ECE5884 Wireless Communications

Week 9 Workshop: Diversity Techniques (Multiple-Antenna Systems)

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September 19, 2022

Course outline

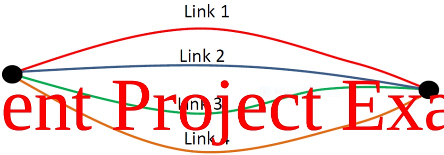
This week: **Ref. Ch. 7 of [Goldsmith, 2005]**

- Week 1: Overview of Wireless Communications
- Week 2: Wireless Channel (Path Loss and Shadowing)
- Week 3: Wireless Channel Models
- Week 4: Capacity of Wireless Channels
- Week 5: Digital Modulation and Detection
- Week 6: Performance Analysis
- Week 7: Equalization
- Week 8: Multicarrier Modulation (OFDM)
- Week 9: Multiple-Antenna Systems: Diversity Techniques
- Week 10: Multiple-Antenna Systems: MIMO Communications
- Week 11: Multiuser Systems
- Week 12: Guest Lecture (Emerging 5G/6G Technologies)

Info.

- Assignment #2 in this week
- Guest lecture on 10th Oct - Week 11 (please attend everyone!)
- More info about the final exam - closed book and testing fundamentals (must know procedures!)

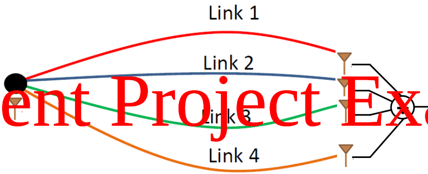
Received signal with single antenna



$$
\begin{aligned}
h_1 &= -1.22 + j0.67 &&= 1.4e^{j2.6} \\
h_2 &= 0.5 + j2.3 &&= 2.3e^{j1.4} \\
h_3 &= 1.2 - j0.7 &&= 1.4e^{-j0.5} \\
h_4 &= 0.45 - j2.2 &&= 2.2e^{-j1.4}
\end{aligned}
$$

- Only Link 1: $r_1 = h_1\, s + n_1 \Rightarrow \gamma_1 = \frac{|h_1|^2 P_s}{N_0} = 1.4^2 \bar{\gamma}$
- All Links: $r = (h_1 + h_2 + h_3 + h_4)\, s + n \Rightarrow \gamma_{all} = \frac{|h_1 + h_2 + h_3 + h_4|^2 P_s}{N_0} = 0.9^2 \bar{\gamma}$
- $\gamma_1 > \gamma_{all}$ - Do we really get benefits of having multiple paths?
- We need a smarter receiving architecture!

Received signal with multiple-antenna co-phasing

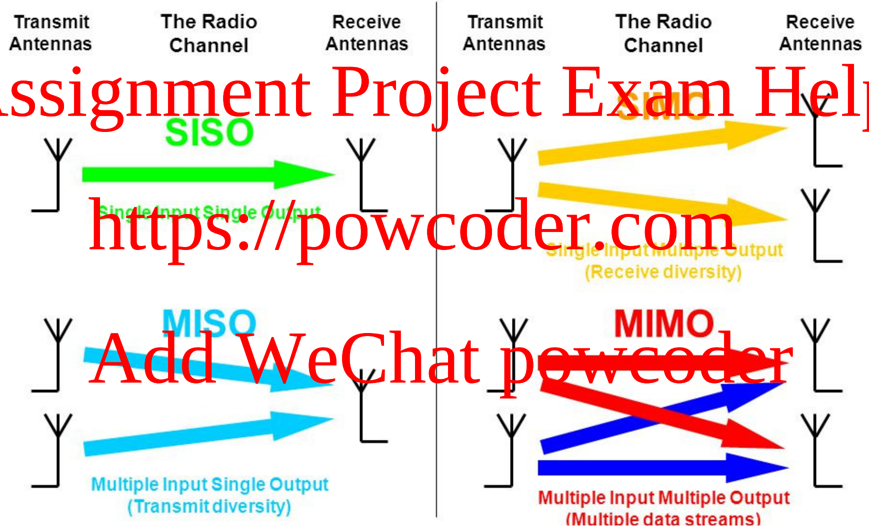


- Co-phasing: The phase θ_i of the ith branch ($h_i = r_i e^{j\theta_i}$) is removed through multiplication by $e^{-j\theta_i}$.

$$|h_1| = 1.4; |h_2| = 2.3; |h_3| = 1.4; |h_4| = 2.2$$

- Only Link 1: $r_1 = h_1 s + n_1 \Rightarrow \gamma_1 = \frac{|h_1|^2 P_s}{N_0} = 1.4^2 \bar{\gamma} = 2\bar{\gamma}$
- All Links (EGC): $r = \sum_{i=1}^{4} |h_i| s + \sum_{i=1}^{4} n_i \Rightarrow \gamma_{EGC} = \frac{(\sum_{i=1}^{4} |h_i|)^2 P_s}{4N_0} = 13.3\bar{\gamma}$
- Selection combining (SC): $\max(|h_i|) \Rightarrow \gamma_{SC} = \frac{2.3^2 P_s}{N_0} = 5.3\bar{\gamma}$
- MRC: $\gamma_{MRC} == \frac{(\sum_{i=1}^{4} |h_i|)^2 P_s}{N_0} = 53.3\bar{\gamma}$

Multiple antennas techniques

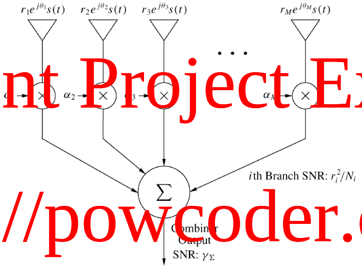


Diversity

- A diversity scheme: a method for improving the reliability of a message signal by using two or more communication channels with different characteristics.
- Diversity techniques mitigate the effect of multipath fading – microdiversity
- We need independent fading paths: use antenna array where the elements of the array are separated in distance – space diversity.
  1. multiple receive antennas - receiver diversity
  2. multiple transmit antennas – transmitter diversity
- Channel state information (CSI) availability:
  1. CSI at Rx (will focus more on this!)
  2. CSI at Tx
- We also have Time Diversity and Frequency Diversity.

Diversity/combining techniques

Techniques entail various trade-offs between performance/complexity.



1. Selection Combining (SC): the combiner outputs the signal on the branch with the highest SNR.
2. Maximal-Ratio Combining (MRC): the output is a weighted sum of all branches, and the weights (α_is) are determined to maximize the SNR.
3. Equal-Gain Combining (EGC): co-phases the signals on each branch and then combines them with equal weighting.
4. Threshold Combining: outputting the first signal whose SNR is above a given threshold γ_T.

Selection Combining (SC)

1. Received signal over the ith channel, $i \in \{1, \cdots, M\}$:

$$y_i(t) = h_i\, s(t) + n_i(t) = r_i e^{j\theta_i}\, s(t) + n_i(t);\; i = 1, \cdots, M \tag{1}$$

2. Received SNR over the ith channel:

$$\gamma_i = \frac{|h_i|^2 P_s}{N_0} = |h_i|^2 \bar{\gamma} = g_i \bar{\gamma}; \quad i = 1, \cdots, M \tag{2}$$

3. Combiner outputs the signal on the branch with the highest SNR.
4. End-to-end SNR of SC:

$$\gamma_{SC} = \max_{i \in \{1, \cdots, M\}} (\gamma_1, \cdots, \gamma_M) \tag{3}$$

5. Selected antenna index

$$i^* = \arg \max_{i \in \{1, \cdots, M\}} (\gamma_1, \cdots, \gamma_M) \tag{4}$$

SC: Outage probability

- The SNR outage is

$$P_{o,SC} = \Pr(\gamma_{SC} < \gamma_{th}) = \Pr(\max(\gamma_1, \cdots, \gamma_M) < \gamma_{th})$$

$$= \prod_{i=1}^{M} \Pr(\gamma_i < \gamma_{th}) = \prod_{i=1}^{M} F_{\gamma_i}(\gamma_{th}) = [F_{\gamma_i}(\gamma_{th})]^M \quad \text{for i.i.d. channels} \tag{5}$$

- $|h_i|$ is the multipath fading channel, e.g., Rayleigh, Rician, Nakagami-m.
- For Nakagami-m fading channels:

$$f_{\gamma}(x) = \left(\frac{m}{\Omega\bar{\gamma}}\right)^m \frac{x^{m-1}}{\Gamma(m)} e^{-\frac{mx}{\Omega\bar{\gamma}}} \text{ and } F_{\gamma_i}(x) = 1 - \frac{\Gamma\left(m, \frac{mx}{\Omega\bar{\gamma}}\right)}{\Gamma(m)} \tag{6}$$

- The SNR outage probability over Nakagami-m fading channels

$$P_{o,SC} = \left[1 - \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\Omega\bar{\gamma}}\right)}{\Gamma(m)}\right]^M \tag{7}$$

Diversity order and array gain

1. For large enough SNR ($\bar{\gamma} \to \infty$), the outage probability P_o as a function of $\bar{\gamma}$ can be written as

$$P_o \approx (G_c \bar{\gamma})^{-G_d} \text{ or } P_o \approx G_c \bar{\gamma}^{-G_d} \tag{8}$$

   - G_c: the coding gain or array gain
   - G_d: the diversity gain, diversity order, or simply diversity.

2. If G_d = the number of independent fading paths that are combined via diversity, the system is said to achieve full diversity order.

3. Asymptotic analysis for SC: By using $\lim_{x \to 0} \Gamma[n, x] \approx \Gamma[n] - \frac{x^n}{n}$,

$$\lim_{\bar{\gamma} \to \infty} P_{o,SC} \approx \left[1 - \frac{\left(\Gamma[m] - \frac{\left(\frac{m\gamma_{th}}{\bar{\gamma}}\right)^m}{m} \right)}{\Gamma(m)} \right]^M = \left(\frac{m^{m-1} \gamma_{th}^m}{\Gamma(m)} \right)^M \bar{\gamma}^{-mM} \tag{9}$$

Maximal-Ratio Combining (MRC)

1. MRC output is a weighted sum of all branches, so the α_i are all nonzero, and the weights are determined to maximize the combiner output's SNR.
2. For a branch with $h_i = r_i e^{j\theta_i}$,
   - The signals are co-phased: $e^{-j\theta_i}$
   - The optimal weight to maximize SNR is: $a_i = r_i$
   - $\alpha_i = r_i e^{-j\theta_i}$
3. End-to-end SNR of MRC: the SNR of the combiner output is the sum of SNRs on each branch.

$$\gamma_{MRC} = \sum_{i=1}^{M} \gamma_i \tag{10}$$

4. The SNR outage is

$$P_{o,MRC} = \Pr\left(\gamma_{MRC} < \gamma_{th}\right) = \Pr\left(\sum_{i=1}^{M} \gamma_i < \gamma_{th}\right) = F_{\gamma_{MRC}}(\gamma_{th}) \tag{11}$$

We need the CDF of γ_{MRC}.

MRC: Outage probability

- For Rayleigh fading channels: i.i.d. Rayleigh fading on each branch with equal average branch SNR $\bar{\gamma}$, the distribution of γ_{MRC} (which is a sum of i.i.d. exponential RVs) is

$$f_{\gamma_{MRC}}(x) = \frac{x^{M-1} e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}^M (M-1)!} \tag{12}$$

$$F_{\gamma_{MRC}}(x) = 1 - \frac{\Gamma\left(M, \frac{x}{\bar{\gamma}}\right)}{\Gamma(M)} = 1 - e^{-\frac{x}{\bar{\gamma}}} \sum_{k=0}^{M-1} \frac{\left(\frac{x}{\bar{\gamma}}\right)^k}{k!} \tag{13}$$

- The SNR outage probability of MRC over i.i.d. Rayleigh fading channels

$$P_{o,MRC} = 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}}} \sum_{k=0}^{M-1} \frac{\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^k}{k!} \tag{14}$$

Equal-Gain Combining (EGC)

1. EGC co-phases the signals on each branch and then combines them with equal weighting.
2. For a branch with $h_i = r_i e^{j\theta_i}$,
   - The signals are co-phased: $e^{-j\theta_i}$
   - The weight is: $a_i = 1$
   - $c_i = e^{j\theta}$
3. End-to-end SNR of EGC: the SNR of the combiner output is

$$\gamma_{EGC} = \frac{P_s}{MN_0}\left(\sum_{i=1}^{M} |h_i|\right)^2 = \frac{\bar{\gamma}}{M}\left(\sum_{i=1}^{M} r_i\right)^2 \quad (15)$$

4. The distribution PDF and CDF of γ_{EGC} do not exist in closed form for $M > 2$.

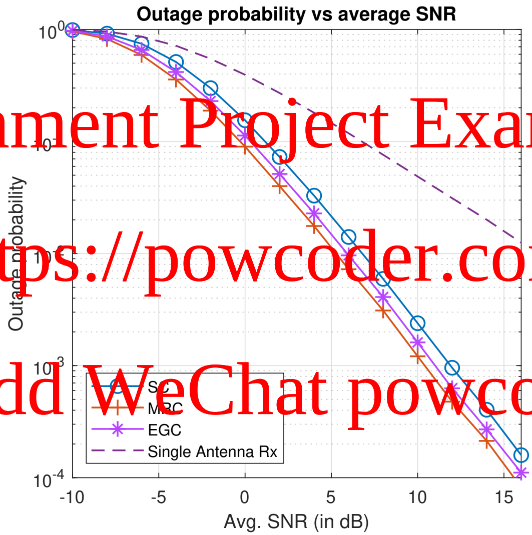


Figure 1: Comparison of SC, MRC, EGC and no diversity.

Numerical results (Diff. number of antennas)

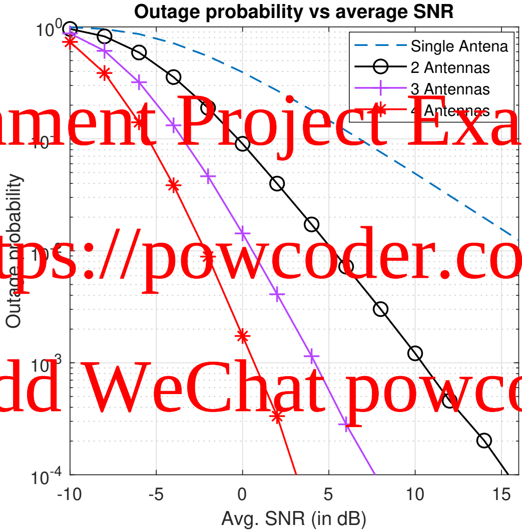


Figure 2: Comparison of multiple antennas receiver for MRC.

References

A. Goldsmith, *Wireless Communications*, Cambridge University Press, USA, 2005.

Thank You!

See you again ☺