ECE5884 Wireless Communications

Week 9: Diversity Techniques (Multiple-Antenna Systems)

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Course outline

This week: **Ref. Ch. 7 of [Goldsmith, 2005]**

- Week 1: Overview of Wireless Communications
- Week 2: Wireless Channel (Path Loss and Shadowing)
- Week 3: Wireless Channel Models
- Week 4: Capacity of Wireless Channels
- Week 5: Digital Modulation and Detection
- Week 6 : Performance Analysis
- Week 7: Equalization
- Week 8: Multicarrier Modulation (OFDM)
- Week 9: Multiple-Antenna Systems: Diversity Techniques
- Week 10: Multiple-Antenna Systems: MIMO Communications
- Week 11: Multiuser Systems
- Week 12: Guest Lecture (Emerging 5G/6G Technologies)

Introduction

- Independent signal paths have a low probability of experiencing deep fades simultaneously.
- Diversity: is to send the same data over independent fading paths/links. These independent paths/links are combined in such a way that the fading of the resultant signal is reduced.

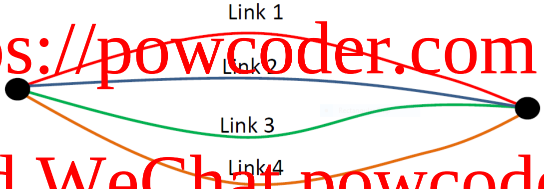
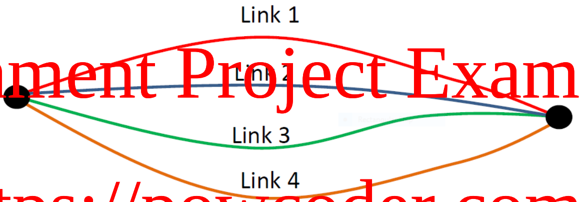


Figure 1: Information symbols are passed through multiple links, each of which fades independently.

- Reliable communication is possible as long as one of the links is strong.

Received signal (with single antenna)



Example: $h_1 = -1.22 + j0.67$; $h_2 = 0.5 + j2.3$; $h_3 = 1.2 - j0.7$; $h_4 = 0.45 - j2.2$

- If only Link 1 is available:

  $r = h_1 s + n \Rightarrow \gamma_1 = \frac{|h_1|^2 P_s}{N_0} = \frac{|1.4|^2 P_s}{N_0}$

- If all Links are available:

  $r = (h_1 + h_2 + h_3 + h_4) s + n \Rightarrow \gamma_{all} = \frac{|h_1 + h_2 + h_3 + h_4|^2 P_s}{N_0} = \frac{|0.9|^2 P_s}{N_0}$

- $\gamma_1 > \gamma_{all}$ - Do we really get benefits of having multiple paths?
- We need a smarter receiving architecture!

Received signal (with multiple antennas)

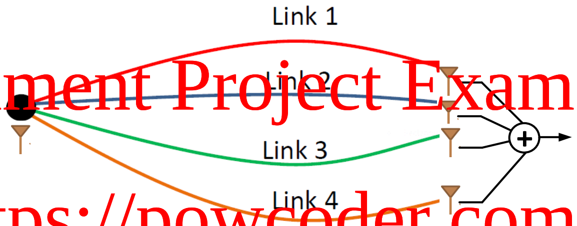
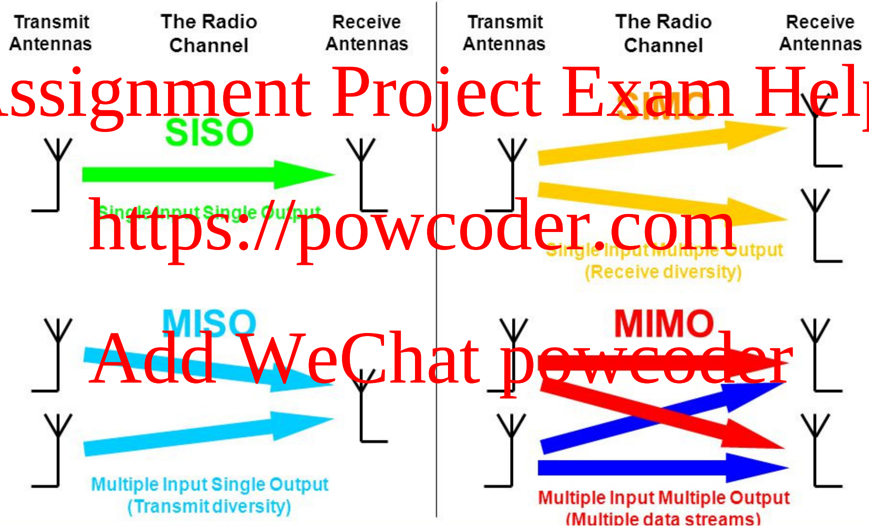


Figure 2: Single antenna Tx with multiple antennas Rx.

- If the antennas are spaced sufficiently far apart, it is unlikely that they all experience deep fades at the same time.
- h_1, h_2, h_3, h_4 are random values which change every coherence time.
- Example: By selecting the antenna with the strongest signal, a technique known as selection combining, we obtain a much better signal than if we had just one (fixed) antenna.

Multiple antennas techniques



Diversity

- A diversity scheme refers to a method for improving the reliability of a message signal by using two or more communication channels with different characteristics.
- Diversity techniques mitigate the effect of multipath fading – microdiversity
- We need independent fading paths: use antenna array where the elements of the array are separated in distance – space diversity.
    1. multiple receive antennas - receiver diversity
    2. multiple transmit antennas – transmitter diversity
- Channel state information (CSI) availability:
    1. CSI at Rx (will focus more on this!)
    2. CSI at Tx
- We also have Time Diversity and Frequency Diversity.

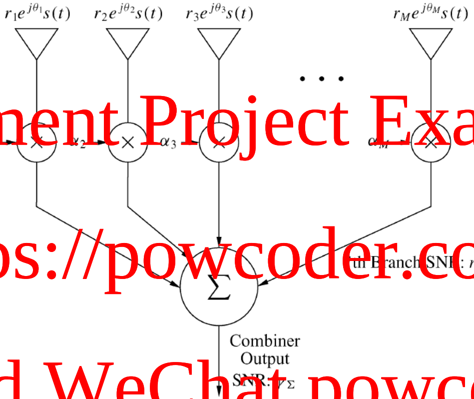


Figure 3: Linear combiner at the receiver with M-branch diversity.

- Linear combiner : the output of the combiner is just a weighted sum of the different fading paths or branches.
- CSI at Rx: The complex fading of the ith branch is $h_i = r_i e^{j\theta_i}$

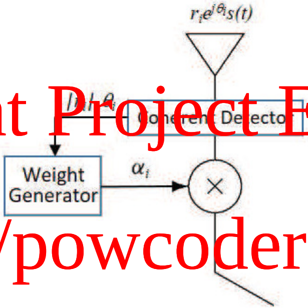
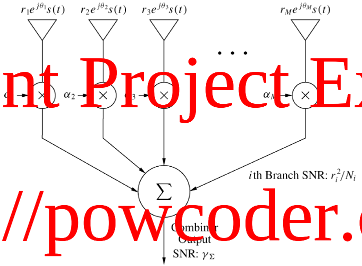


Figure 4: Branch coherent detection.

- The receiver knows h_is, i.e., amplitude: $r_i = |h_i|$ and/or phase: θ_i.
- Combining more than one branch signal requires co-phasing, where the phase θ_i of the ith branch is removed through multiplication by $\alpha_i = a_i e^{-j\theta_i}$ for some real-valued a_i.
- Without co-phasing, the branch signals would not add up coherently in the combiner.

Diversity/combining techniques

Techniques entail various trade-offs between performance/complexity.



1. Selection Combining (SC): the combiner outputs the signal on the branch with the highest SNR.
2. Maximal-Ratio Combining (MRC): the output is a weighted sum of all branches, and the weights (α_is) are determined to maximize the SNR.
3. Equal-Gain Combining (EGC): co-phases the signals on each branch and then combines them with equal weighting.
4. Threshold Combining: outputting the first signal whose SNR is above a given threshold γ_T.

Recap

1. Received signal over the ith channel, $i \in \{1, \cdots, M\}$:

$$y_i(t) = h_i\, s(t) + n_i(t) = r_i e^{j\theta_i}\, s(t) + n_i(t);\, i = 1, \cdots, M \tag{1}$$

2. Received SNR over the ith channel:

$$\gamma_i = \frac{|h_i|^2 P_s}{N_0} = |h_i|^2 \bar{\gamma} = g_i \bar{\gamma}; \quad i = 1, \cdots, M \tag{2}$$

3. Channel distribution: circularly symmetric complex Gaussian random variables with zero mean and unit variance $h_i \sim \mathcal{CN}(0, 1)$
   - $|h_i| \sim$ Rayleigh distribution, i.e.,

$$f_{|h_i|}(x) = 2x\, e^{-x^2} \tag{3}$$

   - $g_i = |h_i|^2 \sim$ Exponential distribution, i.e.,

$$f_{g_i}(x) = e^{-x} \tag{4}$$

   - SNR $\gamma_i = g_i \bar{\gamma} \sim$ Exponential distribution, i.e.,

$$f_{\gamma_i}(x) = \frac{1}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}} \text{ and } F_{\gamma_i}(x) = 1 - e^{-\frac{x}{\bar{\gamma}}} \tag{5}$$

Selection Combining (SC)

1. Selection combiner outputs the signal on the branch with the highest SNR.
2. As only one branch is used at a time, SC requires just one receiver that is switched into the active antenna branch.
3. End-to-end SNR of SC: the path output from the combiner has an SNR equal to the maximum SNR of all the branches.

$$\gamma_{SC} = \max_{i \in \{1,\cdots,M\}} (\gamma_1, \cdots, \gamma_M) \tag{6}$$

4. Selected antenna index

$$i^* = \arg \max_{i \in \{1,\cdots,M\}} (\gamma_1, \cdots, \gamma_M) \tag{7}$$

SC: Outage probability

- The SNR outage is

$$P_{o,SC} = \Pr\left(\gamma_{SC} < \gamma_{th}\right) = \Pr\left(\max\left(\gamma_1, \cdots, \gamma_M\right) < \gamma_{th}\right)$$
$$= \prod_{i=1}^{M} \Pr\left(\gamma_i < \gamma_{th}\right) = \prod_{i=1}^{M} F_{\gamma_i}\left(\gamma_{th}\right) \text{ where } \gamma_i = \frac{|h_i|^2 P_s}{N_0} = |h_i|^2 \bar{\gamma} = g_i \bar{\gamma} \quad (8)$$

- $|h_i|$ is the multipath fading channel, e.g., Rayleigh, Rician, Nakagami-m.
- For Nakagami-m fading channels:

$$f_{|h_i|}(x) = 2\left(\frac{m}{\Omega}\right)^m \frac{x^{2m-1}}{\Gamma(m)} e^{-\frac{mx^2}{\Omega}}; m \geq \frac{1}{2} \quad (9)$$

$$f_{g_i}(x) = \left(\frac{m}{\Omega}\right)^m \frac{x^{m-1}}{\Gamma(m)} e^{-\frac{mx}{\Omega}}; m \geq \frac{1}{2} \quad (10)$$

$$f_{\gamma_i}(x) = \left(\frac{m}{\Omega\bar{\gamma}}\right)^m \frac{x^{m-1}}{\Gamma(m)} e^{-\frac{mx}{\Omega\bar{\gamma}}}; m \geq \frac{1}{2} \quad (11)$$

$$F_{\gamma_i}(x) = 1 - \frac{\Gamma\left(m, \frac{mx}{\Omega\bar{\gamma}}\right)}{\Gamma(m)} \quad (12)$$

SC: Outage probability

- The SNR outage probability over Nakagami-m fading channels

$$P_{o,SC} = \prod_{i=1}^{M} F_{\gamma_i}(\gamma_{th}) \tag{13}$$

$$= \left[F_{\gamma_i}(\gamma_{th})\right]^M \quad \text{for i.i.d. channels} \tag{14}$$

$$= \left[1 - \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\Omega\bar{\gamma}}\right)}{\Gamma(m)}\right]^M \tag{15}$$

- The SNR outage probability behavior at high SNR regime, i.e., $\bar{\gamma} \to \infty$. Let's use $\Omega = 1$.

$$\lim_{\bar{\gamma}\to\infty} P_{o,SC} = \lim_{\bar{\gamma}\to\infty} \left[1 - \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\bar{\gamma}}\right)}{\Gamma(m)} \right]^M \tag{16}$$

- By using $\lim_{x\to 0} \Gamma[n, x] \approx \Gamma[n] - \frac{x^n}{n}$,

$$\lim_{\bar{\gamma}\to\infty} P_{o,SC} \approx \left[1 - \frac{\left(\Gamma[m] - \frac{\left(\frac{m\gamma_{th}}{\bar{\gamma}}\right)^m}{m}\right)}{\Gamma(m)} \right]^M = \left(\frac{m^{m-1}\gamma_{th}^m}{\Gamma(m)} \right)^M \bar{\gamma}^{-mM} \tag{17}$$

Diversity order and array gain

1. For large enough SNR ($\bar{\gamma} \to \infty$), the outage probability P_o as a function of $\bar{\gamma}$ can be written as

$$P_o \approx (G_c\bar{\gamma})^{-G_d} \text{ or } P_o \approx G_c\bar{\gamma}^{-G_d} \tag{18}$$

   where G_c is termed the coding gain or array gain, and G_d is referred to as the diversity gain, diversity order, or simply diversity.

2. The diversity order G_d determines the slope of the outage versus average SNR curve, at high SNR, in a log-log scale.
3. The array gain G_c (in dB) determines the shift of the curve in SNR relative to a benchmark outage curve of $\bar{\gamma}^{-G_d}$.

4. When the diversity order equals the number of independent fading paths that are combined via diversity, the system is said to achieve full diversity order.

SC: Diversity order and array gain

1. For large enough SNR ($\bar{\gamma} \to \infty$), the outage probability P_o as a function of $\bar{\gamma}$ can be written as

$$P_o \approx \left(G_c \bar{\gamma}\right)^{-G_d} \text{ or } P_o \approx G_c \bar{\gamma}^{-G_d} \tag{19}$$

where G_c is termed the coding gain or array gain, and G_d is referred to as the diversity gain, diversity order, or, simply diversity.

2. Previous example:

$$\lim_{\bar{\gamma} \to \infty} P_{o,SC} \approx \left(\frac{m^{m-1} \gamma_{th}^{m}}{\Gamma(m)} \right)^{M} \bar{\gamma}^{-mM} \tag{20}$$

- Diversity order: $G_d = m\,M$ which is full diversity order.
- Array gain: $G_c = \left(\frac{m^{m-1} \gamma_{th}^{m}}{\Gamma(m)} \right)^{M}$

Maximal-Ratio Combining (MRC)

1. MRC output is a weighted sum of all branches, so the α_i are all nonzero, and the weights are determined to maximize the combiner output's SNR.
2. For a branch with $h_i = r_i e^{j\theta_i}$,
   - The signals are co-phased: $e^{-j\theta_i}$
   - The optimal weight to maximize SNR is: $a_i = r_i$
   - $\alpha_i = r_i e^{-j\theta_i}$
3. End-to-end SNR of MRC: the SNR of the combiner output is the sum of SNRs on each branch.

$$\gamma_{MRC} = \sum_{i=1}^{M} \gamma_i \tag{21}$$

MRC: Outage probability

- The SNR outage is

$$P_{o,MRC} = \Pr\left(\gamma_{MRC} < \gamma_{th}\right) = \Pr\left(\sum_{i=1}^{M} \gamma_i < \gamma_{th}\right) \quad (22)$$

- For Rayleigh fading channels: i.i.d. Rayleigh fading on each branch with equal average branch SNR $\bar{\gamma}$, the distribution of γ_{MRC} (which is a sum of i.i.d. exponential RVs) is
  - a chi-square distribution with $2M$ degrees of freedom, expected value $\bar{\gamma}_{MRC} = M\bar{\gamma}$ and variance $2M\bar{\gamma}$.
  - a gamma distribution with shape parameter M and scale parameter $\bar{\gamma}$.

$$f_{\gamma_{MRC}}(x) = \frac{x^{M-1} e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}^M (M-1)!} \quad (23)$$

$$F_{\gamma_{MRC}}(x) = 1 - \frac{\Gamma\left(M, \frac{x}{\bar{\gamma}}\right)}{\Gamma(M)} = 1 - e^{-\frac{x}{\bar{\gamma}}} \sum_{k=0}^{M-1} \frac{\left(\frac{x}{\bar{\gamma}}\right)^k}{k!} \quad (24)$$

MRC: Outage probability

- The SNR outage probability over i.i.d. Rayleigh fading channels

$$P_{o,MRC} = \Pr\left(\sum_{i=1}^{M} \gamma_i < \gamma_{th}\right) \tag{25}$$

$$= 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}}} \sum_{k=0}^{M-1} \frac{\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^k}{k!} \tag{26}$$

Equal-Gain Combining (EGC)

1. EGC co-phases the signals on each branch and then combines them with equal weighting.
2. For a branch with $h_i = r_i e^{j\theta_i}$,
   - The signals are co-phased: $e^{-j\theta_i}$
   - The weight is: $a_i = 1$
   - $c_i = e^{j\theta}$

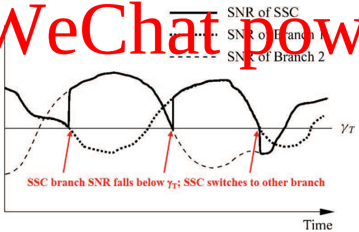
3. End-to-end SNR of EGC: the SNR of the combiner output is

$$\gamma_{EGC} = \frac{P_s}{MN_0}\left(\sum_{i=1}^{M}|h_i|\right)^2 = \frac{\bar{\gamma}}{M}\left(\sum_{i=1}^{M} r_i\right)^2 \tag{27}$$

4. The distribution PDF and CDF of γ_{EGC} do not exist in closed form for $M > 2$.

Threshold Combining

1. Select the the first signal whose SNR is above a given threshold γ_t.
2. Once a branch is chosen, the combiner outputs that signal as long as the SNR on that branch remains above the desired threshold.
3. If the SNR on the selected branch falls below the threshold, the combiner switches to another branch (e.g., switch randomly to another branch).
4. There are several criteria the combiner can use for determining which branch to switch. E.g.,
   - Switch-and-stay combining (SSC): Switching when the SNR falls below a threshold does not always select the branch with the highest SNR.



References

A. Goldsmith, *Wireless Communications*, Cambridge University Press, USA, 2005.

Thank You!

See you again ☺