**ECE5884 Wireless Communications – Assignment 2 19 Sept. 2022**

1. **([2 marks] Digital modulation)** Consider an 4-QAM (quadrature amplitude modulation) constellation signalling with equiprobable message symbols.

   (a) Propose a suitable Gray encoding for the 4-QAM constellation.

   (b) If the average energy of the 4-QAM constellation is $E_s$, write the minimum distance between constellation points, $d_{min}$, in terms of $E_s$.

   (c) Derive an expression of the symbol error rate (SER) for the 4-QAM modulation in terms of the average signal-to-noise power ratio (SNR) $\bar{\gamma}_s$ where $\bar{\gamma}_s = \frac{E_s}{N_0}$ and $N_0$ is the total noise power. All important steps of the derivation should be provided.

   (d) We assume that the channel experiences multipath fading which is denoted as a complex value $h$ and $|h|$ follows Rayleigh distribution with the unit power gain, i.e., $f_{|h|}(x) = 2x\,e^{-x^2}$. Derive an expression of the average symbol error rate (ASER) over Rayleigh fading channel for the 4-QAM modulation in terms of $\bar{\gamma}_s$. All important steps of the derivation should be provided.

   (e) **(MATLAB)** Verify analytical results in Part (c) and Part (d) by using MATLAB simulations. Compare SER of Part (c) and Part (d), and give reasons for your observations.

   (f) In general, what is your comment of the comparison between symbol error rate (SER) and bit error rate (BER) of the same digital modulation scheme? Give reasons. (No need any derivations!)

   (g) What is your comment of the comparison between SER of 4-QAM and SER of 16-QAM? Give reasons. (No need any derivations!)

2. [**2 marks**] An example of an OFDM system in Matlab is given below (in Figure 1).

```
%% CP-OFDM
clear all
numFFT =32; % FFT size define number of narrow subbands the OFDM spectrum is divided
Symbs=100; % Number of  OFDM symbols generated
Data_subcarriers =4;% Out of numFFT subcarriers available only Data_subcarriers are loaded with data
bitsPerSubCarrier=2; % 4-QAM
OverSamps=16; % Oversampling used for accurate estimate of average PSD
numSubbands=1; %number of Subbands
subbandOffset = numFFT/2- (Data_subcarriers*numSubbands)/2;%Center the band
Cyc_pre_len =input('CP length = '); % Cyclic prefix to enable periodicity in the transmission
j = sqrt(-1);
mapping = (1/sqrt(2))* [1+j  1-j  -1+j -1-j];  % Digital Constellation map
Number_bits= bitsPerSubCarrier*Data_subcarriers; % Bits carried by each OFDM symbol
frequency = [-(OverSamps*numFFT/2)  :1: (OverSamps*numFFT/2)-1]./(OverSamps*numFFT);
for bandIdx = 1:numSubbands % Outer loop to initialize the number of bands
for numSymbs = 1:Symbs
       Rand_indx = randi([1,4],Data_subcarriers,1);%  A vector Random numbers in the range 1 to 4 is generated
       symbolsIn = mapping( Rand_indx); % Generate QAM information symbols
       numSamp = length(symbolsIn);
        OFDMSymbol = zeros(numFFT, 1); % OFDM symbol initialized
        StartInd = subbandOffset; % Set the mapping to start at the location that you calculated.
        OFDMSymbol([StartInd+1:Data_subcarriers+StartInd  ]) = symbolsIn.';
        ifftOut = sqrt(numFFT)*ifft(ifftshift(OFDMSymbol)); %Use Matlab's built in function to ifft to calculate numFFt point transformation
        ifftOut = [ifftOut(end-Cyc_pre_len+1:end) ;ifftOut(1:end)]; % % Add CP of length Cyc_pre_len
        OFDMpsd_Avg(:,numSymbs) =(1/(numFFT)) *abs(fft(ifftOut,OverSamps*numFFT)).^2;  % Calculate the power spectral density
end
PSD_mean = mean(OFDMpsd_Avg,2); % Calculate the mean of power spectral density
plot(frequency,fftshift(10*log10(PSD_mean))); % Plot the spectrum
hold on
end
 axis([-0.5 0.5 -60 20]);
  xlabel('Normalized frequency','FontSize',14);
  ylabel('PSD (dBW/Hz)','FontSize',14);
 title(['CP-OFDM'])
```



Figure 1: Example Matlab code to generate the OFDM transmitter output PSD

Please examine the Matlab code comments to follow the steps involved in generating the average power spectral density (PSD) of the OFDM transmitter output. Basing on this code template, generate the following

(a) Set the FFT size equal to 256, generate OFDM spectra for CP length equal to 0, 8 and 64 on a single plot. What effect does increasing the CP length have on the passband of the of the PSD.

(b) For a fixed CP length (use FFT size/4), generate an overlay of spectra for FFTsize equal to 128 and 1024. What is the effect of increasing the FFT size on the out of band spectral characteristics.

3. [**2 marks**] An OFDM based system is operating at a carrier frequency of $2.4GHz, 20MHz$ bandwidth, and FFT size $N = 64$. Local oscillators in the hardware are accurate up to 0.25 ppm(parts per million). A maximum Doppler frequency shift of 250Hz is seen from the mobility of the channel.

   (a) Channel spacing (sub-carrier bandwidth) of the OFDM system?

   (b) Calculate the carrier frequency offset (CFO) introduced from the hardware inaccuracy.

   (c) Calculate the overall CFO from the hardware and the Doppler as a percentage of channel spacing.

   (d) If the bandwidth drops to 1MHz, what is the effect of the overall CFO as a percentage of channel spacing for this OFDM system.

4. **(Optional)** In Week 8 lecture you looked at three system models used for Wireless systems. These are SC-TDE, SC-FDE and OFDM. Compare and contrast the three systems in terms of their transmitter complexity, receiver complexity and suitability for transmission over propagation channels with large multipath delay spread.