ECE5884 Wireless Communications Quiz 8 Due: 19th September 2022

1. A channel $\mathbf{h}$ is defined by its impulse response with $h[0] = 0.5,\ h[1] = j/2, \text{and } h[2] = 0.4exp(j\pi/5)$. Use Matlab to calculate the Least Squares Equalizer of length $L_f = 5$.

   The LS Equalizer is given by

   $$\mathbf{f}_{\mathrm{LS},n_\mathrm{d}} = (\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*\mathbf{e}_{n_\mathrm{d}}$$

   squared error corresponding to a delay n_d is given by

   $$J[n_\mathrm{d}] = \mathbf{e}_{n_\mathrm{d}}^*(\mathbf{I} - \mathbf{H}(\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*)\mathbf{e}_{n_\mathrm{d}}$$

   a. Write down the channel convolution matrix $\mathbf{H}$ formed from the vector channel defined by $\mathbf{h}$.
   b. Generate a plot of the squared error as a function of delay parameter.
   c. What is the delay corresponding to the minimum squared error $\min J[n_d]$.
   d. Write down the f_{LS,n_d} using the delay value calculated in c.
2. Based on the Week 8 Lecture (Slides 54 and 57) discussion on OFDM, derive the spectrum of the OFDM symbol.
3. Use the Matlab code template given below for the OFDM transmitter to understand the effect of modulating data onto individual sub carriers.


```
clear all;
close all;
clc
N=16;      %Total number of sub carriers
k = input('input the index of the sub carrier to modulate');

%Data Mapping
mapping = (1/sqrt(2))*[1+j  1-j  -1+j  -1-j];  %Constellation
Xk = mapping(randi([1,4]));
x = zeros(1,N);
for m = 1:N
x(m)=(1/sqrt(N))*(Xk*exp(j*2*pi*(k-1)*(m-1)/N)); %IDFT equation
end
figure
stem(real(x))
figure
stem(imag(x))
```

   a. Generate plots of $k = 0, (N/2 - 1) \text{ and } N - 1$.
   b. Based on your observation which sub carrier is the fastest (rapid phase changes)?
4. Consider an OFDM system with N = 256 subcarriers in 5MHz of bandwidth, with a carrier of fc = 2GHz and a length L = 16 cyclic prefix. The 4 subcarriers near N/2 are nulled. A digital modulation with M= 64 is used in the modulation of sub-carriers.
   a. What is the subcarrier bandwidth?
   b. What is the length of the guard interval?
   c. Calculate the Number of data bearing sub carriers and number of QAM samples that are carrier by each OFDM symbol.
   d. Calculate the overall OFDM symbol duration including the cyclic prefix.
   e. Calculate the bit rate from the number of information bearing sub carriers and sample period.