ECE 5884/6884
Wireless Communications
Week 8 Lecture

Wireless Channel Equalization techniques for Single Carrier and Multicarrier Systems (OFDM)

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"LOOKS LIKE BERT'S GONE WIRELESS!"

Single Carrier and Multicarrier

Single Carrier systems

- Time domain equalization
- Frequency domain equalization

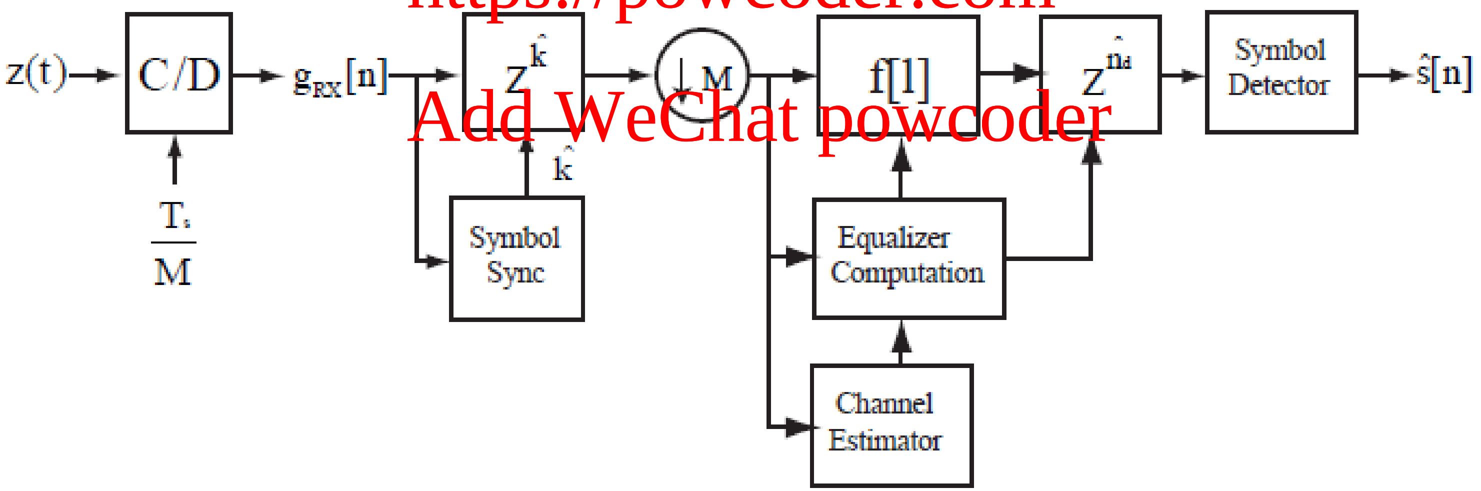
Multicarrier systems

- (Orthogonal Frequency Division Multiplexing OFDM) + Frequency domain equalization

Single Carrier Systems

Least squares time domain equalizer for single carrier systems

- Learning objective: Develop Least Squares based *channel equalizer* to compensate for the effect of wireless channel induced inter symbol interference *(ISI)*



Channel Estimate based time domain equalization (TDE)

Removing ISI using linear equalization

- Consider an FIR linear equalizer with coefficients $\{f[\ell]\}_{\ell=0}^{L_\mathrm{f}}$

$$\sum_{\ell=0}^{L_\mathrm{f}} f[\ell]h[n-\ell] = \delta[n-n_\mathrm{d}] \quad \text{for} \quad n = 0, 1, \ldots, L_\mathrm{f} + L$$

- Write as a linear system

$$\begin{bmatrix} h[0] & 0 & \cdots & \cdots & 0 \\ h[1] & h[0] & 0 & \cdots & \vdots \\ \vdots & \ddots & & & h[0] \\ h[\ell] & & & & h[1] \\ 0 & h[\ell] & \cdots & & \vdots \\ \vdots & & & & h[\ell] \end{bmatrix} \begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[L_\mathrm{f}] \end{bmatrix} = \mathbf{e}_{n_\mathrm{d}} \Rightarrow \mathbf{H}\mathbf{f}_{n_\mathrm{d}} = \mathbf{e}_{n_\mathrm{d}}$$

Toeplitz structure

$L_f + L + 1 \times L_f + 1$
(tall)

$$\mathbf{A}\mathbf{x} = \mathbf{b} \Rightarrow \mathbf{x}_{LS} = (\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{b}$$

$$\mathbf{f}_{\mathrm{LS},n_\mathrm{d}} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{e}_{n_\mathrm{d}}$$

Computation of the LS equalizer

- Toeplitz structure in $\mathbf{H}$ leads to efficient algorithms to solve LS
- $\mathbf{H}$ is full rank as long as at least one coefficient is nonzero
- The LS solution assuming $\mathbf{H}$ is full rank is

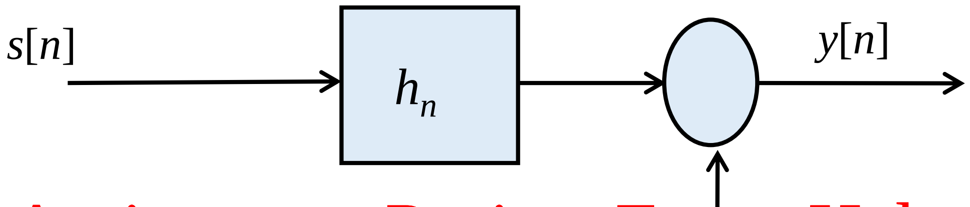
$$\mathbf{f}_{\mathrm{LS},n_{\mathrm{d}}} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{e}_{n_{\mathrm{d}}}$$

  - with squared error

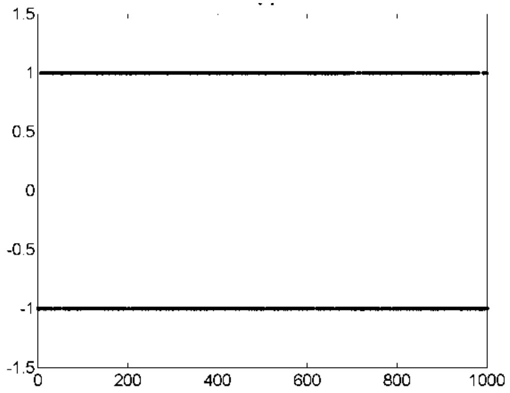
$$J[n_{\mathrm{d}}] = \mathbf{e}_{n_{\mathrm{d}}}^*(\mathbf{I} - \mathbf{H}(\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*)\mathbf{e}_{n_{\mathrm{d}}}$$

- The squared error can be further minimized by choosing n_{d}

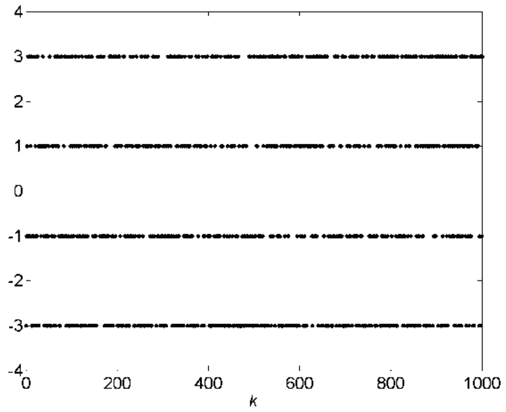
Example: ISI channel

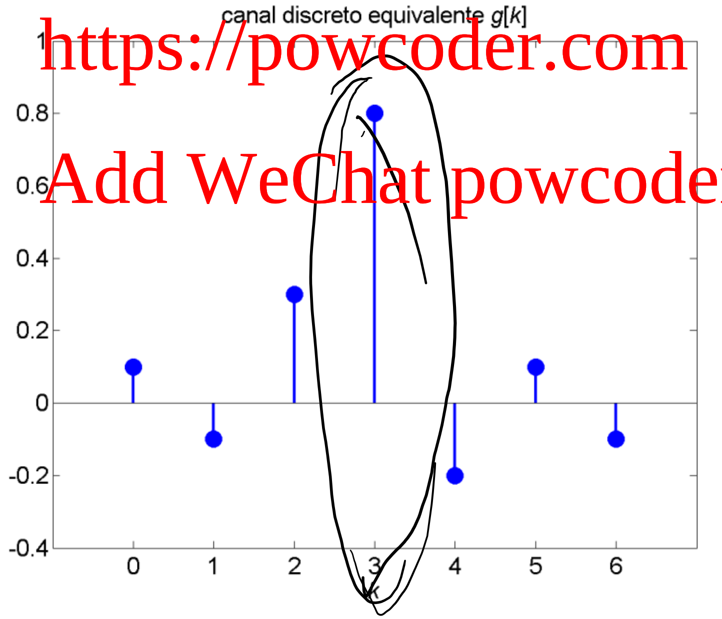


BPSK $s[n]$

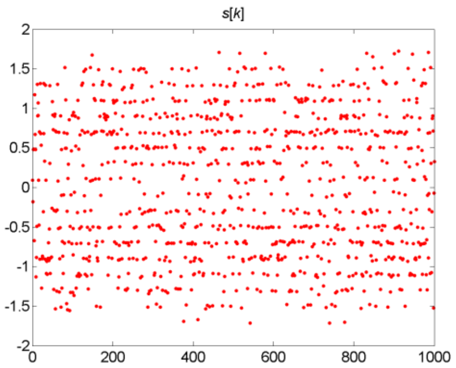


4-PAM $s[n]$

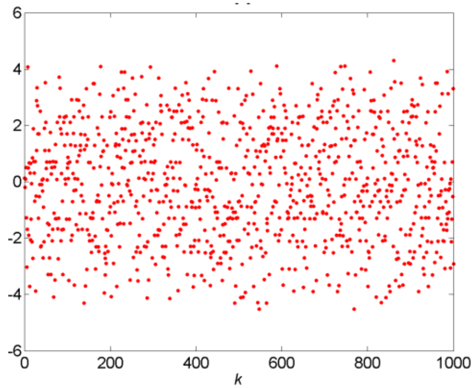




$y[n]$

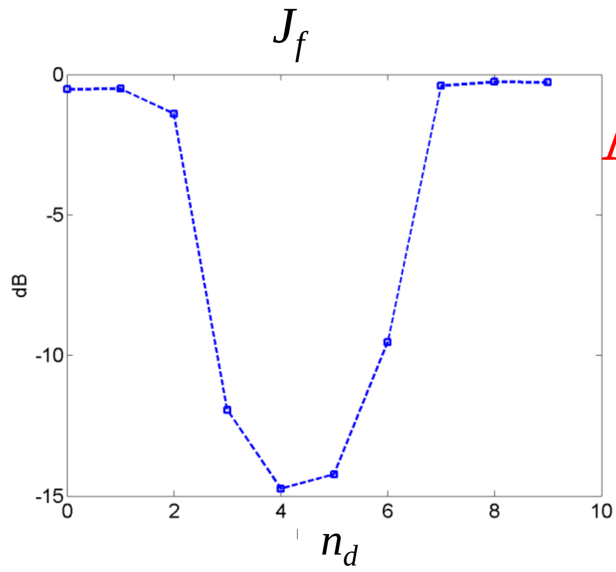


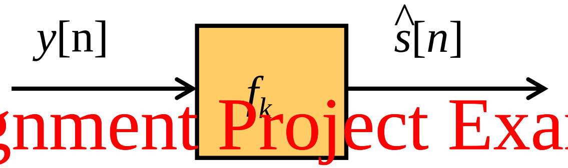
$y[n]$

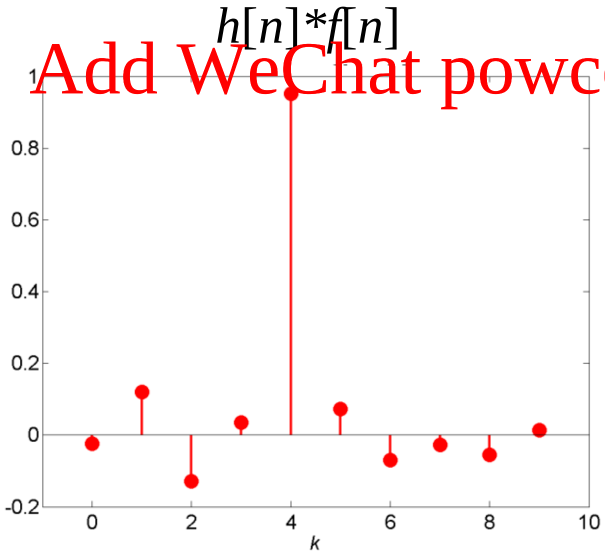


MSE function of filter delay

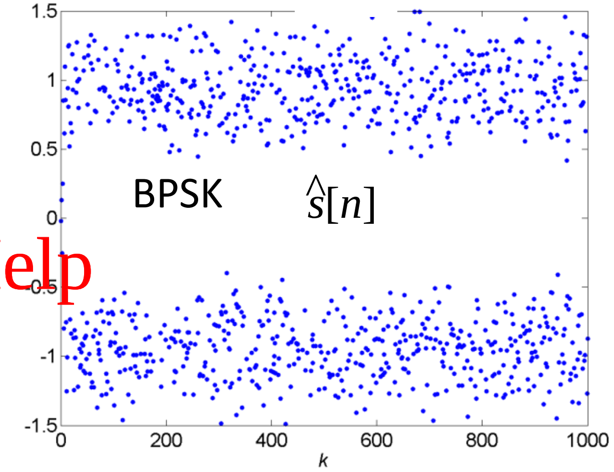
LS equalizer, order 3

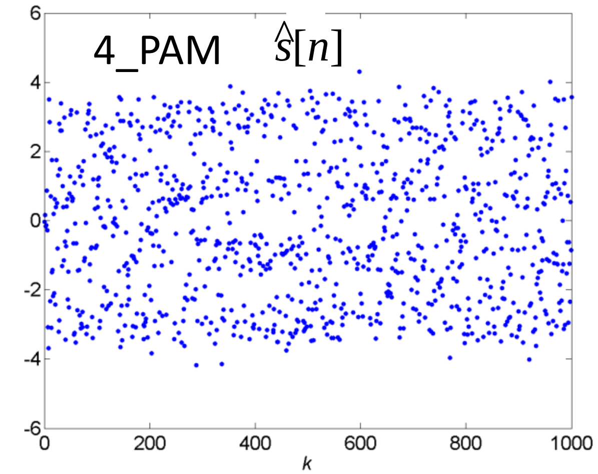






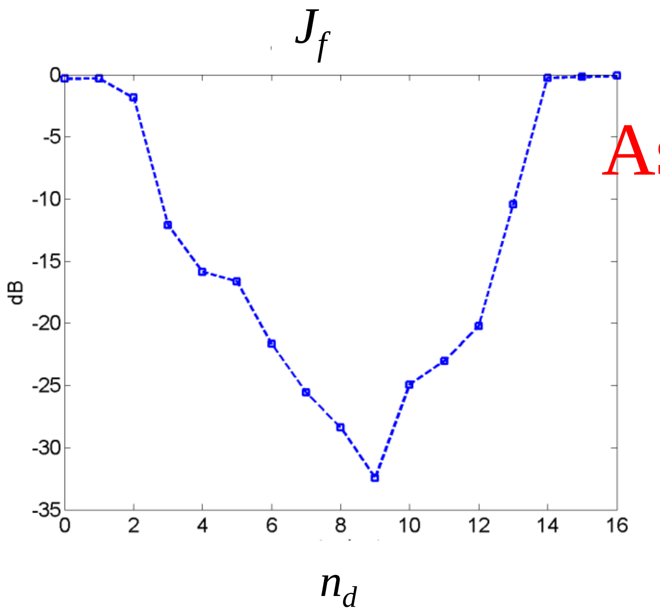
optimal delay
$n_d = 4$



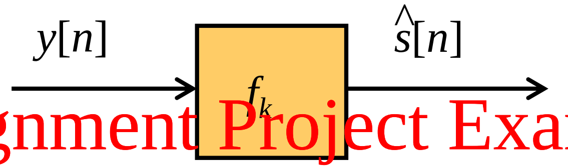


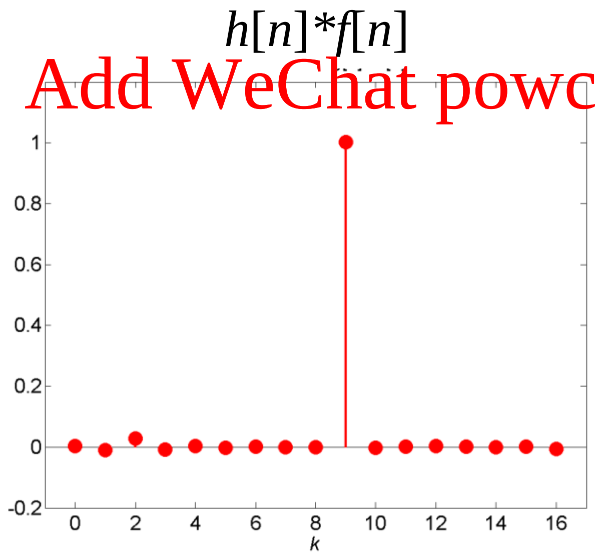
MSE function of filter delay

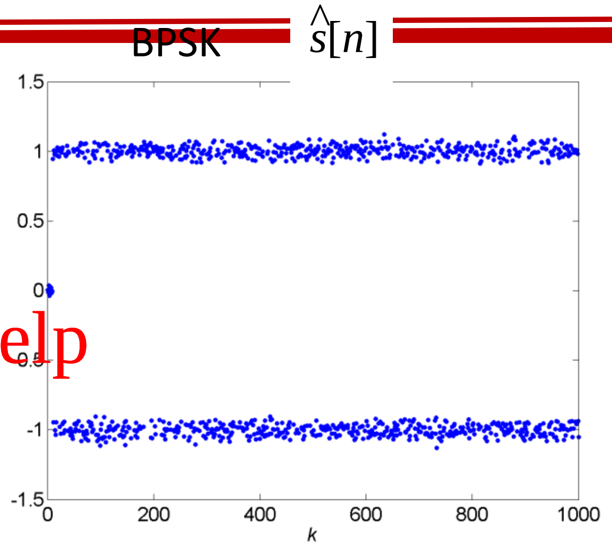
Example: LS equalizer, order 10

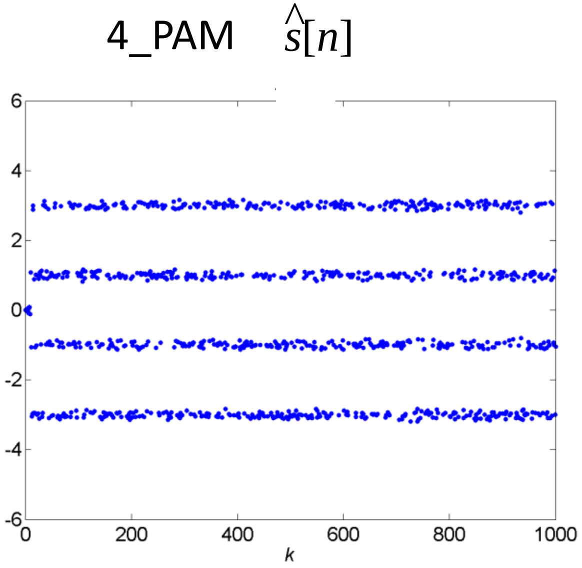


optimal delay
$n_d = 9$

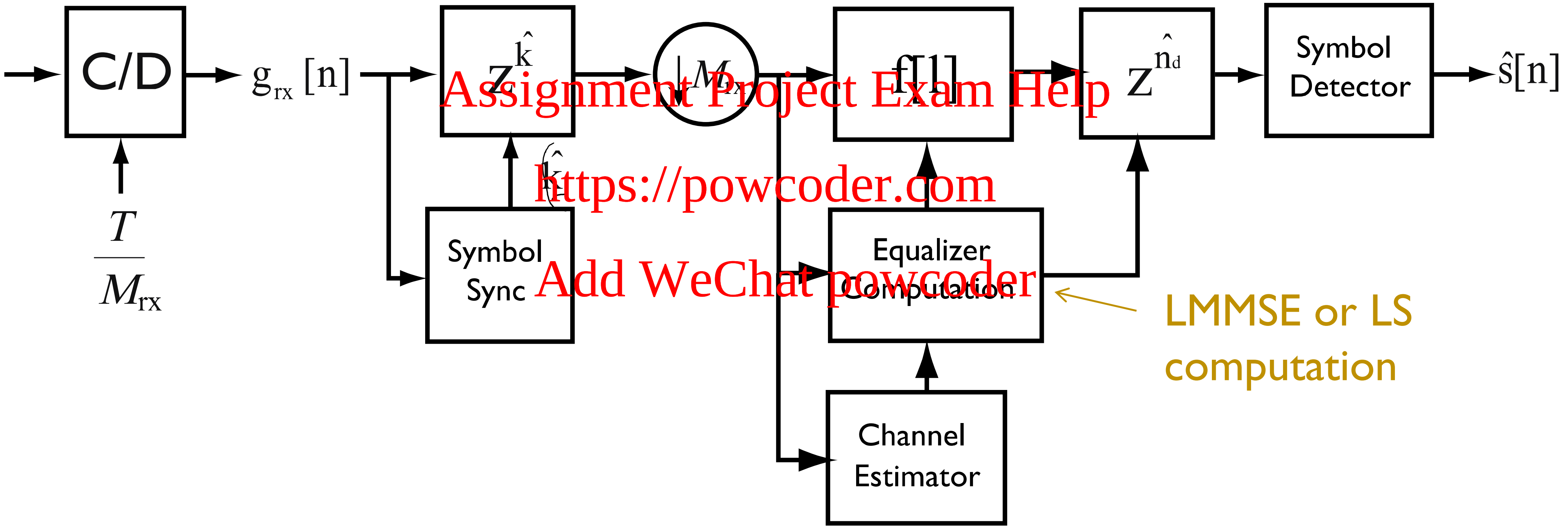








Receiver with channel estimation and linear equalization



LMMSE estimator

- Suppose we want to estimate $\mathbf{x}$ from an observation $\mathbf{y}$
  - Unknown vector $\mathbf{x}$ of size M x 1 with zero mean and covariance $\mathbf{C}_{\mathrm{xx}}$
  - Observation vector $\mathbf{y}$ with zero mean and covariance $\mathbf{C}_{\mathrm{yy}}$
  - $\mathbf{x}$ and $\mathbf{y}$ are jointly correlated with covariance matrix $\mathbf{C}_{\mathrm{yx}}$

- The objective of the LMMSE estimator is to determine a linear transformation such that

$$\mathbf{G}_{\mathrm{MMSE}} = \arg\min_{\mathbf{G}} \mathbb{E}\left[\|\mathbf{x} - \boxed{\mathbf{G}^*\mathbf{y}}\|^2\right]$$

$$\mathbf{x}_m = [\mathbf{x}]_m \qquad \mathbf{g}_m = [\mathbf{G}]_{:,m}$$

- Equivalently

$$\mathbf{G}_{\mathrm{MMSE}} = \arg\min_{\mathbf{G}} \mathbb{E}\left[\sum_{m=1}^{M} |\mathbf{x}_m - \mathbf{g}_m^*\mathbf{y}|^2\right]$$

LMMSE estimator

- Solving for one column of $\mathbf{G}_{\mathrm{MMSE}}$

$$\frac{\mathrm{d}}{\mathrm{d}\mathbf{g}_k^{\mathrm{c}}}\mathbb{E}\left[\sum_{m=1}^{M}|\mathbf{x}_m - \mathbf{g}_m^*\mathbf{y}|^2\right] = \mathbb{E}\left[\frac{\mathrm{d}}{\mathrm{d}\mathbf{g}_k^{\mathrm{c}}}\sum_{m=1}^{M}|\mathbf{x}_m - \mathbf{g}_m^*\mathbf{y}|^2\right]$$

$$= \mathbb{E}\left[\frac{\mathrm{d}}{\mathrm{d}\mathbf{g}_k^{\mathrm{c}}}|\mathbf{x}_k - \mathbf{g}_k^*\mathbf{y}|^2\right]$$

$$= \mathbb{E}\left[\mathbf{y}\left(\mathbf{y}^*\mathbf{g}_k - \mathbf{x}_k^*\right)\right].$$

- MMSE orthogonality equation
  - Taking the expectation and setting the result to zero

$$\mathbf{C}_{\mathbf{yy}}\mathbf{g}_k = [\mathbf{C}_{\mathbf{yx}}]_{:,k}.$$

  - Solution is

$$\mathbf{g}_{k,\mathrm{MMSE}} = \mathbf{C}_{\mathbf{yy}}^{-1}[\mathbf{C}_{\mathbf{yx}}]_{:,k}$$

LMMSE estimator

- Reassembling the column of $\mathbf{G}$ and combining the results together

$$\mathbf{G}_{\text{MMSE}} = \mathbf{C}_{\mathbf{yy}}^{-1}\mathbf{C}_{\mathbf{yx}}$$

- The MMSE estimate of $\mathbf{x}$ is

$$\begin{aligned}\mathbf{x}_{\text{MMSE}} &= \mathbf{G}_{\text{MMSE}}^{*}\mathbf{y} \\ &= \mathbf{C}_{\mathbf{yx}}^{*}\mathbf{C}_{\mathbf{yy}}^{-1}\mathbf{y}.\end{aligned}$$

Reformulating the equalization problem

- Consider the received signal after the equalizer

$h_f > L$

$$\hat{s}[n-n_\mathrm{d}] = \sum_{\ell=0}^{L_\mathrm{f}} f_{n_\mathrm{d}}[\ell]\, y[n-\ell] \quad \Rightarrow \quad \hat{s}[n-n_\mathrm{d}] = \mathbf{f}_{n_\mathrm{d}}^{\mathrm{T}} \mathbf{y}[n]$$

where

$$\mathbf{y}^{\mathrm{T}}[n] = [y[n], y[n-1], \ldots, y[n-L_\mathrm{f}]]^{\mathrm{T}}$$

$L_f + L + 1 \;\; X \;\; L_f + 1$

$$\mathbf{H} = \begin{bmatrix} h[0] & 0 & \cdots & \cdots & 0 \\ h[1] & h[0] & 0 & \cdots & \vdots \\ \vdots & \ddots & & & h[0] \\ h[L] & & & & h[1] \\ 0 & h[L] & \cdots & & \vdots \\ \vdots & & & & h[L] \end{bmatrix}.$$

$$s[n] = [s[n], s[n-1], \ldots s[n-L], 0,0,0 \ldots]$$

$$\mathbf{y}[n] = \mathbf{H}^{\mathrm{T}} \mathbf{s}[n] + \mathbf{v}[n]$$

- The LMMSE equalizer minimizes the error

$\hat{s}$

$$\mathbb{E}\left[\left|s[n-n_\mathrm{d}] - \mathbf{f}_{n_\mathrm{d}}^{\mathrm{T}} \mathbf{y}[n]\right|^2\right]$$

Solving for the LMMSE equalizer

- Assuming that $\mathbf{y}[n] = \mathbf{H}^{\mathrm{T}}\mathbf{s}[n] + \mathbf{v}[n]$
  - $s[n]$ is IID with zero mean and unit variance,
  - $v[n]$ is IID with variance σ_v^2.
  - $s[n]$ and $v[n]$ are independent

- Then the estimation mean squared error is given by
- $$\mathbb{E}\left[\left|s[n-n_{\mathrm{d}}] - \mathbf{f}_{n_{\mathrm{d}}}^{\mathrm{T}}\mathbf{y}[n]\right|^2\right] \qquad \mathbf{H}\mathbf{f}_{n_{\mathrm{d}}} = \mathbf{e}_{n_{\mathrm{d}}}.$$

- The LMMSE equalizer is computed

$$\begin{aligned}\boxed{\mathbf{f}_{n_{\mathrm{d}},\mathrm{MMSE}} = \mathbf{C}_{\mathbf{yy}}^{-\mathrm{c}}\mathbf{C}_{\mathbf{y}s}^{\mathrm{c}}} \\ = \left(\mathbf{H}^*\mathbf{H} + \sigma_v^2\mathbf{I}\right)^{-1}\mathbf{H}^*\mathbf{e}_{n_{\mathrm{d}}}\end{aligned}$$

$$\begin{aligned}\mathbf{C}_{\mathbf{y}s} &= \mathbb{E}\left[\mathbf{y}[n]s^*[n-n_{\mathrm{d}}]\right] \\ &= \mathbf{H}^{\mathrm{T}}\mathbf{e}_{n_{\mathrm{d}}}. \\ \mathbf{C}_{\mathbf{yy}} &= \mathbb{E}\left[\mathbf{y}[n]\mathbf{y}^*[n]\right] \\ &= \mathbf{H}^{\mathrm{T}}\mathbf{H}^{\mathrm{c}} + \sigma_v^2\mathbf{I}\end{aligned}$$

Linear equalization discussion

- Connections between LMMSE and LS solutions

$$\mathbf{f}_{n_d,\mathrm{MMSE}} = \boxed{\left(\mathbf{H}^*\mathbf{H} + \sigma_v^2\mathbf{I}\right)^{-1}\mathbf{H}^*\mathbf{e}_{n_d}}$$

SNR ↑ high.

noise variance becomes large

noise variance goes to zero

SNR is low

$$\frac{1}{\sigma_v^2}\mathbf{H}^*\mathbf{e}_{n_d}$$

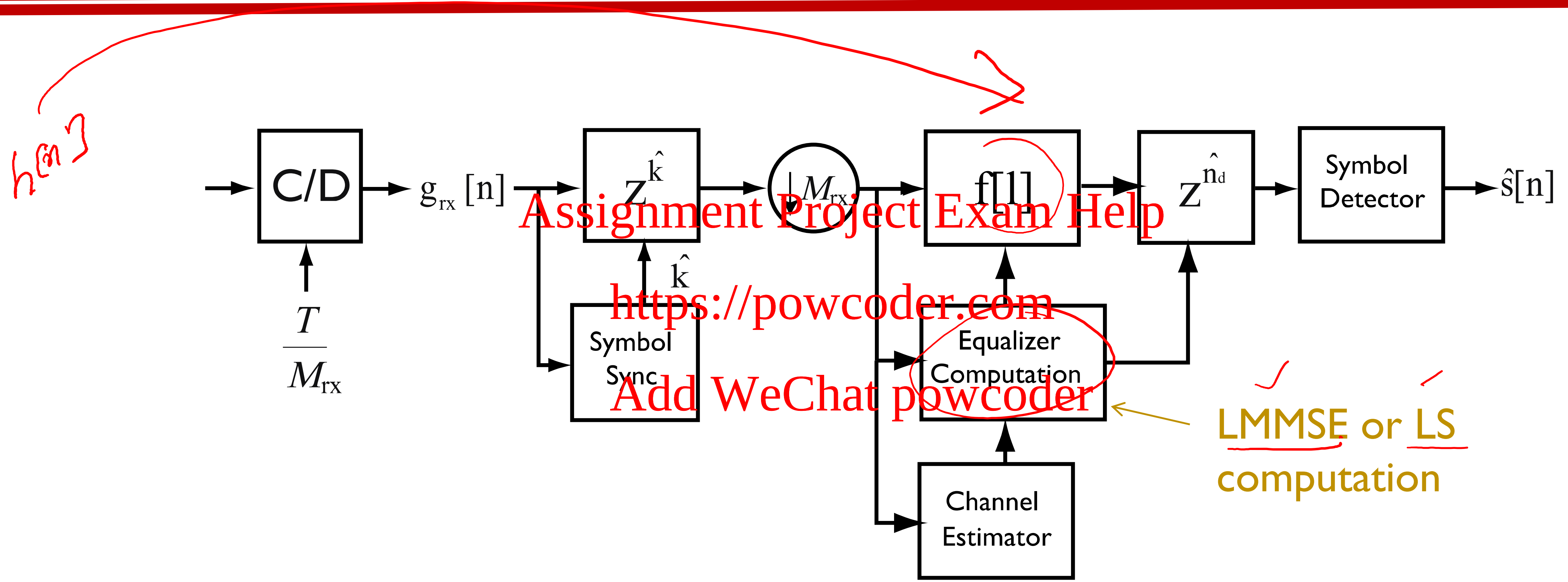
matched filter

$$\mathbf{f}_{\mathrm{LS},n_d} = \left(\mathbf{H}^*\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{e}_{n_d}$$

LS solution

- Complexity of the equalizer depends on the choice of $\mathrm{L_f}$
  - Larger values give better performance but higher complexity
  - $\mathrm{L_f}$ a design parameter that is generally greater than the ISI length L
  - As time domain equalizers need $\boxed{L_f > L}$ complexity grows with the channel introduced ISI

Linear Time domain equalization



Linear time domain equalizer (summary)

- TDE for single carrier systems is calculated using the LS or LMMSE optimization criterion.
- Asymptotically when $\sigma_v^2 \to 0$ LMMSE approaches LS equalizer
- SNR estimation is not required for LS approach, hence less complex.
- Computational complexity of Linear-TDE depends on L and L_f
- The choice of the equalizer length affects the delay n_d.
- Computational **complexity** of **TDE** is $O(L_f^2)$, tolerable for small equalizer lengths but not practical for longer equalizers.

L_f delay
spread in symbol period

10μsec → T 1μsec

10 symbols.

Multipath effect and Equalizer length (TDE)

- Multipath propagation induces ISI (measured in number of symbol periods L) in the time domain. As a consequence of ISI, frequency domain characteristics exhibit frequency selectivity.
- Example, a channel with a maximum delay spread 10μsec on the transmission of symbol rate R_s introduces
  - ISI of $L = 10$ at a R_s =$1M$
  - ISI of $L = 100$ at a R_s =$10M$
  - ISI of $L = 1000$ at a R_s =$100M$
  - ISI of $L = 10000$ at a R_s =$1G$

Is it practical to implement a TDE for transmisión links that support high data rates over multipath (non-line of sight) fading channels?

Frequency Domain Equalization

- TDE require a convolution on the received signal to remove the effects of the channel.
- It is desirable to have receiver complexity that does not change with the channel delay spread.
- An alternative to TDE is to perform equalization completely in the *frequency domain*.
- Frequency Domain Equalization (FDE) is implemented as component wise multiplication.

$$Y[k] = H[k]S[k]$$

Reference: Introduction to Wireless Digital Communications: A Signal Processing Perspective

Frequency domain equalization

Frequency domain equalization (FDE) is channel compensation in the frequency domain.

FDE is based on the discrete Fourier transform (DFT) of signals

The DFT is a basis expansion for finite-length signals

$$\text{Analysis}: X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad k = 0, 1, ..., N-1$$ DFT

$$\text{Synthesis}: x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad n = 0, 1, ..., N-1$$ IDFT

$(N:$ length of signal$)$

Reference: Introduction to Wireless Digital Communications: A Signal Processing Perspective

Circular Shift Property of DFT

The DFT can be computed efficiently with the Fast Fourier transform for N a power of 2 and certain other special cases

Circular shift property of the DFT

Periodic for the memory of the channel

$$\text{If} \quad X_1[k] = e^{j2\pi\left(\frac{k}{N}\right)m} X[n] \Leftrightarrow x_1[n] = \begin{cases} x\left[((n-m))_N\right] & 0 \le n \le N-1 \\ 0 & \text{else} \end{cases}$$

Products in the frequency domain become circular convolution in discrete-time

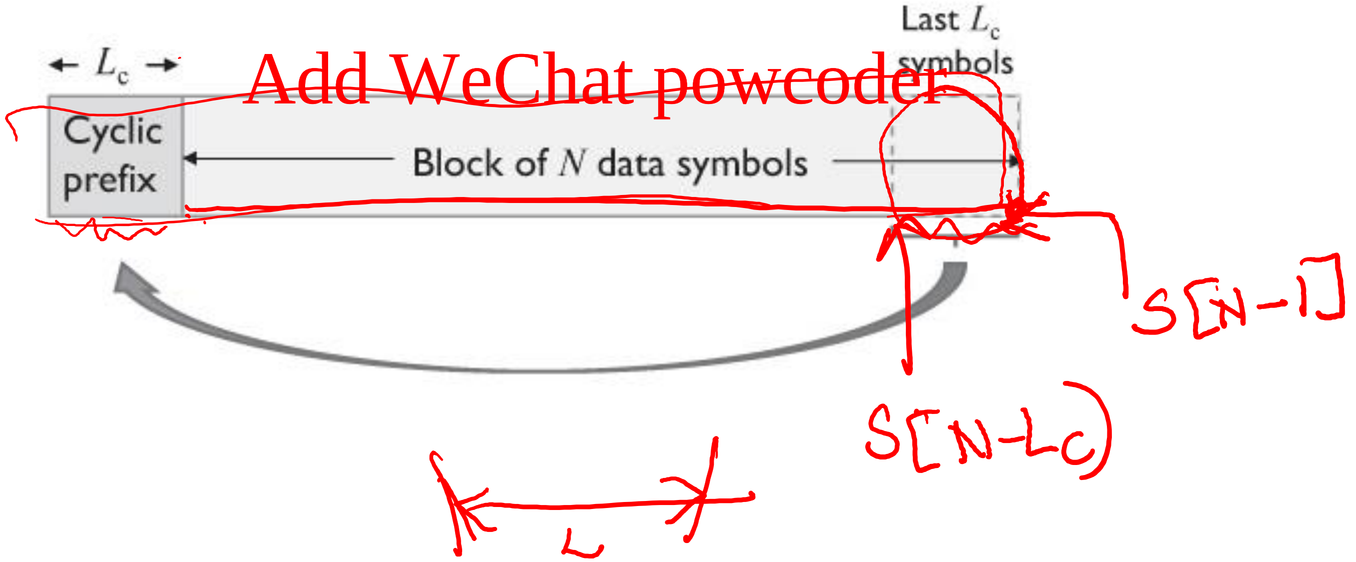
$$\boxed{Y[k] = H[k]S[k] \leftrightarrow y[n] = \boxed{\sum_{l=0}^{N-1} h[l]s\left[((n-l))_N\right]}}$$

Component wise multiplication in the frequency domain is convolution in the time domain

Circular convolution and Linear Convolution

Unfortunately, linear convolution, not circular convolution, is a good model for the effects of wireless propagation.

It is possible to mimic the effects of circular convolution by modifying the transmitted signal with a suitably chosen guard interval. The most common choice is what is called a cyclic prefix,



Reference: Introduction to Wireless Digital Communications: A Signal Processing Perspective

Single Carrier – Cyclic Prefix

Use cyclic prefix to convert a linear convolution to a circular convolution and equalize in the frequency domain. Convolution of signal with channel should appear circular.

Last L_c QAM symbols appended at the start of transmission

Let $s[n]$, $n = 0, \ldots, N-1$ be N QAM symbols for transmission

Create the following signal

– Add a cyclic prefix of length L_c

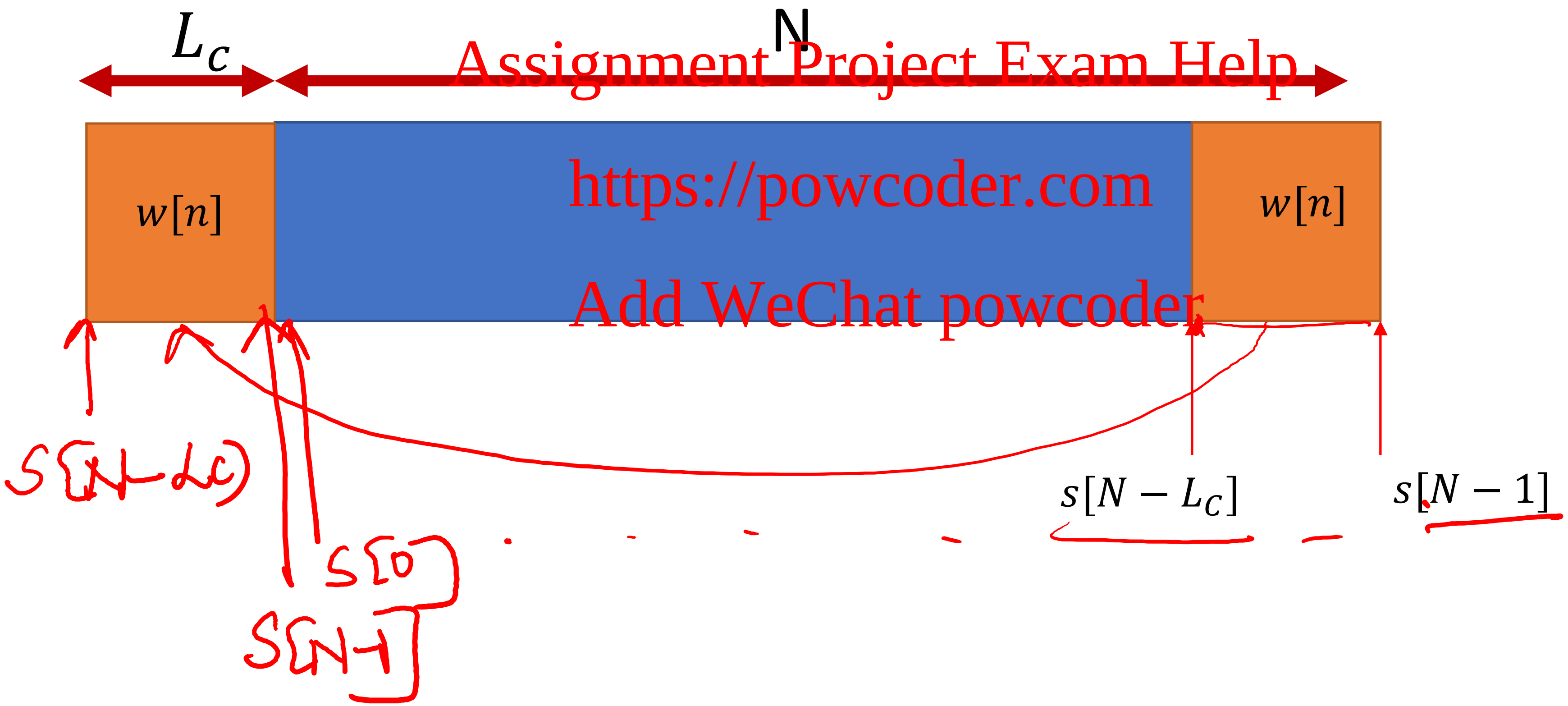
$$w[n] = s[n + N - L_c] \quad n = 0, 1, \ldots, L_c - 1$$

$= s[N - L_c] \; \ldots \; s[N-1]$

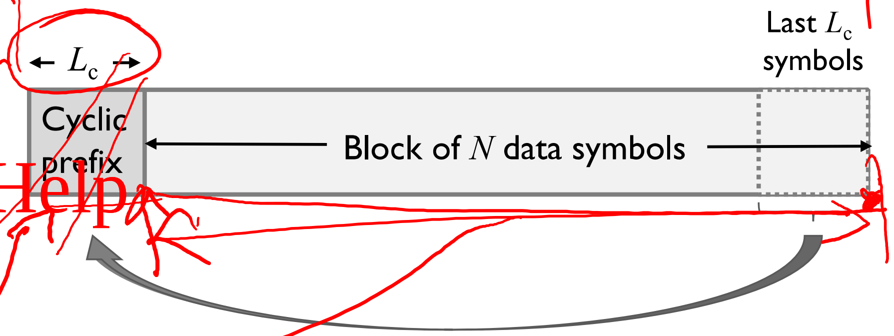
$\boxed{L_c > L}$

Reference: Introduction to Wireless Digital Communications: A Signal Processing Perspective

Single Carrier – Cyclic Prefix



Cyclic prefix



- Organize data into blocks of N symbols
- Assemble a block of N + L_c symbols where
  - Prefixed part
  - Data part

$$w[n] = s[n + N - L_c] \quad n = 0, 1, \ldots, L_c - 1$$

$$w[n] = s[n - L_c] \quad n = L_c, L_c + 1, \ldots, \boxed{L_c + N - 1}$$

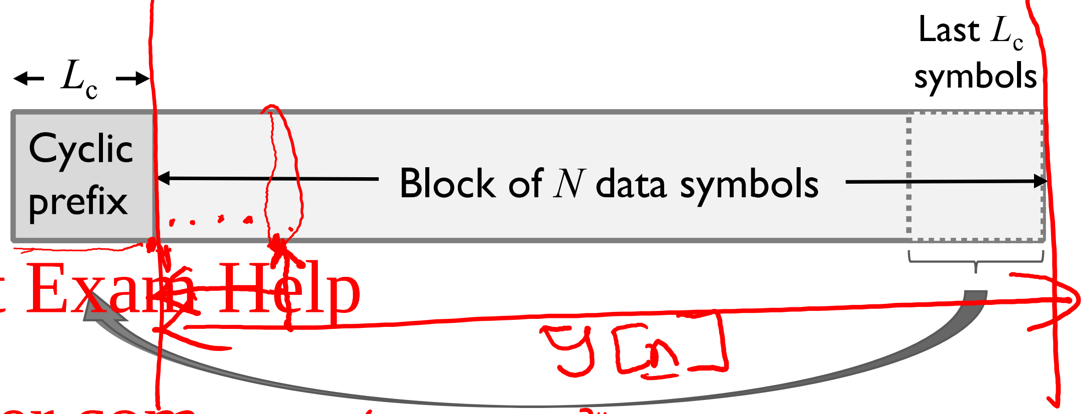
- Neglect the first Lc terms of the convolution

$$\bar{y}[n] = y[n + L_c] \quad n = 0, 1, \ldots, N - 1$$

$$= \sum_{\ell=0}^{L} h[\ell] w[n + L_c - \ell].$$

$h[l]\ w[n-l]$

Exposing the circular convolution 3/3



$$\bar{y}[L] = \sum_{\ell=0}^{L} h[\ell] w[L + L_c - \ell]$$

$$= h[0]w[L + L_c] + h[1]w[L + L_c - 1] + \cdots + h[\ell]w[L + L_c - \ell]$$

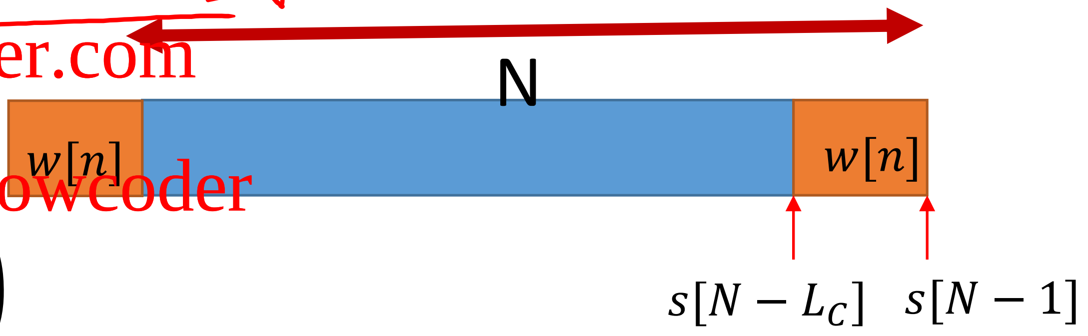
$$= h[0]s[L] + h[1]s[L-1] + \cdots + h[L]s[0]$$

$$\boxed{\bar{y}[L] = \sum_{\ell=0}^{L} h[\ell] s[L - \ell].}$$

Connection to frequency domain equalization

- Conclude that the received signal is equivalently

$$y[n] = \sum_{\ell=0}^{L} h[\ell]\, s[((n-\ell))_N]$$



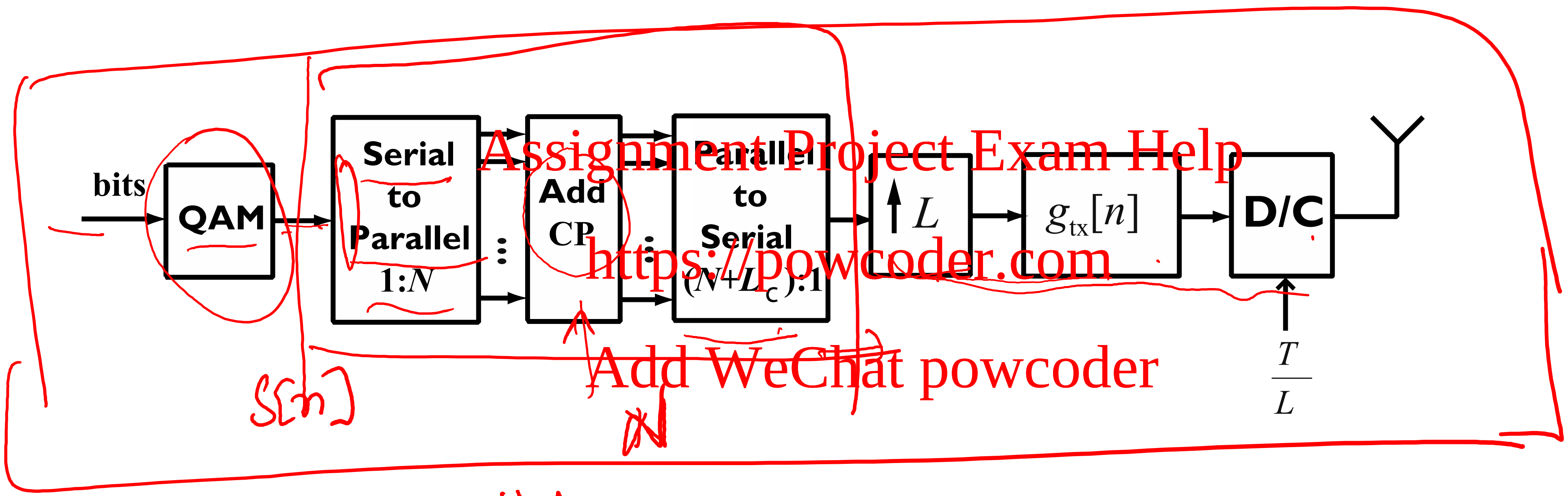
- Using the DFT equalization can proceed as

$$\hat{s}[n] = \mathcal{F}_N^{-1}\left(\frac{\mathrm{y}[k]}{\mathrm{h}[k]}\right)$$

$$= \mathcal{F}_N^{-1}\left(\frac{\mathcal{F}_N(y[n])}{\mathcal{F}_N(h[n])}\right)$$

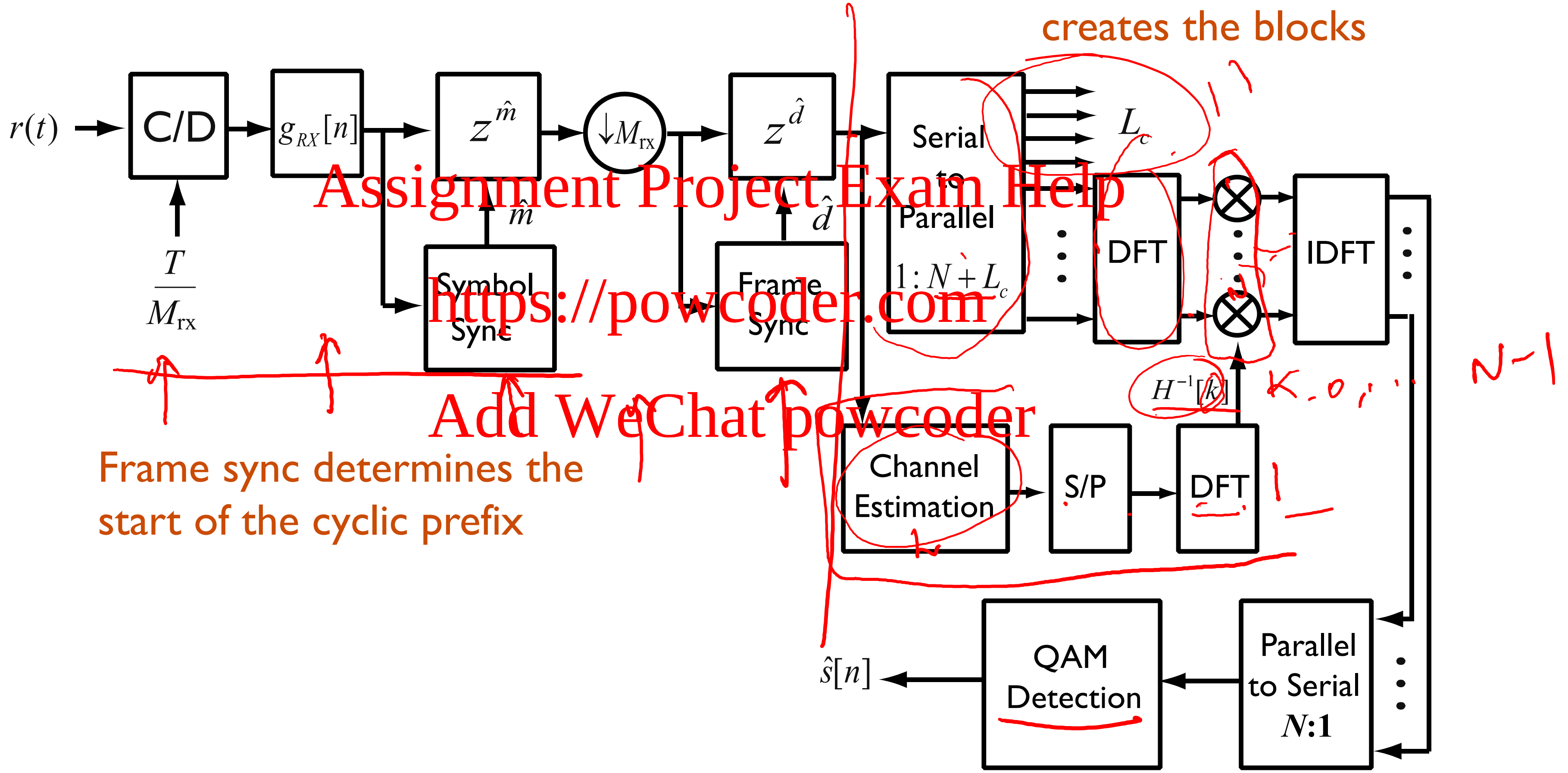
- Can use frequency domain equalization!

SC-FDE transmitter



$n = 0, \ldots, N-1$

SC-FDE receiver



Frame sync determines the start of the cyclic prefix

TDE vs FDE

- In general, the linear time domain equalizer (TDE) length $L_f \geq L$
- FDE of FFT size $N >> L$ offers complexity savings over the TDE for large L.
- As $N >> L$ it increases latency at the receiver for FDE systems.
- An important requirement for FDE is circular transmisión such as a cyclic prefix (CP). The CP overhead makes TDE throughput inefficient compared to the FDE approach.
- More complexity savings can be realized in a MIMO scenario.

$L >> 10^3$ symbols

Multicarrier systems

Why is OFDM so successful?

- Growing demand for increased data rates
- Overall complexity of OFDM is practical on channels with long delay spreads.
- OFDM allows the signal to be tailored to the channel. Large constellations on frequencies (subcarriers) with high signal to noise ratio (SNR)
- Forms the basis of OFDMA (Orthogonal frequency division multiple access)
  - OFDMA is used instead of TDMA, FDMA or CDMA in the LTE fourth generation mobile systems
- Provides spectrum flexibility for example in future cognitive radio systems

What is OFDM?

- OFDM is a particular form of multicarrier system
  - uses discrete Fourier Transform/Fast Fourier Transform (DFT/FFT)
  - sin(x)/x spectra for subcarriers
- Available bandwidth is divided into narrow bands (subcarriers)
  - ~2000-8000 for digital TV
  - ~48 for Hiperlan 2
- Data is transmitted in parallel on these bands

Multicarrier systems

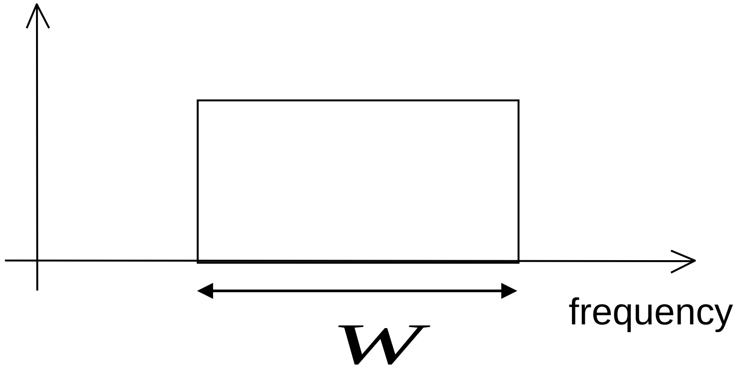
Single carrier system

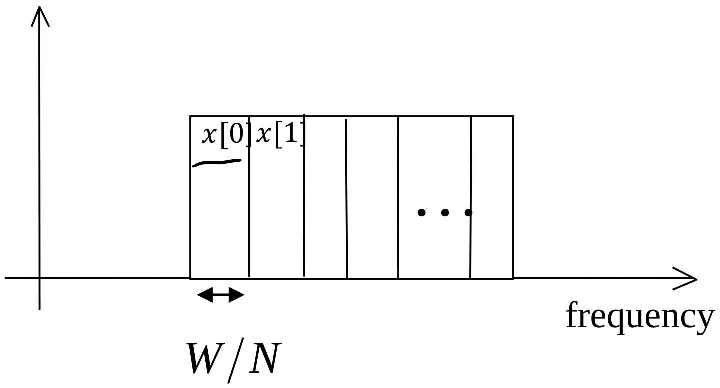
signal representing each bit uses all of the available spectrum

Multicarrier system

available spectrum divided into many narrow bands

data is divided into parallel data streams each transmitted on a separate band





IDFT at the transmitter

The DFT is a basis expansion for finite-length signals

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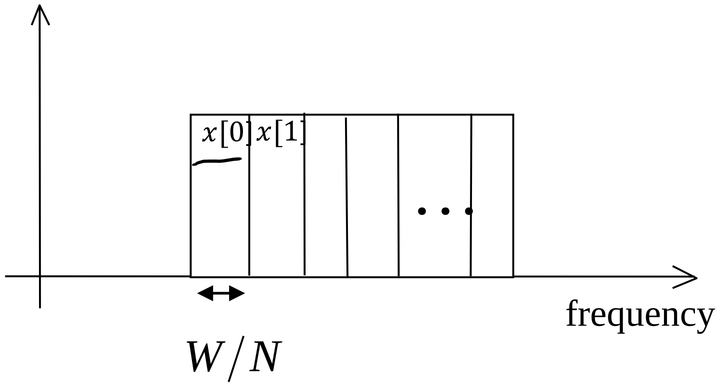
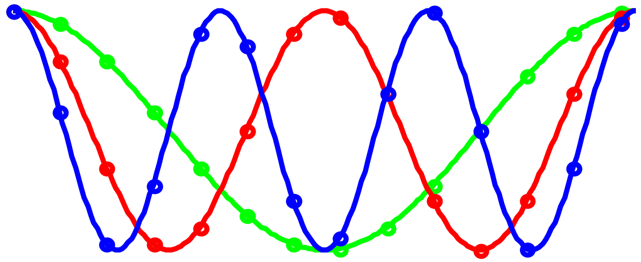
$$\text{Synthesis}: x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad n = 0, 1, ..., N-1$$

$(N: \text{ length of signal})$





Interference free communications?

Each subcarrier has a different frequency

Frequencies chosen so that an integral number of cycles in a symbol period

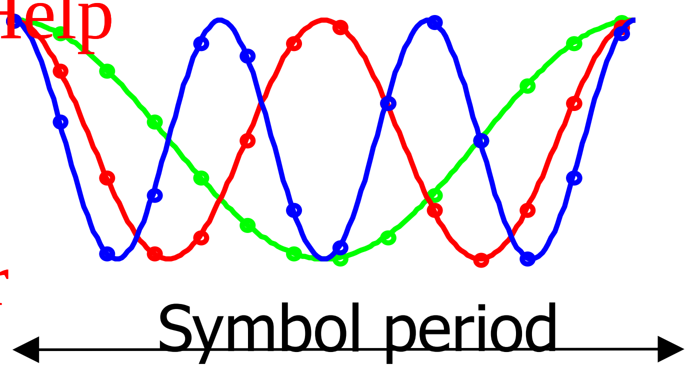
Signals(sub-carriers) are mathematically orthogonal

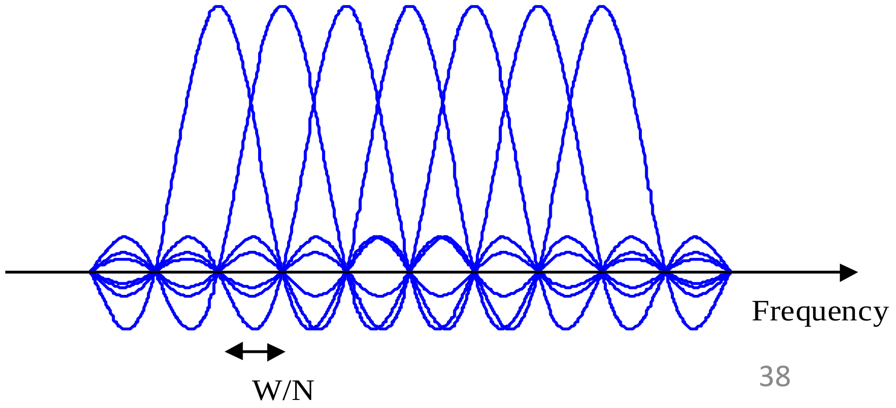
$$\int_0^T \sin\frac{2\pi kt}{T}\sin\frac{-2\pi lt}{T}\,dt = 0, \quad k \neq l$$

$$\int_0^T \sin\frac{2\pi kt}{T}\cos\frac{-2\pi lt}{T}\,dt = 0, \quad \text{all } k \text{ and } l$$

Key is orthogonality of subcarriers

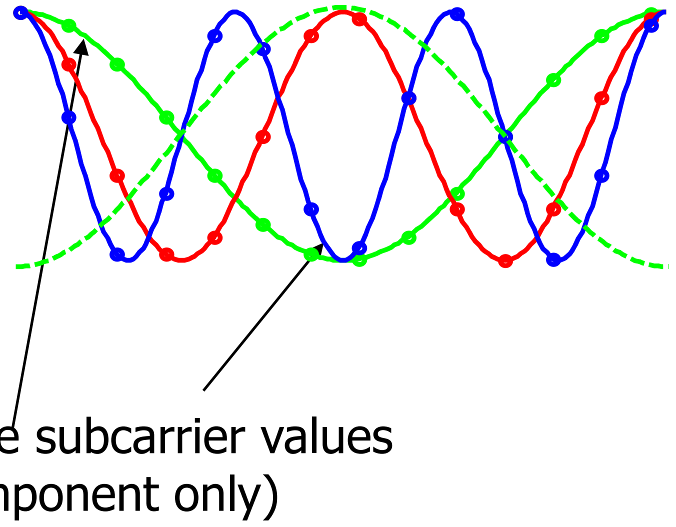
Cosine components of first three subcarriers

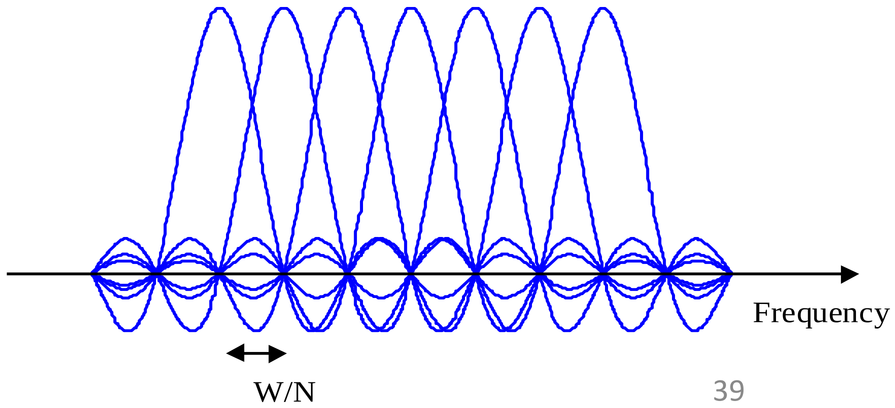




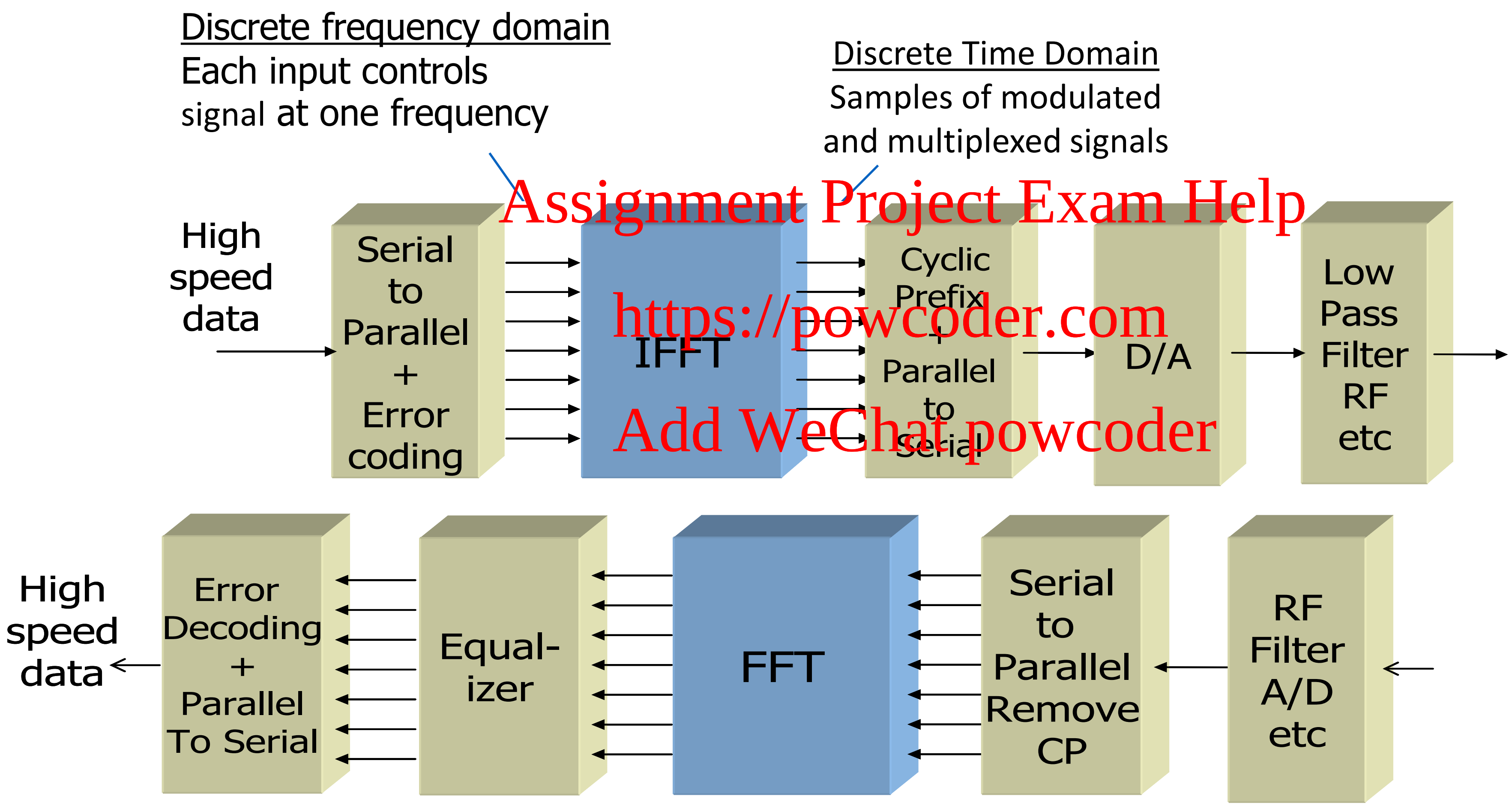
How is data carried on the subcarriers?

- Data is carried by varying the phase or amplitude of each subcarrier
- Quadrature phase shift keying (QPSK), Quadrature Amplitude Modulation (QAM), 4-QAM, 16-QAM, 64-QAM
- The overall OFDM symbol is then formed from the sum of all orthogonal subcarriers.
- The overall OFDM symbol has a finite duration T, hence the effect of windowing in the time domain leading to Sinc shaped spectra of subcarriers.





Baseband OFDM system

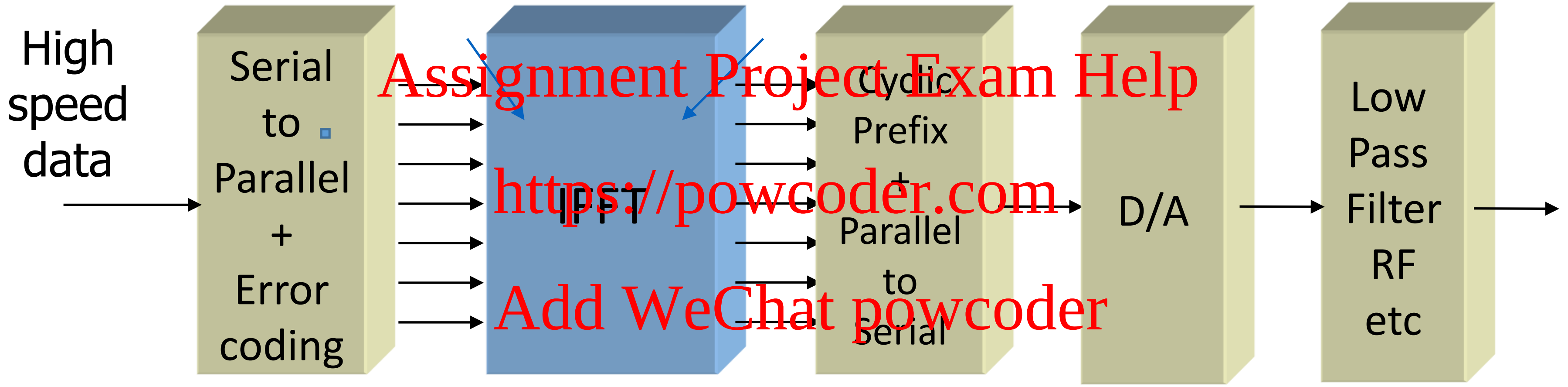


IFFT and FFT are the main components in the transmitter and receiver

Baseband OFDM system

Discrete frequency domain
Each input controls
signal at one frequency

Discrete Time Domain
Samples of modulated
and multiplexed signals



$$b_l = \frac{1}{N} \sum_{k=0}^{N-1} a_k \exp\left(\frac{j2\pi kl}{N} \right)$$

Samples at IFFT output

Samples at IFFT input

How are OFDM signals generated?

- Parallel data streams are used as inputs to an IFFT
- IFFT output is <u>sum</u> of signal samples
- IFFT does modulation and multiplexing in one step
- Filtering and D/A of samples results in baseband signal

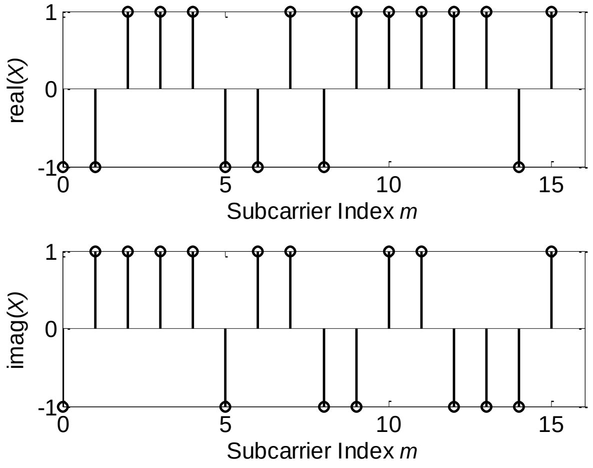
Signal values at the output of the IFFT are the sum of many samples of many sinusoids - looks random

Typical IFFT Output Samples

Reference from ECE 4024 by Prof. Jean Armstrong

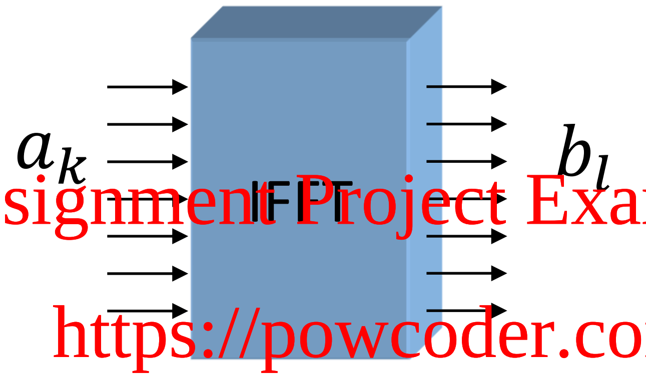
Signals at the input/output of the transmitter IFFT

OFDM Frequency Domain Symbol



Complex numbers representing data to be transmitted in this OFDM symbol

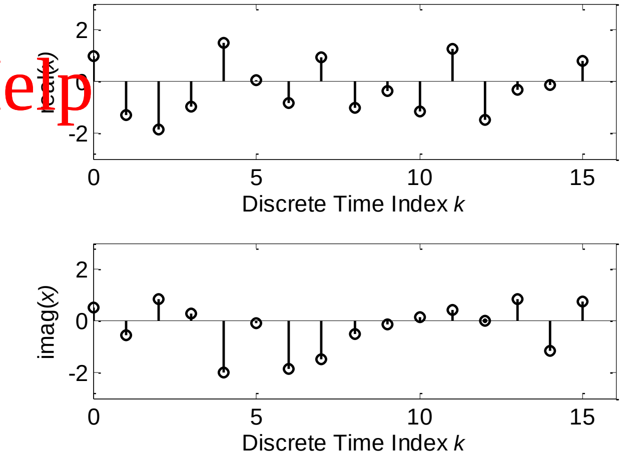
$$N = 16$$



$$b_l = \frac{1}{N} \sum_{k=0}^{N-1} a_k \exp\left(\frac{j2\pi kl}{N} \right)$$

16 subcarriers

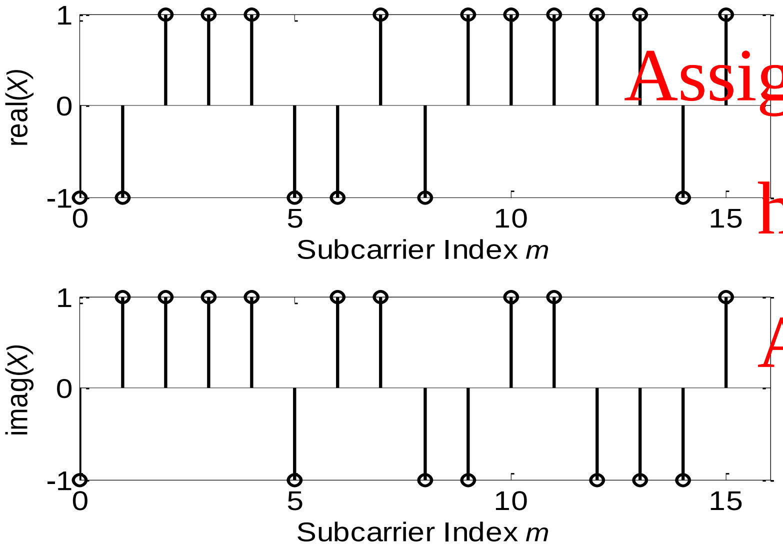
OFDM Time Domain Signal Samples

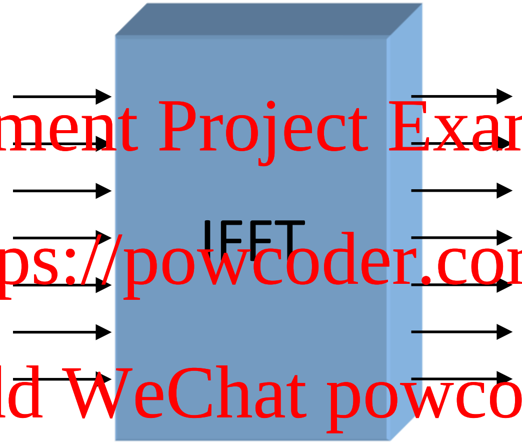


Complex numbers representing samples of signal to be transmitted

Signals at the input/output of the transmitter IFFT

OFDM Frequency Domain Symbol

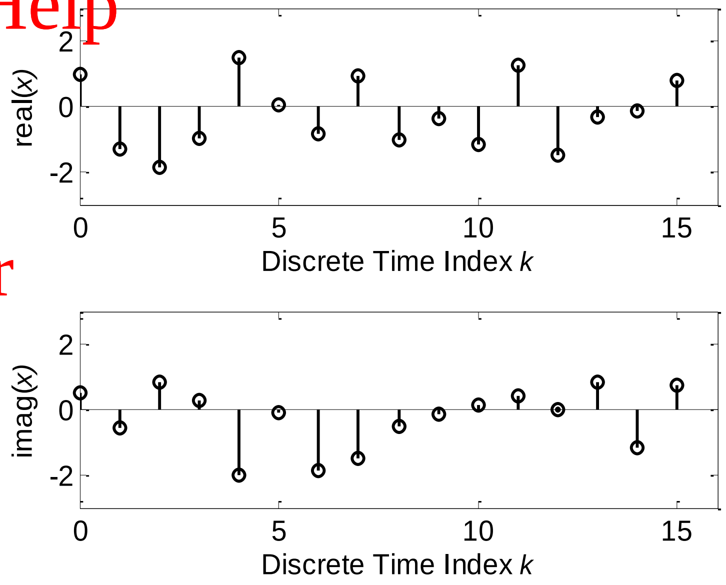




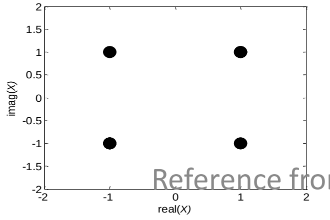
$$N = 16$$

16 subcarriers

OFDM Time Domain Signal Samples



4-QAM constellation

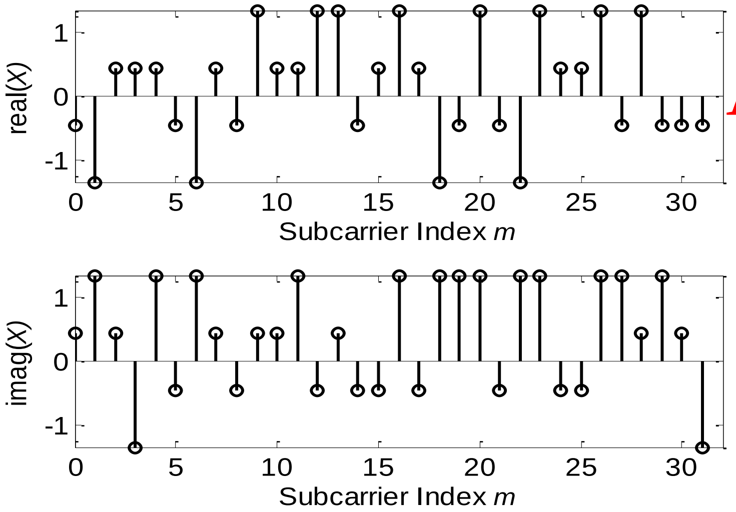


Output has Gaussian distribution for *N* >64

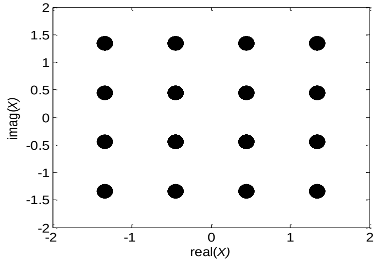
Reference from ECE 4024 by Prof. Jean Armstrong

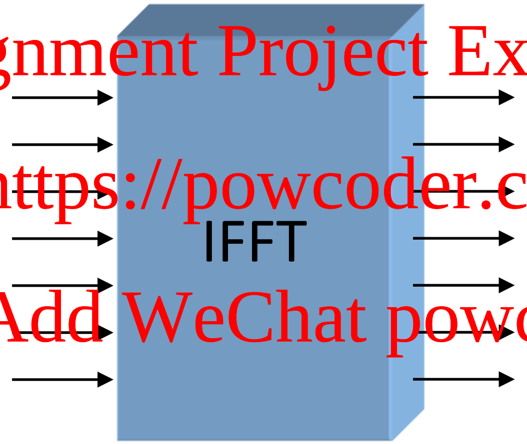
Signals at the input/output of the IFFT, N=32

OFDM Frequency Domain Symbol



16-QAM constellation

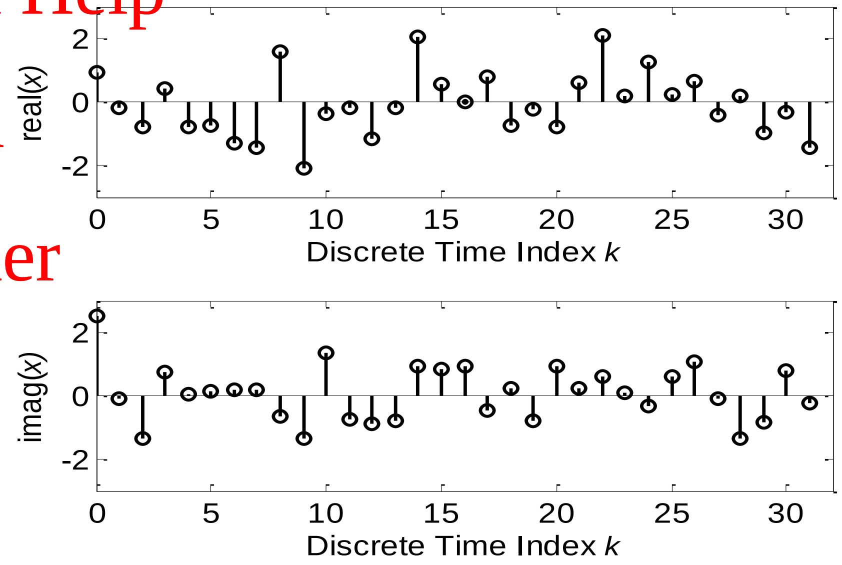




$$N = 32$$

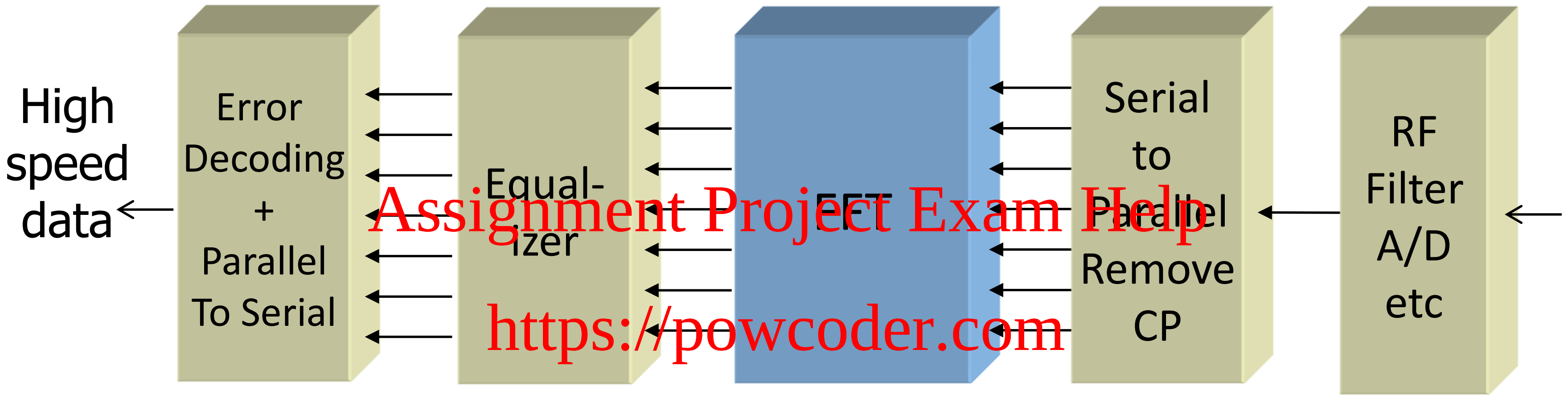
32 subcarriers

OFDM Time Domain Signal Samples



Output has Gaussian distribution for *N* >64

OFDM Receiver

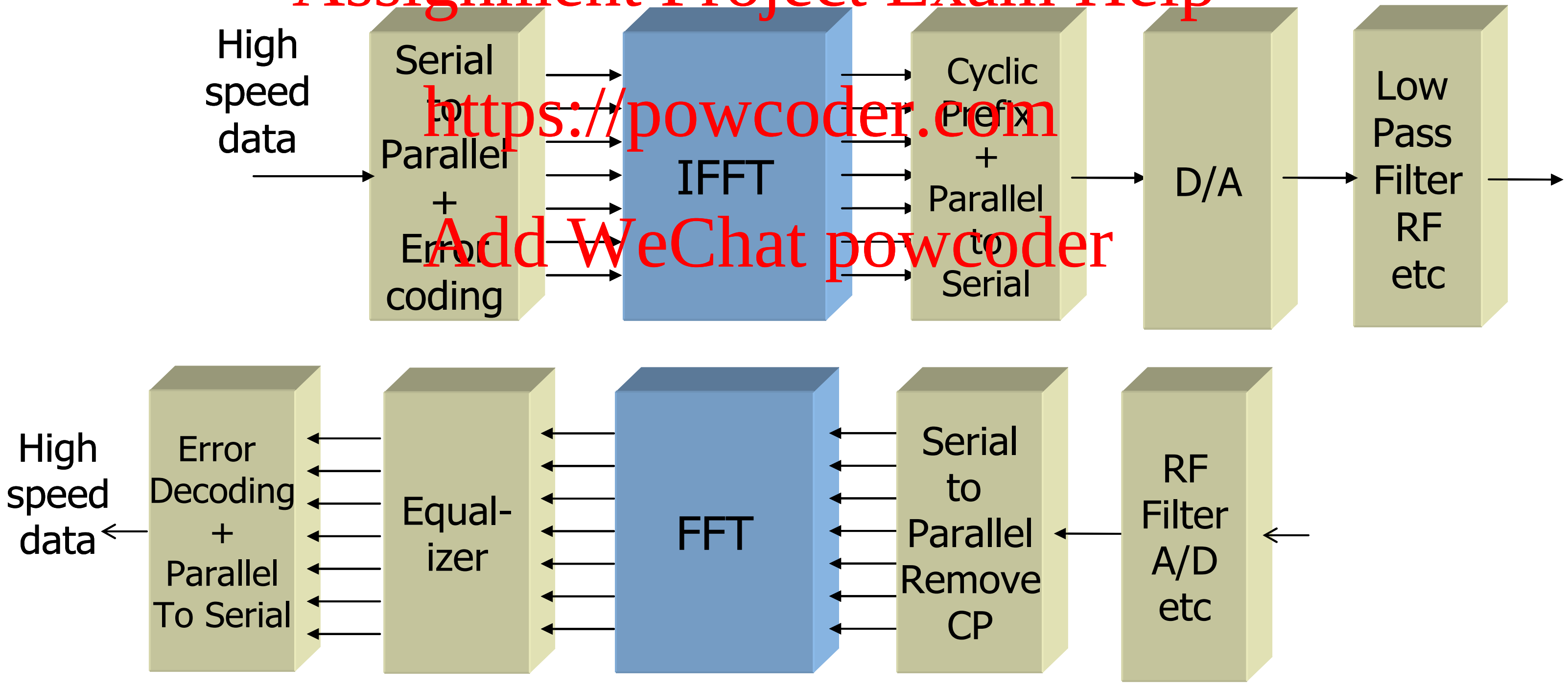


- Key component is the discrete Fourier transform (DFT/FFT)
- It **demultiplexes** the subcarriers and **demodulates** them

Transmitter and receiver

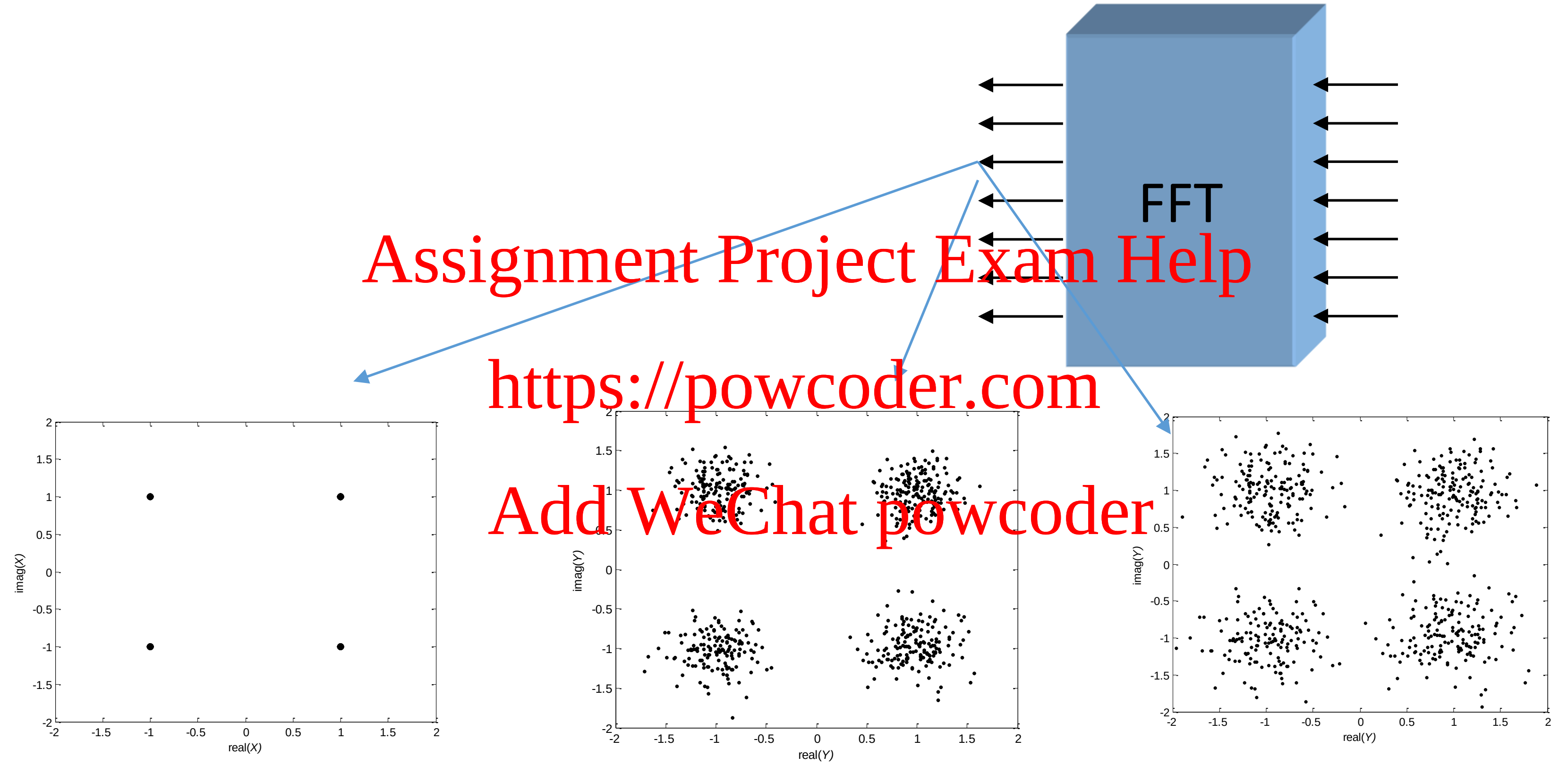
Each block in the receiver performs the ‘inverse’ of the corresponding transmitter function

Ideally what goes in at the transmitter comes out at the receiver



Reference from ECE 4024 by Prof. Jean Armstrong

Received Constellation – 4-QAM No distortion in channel

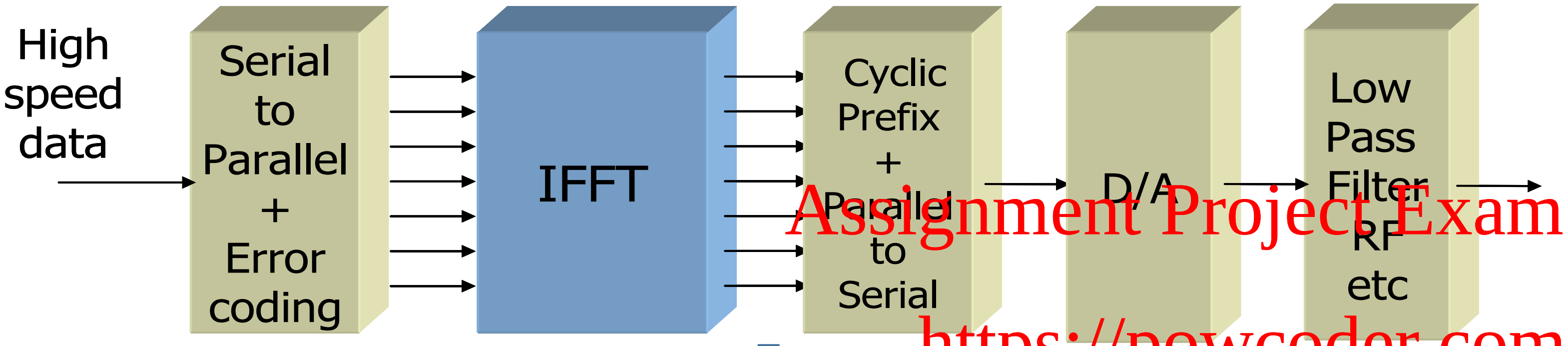


No noise

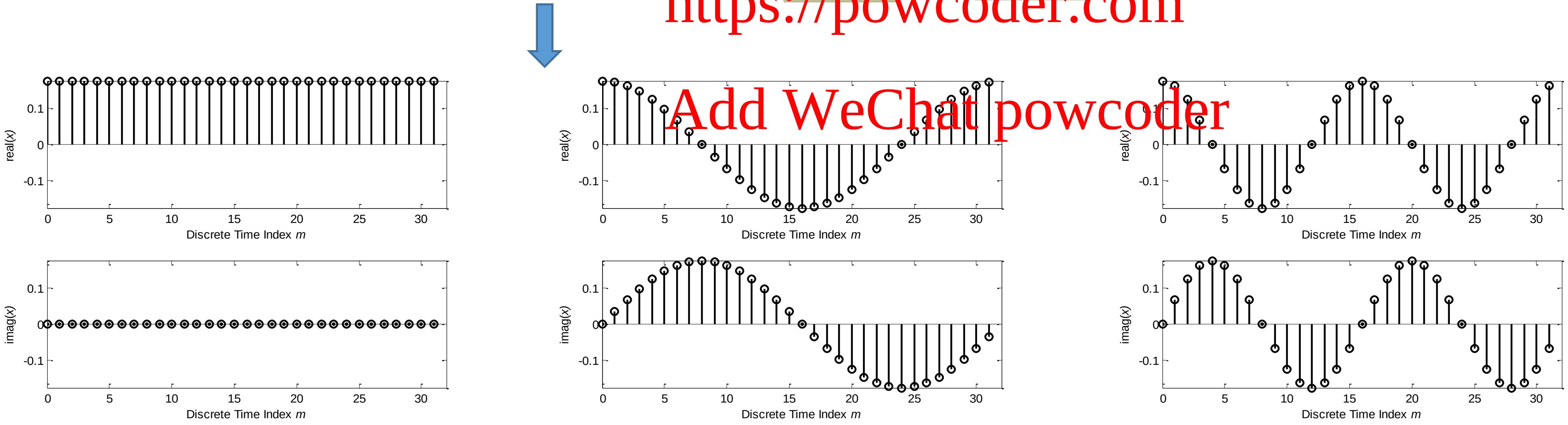
Increasing Noise levels (decreasing SNR)

Reference from ECE 4024 by Prof. Jean Armstrong

The Transmitter IFFT in more detail



Outputs when only one input is non zero and that input = 1 (real only)

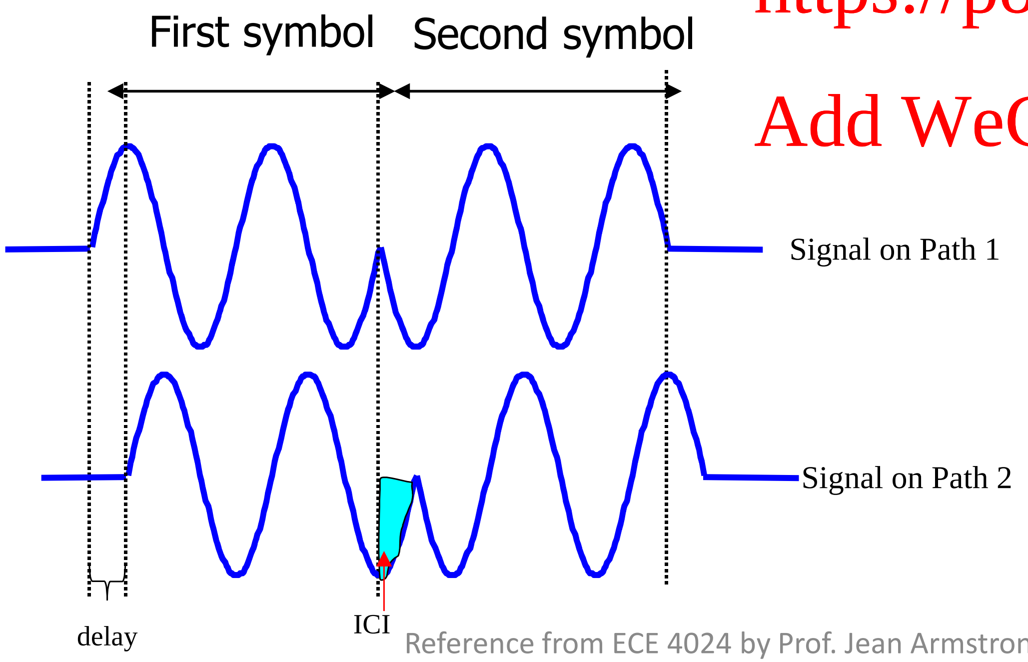


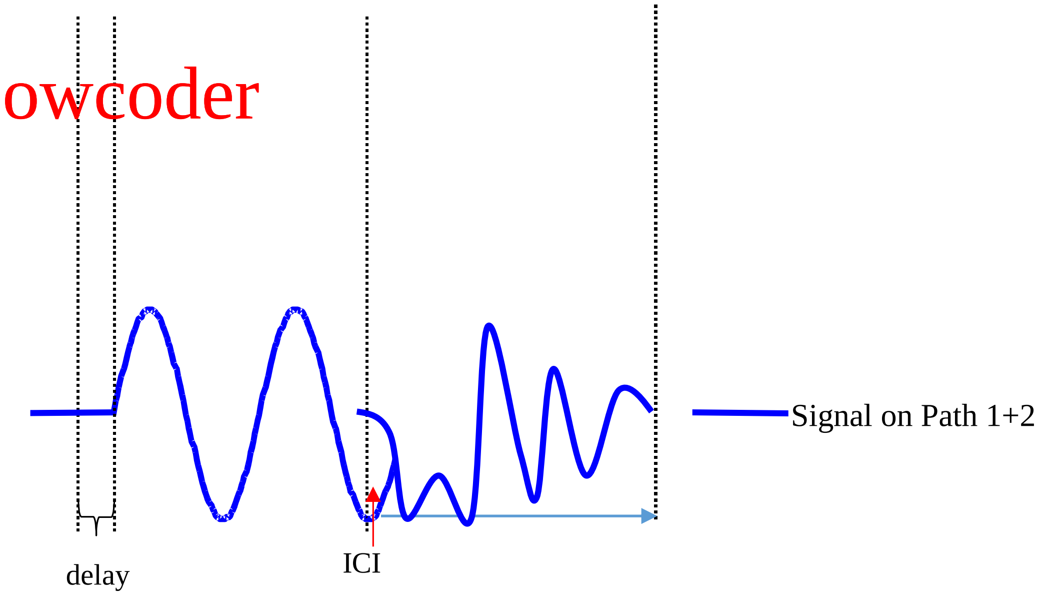
Zeroth Subcarrier **First Subcarrier** **Second Subcarrier**

OFDM in a multipath environment effect on one subcarrier

In Multipath propagation envirornment,

- Received signal in one symbol period is not a sinusoid
- Causes intercarrier interference (ICI)



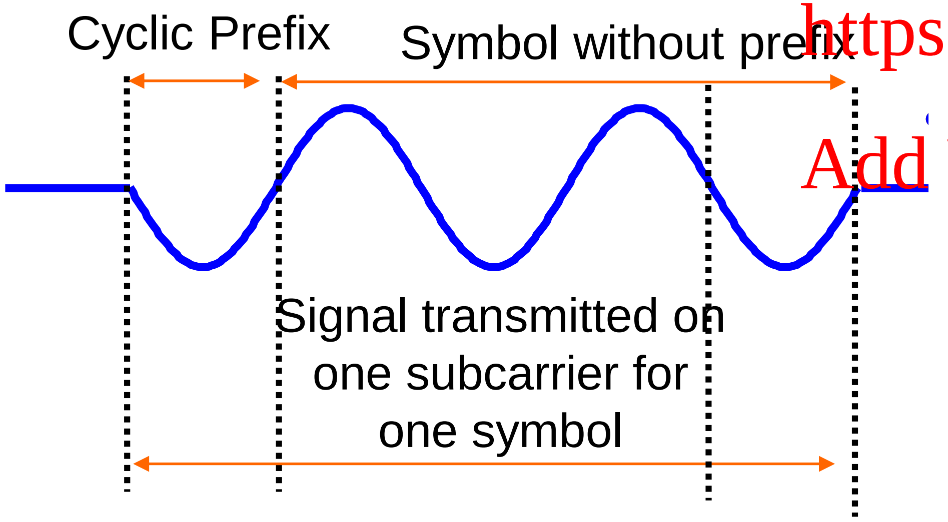


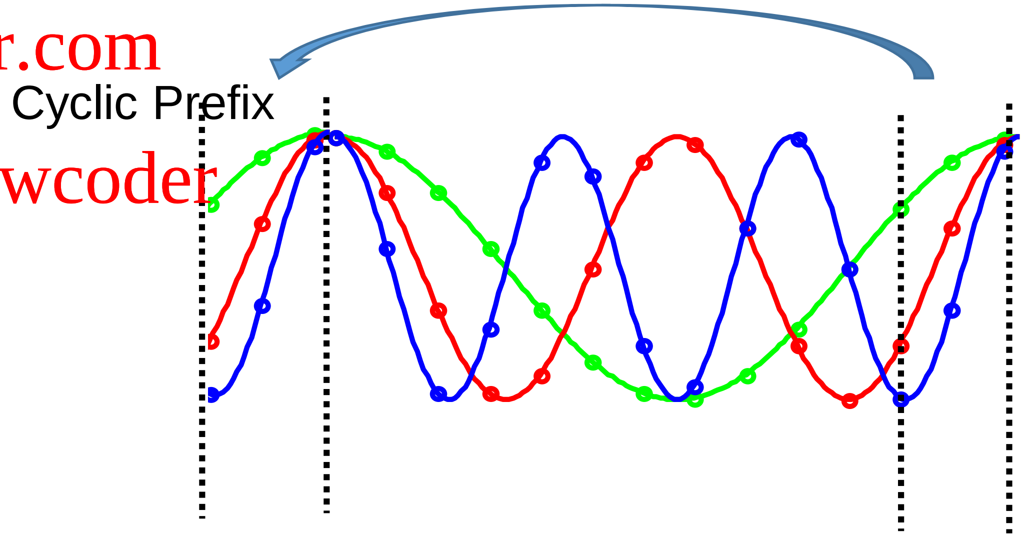
Reference from ECE 4024 by Prof. Jean Armstrong

Cyclic Prefix

Each symbol is cyclically extended

Some loss in efficiency as cyclic prefix carries no new information

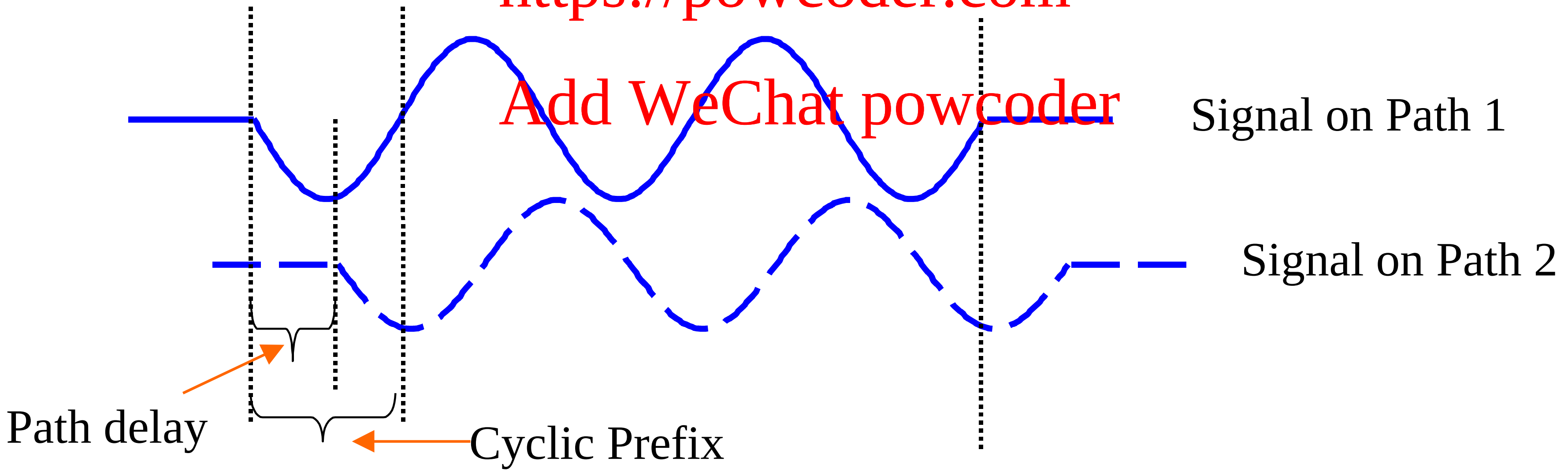




Effect of multipath on symbol with cyclic prefix

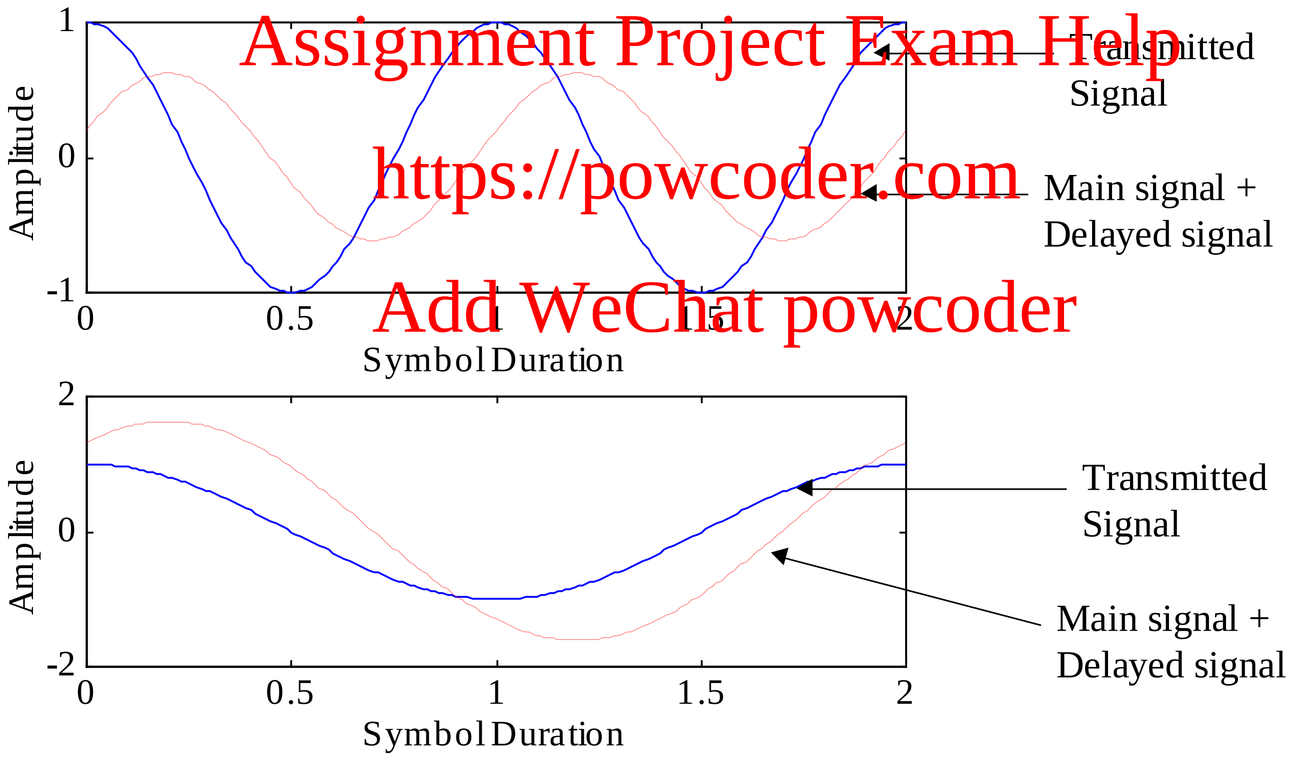
If multipath delay is less than the cyclic prefix

- no intersymbol or intercarrier interference
- amplitude may increase or decrease



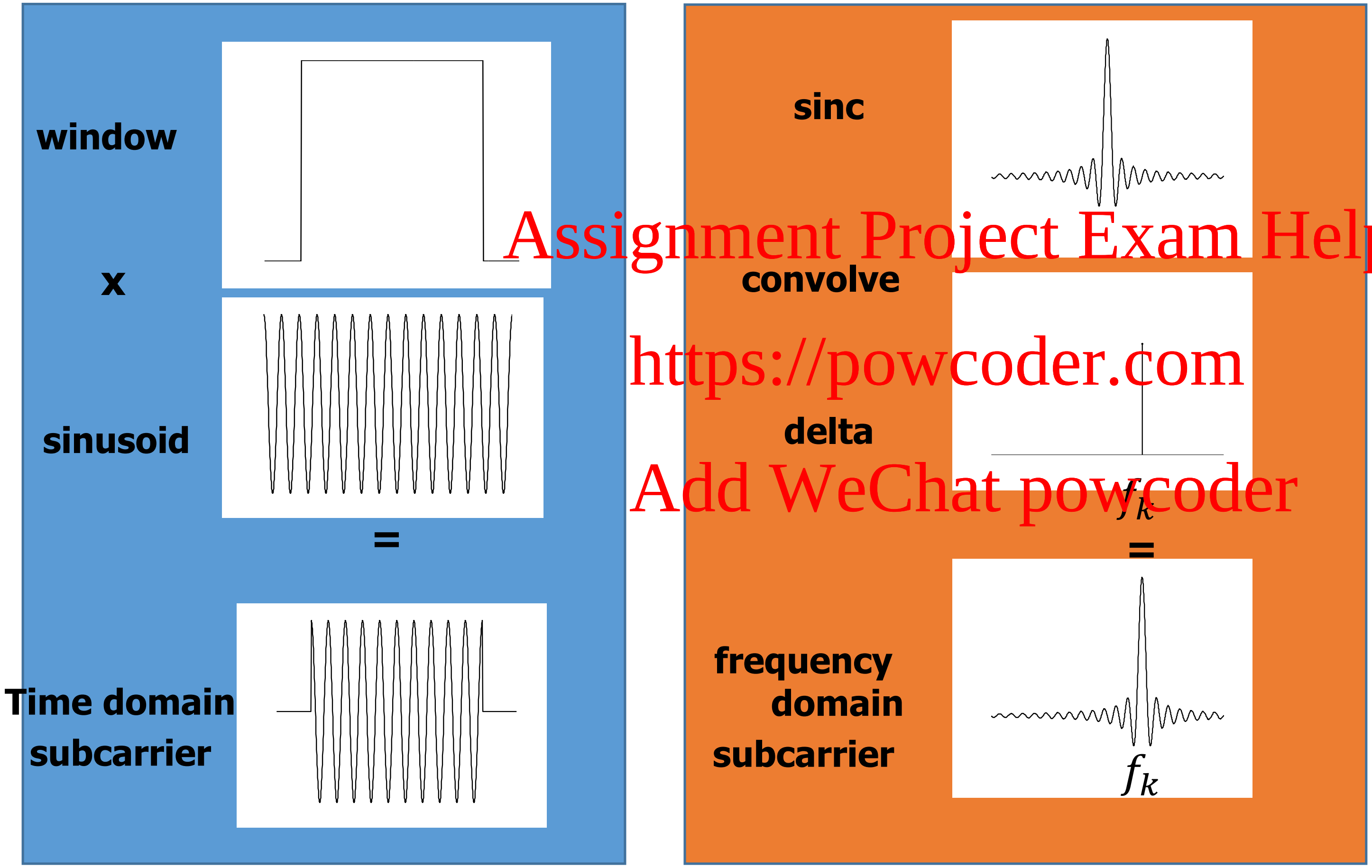
Reference from ECE 4024 by Prof. Jean Armstrong

Frequency selective fading



Reference from ECE 4024 by Prof. Jean Armstrong

Spectrum of an individual OFDM subcarrier



Time domain

Frequency domain

$$\frac{\sin(x)}{x} = \text{sinc}(x)$$

Shape of spectrum depends on shape of window

Width of spectrum depends on time duration of window

Centre frequency of spectrum depends on frequency of sinusoid

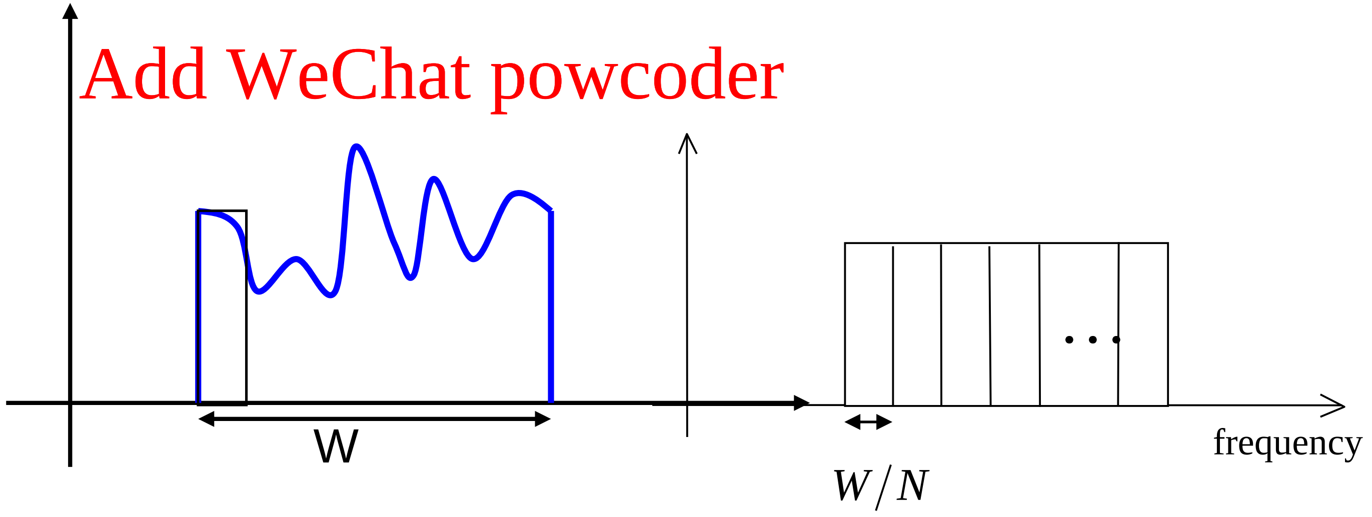
Reference from ECE 4024 by Prof. Jean Armstrong

Spectrum of Received Signal

Multipath fading causes some frequencies to be attenuated

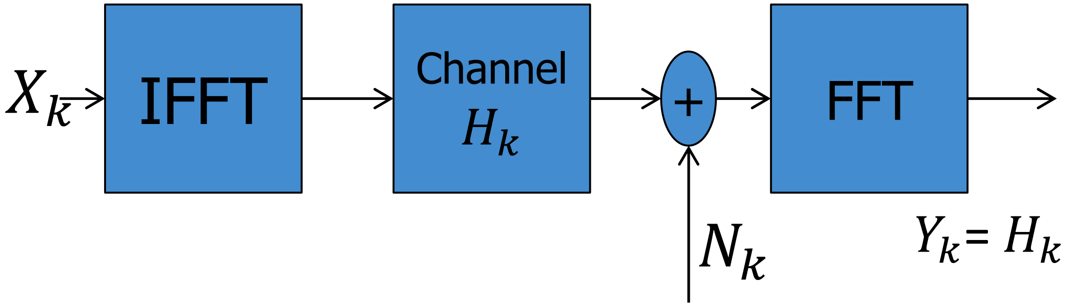
Fading is approximately constant over narrow band (sub-channel)

This is corrected in the receiver

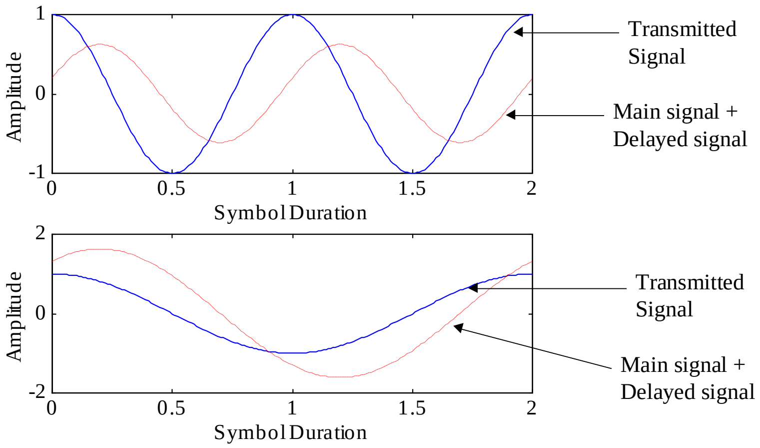


Amplitude and phase change
complex baseband equivalent channel

- Multipath delay causes change in amplitude and phase of each subcarrier
- Change depends on subcarrier frequency
- Can consider the channel 'seen' by the k'th subcarrier as a complex equivalent baseband channel with frequency response H_k
- Corrected in receiver by one complex multiplication per subcarrier
  - Multiply by $\frac{1}{H_k}$



$Y_k = H_k X_k + N_k$



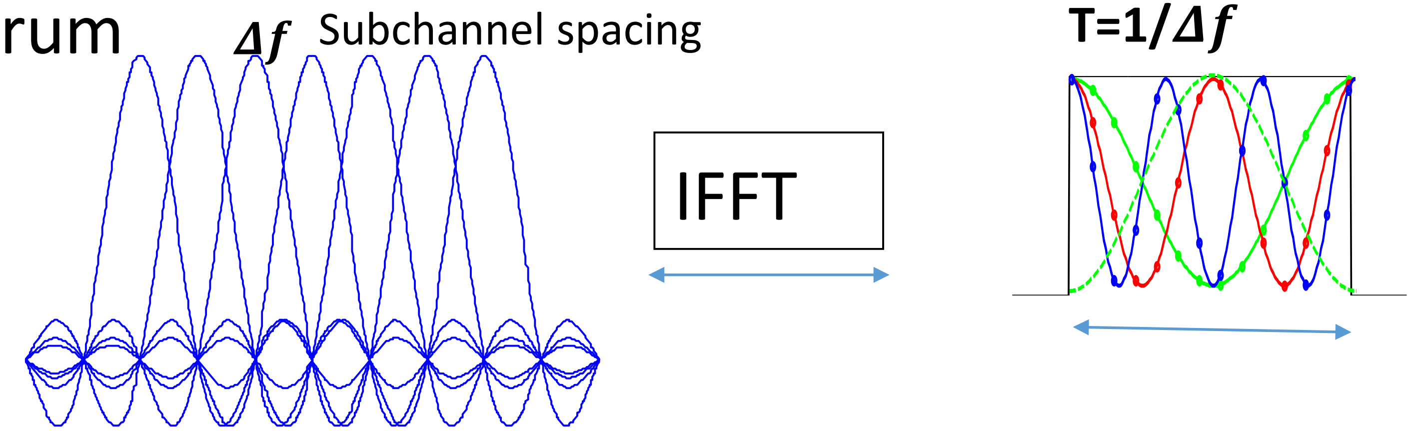
Reference from ECE 4024 by Prof. Jean Armstrong

Complex baseband equivalent of k'th subcarrier

OFDM with N sub carriers

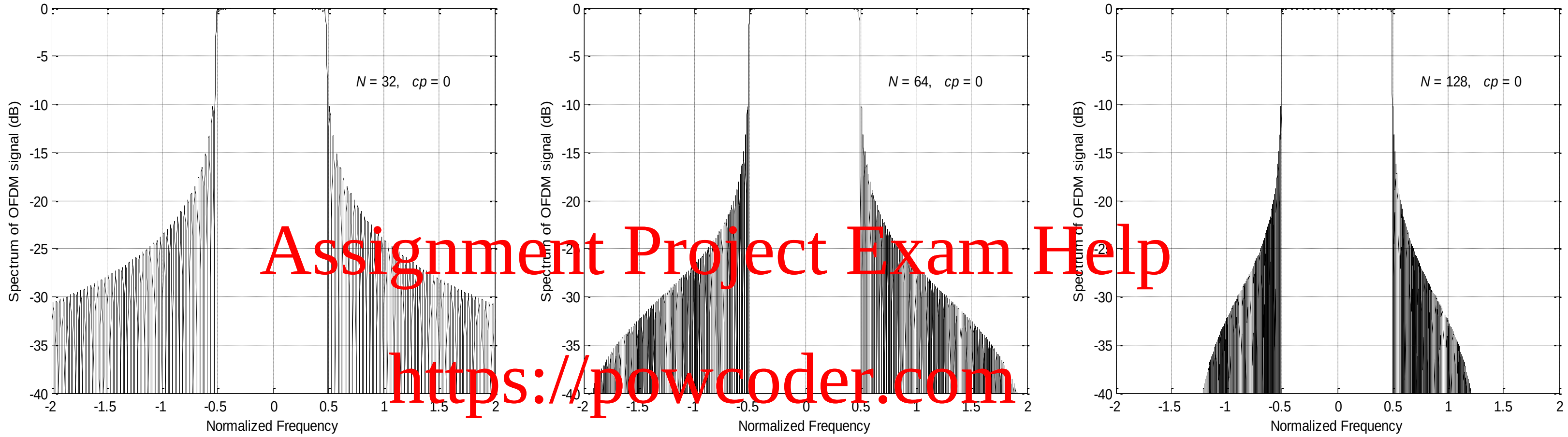
This convolution with delta function at f_0 results in the frequency shift of the Sinc pulse to f_0

OFDM spectrum



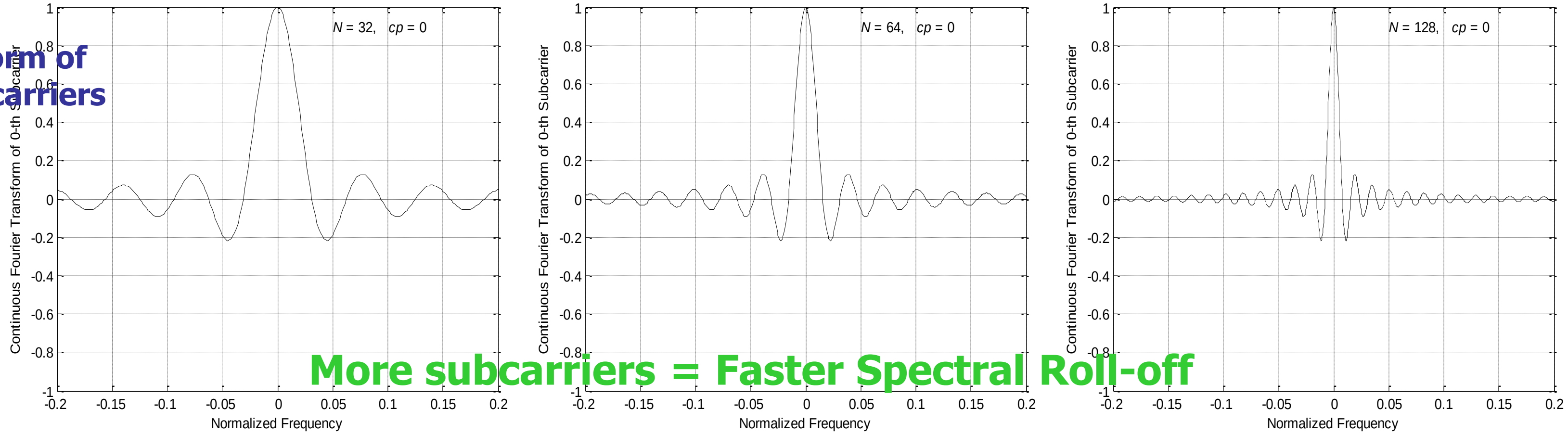
Spectrum – Effect of Number of subcarriers, *N* (no cyclic prefix)

OFDM SPECTRUM



$N = 32$ $N = 64$ $N = 128$

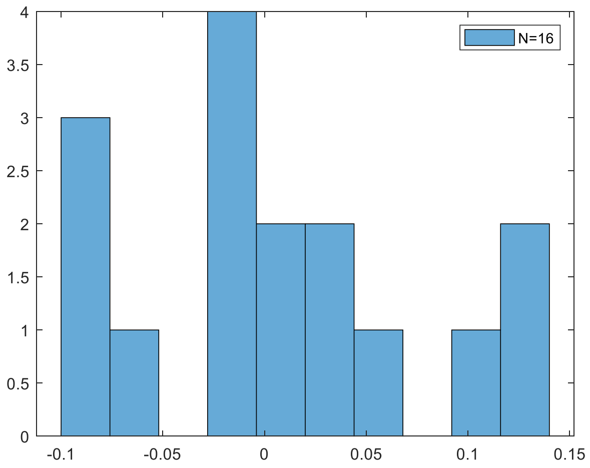
Fourier transform of individual subcarriers

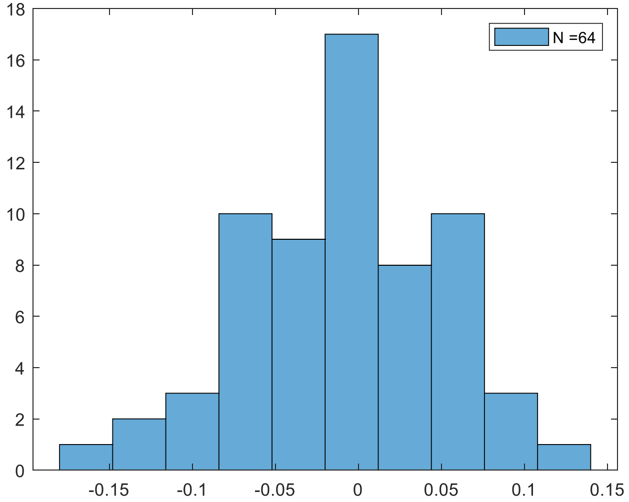


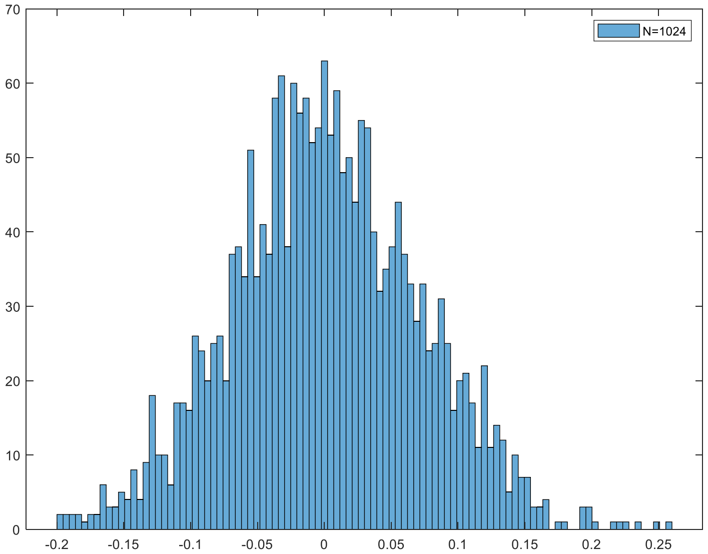
More subcarriers = Faster Spectral Roll-off

Reference from ECE 4024 by Prof. Jean Armstrong

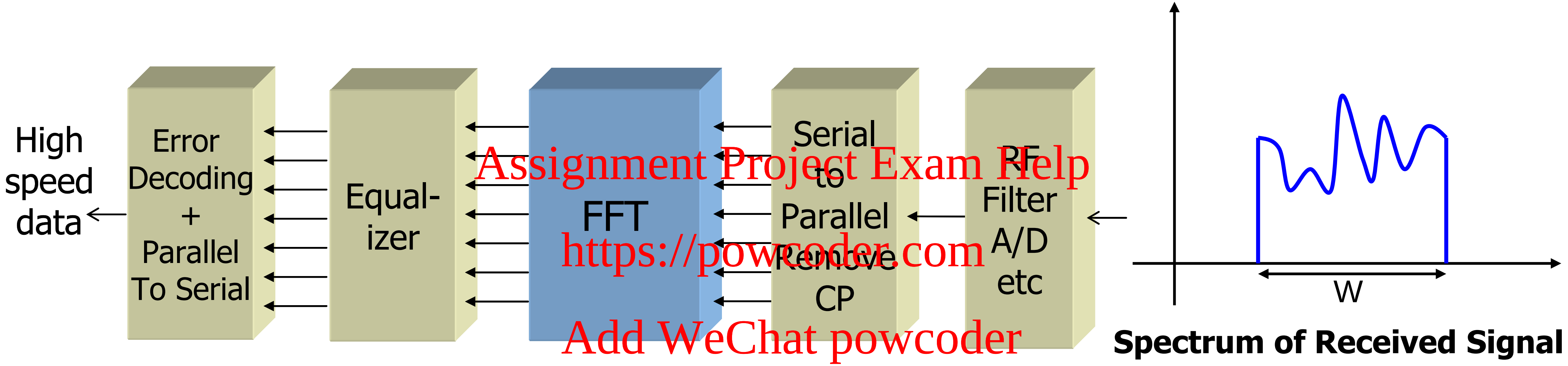
Histograms with N







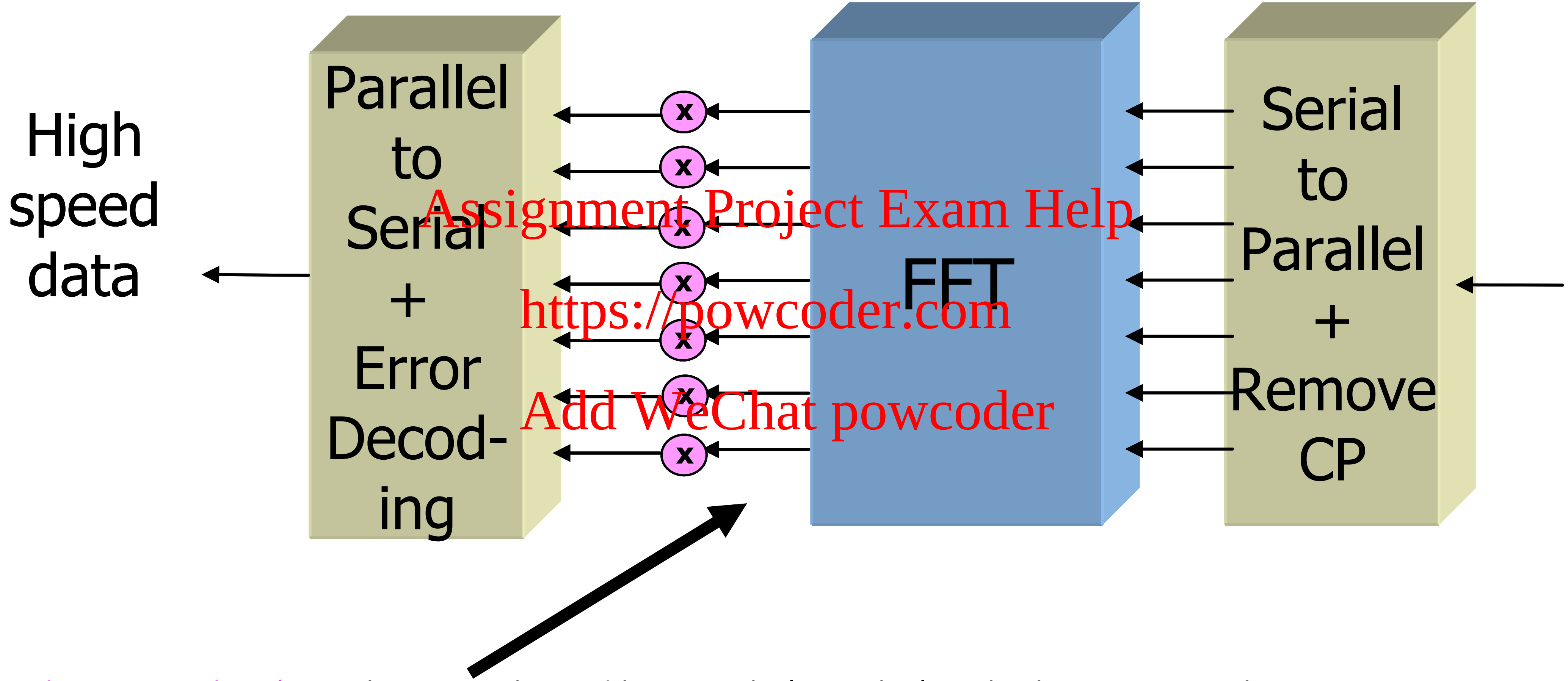
The Receiver Equalizer



Multipath fading affects the gain and phase of each subcarrier

The equalizer corrects the gain and phase of each subcarrier

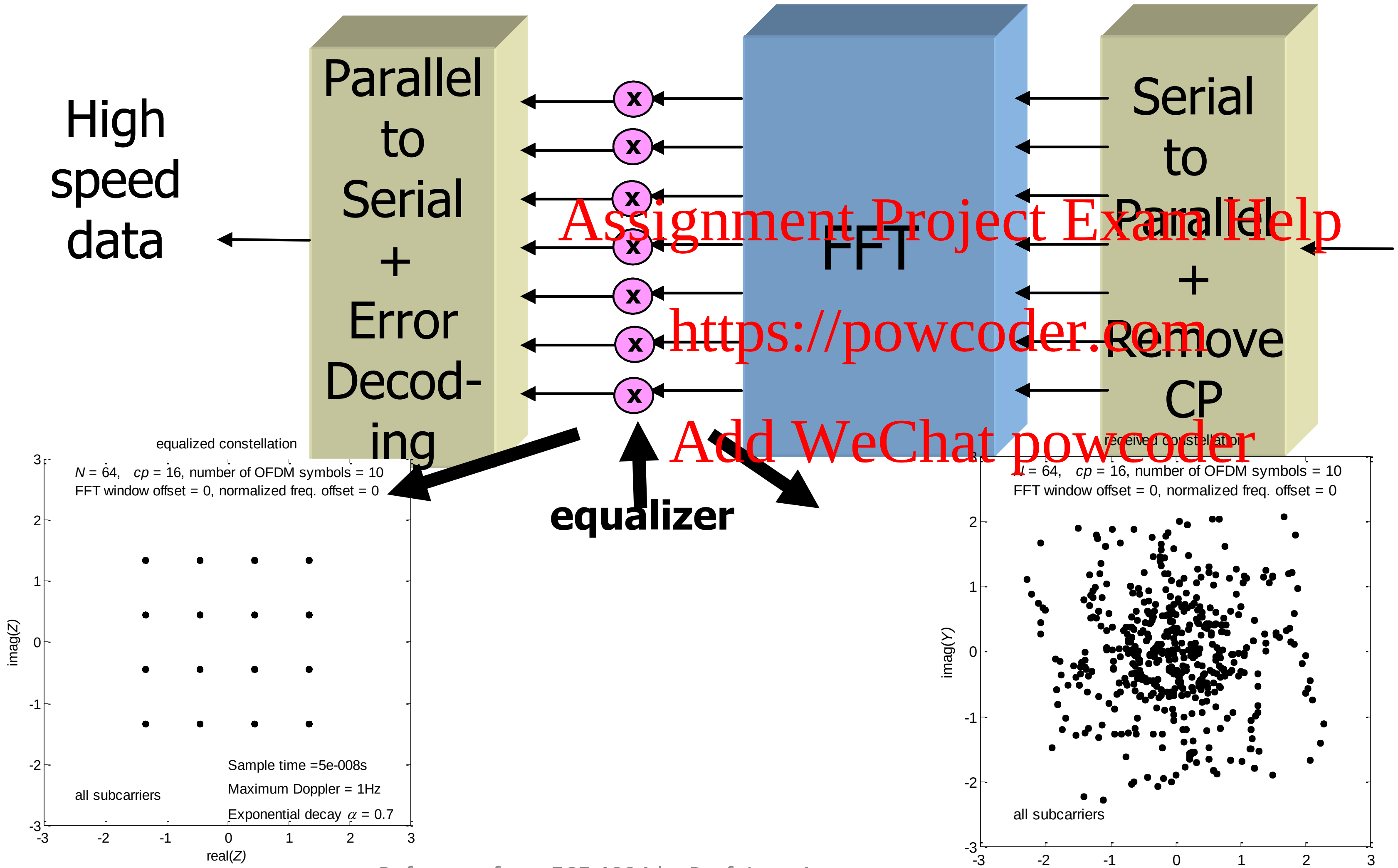
Equalization using Single Tap Equalizer



'Single Tap Equalizer' Equalization achieved by a single (complex) multiplication per subcarrier
Complexity increases linearly with data rate

Reference from ECE 4024 by Prof. Jean Armstrong

Equalization using Single Tap Equalizer



For a noiseless channel the signal is perfectly recovered by the single tap equalizer

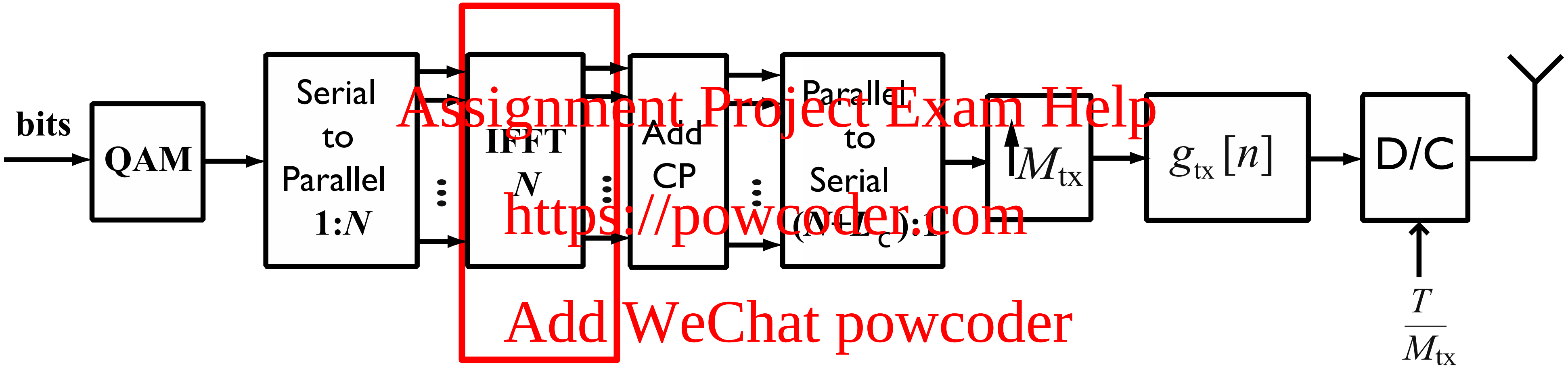
Reference from ECE 4024 by Prof. Jean Armstrong

Advantages of OFDM

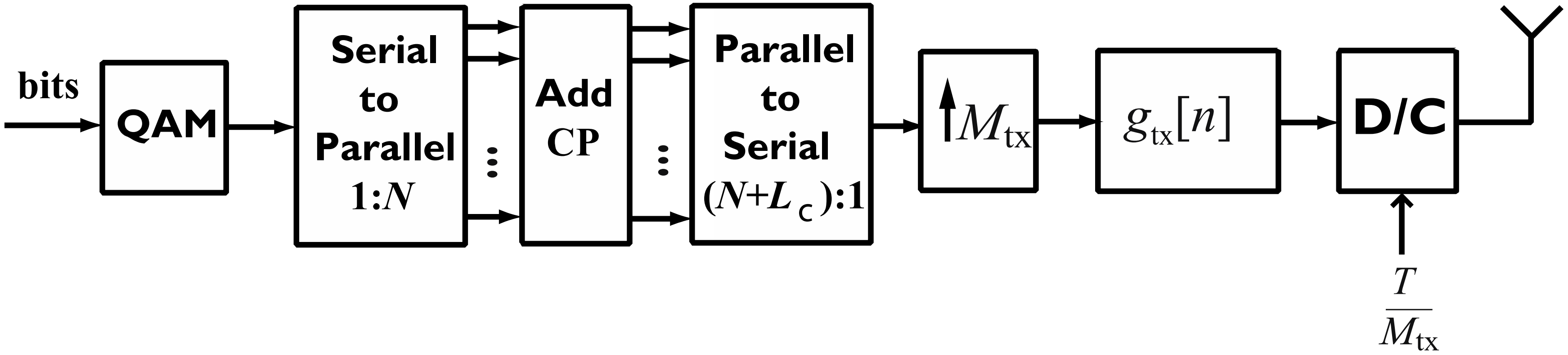
- Allows simple frequency domain equalization at the receiver
- Allows sophisticated types of adaptive modulation where information is adapted to the frequency response of the channel
- Obtains diversity against fading with error control coding

OFDM transmitter block diagram

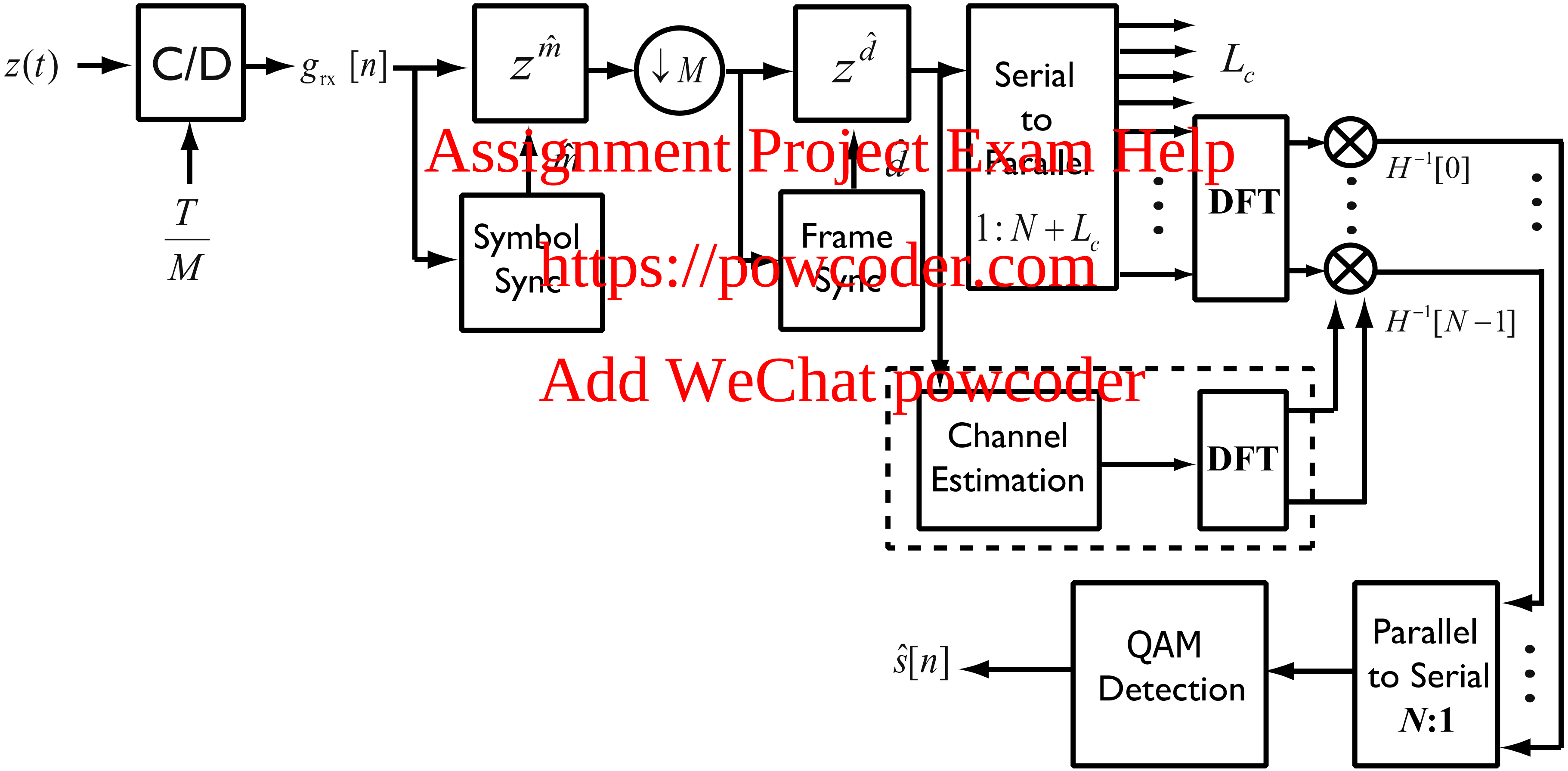
- For OFDM



- Compare with SC-FDE

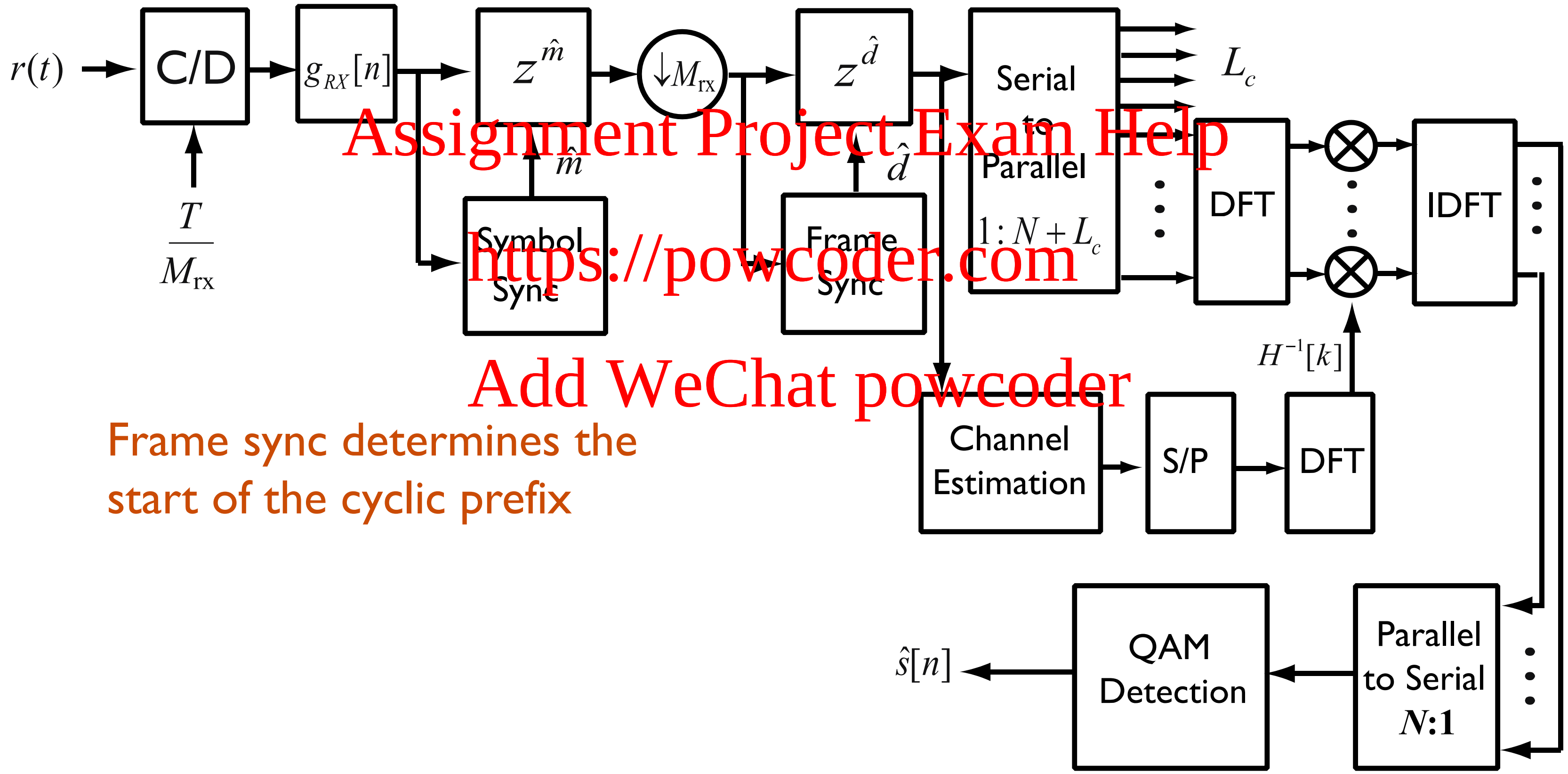


OFDM receiver block diagram



SC-FDE receiver

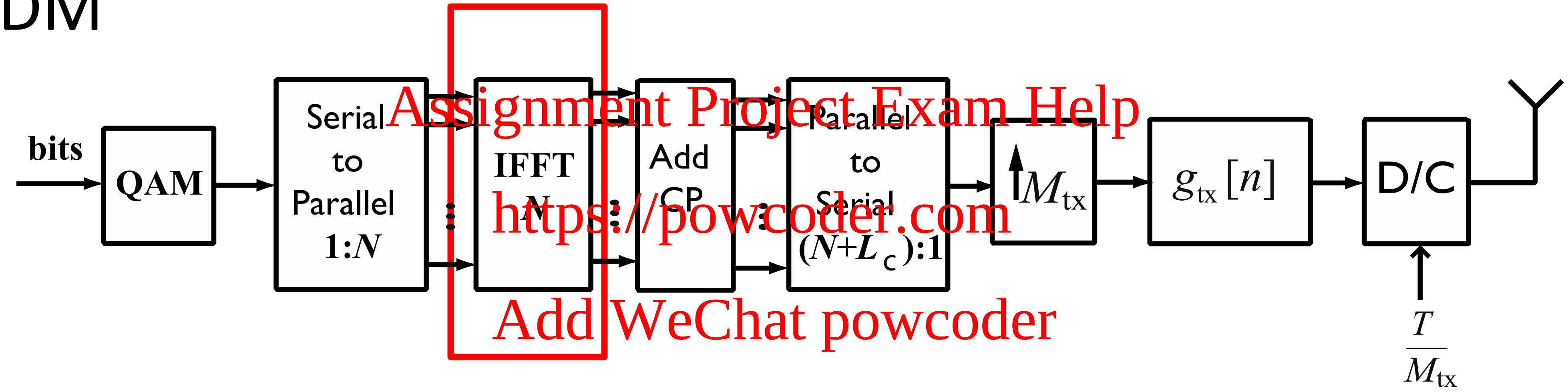
creates the blocks



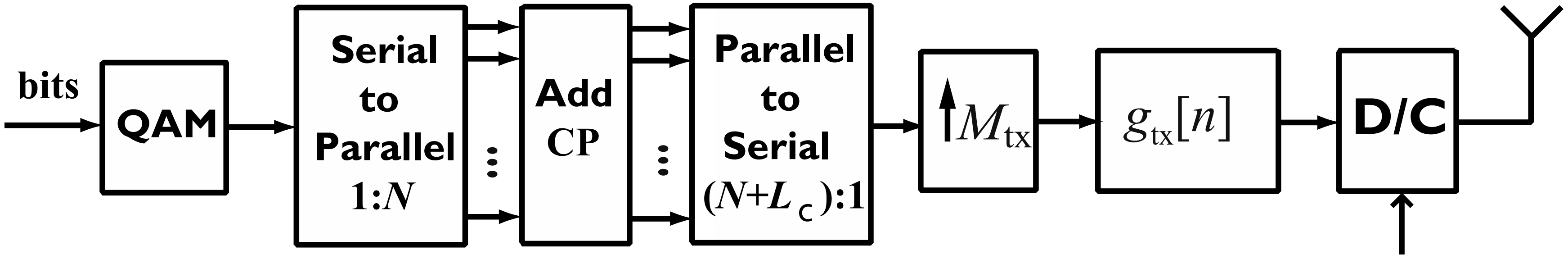
Frame sync determines the start of the cyclic prefix

OFDM transmitter block diagram

- For OFDM



- Compare with SC-FDE



OFDM example

Consider an OFDM system where the OFDM symbol period is 3.2μs, the cyclic prefix has length L_c = 64, and the number of subcarriers is N = 256. Find the sample period, the passband bandwidth (assuming that a sinc pulse-shaping filter is used), the subcarrier spacing, and the guard interval.

Answer:

The sample period T satisfies the relation T (256 + 64) = 3.2μs, so the sample period is T = 10ns.

Then, the bandwidth is $1/T$ = 100MHz. the subcarrier spacing is $1/(NT)$ = 390.625kHz.

Finally, the guard interval is $L_c T$ = 640ns.