{ V/V}$$

9.1 Refer to Fig. 9.2.

(a) $\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$V_{GS} = V_{tn} + V_{OV} = 0.4 + 0.2 = 0.6$ V

(b) $V_{CM} = 0$

$V_S = 0 - V_{GS} = -0.6$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(c) $V_{CM} = +0.4$ V

$V_S = 0.4 - V_{GS} = 0.4 - 0.6 = -0.2$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

Since $V_{CM} = 0.4$ V and $V_D = 0.6$ V, $V_{GD} = -0.2$ V, which is less than V_{tn} (0.4 V), indicating that our implicit assumption of saturation-mode operation is justified.

(d) $V_{CM} = -0.1$ V

$V_S = -0.1 - V_{GS} = -0.1 - 0.6 = -0.7$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(e) The highest value of V_{CM} for which Q_1 and Q_2 remain in saturation is

$V_{CM\,\text{max}} = V_{D1,2} + V_{tn}$

$= 0.6 + 0.4 = 1.0$ V

(f) To maintain the current-source operating properly, we need to keep a minimum voltage of 0.2 V across it, thus

$V_{S\text{min}} = -V_{SS} + V_{CS} = -1 + 0.2 = -0.8$ V

$V_{CM\,\text{min}} = V_{S\text{min}} + V_{GS}$

$= -0.8 + 0.6$

$= -0.2$ V

9.2 Refer to Fig. P9.2.

(a) For $v_{G1} = v_{G2} = 0$ V,

$I_{D1} = I_{D2} = \frac{1}{2} \times 0.5 = 0.25$ mA

$I_{D1,2} = \frac{1}{2}k_p'\left(\frac{W}{L}\right)|V_{OV}|^2$

$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$

$\Rightarrow |V_{OV}| = 0.35$ V

$V_{SG} = |V_{tp}| + |V_{OV}|$

$= 0.8 + 0.35 = 1.15$ V

$V_S = 0 + V_{SG} = +1.15$ V

$V_{D1} = V_{D2} = -V_{SS} + I_D R_D$

$= -2.5 + 0.25 \times 4$

$= -1.5$ V

Since for each of Q_1 and Q_2,

$V_{SD} = 1.15 - (-1.5)$

$= 2.65$ V

which is greater than $|V_{OV}|$, Q_1 and Q_2 are operating in saturation as implicitly assumed.

(b) The highest value of V_{CM} is limited by the need to keep a minimum of 0.4 V across the current source, thus

$V_{CM\,\text{max}} = +2.5 - 0.4 - V_{SG}$

$= +2.5 - 0.4 - 1.15 = +0.95$ V

The lowest value of V_{CM} is limited by the need to keep Q_1 and Q_2 in saturation, thus

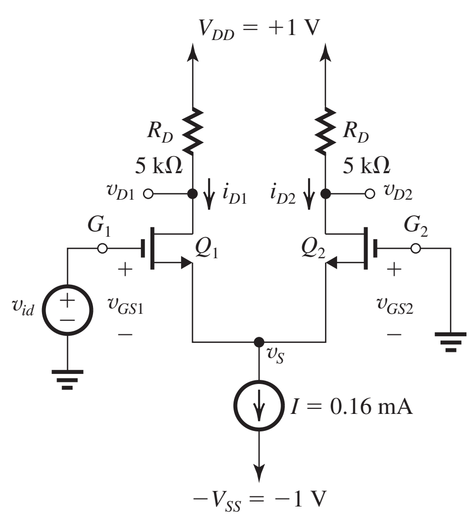
$V_{CM\,\text{min}} = V_{D1,2} - |V_{tp}|$

$= -1.5 - 0.8 = -2.3$ V

Thus,

$-2.3 \text{ V} \leq V_{ICM} \leq +0.95$ V

9.3



(a) For $i_{D1} = i_{D2} = 0.08$ mA,

$v_{G1} = v_{G2}$

Thus,

$$v_{id} = 0 \text{ V}$$

$$i_{D1} = i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$v_{GS1} = v_{GS2} = 0.2 + 0.4 = 0.6 \text{ V}$$

$$v_S = -0.6 \text{ V}$$

$$v_{D1} = v_{D2} = V_{DD} - i_{D1,2}R_D$$

$$= 1 - 0.08 \times 5 = 0.6 \text{ V}$$

$$v_{D2} - v_{D1} = 0 \text{ V}$$

(b) For $i_{D1} = 0.12$ mA and $i_{D2} = 0.04$ mA,

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS2} - V_{tn})^2$$

$$0.04 = \frac{1}{2} \times 0.4 \times 10 \times (v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.541 \text{ V}$$

Thus,

$$v_S = -0.541 \text{ V}$$

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS1} - V_{tn})^2$$

$$0.12 = \frac{1}{2} \times 0.4 \times 10\,(v_{id} - v_S - V_{tn})^2$$

$$= \frac{1}{2} \times 0.4 \times 10\,(v_{id} + 0.541 - 0.4)^2$$

$$\Rightarrow v_{id} = 0.104 \text{ V}$$

$$v_{GS1} = 0.104 - (-0.541) = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_{D2} - v_{D1} = 0.8 - 0.4 = 0.4 \text{ V}$$

(c) $i_{D1} = 0.16$ mA and $i_{D2} = 0$ with Q_2 just cutting off, thus

$$v_{GS2} = V_{tn} = 0.4 \text{ V}$$

$$\Rightarrow v_{S2} = -0.4 \text{ V}$$

$$i_{D1} = \frac{1}{2} \times 0.4 \times 10\,(v_{GS1} - V_{tn})^2$$

$$0.16 = \frac{1}{2} \times 0.4 \times 10\,(v_{id} + 0.4 - 0.4)^2$$

$$\Rightarrow v_{id} = 0.283 \text{ V}$$

which is $\sqrt{2}V_{OV}$, as derived in the text.

$$v_{GS1} = 0.283 - (-0.4) = 0.683 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.16 \times 5 = +0.2 \text{ V}$$

Note that since $v_{G1} = v_{id} = 0.283$ V, Q_1 is still operating in saturation, as implicitly assumed.

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0 \times 5 = 1 \text{ V}$$

$$v_{D2} - v_{D1} = 1 - 0.2 = 0.8 \text{ V}$$

(d) $i_{D1} = 0.04$ mA and $i_{D2} = 0.12$ mA. Since this split of the current I is the complement of that in case (b) above, the value of v_{id} must be the negative of that found in (b). Thus,

$$v_{id} = -0.104 \text{ V}$$

$$v_{GS1} = 0.541 \text{ V}$$

$$v_S = -0.645 \text{ V}$$

$$v_{GS2} = 0.645 \text{ V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D$$

$$= 1 - 0.04 \times 5 = 0.8 \text{ V}$$

$$v_{D2} = 1 - 0.12 \times 5 = 0.4 \text{ V}$$

$$v_{D2} - v_{D1} = -0.4 \text{ V}$$

(e) $i_{D1} = 0$ (Q_1 just cuts off) and $i_{D2} = 0.16$ mA. This case is the complement of that in (c) above, thus

$$v_{GS1} = V_{tn} = 0.4 \text{ V}$$

$$v_{GS2} = 0.683 \text{ V}$$

$$v_S = -0.683 \text{ V}$$

$$v_{id} = -0.683 + 0.4 = -0.283 \text{ V}$$

which is $-\sqrt{2}\,V_{OV}$, as derived in the text.

$$v_{D1} = V_{DD} - i_{D1}R_D = 1 - 0 \times 5 = 1 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1 - 0.16 \times 5 = 0.2 \text{ V}$$

$$v_{D2} - v_{D1} = -0.8 \text{ V}$$

Summary

A summary of the results is shown in the following table on the next page.

Case	i_{D1}(mA)	i_{D2}(mA)	v_{id}(V)	v_S(V)	v_{D1}(V)	v_{D2}(V)	$v_{D2}-v_{D1}$(V)
a	0.08	0.08	0	−0.6	+0.6	+0.6	0
b	0.12	0.04	+0.104	−0.541	+0.4	+0.8	+0.4
c	0.16	0	+0.283	−0.4	+0.2	+1.0	+0.8
d	0.04	0.12	−0.104	−0.645	+0.8	+0.4	−0.4
e	0	0.16	−0.283	−0.683	+1.0	+0.2	−0.8

9.4 Refer to Fig. P9.2.

To determine V_{OV},

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$$\Rightarrow |V_{OV}| = 0.354 \text{ V}$$

With $v_{G2} = 0$ and $v_{G1} = v_{id}$, to steer the current from one side of the differential pair to the other, v_{id} must be the ends of the range

$$-\sqrt{2}\,|V_{OV}| \le v_{id} \le \sqrt{2}\,|V_{OV}|$$

that is,

$$-0.5 \text{ V} \le v_{id} \le +0.5 \text{ V}$$

At $v_{id} = -0.5$ V, Q_2 just cuts off, thus

$$v_S = |V_{tp}| = 0.8 \text{ V}$$

and

$$v_{SG1} = 0.8 - (-0.5) = 1.3 \text{ V}$$

At this value of v_{SG1},

$$i_{D1} = \frac{1}{2} \times 4 \times (1.3 - 0.8)^2$$

$$= 0.5 \text{ mA}$$

which is the entire bias current.

$$v_{D1} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

Observe that since $v_{G1} = v_{D1}$, Q_1 is still operating in saturation, as implicitly assumed.

$$v_{D2} = -2.5 \text{ V}$$

At $v_{id} = +0.5$ V, Q_1 just cuts off, thus $v_{SG1} = |V_{tp}| = 0.8$ V and

$$v_S = +0.5 + 0.8 = +1.3 \text{ V}$$

and thus

$$v_{SG2} = 1.3 \text{ V}$$

which results in

$$i_{D1} = \frac{1}{2} \times 4\,(1.3 - 0.8)^2$$

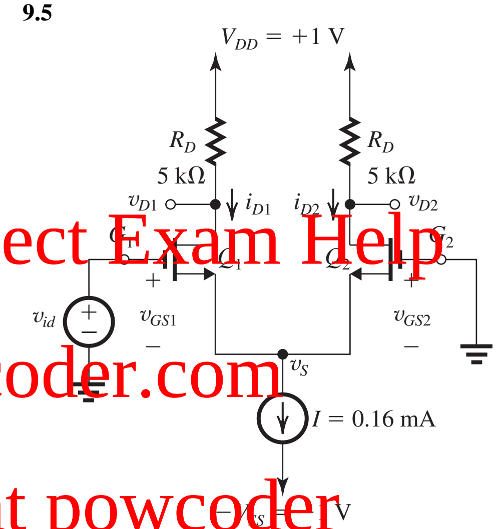
$$= 0.5 \text{ mA}$$

which is the entire bias current. Here,

$$v_{D2} = -2.5 + 0.5 \times 4 = -0.5 \text{ V}$$

which verifies that Q_2 is operating in saturation, as implicitly assumed.

9.5



For $i_{D1} = 0.09$ mA and $i_{D2} = 0.07$ mA,

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS2} - V_{tn})^2$$

$$0.07 = \frac{1}{2} \times 0.4 \times 10(v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.587 \text{ V}$$

and

$$v_S = -0.587 \text{ V}$$

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS1} - V_{tn})^2$$

$$0.09 = \frac{1}{2} \times 0.4 \times 10\,(v_{GS1} - 0.4)^2$$

$$\Rightarrow v_{GS1} = 0.612 \text{ V}$$

$$v_{id} = v_S + v_{GS1} = -0.587 + 0.612$$

$$= 0.025 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0.07 \times 5 = 0.65 \text{ V}$$

$v_{D1} = 1 - 0.09 \times 5 = 0.55$ V

$v_{D2} - v_{D1} = 0.65 - 0.55 = 0.10$ V

$$\text{Voltage gain} = \frac{v_{D2} - v_{D1}}{v_{id}} = \frac{0.10}{0.025} = 4 \text{ V/V}$$

To obtain the complementary split in current, that is, $i_{D1} = 0.07$ mA and $i_{D2} = 0.09$ mA,

$v_{id} = -0.025$ V

9.6 Refer to the circuit in Fig. P9.6.

For $v_{G1} = v_{G2} = 0$ V,

$$I_{D1} = I_{D2} = \frac{0.4}{2} = 0.2 \text{ mA}$$

To obtain

$V_{D1} = V_{D2} = +0.1$ V

$V_{DD} - I_{D1,2}\, R_D = 0.1$

$0.9 - 0.2\, R_D = 0.1$

$\Rightarrow\ R_D = 4$ kΩ

For Q_1 and Q_2,

$$I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4\left(\frac{W}{L}\right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 44.4$$

For Q_3,

$$0.4 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_3 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 88.8$$

Since Q_3 and Q_4 form a current mirror with $I_{D3} = 4 I_{D4}$,

$$\left(\frac{W}{L}\right)_4 = \frac{1}{4}\left(\frac{W}{L}\right)_3 = 22.2$$

$V_{GS4} = V_{GS3} = V_{tn} + V_{OV} = 0.4 + 0.15$

$= 0.55$ V

$$R = \frac{0.9 - (-0.9) - 0.55}{0.1}$$

$= 12.5$ kΩ

The lower limit on V_{CM} is determined by the need to keep Q_3 operating in saturation. For this to happen, the minimum value of V_{DS3} is $V_{OV} = 0.15$ V. Thus,

$V_{ICM\min} = -V_{SS} + V_{OV3} + V_{GS1,2}$

$= -0.9 + 0.15 + 0.4 + 0.15$

$= -0.2$ V

The upper limit on V_{CM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$V_{ICM\max} = V_{D1,2} + V_{tn}$

$= 0.1 + 0.4 = 0.5$ V

Thus,

$-0.2 \text{ V} \le V_{ICM} \le +0.5$ V

9.7 From Exercise 9.3 and the accompanying table, we note that $|v_{id}|_{\max}$ is proportional to V_{OV}:

$$\frac{|v_{id}|_{\max}}{V_{OV}} = \frac{0.126}{0.2} = 0.63$$

Thus, to obtain $|v_{id}|_{\max} = 220$ mV $= 0.22$ V at the same level of linearity, we use

$$V_{OV} = \frac{0.22}{0.63} = 0.35 \text{ V}$$

For this value of V_{OV}, the required (W/L) can be found from

$$0.2 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right) \times 0.35^2$$

$$\Rightarrow \frac{W}{L} = 16.3$$

The value of g_m is

$$g_m = \frac{2\, I_D}{V_{OV}} = \frac{2 \times 0.2}{0.35} = 1.14 \text{ mA/V}$$

9.8 Refer to Eq. (9.23). For

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 \le k$$

$$\Rightarrow \left(\frac{v_{id}/2}{V_{OV}}\right) \le \sqrt{k} \qquad (1)$$

$$\Delta I = I\left(\frac{v_{id}/2}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

$$\Delta I_{\max} = I\ \sqrt{k}\sqrt{1-k}$$

Thus,

$$\frac{\Delta I_{\max}}{I/2} = 2\sqrt{k(1-k)} \qquad \text{Q.E.D.} \qquad (2)$$

and the corresponding value of v_{id} is found from Eq. (2) as

$$v_{id\max} = 2\sqrt{k}\, V_{OV} \qquad \text{Q.E.D.} \qquad (3)$$

Equations (2) and (3) can be used to evaluate $\frac{\Delta I_{\max}}{I/2}$ and $\frac{v_{id\max}}{V_{OV}}$ for various values of k:

k	0.01	0.1	0.2
$\frac{v_{id\max}}{V_{OV}}$	0.2	0.632	0.894
$\frac{\Delta I_{\max}}{I/2}$	0.2	0.6	0.8

9.9 Switching occurs at

$$v_{id} = \sqrt{2} V_{OV}$$

Thus,

$$0.3 = \sqrt{2} V_{OV}$$

$$\Rightarrow V_{OV} = 0.212 \text{ V}$$

Now, to obtain full current switching at $v_{id} = 0.5$ V, V_{OV} must be increased to

$$V_{OV} = 0.212 \times \frac{0.5}{0.3} = 0.353 \text{ V}$$

Since I_D is proportional to V_{OV}^2 the current I_D and hence the bias current I must be increased by the ratio $(0.353/0.212)^2$, then I must be

$$I = 200 \times \left(\frac{0.353}{0.212}\right)^2 = 554.5 \ \mu\text{A}$$

9.10 Refer to Fig. 9.5.

$$g_m = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$1 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

$$\frac{I}{2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) V_{OV}^2$$

$$\frac{1}{2} \times 0.25 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right) 0.25^2$$

$$\Rightarrow \frac{W}{L} = 10$$

9.11 Equations (9.23) and (9.24):

$$i_{D1} = \frac{I}{2} + \frac{I}{2}\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} \qquad (9.23)$$

$$i_{D2} = \frac{I}{2} - \frac{I}{2}\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} \qquad (9.24)$$

(a) For 10% increase above the equilibrium value of $\frac{I}{2}$,

$$\left(\frac{I}{2}\right)\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} = 0.1 \times \frac{I}{2}$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.1$$

$$\Rightarrow \frac{v_{id}}{V_{OV}} \simeq 0.1$$

$$v_{id} \simeq 0.1 V_{OV}$$

(b) In Eqs. (9.23) and (9.24) let

$$i_{D1} = \left(\frac{I}{2}\right) + \left(\frac{I}{2}\right) \times \triangle$$

$$i_{D2} = \left(\frac{I}{2}\right) - \left(\frac{I}{2}\right) \times \triangle$$

where

$$\triangle = \left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

If v_{id} is such that

$$\frac{i_{D1}}{i_{D2}} = m$$

then

$$m = \frac{1 + \triangle}{1 - \triangle}$$

$$\Rightarrow \triangle = \frac{m - 1}{m + 1}$$

For $m = 1$, $\triangle = 0$ and $v_{id} = 0$

For $m = 2$,

$$\triangle = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = \frac{1}{3}$$

Squaring both sides, we obtain a quadratic equation in $\left(\frac{v_{id}}{V_{OV}}\right)^2$ which can be solved to obtain

$$v_{id} = 0.338 V_{OV}$$

For $m = 1.1$,

$$\triangle = \frac{1.1 - 1}{1.1 + 1} = \frac{0.1}{2.1} \simeq 0.05$$

Thus,

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.05$$

$$\Rightarrow v_{id} \simeq 0.05 V_{OV}$$

For $m = 1.01$

$$\triangle = \frac{1.01 - 1}{1.01 + 1} \simeq 0.005$$

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.005$$

$$v_{id} \simeq 0.005 V_{OV}$$

For $m = 20$,

$$\triangle = \frac{m-1}{m+1} = \frac{19}{21} = 0.905 \text{ V}$$

Thus,

$$\left(\frac{v_{id}}{V_{OV}}\right)\sqrt{1 - \frac{1}{4}\left(\frac{v_{id}}{V_{OV}}\right)^2} = 0.905$$

$$\Rightarrow v_{id} = 1.072 V_{OV}$$

9.12 $0.1 = \frac{1}{2} \times 0.2 \times 32 V_{OV}^2$

$$\Rightarrow V_{OV} = 0.18 \text{ V}$$

$$g_m = \frac{2 \times (0.2/2)}{0.18} = 1.11 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_m (R_D \parallel r_o)$$

$$= 1.11 \times (10 \parallel 100) = 10.1 \text{ V/V}$$

9.13 For $v_{id} = 0.1$ V

$$\left(\frac{v_{id}/2}{V_{OV}}\right)^2 = 0.04$$

$$\frac{v_{id}/2}{V_{OV}} = 0.2$$

$$\frac{0.1/2}{V_{OV}} = 0.2$$

$$\Rightarrow V_{OV} = 0.25 \text{ V}$$

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$2 = \frac{I}{0.25}$$

$$\Rightarrow I = 0.5 \text{ mA}$$

$$A_d = \frac{1 \text{ V}}{0.1 \text{ V}} = 10$$

$$g_m R_D = 10$$

$$\Rightarrow R_D = \frac{10}{2} = 5 \text{ k}\Omega$$

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.2 \times \frac{W}{L} \times 0.25^2$$

$$\Rightarrow \frac{W}{L} = 40$$

9.14 To limit the power dissipation to 1 mW,

$$P = (V_{DD} + V_{SS}) I$$

Thus, the maximum value we can use for I is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

Using this value, we obtain

$$V_D = V_{DD} - \frac{I}{2} R_D$$

$$0.2 = 1 - 0.25 \times R_D$$

$$\Rightarrow R_D = 3.2 \text{ k}\Omega$$

$$A_d = g_m R_D$$

$$10 = g_m \times 3.2$$

$$g_m = \frac{10}{3.2} = 3.125 \text{ mA/V}$$

But

$$g_m = \frac{2 \times (I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$3.125 = \frac{0.5}{V_{OV}}$$

$$\Rightarrow V_{OV} = 0.16 \text{ V}$$

To obtain W/L, we use

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.16^2$$

$$\Rightarrow \frac{W}{L} = 48.8 \simeq 50$$

9.15 Since the quiescent power dissipation is

$$P = (V_{DD} + V_{SS}) \times I$$

then the maximum allowable I is

$$I = \frac{1 \text{ mW}}{2 \text{ V}} = 0.5 \text{ mA}$$

We shall utilize this value. The value of V_{OV} can be found from

$$\sqrt{2}\, V_{OV} = 0.25 \text{ V}$$

$$\Rightarrow V_{OV} = \frac{0.25}{\sqrt{2}} = 0.18 \text{ V}$$

The realized value of g_m will be

$$g_m = \frac{2 \times (I/2)}{V_{OV}}$$

$$= \frac{0.5}{0.18} = 2.8 \text{ mA/V}$$

To obtain a differential gain A_d of 10 V/V,

$$A_d = g_m R_D$$

$$10 = 2.8 \times R_D$$

$$\Rightarrow R_D = 3.6 \text{ k}\Omega$$

Finally, the required value of W/L can be determined from

$$I_D = \frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{OV}^2$$

$$0.25 = \frac{1}{2} \times 0.4 \times \frac{W}{L} \times 0.18^2$$

$$\Rightarrow \frac{W}{L} = 38.6$$

9.16 (a) $A_d = g_m R_D$

$$20 = g_m \times 47$$

$$\Rightarrow g_m = \frac{20}{47} = 0.426 \text{ mA/V}$$

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}}$

$$0.426 = \frac{I}{0.2}$$

$$\Rightarrow I = 0.085 \text{ mA} = 85\ \mu\text{A}$$

(c) Across each R_D the dc voltage is

$$\frac{I}{2} R_D = \frac{0.085}{2} \times 47 = 2 \text{ V}$$

(d) The peak sine-wave signal across each gate source is 5 mV, thus at each drain the peak sine wave is

$$A_d \times 5 = 20 \times 5 = 100 \text{ mV} = 0.1 \text{ V}$$

(e) The minimum voltage at each drain will be

$$v_{D\min} = V_{DD} - R_D I_D - V_{\text{peak}}$$

$$= V_{DD} - 2 - 0.1$$

For the transistor to remain in saturation

$$v_{D\min} \geq v_{G\max} - V_{tn}$$

where

$$v_{G\max} = V_{CM} + V_{\text{peak}}(\text{input})$$

$$= 0.5 + 0.005 = 0.505 \text{ V}$$

Thus,

$$V_{DD} - 2.1 \geq 0.505 - 0.5$$

$$V_{DD} \geq 2.105 \text{ V}$$

Thus, the lowest value of V_{DD} is 2.21 V.

9.17 For a CS amplifier biased at a current I_D and utilizing a drain resistance R_D, the voltage gain is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I_D} R_D \qquad (1)$$

For a differential pair biased with a current I and utilizing drain resistances R_D, the differential gain is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} \left(\frac{I}{2}\right)}$$

Thus

$$A_d = \sqrt{2\mu_n C_{ox} \frac{W}{L}} \sqrt{I/2}\, R_D \qquad (2)$$

Equating the gains from Eqs. (1) and (2), we get

$$I = 2I_D$$

That is, the differential pair must be biased at a current twice that of the CS amplifier. Since both circuits use equal power supplies, the power dissipation of the differential pair will be twice that of the CS amplifier.

9.18 Since both circuits use the same supply voltages and dissipate equal powers, then their currents must be equal, that is,

$$I_D = I$$

where I_D is the bias current of the CS amplifier and I is the bias current of the differential pair. The gain of the CS amplifier is

$$|A| = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{CS}} I_D}$$

Thus,

$$|A| = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{CS}} I_D} R_D \qquad (1)$$

The gain of the differential amplifier is

$$A_d = g_m R_D$$

where

$$g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right)}$$

Thus,

$$A_d = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{diff}} \left(\frac{I}{2}\right)} R_D \qquad (2)$$

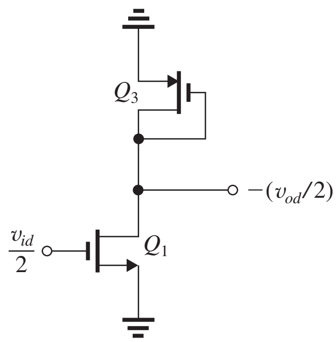
Equating the gains in Eqs. (1) and (2) and substituting $I_D = I$ gives

$$\sqrt{\left(\frac{W}{L}\right)_{\text{CS}}} = \sqrt{\left(\frac{W}{L}\right)_{\text{diff}} \times \frac{1}{2}}$$

$$\Rightarrow \left(\frac{W}{L}\right)_{\text{diff}} = 2\left(\frac{W}{L}\right)_{\text{CS}}$$

If all transistors have the same channel length, each of the differential pair transistors must be twice as wide as the transistor in the CS amplifier.

9.19



(a) The figure shows the differential half-circuit. Recalling that the incremental (small-signal) resistance of a diode-connected transistor is given by $\left(\frac{1}{g_m} \parallel r_o\right)$, the equivalent load resistance of Q_1 will be

$$R_D = \frac{1}{g_{m3}} \parallel r_{o3}$$

and the differential gain of the amplifier in Fig. P9.19 will be

$$A_d \equiv \frac{v_{od}}{v_{id}} = g_{m1}\left[\frac{1}{g_{m3}} \parallel r_{o3} \parallel r_{o1}\right]$$

Since both sides of the amplifier are matched, this expression can be written in a more general way as

$$A_d = g_{m1,2}\left[\frac{1}{g_{m3,4}} \parallel r_{o3,4} \parallel r_{o1,2}\right]$$

(b) Neglecting $r_{o1,2}$ and $r_{o3,4}$ (much larger that $1/g_{m3,4}$),

$$A_d \simeq \frac{g_{m1,2}}{g_{m3,4}}$$

$$= \frac{\sqrt{2\mu_n C_{ox}(W/L)_{1,2}(I/2)}}{\sqrt{2\mu_p C_{ox}(W/L)_{3,4}(I/2)}}$$

$$= \sqrt{\frac{\mu_n (W/L)_{1,2}}{\mu_p (W/L)_{3,4}}}$$

(c) $\mu_n = 4\mu_p$ and all channel lengths are equal,

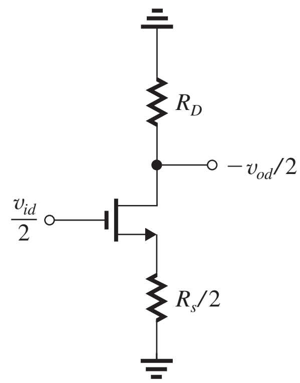
$$A_d = 2\sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

For $A_d = 10$,

$$10 = 2\sqrt{\frac{W_{1,2}}{W_{3,4}}}$$

$$\Rightarrow \frac{W_{1,2}}{W_{3,4}} = 25$$

9.20



From symmetry, a virtual ground appears at the mid point of R_s. Thus, the differential half circuit will be as shown in the figure, and

$$A_d \equiv \frac{v_{od}}{v_{id}} = \frac{R_D}{\frac{1}{g_m} + \frac{R_s}{2}}$$

For $R_s = 0$,

$$A_d = \frac{R_D}{1/g_m} = g_m R_D,$$

as expected.

To reduce the gain to half this value, we use

$$\frac{R_s}{2} = \frac{1}{g_m}$$

$$\Rightarrow R_s = \frac{2}{g_m}$$

9.21 Refer to Fig. P9.21

(a) With $v_{G1} = v_{G2} = 0$,

$$v_{GS1} = v_{GS2} = V_{OV1,2} + V_{tn}$$

Thus

$$V_{S1} = V_{S2} = -(V_{OV1,2} + V_{tn})$$

(b) For the situation in (a), V_{DS} of Q_3 is zero, thus zero current flows in Q_3. Transistor Q_3 will have an overdrive voltage of

$$V_{OV3} = V_C - V_{S1,2} - V_{tn}$$

$$= V_C + (V_{OV1,2} + V_{tn}) - V_{tn}$$

$$= V_C + V_{OV1,2}$$

(c) With $v_{G1} = v_{id}/2$ and $v_{G2} = -v_{id}/2$ where v_{id} is a small signal, a small signal will appear between drain and source of Q_3. Transistor Q_3 will be operating in the triode region and its drain-source resistance r_{DS} will be given by

$$r_{DS} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3 V_{OV3}}$$

Thus,

$$R_s = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3 V_{OV3}}$$

Now,

$$g_{m1,2} = (\mu_n C_{ox})\left(\frac{W}{L}\right)_{1,2} V_{OV1,2}$$

$$g_{m3} = (\mu_n C_{ox})\left(\frac{W}{L}\right)_3 V_{OV3}$$

For $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_{1,2}$,

$$\mu_n C_{ox}\left(\frac{W}{L}\right) = \frac{g_{m1,2}}{V_{OV1,2}}$$

Thus,

$$R_s = \frac{1}{\frac{g_{m1,2}}{V_{OV1,2}} \times V_{OV3}} = \frac{1}{g_{m1,2}}\frac{V_{OV1,2}}{V_{OV3}}$$

(d) (i) $R_s = \dfrac{1}{g_{m1,2}}$

$V_{OV3} = V_{OV1,2}$

But

$V_{OV3} = V_C + V_{OV1,2}$

$\Rightarrow V_C = 0$

(ii) $R_s = \dfrac{0.5}{g_{m1,2}}$

$\Rightarrow V_{OV3} = 2\ V_{OV1,2}$

But

$V_{OV3} = V_C + V_{OV1,2}$

$\Rightarrow V_C = V_{OV1,2}$

9.22 Refer to Fig. P9.22.

(a) With $v_{G1} = v_{G2} = 0$ V,

$V_{S1} = V_{S2} = -V_{GS1,2} = -(V_t + V_{OV})$

The current through Q_3 and Q_4 will be zero because the voltage across them $(v_{DS3} + v_{DS4})$ is zero.

Because the voltages at their gates are zero and at their sources are $-(V_t + V_{OV})$, each of Q_3 and Q_4 will be operating at an overdrive voltage equal to V_{OV}. Thus each of Q_3 and Q_4 will have an r_{DS} given by

$$r_{DS3,4} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4} V_{OV}} \quad (1)$$

Since

$$g_{m1,2} = \mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV} \quad (2)$$

substituting from (2) into (1) gives

$$r_{DS3,4} = \frac{1}{g_{m1,2}}\frac{(W/L)_{1,2}}{(W/L)_{3,4}}$$

and since

$R_s = r_{DS3} + r_{DS4}$

then

$$R_s = \frac{2}{g_{m1,2}}\frac{(W/L)_{1,2}}{(W/L)_{3,4}} \quad (3)$$

(b) With $v_{G1} = v_{id}/2$ and $v_{G2} = -v_{id}/2$ where v_{id} is a small signal,

$$A_d \equiv \frac{v_{od}}{v_{id}}$$

$$= \frac{2\,R_D}{\frac{1}{g_{m1}} + R_s + \frac{1}{g_{m2}}}$$

Using (3), we obtain

$$A_d = \frac{R_D}{\frac{1}{g_{m1,2}} + \frac{1}{g_{m1,2}}\frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

$$= \frac{g_{m1,2}R_D}{1 + \frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

9.23 Refer to Fig. P9.23.

The value of R is found as follows:

$$R = \frac{V_{G6} - V_{G7}}{I_{\text{REF}}}$$

$$= \frac{0.8 - (-0.8)}{0.2} = 8\ \text{k}\Omega$$

Since $I = I_{\text{REF}}$, Q_3 and Q_6 are matched and are operating at

$|V_{OV}| = 1.5 - 0.8 - 0.5 = 0.2$ V

Thus,

$$0.2 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{6,3} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_6 = 100$$

Each of Q_4 and Q_5 is conducting a dc current of $(I/2)$ while Q_7 is conducting a dc current $I_{\text{REF}} = I$. Thus Q_4 and Q_5 are matched and their W/L ratios are equal while Q_7 has twice the (W/L) ratio of Q_4 and Q_5. Thus,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{4,5} V_{OV4,5}^2$$

where

$$V_{OV4,5} = -0.8 - (-1.5) - 0.5 = 0.2 \text{ V}$$

thus,

$$0.1 = \frac{1}{2} \times 0.25 \times \left(\frac{W}{L}\right)_{4,5} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 20$$

and

$$\left(\frac{W}{L}\right)_7 = 40$$

$$r_{o4} = r_{o5} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o4,5})$$

$$50 = g_{m1,2}(100 \parallel 100)$$

$$\Rightarrow g_{m1,2} = 1 \text{ mA/V}$$

But

$$g_{m1,2} = \frac{2(I/2)}{|V_{OV1,2}|}$$

$$1 = \frac{0.2}{|V_{OV1,2}|}$$

$$\Rightarrow |V_{OV1,2}| = 0.2 \text{ V}$$

The (W/L) ratio for Q_1 and Q_2 can now be determined from

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{1,2} \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 50$$

A summary of the results is provided in the table below.

Transistor	***W/L***	**I_D(mA)**	**$\vert V_{GS}\vert$(V)**
Q_1	50	0.1	0.7
Q_2	50	0.1	0.7
Q_3	100	0.2	0.7
Q_4	20	0.1	0.7
Q_5	20	0.1	0.7
Q_6	100	0.2	0.7
Q_7	40	0.2	0.7

9.24 Refer to Fig. P9.24.

(a) Since the dc voltages V_{GS1} and V_{GS2} are equal, Q_1 and Q_2 will be operating at the same value of V_{OV} and their dc currents I_{D1} and I_{D2} will have the same ratio at their (W/L) ratios, that is,

$$I_{D1} = I/3$$

$$I_{D2} = 2I/3$$

(b) Q_1 and Q_2 will be operating at the same V_{OV}, obtained as follows:

$$\frac{I}{3} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$\Rightarrow V_{OV} = \sqrt{\frac{2I}{3\mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

(c) $A_d \equiv \dfrac{v_{od}}{v_{id}}$

$$= \frac{2R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$$

where

$$g_{m1} = \frac{2 \times (I/3)}{V_{OV}} = \frac{2I}{3V_{OV}}$$

$$g_{m2} = \frac{2 \times (2I/3)}{V_{OV}} = \frac{4I}{3V_{OV}}$$

$$A_d = \frac{2R_D}{\left(\dfrac{3}{2} + \dfrac{3}{4}\right)(V_{OV}/I)} = \frac{8}{9}\frac{IR_D}{V_{OV}}$$

9.25 Refer to Fig. 9.13.

All transistors have the same channel length and are carrying a dc current $I/2$. Thus all transistors have the same $r_o = \dfrac{|V_A|}{I/2}$. Also, all transistors are operating at the same $|V_{OV}|$ and have equal dc currents, thus all have the same $g_m = \dfrac{2(I/2)}{|V_{OV}|} = I/|V_{OV}|$. Thus all transistors have equal intrinsic gain $g_m r_o = 2|V_A|/|V_{OV}|$. Now, the gain A_d is given by

$$A_d = g_m(R_{on} \parallel R_{op})$$

$$= \frac{1}{2}g_m R_{on}$$

$$= \frac{1}{2}g_m(g_m r_o)r_o = \frac{1}{2}(g_m r_o)^2$$

Thus,

$$A_d = \frac{1}{2}\left[\frac{2|V_A|}{V_{OV}}\right]^2$$

$$= 2(|V_A|/|V_{OV}|)^2 \qquad \text{Q.E.D.}$$

To obtain $A_d = 500$ V/V while operating all transistors at $|V_{OV}| = 0.2$ V, we use

$$500 = 2\frac{|V_A|^2}{0.04}$$

$$\Rightarrow |V_A| = 3.16 \text{ V}$$

Since $|V_A'| = 5$ V/µm, the channel length L (for all transistors) must be

$$3.16 = 5 \times L$$

$$L = 0.632 \text{ µm}$$

To obtain the highest possible g_m, we operate at the highest possible I consistent with limiting the power dissipation (in equilibrium) to 0.5 mW. Thus,

$$I = \frac{0.5 \text{ mW}}{(0.9 + 0.9)\text{V}} = 0.28 \text{ mA}$$

9.26 Refer to Fig. 9.15(a).

The current I will split equally between Q_1 and Q_2. Thus

$$I_{E1} = I_{E2} = 0.2 \text{ mA}$$

$$I_{C1} = I_{C2} = \alpha \times 0.2 = 0.99 \times 0.2 = 0.198 \text{ mA}$$

$$V_{BE1} = V_{BE2} = 0.7 + 0.025 \ln\left(\frac{0.198}{1}\right)$$

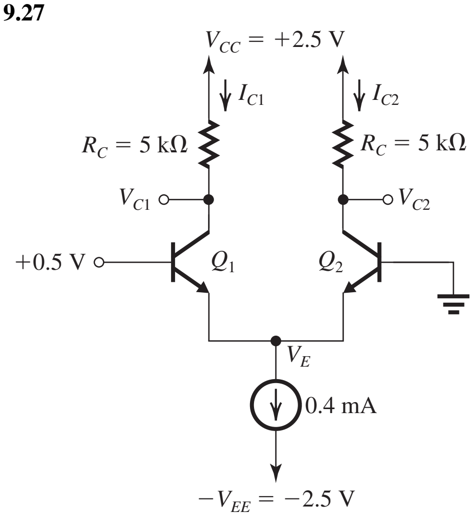
$$= 0.660 \text{ V}$$

$$V_{E1} = V_{E2} = -1 - 0.66 = -1.66 \text{ V}$$

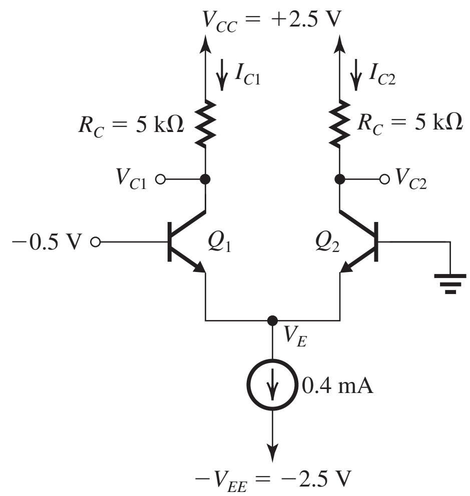
$$V_{C1} = V_{C2} = V_{CC} - I_{C1,2}\, R_C$$

$$= 2.5 - 0.198 \times 5 = +1.51 \text{ V}$$

9.27



(a)



(b)

(a) For $v_{B1} = +0.5$ V, Q_1 conducts all the current I (0.4 mA) while Q_2 cuts off. Thus Q_1 will have a V_{BE} obtained as follows:

$$V_{BE1} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.4}{1}\right)$$

$$= 0.677 \text{ V}$$

Thus,

$$V_E = +0.5 - 0.677 = -0.177 \text{ V}$$

which indicates that $V_{BE2} = +0.177$ V, too small to turn Q_2 on.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0.99 \times 0.4 \times 5$$

$$= +0.52 \text{ V}$$

$$V_{C2} = V_{CC} - I_{C2} \times R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

Observe that Q_1 is operating in the active mode, as implicitly assumed, and the current source has a voltage of 2.323 V across it, more than sufficient for its proper operation.

(b) With $v_{B1} = -0.5$ V, Q_1 turns off and Q_2 conducts all the bias current (0.4 mA) and thus exhibits a V_{BE} of 0.677 V, thus

$$V_E = -0.677 \text{ V}$$

which indicated that $V_{BE1} = +0.177$ V, which is too small to turn Q_1 on. Also, note that the current source has a voltage of $-0.677 + 2.5 = 1.823$ V across it, more than sufficient for its proper operation.

$$V_{C1} = V_{CC} - I_{C1}R_C$$

$$= 2.5 - 0 \times 5 = 2.5 \text{ V}$$

$$V_{C2} = 2.5 - 0.99 \times 0.4 \times 5 = +0.52 \text{ V}$$

9.28 Refer to Fig. 9.15(a) and assume the current source I is implemented with a single BJT that requires a minimum of 0.3 V for proper operation. Thus, the minimum voltage allowed at the emitters of Q_1 and Q_2 is $-2.5\text{ V} + 0.3\text{ V} = -2.2$ V. Now, since each of Q_1 and Q_2 is conducting a current of 0.2 mA, their V_{BE} voltages will be equal:

$$V_{BE1,2} = 0.7 + 0.025 \ln\left(\frac{0.99 \times 0.2}{1}\right)$$

$= 0.660$ V

Thus, the minimum allowable V_{CM} is

$V_{CM\,\text{min}} = -2.2 + 0.660 = -1.54$ V

The upper limit on V_{CM} is dictated by the need to keep Q_1 and Q_2 operating in the active mode, thus

$V_{CM\,\text{max}} = 0.4 + V_{C1,2}$

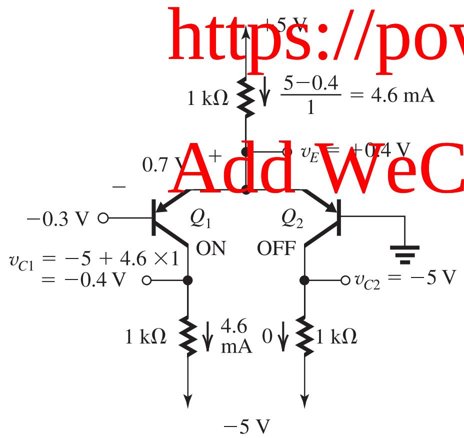
$= 0.4 + (2.5 - 0.99 \times 0.2 \times 5)$

$= +1.91$ V

Thus, the input common-mode range is

$-1.54\text{ V} \le V_{ICM} \le 1.91$ V

9.29



The solution is given on the circuit diagram.

9.30 (a) Refer to Fig. 9.15(a).

$$I_{E1} = I_{E2} = \frac{I}{2} = 10\ \mu\text{A}$$

$I_{C1} = I_{C2} = \alpha \times 10 = 0.98 \times 10 = 9.8\ \mu$A

$$V_{BE1} = V_{BE2} = 0.690 + 0.025 \ln\left(\frac{9.8 \times 10^{-3}}{1}\right)$$

$= 0.574$ V

Thus,

$V_E = -0.574$ V

$V_{C1} = V_{C2} = V_{CC} - I_C R_C$

$= 1.2 - 9.8 \times 10^{-3} \times 82$

$\simeq 0.4$ V

(b) Refer to Fig. 9.15(a).

The maximum value of V_{CM} is limited by the need to keep Q_1 and Q_2 in the active mode. This is achieved by keeping $v_{CE1,2} \ge 0.3$ V.

Since $V_{C1,2} = 0.4$ V,

$V_{E\text{max}} = 0.4 - 0.3 = 0.1$ V

and

$V_{CM\,\text{max}} = V_{BE1,2} + V_{E\text{max}}$

$V_{CM\,\text{max}} = 0.574 + 0.1 = 0.674$ V

The minimum value of V_{CM} is dictated by the need to keep the current source operating properly, i.e. to keep 0.3 V across it, thus

$V_{E\text{min}} = -1.2 + 0.3 = -0.9$ V

and

$V_{CM\,\text{min}} = V_{E\text{min}} + V_{BE1,2}$

$= -0.9 + 0.574 = -0.326$ V

Thus, the input common-mode range is

$-0.326\text{ V} \le V_{ICM} \le +0.674$ V

(c) Refer to Fig. 9.15(d).

$i_{E1} = 11\ \mu$A, $\quad i_{E2} = 9\ \mu$A

$i_{C1} = 10.78\ \mu$A, $\quad i_{C2} = 8.82\ \mu$A

$$v_{BE1} = 0.69 + 0.025 \ln\left(\frac{10.78 \times 10^{-3}}{1}\right)$$

$= 0.5767$ V

$$v_{BE2} = 0.69 + 0.025 \ln\left(\frac{8.82 \times 10^{-3}}{1}\right)$$

$= 0.5717$ V

Thus,

$v_{B1} = v_{BE1} - v_{BE2}$

$= 0.5767 - 0.5717 = 0.005$ V

$= 5$ mV

9.31 Refer to Fig. 9.15(a) with V_{CC} replaced by $(V_{CC} + v_r)$.

$$v_{C1} = (V_{CC} + v_r) - \alpha \frac{I}{2} R_C$$

$$= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r$$

$$v_{C2} = (V_{CC} + v_r) - \alpha \frac{I}{2} R_C$$

$$= (V_{CC} - \alpha \frac{I}{2} R_C) + v_r$$

$v_{od} \equiv v_{C2} - v_{C1} = 0$

Thus, while v_{C1} and v_{C2} will include a ripple component v_r, the difference output voltage v_{od} will be ripple free. Thus, the differential amplifier rejects the undesirable supply ripple.

9.32 Refer to Fig. 9.14.

(a) $V_{CM\max} = V_{CC} - \dfrac{I}{2}R_C$

(b) For $V_{CC} = 2$ V and $V_{CM\max} = 1$ V,

$1 = 2 - \dfrac{1}{2}(IR_C)$

$\Rightarrow IR_C = 2$ V

(c) $I_B = \dfrac{I/2}{\beta + 1} \leq 2\ \mu\text{A}$

$I \leq 2 \times 101 \times 2 = 404\ \mu\text{A}$

Select

$I = 0.4$ mA

then

$R_C = \dfrac{2}{0.4} = 5\ \text{k}\Omega$

9.33 $\dfrac{\Delta i_{E1}}{I} = \dfrac{i_{E1} - (I/2)}{I}$

$= \dfrac{i_{E1}}{I} - 0.5$

Using Eq. (9.48), we obtain

$\dfrac{\Delta i_{E1}}{I} = \dfrac{1}{1 + e^{-v_{id}/V_T}} - 0.5$

Observe that for $v_{id} < 10$ mV the proportional transconductance gain is nearly constant at about 10. The gain decreases as v_{id} further increases, indicating nonlinear operation. This is especially pronounced for $v_{id} > 20$ mV.

9.34 Require $v_{od} = 1$ V when $v_{id} = 10$ mV and $I = 1$ mA.

Using Eq. (9.48), we obtain

$i_{E1} = \dfrac{1\ (\text{mA})}{1 + e^{-10/25}} = 0.599$ mA

$i_{E2} = I - i_{E1} = 1 - 0.599 = 0.401$ mA

$v_{od} = v_{C2} - v_{C1}$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

$= (i_{C1} - i_{C2})R_C$

$\simeq (i_{E1} - i_{E2})R_C$

$= 0.198R_C$

For $v_{od} = 1$ V, we have

$R_C = \dfrac{1}{0.198} = 5.05\ \text{k}\Omega$

$V_{C1} = V_{C2} = V_{CC} - \dfrac{I}{2}R_C$

$= 5 - 0.5 \times 5.05 \simeq 2.5$ V

With a signal of 10 mV applied, the voltage at one collector rises to 3 V and at the other falls to 2 V. To ensure that the transistors remain in the active region, the maximum common-mode input voltage must be limited to $(2 - 0.4) = +1.6$ V.

9.35 Refer to Fig. 9.14.

$v_{od} = v_{C2} - v_{C1}$

$= (V_{CC} - i_{C2}R_C) - (V_{CC} - i_{C1}R_C)$

$= R_C(i_{C1} - i_{C2})$

Using Eqs. (9.48) and (9.49) and assuming $\alpha \simeq 1$, so that $i_{C1} \simeq i_{E1}$ and $i_{C2} \simeq i_{E2}$, we get

$v_{od} = IR_C\left[\dfrac{1}{1 + e^{-v_{id}/V_T}} - \dfrac{1}{1 + e^{v_{id}/V_T}}\right]$

$= 5\left[\dfrac{1}{1 + e^{-v_{id}/V_T}} - \dfrac{1}{1 + e^{v_{id}/V_T}}\right]$

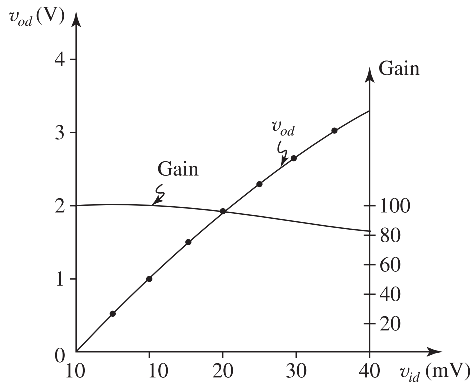
This relationship can be used to obtain the data in the table below.

This table belongs to Problem 9.33.

v_{id} (mV)	2	5	8	10	20	30	40
$\left[\dfrac{\Delta i_{E1}}{I}/v_{id}\right]$ (V^{-1})	9.99	9.97	9.92	9.87	9.50	8.95	8.30

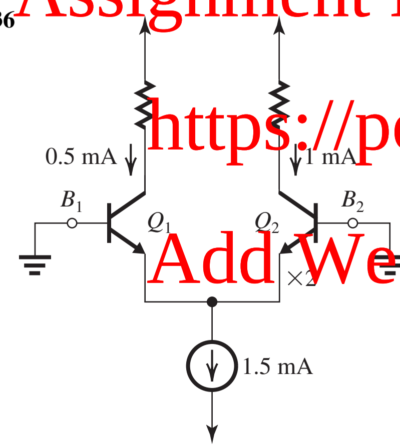
This table belongs to Problem 9.35.

v_{id} (mV)	2	5	10	15	20	25	30	35	40
v_{od} (V)	0.2	0.498	0.987	1.457	1.90	2.311	2.685	3.022	3.320
Gain $= \dfrac{v_{od}}{v_{id}}$	100	99.7	98.7	97.1	95.0	92.4	89.5	86.3	83.0



The figure shows v_{od} versus v_{id} and the gain versus v_{id}. Observe that the transfer characteristic is nearly linear and the gain is nearly constant for $v_{id} \leq 10$ mV. As v_{id} increases, the transfer characteristic bends and the gain is reduced. However, for v_{id} even as large as 20 mV, the gain is only 5% below its ideal value of 100.

9.36



Since Q_2 has twice the EBJ area of Q_1, the 1.5-mA bias current will split in the same ratio, that is,

$i_{E2} = 2\, i_{E2}$

Thus,

$i_{E2} = 1$ mA and $i_{E1} = 0.5$ mA

To equalize the collector currents, we apply a signal

$v_{id} = v_{B1} - v_{B2}$

Now,

$$i_{C1} = \frac{I_S}{\alpha} e^{(v_{B1} - v_E)/V_T} \quad (1)$$

$$i_{C2} = \frac{2I_S}{\alpha} e^{(v_{B2} - v_E)/V_T} \quad (2)$$

where we have denoted the scale current of Q_1 by I_S and that of Q_2 as $2I_S$. Dividing (1) by (2), we get

$$\frac{i_{C1}}{i_{C2}} = \frac{1}{2} e^{(v_{B1} - v_{B2})/V_T}$$

For $i_{C1} = i_{C2}$, we obtain

$v_{B1} - v_{B2} = V_T \ln 2$

$= 25 \ln 2 = 17.3$ mV

9.37 (a) $V_{BE} = 0.69 + 0.025 \ln\left(\frac{0.1}{1}\right)$

$= 0.632$ V

(b) Using Eq. (9.48), we obtain

$$i_{C1} = \alpha i_{E1} \simeq \frac{I}{1 + e^{-v_{id}/V_T}}$$

For $v_{id} = 20$ mV,

$$i_{C1} = \frac{200\ \mu\text{A}}{1 + e^{-20/25}} = 138\ \mu\text{A}$$

$i_{C2} = 200 - 138 = 62\ \mu$A

(c) For $v_{id} = 200$ mV while $i_{C1} = 138\ \mu$A and $i_{C2} = 62\ \mu$A: Since i_{C1} and i_{C2} have not changed, v_{BE1} and v_{BE2} also would not change. Thus,

$v_{B1} - v_{B2} = v_{BE1} + i_{E1}R_e - i_{E2}R_e - v_{BE2}$

$= (v_{BE1} - v_{BE2}) + R_e\,(i_{E1} - i_{E2})$

$200 = 20 + R_e\,(i_{C1} - i_{C2})$

$= 20 + R_e\,(138 - 62)$

$$\Rightarrow R_e = \frac{180\ \text{mV}}{76\ \mu\text{A}} = 2.37\ \text{k}\Omega$$

(d) Without R_e,

$v_{id} = 20$ mV $\rightarrow i_{C1} - i_{C2} = 76\ \mu$A

$$G_m = \frac{76\ \mu\text{A}}{20\ \text{mV}} = 3.8\ \text{mA/V}$$

With R_e,

$v_{id} = 200$ mV $\rightarrow i_{C1} - i_{C2} = 76\ \mu$A

$$G_m = \frac{76\ \mu\text{A}}{200\ \text{mV}} = 0.38\ \text{mA/V}$$

Thus, the effective G_m has been reduced by a factor of 10, which is the same factor by which the allowable input signal has been increased while maintaining the same linearity.

9.38 $g_m = \dfrac{I_C}{V_T} = \dfrac{\alpha \times 0.2}{0.025} \simeq 8$ mA/V

$$R_{id} = 2r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{160}{8} = 40\ \text{k}\Omega$$

9.39 $R_{id} = 2r_\pi = 20\ \text{k}\Omega$

$r_\pi = 10\ \text{k}\Omega$

$\dfrac{\beta}{g_m} = 10\ \text{k}\Omega$

$\dfrac{100}{g_m} = 10$

$\Rightarrow g_m = 10\ \text{mA/V}$

$A_d = 100 = g_m R_C$

$R_C = \dfrac{100}{g_m} = \dfrac{100}{10} = 10\ \text{k}\Omega$

$g_m = \dfrac{I_C}{V_T} \simeq \dfrac{I/2}{V_T}$

$\Rightarrow I = 2V_T g_m$

$= 2 \times 0.025 \times 10 = 0.5\ \text{mA}$

9.40 $v_{id} = 10\ \text{mA/V}$

Input signal to half-circuit = 5 mV. For $I = 200\ \mu\text{A}$, the bias current of the half-circuit is $100\ \mu\text{A}$ and,

$r_e = \dfrac{25\ \text{mV}}{0.1\ \text{mA}} = 250\ \Omega$

Gain of half-circuit $= -\dfrac{R_C}{r_e} = -\dfrac{10}{0.25}$

$= -40\ \text{V/V}$

At each collector we expect a signal of $40 \times 5\ \text{mV} = 200\ \text{mV}$. Between the two collectors, the signal will be 400 mV.

9.41 (a) $r_e = \dfrac{25\ \text{mV}}{0.25\ \text{mA}} = 100\ \Omega$

The 0.1-V differential input signal appears across $(2r_e + 2R_e)$, thus

$i_e = \dfrac{100\ \text{mV}}{200 + 2 \times 400} = 0.1\ \text{mA}$

$v_{be} = 0.1 \times 100 = 10\ \text{mV}$

(b) The total emitter current in one transistor is

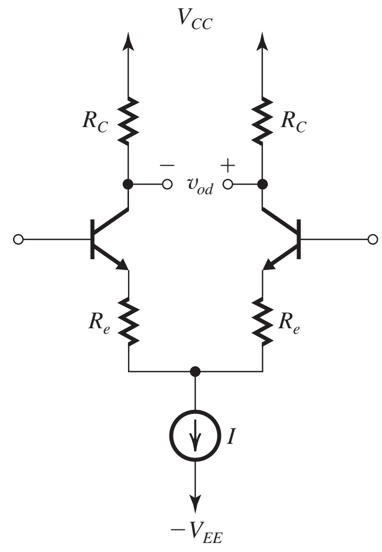
$\dfrac{I}{2} + i_e = 0.35\ \text{mA}$ and in the other transistor

$\dfrac{I}{2} - i_e = 0.15\ \text{mA}$.

(c) At one collector the signal voltage is $-\alpha i_e R_C \simeq -i_e R_C = -0.1 \times 10 = -1\ \text{V}$ and at the other collector the signal voltage is +1 V.

(d) Voltage gain $= \dfrac{2\ \text{V}}{0.1\ \text{V}} = 20\ \text{V/V}$

9.42



$v_{id} = 100\ \text{mV}$ appears across $(2\ r_e + 2\ R_e)$. Thus the signal across $(r_e + R_e)$ is 50 mV. Since the signal across r_e is 5 mV, it follows that the signal across R_e must be $50 - 5 = 45$ mV and thus

$R_e = 9r_e$

The input resistance R_{id} is

$R_{id} = (\beta + 1)(2r_e + 2R_e)$

$= 2(100 + 1)\ (r_e + R_e)$

$= 2 \times 101 \times (r_e + 9r_e)$

$= 2 \times 101 \times 10r_e$

To obtain $R_{id} = 100\ \text{k}\Omega$,

$100 = 2 \times 101 \times 10 \times r_e$

$\Rightarrow r_e \simeq 50\ \Omega$

Since

$r_e = \dfrac{V_T}{I_E}$,

$50 = \dfrac{25\ \text{mV}}{I_E}$

$\Rightarrow I_E = 0.5\ \text{mA}$

$I = 1\ \text{mA}$

$R_e = 9r_e = 9 \times 50 = 450\ \Omega$

Gain $= \dfrac{\alpha \times 2R_C}{2r_e + 2R_e}$

$\simeq \dfrac{R_C}{r_e + R_e}$

But the gain required is

Gain $= \dfrac{v_{od}}{v_{id}} = \dfrac{2\ \text{V}}{0.1\ \text{V}} = 20\ \text{V/V}$

Thus,

$$20 = \frac{R_C}{0.05 + 0.45}$$

$$\Rightarrow R_C = 10 \text{ k}\Omega$$

The determination of a suitable value of V_{CC} requires information on the required input common-mode range (which is not specified). Suffice it to say that the dc voltage drop across R_C is 5 V and that each collector swings ± 1 V. A supply voltage $V_{CC} = 10$ V will certainly be sufficient.

9.43 (a) The maximum allowable value of the bias current I is found as

$$I = \frac{P}{(V_{CC} + V_{EE})} = \frac{1 \text{ mW}}{5 \text{ V}} = 0.2 \text{ mA}$$

We choose to operate at this value of I. Thus

$$g_m = \frac{I_C}{V_T} = \frac{\alpha(0.2/2)}{0.025} \simeq 4 \text{ mA/V}$$

$$A_d = g_m R_C$$

$$60 = 4 \times R_C$$

$$\Rightarrow R_C = 15 \text{ k}\Omega$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 2.5 - \frac{0.2}{2} \times 15$$

$$= +1 \text{ V}$$

(b) $R_{id} = 2r_\pi = 2\frac{\beta}{g_m}$

$$= 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

(c) $v_{od} = A_d \times v_{id}$

$$= 60 \times 10 = 600 \text{ mV} = 0.6 \text{ V}$$

Thus, there will be ± 0.3 V signal swing at each collector. That is, the voltage at each collector will range between 0.7 V and +1.3 V.

(d) To maintain the BJT in the active mode at all times, the maximum allowable V_{CM} is limited to

$$V_{CM\max} = 0.4 + v_{C\min}$$

$$= 0.4 + 0.7 = 1.1 \text{ V}$$

9.44 (a) Consider transistor Q_1,

$$v_{C1\min} = (V_{CC} - \frac{I}{2} R_C) - A_d\left(\frac{\hat{v}_{id}}{2}\right) \qquad (1)$$

where

$$A_d = g_m R_C \simeq \frac{I/2}{V_T} R_C$$

$$= \frac{I R_C}{2V_T}$$

Thus,

$$\frac{I R_C}{2} = A_d V_T \qquad (2)$$

Substituting from (2) into (1), we obtain

$$v_{C1\min} = V_{CC} - A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right) \qquad (3)$$

Since

$$v_{B1} = V_{CM\max} + \frac{\hat{v}_{id}}{2}$$

to keep Q_1 in the active mode,

$$v_{B1} \le 0.4 + v_{C1\min}$$

Thus,

$$V_{CM\max} + \frac{\hat{v}_{id}}{2} = 0.4 + V_{CC} - A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right)$$

$$\Rightarrow V_{CM\max} = V_{CC} + 0.4 - \frac{\hat{v}_{id}}{2} -$$

$$A_d\left(V_T + \frac{\hat{v}_{id}}{2}\right) \qquad \text{Q.E.D.} \qquad (4)$$

(b) $V_{CC} = 2.5$ V, $\hat{v}_{id} = 10$ mV,

$A_d = 50$ V/V,

$$V_{CM\max} = 2.5 + 0.4 - 0.005 - 50(25 + 5) \times 10^{-3}$$

$$\simeq 1.4 \text{ V}$$

$$\hat{v}_{od} = A_d \times \hat{v}_{id} = 50 \times 10 = 500 \text{ mV}$$

$$= 0.5 \text{ V}$$

Using Eq. (2), we obtain

$$I R_C = 2 A_d V_T = 2 \times 50 \times 0.025$$

$$= 2.5 \text{ V}$$

To limit the power dissipation in the quiescent state to 1 mV, the bias current must be limited to

$$I = \frac{P_{\max}}{V_{CC} + V_{EE}} = \frac{1}{5} = 0.2 \text{ mA}$$

Using this value for I, we get

$$R_C = \frac{2.5}{0.2} = 12.5 \text{ k}\Omega$$

(c) To obtain $V_{CM\max} = 1$ V, we use Eq. (4) to determine the allowable value of A_d,

$$1 = 2.5 + 0.4 - 0.005 - A_d(25 + 5) \times 10^{-3}$$

$$\Rightarrow A_d = 63.2 \text{ V/V}$$

Thus, by reducing $V_{CM\max}$ from 1.4 V to 1 V, we are able to increase the differential gain from 50 V/V to 63.2 V/V.

9.45 $A_d = g_m R_C$

$$= \frac{I_C}{V_T} R_C$$

$$\simeq \frac{(I/2)}{V_T} R_C$$

$$= \frac{I R_C}{2V_T}$$

$$= \frac{4}{2 \times 0.025} = 80 \text{ V/V}$$

$$V_{C1} = V_{C2} = V_{CC} - \frac{I}{2} R_C$$

$$= 5 - 2 = 3 \text{ V}$$

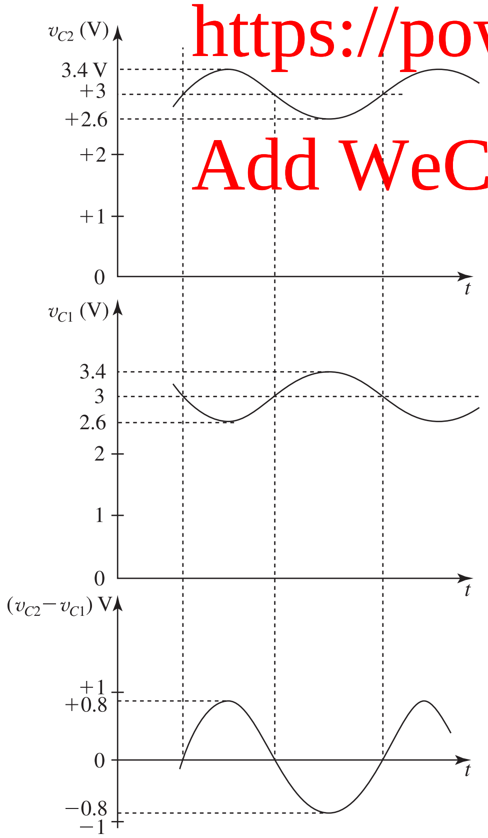
$$v_{C1} = 3 - 80 \times 0.005 \sin(\omega t)$$

$$= 3 - 0.4 \sin(\omega t)$$

$$v_{C2} = 3 + 0.4 \sin(\omega t)$$

$$v_{C2} - v_{C1} = 0.8 \sin(\omega t)$$

The waveforms are sketched in the figure below.



9.46 See figure on next page. The circuit together with its equivalent half-circuit are shown in the figure.

$$A_d = g_{m1,2}(r_{o1,2} \parallel r_{o3,4})$$

For

$$r_{o1,2} = r_{o3,4} = \frac{V_A}{\alpha(I/2)} \simeq \frac{2V_A}{I}$$

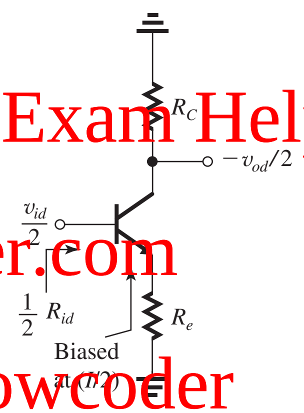
$$g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{I}{2V_T}$$

$$A_d = \frac{I}{2V_T}\left(\frac{2V_A}{I} \parallel \frac{2V_A}{I}\right)$$

$$= \frac{I}{2V_T} \times \frac{V_A}{I} = \frac{V_A}{2V_T}$$

$$= \frac{20}{2 \times 0.025} = 400 \text{ V/V}$$

9.47



Both circuits have the same differential half-circuit shown in the figure. Thus, for both

$$A_d = \frac{\alpha R_C}{r_e + R_e}$$

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

$$= 2(\beta + 1)(r_e + R_e)$$

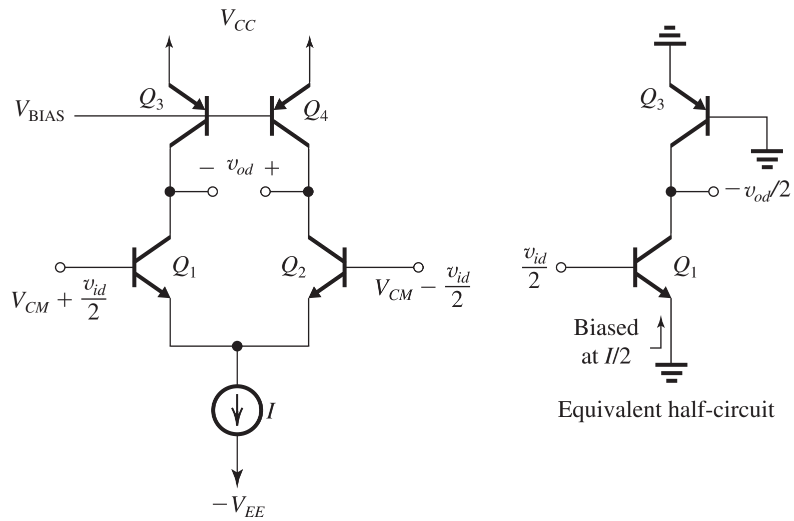
With $v_{id} = 0$, the dc voltage appearing at the top end of the bias current source will be

(a) $V_{CM} - V_{BE} - \left(\frac{I}{2}\right) R_C$

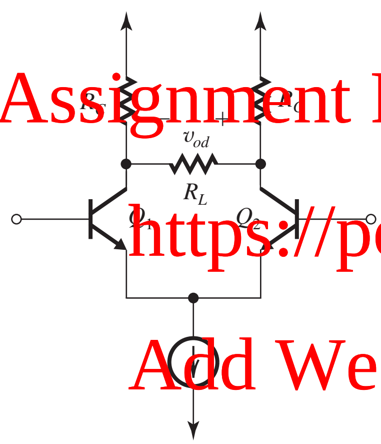
(b) $V_{CM} - V_{BE}$

Since circuit (b) results in a larger voltage across the current source and given that the minimum value of V_{CM} is limited by the need to keep a certain specified minimum voltage across the current source, we see that circuit (b) will allow a larger negative V_{CM}.

This figure belongs to Problem 9.46.



9.48



$$A_d = \alpha \frac{\text{Total resistance between collectors}}{\text{Total resistance in the emitter circuit}}$$

$$= \alpha \frac{(2R_C \parallel R_L)}{2r_e}$$

9.49 Refer to Fig. P9.47(a).

$$\frac{I}{2} R_e = 4V_T$$

$$\Rightarrow R_e = \frac{8V_T}{I} \qquad (1)$$

$$\alpha\left(\frac{I}{2}\right) R_C = 60V_T$$

$$R_C = \frac{120V_T}{\alpha I} \qquad (2)$$

$$A_d = \alpha \frac{\text{Total resistance in collector circuit}}{\text{Total resistance in emitter circuit}}$$

$$A_d = \alpha \frac{2R_C}{2r_e + 2R_e} = \alpha \frac{R_C}{r_e + R_e}$$

Substituting for R_C from (2), for R_e from (1), and for $r_e = V_T/(I/2)$, we obtain

$$A_d = \frac{\alpha(120V_T/\alpha I)}{(2V_T/I) + (8V_T/I)}$$

$$= \frac{120}{2+8} = 12 \text{ V/V}$$

9.50 $$\frac{v_{id}}{v_{\text{sig}}} = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \qquad (1)$$

where

$$R_{id} = (\beta + 1)(2r_e + 2R_e)$$

thus,

$$\frac{v_{id}}{v_{\text{sig}}} = \frac{2(\beta+1)(r_e + R_e)}{2(\beta+1)(r_e+R_e) + R_{\text{sig}}} \qquad (2)$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha \times \text{Total resistance between collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{2\alpha R_C}{2r_e + 2R_e}$$

$$\frac{v_{od}}{v_{id}} = \frac{\alpha R_C}{r_e + R_e} \qquad (3)$$

Using (2) and (3), we get

$$G_v \equiv \frac{v_{od}}{v_{\text{sig}}} = \frac{2\alpha(\beta+1)R_C}{2(\beta+1)(r_e+R_e) + R_{\text{sig}}}$$

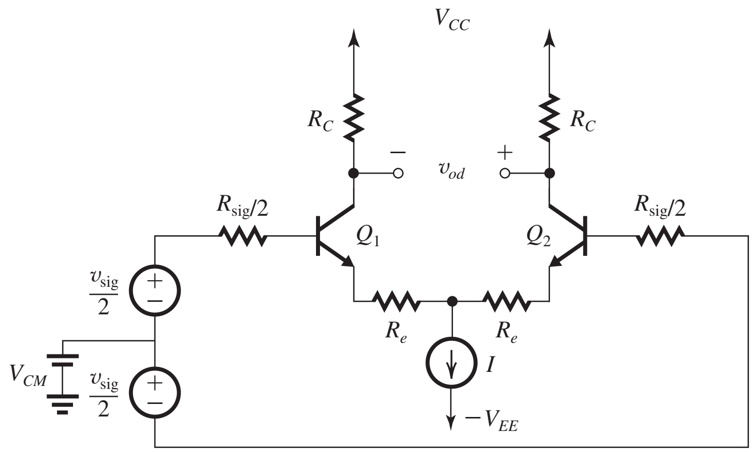
Since $\alpha = \dfrac{\beta}{\beta+1}$, $\alpha(\beta+1) = \beta$, we have

$$G_v = \frac{2\beta R_C}{2(\beta+1)(r_e+R_e) + R_{\text{sig}}} \qquad (4)$$

If $v_{id} = 0.5\ v_{\text{sig}}$, then from (1) we obtain

$$R_{id} = R_{\text{sig}}$$

This figure belongs to Problem 9.50.



Substituting for $R_{\text{sig}} = R_{id} = 2(\beta + 1)(r_e + R_e)$ into Eq. (4) gives

$$G_v = \frac{2\beta R_C}{4(\beta + 1)(r_e + R_e)} = \frac{1}{2}\frac{\alpha R_C}{r_e + R_e} \qquad (5)$$

If β is doubled to 2β while R_{sig} remains at its old value, we get

$$R_{\text{sig}} = 2(\beta + 1)(r_e + R_e) \qquad (6)$$

then the new value of G_v is obtained by replacing β by 2β in Eq. (4) and substituting for R_{sig} from (5):

$$G_v = \frac{4\beta R_C}{2(2\beta + 1)(r_e + R_e) + 2(\beta + 1)(r_e + R_e)}$$

$$\simeq \frac{4R_C}{6(r_e + R_e)} = \frac{2}{3}\frac{R_C}{r_e + R_e}$$

Thus the gain increases from approximately $\frac{1}{2}R_C/(r_e + R_e)$ to $\frac{2}{3}R_C/(r_e + R_e)$.

9.51 $R_{id} = 2r_\pi = 2\frac{\beta}{g_m}$

$$g_m = \frac{I_C}{V_T} \simeq \frac{0.2}{0.025} = 8 \text{ mA/V}$$

$$R_{id} = \frac{2 \times 100}{8} = 25 \text{ k}\Omega$$

$$G_v = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \frac{\alpha(2R_C \parallel R_L)}{2r_e}$$

$$G_v = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \times \frac{1}{2}\, g_m(2R_C \parallel R_L)$$

$$= \frac{25}{25 + 100} \times \frac{1}{2} \times 8 \times (40 \parallel 40)$$

$$= 16 \text{ V/V}$$

9.52 Refer to Fig. P9.52.

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25}{2r_e + 2 \times 0.25}$$

where

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

Thus,

$$\frac{v_o}{v_i} = \frac{0.99 \times 25}{2 \times 0.25 + 2 \times 0.25} \simeq 25 \text{ V/V}$$

$$R_{\text{in}} = (\beta + 1)(2r_e + 2R_e)$$

$$= 2 \times 101 \times (0.25 + 0.25)$$

$$= 101 \text{ k}\Omega$$

9.53 Refer to Fig. P9.53.

$$r_e = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{0.99 \times 25 \text{ k}\Omega}{2r_e + 500\ \Omega}$$

$$= \frac{0.99 \times 25 \text{ k}\Omega}{500\ \Omega + 500\ \Omega} \simeq 25 \text{ V/V}$$

$$R_{\text{in}} = (\beta + 1)(2r_e + 500\ \Omega)$$

$$= 101 \times (2 \times 250\ \Omega + 500\ \Omega)$$

$$= 101 \text{ k}\Omega$$

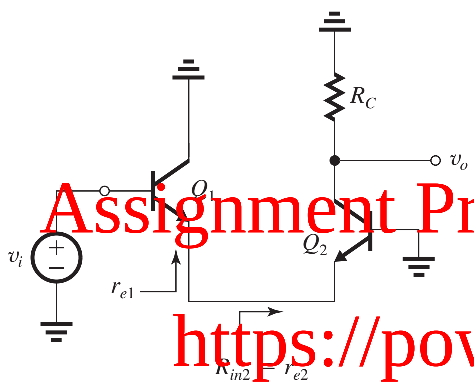
9.54 (a) Refer to the circuit in Fig. P9.54. As a differential amplifier, the voltage gain is found from

$$\frac{v_o}{v_i} = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times R_C}{2r_e}$$

$$= \frac{\alpha R_C}{2r_e}$$

(b) The circuit in Fig. P9.54 can be considered as the cascade connection of an emitter follower Q_1 (biased at an emitter current $I/2$) and a common-gate amplifier Q_2 (also biased at an emitter current of $I/2$). Referring to the figure below:



$$\frac{v_{e1,2}}{v_i} = \frac{r_{e2}}{r_{e1} + r_{e2}} = \frac{1}{2}$$

$$\frac{v_o}{v_{e1,2}} = \frac{\alpha R_C}{r_{e2}}$$

Thus,

$$\frac{v_o}{v_i} = \frac{1}{2} \times \frac{\alpha R_C}{r_{e2}} = \frac{\alpha R_C}{2r_e}$$

which is identical to the expression found in (a) above.

9.55 $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$

$= \sqrt{2 \times 3 \times 0.1} = 0.77$ mA/V

$|A_d| = g_m R_D = 0.77 \times 10 = 7.7$ V/V

$$|A_{cm}| = \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{10}{2 \times 100} \times 0.01 = 5 \times 10^{-4} \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 1.54 \times 10^4 \text{ or } 83.8 \text{ dB}$$

9.56 Refer to Fig. P9.2.

$$I_D = 0.25 \text{ mA} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV}|^2$$

$$0.25 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.353$ V

$$g_m = \frac{2I_D}{|V_{OV}|} = \frac{2 \times 0.25}{0.353} = 1.416 \text{ mA/V}$$

$|A_d| = g_m R_D = 1.416 \times 4 = 5.67$ V/V

$$|A_{cm}| = \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right)$$

$$= \frac{4}{2 \times 30} \times 0.02$$

$= 1.33 \times 10^{-3}$ V/V

CMRR = 4252.5 or 72.6 dB

9.57 Refer to Fig. P9.57

(a) Assume $v_{id} = 0$ and the two sides of the differential amplifier are matched. Thus,

$I_{D1} = I_{D2} = 0.5$ mA

$$I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$0.5 = \frac{1}{2} \times 2.5 \times V_{OV}^2$$

$\Rightarrow V_{OV} = 0.632$ V

$V_{CM} = V_{GS} + 1 \text{ mA} \times R_{SS}$

$= V_t + V_{OV} + 1 \times R_{SS}$

$= 0.7 + 0.632 + 1$

$= 2.332$ V

(b) $g_m = \dfrac{2I_D}{V_{OV}} = \dfrac{2 \times 0.5}{0.632} = 1.58$ mA/V

$A_d = g_m R_D$

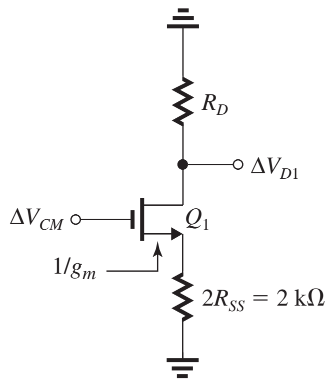
$8 = 1.38 \times R_D$

$\Rightarrow R_D = 5.06$ kΩ

(c) $V_{D1} = V_{D2} = V_{DD} - I_D R_D$

$= 5 - 0.5 \times 5.06 = 2.47$ V

(d)



The figure shows the common-mode half-circuit,

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{R_D}{\dfrac{1}{g_m} + 2\,R_{SS}}$$

$$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{5.06}{\dfrac{1}{1.58} + 2} = -1.92\ \text{V/V}$$

(e) For Q_1 and Q_2 to enter the triode region

$$V_{CM} + \Delta V_{CM} = V_t + V_{D1} + \Delta V_{D1}$$

Substituting $V_{CM} = 2.332$, $V_t = 0.7$ V, $V_{D1} = 2.47$ V, and $\Delta V_{D1} = -1.92\Delta V_{CM}$ results in

$$2.332 + \Delta V_{CM} = 0.7 + 2.47 - 1.92\Delta V_{CM}$$

$$\Rightarrow \Delta V_{CM} = 0.287\ \text{V}$$

With this change, $V_{CM} = 2.619$ V and $V_{D1,2} = 1.919$ V; thus $V_{CM} = V_t + V_{D1,2}$

9.58 The new deliberate mismatch $\Delta R_D/R_D$ cancels the two existing mismatch terms in the expression for A_{cm} given in the problem statement so as to reduce A_{cm} to zero. Thus,

$$\frac{R_D}{2R_{SS}} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\frac{5}{2 \times 25} \times \frac{\Delta R_D}{R_D} = -0.002$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = -0.02 \text{ or } -2\%$$

(Note the sign of the change is usually determined experimentally.)

9.59 $|A_{cm}| = \left(\dfrac{R_D}{2R_{SS}}\right)\ \dfrac{\Delta(W/L)}{W/L}$

$$|A_d| = g_m R_D$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = 2g_m R_{SS} \Big/ \frac{\Delta(W/L)}{W/L}$$

where

$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.1/2)}{0.2} = 0.5\ \text{mA/V}$$

For CMRR of 80 dB, the CMRR is 10^4; thus

$$10^4 = 2 \times 0.5 \times R_{SS}/0.02$$

$$R_{SS} = 200\ \text{k}\Omega$$

For the current source transistor to have $r_o = 200\ \text{k}\Omega$,

$$200 = \frac{V_A' \times L}{0.1\ \text{mA}}$$

$$L = \frac{200 \times 0.1}{5} = 4\ \mu\text{m}$$

9.60 It is required to raise the CMRR by 40 dB, that is, by a factor of 100. Thus, the cascoding of the bias current source must raise its output resistance R_{SS} by a factor of 100. Thus the cascode transistor must have $A_0 = 100$. Since

$$A_0 = g_m r_o = \frac{2I}{V_{OV}}\frac{V_A}{I} = \frac{2V_A}{V_{OV}}$$

$$100 = \frac{2V_A}{0.2}$$

$$\Rightarrow V_A = 10\ \text{V}$$

$$V_A = V_A' \times L$$

$$10 = 5 \times L$$

$$\Rightarrow L = 2\ \mu\text{m}$$

9.61 Refer to Fig. P9.61.

(a) $\dfrac{v_o}{v_{id}} =$

$$\alpha\ \frac{\text{Total resistance across which } v_o \text{ appears}}{\text{Total resistance in the emitter}}$$

$$= \alpha \times \frac{2\ \text{k}\Omega}{r_{e1} + r_{e2}}$$

To determine $r_{e1} = r_{e2} = r_e = \dfrac{V_T}{I_E}$, where I_E is the dc emitter current of each of Q_1 and Q_2, we use

$$V_E = V_B - V_{BE} = 0 - 0.7$$

$$= -0.7\ \text{V}$$

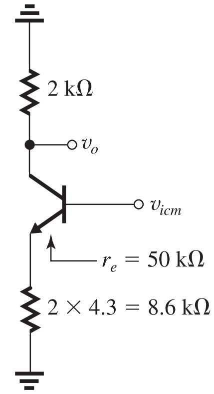
$$2I_E = \frac{-0.7 - (-5)}{4.3} = 1\ \text{mA}$$

$$I_E = 0.5\ \text{mA}$$

$$r_{e1} = r_{e2} = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$$\frac{v_o}{v_{id}} = \alpha \times \frac{2\ \text{k}\Omega}{0.1\ \text{k}\Omega} \simeq 20\ \text{V/V}$$

(b)



The common-mode half-circuit is shown in the figure,

$$\frac{v_o}{v_{icm}} = -\frac{\alpha \times 2\ \text{k}\Omega}{(0.05 + 8.6)\ \text{k}\Omega}$$

$\simeq -0.23$ V/V

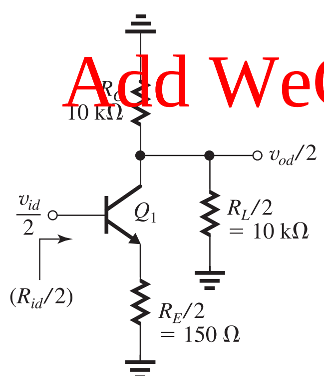
$$\left|\frac{v_o}{v_{icm}}\right| = 0.23\ \text{V/V}$$

(c) $\text{CMRR} = \frac{|v_o/v_{id}|}{|v_o/v_{icm}|} = \frac{20}{0.23} = 86.5$

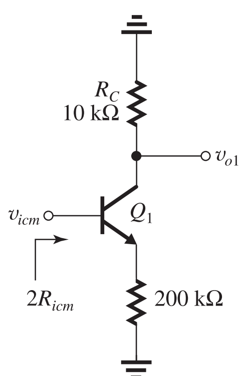
or 38.7 dB

(d) $v_o = -0.023 \sin 2\pi \times 60t$
$+ 0.2 \sin 2\pi \times 1000t$ volts

9.62



(a)



(b)

Figure (a) shows the differential half-circuit.

$I_E = 0.5$ mA, $\quad I_C = \alpha I_E \simeq 0.5$ mA

$$g_m = \frac{I_C}{V_T} = \frac{0.5\ \text{mA}}{0.025\ \text{V}} = 20\ \text{mA/V}$$

$$r_e = \frac{25\ \text{mV}}{0.5\ \text{mA}} = 50\ \Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100}{0.5} = 200\ \text{k}\Omega$$

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$\simeq \frac{10\ \text{k}\Omega \parallel 10\ \text{k}\Omega}{(50 + 150)\ \Omega}$$

$$= \frac{5}{0.2} = 25\ \text{V/V}$$

We have neglected r_o because its equivalent value at the output will be $r_o[1 + (R_e/r_e)] =$ $200[1 + (150/50)] = 800$ kΩ which is much greater than the effective load resistance of 5 kΩ.

$R_{id} = 2 \times (\beta + 1)(50\ \Omega + 150\ \Omega)$

$= 2 \times 101 \times 0.2\ (\text{k}\Omega) = 40.4\ \text{k}\Omega$

$$|A_{cm}| \simeq \left(\frac{R_C}{2R_{SS}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$|A_{cm}| = \frac{10}{200} \times 0.02 = 0.001\ \text{V/V}$$

To obtain R_{icm}, we use Eq. (9.96):

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

where $2R_{EE} = 200$ kΩ, thus $R_{EE} = 100$ kΩ and

$$R_{icm} = 100 \times 100 \frac{1 + (10/(100 \times 200))}{1 + \dfrac{10 + 200}{200}}$$

$= 4.88$ MΩ

9.63 (a) $g_m = \frac{I_C}{V_T} \simeq \frac{0.1\ \text{mA}}{0.025\ \text{V}} = 4$ mA/V

$A_d = g_m R_C = 4 \times 25 = 100$ V/V

(b) $R_{id} = 2r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{100}{4} = 50$ kΩ

(c) $|A_{cm}| = \left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$

$$= \frac{25}{2 \times 500} \times 0.01$$

$= 2.5 \times 10^{-4}$ V/V

(d) $\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{2.5 \times 10^{-4}} = 4 \times 10^5$

or 112 dB

(e) $r_o = \dfrac{V_A}{I_C} \simeq \dfrac{100}{0.1} = 1000\ \text{k}\Omega$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

$$= 100 \times 500 \frac{1 + (25/(100 \times 1000))}{1 + \dfrac{25 + 1000}{1000}}$$

$\simeq 25\ \text{M}\Omega$

9.64 $R_{EE} = \dfrac{V_A}{I} = \dfrac{20}{0.2} = 100\ \text{k}\Omega$

For the transistors in the differential pair, we have

$$r_o = \frac{V_A}{I/2} = \frac{20}{0.1} = 200\ \text{k}\Omega$$

$$R_{icm} \simeq \beta R_{EE} \frac{1 + (R_C/\beta r_o)}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

For $R_C \ll r_o$,

$$R_{icm} \simeq \beta R_{EE} \bigg/ \left(1 + \frac{2R_{EE}}{r_o}\right)$$

$$= \frac{50 \times 100}{1 + \dfrac{2 \times 100}{200}} = 2.5\ \text{M}\Omega$$

9.65 For the differential-pair transistors, we have

$I_C \simeq 0.25$ mA

$$g_m = \frac{0.25}{0.025} = 10\ \text{mA/V}$$

$$r_o = \frac{V_A}{I_C} = \frac{50}{0.25} = 200\ \text{k}\Omega$$

(a) $A_d = g_m R_C = 10 \times 5 = 50$ V/V

where we have neglected the effect of r_o since $r_o \gg R_C$.

(b) If the bias current is realized using a simple current source,

$$R_{EE} = r_o|_{\text{current source}} = \frac{V_A}{I} = \frac{50}{0.5} = 100\ \text{k}\Omega$$

$$|A_{cm}| = \left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$= \left(\frac{5}{2 \times 100}\right) \times 0.1$$

$= 2.5 \times 10^{-3}$ V/V

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{50}{2.5 \times 10^{-3}} = 2 \times 10^4$$

or 86 dB

(c) If the bias current I is generated using a Wilson mirror,

$$R_{EE} = R_o|_{\text{Wilson mirror}}$$

$$= \frac{1}{2}\beta r_o$$

where r_o is that of the transistors in the Wilson mirror, then

$$r_o = \frac{50}{0.5} = 100\ \text{k}\Omega$$

$$R_{EE} = \frac{1}{2} \times 100 \times 100 = 5\ \text{M}\Omega$$

$$|A_{cm}| = \left(\frac{5}{2 \times 5{,}000}\right) \times 0.1$$

$= 5 \times 10^{-5}$ V/V

$$\text{CMRR} = \frac{50}{5 \times 10^{-5}} = 10^6$$

or 120 dB

9.66 See figure on next page.

$v_{be1} = 2.5 \sin(\omega t)$, mV and

$v_{be2} = -2.5 \sin(\omega t)$, mV

$$v_{C1} \simeq V_{CC} - \left(\frac{I}{2}\right)R_C - g_m R_C \times 2.5 \times 10^{-3} \sin(\omega t)$$

where

$$g_m = \frac{I/2}{V_T} = \frac{I\ \text{mA}}{0.05\ \text{V}}$$

Thus

$$v_{C1} = 5 - \frac{I}{2} \times 10 - \frac{I}{0.05} \times 10 \times 2.5 \times 10^{-3} \sin(\omega t)$$

$= 5 - 5I - 0.5I \sin(\omega t)$

Similarly,

$v_{C2} = 5 - 5I + 0.5I \sin(\omega t)$

To ensure operation in the active mode at all times with $v_{CB} = 0$ V, we use

$v_{C1\min} = 0.005$

$5 - 5.5I = 0.005$

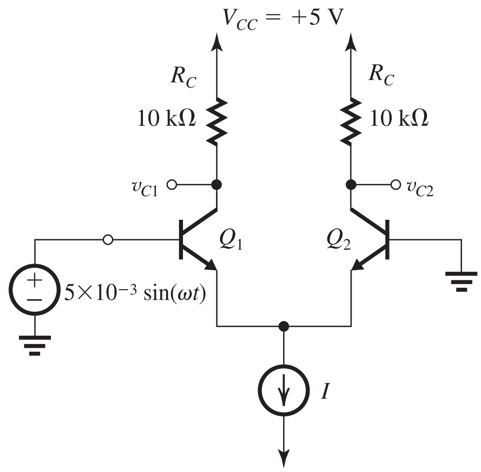
$\Rightarrow I \simeq 0.9$ mA

With this value of bias current, we obtain

$$g_m = \frac{0.9}{0.05} = 18\ \text{mA/V}$$

$A_d = g_m R_C = 18 \times 10 = 180$ V/V

At each collector there will be a sine wave of $180 \times 2.5 = 450$ mV $= 0.45$ V amplitude superimposed on the dc bias voltage of $5 - 0.45 \times 10 = 0.5$ V. Between the two collectors there will be a sine wave with 0.9 V peak amplitude. The figure illustrates the waveforms obtained.



9.67 $\frac{v_{o1}}{v_{id}} = -100$ V/V $\frac{v_{o2}}{v_{id}} = +100$ V/V

$\frac{v_{o1,2}}{v_{icm}} = -0.1$ V/V

$R_{id} = 10$ kΩ

$I = 2$ mA

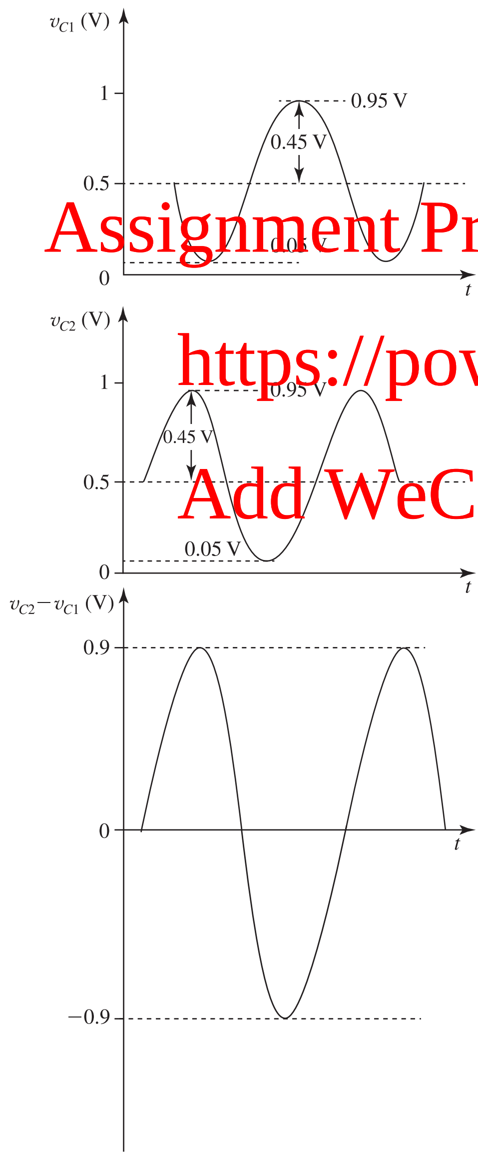
$I_{E1} = I_{E2} = 1$ mA

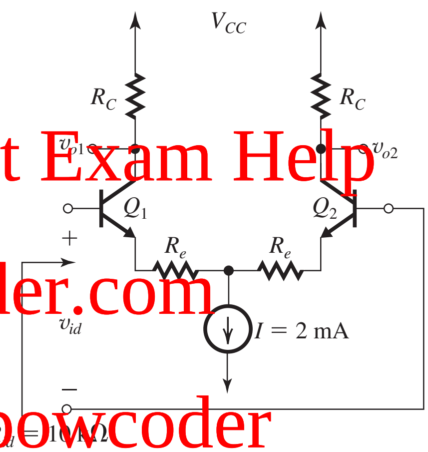
$r_{e1} = r_{e2} = 25\ \Omega$

$g_{m1} = g_{m2} = 40$ mA/V

$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = \frac{100}{40} = 2.5$ kΩ

Since $R_{id} > r_\pi$, we need emitter resistances, as shown in the figure.





$10 = (\beta + 1)\ (2r_e + 2R_e)$

$r_e + R_e = \frac{10}{2 \times 101} \simeq 50\ \Omega$

$R_e = 25\ \Omega$

$\frac{v_{o1}}{v_{id}} = -\frac{\alpha R_C}{2(r_e + R_e)}$

$-100 = \frac{-\alpha R_C}{2(0.025 + 0.025)}$

$\Rightarrow R_C \simeq 10$ kΩ

To allow for ±2 V swing at each collector,

$V_{CC} - \frac{I}{2} R_C - 2 \geq 0$

assuming that $V_{CM} = 0$ V. Thus,

$V_{CC} = \frac{2}{2} \times 10 + 2 = 12$ V

We can use $V_{CC} = 15$ V to allow for V_{ICM} as high as $+3$ V.

$$|A_{cm}| \text{ (to each collector) } \simeq \frac{R_C}{2R_{EE}}$$

For $|A_{cm}| = 0.1$,

$$0.1 = \frac{10}{2R_{EE}}$$

$$\Rightarrow R_{EE} = 50 \text{ k}\Omega$$

This is the minimum value of R_o of the bias current source. If the current source is realized by a simple current mirror, we obtain

$$R_{EE} = r_o = \frac{V_A}{I}$$

Thus,

$$50 = \frac{V_A}{2}$$

$$\Rightarrow V_A = 100 \text{ V}$$

The common-mode input resistance is

$$R_{icm} \simeq \beta R_{EE} \frac{1 + R_C/\beta r_o}{1 + \dfrac{R_C + 2R_{EE}}{r_o}}$$

where r_o is the output resistance of each of Q_1 and Q_2,

$$r_o = \frac{V_A}{I/2} = \frac{100}{1} = 100 \text{ k}\Omega$$

$$R_{icm} = 100 \times 50 \frac{1 + (10/(100 \times 100))}{1 + \dfrac{10 + 100}{100}}$$

$$= 2.4 \text{ M}\Omega$$

9.68 If the output is taken single-endedly, then

$$|A_{cm}| = \frac{R_C}{2R_{EE}}$$

$$|A_d| = \frac{1}{2} g_m R_C$$

$$\text{CMRR}_s = \frac{|A_{cm}|}{|A_d|} = g_m R_{EE}$$

If the output is taken differentially, then

$$|A_{cm}| = \left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right)$$

$$|A_d| = g_m R_C$$

$$\text{CMRR}_d = 2g_m R_{EE} \Big/ \left(\frac{\Delta R_C}{R_C}\right)$$

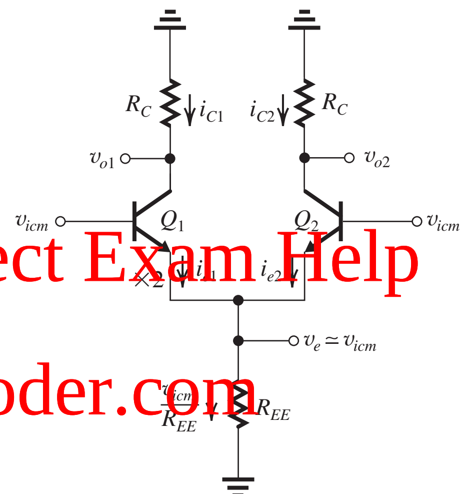
Thus,

$$\frac{\text{CMRR}_d}{\text{CMRR}_s} = \frac{2}{\Delta R_C/R_C}$$

$$20 \log \frac{2}{\Delta R_C/R_C} = 34 \text{ dB}$$

$$\Rightarrow \frac{\Delta R_C}{R_C} = 0.04 = 4\%$$

9.69 If Q_1 has twice the base-emitter junction area of Q_2, the bias current I will split $\frac{2}{3} I$ in Q_1 and $\frac{1}{3} I$ in Q_2. This is because with B_1 and B_2 grounded the two transistors will have equal V_{BE}'s. Thus their currents must be related by the ratio of their scale currents I_S, which are proportional to the junction areas.



With a common-mode input signal v_{icm} applied, as shown in the figure, the current (v_{icm}/R_{EE}) will split between Q_1 and Q_2 in the same ratio as that of their base-emitter junction areas, thus

$$i_{e1} = \frac{2}{3} \frac{v_{icm}}{R_{EE}}$$

and

$$i_{e2} = \frac{1}{3} \frac{v_{icm}}{R_{EE}}$$

Thus,

$$v_{o1} = -i_{c1} R_C \simeq -i_{e1} R_C = -\frac{2}{3} \frac{R_C}{R_{EE}} v_{icm}$$

and

$$v_{o2} = -\frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

With the output taken differentially, we have

$$v_{o2} - v_{o1} = \frac{1}{3} \frac{R_C}{R_{EE}} v_{icm}$$

$$A_{cm} = \frac{1}{3} \frac{R_C}{R_{EE}} = \frac{1}{3} \times \frac{12}{500} = 0.008 \text{ V/V}$$

9.70 $g_m = \sqrt{2\,k_n'(W/L)I_D}$

$= \sqrt{k_n'(W/L)I}$

$A_d = g_m R_D$

$V_{OV} = \dfrac{2\,I_D}{g_m} = \dfrac{I}{g_m}$

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

For $I = 160\ \mu$A, we have

$g_m = \sqrt{4 \times 0.16} = 0.8$ mA/V

$A_d = 0.8 \times 10 = 8$ V/V

$V_{OV} = \dfrac{0.16}{0.8} = 0.2$ V

$V_{OS} = \dfrac{0.2}{2} \times 0.02 = 2$ mV

For $I = 360\ \mu$A, we have

$g_m = \sqrt{4 \times 0.36} = 1.2$ mA/V

$A_d = 1.2 \times 10 = 12$ V/V

$V_{OV} = \dfrac{0.36}{1.2} = 0.3$ V

$V_{OS} = \dfrac{0.3}{2} \times 0.02 = 3$ mV

Thus by increasing the bias current, both the gain and the offset voltage increase, and by the same factor (1.5).

9.71 (a) $g_m = \sqrt{2k_n I_D} = \sqrt{k_n I}$

$$A_d = g_m R_D = \sqrt{k_n I}\ R_D \qquad (1)$$

$V_{OV} = \sqrt{\dfrac{I/2}{\frac{1}{2}k_n}} = \sqrt{\dfrac{I}{k_n}}$

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

Thus,

$$V_{OS} = \frac{1}{2}\sqrt{I/k_n}\left(\frac{\Delta R_D}{R_D}\right) \qquad (2)$$

(b) For each value of V_{OS} we use Eq. (2) to determine I and then Eq. (1) to determine A_d. The results are as follows:

V_{OS} (mV)	1	2	3	4	5
I (mA)	0.04	0.16	0.36	0.64	1.00
A_d (V/V)	4	8	12	16	20

We observe that by accepting a larger offset we are able to obtain a higher gain. Observe that the gain realized is proportional to the offset voltage one is willing to accept.

9.72 The offset voltage due to ΔV_t is

$V_{OS} = \pm 5$ mV

The offset voltage due to ΔR_D is

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right) = \dfrac{0.3}{2} \times 0.02 = 3$ mV

The offset voltage due to $\Delta(W/L)$ is

$V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\dfrac{\Delta(W/L)}{(W/L)} = \dfrac{0.3}{2} \times 0.02 = 3$ mV

The worst-case offset voltage will be when all three components add up,

$V_{OS} = 5 + 3 + 3 = 11$ mV

The major contribution to the total is the variability of V_t.

To compensate for a total offset of 11 mV by appropriately varying R_D, we need to change R_D by ΔR_D obtained from

$11\text{ mV} = \left(\dfrac{V_{OV}}{2}\right) \times \dfrac{\Delta R_D}{R_D}$

$\Rightarrow \dfrac{\Delta R_D}{R_D} = \dfrac{11 \times 2}{300} = 0.0733$

or 7.33%

9.73 $V_{OV} = \sqrt{\dfrac{I/2}{\frac{1}{2}k_n'(W/L)}} = \sqrt{\dfrac{I}{k_n'(W/L)}}$

$= \sqrt{\dfrac{0.1}{0.2 \times 10}} = 0.224$ V

$\dfrac{\Delta R_D}{R_D} = 0.04 \Rightarrow V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta R_D}{R_D}\right)$

$= \dfrac{0.224}{2} \times 0.04 = 4.5$ mV

$\dfrac{\Delta(W/L)}{(W/L)} = 0.04 \Rightarrow V_{OS} = \left(\dfrac{V_{OV}}{2}\right)\left(\dfrac{\Delta(W/L)}{(W/L)}\right)$

$= \dfrac{0.224}{2} \times 0.04 = 4.5$ mV

$\Delta V_t = 5\text{ mV} \Rightarrow V_{OS} = \Delta V_t = 5$ mV

Worst-case $V_{OS} = 4.5 + 4.5 + 5 = 14$ mV

If the three components are independent,

$V_{OS} = \sqrt{4.5^2 + 4.5^2 + 5^2} = 8.1$ mV

9.74 $V_{OS} = V_T\left(\dfrac{\Delta R_C}{R_C}\right)$

$= 25 \times 0.1 = 2.5$ mV

9.75 $V_{OS} = V_T\left(\dfrac{\Delta I_S}{I_S}\right)$

$= 25 \times 0.1 = 2.5$ mV

9.76 With both input terminals grounded, a mismatch $\triangle R_C$ between the two collector resistors gives rise to an output voltage

$$V_O = \alpha\left(\frac{I}{2}\right)\triangle R_C \qquad (1)$$

With a resistance R_E connected in the emitter of each transistor, the differential gain becomes

$$|A_d| = \frac{\alpha \times 2R_C}{2(r_e + R_E)} = \frac{\alpha R_C}{R_E + r_e} \qquad (2)$$

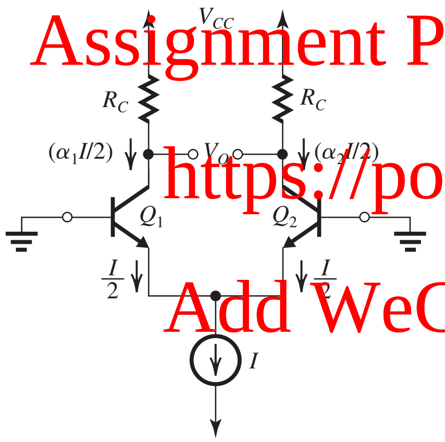
The input offset voltage V_{OS} is obtained by dividing V_O in (1) by $|A_d|$ in (2),

$$V_{OS} = \frac{I}{2}(r_e + R_E)\left(\frac{\triangle R_C}{R_C}\right)$$

Since $r_e = \dfrac{V_T}{I/2}$,

$$V_{OS} = (V_T + \frac{1}{2}IR_E)\left(\frac{\triangle R_C}{R_C}\right)$$

9.77



The current I splits equally between the two emitters. However, the unequal β's will mean unequal α's. Thus, the two collector currents will be unequal,

$$I_{C1} = \alpha_1 I/2$$

$$I_{C2} = \alpha_2 I/2$$

and the collector voltages will be unequal,

$$V_{C1} = V_{CC} - \alpha_1(I/2)R_C$$

$$V_{C2} = V_{CC} - \alpha_2(I/2)R_C$$

Thus a differential output voltage V_O develops:

$$V_O = V_{C2} - V_{C1}$$

$$= \frac{1}{2}IR_C(\alpha_1 - \alpha_2)$$

The input offset voltage V_{OS} can be obtained by dividing V_O by the differential gain A_d:

$$A_d = g_m R_C \simeq \frac{I/2}{V_T}R_C = \frac{IR_C}{2V_T}$$

Thus,

$$V_{OS} = V_T(\alpha_1 - \alpha_2)$$

Substituting, we obtain

$$\alpha_1 = \frac{\beta_1}{\beta_1 + 1}$$

and

$$\alpha_2 = \frac{\beta_2}{\beta_2 + 1}$$

$$V_{OS} = V_T\left(\frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1}\right)$$

$$= V_T\frac{\beta_1\beta_2 + \beta_1 - \beta_1\beta_2 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$= V_T\frac{\beta_1 - \beta_2}{(\beta_1 + 1)(\beta_2 + 1)}$$

$$\simeq V_T\frac{\beta_1 - \beta_2}{\beta_1\beta_2}$$

$$= V_T\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right) \qquad \text{Q.E.D.}$$

For $\beta_1 = 50$ and $\beta_2 = 100$, we have

$$V_{OS} = 25\left(\frac{1}{100} - \frac{1}{50}\right) = -0.25 \text{ mV}$$

9.78 For the MOS amplifier:

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\triangle R_D}{R_D}\right)$$

$$= \frac{200}{2} \times 0.04$$

$$= 4 \text{ mV}$$

For the BJT amplifier:

$$V_{OS} = V_T\left(\frac{\triangle R_C}{R_C}\right)$$

$$= 25 \times 0.04 = 1 \text{ mV}$$

If in the MOS amplifier the width of each device is increased by a factor of 4 while the bias current is kept constant, V_{OV} will be reduced by a factor of 2. Thus V_{OS} becomes

$$V_{OS} = 2 \text{ mV}$$

9.79 Since the only difference between the two sides of the differential pair is the mismatch in V_A, we can write

$$I_{C1} = I_C\left(1 + \frac{V_{CE1}}{V_{A1}}\right)$$

$$I_{C2} = I_C\left(1 + \frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C1} + I_{C2} = \alpha I$$

$$I_C\left(2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}}\right) = \alpha I$$

$$\Rightarrow I_C = \alpha I \Big/ \left(2 + \frac{V_{CE1}}{V_{A1}} + \frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C1} = \frac{\alpha I}{2} \frac{1 + \dfrac{V_{CE1}}{V_{A1}}}{1 + \dfrac{V_{CE1}}{2V_{A1}} + \dfrac{V_{CE2}}{2V_{A2}}}$$

For $\dfrac{V_{CE1}}{V_{A1}} \ll 1$ and $\dfrac{V_{CE2}}{V_{A2}} \ll 1$ we have

$$I_{C1} \simeq \frac{\alpha I}{2}\left(1 + \frac{1}{2}\frac{V_{CE1}}{V_{A1}} - \frac{1}{2}\frac{V_{CE2}}{V_{A2}}\right)$$

$$I_{C2} \simeq \frac{\alpha I}{2}\left(1 + \frac{1}{2}\frac{V_{CE2}}{V_{A2}} - \frac{1}{2}\frac{V_{CE1}}{V_{A1}}\right)$$

The voltage V_O between the two collectors will be

$$V_O = V_{C2} - V_{C1}$$

$$= I_{C1}R_C - I_{C2}R_C$$

$$= \frac{\alpha I}{2} R_C \times \left(\frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}}\right) \qquad (1)$$

Since we still have $I_{C1} \simeq I_{C2} = \alpha \dfrac{I}{2}$, the differential gain is still given by

$$A_d = g_m R_C = \frac{I_C R_C}{V_T} = \frac{\alpha I R_C}{2V_T} \qquad (2)$$

Dividing (1) by (2) gives

$$V_{OS} = V_T\left(\frac{V_{CE1}}{V_{A1}} - \frac{V_{CE2}}{V_{A2}}\right)$$

As a first-order approximation, we can assume

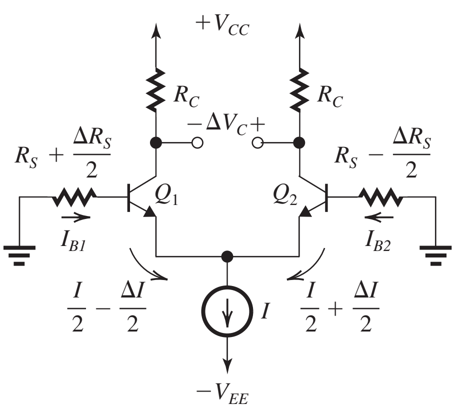
$$V_{CE1} \simeq V_{CE2} = 10 \text{ V}$$

and substitute $V_{A1} = 100$ V and $V_{A2} = 200$ V to determine V_{OS} as

$$V_{OS} = 25\left(\frac{10}{100} - \frac{10}{200}\right)$$

$$= 25 \times 0.05 = 1.25 \text{ mV}$$

9.80



Consider only the incremental currents involved.

Assume the mismatch ΔR_S is split between the two base (source) resistances. The emitter currents will be different, as shown.

Equating the voltage drop from each grounded input to the common emitters, we have

$$I_{B1}\left(R_S + \frac{\Delta R_S}{2}\right) + \left(\frac{I}{2} - \frac{\Delta I}{2}\right) r_e$$

$$= I_{B2}\left(R_S - \frac{\Delta R_S}{2}\right) + \left(\frac{I}{2} + \frac{\Delta I}{2}\right) r_e$$

Subtracting out the $\dfrac{I}{2} r_e$ terms, we have

$$I_{B1}\left(R_S + \frac{\Delta R_S}{2}\right) - \frac{\Delta I}{2} r_e$$

$$= I_{B2}\left(R_S - \frac{\Delta R_S}{2}\right) + \frac{\Delta I}{2} r_e$$

In terms of the emitter currents, this becomes

$$\frac{\left(\dfrac{I}{2} - \dfrac{\Delta I}{2}\right)}{(\beta + 1)}\left(R_S + \frac{\Delta R_S}{2}\right) - \frac{\Delta I}{2} r_e$$

$$= \frac{\left(\dfrac{I}{2} + \dfrac{\Delta I}{2}\right)}{(\beta + 1)}\left(R_S - \frac{\Delta R_S}{2}\right) + \frac{\Delta I}{2} r_e$$

Subtracting $\dfrac{IR_S}{2\,(\beta + 1)}$ and $-\dfrac{\Delta I \Delta R_S}{4(\beta + 1)}$ from each side, we obtain

$$\frac{I \Delta R_S}{4\,(\beta + 1)} - \frac{\Delta I R_S}{2\,(\beta + 1)} - \frac{\Delta I r_e}{2}$$

$$= -\frac{I \Delta R_S}{4\,(\beta + 1)} + \frac{\Delta I R_S}{2\,(\beta + 1)} + \frac{\Delta I r_e}{2}$$

Combining terms, we have

$$\frac{I \Delta R_S}{2\,(\beta + 1)} = \frac{\Delta I R_S}{(\beta + 1)} + \Delta I r_e$$

$$\Delta I\left(\frac{R_S}{(\beta + 1)} + r_e\right) = \frac{I \Delta R_S}{2\,(\beta + 1)} \text{ so that}$$

$$\Delta I = \frac{I \Delta R_S}{2\,(\beta + 1)} \cdot \frac{1}{\dfrac{R_S}{(\beta + 1)} + r_e}$$

$\Delta V_C = \Delta I_C R_C$. If $\dfrac{\beta}{\beta + 1} \approx 1$, we have

$$\Delta V_C = \frac{I \Delta R_S R_C}{2\,(\beta + 1)} \cdot \frac{1}{\dfrac{R_S}{(\beta + 1)} + r_e}$$

Now V_{OS} can be obtained by dividing ΔV_C by $A_d = g_m R_C$,

$$V_{OS} = \frac{\Delta V_C}{A_d} = \frac{\dfrac{I\,\Delta R_S R_C}{2\,(\beta+1)} \cdot \dfrac{1}{\dfrac{R_S}{(\beta+1)} + r_e}}{g_m\, R_C}$$

$$= \frac{I\,\Delta R_S}{2\,(\beta+1)} \cdot \frac{1}{g_m\left[\dfrac{R_S}{(\beta+1)} + r_e\right]}$$

$$V_{OS} = \frac{I\,\Delta R_S}{2} \cdot \frac{1}{g_m\, R_S + (\beta+1)\, r_e g_m}$$

Since $(\beta+1)\, r_e = r_\pi$ and $r_\pi\, g_m = \beta$, we have

$$V_{OS} = \frac{\left(\dfrac{I}{2\beta}\right) \cdot \Delta R_S}{1 + \dfrac{g_m\, R_S}{\beta}} \qquad \text{Q.E.D.}$$

9.81 Refer to Fig. P9.81.

(a) $R_{C1} = 1.04 \times 5 = 5.20\ \text{k}\Omega$

$R_{C2} = 0.96 \times 5 = 4.80\ \text{k}\Omega$

To equalize the total resistance in each collector, we adjust the potentiometer so that

$R_{C1} + x \times 1\ \text{k}\Omega = R_{C2} + (1-x) \times 1\ \text{k}\Omega$

$5.2 + x = 4.8 + 1 - x$

$\Rightarrow x = 0.3\ \text{k}\Omega$

(b) If the area of Q_1 and hence I_{S1} is 5% larger than nominal, then we have

$I_{S1} = 1.05 I_S$

and the area of Q_2 and hence I_{S2} is 5% smaller than nominal,

$I_{S2} = 0.95 I_S$

Thus,

$I_{E1} = 0.5 \times 1.05 = 0.525\ \text{mA}$

$I_{E2} = 0.5 \times 0.95 = 0.475\ \text{mA}$

Assuming $\alpha \simeq 1$, we obtain

$I_{C1} = 0.525\ \text{mA} \quad I_{C2} = 0.475\ \text{mA}$

To reduce the resulting offset to zero, we adjust the potentiometer so that

$V_{C1} = V_{C2}$

$\Rightarrow V_{CC} - (R_{C1} + x) I_{C1} = V_{CC} - (R_{C2} + 1 - x) I_{C2}$

$I_{C1}(R_{C1} + x) = I_{C2}(R_{C2} + 1 - x)$

$0.525(5 + x) = 0.475(5 + 1 - x)$

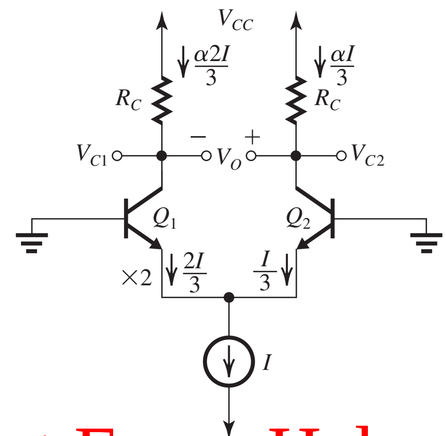
$\Rightarrow x = 0.225$

9.82 $I_{B\max} = \dfrac{400}{2 \times 81} \simeq 2.5\ \mu\text{A}$

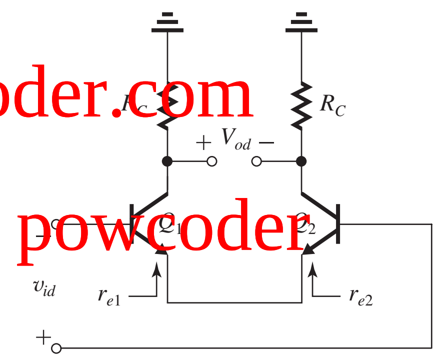
$I_{B\min} = \dfrac{400}{2 \times 201} = 1\ \mu\text{A}$

$I_{OS\max} = \dfrac{200}{81} - \dfrac{200}{201} \simeq 1.5\ \mu\text{A}$

9.83



(a)



(b)

From Fig. (a) we see that the transistor with twice the area (Q_1) will carry twice the current in the other transistor (Q_2). Thus

$I_{E1} = \dfrac{2I}{3}, \qquad I_{E2} = \dfrac{I}{3}$

$I_{C1} = \dfrac{\alpha 2I}{3}, \qquad I_{C2} = \dfrac{\alpha I}{2}$

Thus,

$V_{C1} = V_{CC} - \dfrac{\alpha 2I}{3} R_C$

$V_{C2} = V_{CC} - \dfrac{\alpha I}{3} R_C$

and the dc offset voltage at the output will be

$V_O = V_{C2} - V_{C1}$

$V_O = \dfrac{1}{3} \alpha I R_C$

To reduce this output voltage to zero, we apply a dc input voltage v_{id} in the direction shown in Fig. (b). The voltage v_{id} is required to produce v_{od} in the direction shown which is opposite in direction to V_O and of course $|v_{od}| = |V_O|$, thus

$$A_d v_{id} = \frac{1}{3}\alpha I R_C \qquad (1)$$

The gain A_d is found as follows:

$$A_d = \frac{\alpha \times \text{Total resistance in collectors}}{\text{Total resistance in emitters}}$$

$$= \frac{\alpha \times 2R_C}{r_{e1} + r_{e2}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{V_T}{2I/3} = \frac{3V_T}{2I} = \frac{1.5V_T}{I}$$

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{V_T}{I/3} = \frac{3V_T}{I}$$

thus,

$$A_d = \frac{2\alpha R_C}{4.5\ V_T/I} = \frac{2\alpha I R_C}{4.5\ V_T} \qquad (2)$$

Substituting in Eq. (1) gives

$$v_{id} = 0.75\ V_T = 18.75\text{ mV}$$

Now, using large signal analysis:

$$v_{id} = V_{B2} - V_{B1} = (V_{B2} - V_E) - (V_{B1} - V_E)$$

$$I_{C1} = I_{S1} e^{(V_{B1} - V_E)/V_T} \qquad (3)$$

$$I_{C2} = I_{S2} e^{(V_{B2} - V_E)/V_T} \qquad (4)$$

where $I_{S1} = 2\ I_{S2}$.

To make $I_{C1} = I_{C2}$,

$$I_{S1} e^{(V_{B1} - V_E)/V_T} = I_{S2} e^{(V_{B2} - V_E)/V_T}$$

$$e^{(V_{B2} - V_{B1})/V_T} = 2$$

$$V_{B2} - V_{B1} = V_T \ln 2$$

Thus,

$$v_{id} = 17.3\text{ mV}$$

which is reasonably close to the approximate value obtained using small-signal analysis.

9.84 A 2-mV input offset voltage corresponds to a difference ΔR_C between the two collector resistances,

$$2 = V_T \frac{\Delta R_C}{R_C}$$

$$= 25 \times \frac{\Delta R_C}{20}$$

$$\Rightarrow \Delta R_C = 1.6\text{ k}\Omega$$

Thus a 2-mV offset can be nulled out by adjusting one of the collector resistances by 1.6 kΩ. If the adjustment mechanism raises one R_C and lowers the other, then each need to be adjusted by only $(1.6\text{ k}\Omega/2) = 0.8\text{ k}\Omega$.

If a potentiometer is used (as in Fig. P9.81), the total resistance of the potentiometer must be at least 1.6 kΩ. If specified to a single digit, we use 2 kΩ.

9.85 $G_m = 2$ mA/V

With $R_L = \infty$,

$$A_d = G_m R_o$$

and

$$v_o = G_m R_o v_{id}$$

With $R_L = 20$ kΩ,

$$v_o = G_m R_o v_{id} \frac{R_L}{R_L + R_o}$$

$$= G_m R_o \frac{20}{20 + R_o} v_{id} = \frac{1}{2} G_m R_o v_{id}$$

Thus,

$$R_o = 20\text{ k}\Omega$$

$$A_d\ (\text{with } R_L = \infty) = G_m R_o = 2 \times 20 = 40\text{ V/V}$$

9.86 $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}} = \dfrac{I}{0.25}$

$$R_o = r_{o2} \parallel r_{o4}$$

For

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V_A'|L}{I/2}$$

$$= \frac{2 \times 5 \times 0.5}{I} = \frac{5}{I}$$

$$R_o = \frac{1}{2} \times \frac{5}{I} = \frac{2.5}{I}$$

Thus,

$$A_d = G_m R_o = \frac{I}{0.25} \times \frac{2.5}{I} = 10\text{ V/V}$$

9.87 $\dfrac{I}{2} = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right) V_{OV}^2$

$$0.1 = \frac{1}{2} \times 0.2 \times 50 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.14\text{ V}$$

$$g_{m1,2} = \frac{2 \times (I/2)}{V_{OV}} = \frac{2 \times 0.1}{0.14} = 1.4\text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{|V_A'| \times L}{I/2} = \frac{5 \times 0.5}{0.1}$$

$$= 25\text{ k}\Omega$$

$A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$= 1.4 \times (25 \parallel 25)$

$= 17.5$ V/V

9.88 $A_d = g_{m1,2}(r_{o2} \parallel r_{o4})$

$g_{m1,2} = \sqrt{2k_n' \left(\frac{W}{L}\right) I_D}$

$= \sqrt{4I} = 2\sqrt{I}$

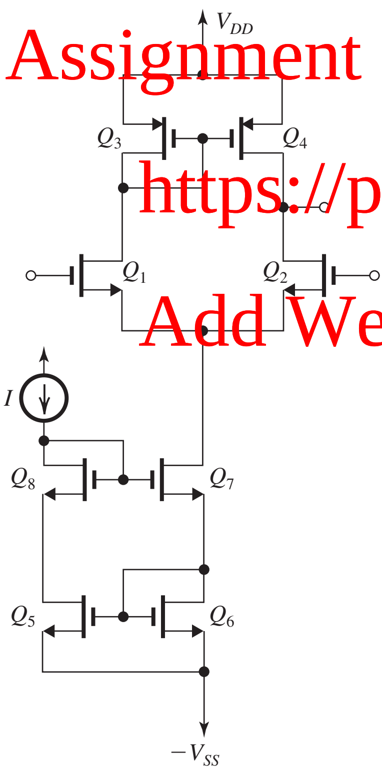
$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I} = \frac{2 \times 5}{I} = \frac{10}{I}$

$A_d = 2\sqrt{I} \times \frac{1}{2} \times \frac{10}{I} = \frac{10}{\sqrt{I}}$

$20 = \frac{10}{\sqrt{I}}$

$\Rightarrow I = 0.25$ mA

9.89



For Q_1, Q_2, Q_3 and Q_4:

$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$

$0.1 = \frac{1}{2} \times 5 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$V_{GS} = V_t + |V_{OV}|$

$= 0.5 + 0.2 = 0.7$ V

For Q_5, Q_6, Q_7, and Q_8:

$I_D = 0.2$ mA

$0.2 = \frac{1}{2} \times 5 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.28$ V

$V_{GS} = 0.5 + 0.28 = 0.78$ V

From the figure we see that for each transistor to operate at V_{DS} at least equal to V_{GS}, the total power supply is given by

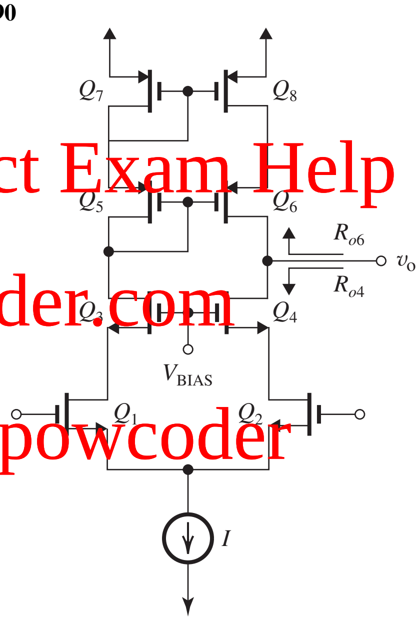
$V_{DD} + V_{SS} = V_{DS4} + V_{DS2} + V_{DS7} + V_{DS6}$

$= V_{GS4} + V_{GS2} + V_{GS7} + V_{GS6}$

$= 0.7 + 0.7 + 0.78 + 0.78$

$= 2.96 \simeq 3.0$ V

9.90



(a) See figure.

(b) $A_d = g_{m1,2}(R_{o4} \parallel R_{o6})$

$g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$

$R_{o6} = g_{m6} r_{o6} r_{o8}$

Since all transistors are operated at a bias current $(I/2)$ and have the same overdrive voltage $|V_{OV}|$ and the same Early voltage, $|V_A|$, all have the same $g_m = I/|V_{OV}|$ and the same

$r_o = \frac{|V_A|}{I/2} = 2|V_A|/I$. Thus,

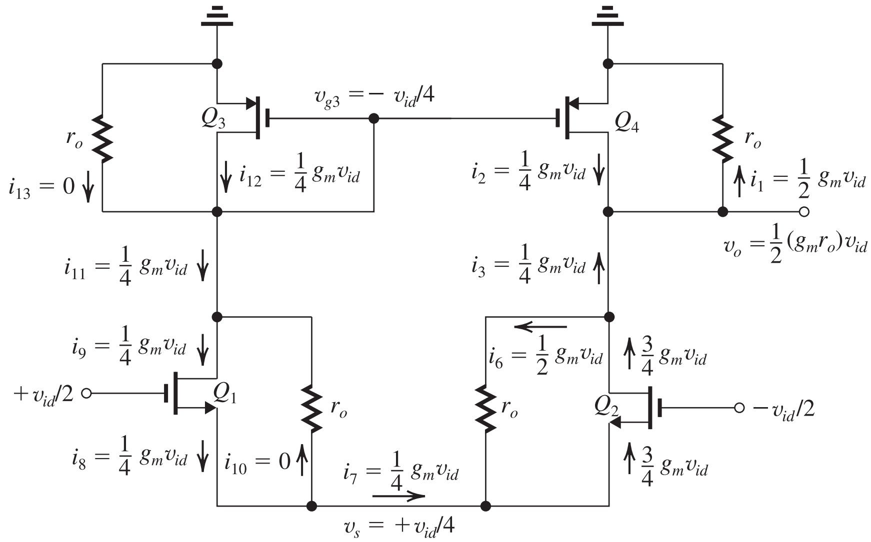
$R_{o6} = g_m r_o^2$

$R_{o4} = g_{m4} r_{o4} r_{o2} = g_m r_o^2$

$A_d = g_m(g_m r_o^2 \parallel g_m r_o^2)$

$= \frac{1}{2}(g_m r_o)^2$

This figure belongs to Problem 9.91.



$$g_m r_o = \frac{I}{|V_{OV}|} \times \frac{2|V_A|}{I} = \frac{2|V_A|}{|V_{OV}|}$$

$A_d = 2(|V_A|/|V_{OV}|)^2$ Q.E.D.

For $|V_{OV}| = 0.2$ V and $|V_A| = 10$ V, we have

$$A_d = 2\left(\frac{10}{0.2}\right)^2 = 5000 \text{ V/V}$$

9.91 The currents i_1 to i_{13} are shown on the circuit diagram. Observe that $i_{11} = i_7 = i$ (the current that enters a transistor exits at the other end!). Also observe that the mirror Q_3 and Q_4 is indeed functioning properly as the drain currents of Q_3 and Q_4 are equal ($i_{12} = i_2 = \frac{1}{4} g_m v_{id}$). However, the currents in their r_o's are far from being equal!

There are some inconsistencies that result from the approximations made to obtain the results shown in Fig. P9.91, namely, $g_m r_o \gg 1$. Note for instance that although we find the current in r_o of Q_2 to be $\frac{1}{2} g_m v_{id}$, the voltages at the two ends of r_o are $\frac{1}{2}(g_m r_o) v_{id}$ and $v_{id}/4$; thus the current must be $v_{id}\left(\frac{1}{2} g_m r_o - \frac{1}{4}\right) \Big/ r_o$, which is approximately $\frac{1}{2} g_m v_{id}$.

The purpose of this problem is to show the huge imbalance that exists in this circuit. In fact, Q_1 has $|v_{gs}| = \frac{1}{4} v_{id}$ while Q_2 has $|v_{gs}| = \frac{3}{4} v_{id}$. This imbalance results from the fact that the current mirror is *not* a balanced load. Nevertheless, we know that this circuit provides a reasonably high common-mode rejection.

9.92 $G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV1,2}} = \frac{0.2}{0.2} = 1$ mA/V

$$r_{o2} = \frac{V_{An}}{I/2} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

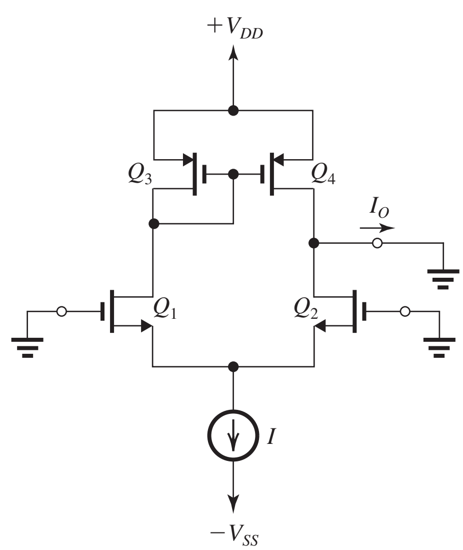
$$r_{o4} = \frac{|V_{Ap}|}{I/2} = \frac{12}{0.1} = 120 \text{ k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 200 \parallel 120 = 75$ kΩ

$A_d = G_m R_o = 1 \times 75 = 75$ V/V

The gain is reduced by a factor of 2 with $R_L = R_o = 75$ kΩ.

9.93



(a) Let

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_A + \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_A - \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A$$

Q_1 and Q_2 have equal values of V_{GS} and thus of V_{OV}, thus

$$I_{D1} = \frac{1}{2}k_n'\left[\left(\frac{W}{L}\right)_A + \frac{1}{2}\Delta\left(\frac{W}{L}\right)_A\right]V_{OV}^2$$

$$= \frac{1}{2}k_n'\left(\frac{W}{L}\right)_A\left[1 + \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]V_{OV}^2$$

Since, in the ideal case

$$I_{D1} = \frac{I}{2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_A V_{OV}^2$$

$$I_{D1} = \frac{I}{2}\left[1 + \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]$$

Similarly, we can show that

$$I_{D2} = \frac{I}{2}\left[1 - \frac{1}{2}\frac{\Delta(W/L)_A}{(W/L)_A}\right]$$

The current mirror causes

$$I_{D4} = I_{D3} = I_{D1}$$

Thus,

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D1} - I_{D2}$$

$$= \frac{I}{2}\frac{\Delta(W/L)_A}{(W/L)_A}$$

The input offset voltage is

$$V_{OS} = \frac{I_O}{G_m}$$

where

$$G_m = g_{m1,2} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

Thus,

$$V_{OS} = (V_{OV}/2)\,\frac{\Delta(W/L)_A}{(W/L)_A} \qquad \text{Q.E.D.}$$

(b) $I_{D1} = I_{D2} = \dfrac{I}{2}$

$$I_{D3} = I_{D1}$$

If the (W/L) ratios of the mirror transistors have a mismatch $\Delta(WL)_M$, the current transfer ratio of the mirror will have an error of $[\Delta(W/L)_M/(W/L)_M]$. Thus

$$I_{D4} = I_{D3}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right]$$

At the output node, we have

$$I_O = I_{D4} - I_{D2}$$

$$= I_{D3}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right] - I_{D2}$$

$$= I_{D1}\left[1 + \frac{\Delta(W/L)_M}{(W/L)_M}\right] - I_{D2}$$

$$= \frac{I}{2}\frac{\Delta(W/L)_M}{(W/L)_M}$$

and the corresponding V_{OS} will be

$$V_{OS} = \frac{I_O}{G_m} = \frac{I_O}{I/V_{OV}}$$

$$= \left(\frac{V_{OV}}{2}\right)\frac{\Delta(W/L)_M}{(W/L)_M} \qquad \text{Q.E.D.}$$

(c) $V_{OS}|_{Q_1,Q_2\text{ mismatch}} = \left(\dfrac{0.2}{2}\right) \times 0.02 = 2\text{ mV}$

$$V_{OS}|_{Q_3,Q_4\text{ mismatch}} = \left(\frac{0.2}{2}\right) \times 0.02 = 2\text{ mV}$$

Worst-case $V_{OS} = 2 + 2 = 4$ mV

9.94 $I_{E1} = I_{E2} = 0.25$ mA

$$I_{C1} = I_{C2} \simeq 0.25\text{ mA}$$

$$g_{m1,2} = \frac{I_{C1,2}}{V_T} = \frac{0.25\text{ mA}}{0.025\text{ V}} = 10\text{ mA/V}$$

$$r_o = \frac{|V_A|}{I_C} = \frac{10\text{ V}}{0.25\text{ mA}} = 40\text{ k}\Omega$$

$$R_{id} = 2\,r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{100}{10} = 20\text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 40 \parallel 40 = 20\text{ k}\Omega$$

$$G_m = g_{m1,2} = 10\text{ mA/V}$$

$$A_d = G_m R_o = 10 \times 20 = 200\text{ V/V}$$

If $R_L = R_{id} = 20\text{ k}\Omega$, then

$$G_v = 200 \times \frac{R_L}{R_L + R_o}$$

$$= 200 \times \frac{20}{20 + 20} = 100\text{ V/V}$$

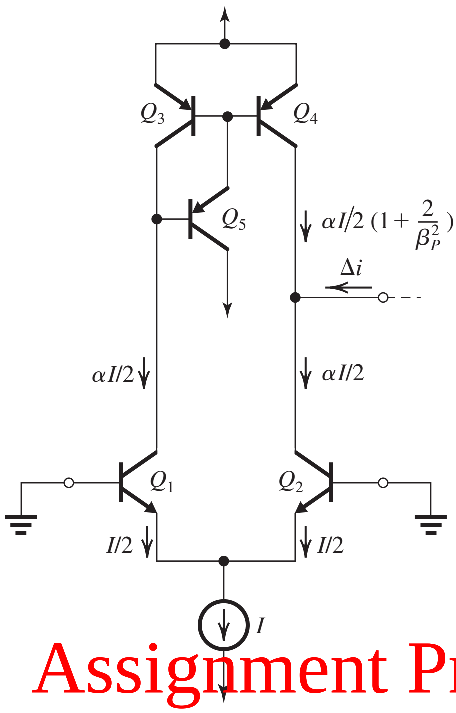
9.95 Using Eq. (9.145), we obtain

$$V_{OS} = -\frac{2V_T}{\beta_p}$$

$$-2 = -\frac{2 \times 25}{\beta_p}$$

$$\Rightarrow \beta_p = 25$$

9.96



The figure shows a BJT differential amplifier loaded in a base-current-compensated current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror, we ground the two input terminals and determine the output current Δi as follows:

$$\Delta i = I_{C2} - I_{C4}$$

$$= \alpha\frac{I}{2} - \alpha\frac{I}{2}\,\frac{1}{1+(2/\beta_p^2)}$$

$$= \alpha\frac{I}{2}\left[1 - \frac{1}{1+(2/\beta_p^2)}\right]$$

$$\simeq -\alpha\frac{I}{2}\,\frac{2}{\beta_p^2} = -\alpha\frac{I}{\beta_p^2}$$

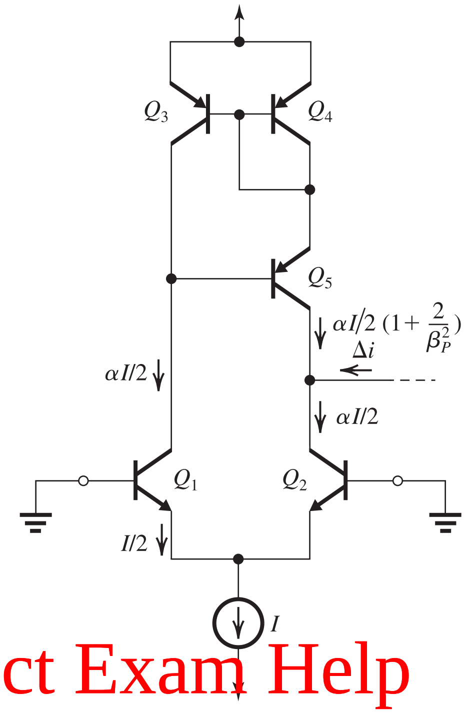
Dividing Δi by $G_m = g_{m1,2} = \dfrac{\alpha I}{2V_T}$ gives

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For $\beta_p = 50$,

$$V_{OS} = -\frac{2\times 25}{50^2} = -20\ \mu\text{V}$$

9.97



The figure shows a BJT differential amplifier loaded with a Wilson current mirror. To determine the systematic input offset voltage resulting from the error in the current-transfer ratio of the mirror, we ground the two input terminals and determine the output current Δi as follows:

$$\Delta i = \alpha\frac{I}{2} - \alpha\frac{I}{2}\,\frac{1}{1+(2/\beta_p^2)}$$

$$\simeq \alpha\frac{I}{2}\,\frac{2}{\beta_p^2} = \frac{\alpha I}{\beta_p^2}$$

Dividing Δi by $G_m = g_{m1,2} = \dfrac{\alpha I/2}{V_T}$ provides the input offset voltage V_{OS}:

$$V_{OS} = -\frac{2V_T}{\beta_p^2}$$

For $\beta_p = 50$,

$$V_{OS} = -\frac{2\times 25}{50^2} = -20\ \mu\text{V}$$

9.98 Refer to Fig. P9.98.

$$A_d = G_m R_o$$

where

$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

and

$$R_o = R_{o4} \parallel R_{o7}$$

Here R_{o4} is the output resistance of the cascode amplifier (looking into the collector of Q_4), thus

$$R_{o4} = g_{m4} r_{o4} (r_{o2} \parallel r_{\pi 4})$$

Usually $r_{\pi 4} \ll r_{o2}$,

$$R_{o4} \simeq g_{m4} r_{\pi 4} r_{o4} = \beta_4 r_{o4}$$

The resistance R_{o7} is the output resistance of the Wilson mirror and is given by

$$R_{o7} = \frac{1}{2}\beta_7 r_{o7}$$

Thus

$$R_o = (\beta_4 r_{o4}) \parallel \left(\frac{1}{2}\beta_7 r_{o7}\right)$$

Since all β and r_o are equal, we obtain

$$R_o = (\beta r_o) \parallel \left(\frac{1}{2}\beta r_o\right)$$

$$= \frac{1}{3}\beta r_o$$

and

$$A_d = \frac{1}{3}\beta g_m r_o \qquad \text{Q.E.D.}$$

For $\beta = 100$ and $V_A = 20$ V, we have

$$g_m r_o = \frac{I_C}{V_T} \frac{V_A}{I_C} = \frac{V_A}{V_T} = \frac{20}{0.025} = 800$$

$$A_d = \frac{1}{3} \times 100 \times 800 = 2.67 \times 10^4 \text{ V/V}$$

9.99 Refer to Fig. P9.98.

(a) $V_{B7} = +5 - V_{EB6} - V_{EB7} = 5 - 0.7 - 0.7$

$= +3.6$ V

$v_{O\max} = V_{B7} + 0.4 = +4$ V

(b) The dc bias voltage should be

$V_O = v_{O\max} - 1.5$

$= 4 - 1.5 = +2.5$ V

(c) For v_O to swing negatively (i.e., below the dc bias value of 2.5 V) by 1.5 V, that is, to +1 V with Q_4 remaining in saturation, V_{BIAS} should be

$V_{\text{BIAS}} = v_{O\min} + 0.4$

$= 1.4$ V

(d) With $V_{\text{BIAS}} = 1.4$ V, the bias voltage at the collectors of Q_1 and Q_2 is

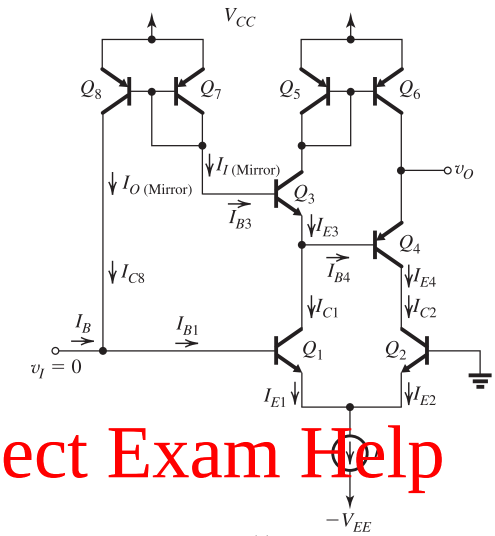
$V_{C1,2} = V_{\text{BIAS}} - V_{BE3,4}$

$= 1.4 - 0.7 = +0.7$ V

The upper limit on V_{CM} is 0.4 V above $V_{C1,2}$:

$V_{CM\max} = 0.7 + 0.4 = +1.1$ V

9.100



(a)

(a) With $v_I = 0$,

$$I_{E1} = I_{E2} = \frac{I}{2}$$

$$I_{C1} = I_{C2} = \frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{E4} = I_{C2} = \frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{B4} = \frac{I_{E4}}{\beta+1} = \frac{1}{\beta+1}\frac{\beta}{\beta+1}\frac{I}{2}$$

$$I_{E3} = I_{C1} + I_{B4}$$

$$= \frac{\beta}{\beta+1}\frac{I}{2} + \frac{1}{\beta+1}\frac{\beta}{\beta+1}\frac{I}{2}$$

$$= \frac{\beta}{\beta+1}\frac{I}{2}\left(1 + \frac{1}{\beta+1}\right)$$

$$I_{B3} = \frac{I_{E3}}{\beta+1} = \frac{\beta}{(\beta+1)^2}\left(1 + \frac{1}{\beta+1}\right)\frac{I}{2}$$

Since I_{B3} is the input current to the $Q_7 - Q_8$ mirror and I_{C8} is its output current, we have

$$\frac{I_{C8}}{I_{B3}} = \frac{1}{1 + \dfrac{2}{\beta}} = \frac{\beta}{\beta+2}$$

Thus,

$$I_{C8} = \frac{\beta^2}{(\beta+1)^2(\beta+2)}\left(1 + \frac{1}{\beta+1}\right)\frac{I}{2}$$

At the input node, we have

$$I_B = I_{B1} - I_{C8}$$

$$= \frac{I/2}{\beta+1} - \frac{\beta^2}{(\beta+1)^2(\beta+2)}\left(1+\frac{1}{\beta+1}\right)\frac{I}{2}$$

$$I_B = \frac{I/2}{\beta+1}\left[1-\frac{\beta^2}{(\beta+1)(\beta+2)}\left(1+\frac{1}{\beta+1}\right)\right]$$

$$= \frac{I/2}{\beta+1}\left[1-\frac{\beta^2}{(\beta+1)(\beta+2)}\,\frac{\beta+2}{\beta+1}\right]$$

$$= \frac{I/2}{\beta+1}\,\frac{(\beta+1)^2-\beta^2}{(\beta+1)^2}$$

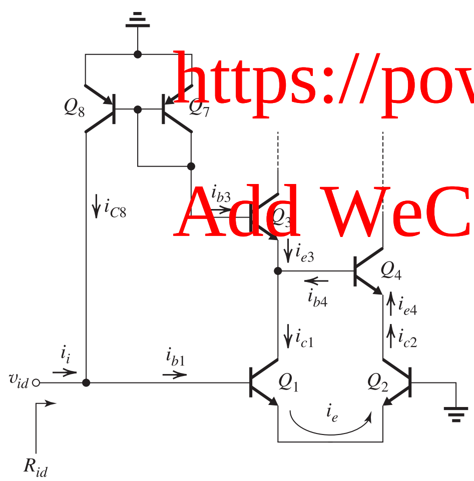
$$= \frac{I/2}{\beta+1}\,\frac{2\beta+1}{(\beta+1)^2}$$

$$\simeq \frac{I/2}{\beta+1}\,\frac{2\beta}{\beta^2}$$

$$= \frac{I/2}{\beta+1}\Big/\left(\frac{\beta}{2}\right)$$

Thus, including the current mirror $Q_7 - Q_8$ reduces the input bias current by a factor equal to $(\beta/2)$, a substantial decrease!

(b)



(b)

The analysis follows the same process used above, except that here we deal with signal quantities.

$$i_e = \frac{v_{id}}{2r_e}$$

where $r_e = r_{e1} = r_{e2}$

$$i_{c1} = i_{c2} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}$$

$$i_{e4} = i_{c2} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}$$

$$i_{b4} = \frac{i_{e4}}{\beta+1} = \frac{\beta}{(\beta+1)^2}\,\frac{v_{id}}{2r_e}$$

$$i_{e3} = i_{c1} - i_{b4} = \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e} - \frac{\beta}{(\beta+1)^2}\,\frac{v_{id}}{2r_e}$$

$$= \frac{\beta}{\beta+1}\,\frac{v_{id}}{2r_e}\left(1-\frac{1}{\beta+1}\right)$$

$$= \left(\frac{\beta}{\beta+1}\right)^2\frac{v_{id}}{2r_e}$$

$$i_{b3} = \frac{i_{e3}}{\beta+1} = \frac{\beta^2}{(\beta+1)^3}\,\frac{v_{id}}{2r_e}$$

$$i_{c8} = i_{b3}\,\frac{1}{1+\dfrac{2}{\beta}} = i_{b3}\,\frac{\beta}{\beta+2}$$

$$= \frac{\beta^3}{(\beta+1)^3}\,\frac{1}{\beta+2}\,\frac{v_{id}}{2r_e}$$

At the input node, we have

$$i_i = i_{b1} - i_{c8}$$

$$= \frac{v_{id}/2r_e}{\beta+1} - \frac{\beta^3}{(\beta+1)^3}\,\frac{1}{\beta+2}\,\frac{v_{id}}{2r_e}$$

$$= \frac{v_{id}}{2(\beta+1)r_e}\left[1-\frac{\beta^3}{(\beta+1)^2}\,\frac{1}{\beta+2}\right]$$

$$\simeq \frac{v_{id}}{2(\beta+1)r_e}\left[1-\frac{\beta^3}{(\beta+1)^3}\right]$$

$$= \frac{v_{id}}{2(\beta+1)r_e}\left[1-\frac{1}{(1+1/\beta)^3}\right]$$

$$\simeq \frac{v_{id}}{2(\beta+1)r_e}\,\frac{3}{\beta}$$

$$= \left[\frac{v_{id}}{2(\beta+1)r_e}\right]\Big/\frac{\beta}{3}$$

Thus the input current is reduced by a factor $(\beta/3)$, which results in R_{id} increasing by a factor $(\beta/3)$.

9.101 To maximize the positive output voltage swing, we select V_{BIAS} as large as possible while maintaining the *pnp* current sources in saturation. For the latter to happen, we need a minimum of 0.3 V across each current source. Thus the maximum allowable voltage at the emitters of Q_3 and Q_4 is $V_{CC} - 0.3 = 5 - 0.3 = +4.7$ V. Then, the maximum allowable value of $V_{\text{BIAS}} = 4.7 - 0.7 = +4$ V. To keep Q_4 in saturation,

$$v_{O\max} = V_{\text{BIAS}} + 0.4 = 4.4 \text{ V}$$

If the dc voltage at the output is 0 V, then the maximum positive voltage swing is 4.4 V. In the negative direction,

$$v_{O\min} = -V_{EE} + V_{BE7} + V_{BE5} - 0.4$$

$= -5 + 0.7 + 0.7 - 0.4$

$= -4$ V

Thus,

$-4 \text{ V} \leq v_O \leq +4.4 \text{ V}$

$$G_m = g_{m1,2} \simeq \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$R_{o4} = \beta_4 r_{o4} = 50 \times \frac{|V_A|}{I/2}$$

$$= 50 \times \frac{100 \text{ V}}{0.25 \text{ mA}} = 20 \text{ M}\Omega$$

$$R_{o5} = \frac{1}{2}\beta_5 r_{o5} = \frac{1}{2} \times 100 \times \frac{100}{0.25}$$

$= 20$ MΩ

$R_o = R_{o4} \parallel R_{o5} = 20 \text{ M}\Omega \parallel 20 \text{ M}\Omega = 10 \text{ M}\Omega$

$A_d = G_m R_o = 10 \times 10{,}000 = 10^5$ V/V

9.102 The overdrive voltage, $|V_{OV}|$, at which Q_1 and Q_2 are operating is found from

$$\frac{I}{2} = \frac{1}{2}k_p'(W/L)|V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 6.4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.18$ V

$$G_m = g_{m1,2} = \frac{2(I/2)}{|V_{OV}|}$$

$$= \frac{0.2}{0.18} = 1.13 \text{ mA/V}$$

$$r_{o2} = \frac{|V_{Ap}|}{I/2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Anpn}|}{I/2} = \frac{30}{0.1} = 300 \text{ k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 100 \text{ k}\Omega \parallel 300 \text{ k}\Omega = 75 \text{ k}\Omega$

$A_d = G_m R_o = 1.13 \times 75 = 85$ V/V

9.103 (a) For Q_1 and Q_2,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$0.1 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 12.5$$

For Q_3 and Q_4,

$$\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$0.1 = \frac{1}{2} \times 0.1 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$$

(b) $G_m = g_{m1,2} = \dfrac{2\,(I/2)}{V_{OV}} = \dfrac{I}{V_{OV}} = \dfrac{0.2}{0.2} =$ 1 mA/V

$A_d = G_m R_o$

$50 = 1 \times R_o$

$\Rightarrow R_o = 50$ kΩ

But

$R_o = r_{o2} \parallel r_{o4}$

and $r_{o2} = r_{o4}$ (Q_2 and Q_4 have the same $I_D = \dfrac{I}{2}$ and the same V_A). Thus

$$r_{o2} = r_{o4} = 100 \text{ k}\Omega = \frac{|V_A|}{I/2}$$

$$|V_A| = \frac{I}{2} \times 100 \text{ k}\Omega = 10 \text{ V}$$

$10 = |V_A'|L = 20\,L$

$\Rightarrow L = 0.5$ μm

(c) $v_{O\min} = V_{ICM} - V_{tn}$

$= 0 - 0.5 = -0.5$ V

$v_{O\max} = V_{DD} - |V_{OV}| = 1 - 0.2 = 0.8$ V

Thus,

$-0.5 \text{ V} \leq v_O \leq 0.8 \text{ V}$

(d) $R_{SS} = \dfrac{|V_A|}{I} = \dfrac{10}{0.2} = 50$ kΩ

The CMRR can be obtained using Eq. (9.159):

$\text{CMRR} = (g_m r_o)(g_m R_{SS})$

$= (1 \times 100)(1 \times 50)$

$= 5000$ or 74 dB

9.104 The CMRR is given by Eq. (9.158):

$\text{CMRR} = [g_{m1,2}(r_{o2} \parallel r_{o4})]\,[2\,g_{m3}R_{SS}]$

(a) Current source is implemented with a simple current mirror:

$$R_{SS} = r_o|_{Q_S} = \frac{|V_A|}{I}$$

$$g_{m1,2} = g_{m3} = \frac{2(I/2)}{V_{OV}} = \frac{I}{V_{OV}}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{2|V_A|}{I}$$

Thus,

$$\text{CMRR} = \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{|V_A|}{I}$$

$$= 2\left(\frac{V_A}{V_{OV}}\right)^2 \qquad \text{Q.E.D.}$$

(b) Current source is implemented with the modified Wilson mirror in Fig. P9.89:

$R_{SS} = g_{m7} r_{o7} r_{o9}$

Transistor Q_7 has the same $k'(W/L)$ as Q_1 and Q_2, but Q_7 carries a current I twice that of Q_1 and Q_2. Thus

$$V_{OV7} = \sqrt{2} V_{OV1,2} = \sqrt{2} V_{OV}$$

and

$$g_{m7} = \frac{2I}{V_{OV7}} = \frac{2I}{\sqrt{2} V_{OV}} = \frac{\sqrt{2} I}{V_{OV}}$$

$$r_{o7} = r_{o9} = \frac{V_A}{I}$$

Thus,

$$R_{SS} = \frac{\sqrt{2} I}{V_{OV}} \left(\frac{V_A}{I}\right)^2 = \frac{\sqrt{2} V_A^2}{V_{OV} I}$$

and

$$\text{CMRR} = \frac{I}{V_{OV}} \times \frac{1}{2} \times \frac{2|V_A|}{I} \times 2 \times \frac{I}{V_{OV}} \times \frac{\sqrt{2} V_A^2}{V_{OV} I}$$

$$= 2\sqrt{2} \left(\frac{|V_A|}{|V_{OV}|}\right)^3 \quad \text{Q.E.D}$$

For $k'(W/L) = 4\ \text{mA/V}^2$ and $I = 160\ \mu\text{A}$,

$$0.080 = \frac{1}{2} \times 4 \times |V_{OV}|^2$$

$\Rightarrow |V_{OV}| = 0.2$ V

For $|V_A| = 5$ V:

For case (a),

$$\text{CMRR} = 2 \times \left(\frac{5}{0.2}\right)^2 = 1250 \text{ or } 62 \text{ dB}$$

For case (b),

$$\text{CMRR} = 2\sqrt{2} \left(\frac{5}{0.2}\right)^3 = 4.42 \times 10^4$$

or 93 dB

9.105 $G_m = g_{m1,2} = \dfrac{2(I/2)}{V_{OV}} = \dfrac{0.2}{0.2} = 1$ mA/V

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{5}{0.1} = 50\ \text{k}\Omega$$

$R_o = r_{o2} \parallel r_{o4} = 50\ \text{k}\Omega \parallel 50\ \text{k}\Omega$

$= 25\ \text{k}\Omega$

$A_d = G_m R_o = 1 \times 25 = 25$ V/V

$$R_{SS} = \frac{|V_A|}{I} = \frac{5}{0.2} = 25\ \text{k}\Omega$$

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 25} = 0.02\ \text{mA/V}$$

$$R_{im} = \frac{1}{g_{m3}} \parallel r_{o3}$$

where

$g_{m3} = g_{m1} = g_{m2} = 1$ mA/V

$r_{o3} = r_{o2} = r_{o4} = 50\ \text{k}\Omega$

$R_{im} = 1\ \text{k}\Omega \parallel 50\ \text{k}\Omega = 0.98\ \text{k}\Omega$

$$A_m = 1 \Big/ \left(1 + \frac{1}{g_m r_{o3}}\right)$$

$$= 1 \Big/ \left(1 + \frac{1}{1 \times 50}\right) = 0.98\ \text{A/A}$$

$R_{om} = r_{o4} = 50\ \text{k}\Omega$

$R_{o2} = r_{o2} + 2R_{SS} + 2g_{m2} r_{o2} R_{SS}$

$= 50 + 50 + 2 \times 1 \times 50 \times 25$

$= 2600\ \text{k}\Omega$

$A_{cm} = -(1 - A_m) G_{mcm} (R_{om} \parallel R_{o2})$

$A_{cm} = -(1 - 0.98) \times 0.02 \times (50 \parallel 2600)$

$= -0.0196$ V/V

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{25}{0.0196} = 1274$$

or 62.1 dB

Alternatively, using the approximate expression in Eq. (9.157), we obtain

$$A_{cm} \simeq -\frac{1}{2g_{m3} R_{SS}} = -\frac{1}{2 \times 1 \times 25} = -0.02\ \text{V/V}$$

and

$$\text{CMRR} = \frac{25}{0.02} = 1250$$

or 61.9 dB

9.106 From Eq. (9.153), we have

$A_{cm} = -(1 - A_m) G_{mcm} (R_{om} \parallel R_{o2})$

where

$$G_{mcm} = \frac{1}{2R_{SS}} = \frac{1}{2 \times 45} = 0.011\ \text{mA/V}$$

Using the fact that $R_{o2} \gg R_{om}$, we obtain

$A_{cm} \simeq -(1 - 0.98) \times 0.011 \times 45$

$= -0.01$ V/V

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right| = \frac{30}{0.01} = 3000$$

or 69.5 dB

9.107 $\text{CMRR} = \left|\dfrac{A_d}{A_{cm}}\right|$

CMRR = 60 dB or equivalently 1000. Thus,

$$1000 = \frac{50}{|A_{cm}|}$$

$$\Rightarrow |A_{cm}| = 0.05 \text{ V/V}$$

But from Eq. (9.153), we obtain

$$|A_{cm}| = (1 - A_m)G_{mcm}(R_{om} \parallel R_{o2})$$

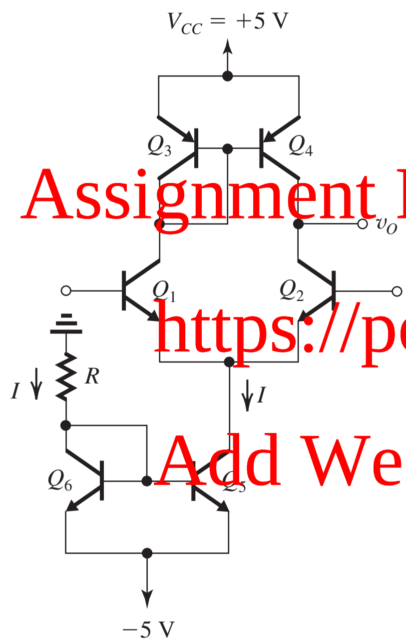
Since $R_{om} \ll R_{o2}$ and $G_{mcm} = 1/2R_{SS}$, we have

$$|A_{cm}| = (1 - A_m)\frac{R_{om}}{2R_{SS}}$$

$$0.05 = (1 - A_m) \times \frac{20}{2 \times 20}$$

$$\Rightarrow (1 - A_m) = 0.1$$

9.108



$$G_m = g_{m1,2} \simeq \frac{I/2}{V_T}$$

$$5 = \frac{I/2}{V_T}$$

$$\Rightarrow I = 0.25 \text{ mA}$$

Utilizing two matched transistors, Q_5 and Q_6, the value of R can be found from

$$I = \frac{0 - (-5) - 0.7}{R} = 0.25 \text{ mA}$$

$$\Rightarrow R = 17.2 \text{ k}\Omega$$

$$R_{id} = 2r_\pi = 2\frac{\beta}{g_m} = 2 \times \frac{100}{5} = 40 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4}$$

where

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

thus

$$R_o = 800 \text{ k}\Omega \parallel 800 \text{ k}\Omega = 400 \text{ k}\Omega$$

$$A_d = G_m R_o = 5 \times 400 = 2000 \text{ V/V}$$

$$I_B = \frac{I/2}{\beta + 1} \simeq \frac{0.125 \text{ mA}}{100} = 1.25 \text{ µA}$$

The lower limit on V_{ICM} is determined by the lowest voltage allowed at the collector of Q_5 while Q_5 is in the active mode. This voltage is $-5 + 0.3 = -4.7$ V. Thus

$$V_{ICM\,\text{min}} = -4.7 + V_{BE1,2} = -4.7 + 0.7$$

$$= -4 \text{ V}$$

The upper limit on V_{ICM} is determined by the need to keep Q_1 in the active mode. Thus

$$V_{ICM\,\text{max}} = V_{C1} + 0.4$$

$$= 4.3 + 0.4 = 4.7 \text{ V}$$

Thus the input common-mode range is

$$-4 \text{ V} \le V_{ICM} \le +4.7 \text{ V}$$

The common-mode gain can be found using Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

Here

$$r_{o4} = \frac{V_A}{I/2} = \frac{100}{0.125} = 800 \text{ k}\Omega$$

$$\beta_3 = 100$$

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

Thus

$$A_{cm} = -\frac{800}{100 \times 400} = -0.02 \text{ V/V}$$

The CMRR can be found as

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{0.02} = 100{,}000$$

or 100 dB

9.109 See figure on next page.

From the solution to Problem 9.108, we know that $I = 0.25$ mA. For the Widlar current source, use $R = 2$ kΩ. Thus

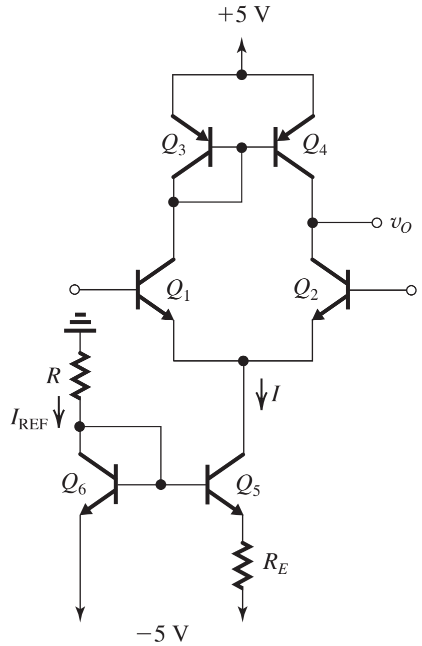
$$I_{\text{REF}} = \frac{5 - 0.7}{2} = 2.15 \text{ mA}$$

The value of R_E can be found from

$$IR_E = V_{BE6} - V_{BE5} = V_T \ln\left(\frac{I_{\text{REF}}}{I}\right)$$

$$0.25 \times R_E = 0.025 \ln\left(\frac{2.15}{0.25}\right)$$

$$R_E = 215 \ \Omega$$



The output resistance of the Widlar current source is given by Eq. (8.102). Thus

$$R_{EE} = [1 + g_{m5}(R_E \parallel r_{\pi 5})]r_{o5}$$

where

$$g_{m5} = \frac{I}{V_T} = \frac{0.25 \text{ mA}}{0.025 \text{ V}} = 10 \text{ mA/V}$$

$$r_{\pi 5} = \frac{\beta}{g_{m5}} = \frac{100}{10} = 10 \text{ k}\Omega$$

$$r_{o5} = \frac{V_A}{I} = \frac{100}{0.25} = 400 \text{ k}\Omega$$

$$R_{EE} = [1 + 10(0.215 \parallel 10)] \times 400$$

$$= 1.24 \text{ M}\Omega$$

R_{id}, R_o, A_d, I_B, and the range of V_{ICM} will be the same as in Problem 9.108. The common-mode gain, however, will be lower:

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

$$= -\frac{800}{100 \times 1240} = 6.45 \times 10^{-3} \text{ V/V}$$

and the CMRR will be

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{2000}{6.45 \times 10^{-3}} = 3.1 \times 10^5$$

or 110 dB

9.110 $G_m = g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{0.2}{0.025} = 8 \text{ mA/V}$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{40}{0.2} = 200 \text{ k}\Omega$$

$$R_o = 200 \text{ k}\Omega \parallel 200 \text{ k}\Omega = 100 \text{ k}\Omega$$

$$A_d = G_m R_o = 8 \times 100 = 800 \text{ V/V}$$

$$R_{id} = 2r_\pi = 2\beta/g_m$$

$$= \frac{300}{8} = 37.5 \text{ k}\Omega$$

$$R_{EE} = \frac{|V_A|}{I} = \frac{40}{0.4} = 100 \text{ k}\Omega$$

The common-mode gain can be found using Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

$$= -\frac{200}{150 \times 100} = -0.013 \text{ V/V}$$

The CMRR can be obtained from

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{800}{0.013} = 60{,}000$$

or 96 dB

$$G_v = \frac{R_{id}}{R_{id} + R_{\text{sig}}} \times A_d$$

$$= \frac{37.5}{37.5 + 30} \times 800 = 444.4 \text{ V/V}$$

9.111 Refer to Fig. P9.111. To determine the bias current I, which is the current in the collector of Q_5, we first find the reference current through the 6.65-kΩ resistor:

$$I_{REF} = \frac{9 - (-5) - 0.7}{6.65} = 2 \text{ mA}$$

Assuming Q_5 and Q_6 are matched, we have

$$I = 2 \text{ mA}$$

(a) $g_{m1,2} \simeq \dfrac{I/2}{V_T} = \dfrac{1 \text{ mA}}{0.025 \text{ V}} = 40 \text{ mA/V}$

$$R_{id} = 2r_\pi = 2\beta/g_{m1,2}$$

$$= \frac{2 \times 100}{40} = 5 \text{ k}\Omega$$

(b) $A_d = G_m R_o$

where

$$G_m = g_{m1,2} = 40 \text{ mA/V}$$

$$R_o = r_{o2} \parallel r_{o4}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{1} = 60 \text{ k}\Omega$$

$$R_o = 60 \text{ k}\Omega \parallel 60 \text{ k}\Omega = 30 \text{ k}\Omega$$

$$A_d = 40 \times 30 = 1200 \text{ V/V}$$

(c) A_{cm} can be found using Eq. (9.165),

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where

$$R_{EE} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{2} = 30\ \text{k}\Omega$$

$$A_{cm} = -\frac{60}{100 \times 30} = -0.02\ \text{V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{0.02} = 60,000$$

or 95.6 dB

9.112 Refer to Fig. P9.112. To determine the bias current I, which is the drain current of Q_7, we analyze the Wilson mirror circuit as follows: All four transistors, $Q_5 - Q_8$, are conducting equal currents (I) and have the same V_{GS},

$$V_{GS} = V_t + V_{OV}$$

Thus

$$IR = 15 - (-5) - 2\ V_{GS}$$

$$144I = 20 - 2\ V_t - 2\ V_{OV}$$

But

$$I = \frac{1}{2} k_n'(W/L)V_{OV}^2$$

$$= \frac{1}{2} \times 2 \times V_{OV}^2 = V_{OV}^2$$

Thus

$$144\ V_{OV}^2 = 20 - 2 \times 0.7 - 2V_{OV}$$

$$144\ V_{OV}^2 + 2V_{OV} - 18.6 = 0$$

$$\Rightarrow V_{OV} = 0.35\ \text{V}$$

and

$$I = 0.35^2 = 0.12\ \text{mA}$$

(a) $R_{id} = 2r_\pi = 2\beta/g_m$

where

$$g_m = g_{m1,2} \simeq \frac{I/2}{V_T} = \frac{0.06}{0.025} = 2.4\ \text{mA}$$

$$R_{id} = \frac{2 \times 100}{2.4} = 83.3\ \text{k}\Omega$$

(b) $A_d = g_{m1,2}R_o$

where

$$R_o = r_{o2} \parallel r_{o4}$$

But

$$r_{o2} = r_{o4} = \frac{|V_A|}{I/2} = \frac{60}{0.06} = 1\ \text{M}\Omega$$

$$R_o = 500\ \text{k}\Omega$$

$$A_d = 2.4 \times 500 = 1200\ \text{V/V}$$

(c) A_{cm} can be found from Eq. (9.165):

$$A_{cm} = -\frac{r_{o4}}{\beta_3 R_{EE}}$$

where R_{EE} is the output resistance of the Wilson mirror,

$$R_{EE} = g_{m7}r_{o7}r_{o5}$$

where

$$g_{m7} = \frac{2I}{V_{OV}} = \frac{2 \times 0.12}{0.35}$$

$$= 0.7\ \text{mA/V}$$

$$r_{o7} = r_{o5} = \frac{|V_A|}{I} = \frac{60}{0.12} = 500\ \text{k}\Omega$$

$$R_{EE} = 0.7 \times 500^2 = 175\ \text{M}\Omega$$

$$A_{cm} = -\frac{1}{100 \times 175} = 5.7 \times 10^{-5}\ \text{V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{1200}{5.7 \times 10^{-5}} = 21 \times 10^6$$

or 146 dB

9.113 Refer to Fig. 9.40.

W_6 can be determined using Eq. (9.172):

$$\frac{(W/L)_6}{(W/L)_4} = 2\,\frac{(W/L)_7}{(W/L)_5}$$

$$\frac{(W/0.5)_6}{(10/0.5)} = 2\,\frac{(60/0.5)}{(60/0.5)}$$

$$\Rightarrow W_6 = 20\ \mu\text{m}$$

For all devices we can evaluate I_D as follows:

$$I_{D8} = I_{\text{REF}} = 225\ \mu\text{A}$$

$$I_{D5} = I_{\text{REF}}\frac{(W/L)_5}{(W/L)_8} = I_{\text{REF}} = 225\ \mu\text{A}$$

$$I = I_{D5} = 225\ \mu\text{A}$$

$$I_{D1} = I_{D2} = \frac{1}{2}I_{D5} = 112.5\ \mu\text{A}$$

$$I_{D3} = I_{D4} = I_{D1} = 112.5\ \mu\text{A}$$

$$I_{D6} = I_{D7} = I_{\text{REF}} = 225\ \mu\text{A}$$

With I_D in each device known, we can use

$$I_{Di} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_i |V_{OVi}|^2$$

to determine $|V_{OVi}|$ and then

$$|V_{GSi}| = |V_{OVi}| + |V_t|$$

The values of g_{mi} and r_{oi} can then be determined from

$$g_{mi} = \frac{2I_{Di}}{|V_{OVi}|}$$

$$r_{oi} = \frac{|V_A|}{I_{Di}}$$

The results are summarized in the following table.

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D (μA)	112.5	112.5	112.5	112.5	225	225	225	225
$\lvert V_{OV}\rvert$ (V)	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$\lvert V_{GS}\rvert$ (V)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
g_m (mA/V)	0.9	0.9	0.9	0.9	1.8	1.8	1.8	1.8
r_o (kΩ)	80	80	80	80	40	40	40	40

$A_1 = -g_{m1}(r_{o2} \parallel r_{o4})$

$= -0.9 \times (80 \parallel 80) = -36$ V/V

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

$= -1.8 \times (40 \parallel 40) = -36$ V/V

$A_0 = A_1 A_2 = -36 \times -36 = 1296$ V/V

The upper limit of V_{ICM} is determined by the need to keep Q_5 in saturation, thus

$V_{ICM\max} = V_{DD} - |V_{OV5}| - |V_{SG1}|$

$= 1.5 - 0.25 - 1 = +0.25$ V

The lower limit of V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$V_{ICM\min} = V_{G3} - |V_t|$

$= -V_{SS} + |V_{GS3}| - |V_t|$

$= -1.5 + 1 - 0.75 = -1.25$ V

Thus

$-1.25 \text{ V} \le V_{ICM} \le +0.25$ V

The output voltage range is

$-V_{SS} + V_{OV6} \le v_O \le V_{DD} - |V_{OV7}|$

that is,

$-1.25 \text{ V} \le v_O \le +1.25$ V

9.114

(a) $I_{D1} = I_{D2} = 100 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$$100 = \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{1,2} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 12.5$$

$$I_{D3} = I_{D4} = 100 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$100 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.04$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$$

$I_{D5} = I_{D7} = I_{D8} = 200$ μA

Thus

$$200 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{5,7,8} V_{OV}^2$$

$$= \frac{1}{2} \times 400 \times \left(\frac{W}{L}\right)_{5,7,8} \times 0.04$$

$$\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_8 = 25$$

$$I_{D6} = 200 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_6 |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.04$$

$$\left(\frac{W}{L}\right)_6 = 100$$

The results are summarized in the following table:

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
W/L	12.5	12.5	50	50	25	100	25	25

Ideally, the dc voltage at the output is zero.

(b) The upper limit of V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$V_{ICM\max} = V_{D1} + V_t$

$= V_{DD} - |V_{SG4}| + V_t$

$= 0.9 - |V_t| - |V_{OV4}| + V_t$

$= 0.9 - 0.2 = +0.7$ V

The lower limit of V_{ICM} is determined by the need to keep Q_5 in saturation,

$V_{ICM\min} = -0.9 + |V_{OV5}| + |V_{GS1}|$

$= -0.9 + 0.2 + 0.2 + 0.4 = -0.1$ V

Thus

$-0.1 \text{ V} \le V_{ICM} \le +0.7$ V

(c) $v_{O\max} = V_{DD} - |V_{OV6}|$

$= 0.9 - 0.2 = +0.7$ V

$v_{O\min} = -V_{SS} + |V_{OV7}|$

$= -0.9 + 0.2 = -0.7$ V

Thus

$-0.7 \text{ V} \le v_O \le +0.7 \text{ V}$

(d) $A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4})$

where

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.1 \text{ mA}} = \frac{6}{0.1} = 60 \text{ k}\Omega$$

$A_1 = -1 \times (60 \parallel 60) = -30$ V/V

$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$

where

$$g_{m6} = \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.2} = \frac{6}{0.2} = 30 \text{ k}\Omega$$

$A_2 = -2 \times (30 \parallel 30) = -30$ V/V

$A_0 = A_1 A_2 = 30 \times 30 = 900$ V/V

9.115 (a) Increasing $(W/L)_1$ and $(W/L)_2$ by a factor of 4 reduces $|V_{OV1,2}|$ by a factor of 2. Thus $g_{m1,2} = 2I_D/|V_{OV1,2}|$ increase by a factor of 2.

(b) A_1 is proportional to $g_{m1,2}$, thus A_1 increases by a factor of 2 and the overall voltage gain increases by a factor of 2.

(c) Since the input offset voltage is proportional to $|V_{OV1,2}|$, it will decrease by a factor of 2. This, however, does not apply to V_{OS} due to ΔV_t.

9.116 If $(W/L)_7$ becomes 48/0.8, I_{D7} will become

$$I_{D7} = I_{D8} \frac{(W/L)_7}{(W/L)_8}$$

$$= I_{\text{REF}} \frac{(48/0.8)}{(40/0.8)}$$

$= 90 \times 1.2 = 108$ µA

Thus I_{D7} will exceed I_{D6} by 18 µA, which will result in a systematic offset voltage,

$V_O = 18 \text{ µA}(r_{o6} \parallel r_{o7})$

where

$r_{o6} = 111$ kΩ

and r_{o7} now becomes

$$r_{o7} = \frac{10}{0.108} = 92.6 \text{ k}\Omega$$

Thus

$V_O = 18 \times 10^{-3} \times (111 \parallel 92.6)$

$= 909$ mV

The corresponding input offset voltage will be

$$V_{OS} = \frac{V_O}{A_0}$$

$$= \frac{909}{1109} = 0.82 \text{ mV}$$

9.117 Refer to Fig. 9.40 and let the two input terminals be grounded. Then,

$$I_{D1} = I_{D2} = \frac{I}{2}$$

If Q_3 has a threshold voltage V_t and Q_4 has a threshold voltage $V_t + \Delta V_t$ then

$$I_{D3} = \frac{I}{2} = \frac{1}{2} k_{n3}(V_{GS3} - V_t)^2$$

$$\Rightarrow V_{GS3} = V_t + \sqrt{I/k_{n3}}$$

$$I_{D4} = \frac{1}{2} k_{n4}(V_{GS4} - V_t - \Delta V_t)^2$$

Since $k_{n4} = k_{n3}$ and $V_{GS4} = V_{GS3}$ we have

$$I_{D4} = \frac{I}{2} k_{n3}(V_{GS3} - V_t - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3}(\sqrt{I/k_{n3}} - \Delta V_t)^2$$

$$= \frac{1}{2} k_{n3} \frac{I}{k_{n3}} \left(1 - \frac{\Delta V_t}{\sqrt{I/k_{n3}}}\right)^2$$

$$= \frac{I}{2}\left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

The output current of the first stage will be

$$I_O = I_{D2} - I_{D4}$$

$$= \frac{I}{2} - \frac{I}{2}\left(1 - \frac{\Delta V_t}{V_{OV3}}\right)^2$$

For $\dfrac{\Delta V_t}{V_{OV3}} \ll 1$ we obtain

$$I_O \simeq \frac{I}{2} - \frac{I}{2}\left(1 - \frac{2\Delta V_t}{V_{OV3}}\right)$$

$$= \frac{2(I/2)}{V_{OV3}} \Delta V_t$$

$= g_{m3}\,\Delta V_t$ Q.E.D.

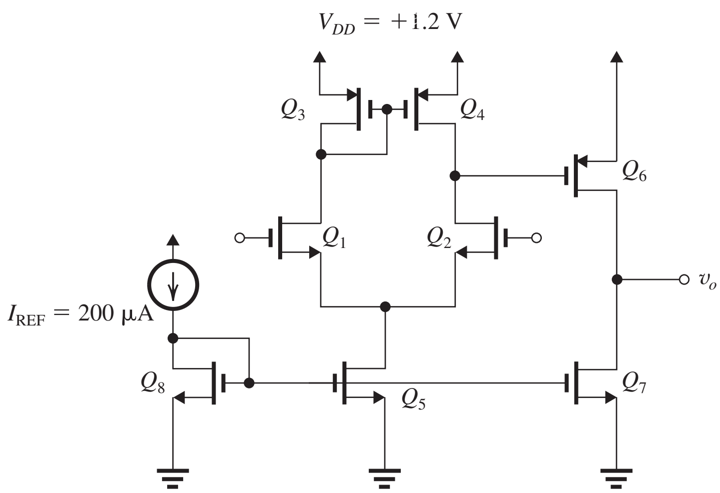
The corresponding input offset voltage will be

$$V_{OS} = \frac{I_O}{g_{m1,2}}$$

$$= \frac{g_{m3}\,\Delta V_t}{g_{m1,2}}$$

$$V_{OS} = \frac{g_{m3}}{g_{m1,2}}\,\Delta V_t$$

9.118



(a) With the two input terminals connected to a dc voltage of $V_{DD}/2 = +0.6$ V and for $Q_1 - Q_4$ to conduct a current of 200 µA, we have

$$I_{D1,2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$200 = \frac{1}{2} \times 540 \times \left(\frac{W}{L}\right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 32.9$$

$$I_{D3,4} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_{3,4} |V_{OV}|^2$$

$$200 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{3,4} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{3,4} = 178$$

Transistor Q_5 must carry a current of 400 µA, thus

$$400 = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_5 V_{OV}^2$$

$$= \frac{1}{2} \times 540 \times \left(\frac{W}{L}\right)_5 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 65.8$$

Similarly, Q_7 is required to conduct a current of 400 µA, thus

$$\left(\frac{W}{L}\right)_7 = \left(\frac{W}{L}\right)_5 = 65.8$$

Transistor Q_8 conducts a current of 200 µA, thus

$$\left(\frac{W}{L}\right)_8 = \frac{1}{2}\left(\frac{W}{L}\right)_5 = 32.9$$

Finally, Q_6 must conduct a current equal to that of Q_7, that is, 400 µA, thus

$$400 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = 356$$

The results are summarized in the following table:

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D (µA)	200	200	200	200	400	400	400	200
W/L	32.9	32.9	178	178	65.8	356	65.8	32.9

(b) The upper limit on V_{ICM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$$V_{ICM\max} = V_{D1,2} + |V_t|$$

$$= V_{DD} - |V_t| - |V_{OV}| + |V_t|$$

$$= 1.2 - 0.15 = 1.05 \text{ V}$$

The lower limit on V_{ICM} is determined by the need to keep Q_5 in saturation, thus

$$V_{ICM} = |V_{OV5}| + V_{GS1,2}$$

$$= 0.15 + 0.15 + 0.35 = 0.65 \text{ V}$$

Thus

$$0.65 \text{ V} \le V_{ICM} \le 1.05 \text{ V}$$

Note that the input dc voltage in part (a) falls outside the allowable range of V_{ICM}! Thus, part (a) should have specified a V_{ICM} greater than 0.65 V. The results of part (a), however, will not change.

(c) $0.15 \text{ V} \le v_O \le (1.2 - 0.15)$

that is,

$$0.15 \text{ V} \le v_O \le 1.05 \text{ V}$$

(d) $g_{m1,2} = \dfrac{2 \times 0.2}{0.15} = 2.67$ mA/V

$$r_{o2} = r_{o4} = \frac{|V_A|}{0.2\text{ mA}} = \frac{1.8}{0.2} = 9\text{ k}\Omega$$

$$A_1 = -g_{m1,2}(r_{o2} \parallel r_{o4}) = 2.67(9 \parallel 9)$$

$$= -12\text{ V/V}$$

$$g_{m6} = \frac{2 \times 0.4}{0.15} = 5.33\text{ mA/V}$$

$$r_{o6} = r_{o7} = \frac{|V_A|}{0.4\ \text{ mA}} = \frac{1.8}{0.4} = 4.5\text{ k}\Omega$$

$$A_2 = -g_{m6}(r_{o6} \parallel r_{o7})$$

$$= -5.33(4.5 \parallel 4.5) = 12\text{ V/V}$$

$$A_0 = A_1 A_2 = -12 \times -12 = 144\text{ V/V}$$

9.119 Refer to Fig. P9.119.

(a) With the inputs grounded and the output at 0 V dc, we have

$$I_{E1} = I_{E2} = \frac{1}{2} \times 0.4 = 0.2\text{ mA}$$

$$I_{E3} = I_{E4} \simeq 0.2\text{ mA}$$

$$I_{E5} \simeq 0.5\text{ mA}$$

$$I_{E6} = 1\text{ mA}$$

(b) The short-circuit transconductance of the first stage is

$$G_m = g_{m1,2} = \frac{I_{C1,2}}{V_T} \simeq \frac{0.2\text{ mA}}{0.025\text{ V}} = 8\text{ mA/V}$$

The voltage gain of the first stage can be obtained by multiplying G_m by the total resistance at the output node of the stage, i.e., the common collectors of Q_2 and Q_4 and the base of Q_5. Since $r_{o2} = r_{o4} = \infty$, the resistance at this node is equal to the input resistance of Q_5 which is $R_{\pi 5}$,

$$r_{\pi 5} = \frac{\beta}{g_{m5}}$$

where

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5}{0.025} = 20\text{ mA/V}$$

thus

$$r_{\pi 5} = \frac{100}{20} = 5\text{ k}\Omega$$

Thus the voltage gain of the first stage is given by

$$A_1 \equiv \frac{v_{b5}}{v_{id}} = -G_m r_{\pi 5}$$

$$= -8 \times 5 = -40\text{ V/V}$$

The voltage gain of the second stage is

$$A_2 \equiv \frac{v_{c5}}{v_{b5}} = -g_{m5} R_C$$

where R_{C5} is the total resistance in the collector of Q_5. Since $r_{o5} = \infty$, R_{C5} is simply the input resistance of the emitter follower Q_6, we have

$$R_{C5} = R_{i6} = (\beta + 1)(r_{e6} + R_L)$$

where

$$r_{e6} = \frac{V_T}{I_{E6}} = \frac{25\text{ mV}}{1\text{ mA}} = 25\ \Omega$$

$$R_{i6} = (100 + 1)(0.025 + 1)$$

$$= 103.5\text{ k}\Omega$$

Thus

$$A_2 = -20 \times 103.5 = -2070\text{ V/V}$$

The gain of the third stage is given by

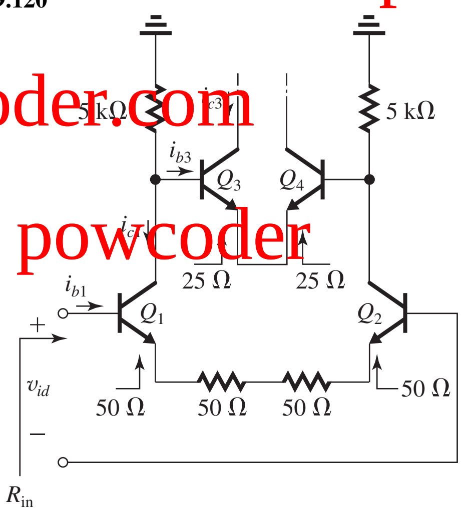
$$A_3 = \frac{v_o}{v_5} = \frac{R_L}{R_L + r_{e6}} = \frac{1}{1 + 0.025} = 0.976\text{ V/V}$$

The overall voltage gain can now be obtained as

$$A_0 \equiv \frac{v_o}{v_{id}} = A_1 A_2 A_3$$

$$= -40 \times -2070 \times 0.976 = 8.07 \times 10^4\text{ V/V}$$

9.120



$$R_{\text{in}2} = 2(\beta + 1)(25 + 25)$$

$$= 2 \times 101 \times 50 \simeq 10\text{ k}\Omega$$

Effective load of first stage $= R_{\text{in}2} \parallel (5 + 5)$

$$= 10 \parallel 10 = 5\text{ k}\Omega$$

$$A_1 = \alpha \frac{\text{Total resistance between collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{5\text{ k}\Omega}{4 \times 50\ \Omega} = 25\text{ V/V}$$

$$R_{\text{in}} = (\beta + 1)(4 \times 50\ \Omega)$$

$$= 101 \times 200 \simeq 20\text{ k}\Omega$$

$$\frac{i_{c1}}{i_{b1}} = \beta_1 = 100$$

$$\frac{i_{b3}}{i_{c1}} = \frac{(5+5)}{(5+5)+R_{\text{in}2}} = \frac{10}{10+10} = 0.5$$

$$\frac{i_{c3}}{i_{b3}} = \beta_3 = 100$$

Thus

$$\frac{i_{c3}}{i_{b1}} = \frac{i_{c3}}{i_{b3}} \times \frac{i_{b3}}{i_{c1}} \times \frac{i_{c1}}{i_{b1}} = 100 \times 0.5 \times 100$$

$$= 5000 \text{ A/A}$$

9.121 Refer to Fig. 9.41. From Example 9.7, we obtain

$$I_{C1} = I_{C2} = 0.25 \text{ mA}$$

$$I_{C4} = I_{C5} = 1 \text{ mA}$$

$$I_{C7} = 1 \text{ mA}$$

$$I_{C8} = 5 \text{ mA}$$

Thus

$$r_{e1} = r_{e2} = \frac{25 \text{ mV}}{0.25 \text{ mA}} = 100 \ \Omega$$

$$r_{e4} = r_{e5} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

With 100-Ω resistance in the emitter of each of Q_1 and Q_2, we have

$$R_{id} = (\beta+1)(2r_{e1,2} + 2R_{e1,2})$$

$$= 101 \times (2 \times 0.1 + 2 \times 0.1)$$

$$= 40.4 \text{ k}\Omega$$

Thus, R_{id} increases by a factor of 2. With 25-Ω resistance in the emitter of each of Q_4 and Q_5, the input resistance of the second stage becomes

$$R_{i2} = (\beta+1)(2r_{e4,5} + 2R_{e4,5})$$

$$= 101 \ (2 \times 0.025 + 2 \times 0.025)$$

$$= 10.1 \text{ k}\Omega$$

Thus, R_{i2} is increased by a factor of 2. The gain of the first stage will be

$$\frac{v_{o1}}{v_{id}} =$$

$$\frac{\alpha \times \text{Total resistance between the collectors of } Q_1 \text{ and } Q_2}{\text{Total resistance in emitters of } Q_1 \text{ and } Q_2}$$

$$\simeq \frac{40 \text{ k}\Omega \parallel 10 \text{ k}\Omega}{2 \times 0.1 + 2 \times 0.1} = 20 \text{ V/V}$$

Thus the gain of the first stage decreases but only slightly. Of course, the two 100-Ω resistances in the emitters reduce the gain but some of the reduction is mitigated by the increase in R_{i2}, which increases the effective load resistance of the first stage.

The gain of the second stage will now be

$$A_2 = \frac{v_{o2}}{v_{o1}} = -\alpha \frac{3 \text{ k}\Omega \parallel R_{i3}}{2 \times 0.025 + 2 \times 0.025}$$

From Example 9.8, $R_{i3} = 234.8 \text{ k}\Omega$, thus

$$A_2 \simeq -\frac{3 \parallel 234.8}{0.1} = -29.6 \text{ V/V}$$

which is about half the value without the two 25-Ω emitter resistances. The gain of the third stage remains unchanged at −6.42 V/V, and the gain of the fourth stage remains unchanged at 1 V/V. Thus the overall voltage gain becomes

$$\frac{v_o}{v_{id}} = A_1 A_2 A_3 A_4$$

$$= 20 \times -29.6 \times -6.42 \times 1$$

$$= 3800.6 \text{ V/V}$$

which is less than half the gain obtained without the emitter resistances. This is the price paid for doubling R_{id}.

9.122 The output resistance is mostly determined by R_5. To reduce R_o by a factor of 2, we use

$$R_o = \frac{152}{2} = R_6 \parallel \left[r_{e6} + \frac{R_5}{\beta+1} \right]$$

$$76 = 3000 \parallel \left[5 + \frac{R_5}{101} \right]$$

$$\Rightarrow R_5 = 7.37 \text{ k}\Omega$$

This change in R_5 will affect the gain of the third stage which will now become

$$A_3 = -\frac{R_5 \parallel (\beta+1)(r_{e8} + R_6)}{R_4 + r_{e7}}$$

$$= -\frac{7.37 \parallel (101)(0.005+3)}{2.3 + 0.025}$$

$$= -3.1 \text{ V/V}$$

which is about half the original value (not surprising since R_5 is about half its original value). To restore the gain of the third stage to its original value, we can reduce R_4. This will, however, change R_{i3} and will reduce the gain of the second stage, though only slightly. For instance, to restore the gain of the third stage to −6.42 V/V, we use

$$\frac{2.3 + 0.025}{R_4 + 0.025} = \frac{6.42}{3.1}$$

$$\Rightarrow R_4 = 1.085 \text{ k}\Omega$$

Now $R_{i3} = 101 \times (1.085 + 0.025) = 112 \text{ k}\Omega$ and the gain of the second stage becomes

$$A_2 = -\frac{3 \text{ k}\Omega \parallel 112 \text{ k}\Omega}{30 \ \Omega} = -58.4 \text{ V/V}$$

which is a slight decrease in magnitude from the original value of −59.2 V/V.

9.123 Refer to Fig. 9.41(a). With R_5 replaced with a 1-mA constant-current source with a high output resistance, the total resistance in the collector of Q_7 now becomes the input resistance of Q_8, which is

$$R_{i4} = (\beta + 1)(r_{e8} + R_6)$$

$$= 101 \times (0.005 + 3) = 303.5\ \text{k}\Omega$$

Thus the gain of the third stage now becomes

$$A_3 = -\alpha \frac{303.5}{2.3 + 0.025}$$

$$\simeq -130.5\ \text{V/V}$$

and the overall voltage gain increases to

$$\frac{v_o}{v_{id}} = 8513 \times \frac{130.5}{6.42} = 1.73 \times 10^5\ \text{V/V}$$

(b) The output resistance now becomes

$$R_o = 3\ \text{k}\Omega \parallel \left(r_{e8} + \frac{\text{very large resistance}}{\beta + 1} \right)$$

$$\simeq 3\ \text{k}\Omega$$

When the amplifier is loaded with $R_L = 100\ \Omega$,

$$G_v = 1.73 \times 10^5 \frac{R_L}{R_L + R_o} =$$

$$1.73 \times 10^5 \times \frac{100}{3000 + 100}$$

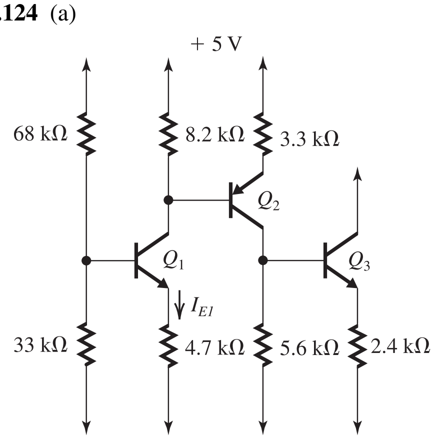
$$G_v = 5581\ \text{V/V}$$

If the original amplifier is loaded in $R_L = 100\ \Omega$,

$$G_v = 8513 \times \frac{100}{152 + 100} = 3378\ \text{V/V}$$

Thus, although the output resistance of the original amplifier is much lower than that of the modified one, the overall voltage gain realized when the original amplifier is loaded in 100-Ω resistance is much lower than that obtained with the modified design. Thus, replacing the 15.7-kΩ resistance with a constant-current source is an excellent modification to make!

9.124 (a)



(a)

Refer to Fig. (a) for the dc analysis. Replacing the 68 kΩ-33 kΩ divider network by its Thévenin equivalent, we obtain

$$V_{BB} = -5\ \text{V} + \frac{33}{33 + 68} \times 10\ \text{V}$$

$$= -1.73\ \text{V}$$

$$R_{BB} = 68\ \text{k}\Omega \parallel 33\ \text{k}\Omega = 22.2\ \text{k}\Omega$$

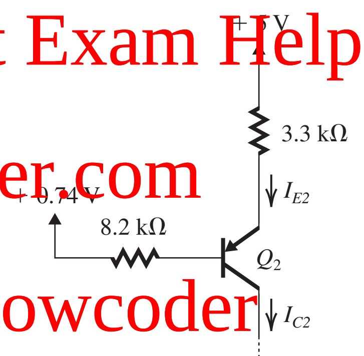
Now, we can determine I_{E1} from

$$I_{E1} = \frac{V_{BB} - (-5) - 0.7}{4.7 + \dfrac{R_{BB}}{\beta + 1}}$$

$$= \frac{-1.73 + 5 - 0.7}{4.7 + \dfrac{22.2}{101}} = 0.52\ \text{mA}$$

$$I_{C1} = \alpha_1 \times 0.52 = 0.99 \times 0.52 \simeq 0.52\ \text{mA}$$

The collector current I_{C1} and the 8.2-kΩ resistor it feeds can be replaced by a Thévenin equivalent as shown in Fig. (b). Thus



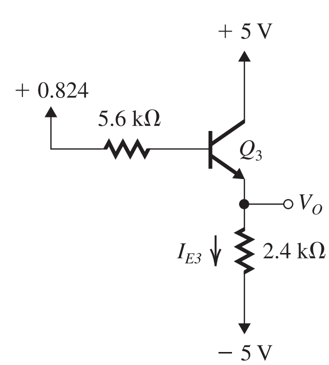
(b)

$$I_{E2} = \frac{5 - 0.74 - 0.7}{3.3 + \dfrac{8.2}{101}}$$

$$= 1.05\ \text{mA}$$

$$I_{C2} \simeq 1.04\ \text{mA}$$

The collector current I_{C2} and the 5.6-kΩ resistance it feeds can be replaced by a Thévenin equivalent as shown in Fig. (c). Thus



(c)

$$I_{E3} = \frac{0.824 - 0.7 - (-5)}{2.4 + \dfrac{5.6}{101}}$$

$$= 2.1 \text{ mA}$$

$$V_O = -5 + 2.1 \times 2.4 = 0 \text{ V}$$

(b) $R_{\text{in}} = 68 \text{ k}\Omega \parallel 33 \text{ k}\Omega \parallel r_{\pi 1}$

where

$$r_{\pi 1} = \frac{\beta}{g_{m1}}$$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.52}{0.025} = 20.8 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{20.8} = 4.81 \text{ k}\Omega$$

$$R_{\text{in}} = 68 \text{ k}\Omega \parallel 33 \parallel 4.81 \simeq 4 \text{ k}\Omega$$

$$R_{\text{out}} = 2.4 \text{ k}\Omega \parallel \left(r_{e3} + \frac{5.6 \text{ k}\Omega}{\beta + 1}\right)$$

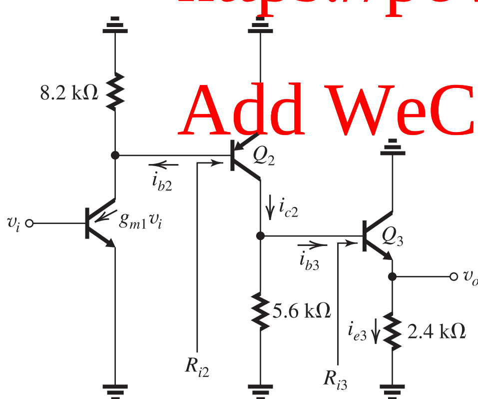
where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{2.1 \text{ mA}} = 11.9 \ \Omega$$

$$R_{\text{out}} = 2.4 \parallel \left(0.0119 + \frac{5.6}{101}\right)$$

$$= 65.5 \ \Omega$$

(c) Refer to Fig. (d)



(d)

$$i_{c1} = g_{m1} v_i = 20.8 v_i$$

$$R_{i2} = r_{\pi 2} = \frac{\beta}{g_{m2}}$$

where

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{1.04 \text{ mA}}{0.025 \text{ V}} = 41.6 \text{ mA}$$

$$r_{\pi 2} = \frac{100}{41.6} = 2.4 \text{ k}\Omega$$

$$i_{b2} = g_{m1} v_i \frac{8.2}{8.2 + 2.4} = 16.1 v_i$$

$$i_{c2} = \beta_2 i_{b2} = 100 \times 16.1 v_i = 1610 v_i$$

$$R_{i3} = (\beta + 1)(r_{e3} + 2.4 \text{ k}\Omega)$$

$$= 101(0.0119 + 2.4) = 243.6 \text{ k}\Omega$$

$$i_{b3} = i_{c2} \times \frac{5.6}{5.6 + 243.6} = 0.0225 i_{c2}$$

$$= 0.0225 \times 1610 v_i = 36.18 v_i$$

$$i_{e3} = (\beta + 1) i_{b3}$$

$$= 101 \times 36.18 = 3654 v_i$$

$$v_o = i_{e3} \times 2.4 \text{ k}\Omega$$

$$= 3654 \times 2.4 v_i = 8770 v_i$$

Thus

$$\frac{v_o}{v_i} = 8770 \text{ V/V}$$

9.125 From the figure we observe that the controlled source $g_{m1}\ v_{gs1}$ can be replaced by a resistance $1/g_{m1}$, thus

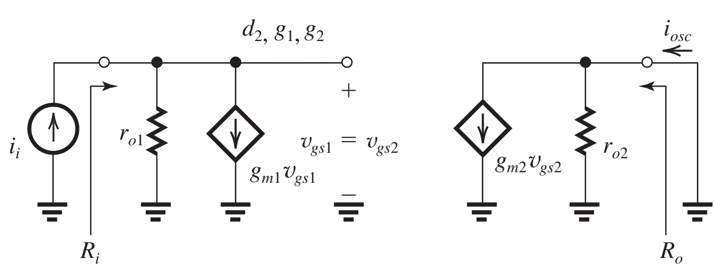
$$v_{gs1} = i_i \left(r_{o1} \parallel \frac{1}{g_{m1}} \right)$$

$$R_i \equiv \frac{v_{gs1}}{i_i} = \frac{1}{g_{m1}} \parallel r_{o1}$$

$$i_{osc} = g_{m2} v_{gs2} = g_{m2} v_{gs1} = g_{m2} \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) i_i$$

$$A_{is} \equiv \frac{i_{osc}}{i_i} = g_{m2} \frac{\dfrac{1}{g_{m1}} r_{o1}}{\dfrac{1}{g_{m1}} + r_{o1}}$$

This figure belongs to Problem 9.125.



$$= \frac{g_{m2}}{g_{m1}} \frac{1}{1 + \dfrac{1}{g_{m1} r_{o1}}}$$

Since $g_{m1} r_o \gg 1$,

$$A_{is} \simeq \frac{g_{m2}}{g_{m1}} \left(1 - \frac{1}{g_{m1} r_{o1}}\right)$$

$$= A_{is}|_{\text{ideal}} \left(1 - \frac{1}{g_{m1} r_{o1}}\right)$$

where

$$A_{is}|_{\text{ideal}} = \frac{g_{m2}}{g_{m1}}$$

Finally, from inspection,

$$R_o = r_{o2}$$

9.126 (a) Refer to Fig. P9.126. The current I_D in each of the eight transistors can be found by inspection. Then, g_m of each transistor can be determined as $2I_D/|V_{OV}|$ and r_o as $|V_A|/I_D$. The results are given in the table below:

(b) See figure below. Observe that at the output node the total signal current is $4i_d$ where

$$i_d = g_{m1,2} \frac{v_{id}}{2}$$

$$= \frac{I}{2|V_{OV}|} v_{id}$$

and since the output resistance is

$$R_o = r_{o6} \parallel r_{o8} = \frac{1}{2} \frac{|V_A|}{I}$$

then

$$v_o = 4\, i_d R_o = 4 \times \frac{I}{2|V_{OV}|} \times \frac{1}{2} \frac{|V_A|}{I} \times v_{id}$$

Thus

$$A_d \equiv \frac{v_o}{v_{id}} = \frac{|V_A|}{|V_{OV}|} \qquad \text{Q.E.D.}$$

(c) See figure on next page. With v_{icm} applied to both input terminals, we can replace each of Q_1 and Q_2 with an equivalent circuit composed of a controlled current, $v_{icm}/2R_{SS}$ in parallel with a very large output resistance (R_{o1} and R_{o2} which are equal). The resistances R_{o1} and R_{o2} will be much larger than the input resistance of each of the mirrors $Q_3 - Q_5$ and $Q_4 - Q_6$ and thus we can neglect R_{o1} and R_{o2} altogether. The short-circuit output current of the $Q_4 - Q_6$ mirror will be

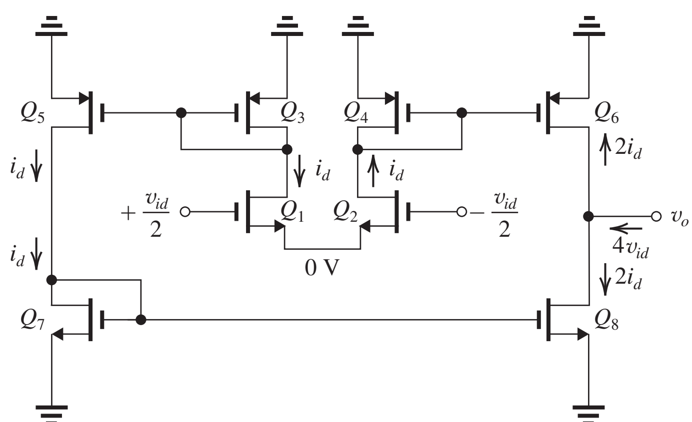
$$i_o = \frac{g_{m6}}{g_{m4}} \left(1 - \frac{1}{g_{m4} r_{o4}}\right) \frac{v_{icm}}{2R_{SS}}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right) \left(\frac{v_{icm}}{R_{SS}}\right)$$

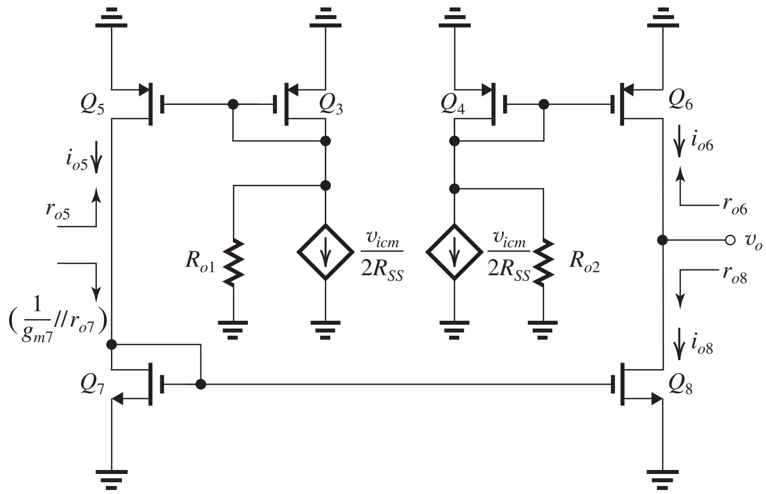
This table belongs to Problem 9.126.

Transistor	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
I_D	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	$\frac{I}{2}$	I	$\frac{I}{2}$	I
g_m	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$	$\frac{I}{\lvert V_{OV}\rvert}$	$\frac{2I}{\lvert V_{OV}\rvert}$
r_o	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$	$\frac{2\lvert V_A\rvert}{I}$	$\frac{\lvert V_A\rvert}{I}$

This figure belongs to Problem 9.126, part (b).



This figure belongs to Problem 9.126, part (c).



and the output resistance will be r_{o6}. The short-circuit output current of the $Q_3 - Q_5$ mirror will be

$$i_{o5} = \frac{g_{m5}}{g_{m3}}\left(1 - \frac{1}{g_{m3}r_{o3}}\right)\frac{v_{icm}}{2R_{SS}}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(\frac{v_{icm}}{2R_{SS}}\right)$$

and the output resistance will be r_{o5}. Since r_{o5} is much larger than the input resistance of the $Q_7 - Q_8$ mirror ($\simeq 1/g_{m7}$), most of i_{o5} will flow into Q_7, resulting in an output short-circuit current i_{o8}:

$$i_{o8} = \frac{g_{m8}}{g_{m7}}\left(1 - \frac{1}{g_{m7}r_{o7}}\right)i_{o5}$$

$$= 2\left(1 - \frac{1}{g_{m7}r_{o7}}\right)i_{o5}$$

$$= \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(1 - \frac{1}{g_{m7}r_{o7}}\right)\frac{v_{icm}}{R_{SS}}$$

and the output resistance is r_{o8}. Thus, at the output node we have a net current

$$i_{o6} - i_{o8} = \left(1 - \frac{|V_{OV}|}{2|V_A|}\right)\left(\frac{1}{g_{m7}r_{o7}}\right)\left(\frac{v_{icm}}{R_{SS}}\right)$$

$$\simeq \left(\frac{1}{g_{m7}r_{o7}}\right)\left(\frac{v_{icm}}{R_{SS}}\right)$$

This current flows into the output resistance ($r_{o6} \parallel r_{o8}$) and thus produces an output voltage

$$v_o = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} v_{icm}$$

and the common-mode gain becomes

$$|A_{cm}| = \frac{r_{o6} \parallel r_{o8}}{R_{SS}} \frac{1}{g_{m7}r_{o7}} \qquad \text{Q.E.D.}$$

(d) $R_{SS} = \frac{|V_A|}{I}$

$$A_d = \left|\frac{V_A}{V_{OV}}\right|$$

$$|A_{cm}| = \frac{\frac{1}{2}|V_A|/I}{|V_A|/I} \frac{1}{[I/|V_{OV}|\,]\,[2|V_A|/I]}$$

$$|A_{cm}| = \frac{1}{2} \times \frac{1}{2}\frac{|V_{OV}|}{|V_A|} = \frac{1}{4}\left|\frac{V_{OV}}{V_A}\right|$$

$$\text{CMRR} = 4\left|\frac{V_A}{V_{OV}}\right|^2 \qquad \text{Q.E.D.}$$

(e) The upper limit on V_{ICM} is determined by Q_1 and Q_2 remaining in saturation, thus

$$V_{ICM_{\max}} = V_{DD} - |V_{SG}| + |V_t|$$

$$= V_{DD} - |V_{OV}|$$

The lower limit on V_{ICM} is determined by the need to keep the bias current source in saturation, i.e. maintaining a minimum voltage across it of $|V_{OV}|$, thus

$$V_{ICM_{\min}} = -V_{SS} + |V_{OV}| + |V_{GS}|$$

$$= -V_{SS} + 2|V_{OV}| + |V_t|$$

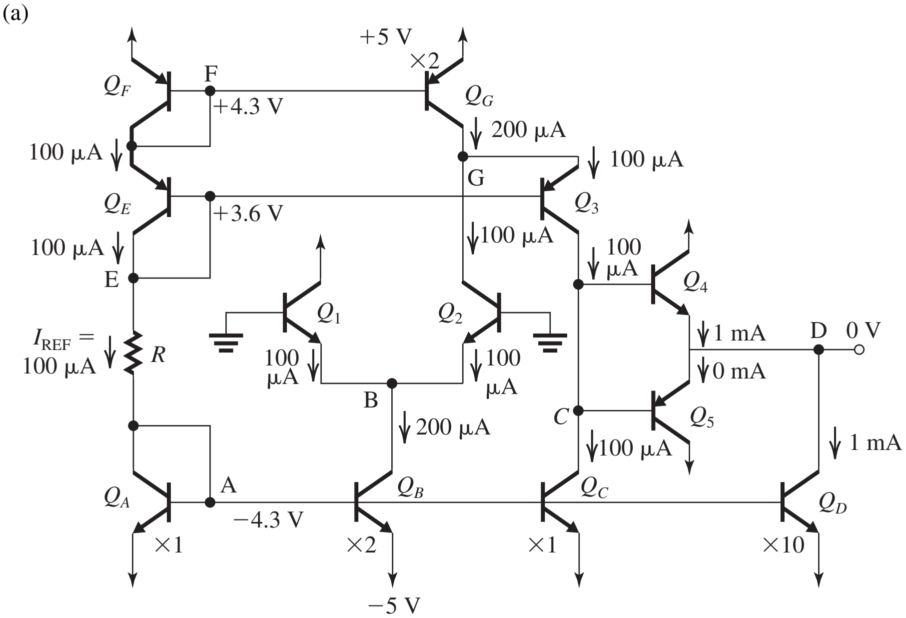
Thus

$$-V_{SS} + |V_t| + 2|V_{OV}| \le V_{ICM} \le V_{DD} - |V_{OV}|$$

The output linear range is

$$V_{DD} - |V_{OV}| \le v_O \le -V_{SS} + |V_{OV}|$$

9.127 (a)



(a)

$$R = \frac{+3.6 - (-4.3)}{0.1 \text{ mA}} = 79 \text{ k}\Omega$$

See Figure (a) for the dc analysis.

$V_A = -4.3$ V $\quad V_B = -0.7$ V $\quad V_C = +0.7$ V

$V_D = 0$ V $\quad V_E = +3.6$ V $\quad V_F = +4.3$ V

$V_G = +4.3$ V

(b) See table below. Results are obtained from Fig. (a) and

$$g_m = \frac{I_C}{V_T}$$

$$r_o = \frac{|V_A|}{I_C}$$

Transistor	I_C (mA)	g_m (mA/V)	r_o (MΩ)
Q_1	0.1	4	2
Q_2	0.1	4	2
Q_3	0.1	4	2
Q_4	1.0	40	0.2
Q_5	0	0	∞
Q_A	0.1		
Q_B	0.2		
Q_C	0.1		2
Q_D	1.0		0.2
Q_E	0.1		
Q_F	0.1		
Q_G	0.2		1

(c) See figure (b) on next page. Total resistance at the collector of Q_3 is

$\beta_3 r_{o3} \parallel r_{oC} \parallel \beta_4(r_{oD} \parallel r_{o4})$

$= (100 \times 2) \parallel 2 \parallel 100 \times (0.2 \parallel 0.2)$

$= 200 \parallel 2 \parallel 10 = 1.65$ MΩ

$$\frac{v_{c3}}{v_{id}} = \frac{1}{2} \times g_{m1,2} \times 1.65 \text{ M}\Omega$$

$$= \frac{1}{2} \times 4 \times 1.65 \times 10^3 = 3300 \text{ V/V}$$

$$\frac{v_o}{v_{c3}} \simeq 1$$

Thus

$$\frac{v_o}{v_{id}} = 3300 \text{ V/V}$$

Observe that the polarity of the two input terminals are correct.

(d) $R_{id} = 2\, r_{\pi 1,2}$

$$= 2 \times \frac{\beta}{g_{m1,2}} = 2 \times \frac{100}{4} = 50 \text{ k}\Omega$$

$$R_o = r_{o4} \parallel r_{oD} \parallel \left[r_{e4} + \frac{r_{oC} \parallel \beta r_{o3}}{\beta + 1} \right]$$

$$= 0.2 \parallel 0.2 \parallel \left[0.025 \times 10^{-3} + \frac{2 \parallel 200}{101} \right]$$

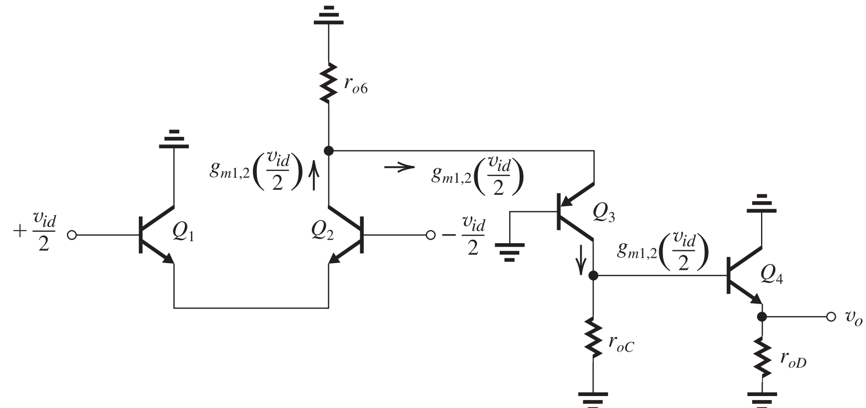
$\simeq (0.2 \parallel 0.2 \parallel 0.02)$ MΩ

$= 16.7$ kΩ

(e) $V_{ICM_{\max}}$ is limited by Q_2 saturating,

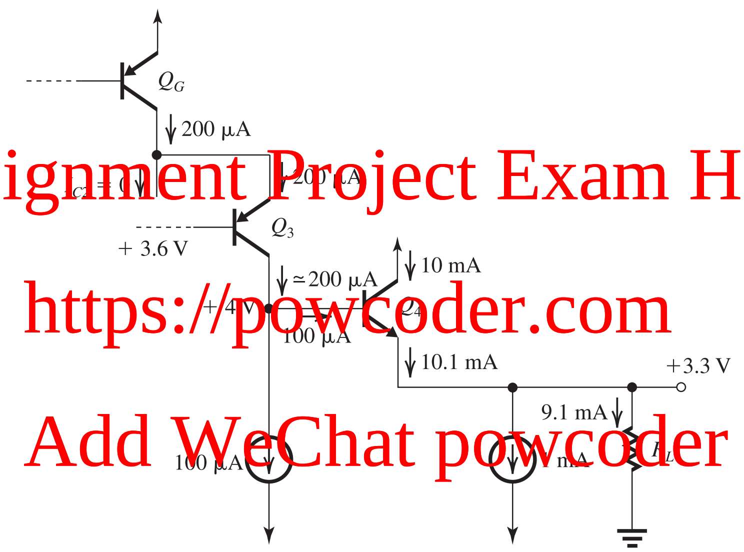
$V_{ICM_{\max}} = V_G + 0.4 = +4.7$ V

This figure belongs to Problem 9.127, part (c).



(b)

This figure belongs to Problem 9.127, part (g).



(c)

$V_{ICM_{\min}}$ is limited by Q_B saturating,

$$V_{ICM_{\min}} = V_A - 0.4 + 0.7$$

$$= -4.3 - 0.4 + 0.7 = -4 \text{ V}$$

(f) The voltage at the base of Q_4 can rise to $(V_{B3} + 0.4)$ before Q_3 saturates, i.e. to $+3.6 + 0.4 = +4$ V. Thus v_O can go to $+4 - V_{BE4} = +3.3$ V. The output voltage can go down to the value that causes the voltage at C to be 0.4 V below the base voltage of Q_C. Thus

$$v_{O_{\min}} = -4.3 - 0.4 + V_{EB5} = -4 \text{ V}$$

Thus

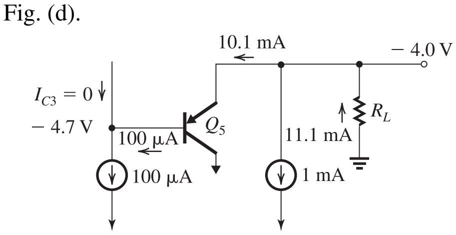
$$-4.0 \text{ V} \le v_O \le +3.3 \text{ V}$$

(g) With v_O at its maximum positive value of +3.3 V, and R_L small enough to cause Q_2 to cut off, the conditions in the circuit become as shown in Fig. (c).

The value of R_L can be found from

$$R_L = \frac{3.3}{9.1} = 363\ \Omega$$

With v_O at its maximum negative value of −4 V and with R_L sufficiently small to cause Q_1 to cut off, Q_2 will conduct 200 μA which leaves Q_3 with zero current (cut off). Transistor Q_4 also cuts off, and the circuit conditions become as shown in Fig. (d).



(d)

Thus

$$R_L = \frac{4}{9.1} = 360\ \Omega$$

9.128 DC analysis

(a) $I_{REF} = 10\ \mu\text{A} = \frac{1}{2} \times 40 \times \frac{5}{5}\left(V_{GS_A} - V_t\right)^2$

$\Rightarrow V_{GS_A} = 1.71\ \text{V} \approx 1.7\ \text{V}$

$10 = \frac{1}{2} \times 20 \times \frac{5}{5}\left(V_{GS_{EF}} - 1\right)^2$

$\Rightarrow V_{GS_{EF}} = 2\ \text{V}$

$R = \dfrac{1 - (-3.3)}{10\ \mu\text{A}} = 430\ \text{k}\Omega$

(b) See figure (a) below.

$V_{GS1} = V_{GS2} = V_{GSA} \approx 1.7\ \text{V}$

$V_{GS3} = \sqrt{\dfrac{2 \times 10}{20 \times \frac{10}{5}}} + 1 = 1.71\ \text{V} \approx 1.7\ \text{V}$

$V_{GS5} = V_{GS3} = 1.7\ \text{V}$

For Q_6: $50 = \frac{1}{2} \times 40 \times \frac{50}{5}(V_{GS6} - V_t)^2$

$\Rightarrow V_{GS6} = 1.50\ \text{V}$

$V_A = -3.3\ \text{V}, \quad V_B = -1.7\ \text{V}$

$V_C = +1.5\ \text{V}, \quad V_D = 0\ \text{V}$

$V_E = +1\ \text{V}, \quad V_F = +3\ \text{V}$

$V_G = +3.3\ \text{V}, \quad V_H = +2.7\ \text{V}$

This figure belongs to Problem 9.128, part (a).

(c)

Transistor	I_D (μA)	V_{GS} (V)	g_m (μA/V)	r_o (MΩ)
Q_1	10	1.7	28.3	5
Q_2	10	1.7	28.3	5
Q_3	10	1.7	28.3	5
Q_4	20	1.7	56.6	2.5
Q_5	10	1.7	28.3	5
Q_6	50	1.5	200	1
Q_7	0	−1.5*	0	∞
Q_A	10	1.7	28.3	5
Q_B	20	1.7	56.6	2.5
Q_C	10	1.7	28.3	5
Q_D	50	1.7	141.4	1
Q_E	10	2	20	5
Q_F	10	2	20	5

* Cut-off.

(d) Refer to Fig. (b). The total resistance at the source of the cascode transistor Q_5 is $(r_{o2} \parallel r_{o4})$. Thus the output resistance of the cascode transistor will be

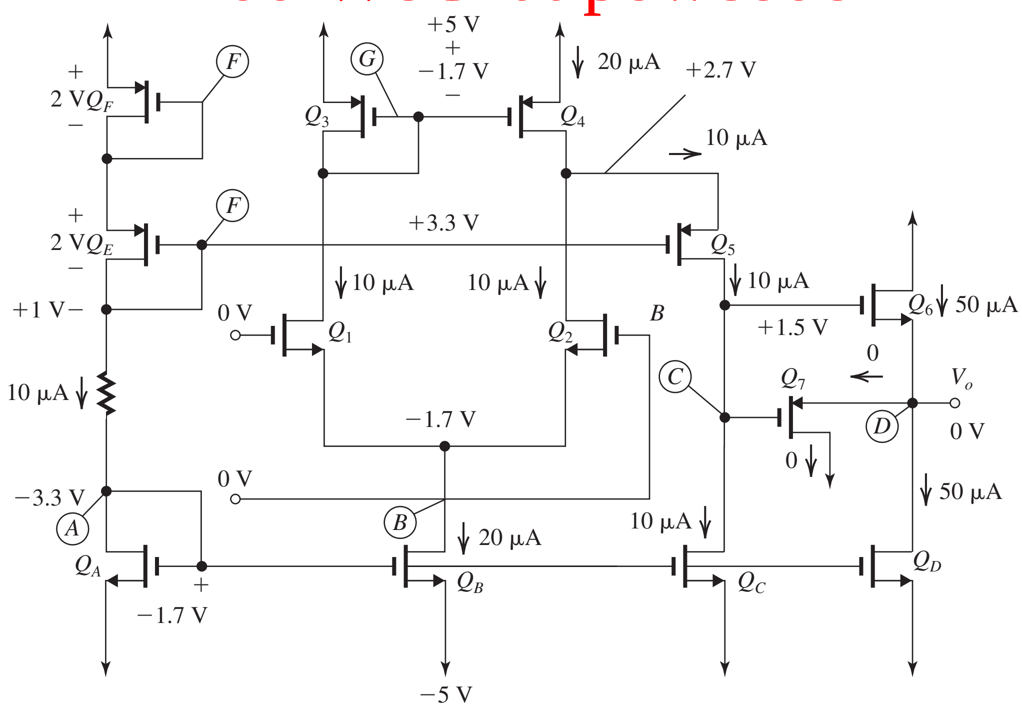
$R_o = g_{m5} r_{o5}(r_{o2} \parallel r_{o4})$

and the total resistance at the drain of Q_5 will be

$R_{total} = R_o \parallel r_{oC} = [g_{m5} r_{o5}(r_{o2} \parallel r_{o4})] \parallel r_{oC}$

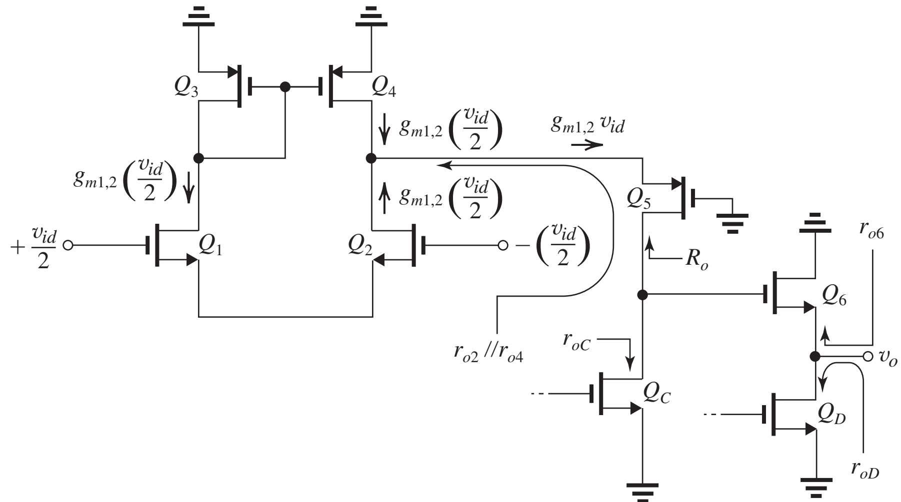
$= [(28.3 \times 5)(5 \parallel 2.5)] \parallel 5$

$= 4.9\ \text{M}\Omega$



(a)

This figure belongs to Problem 9.128, part (d).



(b)

The voltage gain to the drain of Q_5 can be found as

$$\frac{v_{d5}}{v_{id}} = g_{m1,2}R_{\text{total}} = 28.3 \times 4.9 = 138.7 \text{ V/V}$$

The gain of the source-follower output stage is

$$\frac{v_o}{v_{d5}} = \frac{(r_{o6} \parallel r_{oD})}{(r_{o6} \parallel r_{oD}) + \dfrac{1}{g_{m6}}}$$

$$= \frac{1 \parallel 1}{(1 \parallel 1) + \dfrac{1}{200}} \simeq 1 \text{ V/V}$$

and the overall voltage gain is

$$\frac{v_o}{v_{id}} = 138.7 \text{ V/V}$$

$R_{\text{in}} = \infty$

$$R_o = r_{oD} \parallel r_{o6} \parallel \left(\frac{1}{g_{m6}}\right)$$

$$= 1 \parallel 1 \parallel \frac{1}{200}$$

$\simeq 5 \text{ k}\Omega$

(e) $V_{ICM_{\max}} = V_G + |V_t|$

$= 3.3 + 1 = +4.3$ V

$V_{ICM_{\min}} = V_{B\min} + V_{GS1,2}$

$= V_A - |V_t| + V_{GS1,2}$

$= -3.3 - 1 + 1.7 = -2.6$ V

Thus

$-2.6 \text{ V} \le V_{ICM} \le 4.3 \text{ V}$

(f) $v_{O_{\max}} = V_{C_{\max}} - V_{GS6}$

$= V_E + |V_t| - V_{GS6}$

$= 1 + 1 - 1.5 = 0.5$ V

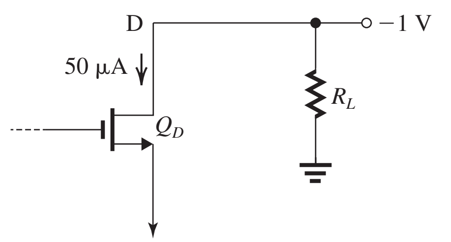
$v_{O_{\min}} = V_A - |V_t|$

$= -3.3 - 1 = -4.3$ V

Thus

$-4.3 \text{ V} \le v_O \le +0.5 \text{ V}$

(g) The circuit conditions are depicted in Fig. (c).



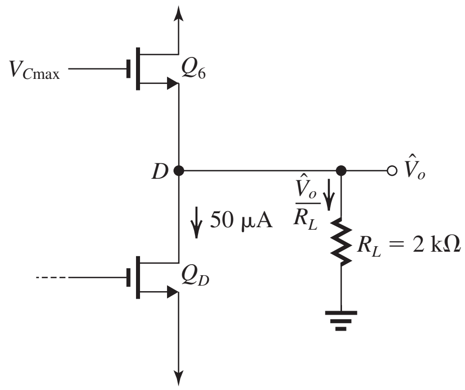
(c)

Observe that Q_6 is cut off and Q_7 has not yet conducted. Thus all the load current is sourced by Q_D. It follows that the maximum negative load current must be 50 µA

$$50 = \frac{1 \text{ V}}{R_L}$$

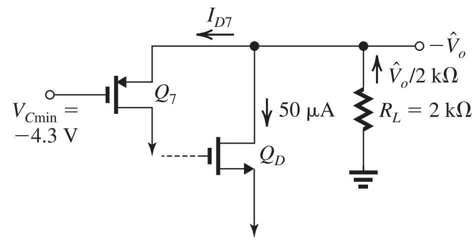
$\Rightarrow R_L = 20 \text{ k}\Omega$

(h) With $R_L = 2 \text{ k}\Omega$ and v_O is at its maximum allowable value (to be determined), the circuit conditions are as indicated in Fig. (d).



(d)

With $R_L = 2$ kΩ and v_O is at its minimum allowable value (to be determined), the circuit conditions become as shown in Fig. (e).



(e)

Here

$$V_{C_{\max}} = V_E + |V_t| = 2\text{ V}$$

Now

$$I_{D6} = \frac{\hat{V}_o}{R_L} + 50\ \mu\text{A}$$

$$= \left(\frac{\hat{V}_o}{2} + 0.05\right)\text{ mA}$$

But

$$I_{D6} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_6 (V_{C_{\max}} - \hat{V}_o - |V_t|)^2$$

$$\frac{\hat{V}_o}{2} + 0.05 = \frac{1}{2} \times 40 \times 10 \times 10^{-3}\ (2 - \hat{V}_o - 1)^2$$

$$\Rightarrow \hat{V}_o = 0.17\text{ V}$$

Here Q_7 turns on and its current becomes

$$I_{D7} = \left(\frac{\hat{V}_o}{2} - 0.05\right)\text{ mA}$$

But

$$I_{D7} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_7 (-\hat{V}_o - V_{C\min} - |V_t|)^2$$

Thus

$$\frac{\hat{V}_o}{2} - 0.05 = \frac{1}{2} \times 20 \times 20 \times 10^{-3}(-\hat{V}_o + 4.3 - 1)^2$$

$$\Rightarrow \hat{V}_o = 1.45\text{ V}$$

Thus

$$-1.45\text{ V} \le v_O \le +0.17\text{ V}$$