Ex: 11.1 (c) $A = 100$ V/V and $A_f = 10$ V/V

Since A is not much greater than A_f, we shall use the exact expression to determine β and hence R_2/R_1,

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{100}{1 + 100\beta}$$

$$\Rightarrow \beta = 0.09 \text{ V/V}$$

Now,

$$\frac{R_1}{R_1 + R_2} = 0.09$$

$$\frac{R_2}{R_1} = \frac{1}{0.09} - 1 = 10.11$$

(d) $A\beta = 100 \times 0.09 = 9$

$1 + A\beta = 10$

$\Rightarrow 20$ dB

(e) $V_o = A_f V_s = 10 \times 1 = 10$ V

$V_f = \beta V_o = 0.09 \times 10 = 0.9$ V

$$V_i = \frac{V_o}{A} = \frac{10}{100} = 0.1 \text{ V}$$

(f) $A \rightarrow 80$ V/V

$$A_f = \frac{80}{1 + 80 \times 0.09} = 9.756$$

a change of $\dfrac{9.756 - 10}{10} \times 100 = -2.44\%$ or a reduction of 2.44%.

Ex. 11.2 (c) $A = 10^4$ V/V and $A_f = 10^3$ V/V

$$A_f = \frac{A}{1 + A\beta}$$

$$10^3 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 9 \times 10^{-4} \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 9 \times 10^{-4}$$

$$\frac{R_2}{R_1} = \frac{1}{9 \times 10^{-4}} - 1 = 1110.1$$

(d) $A\beta = 10^4 \times 9 \times 10^{-4} = 9$

$1 + A\beta = 10$

$\Rightarrow 20$ dB

(e) $V_s = 0.01$ V

$V_o = A_f V_s = 10^3 \times 0.01 = 10$ V

$V_f = \beta V_o = 9 \times 10^{-4} \times 10 = 0.009$ V

$$V_i = \frac{V_o}{A} = \frac{10}{10^4} = 0.001 \text{ V}$$

(f) $A \Rightarrow 0.8 \times 10^4$ V/V

$$A_f = \frac{0.8 \times 10^4}{1 + 0.8 \times 10^4 \times 9 \times 10^{-4}}$$

$= 975.6$ V/V

which is a change of $\dfrac{975.6 - 1000}{1000} \times 100$

$= -2.44\%$ or a reduction of 2.44%.

Ex. 11.3 To constrain the corresponding change in A_f to 0.1%, we need an amount-of-feedback of at least

$$1 + A\beta = \frac{10\%}{0.1\%} = 100$$

Thus the largest obtainable closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta} = \frac{1000}{100} = 10 \text{ V/V}$$

Each amplifier in the cascade will have a nominal gain of 10 V/V and a maximum variability of 0.1%; thus the overall voltage gain will be $(10)^3 = 1000$ V/V and the maximum variability will be 0.3%.

Ex. 11.4 $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{1}{1 + 9} = 0.1$

$A\beta = 10^4 \times 0.1 = 1000$

$1 + A\beta = 1001$

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = \frac{10^4}{1 + 10^4 \times 0.1} = 9.99 \text{ V/V}$$

$f_{Hf} = f_H(1 + A\beta)$

$= 100 \times 1001 = 100.1$ kHz

Ex. 11.5 Signal at output $= V_s \dfrac{A_1 A_2}{1 + A_1 A_2 \beta}$

$$= 1 \times \frac{1 \times 100}{1 + 1 \times 100 \times 1} = 1 \times \frac{100}{101} \simeq 1 \text{ V}$$

Interference at output $= V_n \dfrac{A_1}{1 + A_1 A_2 \beta}$

$$= 1 \times \frac{1}{1 + 1 \times 100 \times 1} \simeq 0.01 \text{ V}$$

Thus S/I at the output becomes 1/0.01

$= 100$ or 40 dB

Since S/I at the input is 1/1=1 or 0 dB, the improvement is 40 dB.

Ex. 11.6 (a) Refer to Fig. 11.8(c).

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b)

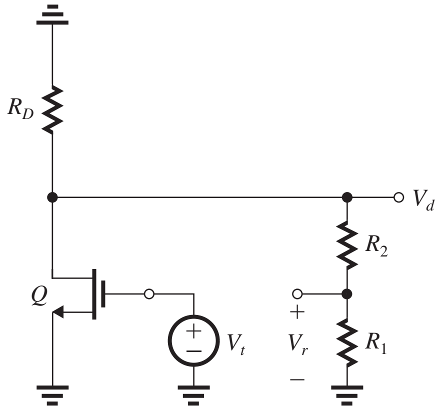


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$. Observe that we have eliminated the input signal V_s, and opened the loop at the gate of Q where the input impedance is infinite obviating the need for a termination resistance at the right-hand side of the break. Now we need to analyze the circuit to determine

$$A\beta \equiv -\frac{V_r}{V_t}$$

First, we write for the gain of the CS amplifier Q,

$$\frac{V_d}{V_t} = -g_m[R_D \parallel (R_1 + R_2)] \quad (1)$$

then we use the voltage-divider rule to find V_r,

$$\frac{V_r}{V_d} = \frac{R_1}{R_1 + R_2} \quad (2)$$

Combining Eqs. (1) and (2) gives

$$A\beta \equiv -\frac{V_r}{V_t} = g_m[R_D \parallel (R_1 + R_2)]\frac{R_1}{R_1 + R_2}$$

which can be simplified to

$$A\beta = g_m \frac{R_D R_1}{R_D + R_1 + R_2}$$

(c) $A = \dfrac{A\beta}{\beta}$

$$= g_m \frac{R_D(R_1 + R_2)}{R_D + R_1 + R_2}$$

(d) $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{20}{20 + 80} = 0.2$ V/V

$$A\beta = 4\frac{10 \times 20}{10 + 20 + 80} = 7.27$$

$$A = \frac{7.27}{0.2} = 36.36 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{36.36}{1 + 36.36 \times 0.2} = 4.4 \text{ V/V}$$

If $A\beta$ were $\gg 1$, then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{0.2} = 5 \text{ V/V}$$

Ex. 11.7 From the solution of Example 11.4,

$A\beta = 6$

$1 + A\beta = 7$

Thus,

$f_{Hf} = (1 + A\beta)f_H$

$= 7 \times 1$

$= 7$ kHz

Ex. 11.8 Refer to Fig. E11.8. The 1-mA bias current will split equally between the emitters of Q_1 and Q_2, thus

$I_{E1} = I_{E2} = 0.5$ mA

Transistor Q_3 will be operating at an emitter current

$I_{E3} = 5$ mA

determined by the 5-mA current source. Since the dc component of $V_s = 0$, the negative feedback will force the dc voltage at the output to be approximately zero. See Fig. 1 on next page.

The β circuit is shown in Fig. 1 together with the determination of β and of the loading effects of the β circuit on the A circuit,

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{1 + 9} = 0.1 \text{ V/V}$$

$R_{11} = R_1 \parallel R_2 = 1 \parallel 9 = 0.9$ kΩ

$R_{22} = R_1 + R_2 = 1 + 9 = 10$ kΩ

The A circuit is shown in Fig. 2. See figure on next page.

$$r_{e1} = r_{e2} = \frac{V_T}{I_{E1,2}} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50\ \Omega$$

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5\ \Omega$$

$$i_e = \frac{V_i}{r_{e1} + r_{e2} + \dfrac{R_s + R_{11}}{\beta + 1}}$$

$$i_e = \frac{V_i}{0.05 + 0.05 + \dfrac{10 + 0.9}{101}}$$

$$\Rightarrow i_e = 4.81 V_i \quad (1)$$

$R_{b3} = (\beta + 1)[r_{e3} + (R_{22} \parallel R_L)]$

$= 101[0.005 + (10 \parallel 2)]$

$= 168.84$ kΩ

These figures belong to Exercise 11.8.

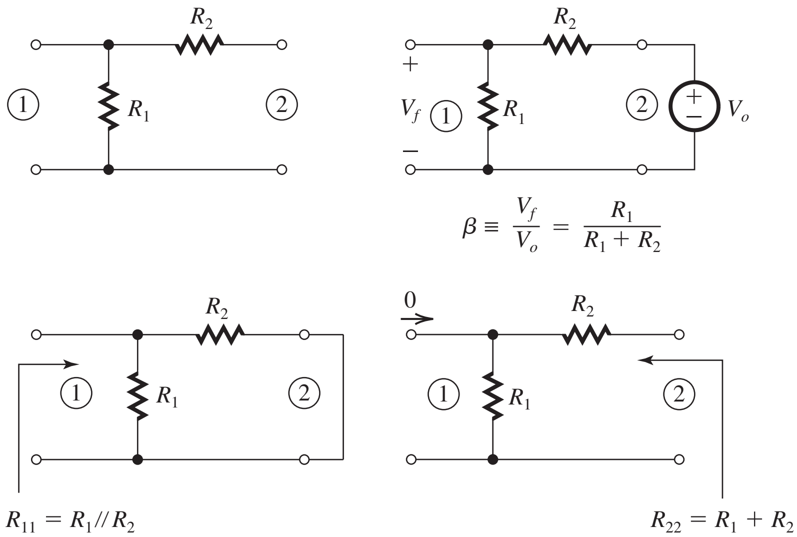


Figure 1

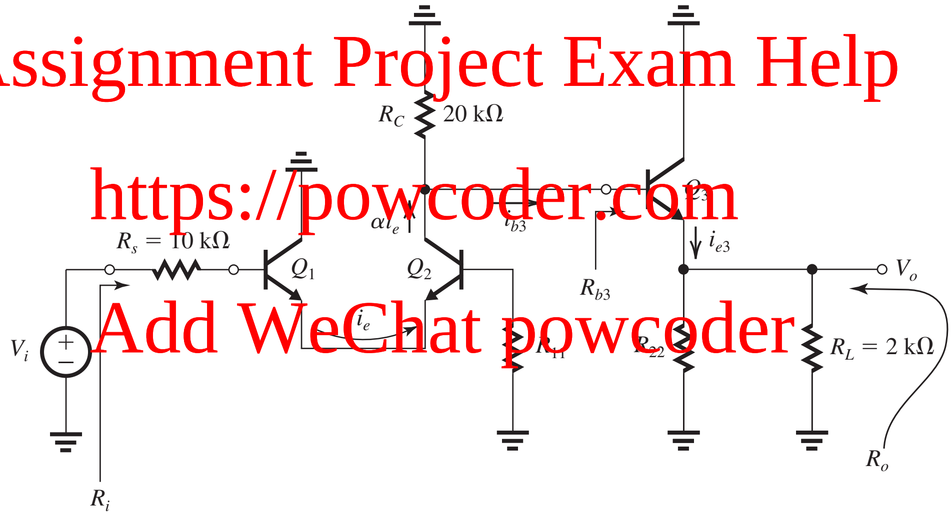


Figure 2

$$i_{b3} = \alpha i_e \frac{R_C}{R_C + R_{b3}}$$

$$= 0.99\, i_e \frac{20}{20 + 168.84}$$

$$\Rightarrow i_{b3} = 0.105 i_e \qquad (2)$$

$$V_o = i_{e3}(R_{22} \parallel R_L)$$

$$= (\beta + 1) i_{b3}(R_{22} \parallel R_L)$$

$$= i_{b3} \times 101(10 \parallel 2)$$

$$\Rightarrow V_o = 168.33 i_{b3} \qquad (3)$$

Combining (1)–(3), we obtain

$$A \equiv \frac{V_o}{V_i} = 85 \text{ V/V}$$

$$\beta = 0.1 \text{ V/V}$$

$$A\beta = 8.5$$

$$1 + A\beta = 9.5$$

$$A_f = \frac{85}{9.5} = 8.95 \text{ V/V}$$

From the A circuit, we have

$$R_i = R_s + R_{11} + (\beta + 1)(r_{e1} + r_{e2})$$

$$= 10 + 0.9 + 101 \times 0.1$$

$$= 21 \text{ k}\Omega$$

$R_{if} = R_i(1 + A\beta)$

$= 21 \times 9.5 = 199.5\ \text{k}\Omega$

$R_{\text{in}} = R_{if} - R_s = 199.5 - 10 = 189.5\ \text{k}\Omega$

From the A circuit, we have

$$R_o = R_L \parallel R_{22} \parallel \left(r_{e5} + \frac{R_C}{\beta + 1}\right)$$

$$= 2 \parallel 10 \parallel \left(0.005 + \frac{20}{101}\right)$$

$= 181\ \Omega$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{181}{9.5} = 19.1\ \Omega$$

$R_{of} = R_L \parallel R_{\text{out}}$

$19.1 = 2\ \text{k}\Omega \parallel R_{\text{out}}$

$\Rightarrow R_{\text{out}} = 19.2\ \Omega$

Ex. 11.9 Figure 1 shows the β circuit together with the determination of β, R_{11} and R_{22}.

$$\beta = \frac{R_1}{R_1 + R_2}$$

$R_{11} = R_1 \parallel R_2$

$R_{22} = R_1 + R_2$

Figure 2 shows the A circuit. We can write

$V_o = g_m(R_D \parallel R_{22})V_i$

Thus,

$$A \equiv \frac{V_o}{V_i} = g_m[R_D \parallel (R_1 + R_2)]$$

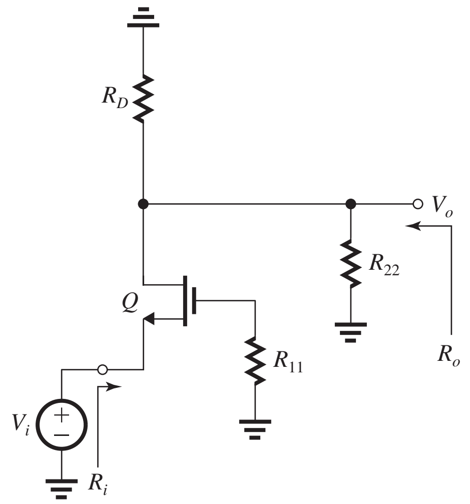


Figure 2

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

From A circuit, we have

$$R_i = \frac{1}{g_m}$$

$R_o = R_D \parallel R_{22}$

$R_{\text{in}} = R_{if} = R_i(1 + A\beta)$

$$R_{\text{in}} = \frac{1}{g_m}(1 + A\beta)$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{\text{out}} = \frac{R_D \parallel (R_1 + R_2)}{1 + A\beta}$$

This figure belongs to Exercise 11.9.

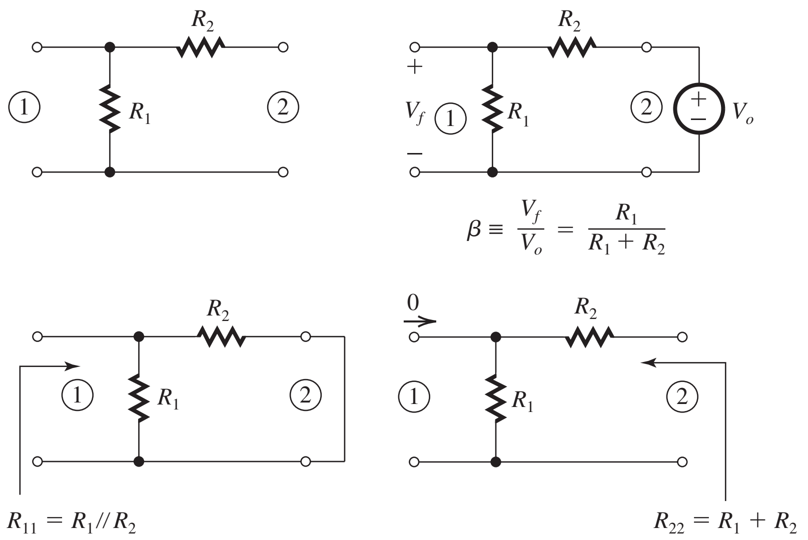


Figure 1

Comparison with the results of Exercise 11.6 shows that the expressions for A and β are identical. However, R_{in} and R_{out} cannot be determined using the method of Exercise 11.6.

Ex. 11.10 From the solution to Example 11.6, we have

$$A\beta = 653.6$$

$$1 + A\beta = 654.6$$

A decrease in the op amp gain by 10% results in a decrease in A by 10% and a corresponding decrease in A_f by

$$\frac{10\%}{1 + A\beta} = \frac{10\%}{654.6} = 0.015\%$$

A more exact solution (not using differential) is as follows:

The open-loop gain A becomes

$$A = 0.9 \times 653.6 = 588.24 \text{ mA/V}$$

$$\beta = R_F = 1 \text{ k}\Omega$$

$$A_f = \frac{588.24}{1 + 588.24 \times 1}$$

$$= 0.9983 \text{ mA/V}$$

Change in $A_f = 0.9983 - 0.9985$

$$= -0.0002$$

Percentage change in $A_f = \dfrac{-0.0002}{0.9983} \times 100$

$$= -0.02\%$$

Ex. 11.11 For a nominal closed-loop transconductance of 2 mA/V, we have

$$R_F = \beta = \frac{1}{2 \text{ mA/V}} = 0.5 \text{ k}\Omega$$

From the solution to Example 12.6, we obtain

$$A = \frac{\mu}{R_F}\,\frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$A = \frac{1000}{0.5}\,\frac{2(0.5 \parallel 100 \parallel 20)}{1 + 2(0.5 \parallel 100 \parallel 20)}$$

$$A = 985.2 \text{ mA/V}$$

$$A_f \equiv \frac{I_o}{V_s} = \frac{985.2}{1 + 985.2 \times 0.5} = 1.996 \text{ mA/V}$$

Ex. 11.12 $A_f \simeq 5$ mA/V

$$\beta \simeq \frac{1}{A_f} = 0.2 \text{ k}\Omega = 200\ \Omega$$

$$R_F = 200\ \Omega$$

Using Eq. (11.36), we obtain

$$A_f \simeq \frac{A_1 g_{m2}}{1 + A_1 g_{m2} R_F}$$

$$= \frac{200 \times 2}{1 + 200 \times 2 \times 0.2} = 4.94 \text{ mA/V}$$

From Eq. (11.32), we have

$$R_i = R_s + R_{id} + R_F$$

$$\simeq R_{id} + R_F$$

$$= 100 + 0.2 = 100.2 \text{ k}\Omega$$

From Eq. (11.35), we get

$$A\beta \simeq A_1 g_{m2} R_F = 200 \times 2 \times 0.2 = 80$$

$$1 + A\beta = 81$$

$$R_{if} = (1 + A\beta) R_i$$

$$= 81 \times 100.2 \simeq 8.1 \text{ M}\Omega$$

From Eq. (11.33), we have

$$R_o = r_{o2} + R_L + R_F$$

$$\simeq r_{o2} + R_F$$

$$= 20 + 0.2 = 20.2 \text{ k}\Omega$$

$$R_{of} = R_o(1 + A\beta) = 20.2 \times 81 = 1.64 \text{ M}\Omega$$

If g_{m2} drops by 50%, A drops by 50% to

$$A = A_1 g_{m2} = 200 \times 1 = 200 \text{ mA/V}$$

and A_f becomes

$$A_f = \frac{200}{1 + 200 \times 0.2} = 4.878 \text{ mA/V}$$

Thus,

$$\Delta A_f = 4.94 - 4.878 = -0.062$$

$$\frac{\Delta A_f}{A_f} \times 100 = -\frac{0.062}{4.94} \times 100 = -1.25\%$$

Ex. 11.13 See figure on next page. Figure 1 shows the circuit for determining the loop gain. The figure also shows the analysis. We start by finding the current in the drain of Q_2 as $g_{m2}V_{g2}$ (this excludes the current in r_{o2}). Since $r_{o2} \gg R_L + R_F$, most of $g_{m2}V_{g2}$ will flow through R_L and $R_F \parallel (R_{id} + R_s)$. Since $R_F \ll R_{id} + R_s$, the voltage across R_F will be approximately $-g_{m2}V_{g2}R_F$. This voltage is amplified by A_1 which provides at its output

$$V_r = -A_1 g_{m2} R_F V_{g2}$$

Thus, we find $A\beta$ as

$$A\beta \equiv -\frac{V_r}{V_{g2}} = A_1 g_{m2} R_F$$

This figure belongs to Exercise 11.13.

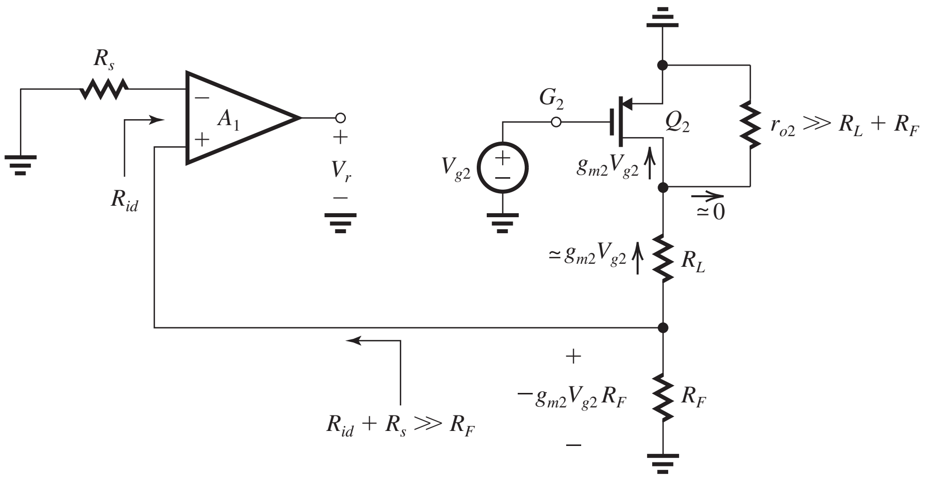


Figure 1

Ex. 11.14 From Eq. (11.34), we obtain

$A\beta =$

$(A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right)\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right)$

From Eq. (11.32), we obtain

$R_i = R_s + R_{id} + R_F$

$R_{if} = R_i(1 + A\beta)$

$= R_i + A\beta R_i$

$= R_s + R_{id} + R_F + (A_1 g_{m2} R_F) R_{id}\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right)$

$R_{\text{in}} = R_{if} - R_s$

$R_{\text{in}} = R_{id} + R_F + (A g_{m2} R_F) R_{id}\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right)$

For $R_F \ll R_{id}$ and $r_{o2} \gg R_L + R_F$, we have

$R_{\text{in}} \simeq R_{id} + A g_{m2} R_F R_{id}$

$= R_{id}(1 + A g_{m2} R_F)$ Q.E.D.

Ex. 11.15 From Eq. (11.34), we obtain

$A\beta =$

$(A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right)\left(\frac{r_{o2}}{r_{o2} + R_L + R_F}\right)$

From Eq. (11.33), we get

$R_o = r_{o2} + R_L + R_F$

$R_{of} = R_o(1 + A\beta)$

$= R_o + A\beta R_o$

$= r_{o2} + R_L + R_F + (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right) r_{o2}$

$R_{\text{out}} = R_{of} - R_L$

$= r_{o2} + R_F + (A_1 g_{m2} R_F)\left(\frac{R_{id}}{R_{id} + R_s + R_F}\right) r_{o2}$

For $R_F \ll r_{o2}$ and $R_{id} \gg R_s + R_F$, we have

$R_{\text{out}} \simeq r_{o2}(1 + A_1 g_{m2} R_F)$ Q.E.D.

Ex. 11.16 To obtain

$A_f \equiv \frac{I_o}{V_s} \simeq 100$ mA/V

when the loop gain is large, we use

$\beta \simeq \frac{1}{A_f} = 10\ \Omega$

But,

$\beta = R_{E1} \times \frac{R_{E2}}{R_{E2} + R_F + R_{E1}}$

For $R_{E1} = R_{E2} = 100\ \Omega$,

$10 = \frac{100 \times 100}{200 + R_F}$

$\Rightarrow R_F = 800\ \Omega$

$\frac{V_o}{V_s} = \frac{-I_o R_{C1}}{V_s}$

$= -A_f R_{C1} = -100 \times 0.6 = -60$ V/V

Ex. 11.17 See figure on next page. Figure 1 shows the circuit prepared for the determination of the loop gain,

$A\beta \equiv -\frac{V_r}{V_t}$

This figure belongs to Exercise 11.17.

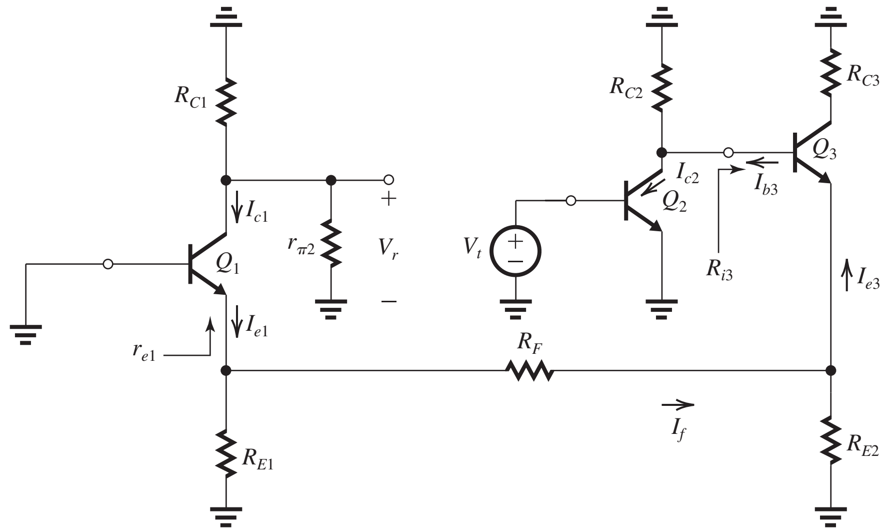


Figure 1

We shall trace the signal around the loop as follows:

$$I_{c2} = g_{m2}V_t \quad (1)$$

$$I_{b3} = I_{c2}\frac{R_{C2}}{R_{C2}+R_{i3}} \quad (2)$$

where

$$R_{i3} = (\beta+1)\{r_{e3} + [R_{E2} \parallel (R_F + (R_{E1} \parallel r_{e1}))]\} \quad (3)$$

$$I_{e3} = (\beta+1)I_{b3} \quad (4)$$

$$I_f = I_{e3}\frac{R_{E2}}{R_{E2}+R_F+(R_{E1} \parallel r_{e1})} \quad (5)$$

$$I_{e1} = I_f\frac{R_{E1}}{R_{E1}+r_{e1}} \quad (6)$$

$$I_{c1} = \alpha I_{e1} \quad (7)$$

$$V_r = -I_{c1}(R_{C1} \parallel r_{\pi 2}) \quad (8)$$

Combining (1)–(7) gives V_r in terms of V_t and hence $A\beta \equiv -V_r/V_t$. We shall do this numerically using the values in Example 11.8:

$g_{m2} = 40$ mA/V, $R_{C2} = 5$ kΩ, $\beta = 100$,

$r_{e3} = 6.25\ \Omega$, $R_{E1} = R_{E2} = 100\ \Omega$, $R_F = 640\ \Omega$,

$r_{e1} = 41.7\ \Omega$, $\alpha_1 = 0.99$, $R_{C1} = 9$ kΩ, and $r_{\pi 2} = 2.5$ kΩ

$$R_{i3} = 101\{0.00625 + [0.1 \parallel (0.64 + (0.1 \parallel 0.0417))]\}$$

$$= 9.42 \text{ k}\Omega$$

$$I_{c2} = 40V_t \quad (9)$$

$$I_{b3} = 0.347I_{c2} \quad (10)$$

$$I_{e3} = 101I_{b3} \quad (11)$$

$$I_f = 0.13I_{e3} \quad (12)$$

$$I_{e1} = 0.706I_f \quad (13)$$

$$I_{c1} = 0.99I_{e1} \quad (14)$$

$$V_r = -1.957I_{c1} \quad (15)$$

Combining (9)–(15), we obtain

$$A\beta = -\frac{V_r}{V_t} = 249.3$$

Ex: 11.18

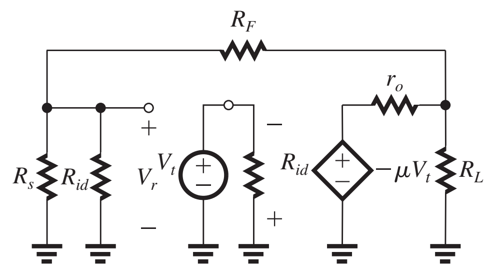


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

Using the voltage-divider rule, we can write by inspection

$$V_r = -\mu V_t \frac{R_L \parallel [R_F + (R_s \parallel R_{id})]}{r_o + \{R_L \parallel [R_F + (R_s \parallel R_{id})]\}}\ \frac{(R_s \parallel R_{id})}{R_F + (R_s \parallel R_{id})}$$

$$V_r = -\mu V_t \frac{R_L(R_s \parallel R_{id})}{r_o[R_L + R_F + (R_s \parallel R_{id})] + R_L[R_F + (R_s \parallel R_{id})]}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} = \frac{\mu R_L(R_{id} \parallel R_s)}{r_o[R_L + R_F + (R_{id} \parallel R_s)] + R_L[R_F + (R_{id} \parallel R_s)]}$$

Q.E.D.

Using the numerical values in Example 11.9, we get

$$A\beta = \frac{10^4 \times 1 \times 1}{0.1(1 + 10 + 1) + 1(10 + 1)}$$

$$= 819.7$$

Ex. 11.19 See figure on next page. Figure 1(a) shows the feedback amplifier circuit. The β circuit is shown in Fig. 1(b), and the determination of β is shown in Fig. 1(c),

$$\beta = -\frac{1}{R_F}$$

(a) For large loop gain, we have

$$A_f \simeq \frac{1}{\beta} = -R_F$$

(b) The determination of R_{11} and R_{22} is illustrated in Figs. 1(d) and (e), respectively:

$$R_{11} = R_{22} = R_F$$

Finally, the A circuit is shown in Fig. 1(f). We can write by inspection

$$R_i = R_s \parallel R_{11} = R_s \parallel R_F$$

$$R_o = r_o \parallel R_{22} = r_o \parallel R_F$$

$$V_{gs} = I_i R_i$$

$$V_o = -g_m V_{gs}(r_o \parallel R_{22})$$

Thus,

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_F)(r_o \parallel R_F)$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{g_m(R_s \parallel R_F)(r_o \parallel R_F)}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

Q.E.D.

(c) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m(r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{in}}$$

thus,

$$\frac{1}{R_{in}} = \frac{1}{R_F}[1 + g_m(r_o \parallel R_F)]$$

$$\Rightarrow R_{in} = \frac{R_F}{1 + g_m(r_o \parallel R_F)}$$

Q.E.D.

(d) $R_{out} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$$= \frac{r_o \parallel R_F}{1 + g_m(R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{out}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m(R_s \parallel R_F)}{R_F}$$

$$\Rightarrow R_{out} = r_o \parallel \frac{R_F}{1 + g_m(R_s \parallel R_F)}$$

Q.E.D.

(e) $A = -5(1 \parallel 10)(20 \parallel 10)$

$$A = -30.3 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10} = -0.1 \text{ mA/V}$$

$$A\beta = 3.03$$

$$1 + A\beta = 4.03$$

$$A_f = \frac{A}{1 + A\beta} = -\frac{30.3}{4.03} = -7.52 \text{ k}\Omega$$

(Compare to the ideal value of -10 kΩ).

$$R_i = R_s \parallel R_F = 1 \parallel 10 = 909\ \Omega$$

$$R_o = r_o \parallel R_F = 20 \parallel 10 = 6.67 \text{ k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{909}{4.03} = 226\ \Omega$$

$$R_{in} = 1 \Big/ \left[\frac{1}{R_{if}} - \frac{1}{R_s}\right] = 291\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{6.67}{4.03} = 1.66 \text{ k}\Omega$$

$$R_{out} = R_{of} = 1.66 \text{ k}\Omega$$

Ex. 11.20 From Eq. (11.54), we obtain

$$A = -\mu \frac{R_i}{R_1 \parallel R_2} \frac{R_1 \parallel R_2 \parallel r_{o2}}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})}$$

For $\mu = 100$, $R_1 = 10$ kΩ, $R_2 = 90$ kΩ, $g_m = 5$ mA/V, $r_{o2} = 20$ kΩ, we have

$$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$$

$$= \infty \parallel \infty \parallel 100 = 100 \text{ k}\Omega$$

$$A = -100 \frac{100}{10 \parallel 90} \frac{10 \parallel 90 \parallel 20}{0.2 + (10 \parallel 90 \parallel 20)}$$

$$= -1076.4 \text{ A/A}$$

$$\beta = -\frac{R_1}{R_1 + R_2} = -\frac{10}{10 + 90} = -0.1 \text{ A/A}$$

$$A_f = -\frac{1076.4}{1 + 107.64} = -9.91 \text{ A/A}$$

This figure belongs to Exercise 11.19.

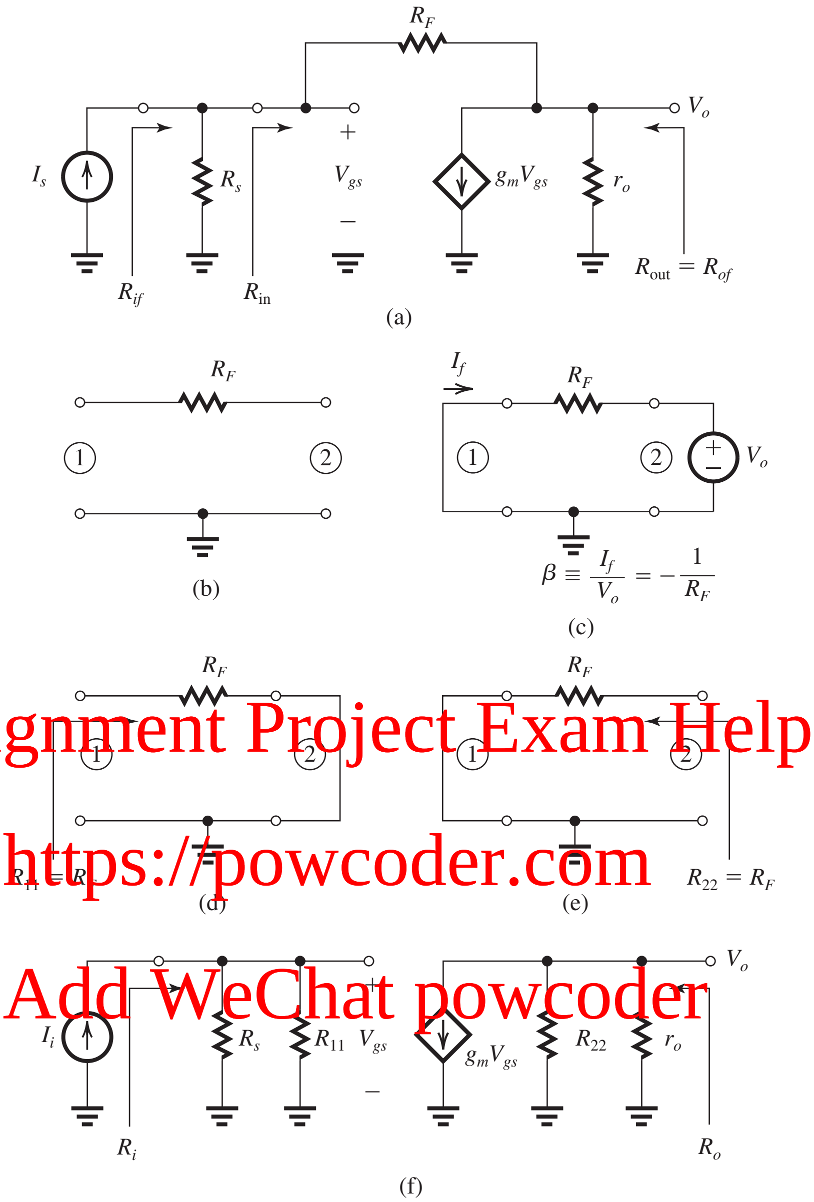


Figure 1

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{100\ \text{k}\Omega}{108.64} = 920\ \Omega$$

$R_{in} = R_{if} = 920\ \Omega$

$R_{out} = R_{of} = R_o(1 + A\beta)$

$R_o = r_{o2} + (R_1 \parallel R_2) + g_m r_{o2}(R_1 \parallel R_2)$

$= 929\ \text{k}\Omega$

$R_{out} = 929 \times 108.64 = 101\ \text{M}\Omega$

Ex. 11.21 With $R_2 = 0$, Eq. (11.48) gives

$\beta = -1$

$A_f = -1$ A/A

Substituting $R_2 = 0$ and $R_s = R_{id} = \infty$ in Eq. (11.50), we obtain

$R_i = R_1$

and in Eq. (11.55), we obtain

$R_o = r_{o2}$

and in Eq. (11.53), we obtain

$$A = -\mu \frac{R_1}{1/g_m} = -\mu g_m R_1$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$A_f = -\frac{\mu g_m R_1}{1 + \mu g_m R_1}$$

$$R_{\text{in}} = R_{if} = R_i/(1 + A\beta)$$

$$= \frac{R_1}{1 + \mu g_m R_1}$$

For $\mu g_m R_1 \gg 1$, we have

$$R_{\text{in}} \simeq 1/\mu g_m$$

$$R_{\text{out}} = R_{of} = (1 + A\beta)R_o$$

$$= (1 + \mu g_m R_1)r_{o2}$$

$$\simeq \mu(g_m r_{o2})R_1$$

Ex. 11.22 Total phase shift will be 180° at the frequency ω_{180} at which the phase shift of each amplifier stage is 60°. Thus,

$$\tan^{-1}\frac{\omega_{180}}{10^4} = 60°$$

$$\omega_{180} = \tan 60° \times 10^4$$

$$= \sqrt{3} \times 10^4 \text{ rad/s}$$

At ω_{180}, we have

$$|A| = \left(\frac{10}{\sqrt{1+3}}\right)^3$$

$$= 125$$

Thus, the loop gain magnitude will be

$$|A\beta| = 125\beta$$

For stable operation, we require

$$125\beta_{cr} < 1$$

$$\Rightarrow \beta_{cr} = \frac{1}{125} = 0.008$$

$\beta \geq \beta_{cr}$ will result in oscillations. Correspondingly, the minimum closed-loop gain for stable operation will be

$$A_f = \frac{10^3}{1 + 10^3\ \beta_{cr}}$$

$$= \frac{10^3}{1 + 1000 \times 0.008} = \frac{1000}{9} = 111.1$$

Ex. 11.23 The feedback shifts the pole by a factor equal to the amount of feedback:

$$1 + A_0\beta = 1 + 10^5 \times 0.01 = 1001$$

The pole will be shifted to a frequency

$$f_{Pf} = f_P(1 + A_0\beta)$$

$$= 100 \times 1001 = 100.1 \text{ kHz}$$

If β is changed to a value that results in a nominal closed-loop gain of 1, then we obtain

$$\beta \simeq 1$$

and

$$1 + A_0\beta = 1 + 10^5 \times 1 \simeq 10^5$$

then the pole will be shifted to a frequency

$$f_{Pf} = 10^5 \times 100 = 10 \text{ MHz}$$

Ex. 11.24 From Eq. (11.68), we see that the poles coincide when

$$(\omega_{P1} + \omega_{P2})^2 = 4(1 + A_0\beta)\omega_{P1}\omega_{P2}$$

$$(10^4 + 10^6)^2 = 4(1 + 100\beta) \times 10^4 \times 10^6$$

$$\Rightarrow 1 + 100\beta = 25.5$$

$$\Rightarrow \beta = 0.245$$

The corresponding value of $Q = 0.5$. This can also be verified by substituting in Eq. (11.70).

A maximally flat response is obtained when $Q = 1/\sqrt{2}$. Substituting in Eq. (11.70), we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{(1 + 100\beta) \times 10^4 \times 10^6}}{10^4 + 10^6}$$

$$\Rightarrow \beta = 0.5$$

In this case, the low-frequency closed-loop gain is

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{100}{1 + 100 \times 0.5} = 1.96 \text{ V/V}$$

Ex. 11.25 The closed-loop poles are the roots of the characteristic equation

$$1 + A(s)\beta = 0$$

$$1 + \left(\frac{10}{1 + \frac{s}{10^4}}\right)^3 \beta = 0$$

To simplify matters, we normalize s by the factor 10^4, thus obtaining the normalized complex-frequency variable $S = s/10^4$, and the characteristic equation becomes

$$(S+1)^3 + 10^3\beta = 0 \quad (1)$$

This equation has three roots, a real one and a pair that can be complex conjugate. The real pole can be found from

$$(S+1)^3 = -10^3\beta$$

$$\Rightarrow S = -1 - 10\beta^{1/3} = -\left(1 + 10\beta^{1/3}\right) \quad (2)$$

Dividing the characteristic polynomial in (1) by $\left(S + 1 + 10\beta^{1/3}\right)$ gives a quadratic whose two roots are the remaining poles of the feedback amplifier. After some straightforward but somewhat tedious algebra, we obtain

$$S^2 + \left(10\beta^{1/3} - 2\right)S + \left(1 + 100\beta^{2/3} - 10\beta^{1/3}\right)$$

$$= 0 \quad (3)$$

The pair of poles can now be obtained as

$$S = \left(-1 + 5\beta^{1/3}\right) \pm j5\sqrt{3}\,\beta^{1/3} \quad (4)$$

Equations (1) and (3) describe the three poles shown in Fig. E11.25.

From Eq. (2) we see that the pair of complex poles lie on the $j\omega$ axis for the value of β that makes the coefficient of S equal to zero, thus,

$$\beta_{cr} = \left(\frac{2}{10}\right)^3 = 0.008$$

Note that this is the same value found in the solution of Exercise 11.22.

Ex. 11.26

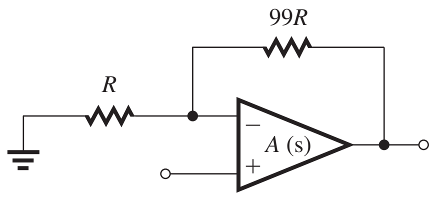


Figure 1

From Fig. 1, we can easily obtain the loop gain as

$$A\beta = A(s) \times 0.01$$

$$= \frac{10^5}{1 + \dfrac{s}{2\pi \times 10}} \times 0.01$$

$$= \frac{1000}{1 + \dfrac{s}{2\pi \times 10}}$$

From this single-pole response (low-pass STC response) we can find the unity-gain frequency by inspection as

$$f_1 = f_P \times 1000$$

$$= 10^4 \text{ Hz}$$

The phase angle at f_1 will be $-90°$ and thus the phase margin is $90°$.

Ex. 11.27 From Eq. (11.82), we obtain

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + e^{-j\theta}|$$

$$= 1/|1 + \cos\theta - j\sin\theta|$$

(a) For PM $= 30°$, $\theta = 180 - 30 = 150°$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 150° - j\sin 150°|$$

$$= 1.93$$

(b) For PM $= 60°$, $\theta = 180 - 60 = 120°$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 120° - j\sin 120°|$$

$$= 1$$

(c) For PM $= 90°$, $\theta = 180 - 90 = 90°$, thus

$$\frac{|A_f(j\omega_1)|}{1/\beta} = 1/|1 + \cos 90° - j\sin 90°|$$

$$= 1/\sqrt{2} = 0.707$$

Ex. 11.28 See figure on next page. To obtain guaranteed stable performance, the maximum rate of closure must not exceed 20 dB/decade. Thus we utilize the graphical construction in Fig. 1 to obtain the

This figure belongs to Exercise 11.28.

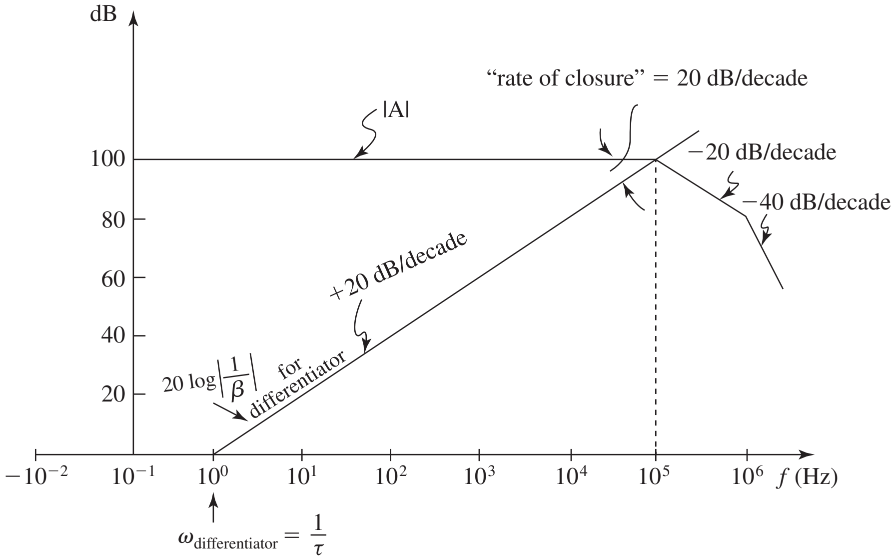


Figure 1

maximum value of the differentiator frequency as 1 Hz. Thus:

$$\frac{1}{\tau} \leq 2\pi \times 1 \text{ Hz}$$

$$\tau \geq \frac{1}{2\pi}\text{s} = 159 \text{ ms}$$

Ex. 11.29 To obtain stable performance for closed-loop gains as low as 20 dB (which is 80 dB below A_0, or equivalently 10^4 below A_0), we must place the new dominant pole at 1 MHz/10^4 = 100 Hz.

Ex. 11.30 The frequency of the first pole must be lowered from 1 MHz to a new frequency

$$f_D' = \frac{10 \text{ MHz}}{10^4} = 1000 \text{ Hz}$$

that is, by a factor of 1000. Thus, the capacitance at the controlling node must be increased by a factor of 1000.

11.1 $A_f = \dfrac{A}{1 + A\beta}$

$$200 = \frac{10^4}{1 + 10^4\beta}$$

$$\Rightarrow \beta = 4.9 \times 10^{-3}$$

If A changes to 10^3, then we get

$$A_f = \frac{1000}{1 + 10^3 \times 4.9 \times 10^{-3}}$$

$$= \frac{1000}{5.9} = 169.5$$

Percentage change in $A_f = \dfrac{169.5 - 200}{200} \times 100$

$$= -15.3\%$$

11.2 (a) Because of the infinite input resistance of the op amp, the fraction of the output voltage V_o that is fed back and subtracted from V_s is determined by the voltage divider (R_1, R_2), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

(b) (i) $A = 1000$ V/V

$$A_f = \frac{A}{1 + A\beta}$$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099 \text{ V/V}$$

$$\frac{R_1}{R_1 + R_2} = 0.099$$

$$1 + \frac{R_2}{R_1} = \frac{1}{0.099}$$

$$R_2 = R_1\left(\frac{1}{0.099} - 1\right)$$

$$= 10\left(\frac{1}{0.099} - 1\right) = 91 \text{ k}\Omega$$

(ii) $A = 200$ V/V

$$10 = \frac{200}{1 + 200\beta}$$

$$\Rightarrow \beta = 0.095 \text{ V/V}$$

$$R_2 = R_1\left(\frac{1}{0.095} - 1\right)$$

$$= 10\left(\frac{1}{0.095} - 1\right) = 95.3 \text{ k}\Omega$$

(iii) $A = 15$ V/V

$$10 = \frac{15}{1 + 15\beta}$$

$$\Rightarrow \beta = 0.033 \text{ V/V}$$

$$R_2 = 10\left(\frac{1}{0.033} - 1\right)$$

$$= 290 \text{ k}\Omega$$

(c) (i) $A = 1000(1 - 0.2) = 800$ V/V

$$A_f = \frac{800}{1 + 800 \times 0.099}$$

$$= 9.975 \text{ V/V}$$

Thus, A_f changes by

$$= \frac{9.975 - 10}{10} \times 100 = -0.25\%$$

(ii) $A = 200(1 - 0.2) = 160$ V/V

$$A_f = \frac{160}{1 + 160 \times 0.095} = 9.877 \text{ V/V}$$

Thus, A_f changes by

$$= \frac{9.877 - 10}{10} \times 100 = -1.23\%$$

(iii) $A = 15(1 - 0.2) = 12$ V/V

$$A_f = \frac{12}{1 + 12 \times 0.033} = 8.574$$

Thus, A_f changes by

$$= \frac{8.575 - 10}{10} \times 100 = -14.3\%$$

We conclude that as A becomes smaller and hence the amount of feedback $(1 + A\beta)$ is lower, the desensitivity of the feedback amplifier to changes in A decreases. In other words, the negative feedback becomes less effective as $(1 + A\beta)$ decreases.

11.3 The direct connection of the output terminal to the inverting input terminal results in $V_f = V_o$ and thus

$$\beta = 1$$

If $A = 1000$, then the closed-loop gain will be

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 1000 \times 1} = 0.999 \text{ V/V}$$

Amount of feedback $= 1 + A\beta$

$$= 1 + 1000 \times 1 = 1001$$

or 60 dB

For $V_s = 1$ V, we obtain

$$V_o = A_f V_s = 0.999 \times 1 = 0.999 \text{ V}$$

$$V_i = V_s - V_o = 1 - 0.999$$

$$= 0.001 \text{ V}$$

If A becomes $1000(1-0.1) = 900$ V/V, then we get

$$A_f = \frac{900}{1+900\times 1} = 0.99889$$

Thus, A_f changes by

$$= \frac{0.99889 - 0.999}{0.999} \times 100 = -0.011\%$$

11.4 $A = \frac{V_o}{V_i} = \frac{5\text{ V}}{10\text{ mV}} = 500$ V/V

$$V_f = V_s - V_i = 1 - 0.01 = 0.99 \text{ V}$$

$$\beta = \frac{V_f}{V_o} = \frac{0.99}{5} = 0.198 \text{ V/V}$$

11.5 (a) $A_f = \frac{A}{1+A\beta}$

Ideally,

$$A_f = \frac{1}{\beta}$$

$$A_f\big|_{\text{ideal}} - A_f = \frac{1}{\beta} - \frac{A}{1+A\beta}$$

$$= \frac{1+A\beta - A\beta}{(1+A\beta)\beta} = \frac{1}{(1+A\beta)\beta}$$

Expressed as a percentage of the ideal gain $1/\beta$, we have

$$\frac{\text{Difference}}{\text{Ideal}} = \frac{1}{1+A\beta} \times 100\%$$

For $A\beta \gg 1$,

$$\frac{\text{Difference}}{\text{Ideal}} \simeq \frac{100}{A\beta}\%$$

(b) For A_f to be within:

(i) 0.1% of ideal value, then

$$\frac{100}{A\beta} \leq 0.1$$

$$\Rightarrow A\beta \geq 1000$$

(ii) 1% of ideal value, then

$$\frac{100}{A\beta} \leq 1$$

$$\Rightarrow A\beta \geq 100$$

(iii) 5% of ideal value, then

$$\frac{100}{A\beta} \leq 5$$

$$\Rightarrow A\beta \geq 20$$

11.6 For each value of A given, we have three different values of β: 0.00, 0.50, and 1.00. To obtain A_f, we use

$$A_f = \frac{A}{1+A\beta}$$

The results obtained are as follows.

Case	A (V/V)	A_f (V/V) for $\beta = 0.00$	A_f (V/V) for $\beta = 0.50$	A_f (V/V) for $\beta = 1.00$
(a)	1	1	0.667	0.500
(b)	10	10	1.667	0.909
(c)	100	100	1.961	0.990
(d)	1000	1000	1.996	0.999
(e)	10,000	10,000	1.9996	0.9999

11.7 $A = \frac{5\text{ V}}{2\text{ mV}} = 2500$ V/V

$$A_f = \frac{5\text{ V}}{100\text{ mV}} = 50 \text{ V/V}$$

Amount of feedback $\equiv 1 + A\beta$

$$= \frac{A}{A_f} = \frac{2500}{50} = 50$$

or 34 dB

$$A\beta = 49$$

$$\beta = \frac{49}{2500} = 0.0196 \text{ V/V}$$

11.8 $A_{\text{nominal}} = 1000$

$$A_{\text{low}} = 500$$

$$A_{\text{high}} = 1500$$

If we apply negative feedback with a feedback factor β, then

$$A_{f,\text{ nominal}} = \frac{1000}{1+1000\beta}$$

$$A_{f,\text{ low}} = \frac{500}{1+500\beta}$$

$$A_{f,\text{ high}} = \frac{1500}{1+1500\beta}$$

It is required that

$$A_{f,\text{ low}} \geq 0.99 A_{f,\text{ nominal}} \tag{1}$$

and

$$A_{f,\text{ high}} \leq 1.01 A_{f,\text{ nominal}} \tag{2}$$

If we satisfy condition (1) with equality, we can determine the required value of β. We must then clock that condition (2) is satisfied. Thus,

$$\frac{500}{1+500\beta} = 0.99 \times \frac{1000}{1+1000\beta}$$

$$\Rightarrow \beta = 0.098$$

For this value of β, we obtain

$$A_{f,\,\text{nominal}} = \frac{1000}{1 + 1000 \times 0.098}$$

$$= 10.101$$

$$A_{f,\,\text{low}} = \frac{500}{1 + 500 \times 0.098} = 10$$

$$A_{f,\,\text{high}} = \frac{1500}{1 + 1500 \times 0.098} = 10.135$$

Thus, the low value of the closed-loop gain is 0.101 below nominal or -1%, and the high value is 0.034 above nominal or 0.34%. Thus, our amplifier meets specification and the nominal value of closed-loop gain is 10.1. This is the highest possible closed-loop gain that can be obtained while meeting specification.

Now, if three closed-loop amplifiers are placed in cascade, the overall gain obtained will be

$$\text{Nominal Gain} = (10.1)^3 = 1030$$

$$\text{Lowest Gain} = 10^3 = 1000$$

$$\text{Highest Gain} = (10.135)^3 = 1041$$

Thus, the lowest gain will be approximately 3% below nominal, and the highest gain will be 1% above nominal.

11.9

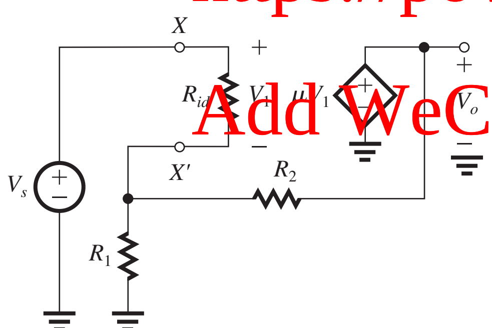


Figure 1

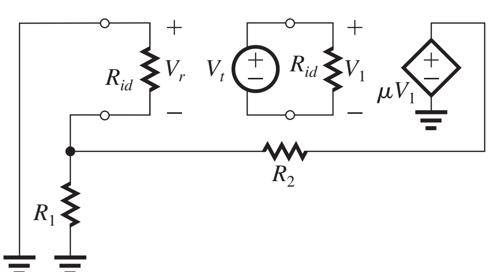


Figure 2

Figure 1 shows the given circuit with the op amp replaced with its equivalent circuit model. To determine the loop gain $A\beta$, we short circuit V_s and break the loop at the input terminals of the op amp. To keep the circuit unchanged, we must place a resistance equal to R_{id} at the left-hand side of the break. This is shown in Fig. 2, where a test signal V_t is applied at the right-hand side of the break. To determine the returned voltage V_r, we use the voltage-divider rule as follows:

$$V_r = -\mu V_1 \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Substituting $V_1 = V_t$ and rearranging, we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2}$$

Since

$$\beta = \frac{R_1}{R_1 + R_2}$$

we get

$$A = \mu \frac{R_1 \parallel R_{id}}{(R_1 \parallel R_{id}) + R_2} \; \frac{R_1 + R_2}{R_1}$$

$$= \mu \frac{R_{id}/(R_1 + R_{id})}{R_1 R_{id}/(R_1 + R_{id}) + R_2} \; (R_1 + R_2)$$

$$= \mu \frac{R_{id}(R_1 + R_2)}{R_1 R_{id} + R_2 R_{id} + R_1 R_2}$$

Thus,

$$A = \mu \frac{R_{id}}{R_{id} + (R_1 \parallel R_2)} \qquad \text{Q.E.D.}$$

11.10 From Eq. (11.10), we have

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{1 + A\beta}$$

Since -40 dB is 0.01, we have

$$0.01 = \frac{1}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

For

$$\frac{dA_f/A_f}{dA/A} = \frac{1}{5}$$

we have

$$1 + A\beta = 5$$

$$\Rightarrow A\beta = 4$$

11.11 For $A = 1000$ V/V, we have

$$A_f = 10 = \frac{1000}{1 + A\beta}$$

$$\Rightarrow \text{ Densensitivity factor} \equiv 1 + A\beta = 100$$

$$A\beta = 99$$

$$\beta = \frac{99}{1000} = 0.099 \text{ V/V}$$

For $A = 500$ V/V, we have

$$A_f = 10 = \frac{500}{1 + A\beta}$$

$\Rightarrow$ Densensitivity factor $\equiv 1 + A\beta = 50$

$$\beta = \frac{49}{500} = 0.098 \text{ V/V}$$

If the $A = 1000$ amplifiers have a gain uncertainty of $\pm 10\%$, the gain uncertainty of the closed-loop amplifiers will be

$$= \frac{\pm 10\%}{100} = \pm 0.1\%$$

If we require a gain uncertainty of $\pm 0.1\%$ using the $A = 500$ amplifiers, then

$$\pm 0.1\% = \frac{\text{Gain uncertainty of } A = 500 \text{ amplifiers}}{50}$$

$\Rightarrow$ Gain uncertainty $= \pm 5\%$

11.12 $A_f = 10$ V/V

$$1 + A\beta = \frac{\pm 10\%}{\pm 0.1\%} = 100$$

$$10 = \frac{A}{100}$$

$\Rightarrow A = 1000$ V/V

$$\beta = \frac{100 - 1}{1000} = 0.099 \text{ V/V}$$

11.13 The open-loop gain varies from A to $10A$ with temperature and time. Correspondingly, A_f varies from $(25 - 1\%)$ i.e. 24.75 V/V to $(25 + 1\%)$ or 25.25 V/V. Substituting these quantities into the formula for the closed-loop gain

$$A_f = \frac{A}{1 + A\beta}$$

we obtain

$$24.75 = \frac{A}{1 + A\beta} \qquad (1)$$

$$25.25 = \frac{10A}{1 + 10A\beta} \qquad (2)$$

Dividing Eq. (2) by Eq. (1), we obtain

$$1.02 = 10\frac{1 + A\beta}{1 + 10A\beta}$$

$$1.02 + 10.2A\beta = 10 + 10A\beta$$

$\Rightarrow A\beta = 44.9$

Substituting in (1) yields

$$A = 24.75 \times 45.9 = 1136$$

and

$$\beta = \frac{44.9}{1136} = 0.0395$$

11.14 Let the gain of the ideal (nonvarying) driver amplifier be denoted μ. Then, the open-loop gain A will vary from 2μ to 12μ. Correspondingly, the closed-loop gain will vary from 95 V/V to 105 V/V. Substituting these quantities into the closed-loop gain expression, we obtain

$$95 = \frac{2\mu}{1 + 2\mu\beta} \qquad (1)$$

$$105 = \frac{12\mu}{1 + 12\mu\beta} \qquad (2)$$

Dividing Eq. (2) by Eq. (1) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$1.105 + 1.105 \times 12\mu\beta = 6 + 12\mu\beta$$

$\Rightarrow \mu\beta = 3.885$

Substituting in Eq. (1) yields

$$\mu = \frac{95(1 + 2 \times 3.885)}{2} = 416.6 \text{ V/V}$$

$$\beta = \frac{3.885}{416.6} = 9.33 \times 10^{-3} \text{ V/V}$$

If A_f is to be held to within $\pm 0.5\%$, Eqs. (1) and (2) are modified to

$$99.5 = \frac{2\mu}{1 + 2\mu\beta} \qquad (3)$$

$$100.5 = \frac{12\mu}{1 + 12\mu\beta} \qquad (4)$$

Dividing (4) by (3) yields

$$1.01 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$\Rightarrow \mu\beta = 49.92$

Substituting into (3) provides

$$\mu = \frac{99.5(1 + 2 \times 49.92)}{2}$$

$= 5016.8$ V/V

which is more than a factor of 10 higher than the gain required in the less constrained case. The value of β required is

$$\beta = \frac{49.92}{5016.8} = 9.95 \times 10^{-3} \text{ V/V}$$

Repeating for $A_f = 10$ V/V (a factor of 10 lower than the original case):

(a) For ±5% maximum variability, Eqs. (1) and (2) become

$$9.5 = \frac{2\mu}{1 + 2\mu\beta} \quad (5)$$

$$10.5 = \frac{12\mu}{1 + 12\mu\beta} \quad (6)$$

Dividing (6) by (5) yields

$$1.105 = \frac{6(1 + 2\mu\beta)}{1 + 12\mu\beta}$$

$$\Rightarrow \mu\beta = 3.885$$

which is identical to the first case considered, and

$$\mu = \frac{9.5(1 + 2 \times 3.885)}{1 + 12\mu\beta} = 41.66 \text{ V/V}$$

which is a factor of 10 lower than the value required when the gain required was 100. The feedback factor β is

$$\beta = \frac{3.885}{41.66} = 9.33 \times 10^{-2} \text{ V/V}$$

which is a factor of 10 higher than the case with $A_f = 10$.

(b) Finally, for the case $A_f = 10 \pm 0.5\%$ we can write by analogy

$$\mu\beta = 49.92$$

$$\mu = 501.68 \text{ V/V}$$

$$\beta = 9.95 \times 10^{-2} \text{ V/V}$$

11.15 If we use one stage, the amount of feedback required is

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{100} = 10$$

Thus the closed-loop amplifier will have a variability of

$$\text{Variability of } A_f = \frac{\pm 30\%}{10} = \pm 3\%$$

which does not meet specifications. Next, we try using two stages. For a nominal gain of 100, each stage will be required to have a nominal gain of 10. Thus, for each stage the amount of feedback required will be

$$1 + A\beta = \frac{1000}{10} = 100$$

Thus, the closed-loop gain of each stage will have a variability of

$$= \frac{\pm 30\%}{100} = \pm 0.3\%$$

and the cascade of two stages will thus show a variability of ±0.6%, well within the required ±1%. Thus two stages will suffice.

We next investigate the design in more detail. Each stage will have a nominal gain of 10 and thus

$$1 + A\beta = \frac{1000}{10} = 100$$

$$\Rightarrow A\beta = 99$$

$$\Rightarrow \beta = 0.099$$

Since A ranges from 700 V/V to 1300 V/V, the gain of each stage will range from

$$A_{f,\text{ low}} = \frac{700}{1 + 700 \times 0.099} = 9.957 \text{ V/V}$$

and a high value of

$$A_{f,\text{ high}} = \frac{1300}{1 + 1300 \times 0.099} = 10.023 \text{ V/V}$$

Thus, the cascade of two stages will have a range of

Lowest gain $= 9.957^2 = 99.14$ V/V

Highest gain $= 10.023^2 = 100.46$ V/V

which is −0.86% to +0.46% of the nominal 100 V/V gain, well within the required ±1%.

11.16 If the nominal open-loop gain is A, then we require that as A drops to $(A/2)$ the closed-loop gain drops from 10 to a minimum of 9.8. Substituting these values in the expression for the closed-loop gain, we obtain

$$10 = \frac{A}{1 + A\beta} \quad (1)$$

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A\beta} \quad (2)$$

Dividing Eq. (1) by Eq. (2) yields

$$1.02 = \frac{2\left(1 + \frac{1}{2}A\beta\right)}{1 + A\beta}$$

$$1.02 = \frac{2 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{1}{1 + A\beta}$$

$$\Rightarrow 1 + A\beta = \frac{1}{0.02} = 50$$

Substituting in Eq. (1) gives

$$A = 10 \times 50 = 500 \text{ V/V}$$

and

$$\beta = \frac{50-1}{500} = 0.098 \text{ V/V}$$

If β is accurate to within $\pm 1\%$, to ensure that the minimum closed-loop gain realized is 9.8 V/V, we have

$$9.8 = \frac{A/2}{1 + \frac{1}{2}A \times 0.098 \times 1.01}$$

$$\Rightarrow A = 653.4 \text{ V/V}$$

11.17 $A_f = \dfrac{A}{1 + A\beta}$

$$100 = \frac{A}{1 + A\beta} \qquad (1)$$

$$99 = \frac{0.1A}{1 + 0.1A\beta} \qquad (2)$$

Dividing Eq. (1) by Eq. (2) gives

$$1.01 = \frac{10(1 + 0.1A\beta)}{1 + A\beta}$$

$$= \frac{10 + A\beta}{1 + A\beta}$$

$$= 1 + \frac{9}{1 + A\beta}$$

$$\Rightarrow \frac{9}{1 + A\beta} = 0.01$$

$$1 + A\beta = 900$$

$$A\beta = 899$$

Substituting $(1 + A\beta) = 900$ into Eq. (1) yields

$$A = 100 \times 900 = 90{,}000 \text{ V/V}$$

The value of β is

$$\beta = \frac{899}{90{,}000} = 9.989 \times 10^{-3} \text{ V/V}$$

If A were increased tenfold, i.e, $A = 900{,}000$, we obtain

$$A_f = \frac{900{,}000}{1 + 8990} = 100.1 \text{ V/V}$$

If A becomes infinite, we get

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1}{\frac{1}{A} + \beta} = \frac{1}{\beta}$$

$$= \frac{1}{9.989 \times 10^{-3}} = 100.11 \text{ V/V}$$

11.18 $A = A_M \dfrac{s}{s + \omega_L}$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{A_M s/(s + \omega_L)}{1 + A_M \beta s/(s + \omega_L)}$$

$$= \frac{A_M s}{s + \omega_L + s A_M \beta}$$

$$= \frac{A_M s}{s(1 + A_M \beta) + \omega_L}$$

$$= \frac{A_M}{1 + A_M \beta} \; \frac{s}{s + \omega_L/(1 + A_M \beta)}$$

Thus,

$$A_{Mf} = \frac{A_M}{1 + A_M \beta}$$

$$\omega_{Lf} = \frac{\omega_L}{1 + A_M \beta}$$

Thus, both the midband gain and the 3-dB frequency are lowered by the amount of feedback, $(1 + A_M \beta)$.

11.19 $1 + A_M \beta = \dfrac{1000}{10} = 100$

Thus,

$$f_{Hf} = (1 + A_M \beta) f_H$$

$$= 100 \times 10 = 1000 \text{ kHz} = 1 \text{ MHz}$$

$$f_{Lf} = \frac{f_L}{1 + A_M \beta}$$

$$= \frac{100}{100} = 1 \text{ Hz}$$

11.20 To capacitively couple the output signal to an 8-Ω loudspeaker and obtain $f_L = 100$ Hz, we need a coupling capacitor C,

$$C = \frac{1}{2\pi f_L \times 8}$$

$$= \frac{1}{2\pi \times 100 \times 8} = 198.9 \ \mu\text{F} \simeq 200 \ \mu\text{F}$$

If closed-loop gain A_{Mf} of 10 V/V is obtained from an amplifier whose open-loop gain $A_M = 1000$ V/V, then

$$1 + A_M \beta = \frac{1000}{10} = 100$$

and

$$f_{Lf} = \frac{f_L}{100} = \frac{100}{100} = 1 \text{ Hz}$$

If the required f_{Lf} is 50 Hz, then

$$f_L = 50 \times (1 + A_M \beta)$$

$$= 50 \times 100 = 5000 \text{ Hz},$$

and the coupling capacitor C will have a value of

$$C = \frac{1}{2\pi \times 5000 \times 8} \simeq 4 \ \mu\text{F}$$

11.21 Let's first try $N = 2$. The closed-loop gain of each stage must be

$$A_f = \sqrt{1000} = 31.6 \text{ V/V}$$

Thus, the amount-of-feedback in each stage must be

$$1 + A\beta = \frac{A}{A_f} = \frac{1000}{31.6} = 31.6$$

The 3-dB frequency of each stage is

$$f_{3dB}|_{\text{stage}} = (1 + A\beta)f_H$$

$$= 31.6 \times 20 = 632 \text{ kHz}$$

Thus, the 3-dB frequency of the cascade amplifier is

$$f_{3dB}|_{\text{cascade}} = 632\sqrt{2^{1/2} - 1} = 406.8 \text{ kHz}$$

which is less than the required 1 MHz.

Next, we try $N = 3$. The closed-loop gain of each stage is

$$A_f = (1000)^{1/3} = 10 \text{ V/V}$$

and thus each stage will have an amount-of-feedback

$$1 + A\beta = \frac{1000}{10} = 100$$

which results in a stage 3-dB frequency of

$$f_{3dB}|_{\text{stage}} = (1 + A\beta)f_H$$

$$= 100 \times 20 = 2000 \text{ kHz}$$

$$= 2 \text{ MHz}$$

The 3-dB frequency of the cascade amplifier will be

$$f_{3dB}|_{\text{cascade}} = 2\sqrt{2^{1/3} - 1}$$

$$= 1.02 \text{ MHz}$$

which exceeds the required value of 1 MHz. Thus, we need three identical stages, each with a closed-loop gain of 10 V/V, an amount-of-feedback of 100, and a loop gain

$$A\beta = 99$$

Thus,

$$\beta = 0.099 \text{ V/V}$$

11.22 $V_{o\,\text{ripple}} = V_n \dfrac{A_1}{1 + A_1 A_2 \beta}$

To reduce $V_{o\,\text{ripple}}$ to 100 mV,

$$0.1 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 9$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{9}$$

$$\Rightarrow A_2 = 100 \text{ V/V}$$

$$\beta = \frac{8}{0.9 \times 100} = 0.089 \text{ V/V}$$

To reduce $V_{o\,\text{ripple}}$ to 10 mV,

$$0.01 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 90$$

$$A_f = \frac{A_1 A_2}{1 + A_1 A_2 \beta}$$

$$10 = \frac{0.9 A_2}{90}$$

$$A_2 = 1000 \text{ V/V}$$

$$\beta = \frac{89}{0.9 \times 1000} = 0.099 \text{ V/V}$$

To reduce $V_{o\,\text{ripple}}$ to 1 mV,

$$0.001 = 1 \times \frac{0.9}{1 + A_1 A_2 \beta}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 900$$

$$10 = \frac{0.9 A_2}{900}$$

$$\Rightarrow A_2 = 10{,}000 \text{ V/V}$$

$$\beta = \frac{899}{0.9 \times 10{,}000} = 0.0999 \text{ V/V}$$

11.23 $A_f = \dfrac{A_1 A_2}{1 + A_1 A_2 \beta}$

$$100 = \frac{10 A_2}{1 + A_1 A_2 \beta} \qquad (1)$$

$$(1 + A_1 A_2 \beta) \times 8 = 40 \text{ kHz}$$

$$\Rightarrow 1 + A_1 A_2 \beta = 5$$

Substituting in (1) gives

$$A_2 = \frac{100 \times 5}{10} = 50 \text{ V/V}$$

$$1 + 10 \times 50 \times \beta = 5$$

$$\Rightarrow \beta = 0.008 \text{ V/V}$$

$$f_{Lf} = \frac{80}{1 + A_1 A_2 \beta}$$

$$= \frac{80}{5} = 16 \text{ Hz}$$

11.24

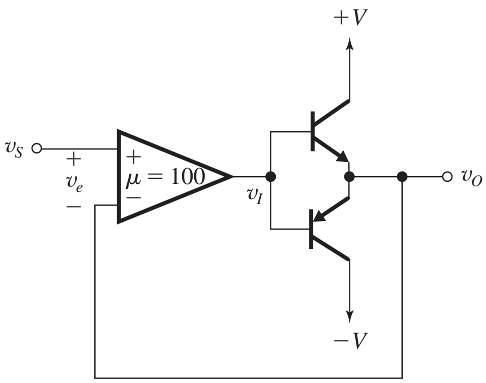


Figure 1

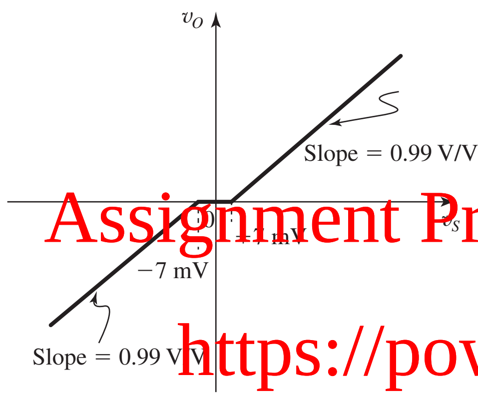


Figure 2

Refer to Fig. 1. For $v_I = +0.7$ V, we have $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{+0.7}{100} = +7 \text{ mV}$$

Similarly, for $v_I = -0.7$ V, we obtain $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{-0.7}{100} = -7 \text{ mV}$$

Thus, the limits of the deadband are now ± 7 mV. Outside the deadband, the gain of the feedback amplifier, that is, v_O/v_S, can be determined by noting that the open-loop gain $A \equiv v_O/v_e = 100$ V/V and the feedback factor $\beta = 1$, thus

$$A_f \equiv \frac{v_O}{v_S} = \frac{A}{1 + A\beta}$$

$$= \frac{100}{1 + 100 \times 1} = 0.99 \text{ V/V}$$

The transfer characteristic is depicted in Fig. 2.

11.25 The closed-loop gain for the first (high-gain) segment is

$$A_{f1} = \frac{1000}{1 + 1000\beta} \quad (1)$$

and that for the second segment is

$$A_{f2} = \frac{100}{1 + 100\beta} \quad (2)$$

We require

$$\frac{A_{f1}}{A_{f2}} = 1.1$$

Thus, dividing Eq. (1) by Eq. (2) yields

$$1.1 = 10\frac{1 + 100\beta}{1 + 1000\beta}$$

$$1.1 + 1100\beta = 10 + 1000\beta$$

$$\Rightarrow \beta = 0.089$$

$$A_{f1} = \frac{1000}{1 + 1000 \times 0.089} = 11.1 \text{ V/V}$$

$$A_{f2} = \frac{100}{1 + 100 \times 0.089} = 10.1 \text{ V/V}$$

The first segment ends at $|v_O| = 10 \text{ mV} \times 1000 = 10$ V. This corresponds to

$$v_S = \frac{10 \text{ V}}{A_{f1}} = \frac{10}{11.1} = 0.9 \text{ V}$$

The second segment ends at $|v_O| = 10 + 0.05 \times 100 = 15$ V. This corresponds to

$$v_S = 0.9 + \frac{15 - 10}{A_{f2}}$$

$$= 0.9 + \frac{5}{10.1} = 1.4 \text{ V}$$

Thus, the transfer characteristic of the feedback amplifier can be described as follows:

For $|v_S| \le 0.9$ V, $\quad v_O/v_S = 11.1$ V/V

For $0.9 \text{ V} \le |v_S| \le 1.4$ V, $\quad v_O/v_S = 10.1$ V/V

For $|v_S| \ge 1.4$ V, $\quad v_O = \pm 15$ V

The transfer characteristic is shown in the figure on next page.

11.26 Because the op amp has an infinite input resistance and a zero output resistance, this circuit is a direct implementation of the ideal feedback structure and thus

$$A = 1000 \text{ V/V}$$

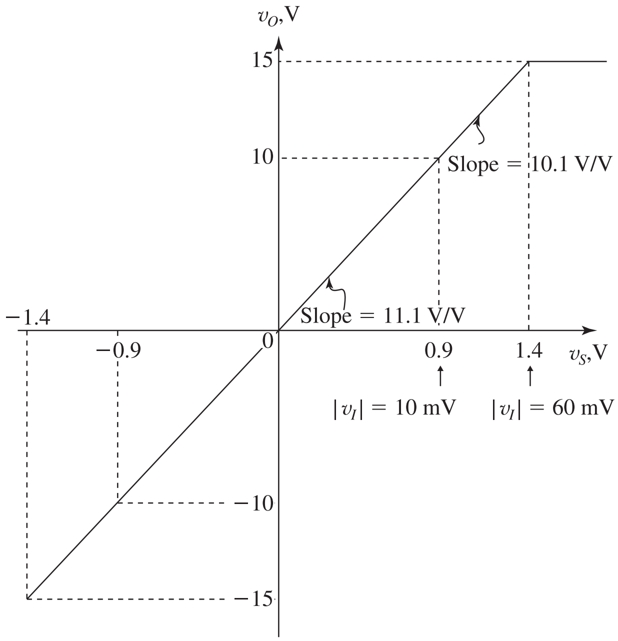
and

$$\beta = \frac{R_1}{R_1 + R_2}$$

The ideal closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

This figure belongs to Problem 11.25.



Thus,

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

$$\beta = \frac{10}{10 + 90} = 0.1 \text{ V/V}$$

$$A\beta = 1000 \times 0.1 = 100$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{1 + 100} = 9.9 \text{ V/V}$$

To obtain A_f that is exactly 10, we use

$$10 = \frac{1000}{1 + A\beta}$$

$$\Rightarrow A\beta = 99$$

$$\beta = 0.099$$

$$0.099 = \frac{R_1}{R_1 + R_2}$$

$$0.099 = \frac{10}{10 + R_2}$$

$$\Rightarrow R_2 = 91 \text{ k}\Omega$$

11.27 Refer to Fig. 11.11.

(a) The ideal closed-loop gain is given by

$$A_f = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

$$10 = 1 + \frac{R_2}{10}$$

$$\Rightarrow R_2 = 90 \text{ k}\Omega$$

(b) From Example 11.3, we obtain

$$A\beta = \mu \frac{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]}{\{R_L \parallel [R_2 + R_1 \parallel (R_{id} + R_s)]\} + r_o} \times \frac{R_1 \parallel (R_{id} + R_s)}{[R_1 \parallel (R_{id} + R_s)] + R_2} \times \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = 1000 \frac{10 \parallel [90 + 10 \parallel (100 + 100)]}{\{10 \parallel [90 + 10 \parallel (100 + 100)]\} + 1} \times \frac{10 \parallel (100 + 100)}{[10 \parallel (100 + 100)] + 90} \times \frac{100}{100 + 100}$$

$$= 1000 \times 0.9009 \times 0.0957 \times 0.5$$

$$= 43.11$$

$$A = \frac{A\beta}{\beta} = \frac{43.11}{0.1} = 431.1 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta} = \frac{431.1}{1 + 43.11} = 9.77 \text{ V/V}$$

(c) To obtain $A_f = 9.9$ V/V, we use

$$9.9 = \frac{A}{1 + A\beta}$$

$$= \frac{A}{1 + A \times 0.1}$$

$$\Rightarrow A = 1010 \text{ V/V}$$

Thus μ must be increased by the factor $\frac{1010}{431.1} = 2.343$ to become

$$\mu = 2343 \text{ V/V}$$

11.28 Refer to Fig. 11.10.

(a) $\beta = \dfrac{R_1}{R_1 + R_2}$

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$5 = 1 + \frac{R_2}{1}$$

$$\Rightarrow R_2 = 4\ \text{k}\Omega$$

(b) From Example 11.2, we have

$$A\beta = (g_{m1}R_{D1})(g_{m2}R_{D2})\frac{1}{1 + g_{m1}R_1} \times$$

$$\frac{R_1}{R_{D2} + R_2 + \left(R_1 \parallel \dfrac{1}{g_{m1}}\right)}$$

$$= (4 \times 10)(4 \times 10)\frac{1}{1 + 4 \times 1} \times \frac{1}{10 + 4 + (1 \parallel 0.25)}$$

$$= 22.54$$

$$A = \frac{A\beta}{\beta} = \frac{22.54}{0.2} = 112.7\ \text{V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{112.7}{1 + 22.54} = 4.79\ \text{V/V}$$

11.29 (a) The feedback network consists of the voltage divider (R_1R_2), thus

$$\beta = \frac{R_1}{R_1 + R_2}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11\ \text{V/V}$$

(b)

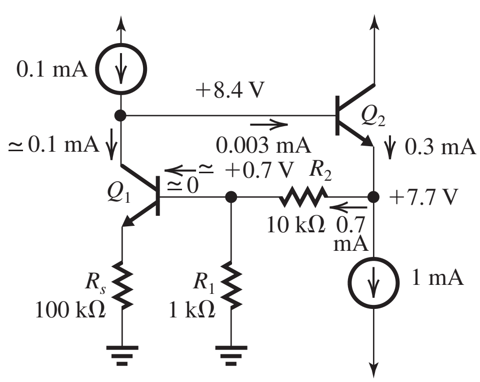


Figure 1

The dc analysis is shown in Fig. 1 from which we see that

$$I_{E1} \simeq 0.1\ \text{mA}$$

$$I_{E2} \simeq 0.3\ \text{mA}$$

$$V_{E2} = +7.7\ \text{V}$$

(c) Setting $V_s = 0$ and eliminating dc sources, the feedback amplifier circuit simplifies to that shown in Fig. 2.

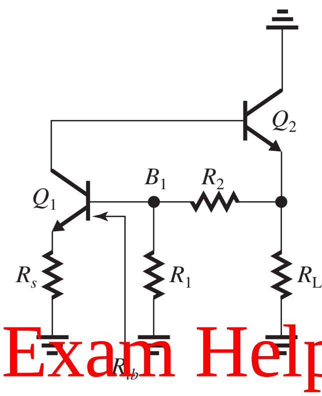


Figure 2

Now breaking the feedback loop at the base of Q_1 while terminating the right-hand side of the circuit (behind the break) in the resistance R_{ib},

$$R_{ib} = (\beta_1 + 1)(r_{e1} + R_s)$$

results in the circuit in Fig. 3 which we can use to determine the loop gain $A\beta$ as follows:

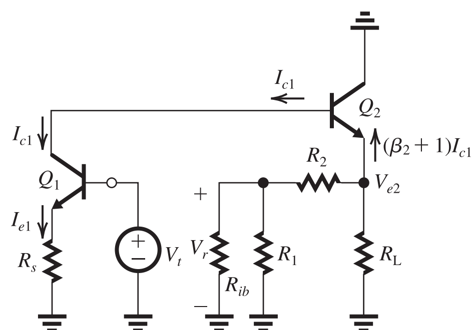


Figure 3

$$I_{e1} = \frac{V_t}{r_{e1} + R_s} \qquad (1)$$

$$I_{c1} = \alpha_1 I_{e1} \qquad (2)$$

$$V_{e2} = -(\beta_2 + 1)I_{c1}\{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\} \qquad (3)$$

$$V_r = V_{e2}\frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2} \qquad (4)$$

Combining (1) to (4), we can determine $A\beta$ as

$$A\beta \equiv -\frac{V_r}{V_t}$$

$$= \alpha_1 \frac{(\beta_2+1)\{R_L \parallel [R_2 + (R_1 \parallel R_{ib})]\}}{r_{e1}+R_s}$$

$$\times \frac{R_1 \parallel R_{ib}}{(R_1 \parallel R_{ib}) + R_2}$$

Substituting

$$\alpha_1(\beta_2+1) = \alpha(\beta+1) = \beta = 100$$

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{0.1 \text{ mA}} = 250\ \Omega$$

$$R_s = 100\ \Omega$$

$$R_L = 1\text{ k}\Omega$$

$$R_1 = 1\text{ k}\Omega$$

$$R_2 = 10\text{ k}\Omega$$

$$R_{ib} = 101(0.25 + 0.1) = 35.35\text{ k}\Omega$$

we obtain

$$A\beta = \frac{100\{1 \parallel [10 + (1 \parallel 35.35)]\}}{0.25+0.1}$$

$$\times \frac{1 \parallel 35.35}{(1 \parallel 35.35) + 10}$$

$$= 23.2$$

(d) $A = \dfrac{A\beta}{\beta} = \dfrac{23.2}{(1/11)} = 255.2$ V/V

$$A_f = \frac{A}{1+A\beta}$$

$$= \frac{255.2}{1+23.2} = 10.5 \text{ V/V}$$

11.30 Refer to Fig. 11.8(c) and to the expressions for β, $A\beta$, and A given in the answer section of Exercise 11.6.

$$A = g_m \frac{R_D(R_1+R_2)}{R_D+R_1+R_2}$$

where

$$g_m = 4 \text{ mA/V}$$

$$R_D = 10\text{ k}\Omega$$

$R_1 + R_2 = 1\text{ M}\Omega$ (the potentiometer resistance)

Thus,

$$A = 4 \times \frac{10 \times 1000}{10+1000} = 39.6 \text{ V/V}$$

$$A_f = \frac{A}{1+A\beta}$$

$$5 = \frac{39.6}{1+39.6\beta}$$

$$\Rightarrow \beta = 0.175 \text{ V/V}$$

$$0.175 = \frac{R_1}{R_1+R_2}$$

$$\Rightarrow R_1 = 0.175 \times 1000 = 175\text{ k}\Omega$$

11.31 (a) The feedback network consists of the voltage divider (R_F, R_{S1}). Thus,

$$\beta = \frac{R_{S1}}{R_{S1}+R_F}$$

and the ideal value of the closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_F}{R_{S1}}$$

$$10 = 1 + \frac{R_F}{0.1}$$

$$\Rightarrow R_F = 0.9\text{ k}\Omega$$

(b)

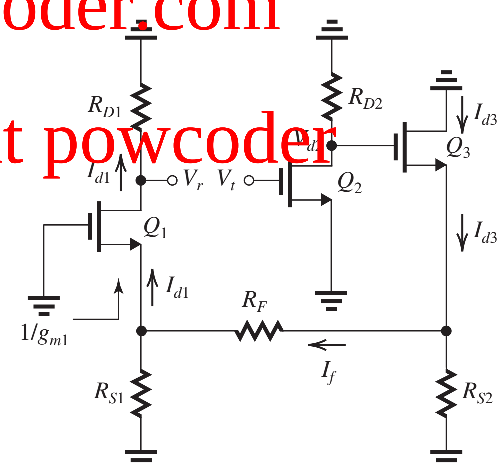


Figure 1

Figure 1 shows the circuit for determining the loop gain. Observe that we have broken the loop at the gate of Q_2 where the input resistance is infinite, obviating the need for adding a termination resistance. Also, observe that as usual we have set $V_s = 0$. To determine the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

we write the following equations:

$$V_{d2} = -g_{m2}R_{D2}V_t \tag{1}$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{g_{m3}} + \left\{R_{S2} \parallel \left[R_F + \left(R_{S1} \parallel \frac{1}{g_{m1}}\right)\right]\right\}} \tag{2}$$

$$I_f = I_{d3}\frac{R_{S2}}{\left[R_F + \left(R_{S1} \parallel \frac{1}{g_{m1}}\right)\right] + R_{S2}} \tag{3}$$

$$I_{d1} = I_f\frac{R_{S1}}{R_{S1} + \frac{1}{g_{m1}}} \tag{4}$$

$$V_r = I_{d1}R_{D1} \tag{5}$$

Substituting the numerical values in (1)–(5), we obtain

$$V_{d2} = -4 \times 10V_t = -40V_t \tag{6}$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{4} + \left\{0.1 \parallel \left[0.9 + \left(0.1 \parallel \frac{1}{4}\right)\right]\right\}}$$

$$I_{d3} = 2.935V_{d2} \tag{7}$$

$$I_f = I_{d3}\frac{0.1}{\left[0.9 + \left(0.1 \parallel \frac{1}{4}\right)\right] + 0.1}$$

$$I_f = 0.0933I_{d3} \tag{8}$$

$$I_{d1} = I_f\frac{0.1}{0.1 + \frac{1}{4}} = 0.286I_f \tag{9}$$

$$V_r = 10I_{d1} \tag{10}$$

Combining (6)–(10) gives

$$V_r = -31.33V_t$$

$$\Rightarrow A\beta = 31.33$$

$$A = \frac{A\beta}{\beta} = \frac{31.33}{0.1} = 313.3 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{313.3}{1 + 31.33} = 9.7 \text{ V/V}$$

Thus, A_f is 0.3 V/V lower than the ideal value of 10 V/V, a difference of -3%. The circuit could be adjusted to make A_f exactly 10 by changing β through varying R_F. Specifically,

$$10 = \frac{313.3}{1 + 313.3\beta}$$

$$\Rightarrow \beta = 0.0968$$

But,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$0.0968 = \frac{0.1}{0.1 + R_F}$$

$$\Rightarrow R_F = 933\ \Omega$$

(an increase of 33 Ω).

11.32 (a) The feedback circuit consists of the voltage divider (R_F, R_E). Thus,

$$\beta = \frac{R_E}{R_E + R_F}$$

and,

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_F}{R_E}$$

Thus,

$$25 = 1 + \frac{R_F}{0.05}$$

$$\Rightarrow R_F = 1.2\ \text{k}\Omega$$

(b) Figure 1 on next page shows the feedback amplifier circuit prepared for determining the loop gain $A\beta$. Observe that we have eliminated all dc sources, set $V_s = 0$, and broken the loop at the base of Q_2. We have terminated the broken loop in a resistance $r_{\pi 2}$. To determine the loop gain

$$A\beta \equiv \frac{V_r}{V_t}$$

we write the following equations:

$$I_{c2} = g_{m2}V_t \tag{1}$$

$$I_{b2} = I_{c2}\frac{R_{C2}}{R_{C2} + (\beta_3 + 1)\left[R_F + (R_E \parallel r_{e1})\right]} \tag{2}$$

$$I_{e3} = (\beta_3 + 1)I_{b2} \tag{3}$$

$$I_{e1} = I_{e3}\frac{R_E}{R_E + r_{e1}} \tag{4}$$

$$V_r = -\alpha_1 I_{e1}(R_{C1} \parallel r_{\pi 2}) \tag{5}$$

Substituting

$$\alpha_1 = 0.99$$

$$R_{C1} = 2\ \text{k}\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ V}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{g_{m2}} = \frac{100}{80} = 1.25\ \text{k}\Omega$$

$$R_E = 0.05\ \text{k}\Omega$$

$$r_{e1} = \frac{V_T}{I_{E1}} \simeq \frac{25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega = 0.025\ \text{k}\Omega$$

$$\beta_3 = 100$$

$$R_{C2} = 1\ \text{k}\Omega$$

$$R_F = 1.2\ \text{k}\Omega$$

This figure belongs to Problem 11.32, part (b).

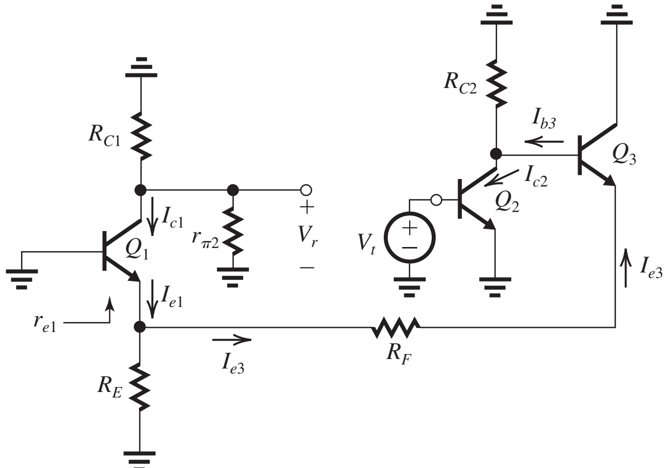


Figure 1

we obtain

$$I_{c2} = 80V_t \quad (6)$$

$$I_{b3} = I_{c2}\frac{1}{1 + 101[1.2 + (0.05 \parallel 0.025)]}$$

$$= 8.072 \times 10^{-3} I_{c2} \quad (7)$$

$$I_{e3} = 101 I_{b3} \quad (8)$$

$$I_{e1} = I_{e3}\frac{50}{50 + 25} = 0.667 I_{e3} \quad (9)$$

$$V_r = -0.99(2 \parallel 1.25) I_{e1}$$

$$V_r = -0.7615 I_{e1} \quad (10)$$

Combining (6)–(10) results in

$$A\beta = 33.13$$

$$A = \frac{A\beta}{\beta} = \frac{33.13}{1/25} = 828.2 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{828.2}{1 + 33.13} = 24.3 \text{ V/V}$$

11.33 All MOSFETs are operating at $I_D = 100\ \mu\text{A} = 0.1$ mA and $|V_{OV}| = 0.2$ V, thus

$$g_{m1,2} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

All devices have

$$r_o = \frac{|V_A|}{I_D} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

(a)

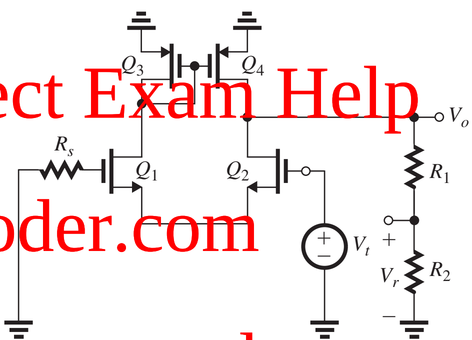


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$.

$$V_o = -g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]V_t \quad (1)$$

$$V_r = \frac{R_2}{R_1 + R_2} V_o = \beta\ V_o$$

Thus,

$$A\beta \equiv -\frac{V_r}{V_t} = g_{m1,2}[r_{o2} \parallel r_{o4} \parallel (R_1 + R_2)]\beta$$

$$= 1(100 \parallel 100 \parallel 1000)\beta$$

$$= 47.62\beta$$

Thus,

$$A = 47.62 \text{ V/V}$$

(b) $A_f = \dfrac{A}{1 + A\beta}$

$$5 = \frac{47.62}{1 + 47.62\beta}$$

$$\Rightarrow \beta = 0.179 \text{ V/V}$$

$$\frac{R_2}{R_1 + R_2} = 0.179$$

$$\Rightarrow R_2 = 179\ \text{k}\Omega$$

$$R_1 = 821\ \text{k}\Omega$$

11.34 $R_i = 2\ \text{k}\Omega$

$R_o = 2\ \text{k}\Omega$

$A = 1000$ V/V

$\beta = 0.1$ V/V

Loop Gain $\equiv A\beta = 1000 \times 0.1 = 100$

$1 + A\beta = 101$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{101} = 9.9\ \text{V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 2 \times 101 = 202\ \text{k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{2}{101} = 19.8\ \Omega$$

11.35 Since the output voltage is sampled, the resistance-with-feedback is lower. The reduction is by the factor $(1 + A\beta)$, thus

$1 + A\beta = 200$

$A\beta = 199$

$$R_{of} = \frac{R_o}{200}$$

$$\Rightarrow R_o = 200 \times 100 = 20,000\ \Omega$$

$$= 20\ \text{k}\Omega$$

11.36 $$A = \frac{A_0}{1 + \frac{s}{\omega_H}}$$

$$1 + A\beta = 1 + \frac{A_0\beta}{1 + \frac{s}{\omega_H}}$$

$$Z_{if} = R_i(1 + A\beta)$$

$$= R_i + \frac{A_0\beta R_i}{1 + \frac{s}{\omega_H}}$$

Thus, Z_{if} consists of a resistance R_i in series with an admittance Y,

$$Y = \frac{1}{A_0\beta R_i} + \frac{s}{A_0\beta R_i\omega_H}$$

which is a resistance $(A_0\beta R_i)$ in parallel with a capacitance $1/A_0\beta R_i\omega_H$. The equivalent circuit is shown in Fig. 1.

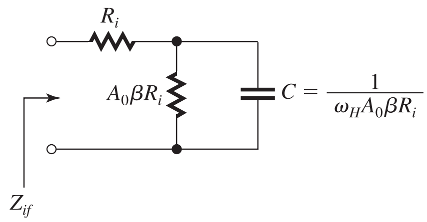


Figure 1

$$Z_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{R_o}{1 + \frac{A_0\beta}{1 + \frac{s}{\omega_H}}}$$

Thus, the output admittance Y_{of} is

$$Y_{of} \equiv \frac{1}{Z_{of}} = \frac{1}{R_o} + \frac{A_0\beta}{R_o\left(1 + \frac{s}{\omega_H}\right)}$$

which consists of a resistance R_o in parallel with an impedance Z given by

$$Z = \frac{R_o}{A_0\beta} + s\frac{R_o}{A_0\beta\omega_H}$$

which consists of a resistance $(R_o/A_0\beta)$ in series with an inductance $L = R_o/A_0\beta\omega_H$. The equivalent circuit of Z_{of} is shown in Fig. 2.

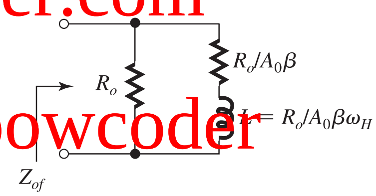


Figure 2

11.37 $A = 1000$ V/V

$R_i = 1\ \text{k}\Omega$

$R_{if} = 10\ \text{k}\Omega$

Thus, the connection at the input is a series one, and

$$1 + A\beta = \frac{10}{1} = 10$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1000}{10} = 100\ \text{V/V}$$

To implement a unity-gain voltage follower, we use $\beta = 1$. Thus the amount of feedback is

$$1 + A\beta = 1 + 1000 = 1001$$

and the input resistance becomes

$$R_{if} = (1 + A\beta)R_i$$

$$= 1001 \times 1 = 1001\ \text{k}\Omega = 1.001\ \text{M}\Omega$$

11.38 (a) $\beta = 1$

$A_f\big|_{\text{ideal}} = 1$ V/V

(b) Substituting $R_1 = \infty$ and $R_2 = 0$ in the expression for A in Example 11.4, we obtain

$$A = \mu \frac{R_L}{R_L + r_o} \frac{R_{id}}{R_{id} + R_s}$$

$$A\beta = A \times 1 = A$$

(c) $A = 10^4 \times \dfrac{2}{2+1} \times \dfrac{100}{100+10}$

$= 6060.6$ V/V

$A\beta = 6060.6$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{6060.6}{1 + 6060.6} = 0.9998 \text{ V/V}$$

From Example 11.4 with $R_1 = \infty$ and $R_L = 0$, we have

$R_i = R_s + R_{id} = 10 + 100 = 110$ kΩ

$R_{if} = R_i(1 + A\beta)$

$= 110 \times 6061.6 = 667$ MΩ

$R_{\text{in}} = R_{if} - R_s \simeq 667$ MΩ

$R_o = r_o \parallel R_L = 1 \parallel 2 = 0.67$ kΩ

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.67 \text{ k}\Omega}{6061.6} = 0.11\ \Omega$$

$R_{of} = R_{\text{out}} \parallel R_L$

$R_{\text{out}} \simeq 0.11\ \Omega$

11.39 Refer to the solution to Problem 11.29.

(a) $\beta = \dfrac{R_1}{R_1 + R_2}$

$$A_f = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

$$= 1 + \frac{10}{1} = 11 \text{ V/V}$$

(b) From the solution to Problem 10.29, we have

$I_{E1} \simeq 0.1$ mA

$I_{E2} \simeq 0.3$ mA

$V_{E2} = +7.7$ V

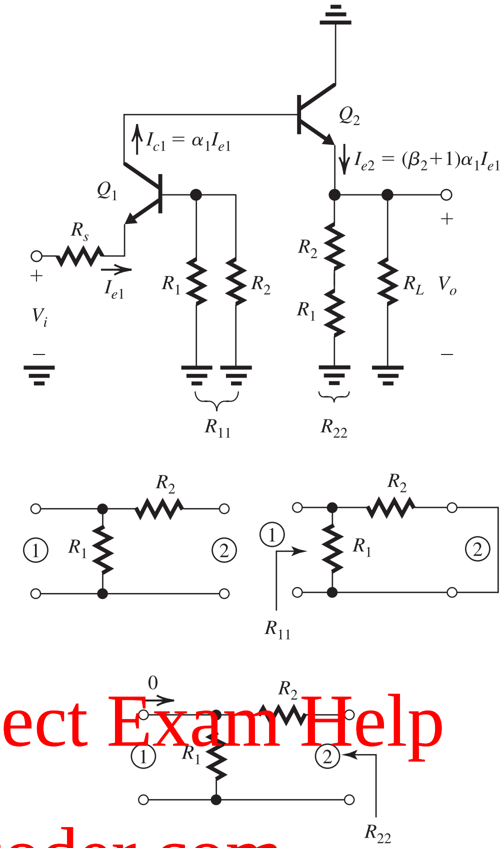


Figure 1

(c) The A circuit is shown in Fig. 1.

$$I_{e1} = \frac{V_i}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

$V_o = I_{e2}[R_L \parallel (R_1 + R_2)]$

$$= (\beta_2 + 1)\alpha_1 V_i \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Since $\beta_1 = \beta_2 = \beta$ and $\alpha = \dfrac{\beta}{\beta + 1}$, we have

$$A \equiv \frac{V_o}{V_i} = \beta \frac{R_L \parallel (R_1 + R_2)}{R_s + r_{e1} + \dfrac{R_1 \parallel R_2}{\beta_1 + 1}}$$

Substituting $\beta = 100$, $R_L = 1$ kΩ, $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $R_s = 0.1$ kΩ, and $r_{e1} = 0.25$ kΩ gives

$$A = 100 \frac{1 \parallel 11}{0.1 + 0.25 + \dfrac{1 \parallel 10}{101}}$$

$= 255.3$ V/V

$$R_i = R_s + r_{e1} + \frac{R_1 \parallel R_2}{\beta_1 + 1}$$

$$= 0.1 + 0.25 + \frac{1 \parallel 10}{101} = 0.359\ \text{k}\Omega$$

$$R_o = R_L \parallel (R_1 + R_2)$$

$$= 1 \parallel 11 = 0.917\ \text{k}\Omega$$

(d) $\beta = \dfrac{R_1}{R_1 + R_2}$

$$= \frac{1}{1+10} = \frac{1}{11}$$

(e) $\dfrac{V_o}{V_s} = A_f = \dfrac{A}{1 + A\beta}$

$$1 + A\beta = 1 + \frac{255.3}{11} = 24.21$$

$$A_f = \frac{255.3}{24.21} = 10.5\ \text{V/V}$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 0.359 \times 24.21 = 8.69\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} - R_s$$

$$= 8.69 - 0.1 = 8.59\ \text{k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{0.917\ \text{k}\Omega}{24.21} = 37.9\ \Omega$$

$$R_{of} = R_{\text{out}} \parallel R_L$$

$$37.9 = R_{\text{out}} \parallel 1000$$

$$\Rightarrow R_{\text{out}} = 39.4\ \Omega$$

The value of A_f (10.5 V/V) is 0.5 less than the ideal value of 11, which is 4.5%.

11.40 (a) Refer to Fig. P11.40. Assume that for some reason v_S increases. This will increase the differential input signal $(v_S - v_O)$ applied to the differential amplifier. The drain current of Q_1 will increase, and this increase will be mirrored in the drain current of Q_4. The increase in i_{D4} will cause the voltage at the gate of Q_5 to rise. Since Q_5 is operating as a source follower, the voltage at its source, v_O, will follow and increase. This will cause the differential input signal $(v_S - v_O)$ to decrease, thus counteracting the originally assumed change. Thus, the feedback is negative.

(b) Figure 1 on the next page shows the circuit prepared for dc analysis. We see that

$$I_{D1} = I_{D2} = 100\ \mu\text{A}$$

$$I_{D3} = 100\ \mu\text{A}$$

$$I_{D4} = 300\ \mu\text{A}$$

$$I_{D5} = 0.8\ \text{mA}$$

For Q_1 and Q_2, use

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$$

$$100 = \frac{1}{2} \times 120 \times \frac{20}{1} V_{OV1,2}^2$$

$$\Rightarrow V_{OV1,2} = 0.29\ \text{V}$$

For Q_3, use

$$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)|V_{OV3}|^2$$

$$100 = \frac{1}{2} \times 60 \times \frac{40}{1}|V_{OV3}|^2$$

$$\Rightarrow |V_{OV3}| = 0.29\ \text{V}$$

Since $V_{SG4} = V_{SG3}$, we have

$$|V_{OV4}| = |V_{OV3}| = 0.29\ \text{V}$$

Finally for Q_5, use

$$800 = \frac{1}{2} \times 120 \times \frac{20}{1} \times V_{OV5}^2$$

$$\Rightarrow V_{OV5} = 0.82\ \text{V}$$

If perfect matching pertains, then

$$V_{D4} = V_{D3} = V_{DD} - V_{SG3}$$

$$= 2.5 - |V_t| - |V_{OV}|$$

$$= 2.5 - 0.7 - 0.29 = 1.51\ \text{V}$$

$$V_O = V_{D4} - V_{GS5}$$

$$= V_{D4} - V_t - V_{OV5}$$

$$= 1.51 - 0.7 - 0.82 = -0.01\ \text{V}$$

which is approximately zero, as stated in the Problem statement.

(c) $g_{m1} = g_{m2} = g_{m3} = \dfrac{2I_D}{|V_{OV}|}$

$$= \frac{2 \times 0.1}{0.29} = 0.7\ \text{mA/V}$$

$$g_{m4} = \frac{2 \times 0.3}{0.29} \simeq 2\ \text{mA/V}$$

$$g_{m5} = \frac{2 \times 0.8}{0.82} \simeq 2\ \text{mA/V}$$

$$r_{o1} = r_{o2} = r_{o3} = \frac{|V_A|}{I_D} = \frac{|V_A'| \times L}{I_D}$$

$$= \frac{24 \times 1}{0.1} = 240\ \text{k}\Omega$$

$$r_{o4} = \frac{24}{0.3} = 80\ \text{k}\Omega$$

$$r_{o5} = \frac{24}{0.8} = 30\ \text{k}\Omega$$

(d) Figure 2 on the next page shows the A circuit. Observe that since the β network is simply a wire connecting the output node to the gate of Q_2, we have $R_{11} = 0$ and $R_{22} = \infty$. To determine A, we write

This figure belongs to Problem 11.40, part (b).

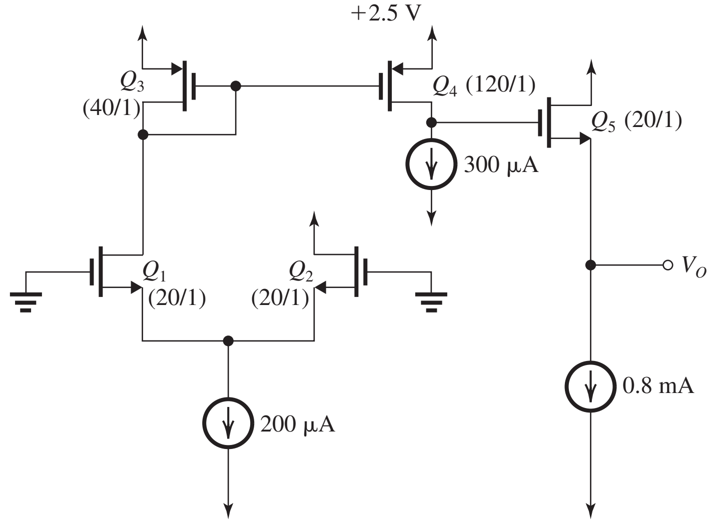


Figure 1

This figure belongs to Problem 11.40, part (d).

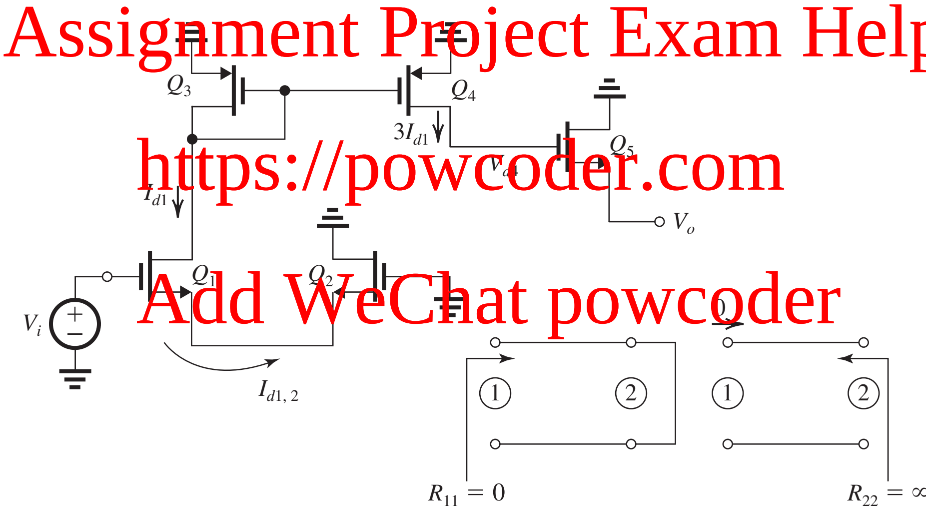


Figure 2

$$I_{d1,2} = \frac{V_i}{2/g_{m1,2}} = \frac{1}{2} g_{m1,2} V_i$$

Since $\left(\frac{W}{L}\right)_4 = 3\left(\frac{W}{L}\right)_3$, the drain current of Q_4 will be

$$I_{d4} = 3I_{d1} = \frac{3}{2} g_{m1,2} V_i$$

The voltage at the drain of Q_4 will be

$$V_{d4} = I_{d4} r_{o4}$$

$$= \frac{3}{2} g_{m1,2} r_{o4} V_i$$

Finally, V_o is related to V_{d4} as

$$\frac{V_o}{V_{d4}} = \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = \frac{3}{2} g_{m1,2} r_{o4} \frac{r_{o5}}{r_{o5} + \frac{1}{g_{m5}}}$$

Substituting numerical values, we obtain

$$A = \frac{3}{2} \times 0.7 \times 80 \times \frac{30}{30 + 0.5}$$

$$= 82.6 \text{ V/V}$$

The output resistance R_o is

$$R_o = r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= 30 \parallel 0.5 = 0.492 \text{ k}\Omega$$

$$= 492\ \Omega$$

(e) $A_f = \dfrac{A}{1 + A\beta}$

$$= \frac{82.6}{1 + 82.6} = 0.988 \text{ V/V}$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{492}{1 + 82.6} = 5.9\ \Omega$$

$$R_{\text{out}} = R_{of} = 5.9\ \Omega$$

(f) To obtain a closed-loop gain of 5 V/V, we connect a voltage divider in the feedback loop, as shown in Fig. 3.

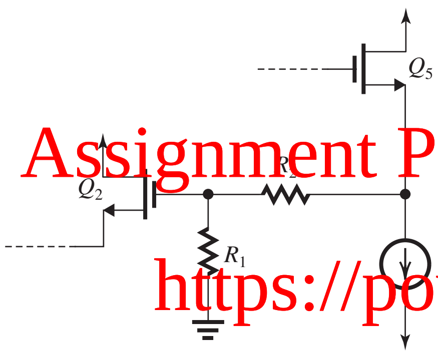


Figure 3

$$\beta = \frac{R_1}{R_1 + R_2}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$5 = \frac{82.6}{1 + 82.6\beta}$$

$$\Rightarrow \beta = 0.188$$

Selecting $R_1 = 1$ MΩ, we obtain

$$0.188 = \frac{1}{1 + R_2}$$

$$\Rightarrow R_2 = 4.319 \text{ M}\Omega$$

Note that by selecting large values for R_1 and R_2, we have ensured that their loading effect on the A circuit would be negligible.

11.41 (a) Let V_s increase by a small increment. Since Q_1 is operating in effect as a CS amplifier, a negative incremental voltage will appear at its drain. Transistor Q_2 is also operating as a CS amplifier; thus a positive incremental voltage will appear at its drain. Transistor Q_3 is operating as a source follower; thus the signal at its source (which is the output voltage) will follow that at its gate and thus will be positive. The end result is that we are feeding back through the voltage divider (R_2, R_1) a positive incremental signal that will appear across R_1 and thus at the source of the Q_1. This signal, being of the same polarity as the originally assumed change in the signal at the gate of $Q_1(V_s)$, will *subtract* from the original change, causing a *smaller* signal to appear across the gate-source terminals of Q_1. Hence, the feedback is negative.

(b) $\beta = \dfrac{R_1}{R_1 + R_2}$

Thus,

$$\beta = \frac{2}{2 + 18} = 0.1 \text{ V/V}$$

If the loop gain is large, the closed-loop gain approaches the ideal value

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Thus,

$$A_f\big|_{\text{ideal}} = 1 + \frac{18}{2} = 10 \text{ V/V}$$

(c) $V_{G1} = 0.9$ V

$$V_{S1} = V_{G1} - V_{GS1}$$

$$= V_{G1} - V_{t1} - V_{OV1}$$

$$= 0.9 - 0.5 - 0.2 = 0.2 \text{ V}$$

$$V_{G2} = V_{DD} - V_{SG2}$$

$$= V_{DD} - |V_{t2}| - |V_{OV2}|$$

$$= 1.80 - 0.5 - 0.2 = 1.1 \text{ V}$$

Thus, current source I_1 will have 0.7-V drop across it, more than sufficient for its proper operation. Since $V_{S1} = 0.2$ V the dc current through R_1 will be

$$I_{R1} = \frac{V_{S1}}{R_1} = \frac{0.2 \text{ V}}{2 \text{ k}\Omega} = 0.1 \text{ mA}$$

Now, a node equation at S_1 reveals that because $I_{D1} = 0.1$ mA and $I_{R1} = 0.1$ mA, the dc current in R_2 will be zero. Thus, it will have a zero voltage drop across it and

$$V_{S3} = V_{S1} = 0.2 \text{ V}$$

Thus, current source I_3 will have across it, the minimum voltage required to keep it operating properly. Finally,

$$V_{G3} = V_{S3} + V_{GS3}$$

$$= V_{S3} + V_{t3} + V_{OV3}$$

$$= 0.2 + 0.5 + 0.2 = 0.9 \text{ V}$$

Thus, current source I_2 will have across it a voltage more than sufficient to keep it operating properly.

(d)

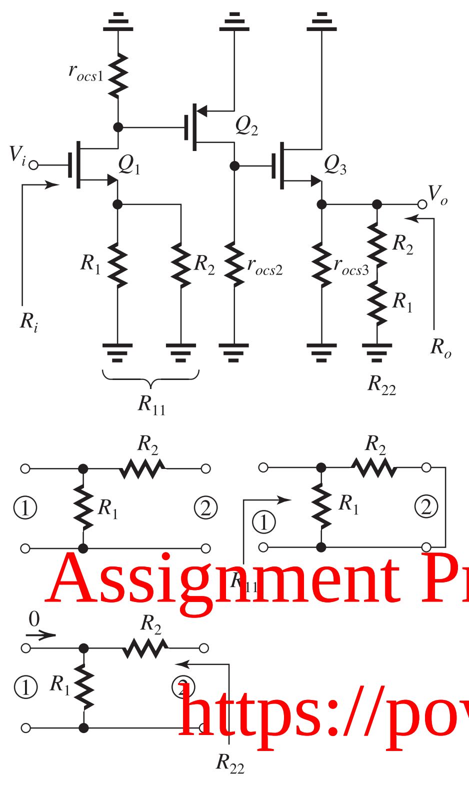


Figure 1

Figure 1 shows the A circuit as well as the β circuit and how the loading-effect resistances R_{11} and R_{22} are determined.

To determine A, let's first determine the small-signal parameters of all transistors as well as r_o of each of the three current sources.

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{|V_A|}{I_{D1}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs1} = \frac{|V_A|}{I_1} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = \frac{|V_A|}{I_{D2}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs2} = \frac{|V_A|}{I_2} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$g_{m3} = \frac{2I_{D3}}{V_{OV3}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o3} = \frac{|V_A|}{I_{D3}} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

$$r_{ocs3} = \frac{|V_A|}{I_3} = \frac{10}{0.1} = 100 \text{ k}\Omega$$

Refer to the A circuit.

Transistor Q_1 is a CS amplifier with a resistance R_{11} in its source:

$$R_s = R_{11} = R_1 \parallel R_2 = 2 \parallel 18 = 1.8 \text{ k}\Omega$$

Transistor Q_1 will have an effective transconductance:

$$G_{m1} = \frac{g_{m1}}{1 + g_m R_s} = \frac{2}{1 + 2 \times 1.8} = 0.43 \text{ mA/V}$$

The output resistance of Q_1 will be

$$R_{o1} = (1 + g_{m1} R_s) r_{o1}$$

$$= (1 + 2 \times 1.8) \times 100 = 460 \text{ k}\Omega$$

The total resistance at the drain of Q_1 is

$$R_{d1} = r_{ocs1} \parallel R_{o1}$$

$$= 100 \parallel 460 = 82.1 \text{ k}\Omega$$

Thus, the voltage gain of the first stage is

$$A_1 = -G_{m1} R_{d1}$$

$$= -0.43 \times 82.1 = -35.3 \text{ V/V}$$

The gain of the second stage is

$$A_2 = -g_{m2}(r_{ocs2} \parallel r_{o2})$$

$$= -1(100 \parallel 100) = -50 \text{ V/V}$$

To determine the gain of the third stage, we first determine the total resistance between the source of Q_3 and ground:

$$R_{s3} = r_{ocs3} \parallel r_{o3} \parallel (R_1 + R_2)$$

$$R_{s3} = 100 \parallel 100 \parallel 20$$

$$= 14.3 \text{ k}\Omega$$

Thus,

$$A_3 = \frac{R_{s3}}{R_{s3} + \dfrac{1}{g_{m3}}}$$

$$= \frac{14.3}{14.3 + \dfrac{1}{1}} = 0.935 \text{ V/V}$$

The overall voltage gain A can now be found as

$$A = A_1 A_2 A_3$$

$$= -35.3 \times -50 \times 0.935 = 1650 \text{ V/V}$$

(e) We already found β in (b) as

$$\beta = 0.1 \text{ V/V}$$

(f) $1 + A\beta = 1 + 1650 \times 0.1 = 166$

$$A_f = \frac{A}{1 + A\beta} = \frac{1650}{166} = 9.94 \text{ V/V}$$

which is lower by 0.06 or 0.6% than the ideal value obtained in (b).

(g) $R_{of} = \dfrac{R_o}{1 + A\beta}$

To obtain R_o refer to the output part of the A circuit.

$$R_o = (R_1 + R_2) \parallel r_{ocs3} \parallel r_{o3} \parallel \frac{1}{g_{m3}}$$

$$= 20 \parallel 100 \parallel 100 \parallel 1$$

$$= 935\ \Omega$$

$$R_{of} = \frac{935}{166} = 5.6\ \Omega$$

Note: This problem, though long, is extremely valuable as it exercises the student's knowledge in many aspects of amplifier design.

11.42 (a) Refer to Fig. P11.33. Let V_s increase by a positive increment. This will cause the drain current of Q_1 to increase. The increase in I_{d1} will be fed to the $Q_3 - Q_4$ mirror, which will provide a corresponding increase in the drain current of Q_4. The latter current will cause the voltage at the output node to rise. A fraction of the increase in V_o is applied through the divider (R_1, R_2) to the gate of Q_2. The increase in the voltage of the gate of Q_2 will *subtract* from the initially assumed increase of the voltage of the gate of Q_1, resulting in a smaller increase in the differential voltage applied to the (Q_1, Q_2) pair. Thus, the feedback counter acts the originally assumed change, verifying that it is negative.

(b) The negative feedback will cause the dc voltage at the gate of Q_2 to be approximately equal to the dc voltage at the gate of Q_1, that is, zero. Now, with $V_{G2} \simeq 0$, the dc current in R_2 will be zero and similarly the dc current in R_1 will be zero, resulting in $V_O = 0$ V dc.

(c) Figure 1 shows the A circuit. It also shows how the loading effect of the β network on the A circuit, namely R_{11} and R_{22}, are found. The gain of the A circuit can be written by inspection as

$$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22})$$

where

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

This figure belongs to Problem 11.42, part (c).

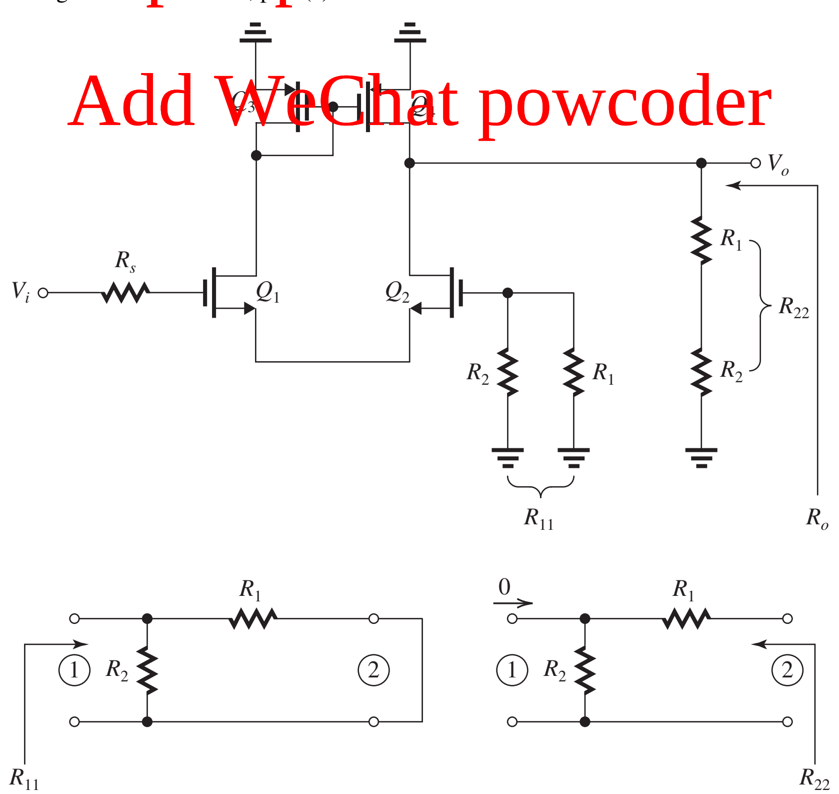


Figure 1

$r_{o2} = r_{o4} = \dfrac{|V_A|}{I_{D3,4}} = \dfrac{10}{0.1} = 100\ \text{k}\Omega$

$R_{22} = R_1 + R_2 = 1\ \text{M}\Omega$

$A = 1(100 \parallel 100 \parallel 1000) = 47.62\ \text{V/V}$

This is identical to the value found in the solution to Problem 11.33.

(d) $\dfrac{V_o}{V_s} = A_f = \dfrac{A}{1 + A\beta}$

$5 = \dfrac{47.62}{1 + 47.62\beta}$

$\Rightarrow \beta = 0.179$

Thus,

$\dfrac{R_2}{R_1 + R_2} = 0.179$

$R_2 = 0.179\ \text{M}\Omega = 179\ \text{k}\Omega$

$R_1 = 1000 - 179 = 821\ \text{k}\Omega$

Again, these values are identical to those found in Problem 11.33.

(e) Refer to Fig. 1.

$R_o = R_{22} \parallel r_{o2} \parallel r_{o4}$

$= 1000 \parallel 100 \parallel 100 = 47.62\ \text{k}\Omega$

$R_{\text{out}} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$= \dfrac{47.62}{1 + 47.62 \times 0.179}$

$= 5\ \text{k}\Omega$

This value cannot be found using the loop-gain analysis method of Problem 11.33.

(f) With $R_L = 10\ \text{k}\Omega$,

$\dfrac{V_o}{V_s} = 5 \times \dfrac{R_L}{R_L + R_{\text{out}}}$

$5 \times \dfrac{10}{10 + 5} = 3.33\ \text{V/V}$

(g) As an alternative to (f), we shall redo the analysis of the A circuit in (c) above with $R_L = 10\ \text{k}\Omega$ included:

$A = g_{m1,2}(r_{o2} \parallel r_{o4} \parallel R_{22} \parallel R_L)$

$= 1(100 \parallel 100 \parallel 1000 \parallel 10)$

$= 8.26\ \text{V/V}$

Using $\beta = 0.179$, we obtain

$A_f = \dfrac{8.26}{1 + 8.26 \times 0.179} = 3.33\ \text{V/V}$

which is identical to the value found in (f) above.

11.43 All transistors have $L = 1\ \mu\text{m}$, thus all have $|V_A| = |V_A'| \times L = 10 \times 1 = 10$ V. Also, all have $|V_t| = 0.75$ V.

(a) Figure 1 shows the circuit prepared for dc design. We have also indicated some of the current and voltage values. We now find the (W/L) ratios utilizing

$I_D = \dfrac{1}{2}\mu_n C_{ox}\left(\dfrac{W}{L}\right)V_{OV}^2$

for the NMOS transistors, and

$I_D = \dfrac{1}{2}\mu_p C_{ox}\left(\dfrac{W}{L}\right)|V_{OV}|^2$

for the PMOS devices.

This figure belongs to Problem 11.43, part (a).

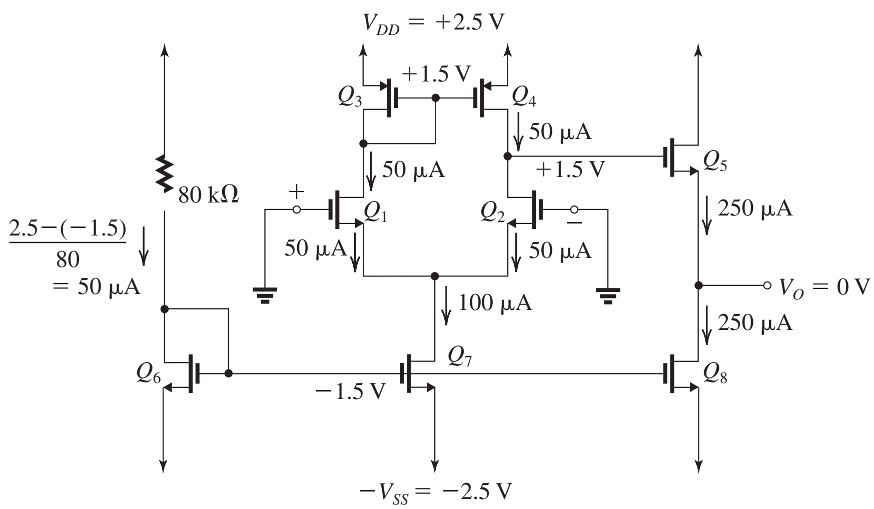


Figure 1

For Q_6,

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_6 \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_6 = 16$$

For Q_7,

$$\frac{(W/L)_7}{(W/L)_6} = \frac{100\ \mu\text{A}}{50\ \mu\text{A}} = 2$$

$$\Rightarrow (W/L)_7 = 2 \times 16 = 32$$

For Q_8,

$$\frac{(W/L)_8}{(W/L)_6} = \frac{250\ \mu\text{A}}{50\ \mu\text{A}} = 5$$

$$\Rightarrow \left(\frac{W}{L}\right)_8 = 5 \times 16 = 80$$

For Q_1 and Q_2,

$$50 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_{1,2} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 16$$

For Q_3 and Q_4,

$$50 = \frac{1}{2} \times 50 \times \left(\frac{W}{L}\right)_{3,4} \times 0.25^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 32$$

Finally, since $V_{G3} = V_{D4} = V_{D3} = 1.5$ V and we require $V_O = 0$ V, we have

$$V_{GS5} = 1.5\ \text{V}$$

$$V_{OV5} = 1.5 - 0.75 = 0.75\ \text{V}$$

$$250 = \frac{1}{2} \times 100 \times \left(\frac{W}{L}\right)_5 \times 0.75^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = 8.9$$

(b) The maximum value of V_{ICM} is limited by Q_1 leaving the saturation region,

$$V_{ICM\max} = V_{D1} + V_t$$

$$= 1.5 + 0.75 = 2.25\ \text{V}$$

The minimum value of V_{ICM} is limited by the need to keep Q_7 in saturation. This is achieved by keeping V_{D7} at a minimum voltage of

$$-2.5 + |V_{OV7}| = -2.5 + 0.25 = -2.25\ \text{V}$$

Thus,

$$V_{ICM\min} = -2.25 + V_{GS1}$$

$$= -2.25 + 1 = -1.25\ \text{V}$$

Thus,

$$-1.25\ \text{V} \le V_{ICM} \le +2.25\ \text{V}$$

(c) $g_{m1,2} = \dfrac{2I_{D1,2}}{V_{OV1,2}}$

$$= \frac{2 \times 0.05}{0.2} = 0.5\ \text{mA/V}$$

$$g_{m5} = \frac{2I_D}{|V_{OV5}|} = \frac{2 \times 0.25}{0.75} = 0.67\ \text{mA/V}$$

(d) $r_{o1} = r_{o2} = r_{o3} = r_{o4} = r_{o6} = \dfrac{|V_A|}{I_D} =$

$$\frac{10}{0.05} = 200\ \text{k}\Omega$$

$$r_{o7} = \frac{10}{0.01} = 100\ \text{k}\Omega$$

$$r_{o5} = r_{o8} = \frac{10}{0.25} = 40\ \text{k}\Omega$$

(e) Figure 2 on the next page shows the A circuit, the β circuit, and how the loading effects of the β circuit on the A circuit, namely R_{11} and R_{22}, are determined.

$$\frac{V_{g5}}{V_i} = g_{m1,2}(r_{o2} \parallel r_{o4})$$

$$= 0.5(200 \parallel 200) = 50\ \text{V/V}$$

$$\frac{V_o}{V_{g5}} = \frac{R_s}{R_s + \dfrac{1}{g_{m5}}}$$

where

$$R_s = r_{o8} \parallel r_{o5} \parallel (R_1 + R_2) \parallel R_L$$

$$= 40 \parallel 40 \parallel 100 \parallel 100 = 14.3\ \text{k}\Omega$$

Thus,

$$\frac{V_o}{V_{g5}} = \frac{14.3}{14.3 + (1/0.67)} = 0.905\ \text{V/V}$$

$$A = \frac{V_o}{V_i} = \frac{V_{g5}}{V_i} \times \frac{V_o}{V_{g5}}$$

$$= 50 \times 0.905 = 45.3\ \text{V/V}$$

$$A_f = 10 = \frac{A}{1 + A\beta}$$

$$10 = \frac{45.3}{1 + 45.3\beta}$$

$$\Rightarrow \beta = 0.078$$

$$\frac{R_2}{R_1 + R_2} = 0.078$$

$$R_2 = 7.8\ \text{k}\Omega$$

$$R_1 = 100 - 7.8 = 92.2\ \text{k}\Omega$$

This figure belongs to Problem 11.43, part (e).

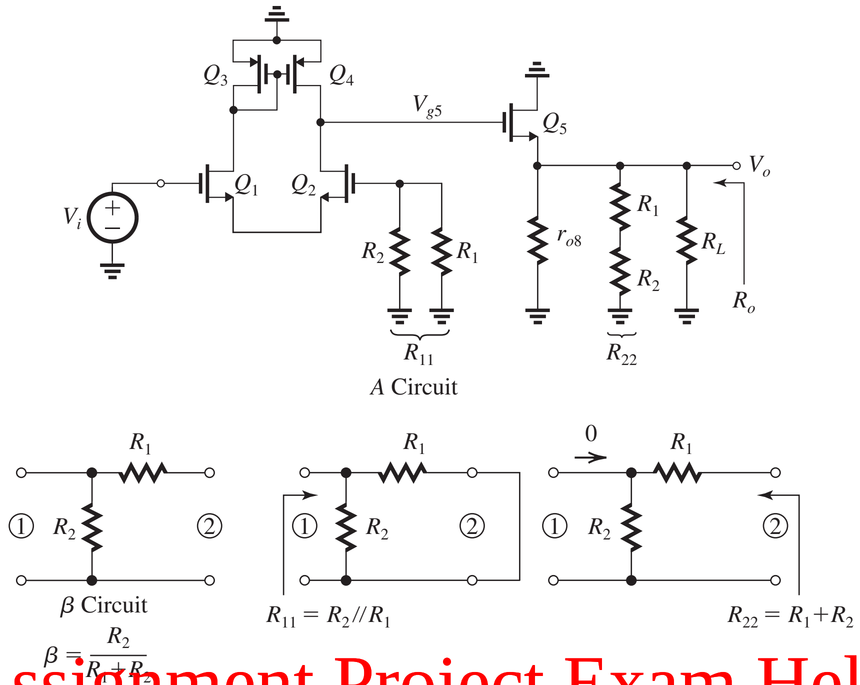


Figure 2

(f) Refer to Fig. 2.

$$R_o = R_L \parallel (R_1 + R_2) \parallel r_{o8} \parallel r_{o5} \parallel \frac{1}{g_{m5}}$$

$$= R_s \parallel \frac{1}{g_{m5}} = 14.3 \parallel (1/0.67)$$

$$= 1.36 \text{ k}\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{1.36 \text{ k}\Omega}{1 + 45.3 \times 0.078} \simeq 300 \ \Omega$$

$$R_{\text{out}} \parallel R_L = R_{of}$$

$$\Rightarrow R_{\text{out}} \simeq 300 \ \Omega$$

11.44 (a) Figure 1 on the next page shows the A circuit and the circuit for determining β as well as the determination of the loading effects of the β circuit.

(b) If $A\beta$ is large, then

$$A_f \equiv \frac{V_o}{V_s} \simeq \frac{1}{\beta}$$

Since

$$\beta = \frac{R_E}{R_F + R_E}$$

we have

$$A_f = \frac{R_F + R_E}{R_E} \qquad \text{Q.E.D.}$$

(c) $25 = 1 + \dfrac{R_F}{50 \ \Omega}$

$$\Rightarrow R_F = 1.2 \text{ k}\Omega$$

(d) Refer to the A circuit in Fig. 1. The voltage gain of Q_1 is given by

$$\frac{V_{c1}}{V_i} = -\alpha_1 \frac{R_{C1} \parallel r_{\pi 2}}{r_{e1} + R_{11}}$$

where

$$r_{e1} = \frac{V_T}{I_{E1}} = \frac{25 \text{ mV}}{1 \text{ mA}} = 25 \ \Omega$$

$$R_{11} = R_E \parallel R_F = 50 \ \Omega \parallel 1200 \ \Omega = 48 \ \Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} \simeq \frac{I_{E2}}{V_T} = \frac{2 \text{ mA}}{0.025 \text{ mA}} = 80 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta_2}{80} = \frac{100}{80} = 1.5 \text{ k}\Omega$$

$$\alpha_1 = 0.99 \simeq 1$$

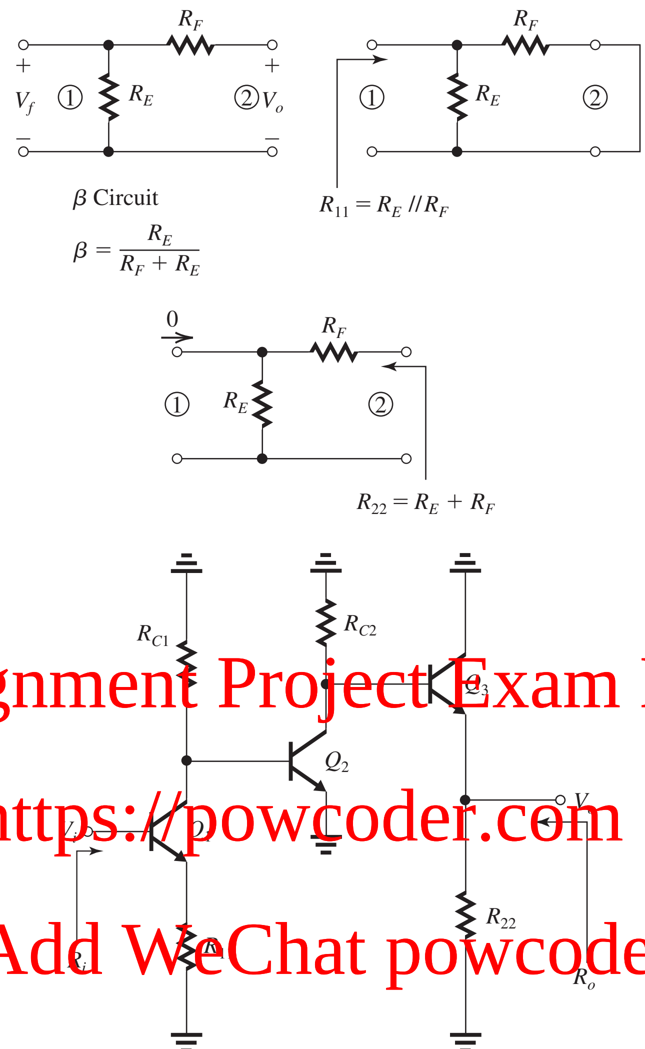
$$\frac{V_{c1}}{V_i} = -10 = -\frac{R_{C1} \parallel 1.5}{0.025 + 0.048}$$

$$\Rightarrow R_{C1} = 1.42 \text{ k}\Omega$$

Next consider the second stage composed of the CE transistor Q_2. The load resistance of the second stage is composed of R_{C2} in parallel with the input resistance of emitter-follower Q_3. The latter resistance is given by

$$R_{i3} = (\beta_3 + 1)(r_{e3} + R_{22})$$

This figure belongs to Problem 11.44, part (a).



A Circuit

Figure 1

where

$$r_{e3} = \frac{V_T}{I_{E3}} = \frac{25 \text{ mV}}{5 \text{ mA}} = 5\ \Omega$$

$$R_{22} = R_F + R_E = 1.2 + 0.05 = 1.25 \text{ k}\Omega$$

Thus,

$$R_{i3} = 101 \times 1.25 = 126.3 \text{ k}\Omega$$

$$A_2 \equiv \frac{V_{c2}}{V_{b2}} = -g_{m2}(R_{C2} \parallel R_{i3})$$

$$-50 = -80(R_{C2} \parallel 126.3)$$

$$\Rightarrow R_{C2} = 628\ \Omega$$

(e) $A = A_1 A_2 A_3$

where

$$A_3 = \frac{R_{22}}{R_{22} + r_{e3}} = \frac{1.25}{1.25 + 0.005} = 0.996 \text{ V/V}$$

$$A \equiv -10 \times -50 \times 0.996$$

$$= 498 \text{ V/V}$$

$$A_f \equiv \frac{V_o}{V_s} = \frac{498}{1 + 498 \times \dfrac{50}{1250}}$$

$$= 23.8 \text{ V/V}$$

(f) Refer to the *A* circuit in Fig. 1.

$$R_i = (\beta_1 + 1)(r_{e1} + R_{11})$$

$$R_i = 101(0.025 + 0.048)$$

$$= 7.37 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

where

$$1 + A\beta = 1 + \frac{498}{25} = 20.92$$

$R_{if} = 7.37 \times 20.92 = 154\ \text{k}\Omega$

$R_o = R_{22} \parallel \left[r_{e3} + \dfrac{R_{C2}}{\beta_3 + 1} \right]$

$= 1.25 \parallel \left[0.005 + \dfrac{0.628}{101} \right]$

$= 11.1\ \Omega$

$R_{\text{out}} = R_{of} = \dfrac{R_o}{1 + A\beta}$

$= \dfrac{11.1}{20.92} = 0.53\ \Omega$

11.45 (a) Refer to Fig. P11.45. If V_s increases, the output of A_1 will decrease and this will cause the output of A_2 to increase. This, in turn, causes the output of A_3, which is V_o, to increase. A portion of the positive increment in V_o is fed back to the positive input terminal of A_1 through the voltage divider (R_2, R_1), The increased voltage at the positive input terminal of A_1 counteracts the originally assumed increase at the negative input terminal, verifying that the feedback is negative.

(b) $A_f\big|_{\text{ideal}} = \dfrac{1}{\beta}$

where

$\beta = \dfrac{R_1}{R_1 + R_2}$

Thus, to obtain an ideal closed-loop gain of 5 V/V we need $\beta = 0.2$:

$0.2 = \dfrac{20}{20 + R_2}$

$\Rightarrow R_2 = 80\ \text{k}\Omega$

(c) Figure 1 shows the small-signal equivalent circuit of the feedback amplifier.

(d) Figure 2 on the next page shows the A circuit and the β circuit together with the determination of its loading effects, R_{11}, and R_{22}. We can write

$\dfrac{V_1}{V_i} = -\dfrac{82}{82 + 9 + 16} = -0.766\ \text{V/V}$

$V_2 = 20V_1 \times \dfrac{5}{3.2 + 5} = 12.195V_1$

$V_3 = -20V_2(20 \parallel 20) = -200V_2$

$V_o = V_3 \dfrac{1 \parallel 100}{(1 \parallel 100) + 1} = 0.497V_3$

Thus,

$A \equiv \dfrac{V_o}{V_i} = 0.497 \times -200 \times 12.195 \times -0.766$

$= 928.5\ \text{V/V}$

(e) $\beta = \dfrac{20}{20 + 80} = 0.2\ \text{V/V}$

$1 + A\beta = 1 + 928.5 \times 0.2 = 186.7$

(f) $A_f \equiv \dfrac{V_o}{V_s} = \dfrac{A}{1 + A\beta}$

$= \dfrac{928.5}{186.7} = 4.97\ \text{V/V}$

which is nearly equal to the ideal value of 5 V/V.

(g) From the A circuit,

$R_i = 9 + 82 + 16 = 107\ \text{k}\Omega$

$R_{if} = R_i(1 + A\beta)$

$= 107 \times 186.7 = 19.98\ \text{M}\Omega$

$R_{\text{in}} = R_{if} - R_s \simeq 19.98\ \text{M}\Omega$

(h) From the A circuit,

$R_o = R_L \parallel R_{22} \parallel 1\ \text{k}\Omega$

$= 1 \parallel 100 \parallel 1 = 497.5\ \Omega$

This figure belongs to Problem 11.45, part (c).

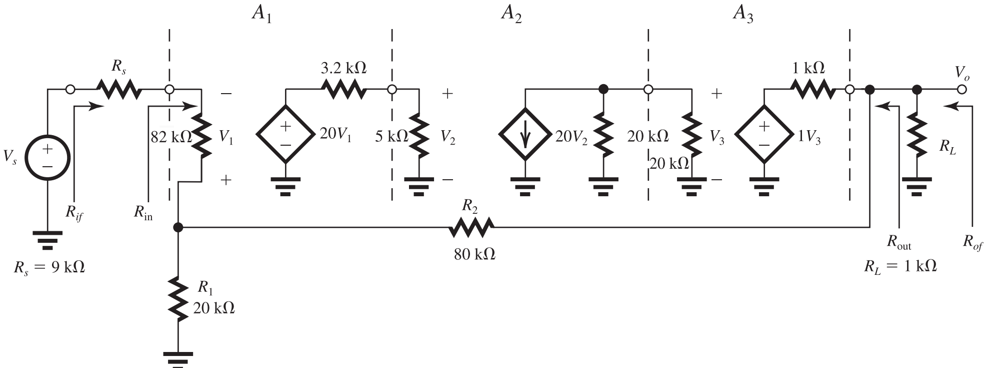


Figure 1

This figure belongs to Problem 11.45, part (d).

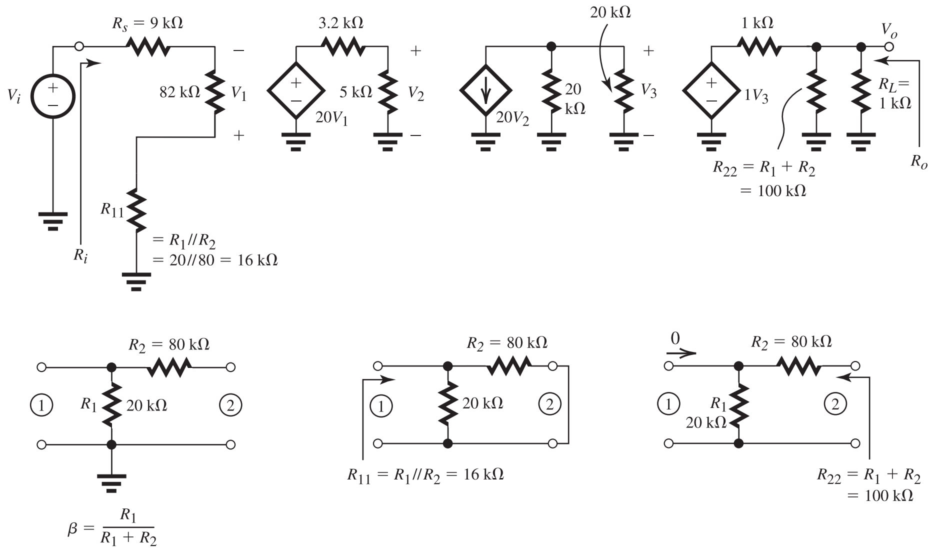


Figure 2

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{497.5}{186.7} = 2.66\ \Omega$$

$R_{\text{out}} \parallel R_L = R_{of}$

$R_{\text{out}} \parallel 1000 = 2.66\ \Omega$

$R_{\text{out}} \simeq 2.66\ \Omega$

(i) $f_{Hf} = f_H(1 + A\beta)$

$= 100 \times 186.7$

$= 18.67$ kHz

(j) If A_1 drops to half its nominal value, A will drop to half its nominal value:

$$A = \frac{1}{2} \times 928.5 = 464.25$$

and A_f becomes

$$A_f = \frac{464.25}{1 + 464.25 \times 0.2} = 4.947\ \text{V/V}$$

Thus, the percentage change in A_f is

$$= \frac{4.947 - 4.97}{4.97} = -0.47\%$$

11.46 To obtain $A_f \equiv \frac{I_o}{V_s} \simeq 10$ mA/V, we select

$$R_F = \beta = \frac{1}{A_f} = 100\ \Omega$$

From Example 11.6, we obtain

$$A = \frac{\mu}{R_F} \frac{g_m(R_F \parallel R_{id} \parallel r_{o2})}{1 + g_m(R_F \parallel R_{id} \parallel r_{o2})}$$

$$\equiv \frac{1000}{0.1\ \text{k}\Omega} \frac{2(0.1 \parallel 100 \parallel 20)}{1 + 2(0.1 \parallel 100 \parallel 20)}$$

$= 10{,}000 \times 0.1658$

$= 1658$ A/V

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{1658}{1 + 1658 \times 0.1} = 9.94\ \text{mA/V}$$

11.47 (a)

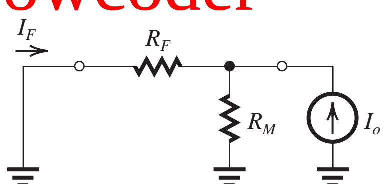


Figure 1

Figure 1 shows the β network with the input port short-circuited. Thus,

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_M}{R_M + R_F}$$

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = -\left(1 + \frac{R_F}{R_M}\right)$$

(b) Figure 2 on the next page shows the circuit for determining the loop gain $A\beta$,

$$A\beta = -\frac{V_r}{V_t}$$

First, we express I_{d2} in terms of V_t:

$$I_{d2} = -g_{m2}V_t \qquad (1)$$

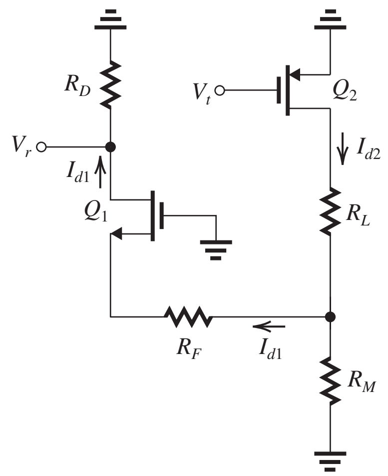


Figure 2

Then we determine I_{d1}:

$$I_{d1} = I_{d2} \frac{R_M}{R_M + R_F + \frac{1}{g_{m1}}} \qquad (2)$$

The returned voltage V_r can now be obtained as

$$V_r = I_{d1} R_D \qquad (3)$$

Combining Eqs. (1)–(3), we find V_r/V_t:

$$\frac{V_r}{V_t} = -\frac{g_{m2} R_D R_M}{R_M + R_F + \frac{1}{g_{m1}}}$$

Thus,

$$A\beta = \frac{g_{m2} R_D R_M}{R_M + R_F + \frac{1}{g_{m1}}}$$

Dividing the expression for $A\beta$ by

$\beta = -\dfrac{R_M}{R_M + R_F}$ yields

$$A = -\frac{g_{m2} R_D}{1 + 1/[g_{m1}(R_M + R_F)]} \qquad \text{Q.E.D.}$$

(c) $A = -\dfrac{4 \times 10}{1 + 1/[4 \times 1]}$

$= -32$ A/A

$A_f = -5 = -\dfrac{32}{1 - 32 \times \beta}$

$\beta = -0.169$ A/A

$-\dfrac{R_M}{R_M + R_F} = -0.169$

$R_M = 0.169 \times 1 = 0.169$ kΩ

$= 169\ \Omega$

11.48 (a) Refer to Fig. P11.48(b).

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = -R_F$$

For $A_f\big|_{\text{ideal}} = -1$ kΩ, we have

$R_F = 1$ kΩ

(b)

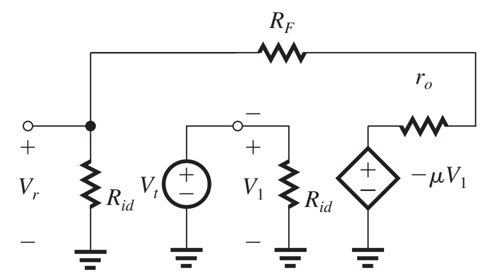


Figure 1

Figure 1 shows the circuit for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

Writing V_r in terms of $V_1 = V_t$ yields

$$V_r = -\mu V_t \frac{R_{id}}{R_{id} + R_F + r_o}$$

Thus,

$$A\beta = -\frac{V_r}{V_t} = \mu \frac{R_{id}}{R_{id} + R_F + r_o} \qquad \text{Q.E.D.}$$

(c) $A\beta = 1000 \dfrac{100}{100 + 1 + 1}$

$= 980.4$

$A = \dfrac{980.4}{\beta} = \dfrac{980.4}{-1/R_F}$

$= -980.4$ kΩ

$A_f = \dfrac{A}{1 + A\beta}$

$= -\dfrac{980.4}{1 + 980.4} = -0.999$ kΩ

11.49

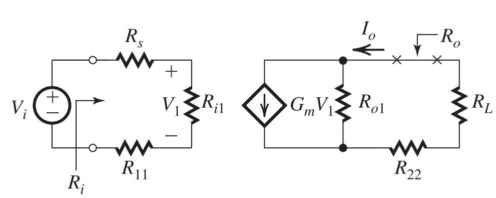


Figure 1

Figure 1 shows the A circuit where

$R_{i1} = 10$ kΩ

$R_{o1} = 100$ kΩ

$\beta = 200\ \Omega$

$R_{22} = 200\ \Omega$

$R_{11} = 10\ \text{k}\Omega$

$R_s = 10\ \text{k}\Omega$

$R_L = 10\ \text{k}\Omega$

To determine A,

$$A \equiv \frac{I_o}{V_i}$$

we write

$$V_1 = V_i \frac{R_{i1}}{R_{i1} + R_s + R_{11}}$$

$$= V_i \frac{10}{10 + 10 + 10} = \frac{1}{3} V_i \qquad (1)$$

$$I_o = G_m V_1 \frac{R_{o1}}{R_{o1} + R_L + R_{22}}$$

$$= 0.6 \times \frac{100}{100 + 10 + 0.2} V_i$$

$$= 0.544\ V_i \qquad (2)$$

Combining (1) and (2), we obtain

$$I_o = 0.544 \times \frac{1}{3} V_i = 0.1815\ V_i$$

$A = 0.1815$ A/V

$$A_f = \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{0.1815}{1 + 0.1815 \times 200} = \frac{0.1815}{1 + 36.2} = 4.88\ \text{mA/V}$$

$$R_{if} = R_i(1 + A\beta)$$

R_i is obtained from the A circuit as

$$R_i = R_s + R_{i1} + R_{11}$$

$$= 10 + 10 + 10 = 30\ \text{k}\Omega$$

Thus,

$$R_{if} = 30 \times 37.2 = 1.116\ \text{M}\Omega$$

$$R_{\text{in}} = R_{if} - R_s$$

$$= 1.116 - 0.010 = 1.006\ \text{M}\Omega$$

$$\simeq 1\ \text{M}\Omega$$

$$R_{of} = R_o(1 + A\beta)$$

where R_o is obtained from the A circuit as

$$R_o = R_L + R_{o1} + R_{22}$$

$$= 10 + 100 + 0.2 = 110.2\ \text{k}\Omega$$

$$R_{of} = 110.2 \times 37.2 = 4.1\ \text{M}\Omega$$

$$R_{\text{out}} = R_{of} - R_L = 4.1 - 0.01 = 4.09\ \text{M}\Omega$$

11.50 (a)

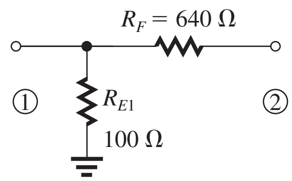


Figure 1

Figure 1 shows the β circuit from which we obtain

$$\beta = \frac{R_{E1}}{R_{E1} + R_F}$$

$$= \frac{100}{100 + 640} = 0.135\ \text{V/V}$$

(b) For $A\beta \gg 1$,

$$\frac{V_{e3}}{V_s} = A_f\big|_{\text{ideal}} = \frac{1}{\beta} = 7.4\ \text{V/V}$$

(c)

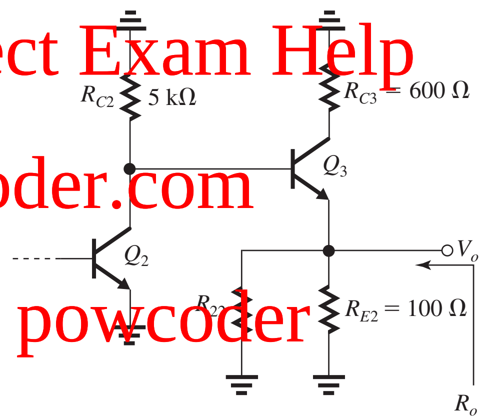


Figure 2

Figure 2 shows the portion of the A circuit relevant for calculating R_o:

$$R_o = R_{E2} \parallel R_{22} \parallel \left[r_{e3} + \frac{R_{C2}}{\beta_3 + 1} \right]$$

where $R_{E2} = 100\ \Omega$, R_{22} (from β circuit) $= 740\ \Omega$, $r_{e3} = 5\ \Omega$, $R_{C2} = 5\ \text{k}\Omega$, $\beta_3 = 100$;

thus,

$$R_o = 100 \parallel 740 \parallel \left[5 + \frac{5000}{101} \right]$$

$$= 33.7\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{33.7}{1 + 246.3} = 0.14\ \Omega$$

11.51 (a)

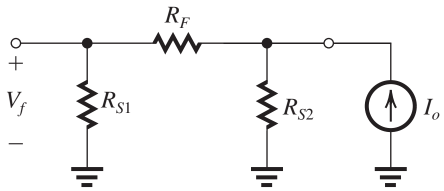


Figure 1

Figure 1 shows the β network. The value of β can be obtained from

$$\beta \equiv \frac{V_f}{I_o}$$

$$= \frac{R_{S1}R_{S2}}{R_{S2}+R_F+R_{S1}}$$

If $A\beta \gg 1$, then

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_{S1}} + \frac{1}{R_{S2}} + \frac{R_F}{R_{S1}R_{S2}} \qquad (1)$$

For $A_f \simeq 100$ mA/V,

$$100 = \frac{1}{0.1} + \frac{1}{0.1} + \frac{R_F}{0.1 \times 0.1}$$

$$\Rightarrow R_F = 0.8 \text{ k}\Omega$$

(b) Figure 2 shows the A circuit and the determination of the loading effects of the β circuit, namely R_{11} and R_{22}.

$$R_{11} = R_{S1} \parallel (R_F + R_{S2})$$

$$= 100 \parallel (800 + 100) = 80 \ \Omega$$

$$R_{22} = R_{S2} \parallel (R_F + R_{S1}) = 80 \ \Omega$$

The value of A is determined as follows:

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{(1/g_{m1}) + R_{11}}$$

$$= -\frac{10}{(1/4) + 0.08} = -30.3 \text{ V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2}R_{D2} = -4 \times 10 = -40 \text{ V/V}$$

$$\frac{I_o}{V_{d2}} = \frac{1}{1/g_{m5} + R_{22}}$$

$$= \frac{1}{0.25 + 0.08} \simeq 3 \text{ mA/V}$$

Thus,

$$A = \frac{I_o}{V_i} = 3 \times -40 \times -30.3 = 3636 \text{ mA/V}$$

(c) $\beta = 0.01$ kΩ

$$1 + A\beta = 1 + 3636 \times 0.01$$

$$= 37.36$$

$$A_f = \frac{I_o}{V_i} = \frac{3636}{37.36} = 97.3 \text{ mA/V}$$

Difference from design value

$$= \frac{97.3 - 100}{100} \times 100$$

$$= -2.7\%$$

This figure belongs to Problem 11.51, part (b).

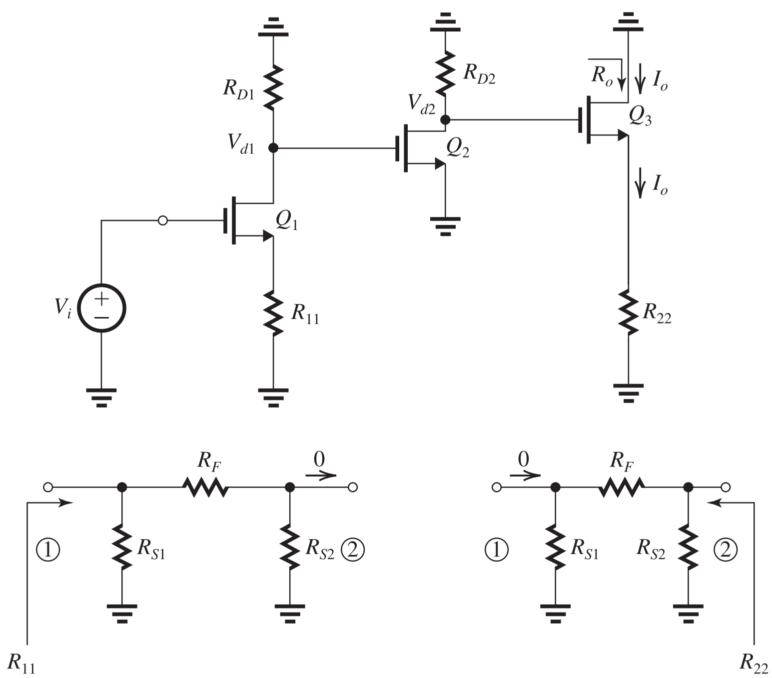


Figure 2

To make A_f exactly 100 mA/V, we can increase R_F (see Eq. (1) to appreciate why we need to increase R_F).

(d) From the A circuit in Fig. 2, we have

$R_o = r_{o3} + R_{22} + g_{m3} r_{o3} R_{22}$

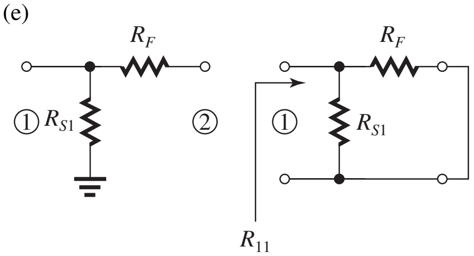
$= 20 + 0.08 + 4 \times 20 \times 0.08$

$= 26.48\ \text{k}\Omega$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 26.48 \times 37.36 = 989.3\ \text{k}\Omega$

(e)



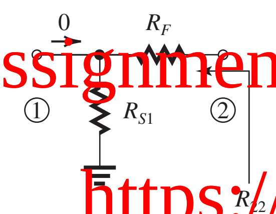


Figure 3

Figure 3 shows the β circuit for the case the output is V_o.

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$= \frac{100}{100 + 800} = \frac{1}{9}$$

Also shown is how R_{11} and R_{22} are determined in this case:

$R_{11} = R_{S1} \parallel R_F = 100 \parallel 800 = 88.9\ \Omega$

$R_{22} = R_F + R_{S1} = 800 + 100 = 900\ \Omega$

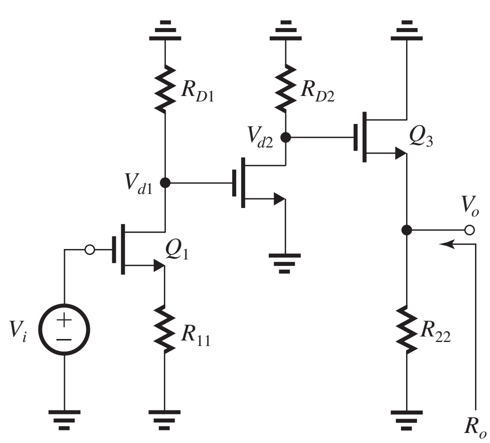


Figure 4

Figure 4 shows the A circuit for this case. To determine A, we write

$$\frac{V_{d1}}{V_i} = -\frac{R_{D1}}{1/g_{m1} + R_{11}}$$

$$\frac{V_{d1}}{V_i} = -\frac{10}{0.25 + 0.0889} = -29.5\ \text{V/V}$$

$$\frac{V_{d2}}{V_{d1}} = -g_{m2} R_{D2} = -4 \times 10 = -40\ \text{V/V}$$

$$\frac{V_o}{V_{d2}} = \frac{R_{22}}{R_{22} + \dfrac{1}{g_{m5}}} = \frac{88.9}{88.9 + 250} = 0.26\ \text{V/V}$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.26 \times -40 \times -29.5 = 306.9\ \text{V/V}$$

$$1 + A\beta = 1 + 306.9 \times \frac{1}{9} = 35.1$$

which is a little lower than the value (37.36) found when we analyzed the amplifier as a transconductance amplifier.

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{306.9}{35.1} = 8.74\ \text{V/V}$$

(f) From the A circuit in Figure 4, we have

$$R_o = R_{22} \parallel \frac{1}{g_{m5}}$$

$R_o = 900\ \Omega \parallel 250\ \Omega$

$= 195.7\ \Omega$

$$R_{\text{out2}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{195.7}{35.1} = 5.6\ \Omega$$

11.52 (a)

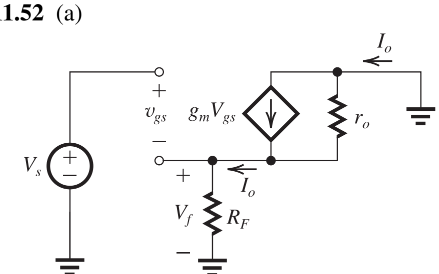


Figure 1

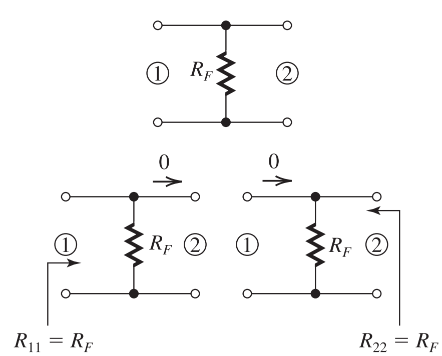


Figure 2

Figure 1 shows the small-signal equivalent circuit of the feedback amplifier. Observe that the resistance R_F senses the output current I_o and provides a voltage I_oR_F that is subtracted from V_s. Thus the feedback network is composed of the resistance R_F, as shown in Fig. 2. Because the feedback is of the series-series type, the loading resistances R_{11} and R_{22} are determined as indicated in Fig. 2,

$R_{11} = R_F$

$R_{22} = R_F$

(b) The β circuit is shown in Fig. 2 and

$\beta = R_F$

Figure 3 shows the A circuit.

(c)

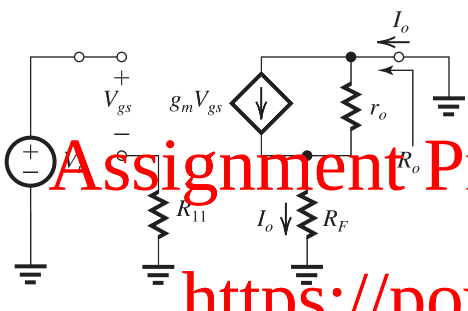


Figure 3

To determine $A = I_o/V_i$, we write

$V_{gs} = V_i$

$$I_o = g_m V_{gs}\frac{r_o}{r_o + R_F}$$

$$A \equiv \frac{I_o}{V_i} = g_m\frac{r_o}{r_o + R_F}$$

$$1 + A\beta = 1 + \frac{g_m r_o R_F}{r_o + R_F}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{g_m r_o/(r_o + R_F)}{1 + g_m r_o R_F/(r_o + R_F)}$$

$$= \frac{g_m}{1 + g_m R_F + \dfrac{R_F}{r_o}}$$

From the A circuit in Fig. 3, we have

$R_o = r_o + R_F$

$R_{of} = (1 + A\beta)R_o$

$$= \left(1 + \frac{g_m r_o R_F}{r_o + R_F}\right)(r_o + R_F)$$

$= r_o + R_F + g_m r_o R_F$

which is a familiar relationship!

11.53 (a) The β circuit is shown in Fig. 1:

$\beta = R_F$

For $A\beta \gg 1$, $A_f \equiv I_o/V_s$ approaches the ideal value

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = \frac{1}{R_F}$$

To obtain $A_f \simeq 5$ mA/V, we use

$$R_F = \frac{1}{5} = 0.2\ \text{k}\Omega = 200\ \Omega$$

This figure belongs to Problem 11.53, part (a).

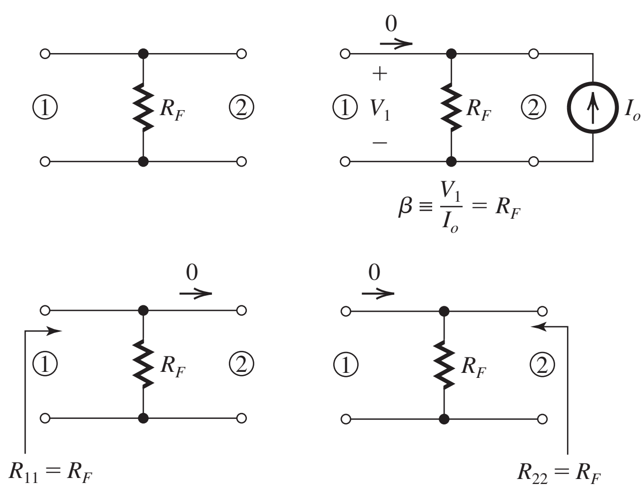


Figure 1

This figure belongs to Problem 11.53, part (b).

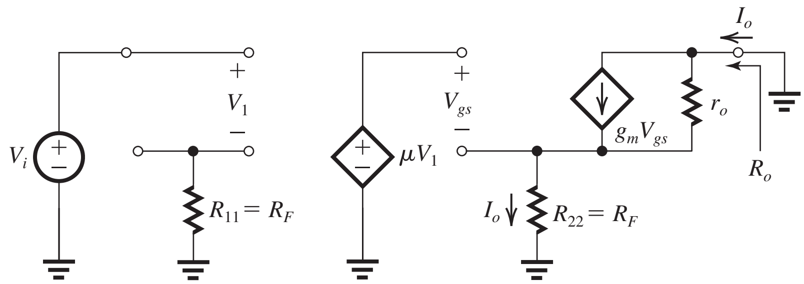


Figure 2

(b) Determining the loading effects of the β network is illustrated in Fig. 1:

$R_{11} = R_{22} = R_F$

Figure 2 shows the A circuit. An expression for $A \equiv I_o/V_i$ can be derived as follows:

$$V_1 = V_i \quad (1)$$

$$V_{gs} = \mu V_1 - I_o R_F \quad (2)$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F} \quad (3)$$

Combining Eqs. (1)–(3) yields

$$A \equiv \frac{I_o}{V_i} = \frac{\mu g_m r_o}{r_o + R_F + g_m r_o R_F}$$

For $\mu = 1000$ V/V, $g_m = 2$ mA/V, $r_o = 20$ kΩ, and $R_F = 0.2$ kΩ, we have

$$A = \frac{1000 \times 2 \times 20}{20 + 0.2 + 2 \times 20 \times 0.2}$$

$= 1418.4$ mA/V

(c) $$A\beta = \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$A\beta = 283.7$

$1 + A\beta = 284.7$

(d) $$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{1418.4}{284.7} = 4.982 \text{ mA/V}$$

which is very close to the ideal value of 5 mA/V.

(e) From the A circuit in Fig. 2, we have

$R_o = r_o + R_F + g_m r_o R_F$

$$1 + A\beta = 1 + \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$R_{of} = (1 + A\beta) R_o$

$= r_o + R_F + g_m r_o R_F + \mu g_m r_o R_F$

$= r_o + R_F + (\mu + 1) g_m r_o R_F$

$\simeq \mu g_m r_o R_F$

$R_o = 20 + 0.2 + 2 \times 20 \times 0.2$

$= 28.2$ kΩ

$R_{of} = 20 + 0.2 + 1001 \times 2 \times 20 \times 0.2$

$= 20 + 0.2 + 8008 = 8028.2$ kΩ

$\simeq 8$ MΩ

11.54 Figure 1 on the next page shows the equivalent circuit with $V_s = 0$ and a voltage V_x applied to the collector for the purpose of determining the output resistance R_o,

$$R_o \equiv \frac{V_x}{I_x}$$

Some of the analysis is displayed on the circuit diagram. Since the current entering the emitter node is equal to I_x, we can write for the emitter voltage

$$V_e = I_x [R_e \parallel (r_\pi + R_b)] \quad (1)$$

The base current can be obtained using the current-divider rule applied to R_e and $(r_\pi + R_b)$ as

$$I_b = -I_x \frac{R_e}{R_e + r_\pi + R_b} \quad (2)$$

The voltage from collector to ground is equal to V_x and can be expressed as the sum of the voltage drop across r_o and V_e,

$$V_x = (I_x - \beta I_b) r_o + V_e$$

Substituting for V_e from (1) and for I_b from (2), we obtain

$$R_o = \frac{V_x}{I_x} = r_o + [R_e \parallel (r_\pi + R_b)]$$

$$+ \frac{R_e \beta r_o}{R_e + r_\pi + R_b}$$

$$= r_o + [R_e \parallel (r_\pi + R_b)] \left[1 + r_o \frac{\beta}{r_\pi + R_b}\right]$$

This figure belongs to Problem 11.54.

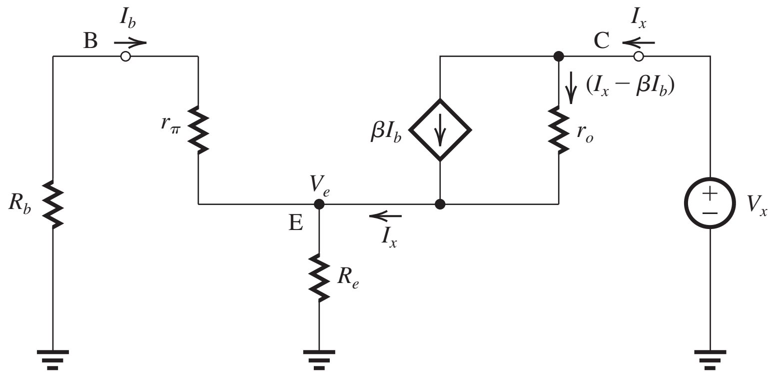


Figure 1

Since $\beta = g_m r_\pi$, we obtain

$R_o = r_o + [R_e \parallel$

$(r_\pi + R_b)]\left[1 + g_m r_o \dfrac{r_\pi}{r_\pi + R_b}\right]$ Q.E.D.

For $R_b = 0$,

$R_o = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$

The maximum value of R_o will be obtained when $R_e \gg r_\pi$. If R_e approaches infinity (zero signal current in the emitter), R_o approaches the theoretical maximum:

$R_{o\max} = r_o + r_\pi(1 + g_m r_o)$

$= r_o + r_\pi + \beta r_o$ (3)

$\simeq \beta r_o$

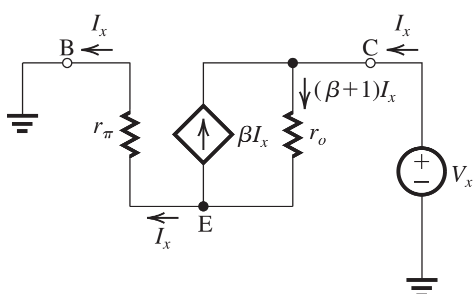


Figure 2

The situation that pertains in the circuit when $R_e = \infty$ is illustrated in Fig. 2. Observe that since the signal current in the emitter is zero, the base current will be equal to the collector current (I_x) and in the direction indicated. The controlled-source current will be βI_x, and this current adds to I_x to provide a current $(\beta + 1)I_x$ in the output resistance r_o. A loop equation takes the form

$V_x = (\beta + 1)I_x r_o + I_x r_\pi$

and thus

$$R_o \equiv \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

which is identical to the result in Eq. (3).

11.55

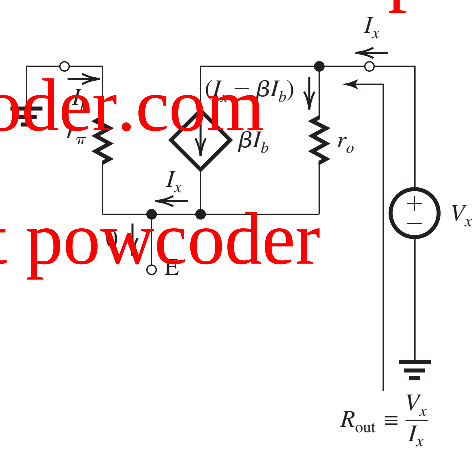


Figure 1

Figure 1 shows the situation that pertains in the transistor when μ is so large that $V_b \simeq 0$ and $I_e \simeq 0$. Observe that

$I_b = -I_x$

Writing a loop equation for the C-E-B, we obtain

$V_x = (I_x - \beta I_b)r_o - I_b r_\pi$

Substituting $I_b = -I_x$, we obtain

$$R_{out} = \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

or if β is denoted h_{fe},

$R_{out} = r_\pi + (h_{fe} + 1)r_o$ Q.E.D.

This figure belongs to Problem 11.56.

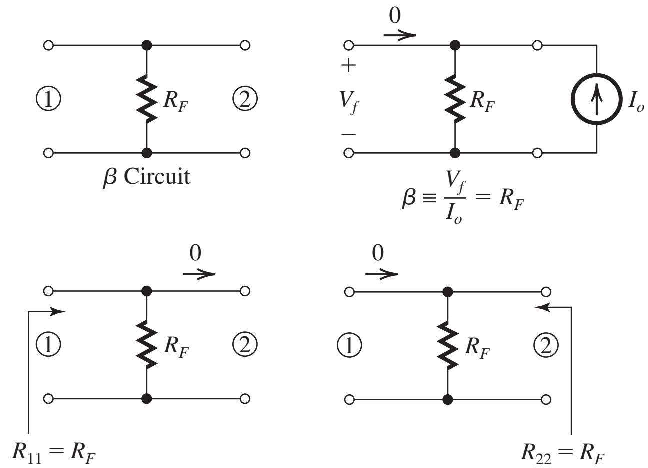


Figure 1

Thus, for large amounts of feedback, R_{out} is limited to this value, which is approximately $h_{fe}r_o$ independent of the amount of feedback. This phenomenon does not occur in MOSFET circuits where $h_{fe} = \infty$.

11.56 Figure 1 above shows the β circuit together with the determination of β, R_{11} and R_{22}.

$\beta = R_F$

$R_{11} = R_{22} = R_F$

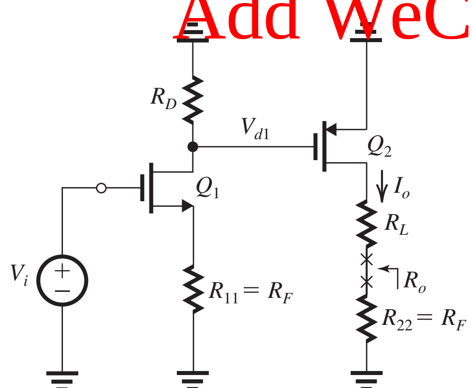


Figure 2

Figure 2 shows the A circuit. To determine $A \equiv I_o/V_i$, we write for Q_1

$$\frac{V_{d1}}{V_i} = -\frac{R_D}{(1/g_{m1}) + R_F} \qquad (1)$$

and for Q_2

$$I_o = -g_{m2}V_{d1} \qquad (2)$$

Combining (1) and (2) results in

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F}$$

$$A\beta = \frac{g_{m2}R_D R_F}{(1/g_{m1}) + R_F}$$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$\Rightarrow A_f = \frac{g_{m2}R_D}{(1/g_{m1}) + R_F + g_{m2}R_D R_F}$$

From the A circuit, breaking the loop at XX gives

$$R_o = R_F + R_L + r_{o2}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= \left[1 + \frac{g_{m2} + R_D R_F}{(1/g_{m1}) + R_F}\right][R_F + R_L + r_{o2}]$$

For

$g_{m1} = g_{m2} = 4$ mA/V, $R_D = 20$ kΩ,

$r_{o2} = 20$ kΩ, $R_F = 100$ Ω, and $R_L = 1$ kΩ,

we obtain

$$A = \frac{4 \times 20}{0.25 + 0.1} = 228.6 \text{ mA/V}$$

$$\beta = 0.1 \text{ k}\Omega$$

$$A\beta = 22.86$$

$$1 + A\beta = 23.86$$

$$A_f = \frac{228.6}{23.86} = 9.56 \text{ mA/V}$$

$$R_o = 0.1 + 1 + 20 = 21.1 \text{ k}\Omega$$

$$R_{of} = 23.86 \times 21.1 = 503.4 \text{ k}\Omega$$

11.57

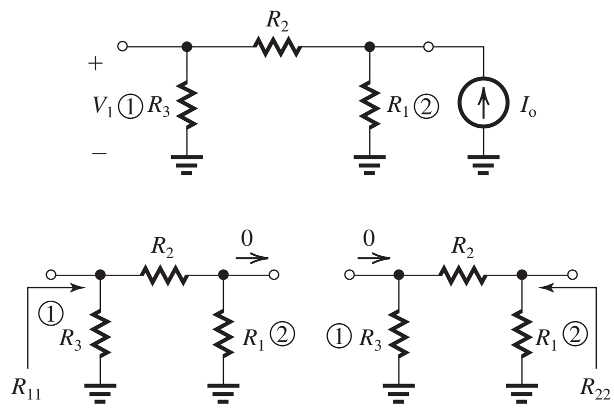


Figure 1

Figure 1 shows the feedback network fed with a current I_o to determine β:

$$\beta \equiv \frac{V_f}{I_o} = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

For $A\beta \gg 1$,

$$A_f = \frac{I_o}{V_s} \simeq \frac{1}{\beta}$$

Thus,

$$A_f = \frac{1}{R_3} + \frac{R_2}{R_1 R_3} + \frac{1}{R_1}$$

For $R_1 = R_3 = 0.1\ \text{k}\Omega$ and $A_f = 100\ \text{mA/V}$,

$$100 = 10 + \frac{R_2}{0.01} + 10$$

$$\Rightarrow R_2 = 0.8\ \text{k}\Omega$$

To obtain the loading effects of the feedback network, refer to Fig. 1.

$$R_{11} = R_3 \parallel (R_2 + R_1)$$

$$= 100\ \Omega \parallel (800 + 100)\ \Omega = 90\ \Omega$$

$$R_{22} = R_1 \parallel (R_2 + R_3)$$

$$= 100 \parallel (800 + 100) = 90\ \Omega$$

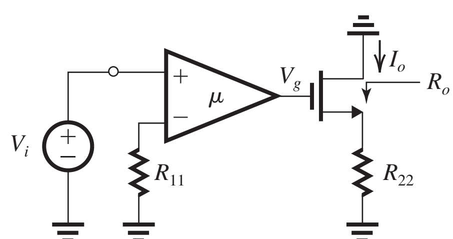


Figure 2

The A circuit is shown in Fig. 2. We can write

$$V_g = \mu V_i \qquad (1)$$

$$I_o = \frac{V_g}{(1/g_m) + R_{22}} \qquad (2)$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{\mu}{(1/g_m) + R_{22}}$$

Since $\beta = 0.01$, we have

$$A\beta = \frac{0.01\mu}{1/g_m + R_{22}}$$

$$= \frac{0.01\mu}{1 + 0.09} = 9.17 \times 10^{-3}\mu$$

For a 60-dB amount of feedback,

$$1 + A\beta = 1000$$

$$A\beta = 999$$

$$9.17 \times 10^{-3}\mu = 999$$

$$\Rightarrow \mu = 1.09 \times 10^5\ \text{V/V}$$

$$R_{\text{out}} = R_{of} = (1 + A\beta)R_o = 1000R_o$$

where R_o can be obtained from the A circuit as

$$R_o = r_o + R_{22} + g_m r_o R_{22}$$

$$= 50 + 0.09 + 1 \times 50 \times 0.09$$

$$= 54.6\ \text{k}\Omega$$

Thus,

$$R_{\text{out}} = 1000 \times 54.6 = 54.6\ \text{M}\Omega$$

11.58 (a) Since V_s has a zero dc component, the gate of Q_1 is at zero dc voltage. The negative feedback will force the gate of Q_2 to be approximately at the same dc voltage as that at the gate of Q_1, thus

$$V_O = 0$$

$$V_{D1} = 1.2 - V_{SG3}$$

$$= 1.2 - |V_t| - |V_{OV3}|$$

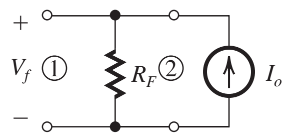
$$= 1.2 - 0.4 - 0.2 = +0.6\ \text{V}$$

$V_{D2} = V_O + V_{GS5}$

$= 0 + V_t + V_{OV5}$

$= 0.6 \text{ V}$

(b)



$$\beta \equiv \frac{V_f}{I_o} = R_F$$

Figure 1

The feedback network is shown in Fig. 1, from which we find

$\beta = R_F = 10 \text{ k}\Omega$

For $A\beta \gg 1$,

$$A_f \simeq \frac{1}{\beta} = \frac{1}{R_F}$$

$$A_f = \frac{1}{10 \text{ k}\Omega} = 0.1 \text{ mA/V}$$

(c) From the β circuit in Fig. 1 and noting that the feedback topology in series-series, the loading effects of the feedback network are

$R_{11} = R_{22} = R_F = 10 \text{ k}\Omega$

Figure 2 shows the A circuit. We can write

$$\frac{V_{g5}}{V_i} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \qquad (1)$$

$$I_o = \frac{V_{g5}}{(1/g_{m5}) + R_{22}}$$

Thus,

$$A \equiv \frac{I_o}{V_i} = \frac{g_{m1,2}(r_{o2} \parallel r_{o4})}{(1/g_{m5}) + R_{22}}$$

$$g_{m1,2} = \frac{2I_{D1,2}}{V_{OV1,2}}$$

$$= \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{|V_A|}{I_{D2,4}}$$

$$= \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$g_{m5} = \frac{2I_{D5}}{V_{OV5}} = \frac{2 \times 0.8}{0.2} = 8 \text{ mA/V}$$

$$A = \frac{1 \times (200 \parallel 200)}{0.125 + 10}$$

$= 9.88 \text{ mA/V}$

$$\frac{I_o}{V_s} = A_f = \frac{A}{1 + A\beta}$$

$$= \frac{9.88}{1 + 9.88 \times 10} = 0.099 \text{ mA/V}$$

(d) From the A circuit, we have

$R_o = r_{o5} + R_{22} + g_{m5} r_{o5} R_{22}$

where

$$r_{o5} = \frac{|V_A|}{I_{D5}} = \frac{20}{0.8} = 25 \text{ k}\Omega$$

$R_o = 25 + 10 + 8 \times 25 \times 10 = 2035 \text{ k}\Omega$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 2.035 \times (1 + 9.88 \times 10)$

$= 203 \text{ M}\Omega$

(e) $V_o = I_o R_F$

$= A_f V_s R_F$

$$\frac{V_o}{V_s} = A_f R_F = 0.099 \times 10 = 0.99 \text{ V/V}$$

This figure belongs to Problem 11.58, part (b).

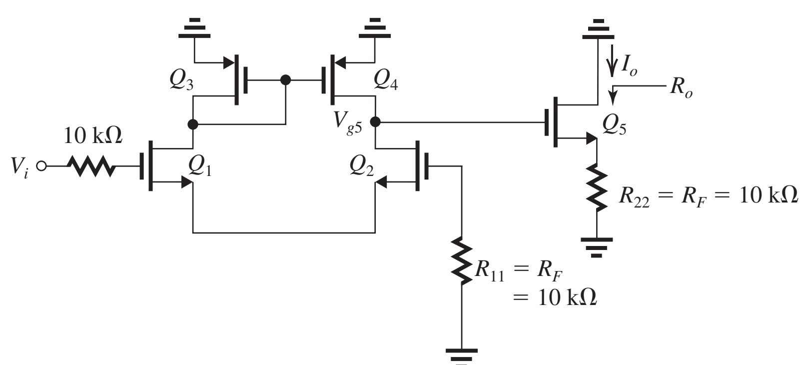


Figure 2

$$R_{out} = \frac{\text{Output resistance at source of } Q_5}{1 + A\beta}$$

$$\simeq \frac{1/g_{m5}}{1 + A\beta}$$

$$= \frac{125\ \Omega}{1 + 9.88 \times 10} = 1.25\ \Omega$$

11.59

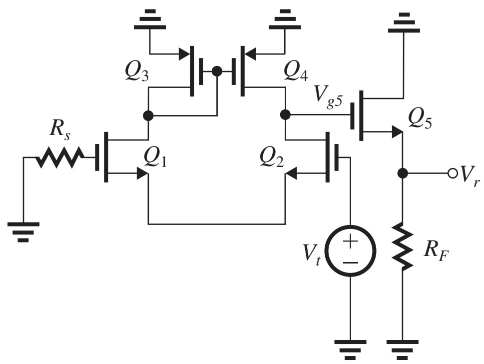


Figure 1

Figure 1 shows the circuit prepared for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

First we write for the gain of differential amplifier

$$\frac{V_{g5}}{V_t} = -g_{m1,2}(r_{o2} \parallel r_{o4}) \tag{1}$$

Next we write for the source follower,

$$\frac{V_r}{V_{g5}} = \frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})} \tag{2}$$

Combining (1) and (2) yields

$$A\beta = -\frac{V_r}{V_t} = g_{m1,2}(r_{o2} \parallel r_{o4})\frac{R_F \parallel r_{o5}}{(R_F \parallel r_{o5}) + (1/g_{m5})}$$

Q.E.D.

11.60 $\mu = 10^3$ V/V, $R_{id} = \infty$, $r_o = 100\ \Omega$, $R_F = 10$ kΩ, and $R_s = R_L = 1$ kΩ. From Example 11.9 Eqs. (11.37) and (11.41),

$$\beta = -\frac{1}{R_F} = -\frac{1}{10\ \text{k}\Omega} = -0.1\ \text{mA/V}$$

$$A = -\mu R_i \frac{(R_F \parallel R_L)}{r_o + R_F \parallel R_L)}$$

where

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$R_i = \infty \parallel 10 \parallel 1 = 0.909\ \text{k}\Omega$$

$$A = -10^3 \times 0.909 \times \frac{(10 \parallel 1)}{0.1 + (10 \parallel 1)}$$

$$= -818.9\ \text{k}\Omega$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{818.9}{1 - 818.9 \times -0.1}$$

$$= -\frac{818.9}{1 + 81.89} = -9.88\ \text{k}\Omega$$

$$R_{if} = R_i/(1 + A\beta)$$

$$= \frac{0.909\ \text{k}\Omega}{1 + 81.89} = 11\ \Omega$$

$$R_{if} = R_{in} \parallel R_s$$

$$= R_{in} \parallel 1\ \text{k}\Omega$$

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{1000}} = \frac{1}{\frac{1}{11} - \frac{1}{1000}}$$

$$= 11.1\ \Omega$$

From Eq. (11.42), we have

$$R_o = r_o \parallel R_F \parallel R_L$$

$$= 0.1 \parallel 10 \parallel 1 = 90.1\ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{90.1}{1 + 81.89} = 1.1\ \Omega$$

$$R_{of} = R_{out} \parallel R_L$$

$$= R_{out} \parallel 1\ \text{k}\Omega$$

$$\Rightarrow R_{out} \simeq 1.1\ \Omega$$

Comparison to the values in Example 11.9:

	$\mu = 10^4$ V/V	$\mu = 10^3$
A_f	-9.99 kΩ	-9.88 kΩ
R_{in}	1.11 Ω	11.1 Ω
R_{out}	0.11 Ω	1.1 Ω

11.61 Comparing the circuit of Fig. E11.19 and that of Fig. 11.24(a), we note the following:

$$\mu = g_m r_{o,Q}$$

$$r_o = r_{o,Q}$$

(This is based on representing the transistor output circuit ($g_m V_{gs}$, $r_{o,Q}$) by its Thévenin equivalent.)

$$R_{id} = \infty$$

$$R_L = \infty$$

Using Eq. (11.39), we obtain

$$R_i = R_{id} \parallel R_F \parallel R_s = R_F \parallel R_s$$

Using Eq. (11.44), we obtain

$$A_f = -\frac{g_m r_o (R_F \parallel R_s)\dfrac{R_F}{r_o + R_F}}{1 + g_m r_o (R_F \parallel R_s)\dfrac{1}{r_o + R_F}}$$

$$= -\frac{(R_s \parallel R_F) g_m (r_o \parallel R_F)}{1 + (R_s \parallel R_F) g_m (r_o \parallel R_F)/R_F}$$

which is the expression given in the answer to Exercise 11.19(b).

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{R_s \parallel R_F}{1 + A\beta}$$

Substituting for $A\beta$ from Eq. (11.43), we obtain

$$R_{if} = \frac{R_s \parallel R_F}{1 + g_m (R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_F} + \frac{g_m (r_o \parallel R_F)}{R_F}$$

But,

$$\frac{1}{R_{if}} = \frac{1}{R_s} + \frac{1}{R_{\text{in}}}$$

Thus,

$$\frac{1}{R_{\text{in}}} = \frac{1}{R_F}[1 + g_m (r_o \parallel R_F)]$$

$$\Rightarrow R_{\text{in}} = \frac{R_F}{[1 + g_m (r_o \parallel R_F)]}$$

which is the answer given to Exercise 11.19(c). Using Eq. (11.42), we obtain

$$R_o = r_o \parallel R_F \parallel R_L = r_o \parallel R_F$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

Substituting for $A\beta$ from Eq. (11.43), we obtain

$$R_{of} = \frac{(r_o \parallel R_F)}{1 + g_m (R_s \parallel R_F)(r_o \parallel R_F)/R_F}$$

$$\frac{1}{R_{of}} = \frac{1}{r_o} + \frac{1}{R_F} + \frac{g_m (R_s \parallel R_F)}{R_F}$$

$$R_{of} = r_o \parallel \frac{R_F}{1 + g_m (R_s \parallel R_F)}$$

$$R_{\text{out}} = R_{of} = r_o \parallel \frac{R_F}{1 + g_m (R_s \parallel R_F)}$$

which is identical to the result given in the answer to Exercise 11.19(d).

11.62

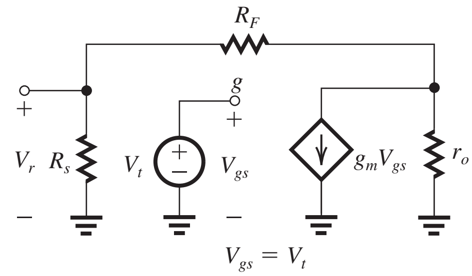


Figure 1

Figure 1 shows the circuit prepared for the determination of the loop gain $A\beta$:

$$A\beta = -\frac{V_r}{V_t}$$

An expression for V_r can be written by inspection as

$$V_r = -g_m V_t [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F}$$

Thus,

$$A\beta = g_m [r_o \parallel (R_s + R_F)] \frac{R_s}{R_s + R_F} \qquad (1)$$

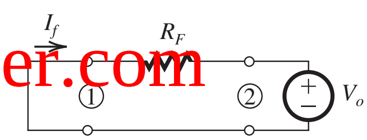


Figure 2

The feedback network (β circuit) is shown in Fig. 2 fed with V_o at port 2 and with port 1 short-circuited to determine β:

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F} \qquad (2)$$

Equations (1) and (2) can how be used to determine A:

$$A = -g_m [r_o \parallel (R_s + R_F)](R_s \parallel R_F)$$

Using the numerical values given in Exercise 11.19(c), we obtain

$$A = -5[20 \parallel (1 + 10)](1 \parallel 10)$$

$$= -32.3 \text{ k}\Omega$$

$$\beta = -\frac{1}{R_F} = -0.1 \text{ mA/V}$$

$$A\beta = 3.23$$

$$A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{32.3}{1 + 3.23} = -7.63 \text{ k}\Omega$$

Compare these results to those found in Exercise 11.19: $A = -32.3\ \text{k}\Omega\ (-30.3\ \text{k}\Omega)$, $\beta = -0.1\ \text{mA/V}\ (-0.1\ \text{mA/V})$, $A\beta = -3.23\ (-3.03)$, and $A_f = -7.63\ \text{k}\Omega\ (-7.52\ \text{k}\Omega)$. The slight differences are due to the approximation used in the systematic analysis method.

11.63 (a)

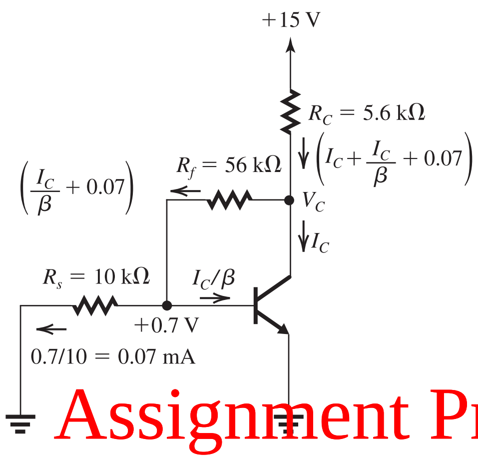


Figure 1

Figure 1 illustrates the dc analysis. We can express the dc collector voltage V_C in two alternative ways:

$$V_C = +15 - R_C\left(I_C + \frac{I_C}{\beta} + 0.07\right)$$

and

$$V_C = 0.7 + R_f\left(\frac{I_C}{\beta} + 0.07\right)$$

Equating these two expressions yields

$$15 - 5.6(I_C + 0.01I_C + 0.07)$$

$$= 0.7 + 56(0.01I_C + 0.07)$$

$$\Rightarrow I_C = 1.6\ \text{mA}$$

$$V_C \simeq 5.5\ \text{V}$$

(b) Figure 2 on the next page show the small-signal equivalent circuit of the amplifier where

$$g_m = \frac{1.6\ \text{mA}}{0.025\ \text{V}} = 64\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{64} = 1.56\ \text{k}\Omega$$

(c) Figure 3 on the next page shows the A circuit. It includes the loading effects of the feedback network:

$$R_{11} = R_{22} = R_f$$

Also, observe that

$$\beta = -\frac{1}{R_f}$$

From the A circuit in Fig. 3, we have

$$R_i = R_s \parallel R_f \parallel r_\pi \qquad (1)$$

$$V_\pi = I_i R_i \qquad (2)$$

$$V_o = -g_m V_\pi (R_C \parallel R_f) \qquad (3)$$

Combining (1), (2) and (3) gives

$$A \equiv \frac{V_o}{I_i} = -g_m(R_s \parallel R_f \parallel r_\pi)(R_C \parallel R_f)$$

$$R_i = 10\ \text{k}\Omega \parallel 56\ \text{k}\Omega \parallel 1.56\ \text{k}\Omega$$

$$= 1.32\ \text{k}\Omega$$

$$A = -64 \times 1.32 \times (5.6 \parallel 56)$$

$$= -429\ \text{k}\Omega$$

From the A circuit, we have

$$R_o = R_C \parallel R_f$$

$$= 5.6\ \text{k}\Omega \parallel 56\ \text{k}\Omega$$

$$= 5.1\ \text{k}\Omega$$

(d) $\beta = -\frac{1}{R_f} = -\frac{1}{56\ \text{k}\Omega}$

$$A\beta = \frac{429}{56} = 7.67$$

$$1 + A\beta = 8.67$$

(e) $A_f = \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$

$$= -\frac{429}{8.67} = -49.5\ \text{k}\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{1.32\ \text{k}\Omega}{8.67} = 152\ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$152\ \Omega = 10\ \text{k}\Omega \parallel R_{\text{in}}$$

$$R_{\text{in}} = 155\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5.1\ \text{k}\Omega}{8.67} = 588\ \Omega$$

This figure belongs to Problem 11.63, part (b).

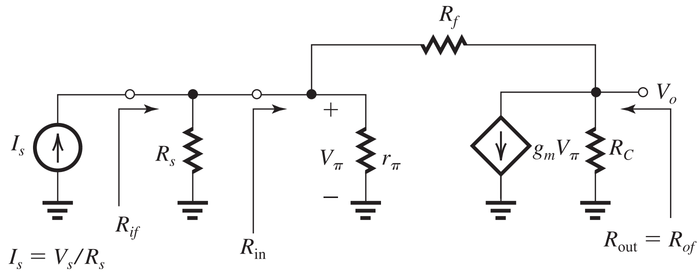


Figure 2

The below two figures belong to Problem 11.63, part (c).

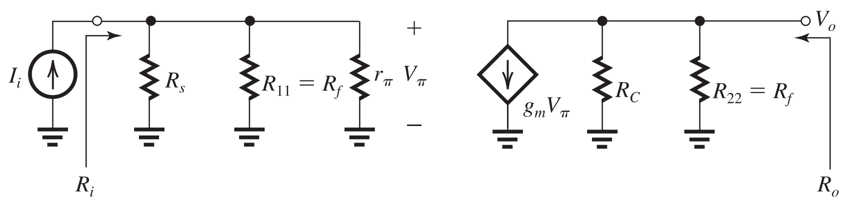


Figure 3

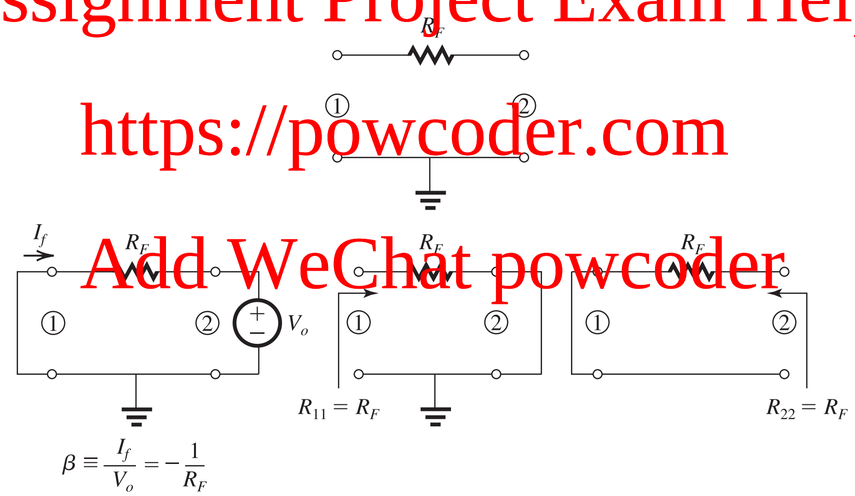


Figure 4

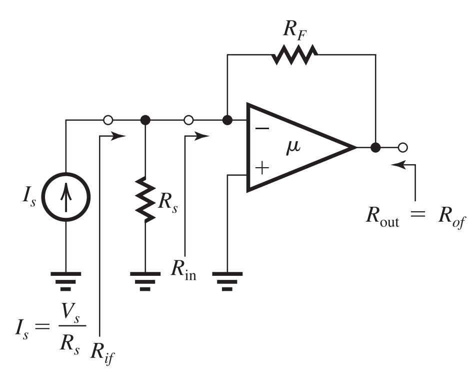
(f) $\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{49.5}{10} = -4.95$ V/V

Refer to Fig. P11.63 and assume the gain of the BJT to be infinite so that the signal voltage at its base is zero (virtual ground). In this case, we have

$$\frac{V_o}{V_s} = -\frac{R_f}{R_s} = -\frac{56}{10} = -5.6 \text{ V/V}$$

Thus, the actual gain magnitude ($\simeq$ 5 V/V) is only about 12% below the ideal value; not bad for a single transistor inverting op amp!

11.64 (a)



Refer to the feedback network shown in Fig. 11.24(b) and to the determination of β illustrated in Fig. 11.24(c). Thus,

$$\beta = -\frac{1}{R_F}$$

If $A\beta \gg 1$, then we have

$$A_f = \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_F$$

and the voltage gain realized will be

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} \simeq -\frac{R_F}{R_s}$$

If $R_s = 2\text{ k}\Omega$, to obtain $V_o/V_s \simeq -10$ V/V, we required

$$R_F = 10 \times R_s = 20\text{ k}\Omega$$

(b) Refer to the solution to Example 11.9.

$$\beta = -\frac{1}{R_F} = -\frac{1}{20\text{ k}\Omega} = -0.05\text{ mA/V}$$

Using Eq. (11.39), we obtain

$$R_i = R_{id} \parallel R_F \parallel R_s$$

$$= 100 \parallel 20 \parallel 2 = 1.786\text{ k}\Omega$$

Using Eq. (11.41) with $R_L = \infty$, we get

$$A \equiv \frac{V_o}{I_i} = -10^3 \times 1.786 \times \frac{20}{20+2}$$

$$= -1623.6\text{ k}\Omega$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1+A\beta}$$

where

$$1 + A\beta = 1 + 1623.6 \times 0.05$$

$$= 82.18$$

$$A_f = \frac{V_o}{I_s} = -\frac{1623.6}{82.18}$$

$$= -19.76\text{ k}\Omega$$

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s} = -\frac{19.76}{2}$$

$$= -9.88\text{ V/V}$$

$$R_{if} = \frac{R_i}{1+A\beta}$$

$$= \frac{1.786}{82.18} = 21.7\ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$21.7\ \Omega = 2000\ \Omega \parallel R_{\text{in}}$$

$$R_{\text{in}} \simeq 21.7\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1+A\beta}$$

where from Eq. (11.42) with $R_L = \infty$ we get

$$R_o = r_o \parallel R_F = 2 \parallel 20 = 1.818\text{ k}\Omega$$

$$R_{\text{out}} = R_{of} = \frac{1.818}{82.18} = 22.1\ \Omega$$

(c) $f_{Hf} = f_H(1 + A\beta)$

$$= 1 \times 82.18$$

$$= 82.18\text{ kHz}$$

11.65

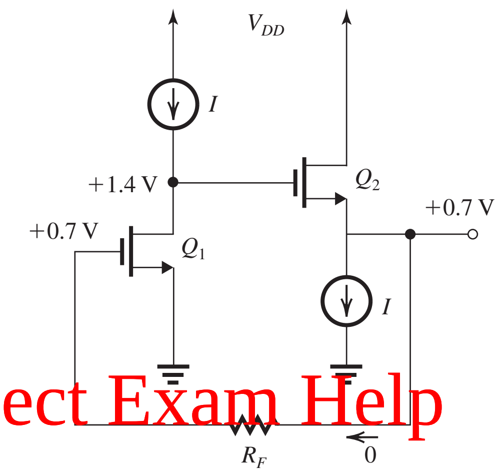


Figure 1

(a) See Figure 1.

$$V_{G1} = V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.2 = +0.7\text{ V}$$

(because the dc voltage across R_F is zero)

$$V_O = V_{G1}$$

$$V_O = +0.7\text{ V}$$

$$V_{D1} = V_O + V_{GS2}$$

$$= 0.7 + 0.5 + 0.2$$

$$= +1.4\text{ V}$$

(b) $g_{m1,2} = \dfrac{2I}{V_{OV}} = \dfrac{2 \times 0.4}{0.2} = 4\text{ mA/V}$

$$r_{o1,2} = \frac{V_A}{I} = \frac{16\text{ V}}{0.4\text{ mA}} = 40\text{ k}\Omega$$

(c) Figure 2 on the next page shows the β circuit and the determination of its loading effects,

$$R_{11} = R_{22} = R_F$$

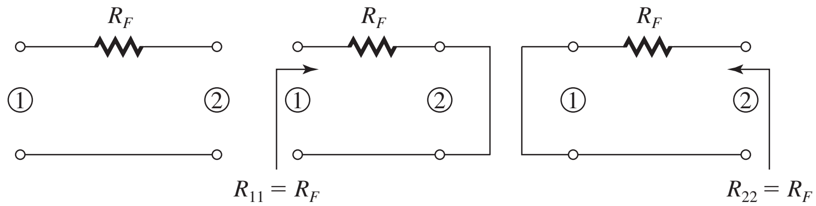
Figure 2 shows also the A circuit. We can write

$$V_{g1} = I_i R_i \tag{1}$$

where

$$R_i = R_{11} = R_F \tag{2}$$

This figure belongs to Problem 11.65, part (c).



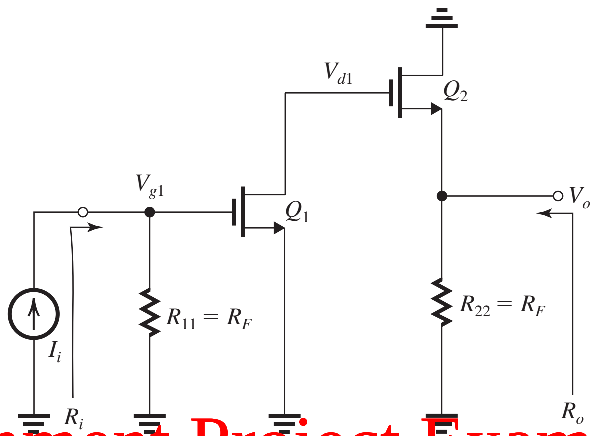


Figure 2

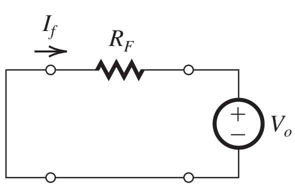
$$V_{d1} = -g_{m1} r_{o1} \tag{3}$$

$$\frac{V_o}{V_{d1}} = \frac{R_{22} \parallel r_{o2}}{(R_{22} \parallel r_{o2}) + \dfrac{1}{g_{m2}}} \tag{4}$$

Combining Eqs. (1)–(4) results in

$$A \equiv \frac{V_o}{I_i} = -g_{m1} r_{o1} R_F \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(d)



$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

Figure 3

From Fig. 3 we see that

$$\beta = -\frac{1}{R_F}$$

$$A\beta = g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$1 + A\beta = 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(e) $A_f \equiv \dfrac{V_o}{I_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{g_{m1} r_{o1} R_F (R_F \parallel r_{o2})}{(R_F \parallel r_{o2}) + 1/g_{m2} + (g_{m1} r_{o1})(R_F \parallel r_{o2})}$$

(f) $R_i = R_F$

$$R_{in} = R_{if} = R_i/(1 + A\beta)$$

$$= R_F \Bigg/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

$$R_{\text{out}} = R_{of} = R_o/(1 + A\beta)$$

where from the A circuit we have

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{\text{out}} = \left(R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}\right) \Bigg/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

(g) $A = -4 \times 40 \times 10 \dfrac{10 \parallel 40}{(10 \parallel 40) + 0.25}$

$= -1551.5$ kΩ

$$\beta = -\frac{1}{R_F} = -\frac{1}{10\ \text{k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = 155.15$$

$$1 + A\beta = 156.15$$

$$A_f = -\frac{1551.5}{156.15} = -9.94\ \text{k}\Omega$$

$$R_i = R_F = 10\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_F}{1 + A\beta} = \frac{10,000\ \Omega}{156.15} = 64\ \Omega$$

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_o = 10 \parallel 40 \parallel 0.25 = 242\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{242}{156.15} = 1.55\ \Omega$$

11.66

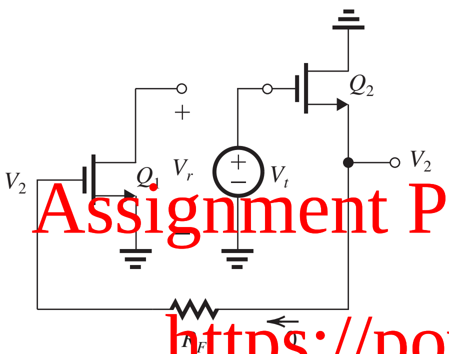


Figure 1

Figure 1 shows the circuit of Fig. P11.65 prepared for determining the loop gain $A\beta$:

$$A\beta \equiv -\frac{V_r}{V_t}$$

The gain of the source-follower Q_2 can be found as

$$\frac{V_2}{V_t} = \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \qquad (1)$$

The gain of the CS transistor Q_1 can be found as

$$V_r = -g_{m1} r_{o1} V_2 \qquad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + 1/g_{m2}} \qquad (3)$$

This expression differs from that obtained in Problem 11.65 utilizing the general feedback analysis method. To determine the numerical difference, we evaluate $A\beta$ in Eq. (3) using

$$g_{m1} = g_{m2} = 4\ \text{mA/V},\ \ r_{o1} = r_{o2} = 40\ \text{k}\Omega$$

$$A\beta = 4 \times 40 \times \frac{40}{40 + 0.25}$$

$$= 159$$

which is larger than the value found in Problem 11.65 (155.15) by about 2.5%. This small difference is a result of the approximations involved in the general method. The more accurate result for $A\beta$ is the one obtained here. However, the loop-gain method does not make it possible to determine the input and output resistances of the feedback amplifier.

11.67 Figure 1 shows the small-signal equivalent circuit of the feedback amplifier of Fig. P11.19. Analysis to determine V_o/I_s proceeds as follows:

Writing a node equation at the output node provides

$$g_m V_{gs} + \frac{V_o}{r_o} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$\Rightarrow V_{gs} = -V_o \frac{\frac{1}{r_o} + \frac{1}{R_F}}{g_m - \frac{1}{R_F}}$$

$$\Rightarrow V_{gs} = -V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)} \qquad (1)$$

Writing a node equation at node G provides

$$I_s - \frac{V_{gs}}{R_s} + \frac{V_o - V_{gs}}{R_F} = 0$$

$$I_s - \frac{V_{gs}}{(R_s \parallel R_F)} + \frac{V_o}{R_F} = 0 \qquad (2)$$

This figure belongs to Problem 11.67.

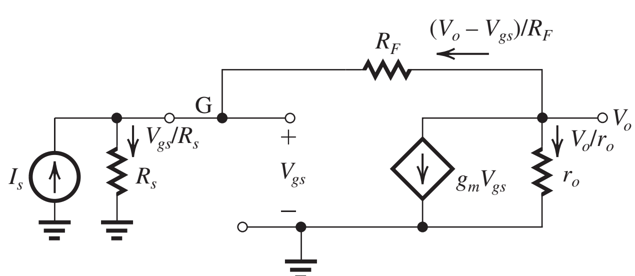


Figure 1

This figure belongs to Problem 11.68, part (a).

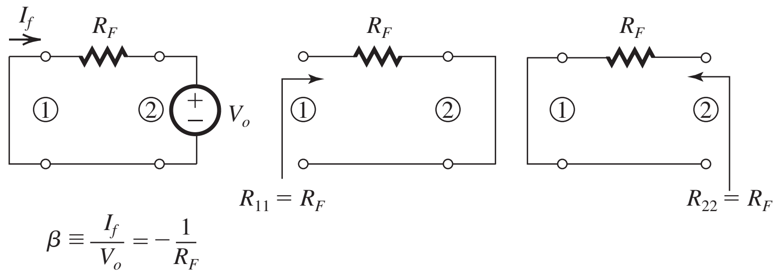


Figure 1

Substituting for V_{gs} from (1) into (2), we obtain

$$I_s + V_o \frac{1}{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)} + \frac{V_o}{R_F}$$

$$= 0 \Rightarrow \frac{V_o}{I_s}$$

$$= \frac{\left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)}{1 + \left(g_m - \frac{1}{R_F}\right)(r_o \parallel R_F)(R_s \parallel R_F)/R_F}$$

For the feedback analysis to be reasonably accurate, we use

$$g_m \gg \frac{1}{R_F}$$

11.68 Figure 1 shows the feedback network with a voltage V_o applied to port 2 to determine β:

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

For $A\beta \gg 1$, we have

$$\frac{V_o}{I_s} \equiv A_f \simeq \frac{1}{\beta} = -R_F$$

Thus, for $\frac{V_o}{I_s} \simeq -10\ \text{k}\Omega$, we select

$$R_F = 10\ \text{k}\Omega$$

The loading of the feedback network on the A circuit can be determined as shown in Fig. 1:

$$R_{11} = R_{22} = R_F$$

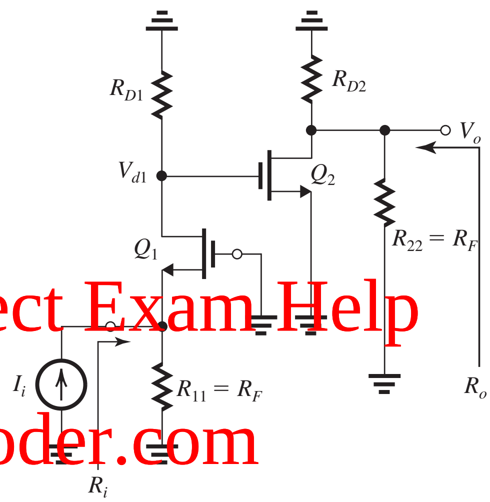


Figure 2

Figure 2 shows the A circuit. For the CG amplifier Q_1, we can write

$$R_i = R_F \parallel \frac{1}{g_{m1}} \qquad (1)$$

$$V_{sg} = I_i R_i \qquad (2)$$

$$V_{d1} = g_{m1} V_{sg} R_{D1} \qquad (3)$$

Combining (1)–(3) yields

$$V_{d1} = (g_{m1} R_{D1}) \left(R_F \parallel \frac{1}{g_{m1}}\right) I_i \qquad (4)$$

For the CS stage Q_2, we can write

$$\frac{V_o}{V_{d1}} = -g_{m2}(R_{D2} \parallel R_F) \qquad (5)$$

Combining (4) and (5), we obtain the open-loop gain A:

$$A \equiv \frac{V_o}{I_s} = -(g_{m1} R_{D1}) \left(R_F \parallel \frac{1}{g_{m1}}\right) g_{m2}(R_{D2} \parallel R_F)$$

This figure belongs to Problem 11.69, part (a).

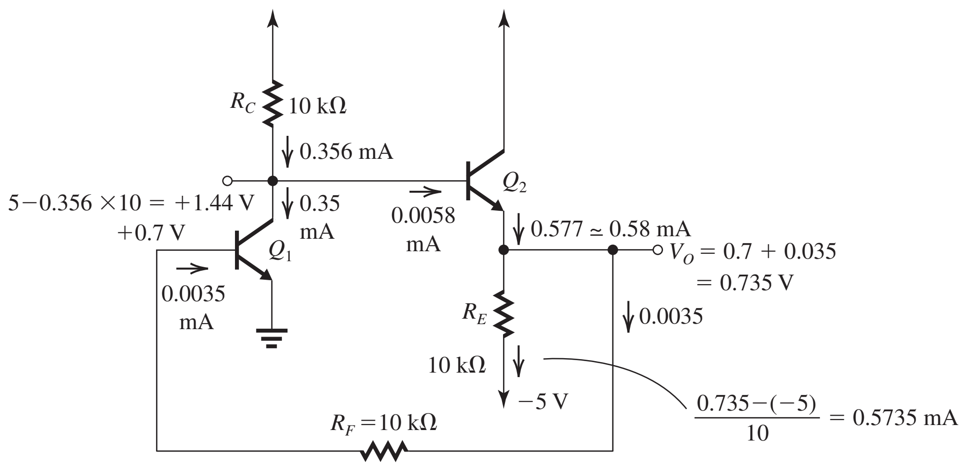


Figure 1

Substituting $g_{m1} = g_{m2} = 4$ mA/V, $R_{D1} = R_{D2} = 10$ kΩ, and $R_F = 10$ kΩ gives

$$A = -(4 \times 10) \times (10 \parallel 0.25) \times 4 \times (10 \parallel 10)$$

$$A = -195 \text{ k}\Omega$$

$$A\beta = \frac{195}{10} = 19.5$$

$$1 + A\beta = 20.5$$

$$A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta}$$

$$= -\frac{195}{20.5} = -9.52 \text{ k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

From Eq. (1), we obtain

$$R_i = 10 \parallel 0.25 = 244 \ \Omega$$

$$R_{\text{in}} = \frac{244}{20.5} = 11.9 \ \Omega$$

From the A circuit,

$$R_o = R_{D2} \parallel R_F$$

$$= 10 \parallel 10 = 5 \text{ k}\Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{5000}{20.5} = 244 \ \Omega$$

11.69 (a) Figure 1 (see figure above) shows the dc analysis. We assumed $I_{C1} = 0.35$ mA and found that $I_{C2} = 0.58$ mA, thus verifying the given values. The dc voltage at the output is

$$V_O = +0.735 \text{ V}$$

(b)

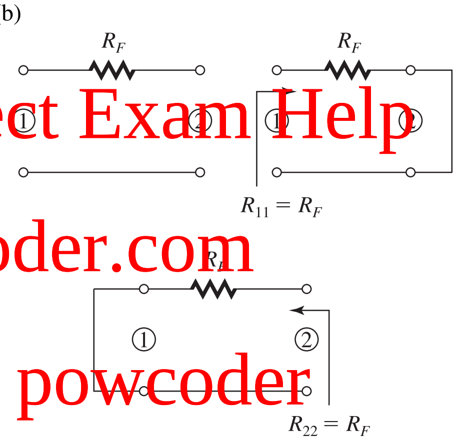


Figure 2

Figure 2 shows the β circuit and the determination of its loading effects on the A circuit:

$$R_{11} = R_{22} = R_F = 10 \text{ k}\Omega$$

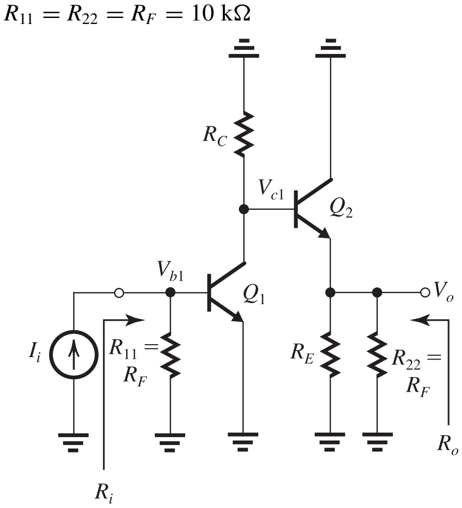


Figure 3

Figure 3 shows the A circuit. The input resistance is given by

$$R_i = R_F = R_F \parallel r_{\pi 1}$$

where

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.35}{0.025} = 14 \text{ mA/V}$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{14} = 7.14 \text{ k}\Omega$$

Thus,

$$R_i = 10 \text{ k}\Omega \parallel 7.14 \text{ k}\Omega = 4.17 \text{ k}\Omega$$

The input voltage V_{b1} is given by

$$V_{b1} = I_i R_i = 4.17 I_i \qquad (1)$$

The collector voltage of Q_1 is given by

$$V_{c1} = -g_{m1} V_{b1} \{ R_C \parallel (\beta_2 + 1)[r_{e2} + (R_E \parallel R_F)] \}$$

where

$$r_{e2} = \frac{V_T}{I_{E2}} = \frac{25 \text{ mV}}{0.58 \text{ mA}} = 43.1 \ \Omega$$

$$V_{c1} = -14 V_{b1} \{ 10 \parallel 101[0.0431 + (10 \parallel 10)] \}$$

$$= -137.3 V_{b1} \qquad (2)$$

The gain of the emitter-follower Q_2 is given by

$$\frac{V_o}{V_{c1}} = \frac{R_E \parallel R_F}{(R_E \parallel R_F) + r_{e2}}$$

$$= \frac{5}{5 + 0.0431} = 0.99 \text{ V/V} \qquad (3)$$

Combining (1)–(3) gives

$$A \equiv \frac{V_o}{I_i} = -0.99 \times 137.3 \times 4.17$$

$$= -567.6 \text{ k}\Omega$$

(c) The value of β can be obtained from the β circuit as shown in Fig. 4:

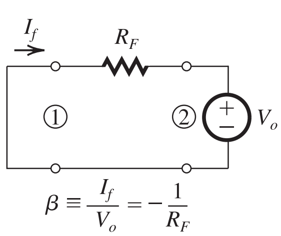


Figure 4

$$\beta = -\frac{1}{R_F} = -\frac{1}{10 \text{ k}\Omega} = -0.1 \text{ mA/V}$$

$$A\beta = -567.6 \times -0.1$$

$$= 56.76$$

$$1 + A\beta = 57.76$$

(d) $A_f \equiv \dfrac{V_o}{I_s} = \dfrac{A}{1 + A\beta}$

$$A_f = -\frac{567.6}{57.76} = -9.83 \text{ k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4.17 \text{ k}\Omega}{57.76} = 72.2 \ \Omega$$

From the A circuit, we have

$$R_o = R_F \parallel R_E \parallel \left[r_{e2} + \frac{R_C}{\beta_2 + 1} \right]$$

$$= 10 \parallel 10 \parallel \left[0.0431 + \frac{10}{101} \right]$$

$$= 138.2 \ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

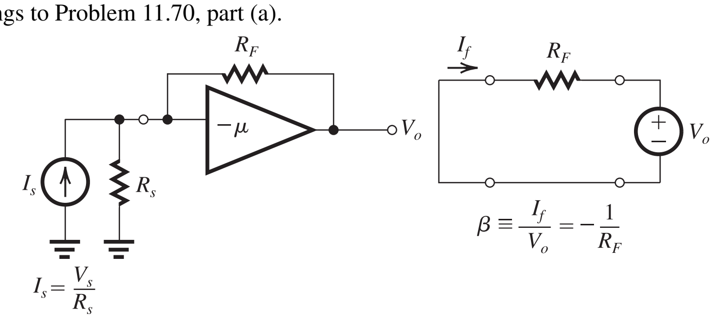
$$= \frac{138.2}{57.76} = 2.4 \ \Omega$$

11.70 (a) Converting the signal source to its Norton's form, we obtain the circuit shown in Fig. 1(a).

This is a shunt–shunt feedback amplifier with the feedback network consisting of the resistor R_f. To determine β, we use the arrangement shown in Fig. 1(b),

$$\beta = -\frac{1}{R_f}$$

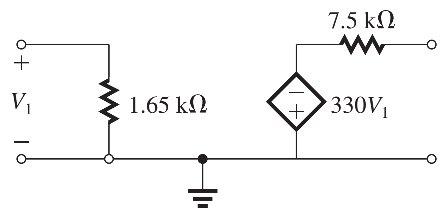
This figure belongs to Problem 11.70, part (a).



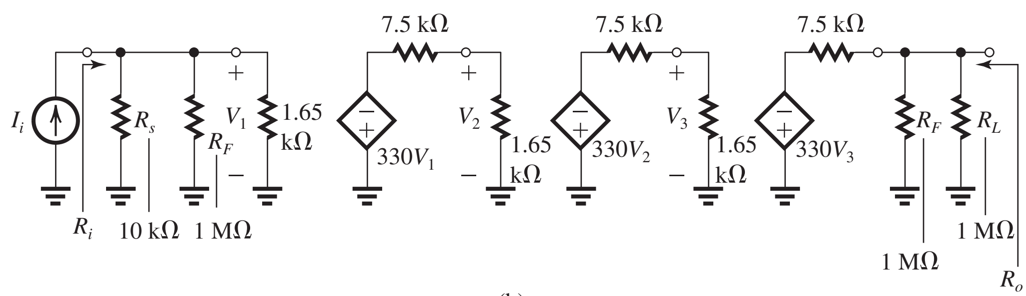
(a) (b)

Figure 1

This figure belongs to Problem 11.70, part (b).



(a)



(b)

Figure 2

Now, for $A\beta \gg 1$, the closed-loop gain becomes

$$A_f \equiv \frac{V_o}{I_s} \simeq \frac{1}{\beta} = -R_f$$

The voltage gain $\frac{V_o}{V_s}$ is obtained as

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$\frac{V_o}{V_s} \simeq -\frac{R_f}{R_s} \qquad \text{Q.E.D.}$$

(b) To obtain a closed-loop voltage gain of approximately −100 V/V, we use

$$-100 = -\frac{R_f}{R_s}$$

For $R_s = 10$ kΩ, we obtain

$$R_f = 1 \text{ M}\Omega$$

Now consider the amplifier stage shown in Fig. P11.70(b). First, we determine the dc bias point as follows:

$$I_E = \frac{15 \times \frac{10}{10+15} - 0.7}{4.7 + \frac{10 \parallel 15}{101}} = 1.11 \text{ mA}$$

$$I_C = 1.11 \times 0.99 = 1.1 \text{ mA}$$

$$g_m = \frac{I_C}{V_T} = \frac{1.1}{0.025} = 44 \text{ mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{44} = 2.27 \text{ k}\Omega$$

$$R_{\text{in}} = 10 \parallel 15 \parallel 2.27 = 1.65 \text{ k}\Omega$$

$$R_{\text{out}} = 7.5 \text{ k}\Omega$$

$$A_{vo} = -g_m \times 7.5 \text{ k}\Omega$$

$$= -44 \times 7.5 = -330 \text{ V/V}$$

Figure 2(a) shows the equivalent circuit of the amplifier stage. Figure 2(b) shows the A circuit of the feedback amplifier made up of the cascade of three stage. Observe that we have included R_s and R_L as well as R_{11} and R_{22}. The overall gain $A \equiv V_o/I_s$ cam be obtained as follows:

$$R_i = R_s \parallel R_F \parallel 1.65 \text{ k}\Omega$$

$$= 10 \parallel 1000 \parallel 1.65 = 0.623 \text{ k}\Omega$$

$$V_1 = I_i R_i = 0.623 I_i \qquad (1)$$

$$V_2 = -330 V_1 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_1 \qquad (2)$$

$$V_3 = -330 V_2 \times \frac{1.65}{1.65 + 7.5} = -59.5 V_2 \qquad (3)$$

$$V_o = -330 V_3 \times \frac{1 \parallel 1000}{(1 \parallel 1000) + 7.5}$$

$$= -38.8 V_3 \qquad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{V_o}{I_i} = -8.558 \times 10^4 \text{ k}\Omega$$

Since

$$\beta = -\frac{1}{R_f} = -\frac{1}{1 \text{ M}\Omega}$$

we have

$A\beta = 85.58$

and

$1 + A\beta = 86.58$

Thus,

$$A_f \equiv \frac{V_o}{V_s} = -\frac{8.558 \times 10^4}{86.58}$$

$= -988\ \text{k}\Omega$

and the voltage gain realized is

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = \frac{-988}{10} = -98.8\ \text{V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{623\ \Omega}{86.58} = 7.2\ \Omega$$

$R_{if} = R_s \parallel R_{\text{in}}$

$R_{\text{in}} \simeq 7.2\ \Omega$

From the A circuit, we have

$R_o = R \parallel R_f \parallel 7.5\ \text{k}\Omega$

$R_o = 1 \parallel 1000 \parallel 7.5 = 881.6\ \Omega$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$= \frac{881.6}{86.58} = 10.2\ \Omega$$

$R_{of} = R_{\text{out}} \parallel R_L$

$\Rightarrow R_{\text{out}} = 10.3\ \Omega$

11.71 (a) Shunt-Series

(b) Series-Series

(c) Shunt-Shunt

11.72

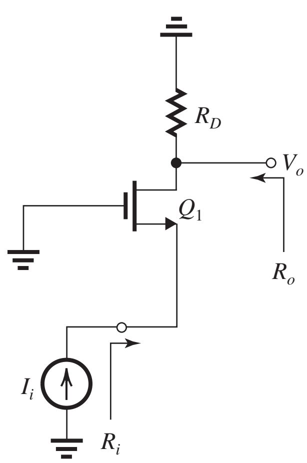


Figure 1

The A circuit is shown in Fig. 1.

$$R_i = \frac{1}{g_{m1}} = \frac{1}{5} = 0.2\ \text{k}\Omega$$

$$A \equiv \frac{V_o}{I_i} = R_D = 10\ \text{k}\Omega$$

$R_o = R_D = 10\ \text{k}\Omega$

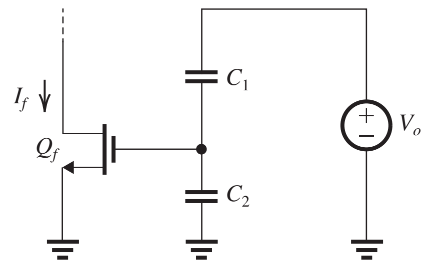


Figure 2

The β circuit is shown in Fig. 2.

$$\beta \equiv \frac{I_f}{V_o} = \frac{C_1}{C_1 + C_2} g_{mf} = \frac{0.9}{0.9 + 0.1} \times 2$$

$= 1.8\ \text{mA/V}$

$$A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{10}{1 + 10 \times 1.8} = 0.53\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1 + A\beta} = \frac{200\ \Omega}{19} = 10.5\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta} = \frac{10\ \text{k}\Omega}{19} = 526\ \Omega$$

11.73 (a)

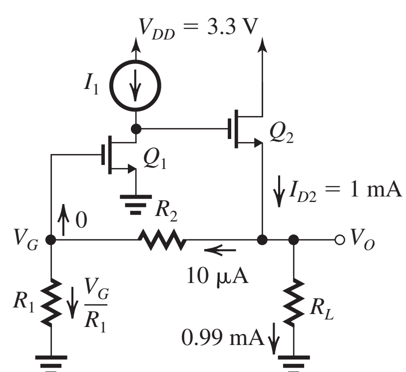


Figure 1

Figure 1 shows the circuit for the purpose of performing a dc design.

$I_{D1} = 100\ \mu\text{A} \Rightarrow I_1 = 100\ \mu\text{A}$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV1}^2$$

$$100 = \frac{1}{2} \times 200 \times \left(\frac{W}{L}\right)_1 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

$$V_{G1} = V_{tn} + V_{OV1}$$

$$= 0.6 + 0.2 = 0.8 \text{ V}$$

Since $I_{R2,R1} = 10\ \mu\text{A}$, we have

$$R_1 = \frac{0.8 \text{ V}}{0.01 \text{ mA}} = 80 \text{ k}\Omega$$

$$I_{RL} = I_{D2} - I_{R2,R1}$$

$$= 1 - 0.01 = 0.99 \text{ mA}$$

$$V_O = 0.99 \times 2 = 1.98 \text{ V}$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 V_{OV2}^2$$

$$1 = \frac{1}{2} \times 0.2 \times \left(\frac{W}{L}\right)_2 \times 0.2^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 250$$

$$R_2 = \frac{V_O - V_G}{0.01 \text{ mA}}$$

$$= \frac{1.98 - 0.8}{0.01} = 118 \text{ k}\Omega$$

$$V_{GS2} = V_{tn} + V_{OV2} = 0.8 \text{ V}$$

$$V_{D1} = V_{G2} = 1.98 + 0.8 = 2.78 \text{ V}$$

(b) The β circuit consists of resistance R_2. The value of β can be determined as shown in Fig. 2.

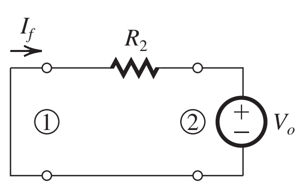


Figure 2

$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_2} = -\frac{1}{118 \text{ k}\Omega}$$

$$= -8.47 \times 10^{-3} \text{ mA/V}$$

Thus,

$$A_f\big|_{\text{ideal}} \equiv \frac{1}{\beta} = -118 \text{ k}\Omega$$

(c) Converting the signal source to its Norton's form, the feedback amplifier takes the form shown in Fig. 3.

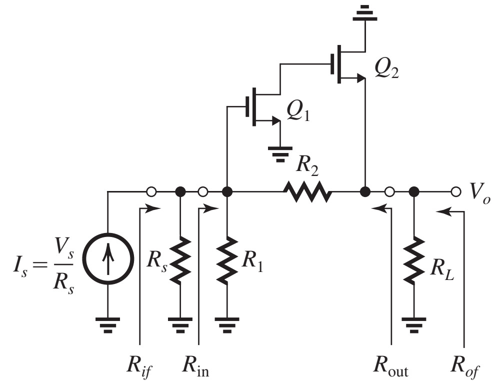


Figure 3

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}$$

Thus,

$$-6 = -\frac{118}{R_s}$$

$$\Rightarrow R_s = 19.7 \text{ k}\Omega$$

(d)

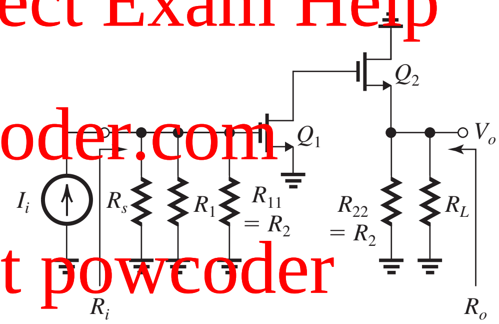


Figure 4

The A circuit is shown in Fig. 4.

$$R_i = R_s \parallel R_1 \parallel R_2$$

$$= 19.7 \parallel 80 \parallel 118 = 13.92 \text{ k}\Omega$$

$$V_{gs1} = I_i R_i = 13.92 I_i \qquad (1)$$

$$V_{d1} = -g_{m1} V_{gs1} r_{o1}$$

where

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.1}{0.2} = 1 \text{ mA/V}$$

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

Thus,

$$V_{d1} = -200 V_{gs1} \qquad (2)$$

$$\frac{V_o}{V_{d1}} = \frac{R_L \parallel R_2 \parallel r_{o2}}{(R_L \parallel R_2 \parallel r_{o2}) + 1/g_{m2}}$$

where

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 1}{0.2} = 10 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_{D2}} = \frac{20}{1} = 20 \text{ k}\Omega$$

Thus,

$$\frac{V_o}{V_{d1}} = \frac{(2 \parallel 118 \parallel 20)}{(2 \parallel 118 \parallel 20) + 0.1} = 0.947 \text{ V/V} \qquad (3)$$

Combining (1)–(3), we obtain

$$A = \frac{V_o}{I_i} = -13.92 \times 200 \times 0.947$$

$$= -2636.7 \text{ k}\Omega$$

$$R_o = R_L \parallel R_2 \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$= 2 \parallel 118 \parallel 20 \parallel 0.1$$

$$= 94.7 \ \Omega$$

(e) $A_f = \dfrac{V_o}{V_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{2636.7}{1 + (2636.7/118)}$$

$$= -\frac{2636.7}{23.34} = -113 \text{ k}\Omega$$

These figures belong to Problem 11.74, part (a).

$$\frac{V_o}{V_s} = \frac{A_f}{R_s} = -\frac{113}{19.7} = -5.73 \text{ V/V}$$

(f) $R_{if} = \dfrac{R_i}{1 + A\beta} = \dfrac{13.92}{23.34} = 0.596 \text{ k}\Omega$

$R_{if} = R_s \parallel R_{\text{in}}$

$0.596 = 19.7 \parallel R_{\text{in}}$

$\Rightarrow R_{\text{in}} = 615 \ \Omega$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{94.7}{23.34} = 4.06 \ \Omega$$

$R_{of} = R_{\text{out}} \parallel R_L$

$4.06 = R_{\text{out}} \parallel 2000 \Rightarrow R_{\text{out}} = 4.1 \ \Omega$

11.74 (a) Figure 1 shows the β network as well as the determination of its loading effects on the A circuit:

$R_{11} = R_F + R_M$

$R_{22} = R_M \parallel R_F$

Figure 2 shows the A circuit. Some of the analysis is shown on the diagram.

$$R_i = R_{11} \parallel \frac{1}{g_{m1}} \qquad (1)$$

$$V_{sg1} = I_i R_i \qquad (2)$$

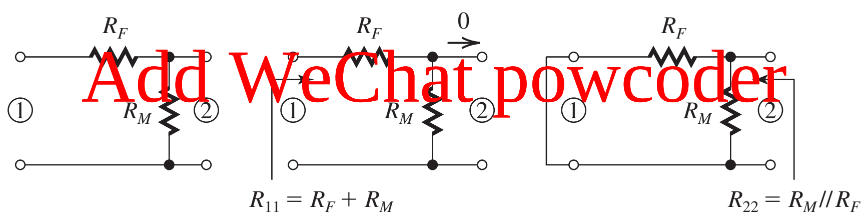


Figure 1

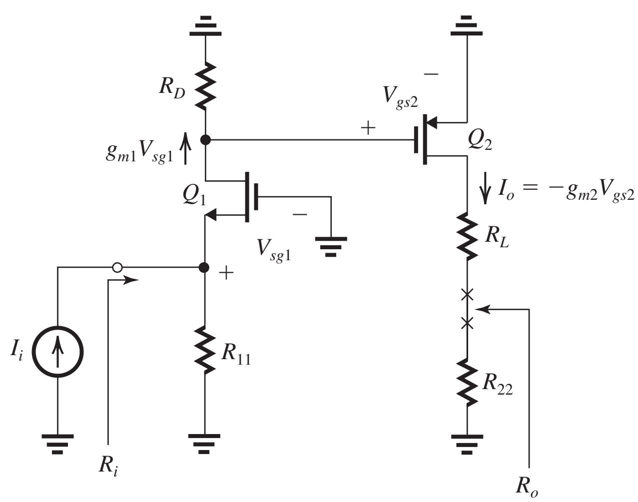


Figure 2

$$V_{gs2} = g_{m1}V_{sg1}R_D \quad (3)$$

$$I_o = -g_{m2}V_{gs2} \quad (4)$$

Combining (1)–(4) gives

$$A \equiv \frac{I_o}{I_i} = -\left(R_{11} \parallel \frac{1}{g_{m1}}\right)(g_{m1}R_D)g_{m2} \quad (5)$$

(b)

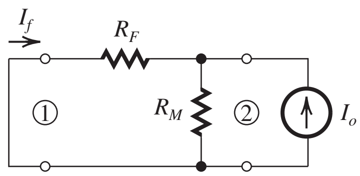


Figure 3

The β circuit prepared for the determination of β is shown in Fig. 3.

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_M}{R_M + R_F} \quad (6)$$

(c) From (5)–(6), we obtain

$$A\beta = \frac{R_M}{R_M + R_F}\left(R_{11} \parallel \frac{1}{g_{m1}}\right)(g_{m1}R_D)g_{m2}$$

(d) $g_{m1} = g_{m2} = 5$ mA/V, $R_D = 20$ kΩ

$R_M = 10$ kΩ, and $R_F = 90$ kΩ, thus

$R_{11} = R_F + R_M = 90 + 10 = 100$ kΩ

$R_{22} = R_M \parallel R_F = 10 \parallel 90 = 9$ kΩ

$A = -(100 \parallel 0.2) \times (5 \times 20) \times 5 = -99.8$ A/A

$$\beta = -\frac{10}{10+90} = -0.1 \text{ A/A}$$

$A\beta = 9.98$

$1 + A\beta = 10.98$

$$A_f \equiv \frac{I_o}{I_s} = \frac{A}{1+A\beta} = -\frac{99.8}{10.98} = -9.1 \text{ A/A}$$

$$R_{\text{in}} = R_{if} = \frac{R_i}{1+A\beta}$$

where

$R_i = 100$ k$\Omega \parallel 0.2$ k$\Omega \simeq 0.2$ kΩ

$$R_{\text{in}} = \frac{200\ \Omega}{10.98} = 18.2\ \Omega$$

(e) Breaking the output loop of the A circuit between XX, we find

$R_o = R_{22} + R_L + r_{o2}$

$= (R_M \parallel R_F) + R_L + r_{o2}$

$= (10 \parallel 90) + 1 + 20$

$= 30$ kΩ

$R_{of} = R_o(1 + A\beta) = 30 \times 10.98$

$= 329.4$ kΩ

$R_{\text{out}} = R_{of} - R_L = 328.4$ kΩ

11.75 Refer to Fig. 11.27(c), which shows the determination of β,

$$\beta = \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2} \quad (1)$$

Refer to Fig. 11.27(e), which shows the A circuit. The input resistance R_i is given by

$R_i = R_s \parallel R_{id} \parallel (R_1 + R_2)$

For our case here, $R_s = R_{id} = \infty$, thus

$R_i = R_1 + R_2$

For $R_{\text{in}} = R_{if} = 1$ kΩ, we have

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$\Rightarrow R_i = R_{if}(1 + A\beta)$

Thus

$R_1 + R_2 = 1$ k$\Omega \times (1 + A\beta)$

Since $1 + A\beta$ is 40 dB, that is,

$1 + A\beta = 100$

we have

$$R_1 + R_2 = 1 \times 100 = 100 \text{ k}\Omega \quad (2)$$

Now,

$$A_f = \frac{A}{1 + A\beta}$$

$$-100 = \frac{A}{100}$$

$\Rightarrow A = -10^4$ A/A

$$\beta = \frac{A\beta}{A} = \frac{99}{-10^4} = -0.0099$$

Using Eqs. (1) and (2), we obtain

$$-0.0099 = -\frac{R_1}{R_1 + R_2}$$

$\Rightarrow R_1 = 0.0099 \times 100 = 0.99$ kΩ

$R_2 = 100 - 0.99 = 99.01$ kΩ

Now, using Eq. (11.53) (page 869), we obtain

$$A = -\mu \frac{R_i}{1/g_m + (R_1 \parallel R_2 \parallel r_{o2})} \frac{r_{o2}}{r_{o2} + (R_1 \parallel R_2)}$$

$$-10^4 = -\mu \frac{100}{0.2 + (0.99 \parallel 99.01 \parallel 20)} \frac{20}{20 + (0.99 \parallel 99.01)}$$

$\mu = 119$ V/V

From Example 11.10, we have

$R_o = r_{o2} + (R_1 \parallel R_2) + g_{m2}r_{o2}(R_1 \parallel R_2)$

$R_o = 20 + (0.99 \parallel 99.01)(1 + 5 \times 20)$

$= 119$ kΩ

$R_{\text{out}} = R_{of} = R_o(1+A\beta) = 119 \times 100 = 11.9$ MΩ

11.76

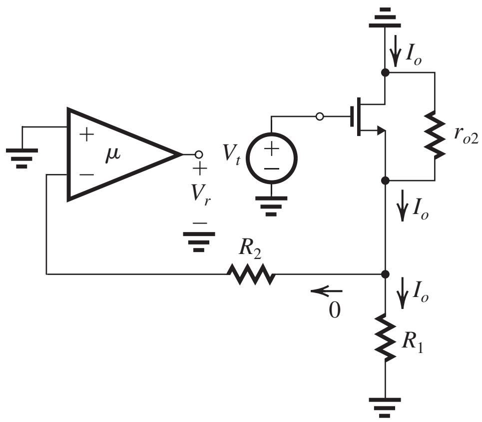


Figure 1

Figure 1 shows the shunt-series feedback amplifier circuit of Fig. 11.27(a) prepared for determining the loop gain,

$$A\beta \equiv -\frac{V_r}{V_t}$$

observe that here $R_s = R_{id} = \infty$. Thus I_o can be obtained as

$$I_o = \frac{V_t}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1} \qquad (1)$$

The voltage V_r can be obtained as

$$V_r = I_o R_1 \times -\mu = -\mu R_1 I_o \qquad (2)$$

Combining Eqs. (1) and (2), we obtain

$$A\beta \equiv -\frac{V_r}{V_t} = \mu \frac{R_1}{\frac{1}{g_m} + (r_{o2} \parallel R_1)} \frac{r_{o2}}{r_{o2} + R_1}$$

For

$\mu = 1000$ V/V, $R_1 = 10$ kΩ,

$g_m = 5$ mA/V, and $r_{o2} = 20$ kΩ

we obtain

$$A\beta = 1000 \times \frac{10}{0.2 + (20 \parallel 10)} \frac{20}{20 + 10}$$

$= 970.9$

which is slightly lower than the value found in Example 11.10 (1076.4), the difference being about -10%. This is a result of the assumptions and approximations made in the general feedback analysis method.

From Example 11.10 (or directly from the β circuit) we have

$\beta = -0.1$ A/A

Thus,

$A = -9709$

and

$$A_f = -\frac{9709}{971.9} = -9.99 \text{ A/A}$$

which is identical to the value obtained in Example 11.10. Thus while $A\beta$ and A differ slightly for the earlier results, A_f is identical; an illustration of the power of negative feedback!

11.77 (a)

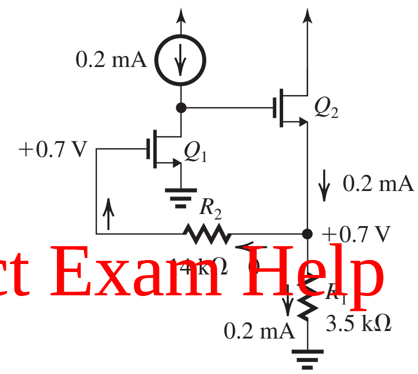


Figure 1

Figure 1 shows the dc analysis. It starts by noting that $I_{D1} = 0.2$ mA. Thus $V_{OV1} = 0.2$ V and

$V_{G1} = V_{GS1} = V_t + V_{OV1} = 0.5 + 0.2$

$= 0.7$ V

Since the dc current through R_2 is zero, the dc voltage drop across it will be zero, thus

$V_{S2} = +0.7$ V

and

$$I_{R1} = \frac{0.7 \text{ V}}{3.5 \text{ k}\Omega} = 0.2 \text{ mA}$$

Thus, Q_2 is operating at

$I_D = 0.2$ mA Q.E.D.

(b) $g_{m1} = g_{m2} = \dfrac{2I_D}{V_{OV}}$

$$= \frac{2 \times 0.2}{0.2} = 2 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{10}{0.2} = 50 \text{ k}\Omega$$

(c)

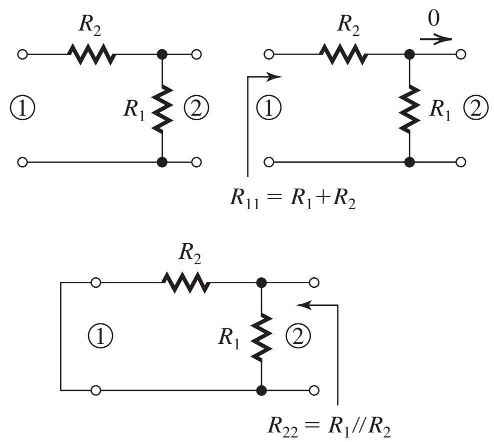


Figure 2

Figure 2 shows the β circuit and the determination of its loading effects, R_{11} and R_{22},

$R_{11} = R_1 + R_2 = 3.5 + 14 = 17.5\ \text{k}\Omega$

$R_{22} = R_1 \parallel R_2 = 3.5 \parallel 14 = 2.8\ \text{k}\Omega$

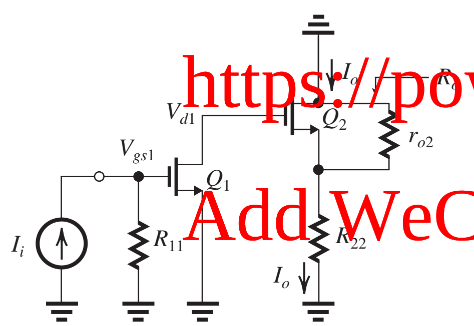


Figure 3

Figure 3 shows the A circuit. To obtain $A = I_o/I_i$, we write

$$V_{gs1} = I_i R_{11} \qquad (1)$$

$$V_{d1} = -g_{m1} r_{o1} V_{gs1} \qquad (2)$$

$$I_o = \frac{V_{d1}}{\dfrac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} \frac{r_{o2}}{r_{o2} + R_{22}} \qquad (3)$$

Combining (1)–(3) yields

$$A = \frac{I_o}{I_i} = -\frac{R_{11}}{\dfrac{1}{g_{m2}} + (r_{o2} \parallel R_{22})} (g_{m1} r_{o1}) \frac{r_{o2}}{r_{o2} + R_{22}}$$

$$A = -\frac{17.5}{0.5 + (50 \parallel 2.8)} \times 2 \times 50 \times \frac{50}{50 + 2.8}$$

$A = -525.8$ A/A

$R_i = R_{11} = 17.5\ \text{k}\Omega$

$R_o = r_{o2} + R_{22} + g_{m2} r_{o2} R_{22}$

$= 50 + 2.8 + 2 \times 50 \times 2.8$

$= 332.8\ \text{k}\Omega$

(d)

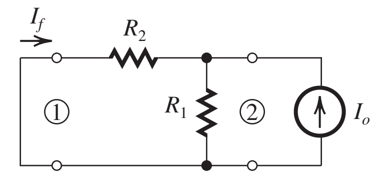


Figure 4

Figure 4 shows the determination of the value of β:

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_1}{R_1 + R_2}$$

Thus,

$$\beta = -\frac{3.5}{3.5 + 14} = -0.2 \text{ A/A}$$

(e) $A\beta = -525.8 \times -0.2$

$= 105.16$

$1 + A\beta = 106.16$

$$A_f = -\frac{525.8}{106.16} = -4.95 \text{ A/A}$$

(f) $R_{\text{in}} = R_{if} = \dfrac{R_i}{1 + A\beta}$

$$= \frac{17.5\ \text{k}\Omega}{106.16} = 164.8\ \Omega$$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= 332.8 \times 106.16 = 35.3\ \text{M}\Omega$

11.78 (a) If μ is a very large, a virtual ground will appear at the input terminal. Thus the input resistance $R_{\text{in}} = V_-/I_i = 0$. Since no current flows in R_s, or into the amplifier input terminal, all the current I_s will flow in the transistor source terminal and hence into the drain, thus

$I_o = I_s$

and

$$\frac{I_o}{I_s} = 1$$

(b) This is a shunt-series feedback amplifier in which the feedback circuit consists of a wire, as shown in Fig. 1. As indicated,

$R_{11} = \infty$

$R_{22} = 0$

The A circuit is shown in Fig. 2, for which we can write

$V_{id} = -I_i(R_s \parallel R_{id})$

$$\simeq -I_i R_s \qquad (1)$$

These figures belong to Problem 11.78, part (b).

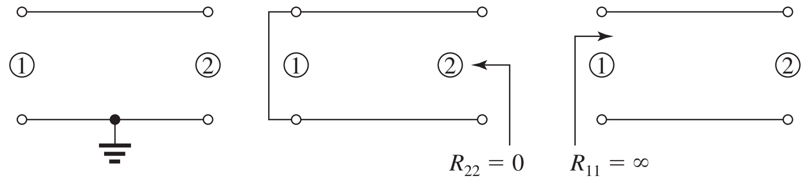


Figure 1

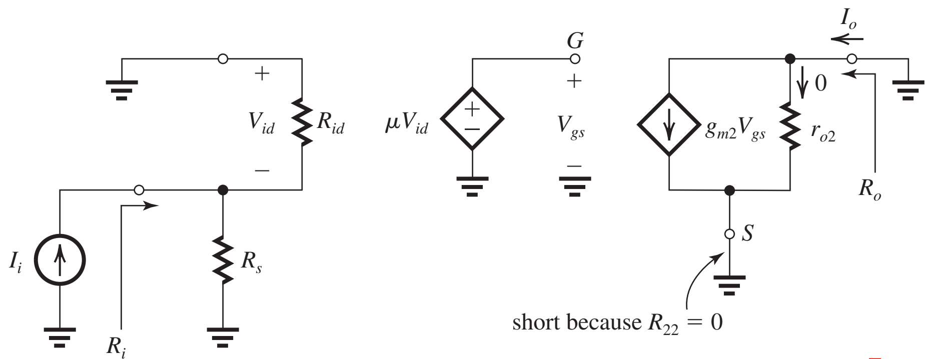


Figure 2

(since R_{id} is very large)

$V_{gs} = \mu V_{id}$ (2)

$I_o = g_{m2} V_{gs}$ (3)

Combining (1)–(3), we obtain

$$A \equiv \frac{I_o}{I_i} = -\mu g_{m2} R_s$$

$R_i = R_s$

$R_o = r_{o2}$

(c)

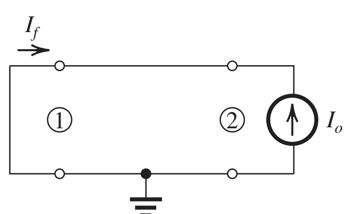


Figure 3

From Fig. 3 we find

$$\beta \equiv \frac{I_f}{I_o} = -1$$

(d) $A\beta = \mu g_{m2} R_s$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

Note that the negative sign is due to our assumption that I_s flows into the input node (see Fig. 2 for the way I_i is applied). If instead I_s is flowing out of the input node, as indicated in Fig. P11.78, then

$$A_f \equiv \frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

If μ is large so that $\mu g_{m2} R_s \gg 1$,

$A_f \simeq 1$

(e) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$= \frac{R_s}{\mu g_{m2} R_s}$$

$R_{if} = R_{\text{in}} \parallel R_s$

$$\frac{1}{R_{if}} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\frac{1}{R_s} + \mu g_{m2} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\Rightarrow R_{\text{in}} = \frac{1}{\mu g_{m2}}$$

$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$

$= r_{o2}(1 + \mu g_{m2} R_s)$

(f)

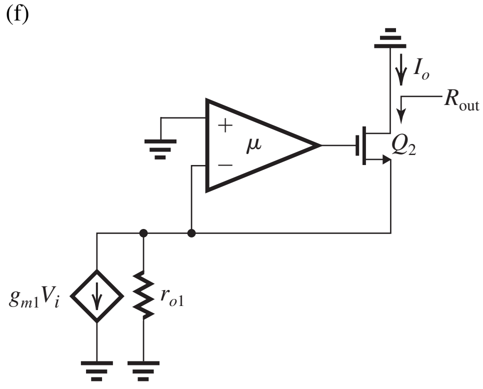


Figure 4

Figure 4 shows the circuit of the cascode amplifier in Fig. P11.78 with V_G replaced by signal ground, and Q_1 replaced by its equivalent circuit at the drain. This circuit looks identical to that of Fig. 11.78(a). Thus we can write

$$I_o \simeq g_{m1} v_i$$

$$R_{out} = r_{o2}(1 + \mu g_{m2} r_{o1}) = r_{o2} + \mu(g_{m2}r_{o2})r_{o1}$$

$$\simeq \mu(g_{m2}r_{o2})r_{o1}$$

Compared to the case of a regular cascode, we see that while $I_o \simeq g_{m1}V_i$ as in the regular cascode, utilizing the "super" CG transistor results in increasing the output resistance by the additional factor μ!

11.79 (a) Refer to Fig. P11.79. Let I_s increase. This will cause the voltage at the input node, which is the voltage at the positive input of the amplifier μ, to increase. The amplifier output voltage will correspondingly increase. Thus, V_{gs} of Q_1 increases and I_{o1} increases. This counteracts the originally assumed change of increased current into the input node. Thus, the feedback is negative.

(b) The negative feedback will cause the dc voltage at the positive input terminal of the amplifier to be equal to V_{BIAS}. Thus, the voltage at the drain of Q_1 will be equal to V_{BIAS}. For Q_1 to operate in the active mode, the minimum voltage at the drain must be equal to V_{OV} which is 0.2 V. Thus the minimum value of V_{BIAS} must be 0.2 V.

(c)

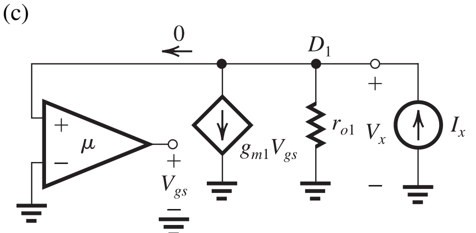


Figure 1

Figure 1 shows the circuit for determining R_{in},

$$R_{in} \equiv \frac{V_x}{I_x}$$

A node equation at D_1 gives

$$V_x = (I_x - g_{m1}V_{gs})r_{o1}$$

$$\Rightarrow V_x = I_x r_{o1} - g_{m1}V_{gs}r_{o1}$$

But,

$$V_{gs} = \mu V_x,$$

thus

$$V_x = I_x r_{o1} - \mu g_{m1} r_{o1} V_x$$

$$\Rightarrow R_{in} \equiv \frac{V_x}{I_x} = \frac{r_{o1}}{1 + \mu g_{m1} r_{o1}}$$

Since $\mu g_{m1} r_{o1} \gg 1$, we have

$$R_{in} \simeq \frac{1}{\mu g_{m1}}$$

(d) Since the drain of Q_2 is outside the feedback loop, we have

$$R_{out} = r_{o2}$$

11.80 See figures on the next two pages.

(a) Refer to Fig. (a).

(b) Refer to Fig. (b).

(c) $R_{outz} = r_{o2} \parallel r_{o4}$

11.81

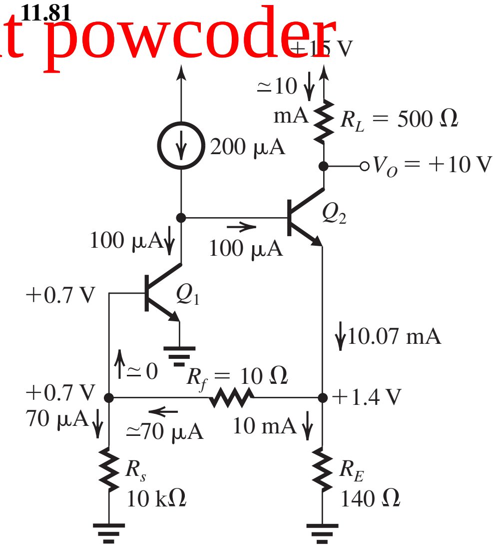


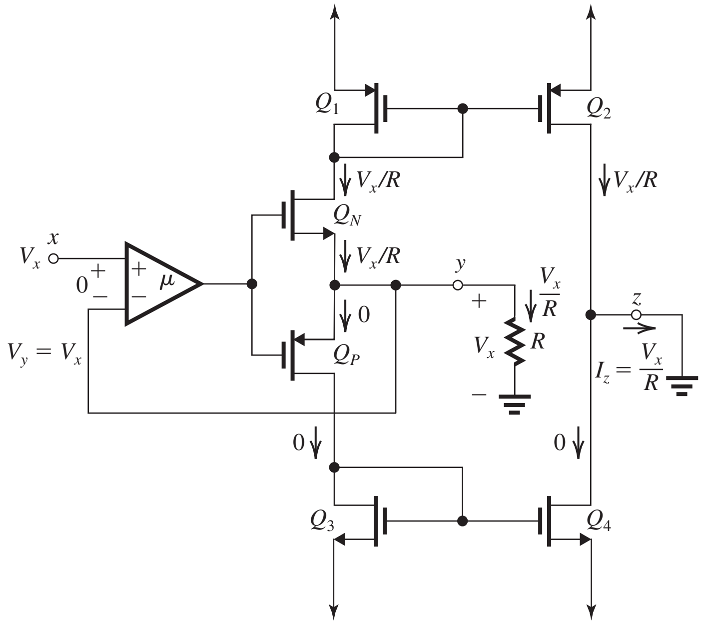
Figure 1

The dc analysis is shown in Fig. 1, from which we see that

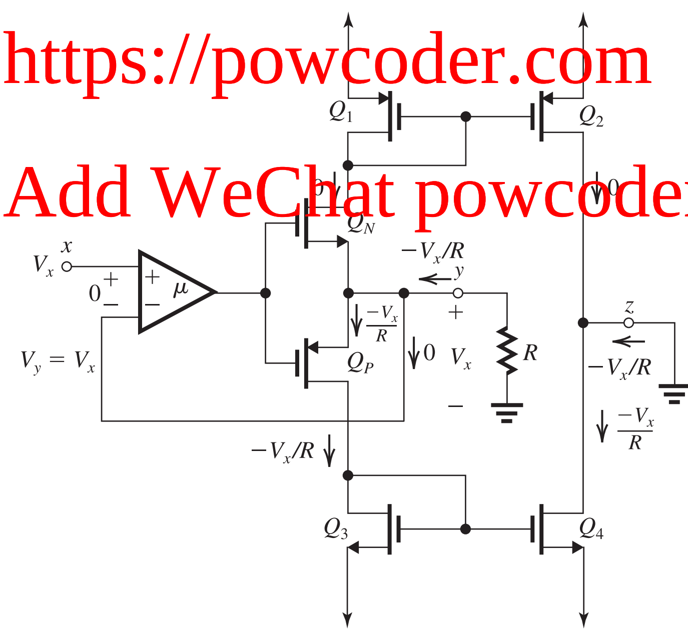
$$I_{C1} = 0.1 \text{ mA}$$

$$I_{C2} = 10 \text{ mA}$$

These figures belong to Problem 11.80.



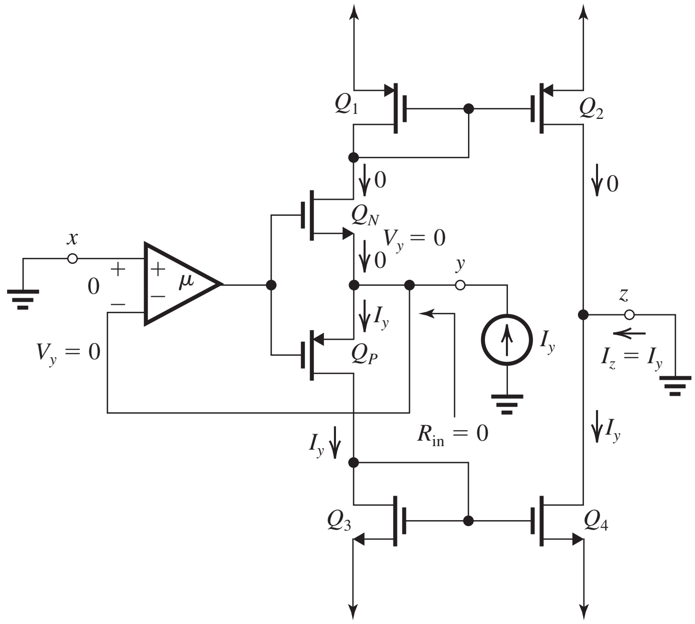
(a) V_x positive



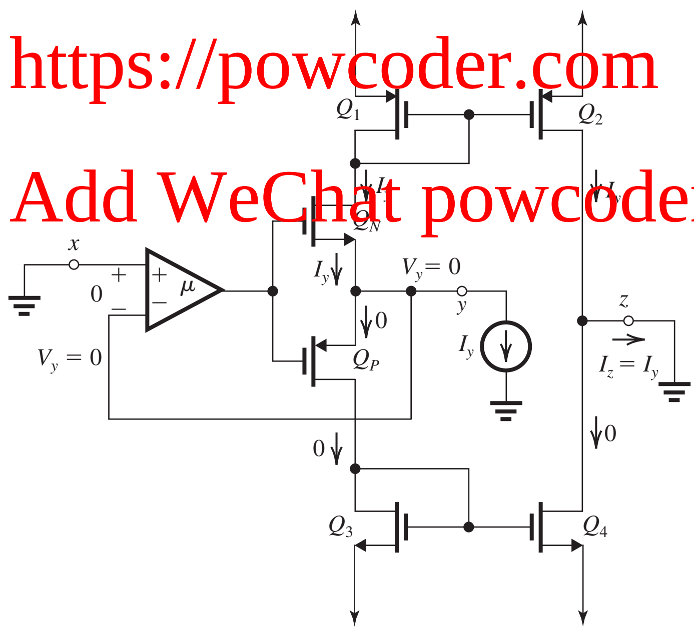
(a) V_x negative

Figure 1

These figures belong to Problem 11.80.



(b) I_y positive



(b) I_y negative

Figure 2

These figures belong to Problem 11.81.

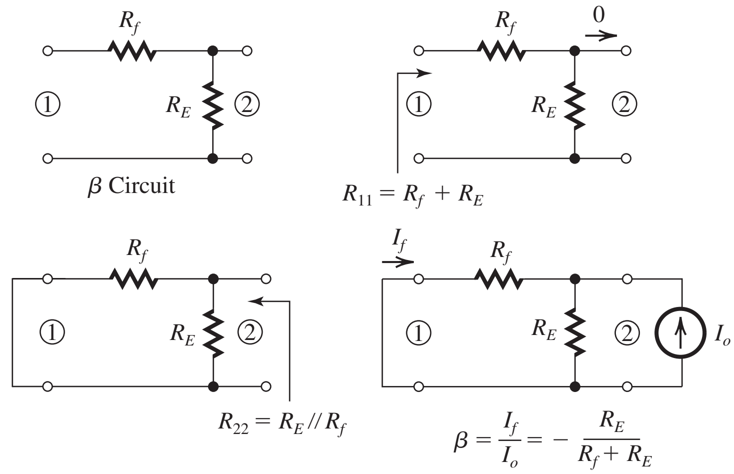


Figure 2

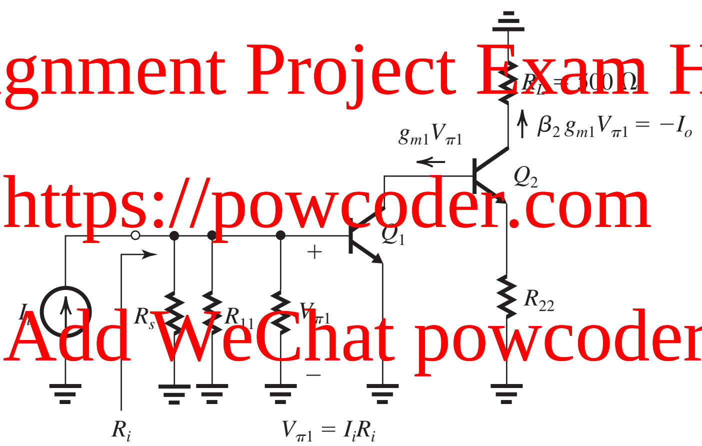


Figure 3

Thus,

$g_{m1} = 4$ mA/V

$r_{\pi 1} = 25$ kΩ

$g_{m2} = 400$ mA/V

The β circuit is shown in Fig. 2 together with the determination of its loading effects and of β.

$R_{11} = R_f + R_E = 10.14$ kΩ

$R_{22} = R_f \parallel R_E = 10 \parallel 0.14 = 0.138$ kΩ

$$\beta = -\frac{R_E}{R_f + R_E} = -\frac{0.14}{10 + 0.4} = 0.0138 \text{ A/A}$$

The A circuit is shown in Fig. 3.

$R_i = R_s \parallel R_{11} \parallel r_{\pi 1}$

$= 10 \parallel 10.14 \parallel 25$

$= 4.19$ kΩ

$V_{\pi 1} = I_i R_i$

$I_o = -\beta_2 g_{m1} V_{\pi 1}$

$$\Rightarrow A = \frac{I_o}{I_i} = -\beta_2 g_{m1} R_i$$

$A = -100 \times 4 \times 4.19 = -1676$ A/A

$A\beta = -1676 \times -0.0138 = 23.13$

$1 + A\beta = 24.13$

$$A_f = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}$$

where

$$I_s = \frac{V_s}{R_s}$$

$$A_f = -\frac{1676}{24.13} = -72.5 \text{ A/A}$$

$$\frac{V_o}{V_s} = \frac{-I_o R_L}{-I_s R_s} = 72.5 \times \frac{0.5}{10} = 3.62 \text{ V/V}$$

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$= \frac{4190\ \Omega}{24.13} = 173.6\ \Omega$$

$$R_{if} = R_s \parallel R_{\text{in}}$$

$$173.6\ \Omega = 10,000\ \Omega \parallel R_{\text{in}}$$

$$\Rightarrow R_{\text{in}} = 176.7\ \Omega$$

11.82 (a) Refer to the circuit in Fig. P11.82. Observe that the feedback signal is capacitively coupled and so are the signal source and R_L; thus, these do not enter into the dc bias calculations and the feedback does not affect the bias. The dc emitter current in Q_1 can be determined from

$$I_{E1} = \frac{12 \times \dfrac{15}{100 + 15} - 0.7}{0.870 + \dfrac{100 \parallel 15}{101}} = 0.865 \text{ mA}$$

$$I_{C1} = 0.99 \times 0.865 = 0.86 \text{ mA}$$

Next consider Q_2 and let its emitter current be I_{E2}. The base current of Q_2 will be $I_{E2}/(\beta + 1) \simeq 0.01\ I_{E2}$. The current through R_{C1} will be $(I_{C1} + I_{B2}) = (0.86 + 0.01\ I_{E2})$. We can thus write the following equation:

$$12 = (0.86 + 0.01\ I_{E2}) \times 10 + 0.7 + 3.4 \times I_{E2}$$

$$\Rightarrow I_{E2} = \frac{12 - 8.6 - 0.7}{3.4 + 0.1} = \frac{2.7}{3.5} = 0.77 \text{ mA}$$

$$I_{C2} = 0.76 \text{ mA}$$

The small-signal parameters of Q_1 and Q_2 can now be obtained as

$$g_{m1} = \frac{0.86}{0.025} = 34.4 \text{ mA/V}$$

$$r_{\pi 1} = \frac{100}{34.4} = 2.91 \text{ k}\Omega$$

$$g_{m2} = \frac{0.76}{0.025} = 30.4 \text{ mA/V}$$

$$r_{\pi 2} = \frac{100}{30.4} = 3.3 \text{ k}\Omega$$

(b) The equivalent circuit of the feedback amplifier is shown in Fig. 1, where

$$R_s = 10 \text{ k}\Omega$$

$$I_s = \frac{V_s}{R_s}$$

$$R_B = R_{B1} \parallel R_{B2} = 13 \text{ k}\Omega$$

(c) See figure on the next page. The determination of the loading effects of the β circuit on the A circuit is shown in Fig. 2:

$$R_{11} = R_f + R_{E2} = 10 + 3.4 = 13.4 \text{ k}\Omega$$

$$R_{22} = R_{E2} \parallel R_f = 3.4 \parallel 10 = 2.54 \text{ k}\Omega$$

The A circuit is shown in Fig. 3 on the next page.

Analysis of the A circuit to determine $A \equiv I_o/I_s$ proceeds as follows:

$$R_i = R_s \parallel R_{11} \parallel R_B \parallel r_{\pi 1}$$

$$= 10 \parallel 13.4 \parallel 13 \parallel 2.91 = 1.68 \text{ k}\Omega$$

$$V_{\pi 1} = I_i R_i \tag{1}$$

$$I_{b2} = -g_{m1} V_{\pi 1} \frac{R_{C1}}{R_{C1} + r_{\pi 2} + (\beta + 1) R_{22}} \tag{2}$$

$$I_o = I_{e2} = (\beta + 1) I_{b2} \tag{3}$$

This figure belongs to Problem 11.82, part (b).

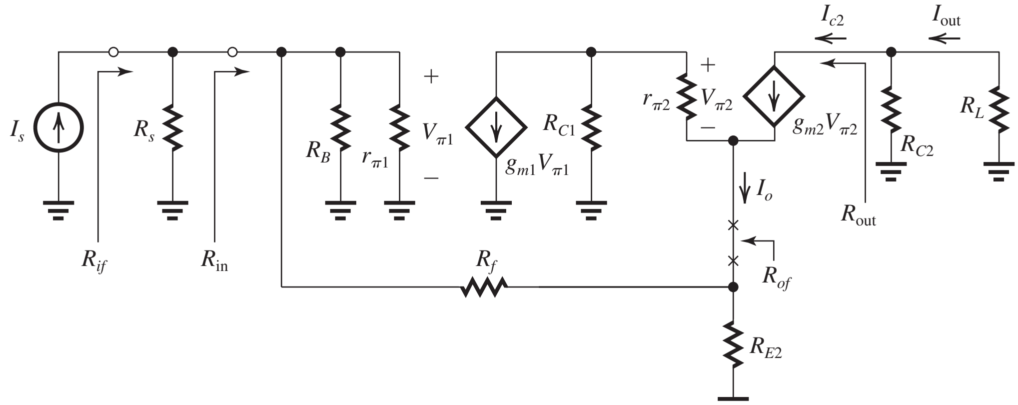


Figure 1

These figures belong to Problem 11.82, part (c).

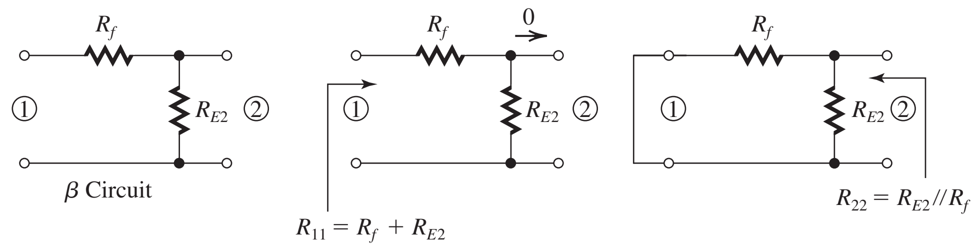


Figure 2

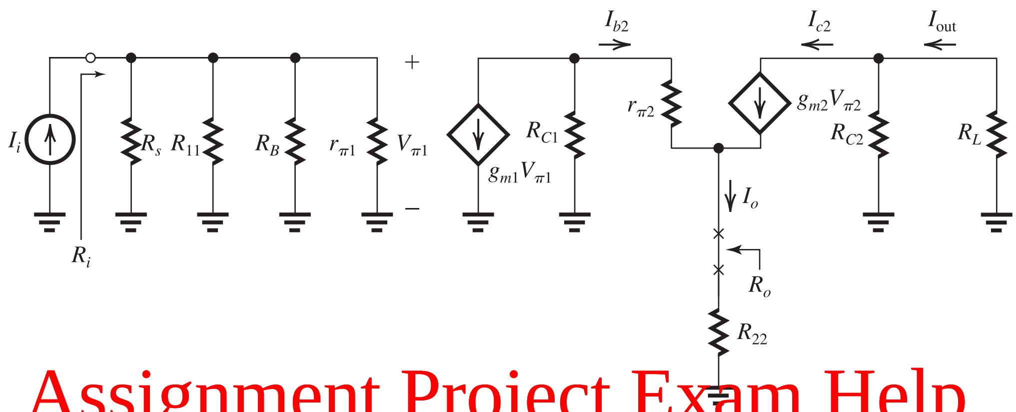


Figure 3

Combining Eqs. (1)–(3) results in

$$A \equiv \frac{I_o}{I_i} = -\frac{(\beta+1)R_i g_{m1} R_{C1}}{R_{C1} + r_{\pi 2} + (\beta+1)R_{22}}$$

$$= \frac{101 \times 1.68 \times 34.4 \times 10}{10 + 3.3 + 101 \times 2.54}$$

$$= -216.3 \text{ A/A}$$

Breaking the emitter loop of Q_2 at XX gives

$$R_o = R_{22} + \frac{r_{\pi 2} + R_{C1}}{\beta + 1}$$

$$= 2.54 + \frac{3.3 + 10}{101} = 2.67 \text{ k}\Omega$$

(d)

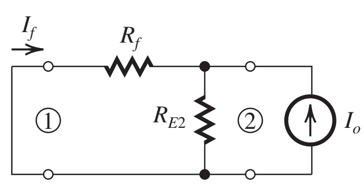


Figure 4

$$\beta \equiv \frac{I_f}{I_o} = -\frac{R_{E2}}{R_{E2} + R_f}$$

$$= -0.254 \text{ A/A}$$

(e) $A\beta = -216.3 \times -0.254 = 54.88$

$1 + A\beta = 55.88$

$$A_f = \frac{I_o}{I_s} = -\frac{216.3}{55.88} = -3.87 \text{ A/A}$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{1.68 \text{ k}\Omega}{55.88} = 30.1 \ \Omega$$

$R_{of} = R_o(1 + A\beta) = 2.67 \times 55.88$

$= 149.2 \text{ k}\Omega$

(f) $R_{if} = R_s \parallel R_{in}$

$30.1 \ \Omega = 10 \text{ k}\Omega \parallel R_{in}$

$R_{in} = 30.2 \ \Omega$

$$I_{in} = I_s \frac{R_s}{R_s + R_{in}} \simeq I_s$$

$$I_{out} = I_{C2} \frac{R_{C2}}{R_{C2} + R_L} = \alpha I_o \frac{R_{C2}}{R_{C2} + R_L}$$

$$\frac{I_{out}}{I_{in}} \simeq \frac{I_{out}}{I_s} = \frac{I_o}{I_s} \times \alpha \frac{R_{C2}}{R_{C2} + R_L}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = -3.87 \times 0.99 \times 0.99 \times \frac{8}{8+1}$$

$$= -3.41 \text{ A/A}$$

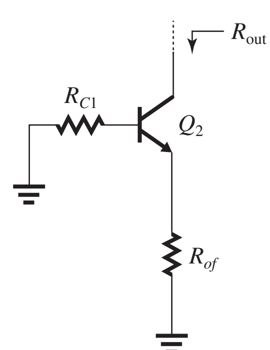


Figure 5

To determine R_{out}, consider the circuit in Fig. 5. Using the formula given at the end of Example 11.8, adapted to our case here, we get

$$R_{\text{out}} = r_{o2} + [R_{of} \parallel (r_{\pi 2} + R_{C1})]\left[1 + g_{m2} r_{o2} \frac{r_{\pi 2}}{r_{\pi 2} + R_{C1}}\right]$$

where

$$r_{o2} = \frac{75 \text{ V}}{0.76 \text{ mA}} = 98.7 \text{ k}\Omega$$

$$R_{\text{out}} = 98.7 + [149.2 \parallel (3.3 + 10)]\left[1 + 30.3 \times 98.7 \times \frac{3.3}{3.3 + 10}\right]$$

$$= 98.7 + 12.21 \times 743 = 9.17 \text{ M}\Omega$$

Check:

Maximum possible $R_{\text{out}} \simeq \beta r_o \simeq 10 \text{ M}\Omega$

So, our result is reasonable.

At $\omega = \omega_{180}$, the magnitude of A becomes

$$|A(j\omega_{180})| = \frac{10^5}{\sqrt{\left[1 + \left(\frac{20,000}{100}\right)^2\right]}\left[1 + \left(\frac{20,000}{20,000}\right)^2\right]^2}$$

$$|A(j\omega_{180})| = \frac{10^5}{200 \times 2} = 250 \text{ V/V}$$

$$|A(j\omega_{180})|\beta_{cr} = 1$$

$$\beta_{cr} = \frac{1}{250} = 4 \times 10^{-3} \text{ V/V}$$

Correspondingly,

$$A_f = \frac{10^5}{1 + 10^5 \times 4 \times 10^{-3}}$$

$$= \frac{10^5}{1 + 400} \simeq 250 \text{ V/V}$$

11.83 $A(s) = \dfrac{10^5}{\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{20,000}\right)^2}$

$$\phi = -\tan^{-1}\frac{\omega}{100} - 2\tan^{-1}\frac{\omega}{20,000}$$

$$180^\circ = \tan^{-1}\frac{\omega_{180}}{100} + 2\tan^{-1}\frac{\omega_{180}}{20,000}$$

Since ω_{180} will be much greater than 100 rad/s, we can assume that at ω_{180}, $\tan^{-1}(\omega_{180/100})$ is approximately 90°, thus

$$2\tan^{-1}\frac{\omega_{180}}{20,000} = 90^\circ$$

$$\Rightarrow \tan^{-1}\frac{\omega_{180}}{20,000} = 45^\circ$$

$$\Rightarrow \omega_{180} = 20,000 \text{ rad/s}$$

which is indeed much greater than 100 rad/s, justifying our original assumption.

11.84 $A(s) = \dfrac{10^5}{\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{20,000}\right)^2}$

$$A(j\omega) = \frac{10^5}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{20,000}\right)^2}$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{100}\right) - 2\tan^{-1}\left(\frac{\omega}{20,000}\right) \quad (1)$$

$$|A(j\omega)| = \frac{10^5}{\sqrt{1 + \left(\frac{\omega}{100}\right)^2}\left[1 + \left(\frac{\omega}{20,000}\right)^2\right]} \quad (2)$$

Using Eqs. (1) and (2), we can obtain the data required to construct Nyquist plots for the two cases: $\beta = 1$ and $\beta = 10^{-3}$. The results are given in the following table.

This table belongs to Problem 11.84.

ω rad/s	$-\tan^{-1}\left(\frac{\omega}{100}\right)$	$-2\tan^{-1}\left(\frac{\omega}{20,000}\right)$	ϕ	$\lvert A\rvert$	$\lvert A\beta\rvert$ $\beta = 1$	$\lvert A\beta\rvert$ $\beta = 10^{-3}$
0	0	0	0	10^5	10^5	100
10^2	-45°	-0.6°	-45.6°	0.7×10^5	0.7×10^5	70
10^3	-84.3°	-5.7°	-90°	10^4	10^4	10
10^4	-89.4°	-53.1°	-142.5°	800	800	0.8
2×10^4	-89.7°	-90°	$\simeq -180^\circ$	250	250	0.25
∞	-90°	-180°	-270°	0	0	0

This figure belongs to Problem 11.84.

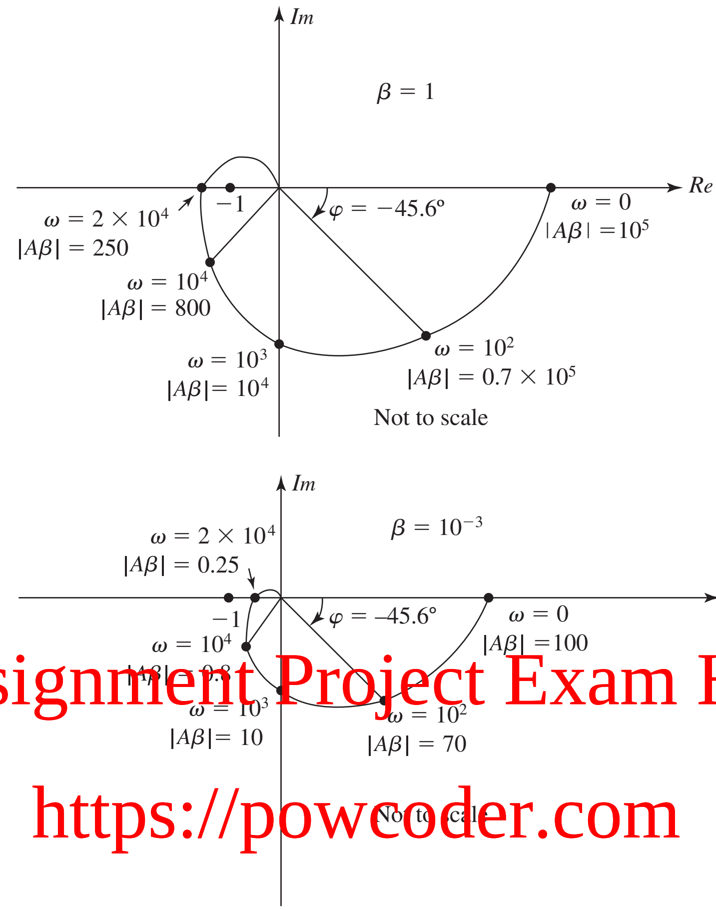


Figure 1

Using these data, we obtain the two Nyquist plots shown in Fig. 1.

We observe that the amplifier with $\beta = 1$ will be unstable and that with $\beta = 10^{-3}$ will be stable.

11.85 $A(s)\beta(s) = \dfrac{10^4 k}{\left(1 + \dfrac{s}{10^3}\right)^3}$

$$A(j\omega)\beta(j\omega) = \frac{10^4 k}{\left(1 + j\dfrac{\omega}{10^3}\right)^3}$$

$$|A(j\omega)\beta(j\omega)| = \frac{10^4 k}{\left(1 + \dfrac{\omega^2}{10^6}\right)^{3/2}}$$

$$\phi(\omega) = -3\tan^{-1}\left(\frac{\omega}{10^3}\right)$$

$$-180° = -3\tan^{-1}\frac{\omega_{180}}{10^3}$$

$$\Rightarrow \omega_{180} = 10^3 \tan 60 = 1732 \text{ rad/s}$$

$$|A(j\omega_{180})\beta(j\omega_{180})| = \frac{10^4 k}{\left(1 + \dfrac{1732^2}{10^6}\right)^{3/2}}$$

$$= 0.125 \times 10^4 k$$

For stable operation,

$$|A(j\omega_{180})\beta(j\omega_{180})| < 1$$

$$\Rightarrow 0.125 \times 10^4 k < 1$$

$$k < 8 \times 10^{-4}$$

11.86 $A(s) = \dfrac{10^4}{\left(1 + \dfrac{s}{10^4}\right)\left(1 + \dfrac{s}{10^5}\right)^2}$

$$A(j\omega) = \frac{10^4}{\left(1 + j\dfrac{\omega}{10^4}\right)\left(1 + j\dfrac{\omega}{10^5}\right)^2}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

$$-180 = -\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega_{180}}{10^5}\right)$$

By trial and error we find

$\omega_{180} = 1.095 \times 10^5$ rad/s

At this frequency,

$$|A| = \frac{10^4}{\sqrt{1 + 10.95^2}\sqrt{1 + 1.095^2}}$$

$= 413.6$

For stable operation,

$|A|\beta_{cr} < 1$

$\beta_{cr} < 2.42 \times 10^{-3}$

Thus, oscillation will commence for

$\beta \geq 2.42 \times 10^{-3}$

11.87 $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

where

$$A_0 = \frac{1\text{ MHz}}{10\text{ Hz}} = 10^5\text{ V/V}$$

Thus,

$$A_f(0) = \frac{10^5}{1 + 10^5 \times 0.1} \simeq 10\text{ V/V}$$

$f_{3dB} = 10(1 + A_0\beta)$

$= 10(1 + 10^5 \times 0.1) \simeq 10^5$ Hz

Unity-gain frequency of closed-loop amplifier

$= A_f(0) \times f_{3dB}$

$= 10 \times 10^5 = 10^6$ Hz $= 1$ MHz

Thus, the pole shifts by a factor equal to the amount-of-feedback, $(1 + A_0\beta)$.

11.88 $A_0 = 10$ V/V

$f_P = 100$ Hz

$f_{Hf} = f_P(1 + A_0\beta) = 10$ kHz

$$\Rightarrow 1 + A_0\beta = \frac{10 \times 10^3}{100} = 100$$

$$\Rightarrow \beta = \frac{99}{10^4} = 0.0099\text{ V/V}$$

$$A_f(0) = \frac{A_0}{1 + A_0\beta}$$

$$= \frac{10^4}{100} = 100\text{ V/V}$$

$$A_f(s) = \frac{A_f(0)}{1 + \dfrac{s}{\omega_{Hf}}}$$

$$A_f(s) = \frac{100}{1 + s/2\pi \times 10^4}$$

11.89 (a) The closed-loop poles become coincident when $Q = 0.5$. Using Eq. (11.70), we obtain

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$0.5 = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

$$\Rightarrow 1 + A_0\beta = 0.5^2\,\frac{(\omega_{P1} + \omega_{P2})^2}{\omega_{P1}\omega_{P2}}$$

$$= 0.5^2 \times \frac{(2\pi)^2(10^4 + 10^5)^2}{(2\pi)^2 \times 10^4 \times 10^5}$$

$$= 0.5^2 \times \frac{11^2}{10} = 3.025$$

$\beta = 2.025 \times 10^{-4}$

$$\omega_c = \frac{1}{2}(\omega_{P1} + \omega_{P2})$$

$$= \frac{1}{2} \times 2\pi(f_{P1} + f_{P2})$$

$$f_c = \frac{1}{2} \times (10^4 + 10^5) = 5.5 \times 10^4\text{ Hz}$$

(b) $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

$$= \frac{10^4}{1 + 2.205 \times 10^{-4} \times 10^4} = 3306\text{ V/V}$$

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta}$$

where

$$A(s) = \frac{A_0}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right)}$$

$$A_f(s) = \frac{A_0}{\left(1 + \dfrac{s}{\omega_{P1}}\right)\left(1 + \dfrac{s}{\omega_{P2}}\right) + A_0\beta}$$

$$= \frac{A_0}{(1 + A_0\beta) + s\left(\dfrac{1}{\omega_{P1}} + \dfrac{1}{\omega_{P2}}\right) + \dfrac{s^2}{\omega_{P1}\omega_{P2}}}$$

$A_f(j\omega) =$

$$\frac{A_0}{(1 + A_0\beta) + j\left(\dfrac{\omega}{\omega_{P1}} + \dfrac{\omega}{\omega_{P2}}\right) - \left(\dfrac{\omega}{\omega_{P1}}\right)\left(\dfrac{\omega}{\omega_{P2}}\right)}$$

$$A_f(j\omega_c) = \frac{10^4}{3.025 + j(5.5 + 0.55) - 5.5 \times 0.55}$$

$$A_f(j\omega_c) = \frac{10^4}{j\,6.05}$$

$$|A_f|(j\omega_c) = \frac{10^4}{6.05} = 1653\text{ V/V}$$

(c) $Q = 0.5$.

(d) If $\beta = 2.025 \times 10^{-3}$ V/V. Using Eq. (11.68), we obtain

$$s = -\frac{1}{2} \times 2\pi(10^4 + 10^5)$$

$$\pm\frac{1}{2} \times$$

$$2\pi\sqrt{(10^4 + 10^5)^2 - 4(1 + 10^4 \times 2.025 \times 10^{-3}) \times 10^4 \times 10^5}$$

$$\frac{s}{2\pi} = -5.5 \times 10^4$$

$$\pm 0.5\sqrt{121 \times 10^8 - 4 \times 21.25 \times 10^9}$$

$$= -5.5 \times 10^4 \pm 0.5 \times 10^4\sqrt{121 - 40 \times 21.25}$$

$$= -5.5 \times 10^4 \pm j0.5 \times 10^4 \times 27$$

$$= (-5.5 \pm j13.25) \times 10^4 \text{ Hz}$$

Using Eq. (11.70), we obtain

$$Q = \frac{\sqrt{(1 + 10^4 \times 2.025 \times 10^{-3})10^4 \times 10^5}}{10^4 + 10^5}$$

$$= 1.325$$

11.90 $A_f(0) = \dfrac{A_0}{1 + A_0\beta}$

$$10 = \frac{1000}{1 + 1000\beta}$$

$$\Rightarrow \beta = 0.099$$

To obtain a maximally flat response,

$$Q = 0.707$$

Using Eq. (11.70), we obtain

$$0.707 = \frac{\sqrt{100 \times 1 \times f_{P2}}}{1 + f_{P2}}$$

$$\frac{1}{2} = \frac{100\,f_{P2}}{(1 + f_{P2})^2}$$

$$f_{P2}^2 + 2\,f_{P2} + 1 = 200 f_{P2}$$

$$f_{P2}^2 - 198\,f_{P2} + 1 = 0$$

$$f_{P2} \simeq 198 \text{ kHz}$$

(the other solution is a very low frequency which obviously does not make physical sense).

The 3-dB frequency of the closed-loop amplifier is f_0, which can be obtained from Eq. (11.68) and the graphical construction of Fig. 11.32:

$$\frac{f_0}{2Q} = \frac{1}{2}(f_{P1} + f_{P2})$$

$$f_0 = \frac{1}{\sqrt{2}}(1 + 198) = 140.7 \text{ kHz}$$

11.91

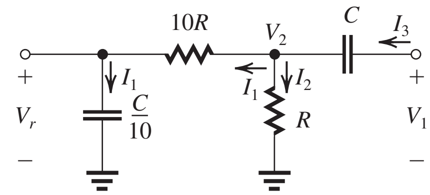


Figure 1

Figure 1 shows the feedback circuit modified according to the specifications in this problem. This circuit replaces that in the feedback path in Figs. 13.34 (a) and (b). Its transfer function $T(s)$,

$$T(s) = \frac{V_r}{V_1}$$

can be determined as indicated in Fig. 1. We start at the left-hand side where the voltage is V_r. The current through $(C/10)$ will be

$$I_1 = s\frac{C}{10}V_r$$

This is the same current that flows through $(10R)$; thus

$$V_2 = I_1 \times 10R + V_r$$

$$= s\frac{C}{10} \times 10RV_r + V_r$$

$$= V_r(1 + sCR)$$

The current I_2 through R can now be found as

$$I_2 = \frac{V_2}{R} = \frac{V_r}{R}(1 + sCR)$$

The input current I_3 is the sum of I and I_2:

$$I_3 = s\frac{C}{10}V_r + \frac{V_r}{R}(1 + sCR)$$

$$I_3 = \frac{V_r}{R}\left(1 + sCR + s\frac{C}{10}R\right)$$

$$= \frac{V_r}{R}(1 + 1.1sCR)$$

The input voltage V_1 can now be found as

$$V_1 = V_2 + \frac{1}{sC}I_3$$

$$= V_r(1 + sCR) + V_r\frac{1}{sCR}(1 + 1.1\,sCR)$$

Finally, $T(s)$ can be obtained as

$$T(s) \equiv \frac{V_r}{V_1} = \frac{s(1/CR)}{s^2 + s\dfrac{2.1}{CR} + \left(\dfrac{1}{CR}\right)^2}$$

Thus,

$$L(s) = \frac{-s(K/CR)}{s^2 + s(2.1/CR) + (1/CR)^2}$$

The characteristic equation is

$$1 + L(s) = 0$$

that is,

$$s^2 + s\frac{2.1}{CR} + \left(\frac{1}{CR}\right)^2 - s\frac{K}{CR} = 0$$

$$s^2 + s\frac{2.1 - K}{CR} + \left(\frac{1}{CR}\right)^2 = 0$$

Thus,

$$\omega_0 = \frac{1}{CR}$$

$$Q = \frac{1}{2.1 - K}$$

For the poles to coincide, $Q = 0.5$, thus

$$0.5 = \frac{1}{2.1 - K}$$

$$\Rightarrow K = 0.1$$

For the response to become maximally flat, $Q = 0.707$:

$$0.707 = \frac{1}{2.1 - K}$$

$$\Rightarrow K = 0.686$$

The circuit oscillates for $K = 2.1$

11.92 $A_f = 10$

Maximally flat response with $f_{3dB} = f_0 = 1$ kHz.

Let each stage in the cascade have a dc gain K and a 3-dB frequency f_P, thus

$$A(s) = \frac{K^2}{\left(1 + \frac{s}{\omega_P}\right)^2}$$

Using the expression for Q in Eq. (11.70), we get

$$Q = \frac{\sqrt{(1 + A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1} + \omega_{P2}}$$

and substituting $Q = 1/\sqrt{2}$, $\omega_{P1} = \omega_{P2} = \omega_P$, and $A_0 = K^2$, we obtain

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{1 + K^2\beta}}{2}$$

$$1 + K^2\beta = 2$$

and

$$K^2\beta = 1$$

Now,

$$A_{f0} = \frac{A_0}{1 + A_0\beta}$$

$$10 = \frac{K^2}{1 + K^2\beta}$$

$$= \frac{K^2}{2}$$

$$\Rightarrow K^2 = 20 \Rightarrow K = \sqrt{20} = 4.47 \text{ V/V}$$

$$\beta = \frac{1}{20} = 0.05 \text{ V/V}$$

Using Eq. (11.68), we obtain

$$\frac{\omega_0}{Q} = \frac{1}{2}(\omega_{P1} + \omega_{P2})$$

$$\frac{f_0}{1/\sqrt{2}} = \frac{1}{2} \times 2\,f_P$$

$$\Rightarrow f_P = \sqrt{2}f_0 = \sqrt{2} \times 1 = 1.414 \text{ kHz}$$

$$A_f(s) = \frac{10\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$A_f(s) = \frac{10(2\pi \times 10^3)^2}{s^2 + s\frac{2\pi \times 10^3}{1/\sqrt{2}} + (2\pi \times 10^3)^2}$$

11.93 Let each stage have the transfer function

$$T(s) = \frac{K}{1 + \frac{s}{\omega_P}}$$

where $\omega_P = 2\pi \times 100 \times 10^3$ rad/s

$$A(s) = \left(\frac{K}{1 + \frac{s}{\omega_P}}\right)^3$$

$$\beta = 1$$

Thus the characteristic equation is given by

$$1 + A(s)\beta = 0$$

$$1 + \frac{K^3}{\left(1 + \frac{s}{\omega_P}\right)^3} = 0$$

To simplify matters, let $\frac{s}{\omega_P} = S$, where S is a normalized frequency variable, thus

$$(1 + S)^3 + K^3 = 0 \qquad (1)$$

This equation has three roots, which are the poles of the feedback amplifier. One of the roots will be real and the other two can be complex conjugate depending on the value of K. The real pole can be directly obtained from Eq. (1) as

$$(1 + S_1)^3 = -K^3$$

$$(1 + S_1) = -K$$

$$S_1 = -(1 + K) \qquad (2)$$

Now we need to obtain the two other poles. The characteristic equation in (1) can be written as

$$S^3 + 3\,S^2 + 3\,S + (1 + K^3) = 0 \qquad (3)$$

Equivalently it can be written as

$$(S + 1 + K)(S^2 + aS + b) = 0 \qquad (4)$$

Equating the coefficients of corresponding terms in (3) and (4), we can find a and b and thus obtain the quadratic factor

$$S^2 + (2 - K)S + (1 - K + K^2) = 0 \qquad (5)$$

This equation can now be easily solved to obtain the pair of complex conjugate poles as

$$S_{2,3} = -1 + \frac{K}{2} \pm j\frac{\sqrt{3}}{2}K$$

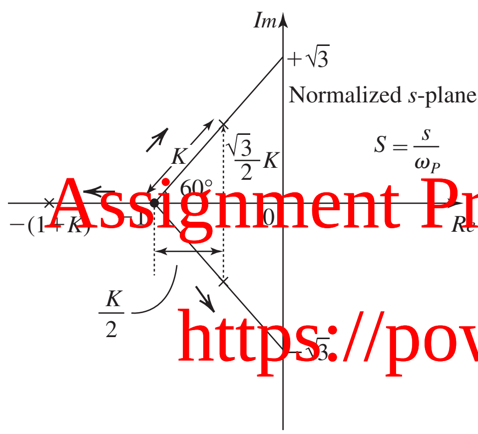


Figure 1

Figure 1 shows the root locus of the three poles in the normalized s plane (normalized relative to $\omega_P = 2\pi \times 10^5$). Observe that as K increases the real pole S_1 moves outwardly on the negative real axis. The pair of conjugate poles move on straight lines with 60° angles to the horizontal. These two poles reach the $j\omega$ axis (which is the boundary for stable operation) at $K = 2$ at which point

$$S_{2,3} = \pm j\sqrt{3}$$

Thus the minimum value of K from which oscillations occur is $K = 2$. Oscillations will be at

$$\omega = \sqrt{3} \times 2\pi f_P$$

or

$$f = \sqrt{3} \times 100 \text{ kHz}$$

$$= 173.2 \text{ kHz}$$

11.94 $A_0 = 10^5$ with a single pole at

$$f_P = 10 \text{ Hz}$$

For a unity-gain buffer, $\beta = 1$, thus

$$A_0\beta = 10^5 \text{ and } f_P = 10 \text{ Hz}$$

Since the loop gain rolls off at a uniform slope of -20 dB/decade, it will reach the 0 dB line ($|A\beta| = 1$) five decades beyond 10 Hz. Thus the unity-gain frequency will be

$$f_1 = 10^5 \times 10 = 10^6 \text{ Hz} = 1 \text{ MHz}$$

The phase shift will be that resulting from a single pole, $-90°$, resulting in a phase margin:

$$\text{Phase margin} = 180° - 90° = 90°$$

11.95

$$A(s) = \frac{10^5}{\left(1 + \frac{s}{2\pi \times 10}\right)\left(1 + \frac{s}{2\pi \times 10^3}\right)}$$

$$\beta = 0.01$$

$$A\beta(j\omega) = \frac{1000}{\left(1 + j\frac{\omega}{2\pi \times 10}\right)\left(1 + j\frac{\omega}{2\pi \times 10^3}\right)} \qquad (1)$$

$$|A\beta| = \frac{1000}{\sqrt{\left[1 + \left(\frac{\omega}{2\pi \times 10}\right)^2\right]\left[1 + \left(\frac{\omega}{2\pi \times 10^3}\right)^2\right]}} \qquad (2)$$

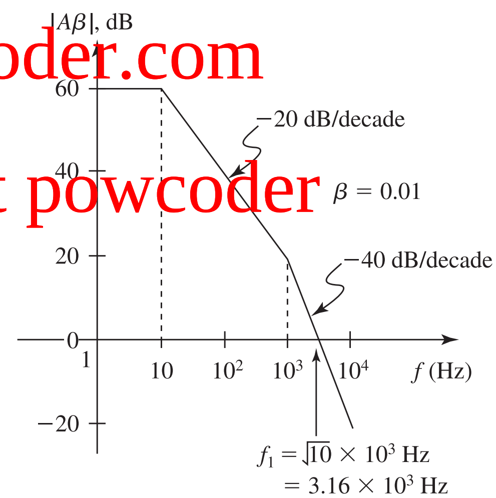


Figure 1

Figure 1 shows a sketch of the Bode plot for $|A\beta|$. Observe that the unity-gain frequency will occur on the -40 dB/decade line. The value of f_1 can be obtained from the Bode plot as follows: The -40 dB/decade line represents gain drop proportional to $10^3/f^2$. For a drop by only 20 dB (a factor of 10) the change in frequency is

$$\frac{10^3}{f_1^2} = \frac{1}{10}$$

$$f_1 = \sqrt{10} \times 10^3 \text{ Hz} = 3.16 \times 10^3 \text{ Hz}$$

However, the Bode plot results are usually approximate. If we require a more exact value for

f_1 we need to iterate a couple of times using the exact equation in (2). The result is

$f_1 = 3.085 \times 10^3$ Hz

The phase angle can now be obtained using (1) as follows:

$$\phi = -\tan^{-1}\frac{\omega_1}{2\pi \times 10} - \tan^{-1}\frac{\omega_1}{2\pi \times 10^3}$$

$$= -\tan^{-1}\frac{f_1}{10} - \tan^{-1}\frac{f_1}{10^3}$$

$$= -\tan^{-1}(3.085 \times 10^2) - \tan^{-1}(3.085)$$

$$= -89.81^\circ - 72.03^\circ = -161.84$$

Thus,

Phase margin $= 180^\circ - 161.84 = 18.15^\circ$

To obtain a phase margin of 45°:

The phase shift due to the first pole will be $\simeq 90^\circ$. Thus, the phase shift due to the second pole must be $\simeq -45^\circ$, thus

$$-45^\circ = -\tan^{-1}\frac{f_1}{10^3}$$

$f_1 \simeq 10^3$ rad/s.

Since f_1 is two decades above f_P we need $A_0\beta$ to be 100. Thus, β will now be

$\beta = 100/10^5 = 10^{-3}$

Figure 2 shows a sketch of the Bode plot for $|A\beta|$ in this case.

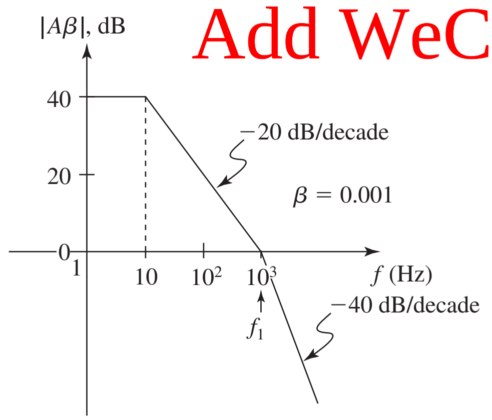


Figure 2

11.96 Using Eq. (11.82), we obtain

$$|A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|}$$

where

$\theta = 180^\circ - \phi$

$\phi \equiv$ Phase margin

Peaking, $P \equiv \dfrac{|A_f(j\omega)|}{1/\beta}$

Thus,

$$P = 1/|1 + e^{-j\theta}|$$

$$= 1/|1 + \cos\theta - j\sin\theta|$$

$$= 1/\sqrt{(1+\cos\theta)^2 + \sin^2\theta}$$

$$= 1/\sqrt{2 + 2\cos\theta}$$

$$= 1/\sqrt{2 + 2\cos(180^\circ - \phi)}$$

$$P = 1/\sqrt{2(1 - \cos\phi)}$$

$$\Rightarrow \phi = \cos^{-1}\left(1 - \frac{1}{2P^2}\right)$$

We use this equation to obtain the following results:

P	ϕ
1.05	56.9°
1.10	54.1°
0.1 dB ≡ 1.0115	59.2°
1.0 dB ≡ 1.122	52.9°
3 dB ≡ 1.414	41.4°

11.97 Figure 1 on the next page shows magnitude and phase Bode plots for the amplifier specified in this problem. From the phase plot we find that $\theta = -135^\circ$ (which corresponds to a phase margin of 45°) occurs at

$f = 3.16 \times 10^5$ Hz

At this frequency, $|A|$ is 70 dB. The β horizontal straight line drawn at 70-dB level gives

$$20\log\left(\frac{1}{\beta}\right) = 70 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-4}$$

Correspondingly,

$$A_f = \frac{10^4}{1 + 10^4 \times 3.16 \times 10^{-4}} = 2.4 \times 10^3 \text{ V/V}$$

or 67.6 dB

11.98 Figure 1 on the next page shows the Bode plot for the amplifier gain and for a differentiator. Observe that following the rate-of-closure rule the intersection of the two graphs is arranged so that the maximum difference in slopes is 20 dB/decade.

This figure belongs to Problem 11.97.

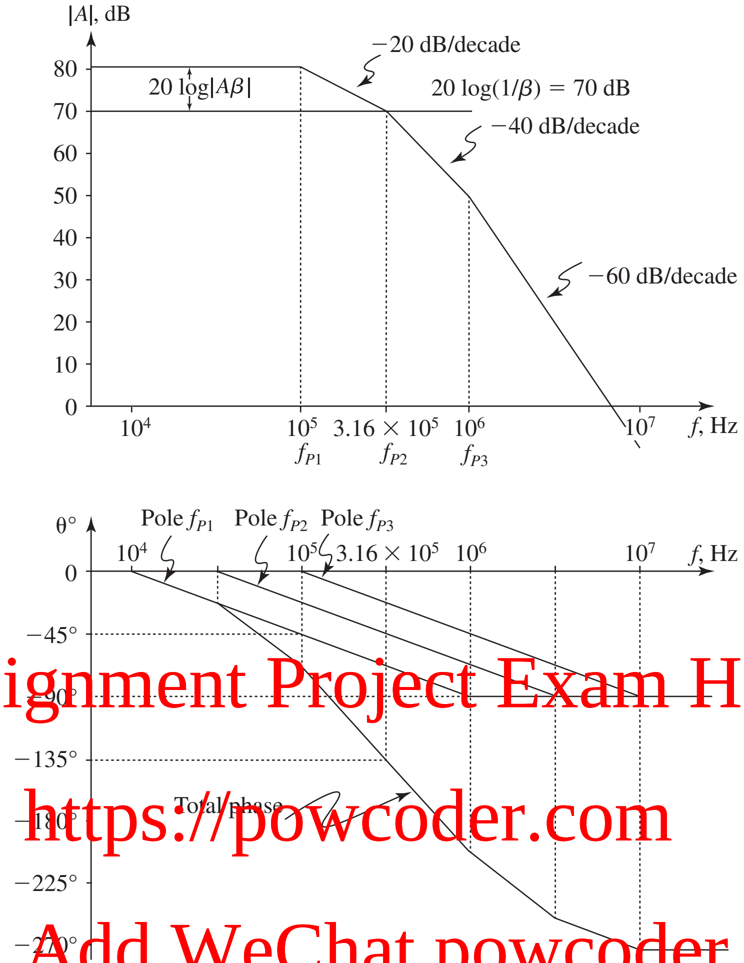


Figure 1

This figure belongs to Problem 11.98.

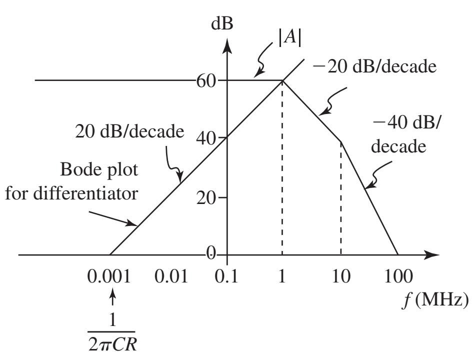


Figure 1

Thus, to guarantee stability,

$$\frac{1}{2\pi CR} \leq 0.001 \text{ MHz or } 1 \text{ kHz}$$

$$\Rightarrow CR \geq \frac{1}{2\pi \times 10^3} = 0.159 \text{ ms}$$

11.99 Figure 1 is a replica of Fig. 11.37 except here we locate on the phase plot the points at which the phase margin is 90° and 135°, respectively. Drawing a vertical line from each of those points and locating the intersection with the $|A|$ line enables us to determine the maximum β that can be used in each case. Thus, for PM = 90°, we have

$$20 \log \frac{1}{\beta} = 90 \text{ dB}$$

$$\Rightarrow \beta = 3.16 \times 10^{-5}$$

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 3.16 \times 10^{-5}}$$

$$= 2.4 \times 10^4 \text{ V/Vor } 87.6 \text{ dB}$$

and for PM = 45°, we have

$$20 \log \frac{1}{\beta} = 80 \text{ dB}$$

$$\Rightarrow \beta = 10^{-4} \text{ V/V}$$

This figure belongs to Problem 11.99.

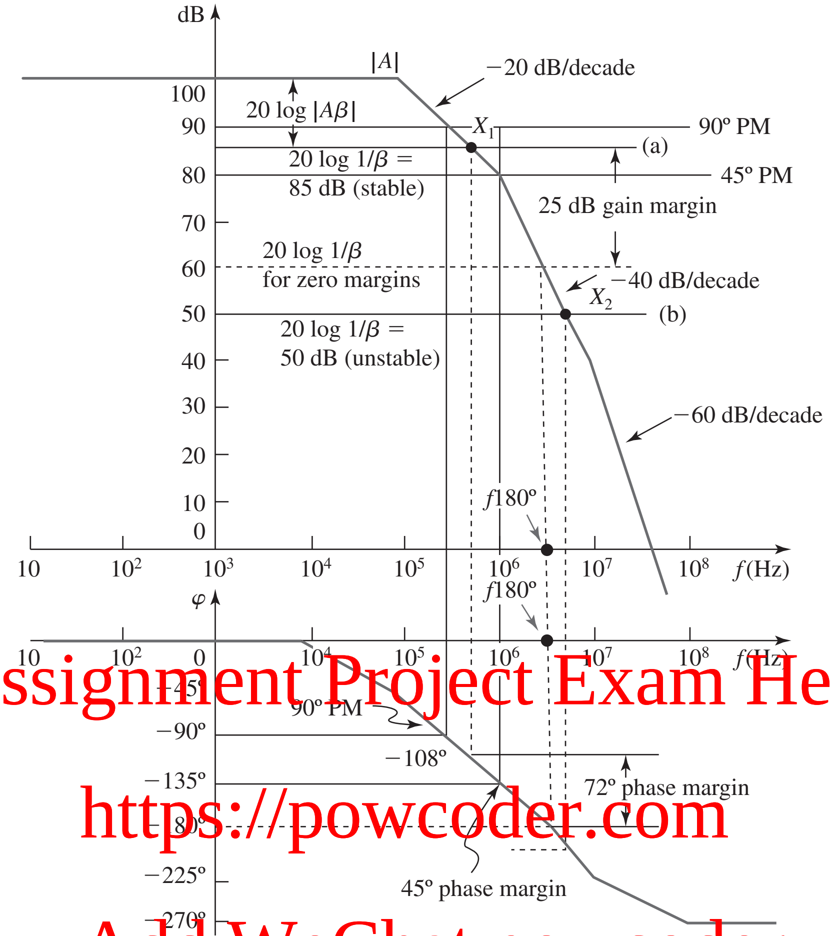


Figure 1

and the corresponding closed-loop gain is

$$A_f = \frac{A_0}{1 + A_0\beta} = \frac{10^5}{1 + 10^5 \times 10^{-4}}$$

$= 9.09 \times 10^3$ V/V

or 79.2 dB

11.100 The new pole must be placed at

$$f_D = \frac{1 \text{ MHz}}{10^4} = 100 \text{ Hz}$$

In this way the modified open-loop gain will decrease at the uniform rate of −20 dB/decade, thus reaching 0 dB in four decades, that is at 1 MHz where the original pole exists. At 1 MHz, the slope changes to −40 dB/decade, but our amplifier will be guaranteed to be stable with a closed-loop gain at low as unity.

11.101 We must move the 1 MHz pole to a new location,

$$f_D = \frac{20 \text{ MHz}}{10^4} = 2 \text{ kHz}$$

This reduction in frequency by a factor of

$\dfrac{1 \text{ MHz}}{2 \text{ kHz}} = 500$ will require that the total capacitance at the controlling node must become 500 times what it originally was.

11.102 Refer to Fig. 11.38.

(a) For $\beta = 0.001$,

$$20 \log \frac{1}{\beta} = 60 \text{ dB}$$

A horizontal line at the 60-dB level will intersect the vertical line at $f_{P2} = 10^6$ Hz at a point Z_1. Drawing a line with a slope of −20 dB/decade from Z_1 will intersect the 100-dB horizontal line at a frequency two decade lower than f_{P2}, thus the frequency to which the 1st pole must be moved is

$$f_D' = \frac{f_{P2}}{100} = \frac{10^6}{100} = 10 \text{ kHz}$$

(b) For $\beta = 0.1$,

$$20 \log \frac{1}{\beta} = 20 \text{ dB}$$

Following a process similar to that for (a) above, the first pole must be lowered to

$$f_D' = \frac{10^6}{10^4} = 100 \text{ kHz}$$

11.103 $R_1 = R_2 = R$

$C_1 = 10C$

$C_2 = C$

$C_f \gg C$

$g_{m1}R = 100$

$$\omega_{P1} = \frac{1}{10CR} = \frac{0.1}{CR} \quad (1)$$

$$\omega_{P2} = \frac{1}{CR} \quad (2)$$

$$\omega_{P1}' = \frac{1}{g_m R C_f R} = \frac{1}{100 C_f R}$$

Thus,

$$\omega_{P1}' = \frac{0.01}{C_f R} \quad (3)$$

$$\omega_{P2}' = \frac{g_m C_f}{C_1 C_2 + C_f(C_1 + C_2)}$$

$$= \frac{g_m C_f}{10C^2 + 11CC_f} = \frac{g_m C_f}{C(10C + 11C_f)}$$

Since $C_f \gg C$, we have

$$\omega_{P2}' \simeq \frac{g_m C_f}{11CC_f} = \frac{g_m}{11C}$$

Substituting $g_m = 100/R$, we obtain

$$\omega_{P2}' = \frac{100}{11CR} \simeq \frac{10}{CR} \quad (4)$$

Equations (1)–(4) provide a summary of pole splitting: The two initial poles with frequencies $\frac{0.1}{CR}$ and $\frac{1}{CR}$, a decade apart in frequency, are split further apart. The lower frequency pole is moved to a frequency $\frac{0.01}{C_f R}$ which is more than a decade lower (because $C_f \gg C$) and the higher frequency pole is moved to a frequency $\frac{10}{CR}$ which is a decade higher. This is further illustrated in Fig. 1.

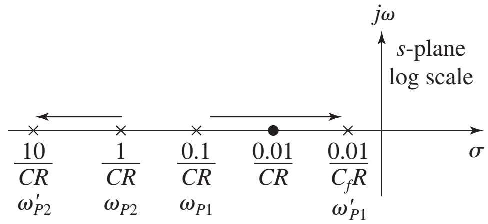


Figure 1

11.104 The fourth, dominant pole must be at

$$f_D = \frac{f_{P1}}{A_0}$$

$$= \frac{10^6}{10^5} = 10 \text{ Hz}$$

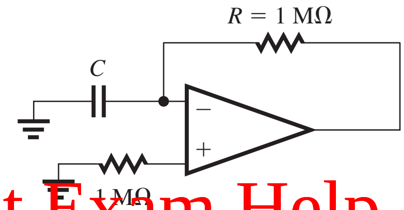


Figure 1

Refer to Fig. 1.

$$f_D = \frac{1}{2\pi RC}$$

$$10 = \frac{1}{2\pi \times 1 \times 10^6 \times C}$$

$\Rightarrow C = 15.9$ nF

11.105 $f_{P1} = \dfrac{1}{2\pi C_1 R_1}$

$$10^5 = \frac{1}{2\pi \times 150 \times 10^{-12} \times R_1}$$

$\Rightarrow R_1 = 10.61$ kΩ

$$f_{P2} = \frac{1}{2\pi C_2 R_2}$$

$$10^6 = \frac{1}{2\pi \times 5 \times 10^{-12} \times R_2}$$

$\Rightarrow R_2 = 31.83$ kΩ

First, we determine an approximate value of f_{P2}' from Eq. (11.94)

$$f_{P2}' = \frac{g_m C_f}{2\pi[C_1 C_2 + C_f(C_1 + C_2)]}$$

Assume that $C_f \gg C_2$, then

$$f_{P2}' \simeq \frac{g_m}{2\pi(C_1 + C_2)}$$

$$= \frac{40 \times 10^{-3}}{2\pi(150 + 5) \times 10^{-12}}$$

$= 41.1$ MHz

which is much greater than f_{P3}. Thus, we use f_{P3} to determine the new location of f_{P1},

$$f'_{P1} = \frac{2 \times 10^6}{10^4} = 200 \text{ Hz}$$

Using Eq. (11.93), we obtain

$$f'_{P1} = \frac{1}{2\pi g_m R_2 C_f R_1}$$

$$200 = \frac{1}{2\pi \times 40 \times 10^{-3} \times 31.83 \times 10^3 \times C_f \times 10.61 \times 10^3}$$

$$\Rightarrow C_f = 58.9 \text{ pF}$$

Since C_f is indeed much greater than C_2, the pole at the output will have the frequency already calculated:

$$f'_{P2} \simeq 41.1 \text{ MHz}$$

11.106

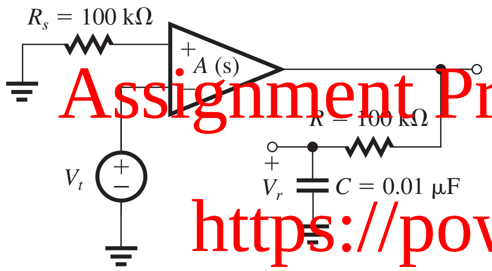


Figure 1

This figure belongs to Problem 11.106, part (a).

$$A(s)\beta(s) = -\frac{V_r}{V_t}$$

$$= A(s)\frac{1/sC}{R + 1/sC}$$

$$A(s)\beta(s) = \frac{10^5}{1 + \frac{s}{10}} \; \frac{1}{1 + sCR}$$

$$CR = 0.01 \times 10^{-6} \times 100 \times 10^3 = 10^{-3} \text{ s}$$

$$A(s)\beta(s) = \frac{10^5}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}$$

(a) Bode plots for the magnitude and phase of $A\beta$ are shown in Fig. 2. From the magnitude plot we find the frequency f_1 at which $|A\beta| = 1$ is

$$f_1 = 3.16 \times 10^4 \text{ Hz}$$

(b) From the phase plot we see that the phase at f_1 is 180° and thus the phase margin is zero. A more exact value for the phase margin can be obtained as follows:

$$\theta(f_1) = -\tan^{-1}\frac{3.16 \times 10^4}{10} - \tan^{-1}\frac{3.16 \times 10^4}{10^3}$$

$$= -89.98 - 88.19 = -178.2$$

Thus the phase margin is 1.8°

(c) $$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

$$= \frac{10^5 \Big/ \left(1 + \frac{s}{10}\right)}{1 + \dfrac{10^5}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}}$$

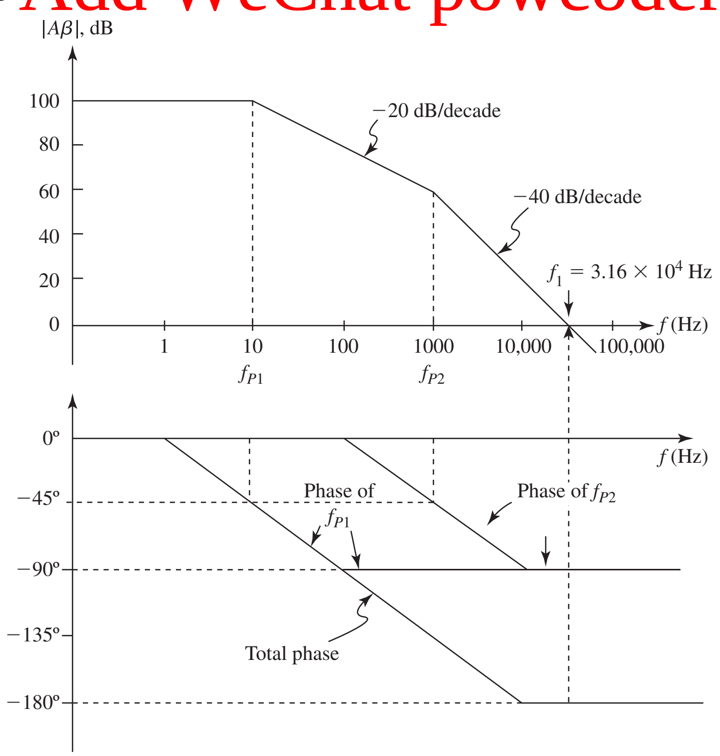


Figure 2

$$= \frac{10^5\left(1 + \frac{s}{10^3}\right)}{10^5 + \left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{10^3}\right)}$$

$$= \frac{\left(1 + \frac{s}{10^3}\right)}{1 + 10^{-5}(1 + 0.101s + 0.0001s^2)}$$

At $s = 0$,

$A_f \simeq 1$

The transmission zero is

$s_Z = -10^3$ rad/s

The poles are the roots of

$10^{-9}s^2 + 1.01 \times 10^{-6}s + 1 = 0$

which are

$s = (-0.505 \pm j31.62) \times 10^3$ rad/s

The poles and zero are shown in Fig. 3.

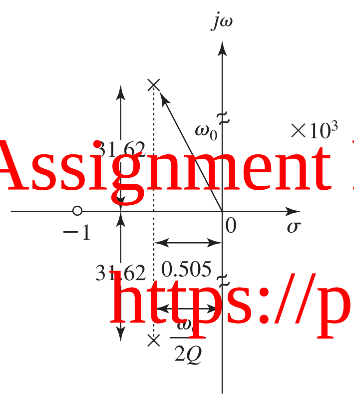


Figure 3

The pair of complex-conjugate poles have

$\omega_0 \simeq 31.62$ krad/s

$Q = 31.3$

Thus, the response is very peaky, as shown in Fig. 4.

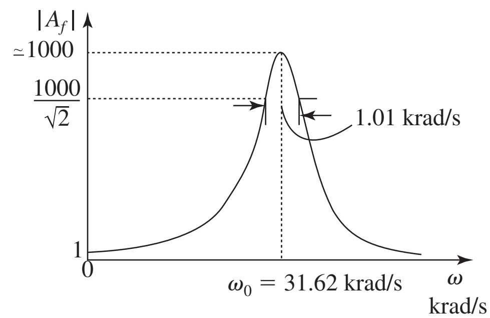


Figure 4