ECE5884 Wireless Communications

Week 3 Workshop: Wireless Channel Models

Saman Atapattu

ARC Future Fellow at The University of Melbourne
Sessional Lecturer at Monash University

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Course outline

This week: Ref. Ch. 3 of [Goldsmith, 2005]

- Week 1: Overview of Wireless Communications
- Week 2: Wireless Channel (Path Loss and Shadowing)
- Week 3: Wireless Channel Models
- Week 4: Capacity of Wireless Channels
- Week 5: Digital Modulation and Detection
- Week 6: Performance Analysis
- Week 7: Equalization
- Week 8: Multicarrier Modulation (OFDM)
- Week 9: Diversity Techniques
- Week 10: Multiple-Antenna Systems (MIMO Communications)
- Week 11: Multiuser Systems
- Week 12: Guest Lecture (Emerging 5G/6G Technologies)

Doppler effect and delay

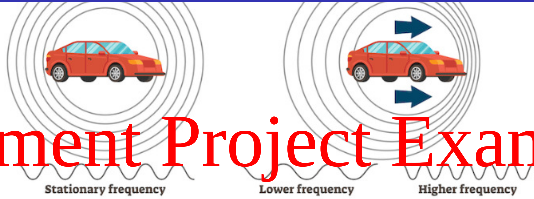


Figure 1: Illustration of the Doppler effect.

$$\text{Doppler frequency: } f_D = \frac{v}{\lambda}\cos\theta \quad \text{where } \lambda = \frac{c}{f_c} \text{ and } c = 3\times 10^8 m/s \tag{1}$$

1. Doppler effect.
2. Scatters.

$$\text{Received signal} \quad r(t) = \Re\left[\left(\sum_{i=0}^{N(t)-1} \alpha_i(t) e^{-j\phi_i(t)} u(t-\tau_i(t))\right) e^{j2\pi f_c t}\right] \tag{2}$$

$\alpha_i(t)$ is fading (also a function of path loss and shadowing). $\phi_i(t)$ depends on delay and Doppler. These two random processes are independent.

Fast/slow fading (w.r.t. time)

Figure 2: Received signal.

- In Coherence time (T_c), channel is not varying.

1. Fast fading: $T_c < T_s$ where T_s is the transmitted symbol duration.
2. Slow fading: $T_c \gg T_s$, e.g. Shadowing (Log-normal model).

Flat/frequency-selective fading (w.r.t. frequency)

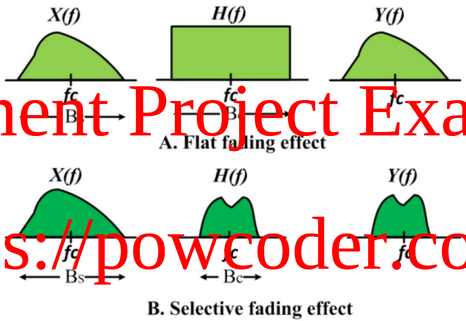


Figure 3: Wireless channel as a filter.

- In coherence bandwidth (B_c), channel response is not varying.

1. Flat fading: $B_s \ll B_c$ where B_s is the signal bandwidth.
2. Frequency-selective fading: $B_s \gg B_c$, OFDM (Week 8).

Intersymbol interference (ISI)

ISI is a form of distortion of a signal in which one symbol interferes with subsequent symbols.

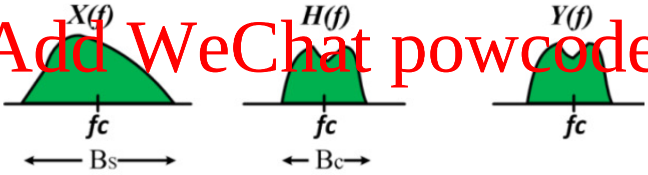
(a) One symbol Tx (No ISI). (b) Four symbols Tx (ISI).

Figure 4: Illustration of ISI effect.

- Send the next symbol after the delay spread, T_m, to avoid ISI.

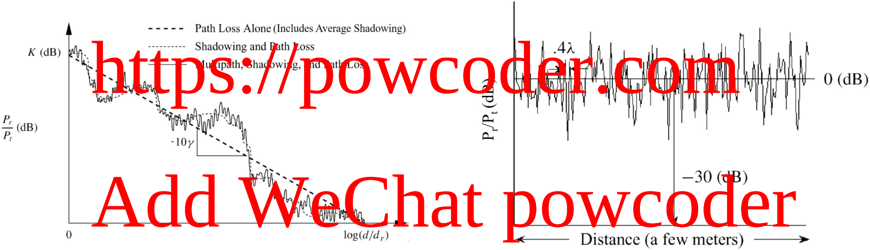
https://www.telecomhall.net/t/what-is-isi-inter-symbol-interference-in-lte/6370

1. Narrowband communications use a narrow bandwidth; are used in a slower form of communication as we allow a longer time for a symbol.
   - $T_m \ll T_s$.
2. Wideband communications use a higher bandwidth; apply Wifi, 4G LTE and beyond, HSPA.
   - $T_m \gg T_s$
   - OFDM (Week 8)



Multipath fading

1. Fast fading: $T_s \ll T_c$
2. Flat fading: $B_s \ll B_D$
3. Narrowband comm.: $T_m \ll T_s$



(a) Combined all.

(b) Narrowband fading.

Figure 5: Ref. Ch. 3 of [Goldsmith, 2005].

System model

$$\text{The received signal:} \quad r(t) = h\,s(t) + n(t) \tag{3}$$

- h – the multipath channel gain, usually a complex number; $s(t)$ – the transmit signal with P_s power; and $n(t)$ – the additive noise.

$$\text{The received signal power:} \quad P_r = |h|^2 P_s \tag{4}$$

$$\text{Multipath channel gain:} \quad h = h_r + jh_i = z\,e^{j\theta} \tag{5}$$

$$\text{Channel envelop:} \quad |h| = z = \sqrt{h_r^2 + h_i^2} \tag{6}$$

- Additive white Gaussian noise: $n(t) = n_r + jn_i$; noise power is constant for all frequencies; $n(t)$ follows circularly symmetric complex Gaussian distribution with zero mean and N_0 variance, i.e., $n(t) \sim \mathcal{CN}(0, N_0)$ where $n_r \sim \mathcal{N}(0, N_o/2)$ and $n_i \sim \mathcal{N}(0, N_o/2)$.

$$\text{Instantaneous SNR:} \quad \gamma = \frac{\text{Signal power}}{\text{Noise power}} = \frac{|h|^2 P_s}{N_0} \tag{7}$$

We need distributions of $|h|$ and $|h|^2$ – Multipath fading models!!!

Rayleigh distribution

- Rayleigh fading is a model that can be used to describe the form of fading that occurs when multipath propagation exists with no Los component.

$$h = h_r + jh_i = z e^{j\theta} \quad \text{and} \quad |h| = z = \sqrt{h_r^2 + h_i^2} \tag{8}$$

- When h_r and h_i are two independent and identical distributed (i.i.d.) Gaussian random variables with mean zero and variance σ^2, i.e., $h_r, h_i \sim \mathcal{N}(0, \sigma^2)$,

1. The average envelope power is $\Omega_p = 2\sigma^2$.
2. the envelop $|h| = z = \sqrt{h_r^2 + h_i^2}$ is Rayleigh distributed;

$$f_Z(z) = \frac{2z}{\Omega_p} e^{-\frac{z^2}{\Omega_p}} \quad \text{and} \quad F_Z(z) = 1 - e^{-\frac{z^2}{\Omega_p}} \tag{9}$$

3. the power $|h|^2$ is Exponentially distributed;

$$f_{Z^2}(t) = \frac{1}{\Omega_p} e^{-\frac{t}{\Omega_p}} \quad \text{and} \quad F_{Z^2}(t) = 1 - e^{-\frac{t}{\Omega_p}} \tag{10}$$

- The channel has a LOS component with a much larger signal power than the other multipath components.
- $h_r \sim \mathcal{N}(m_r, \sigma^2)$ and $h_i \sim \mathcal{N}(m_i, \sigma^2)$;
- $2\sigma^2$ is the average power in the non-LOS multipath components and $s^2 = m_r^2 + m_i^2$ is the power in the LOS component.

1. Average envelope power: $\Omega_p = s^2 + 2\sigma^2$
2. the envelopis Rician/Ricean/Rice distributed;

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right) \tag{11}$$

3. The Rice factor K (fading parameter): $K = \frac{s^2}{2\sigma^2}$ where $K = 0$ for no LoS; $K \to \infty$ for no scatter, and a small K implies severe fading.

$$f_Z(z) = \frac{2(K+1)z}{\Omega_p} e^{-K-\frac{(K+1)z^2}{\Omega_p}} I_0\left(2z\sqrt{\frac{K(K+1)}{\Omega_p}}\right) \tag{12}$$

where $s^2 = \frac{K\Omega_p}{K+1}$ and $\sigma^2 = \frac{\Omega_p}{2(K+1)}$

Nakagami-m distribution

1. The Nakagami distribution was selected to fit empirical data and is known to provide a closer match to some measurement data than either the Rayleigh, Ricean, or log-normal distributions.
2. the envelop $|h|$ is Nakagami-m distributed;

$$f_Z(z) = 2\left(\frac{m}{\Omega_p}\right)^m \frac{z^{2m-1}}{\Gamma(m)} e^{-\frac{mz^2}{\Omega_p}};\ m \geq \frac{1}{2} \tag{13}$$

3. Average envelope power: Ω_p
   - $m = 1$: Rayleigh distribution.
   - $m = 1/2$: a one-sided Gaussian distribution
   - $m \to \infty$: approaches an impulse (no fading).
   - $m \approx \frac{(K+1)^2}{(2K+1)}$: approximation for Rician distribution.
4. the power $|h|^2$ is Gamma distributed;

$$f_{Z^2}(z) = \left(\frac{m}{\Omega_p}\right)^m \frac{z^{m-1}}{\Gamma(m)} e^{-\frac{mz}{\Omega_p}};\ m \geq \frac{1}{2} \tag{14}$$

SNR outage probability

- The SNR outage probability is the probability that the SNR γ falls below a certain predetermined threshold SNR γ_{th}

$$P_{out} = \Pr[\gamma < \gamma_{th}] \tag{15}$$

$$= \Pr\left[\frac{|h|^2 P_s}{N_0} < \gamma_{th}\right] = \Pr\left[|h|^2 < \frac{N_0 \gamma_{th}}{P_s}\right] = F_{|h|^2}\left(\frac{N_0 \gamma_{th}}{P_s}\right) \tag{16}$$

- For Rayleigh fading (use (10))

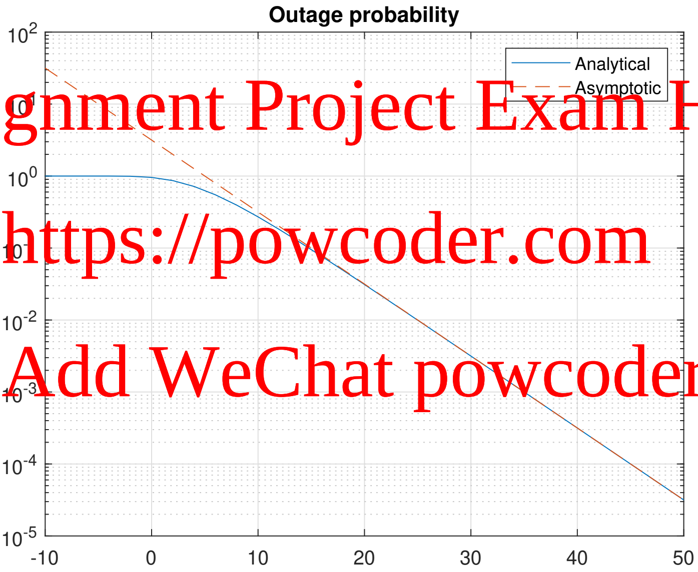
$$P_{out} = 1 - e^{-\frac{\left(\frac{N_0 \gamma_{th}}{P_s}\right)}{2\sigma^2}} = 1 - e^{-\left(\frac{\gamma_{th}}{2\sigma^2}\frac{N_0}{P_s}\right)} = 1 - e^{-\left(\frac{\gamma_{th}}{2\sigma^2}\frac{1}{\bar{\gamma}}\right)} \tag{17}$$

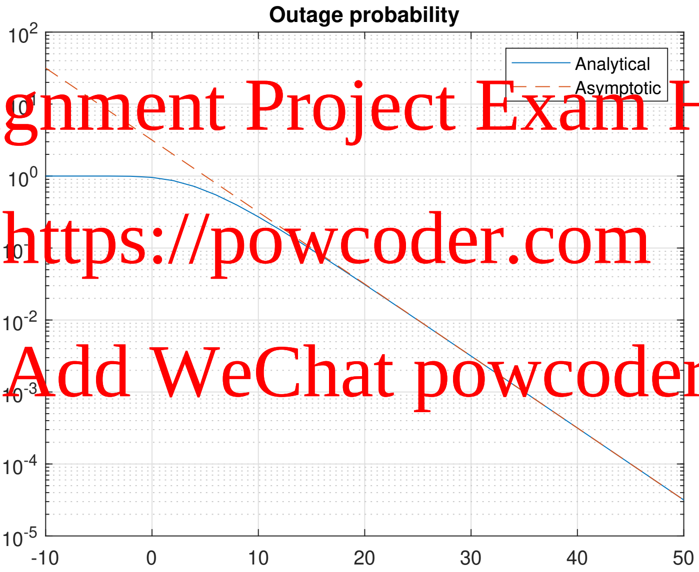
$$\text{When } \bar{\gamma} \to \infty;\ P_{out} \to 1 - \left(1 - \frac{\gamma_{th}}{2\sigma^2}\frac{1}{\bar{\gamma}}\right) = \frac{\gamma_{th}}{2\sigma^2}\frac{1}{\bar{\gamma}} \quad \text{Asymptotic analysis} \tag{18}$$

where $\bar{\gamma} = \frac{P_s}{N_0}$ (we sometime call this as the average transmit SNR!).

- Similarly, you can evaluate the SNR outage probabilities for Rician and Nakagami-m fading channels!

SNR outage probability




Outage probability
Analytical
Asymptotic
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
-10
0
10
20
30
40
50

References

A. Goldsmith, *Wireless Communications*, Cambridge University Press, USA, 2005.

Thank You!

See you again ☺