ECE5884 Wireless Communications – Quiz 9   19 September 2022

1. **((Independent and identical (i.i.d.) variables)** Say we have independent random variables X and Y, and we know their probability density functions (PFDs) $f_X(t) = 2t\,e^{-t^2}$ and $f_Y(t) = 2t\,e^{-t^2}$ which are Rayleigh distributions. For a fixed value $a > 0$, find cumulative distribution functions (CDFs) of Z where (please provide all steps of your derivations)

   (a) $Z = a(X + Y)^2$

   (b) $Z = aX^2 + aY^2$

   (c) $Z = \max(aX^2, aY^2)$

2. **(Multi-antenna systems)** A wireless system consists of a single-antenna transmitter and two antennas receiver. The multipath channel gain between the transmitter and the receiver antennas are h_1 and h_2 where their $|h_1|$ and $|h_2|$ follows i.i.d. Rayleigh distributions with the unit power gain, i.e., $f_{|h_i|}(x) = 2x\,e^{-x^2}$. The average SNR of each branch (channel) is $\bar{\gamma}$.

   (a) Write the end-to-end SNR at the receiver with maximal ratio combining and selection combining (SC).

   (b) Derive closed-form expressions for the SNR outage probabilities for both MRC and SC, when the received SNR falls below a threshold γ_{th}. Please provide details of your derivation.

   (c) Verify analytical outage probability expressions by using MATLAB simulations. You may plot outage probability vs average SNR $\bar{\gamma}$ where $\bar{\gamma}$ varies from -10 dB to 16 dB.

   (d) Find the diversity orders and array gains for both MRC and SC. You may use asymptotic analysis for $\bar{\gamma} \to \infty$.

   (e) **(Optional)** Plot asymptotic expressions on the same plot in (c).