ECE5884 Wireless Communications

Week 3: Wireless Channel Models

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Course outline

This week: Ref. Ch. 3 of [Goldsmith, 2005]

- Week 1: Overview of Wireless Communications
- Week 2: Wireless Channel (Path Loss and Shadowing)
- Week 3: Wireless Channel Models
- Week 4: Capacity of Wireless Channels
- Week 5: Digital Modulation and Detection
- Week 6: Performance Analysis
- Week 7: Equalization
- Week 8: Multicarrier Modulation (OFDM)
- Week 9: Diversity Techniques
- Week 10: Multiple-Antenna Systems (MIMO Communications)
- Week 11: Multiuser Systems
- Week 12: Guest Lecture (Emerging 5G/6G Technologies)

Doppler effect

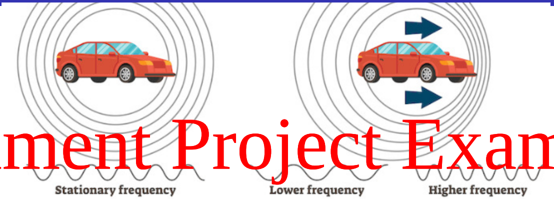


Figure 1: Illustration of the Doppler effect.

Doppler frequency: When θ is the arrival angle of the received signal relative to the direction of motion, v is the receiver velocity toward the transmitter in the direction of motion, and λ is the signal wavelength

$$f_D = \frac{v}{\lambda}\cos\theta \quad \text{where } \lambda = \frac{c}{f_c} \text{ and } c = 3 \times 10^8 \, m/s \tag{1}$$

So, f_D is positive when the Rx is moving toward the Tx (i.e., $-\pi/2 \leq \theta \leq \pi/2$).

$$\text{Max. Doppler spread} \quad B_D = 2v/\lambda \tag{2}$$

$$\text{Channel coherence time} \quad T_c \approx 1/B_D \tag{3}$$

Coherence time (T_c) is the time duration over which the channel impulse response is considered to be not varying.

Delay

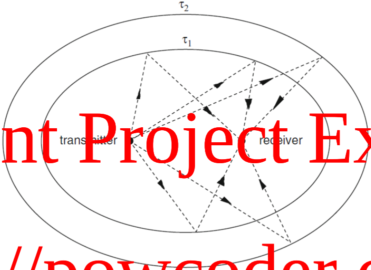


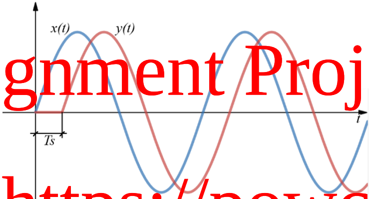
Figure 2: Concentric ellipses model for fading channels (Tx and Rx are located at the foci of the ellipses. Considering only single bounce reflections, all paths that are associated with scatterers on the *n*th elliptical contour have the same delay).

1. Doppler effect.
2. Scatters.

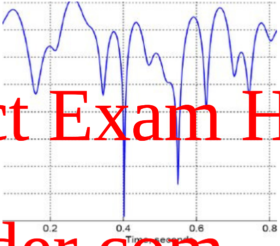
Received signal $$r(t) = \Re\left[\left(\sum_{i=0}^{N(t)-1} \alpha_i(t) e^{-j\phi_i(t)} u(t - \tau_i(t))\right) e^{j2\pi f_c t}\right] \quad (4)$$

$\alpha_i(t)$ is fading (also a function of path loss and shadowing). $\phi_i(t)$ depends on delay and Doppler. These two random processes are independent.

Fast and slow fading



(a) A delayed signal.

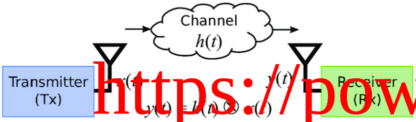


(b) Resultant signal.

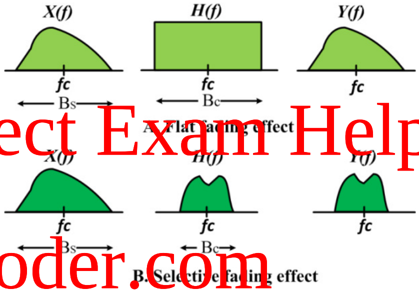
Figure 3: How signal looks like.

1. Fast fading
   - No.of paths $N \uparrow$;
   - Carrier frequency $f_c \uparrow \Rightarrow \lambda_c \downarrow$ Larger phase spread;
   - Velocity $v \uparrow \Rightarrow$ Doppler spread $\uparrow$;
   - $T_c < T_s$ where T_s is the transmitted symbol duration (severe frequency dispersion into the received signal).
2. Slow fading: $T_c \gg T_s$ (little frequency dispersion into the Rx signal).

Flat and frequency-selective fading



(a) Wireless channel response.

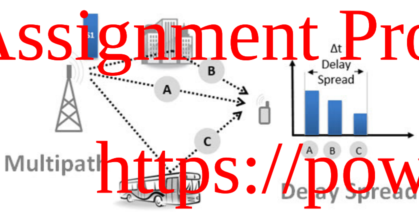


(b) Wireless channel as a filter.

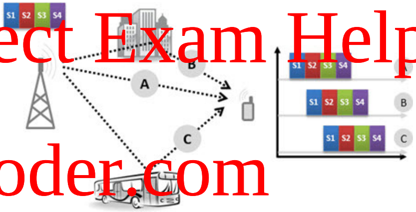
1. Flat fading: the magnitude of the time-variant channel transfer function is constant (or flat) with respect to frequency; $B_s \ll B_c$ where B_c is the coherence bandwidth where the channel can be considered "flat".
2. Frequency-selective fading: the magnitude of the time-variant channel transfer function is no longer flat with respect to frequency $B_s \gg B_c$.
   - the differential path delays $|\tau_i - \tau_j|$ (Figure 2) for some i, j are sufficiently large compared to the modulation symbol period T_s.

Intersymbol interference (ISI)

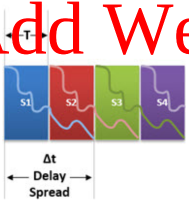
ISI is a form of distortion of a signal in which one symbol interferes with subsequent symbols.



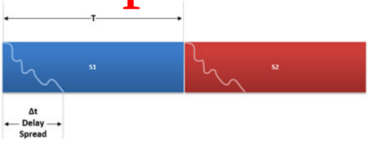
(c) One symbol Tx (No ISI).



(d) Four symbols Tx (ISI).



(e) Shorter T_s (ISI).

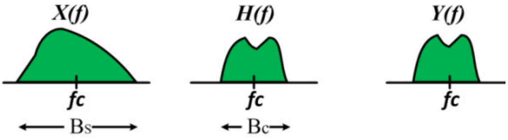


(f) Longer T_s (No ISI).

https://www.telecomhall.net/t/what-is-isi-inter-symbol-interference-in-lte/6370

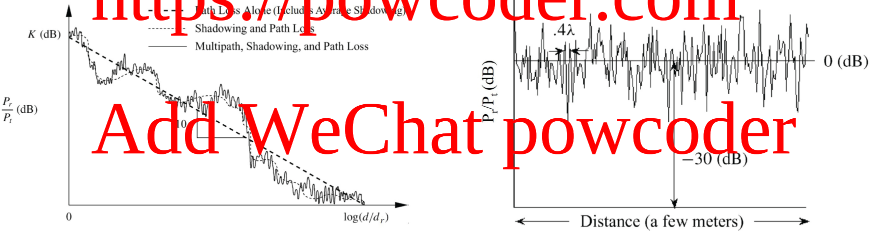
Narrowband and wideband communications

1. Narrowband communications use a narrow bandwidth; are used in a slower form of communication (voice, RFID, GSM 900 satellite downlinks, GPS signals); have a far greater range of reception as narrower filters can be used and therefore cancel out unwanted wideband noise; the transmitted energy also concentrates on a smaller portion of the spectrum.
   - The delay spread T_m of a channel is small relative to the inverse baseband signal bandwidth B_s of the transmitted signal, i.e., $T_m \ll T_s$ where $B_s \approx 1/T_s$ for T_s the signal duration.
2. Wideband communications use a higher bandwidth; the energy of the signal is distributed across the width of the spectrum which makes the signal weaker the wider it gets; is almost exclusively done in higher frequencies (>500MHz+); common modulation technique is OFDM (Week 8); apply WiFi, 4G LTE, HSPA.
   - $T_m \gg T_s$.



Fast/slow Flat/selective Narrowband/wideband

1. Slow fading: $T_s > T_c$, Shadowing, Log-normal
2. Fast fading: $T_s \ll T_c$, Multipath fading, Next
3. Flat fading: $B_s \ll B_D$
4. Selective fading: $B_s \gg B_D$ OFDM
5. Narrowband comm.: $T_m \ll T_s$, Multipath fading, Next
6. Wideband comm.: $T_m \gg T_s$, OFDM



(g) Combined all.

(h) Narrowband fading.

Figure 4: Ref. Ch. 3 of [Goldsmith, 2005].

System model

- The received signal

$$r(t) = h\,s(t) + n(t) \qquad \left(\text{or } = \sqrt{\frac{P_t}{d^\alpha}}\,h\,s(t) + n(t)\right) \tag{5}$$

  - P_t– the transmit power;
  - h– the multipath channel gain (usually a complex number);
  - $s(t)$– the transmit signal;
  - $n(t)$– the additive noise.

- The received signal power, where P_s is the signal power,

$$P_r = |h|^2 P_s \tag{6}$$

- Multipath channel gain

$$h = h_r + jh_i = z\,e^{j\theta} \tag{7}$$

- Fading channel envelop

$$|h| = z = \sqrt{h_r^2 + h_i^2} \tag{8}$$

We need Envelope ($|h|$) and Power Distributions ($|h|^2$)!!!

Multipath fading: Rayleigh distribution

When h_r and h_i are two independent and identical distributed (i.i.d.) Gaussian random variables with mean zero and variance σ^2, i.e., $h_r, h_i \sim \mathcal{N}(0, \sigma^2)$,

1. the envelop $|h| = z = \sqrt{h_r^2 + h_i^2}$ is Rayleigh distributed; (Derive in Quiz 3!)

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} \tag{9}$$

$$F_Z(z) = 1 - e^{-\frac{z^2}{2\sigma^2}} \tag{10}$$

The average envelope power is $\Omega_p = 2\sigma^2$.

2. the power $|h|^2$ is Exponentially distributed; (Derive in Quiz 3!)

$$f_{Z^2}(t) = \frac{1}{2\sigma^2} e^{-\frac{t}{2\sigma^2}} \tag{11}$$

$$F_{Z^2}(t) = 1 - e^{-\frac{t}{2\sigma^2}} \tag{12}$$

Verify these expressions by using MATLAB simulations in Quiz 3! Use $\sigma = 0.5$

Multipath fading: Rician distribution

- The channel has a LOS component with a much larger signal power than the other multipath components.

- Then h_r and h_i are two independent Gaussian RVs with non-zero mean and equal variance σ^2, i.e., $h_r \sim \mathcal{N}(m_r, \sigma^2)$ and $h_i \sim \mathcal{N}(m_i, \sigma^2)$;

1. the envelop $|h| = z = \sqrt{h_r^2 + h_i^2}$ is Rician/Ricean/Rice distributed;

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right) \tag{13}$$

  - $s^2 = m_r^2 + m_i^2$ and $I_0(x)$ is the modified Bessel function of zeroth order.
  - $2\sigma^2$ is the average power in the non-LOS multipath components and s^2 is the power in the LOS component.

Verify this expression by using MATLAB simulations in Quiz 3! Use $\sigma = 0.5$ and $s^2 = 0.9$

Multipath fading: Rician distribution

1. the envelop $|h| = z = \sqrt{h_r^2 + h_i^2}$ is Rician/Ricean/Rice distributed;

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right) \tag{14}$$

2. Average envelope power: $\Omega_p = s^2 + 2\sigma^2$
3. The Rice factor K (fading parameter): $K = \frac{s^2}{2\sigma^2}$
   - $K = 0$: no LoS and the envelope exhibits Rayleigh fading.
   - $K \to \infty$: no scatter and the channel does not exhibit any fading.
   - K is a measure of the severity of the fading: a small K implies severe fading, a large K implies relatively mild fading.

$$f_Z(z) = \frac{2(K+1)z}{\Omega_p} e^{-K-\frac{(K+1)z^2}{\Omega_p}} I_0\left(2z\sqrt{\frac{K(K+1)}{\Omega_p}}\right) \tag{15}$$

where $s^2 = \frac{K\Omega_p}{K+1}$ and $\sigma^2 = \frac{\Omega_p}{2(K+1)}$

Multipath fading: Nakagami distribution

1. The Nakagami distribution was selected to fit empirical data and is known to provide a closer match to some measurement data than either the Rayleigh, Ricean, or log-normal distributions.
2. the envelop $|h|$ is Nakagami-m distributed;

$$f_Z(z) = 2\left(\frac{m}{\Omega_p}\right)^m \frac{z^{2m-1}}{\Gamma(m)} e^{-\frac{mz^2}{\Omega_p}}; \, m \geq \frac{1}{2} \tag{16}$$

3. Average envelope power: Ω_p
   - $m = 1$: Rayleigh distribution.
   - $m = 1/2$: a one-sided Gaussian distribution
   - $m \to \infty$: approaches an impulse (no fading).
   - $m \approx \frac{(K+1)^2}{(2K+1)}$: approximation for Rician distribution.
4. the power $|h|^2$ is Gamma distributed;

$$f_{Z^2}(z) = \left(\frac{m}{\Omega_p}\right)^m \frac{z^{m-1}}{\Gamma(m)} e^{-\frac{mz}{\Omega_p}}; \, m \geq \frac{1}{2} \tag{17}$$

Derive CDF $F_{Z^2}(z)$ expression in Quiz 3!

Channel phase

1. The phase of the received complex envelope $h = h_r + jh_i$ is

$$\phi = \tan^{-1}\left(\frac{h_i}{h_r}\right) \tag{18}$$

2. For Rayleigh fading; the phase ϕ is uniformly distributed over the interval $[-\pi, \pi)$,

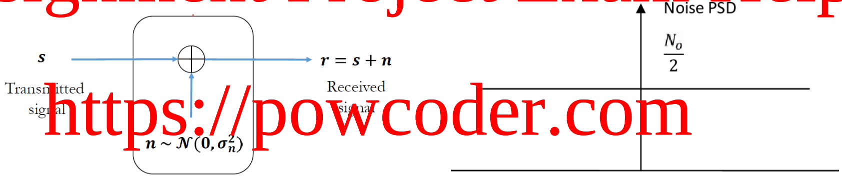
$$f_\phi(x) = \frac{1}{2\pi}; -\pi \leq x \leq \pi \tag{19}$$

3. For Ricean fading channels, the phase ϕ is not uniformly distributed and takes on a more complicated integral form.

Additive white Gaussian noise (AWGN)

- The received signal

$$r(t) = h\,s(t) + n(t) \quad \text{where } n(t) \text{ is the noise} \tag{20}$$



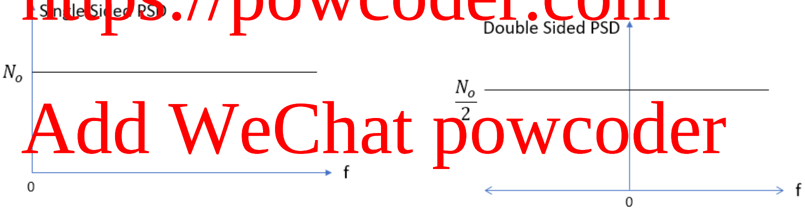
(a) Additive noise. (b) Power spectral density.

Figure 5: AWGN.

- "White" light contains components at all wavelengths (all frequencies) across the visible spectrum. The Power Spectral Density (PSD) of white noise is constant for all frequencies.
- The probability distribution of the noise samples is Gaussian with a zero mean.

Additive white Gaussian noise (AWGN)

- The noise power is N_0 Watt/Hz
- $N_0 = kT$, k is Boltzmann's Constant and T is the temperature in Kelvin.
- For a complex baseband signal, the thermal noise signal is a complex, white Gaussian distributed noise, with half of the power (N_0) in the real component and half the power (N_0) in the imaginary component (but power in itself is not a complex quantity).



(a) Single-sided PSD. (b) Double-sided PSD.

Figure 6: Representations of PSD.

https://dsp.stackexchange.com/questions/54251/psd-of-complex-white-gaussian-noise

Signal-to-noise ratio (SNR)

- The received signal

$$r(t) = h\,s(t) + n(t) \tag{21}$$

- SNR is the ratio between the power of the received signal (desired signal) to the power of background noise (undesired signal)

$$\text{SNR:} \quad \gamma = \frac{\text{Signal power}}{\text{Noise power}} \tag{22}$$

1. For the AWGN channel ($h = 1$)

$$\gamma = \frac{P_s}{N_0} \tag{23}$$

2. For a fading channel

$$\gamma = \frac{|h|^2 P_s}{N_0}; \quad \text{called the } \textit{instantaneous} \text{ received SNR} \tag{24}$$

- If the channel bandwidth is B, the total noise power is BN_0.

SNR outage probability

- The SNR outage probability is the probability that the SNR γ falls below a certain predetermined threshold SNR γ_{th}

$$
\begin{aligned}
P_{out} &= \Pr\left[\gamma < \gamma_{th}\right] && (25)\\
&= \Pr\left[\frac{|h|^2 P_s}{N_0} < \gamma_{th}\right] = \Pr\left[|h|^2 < \frac{N_0\gamma_{th}}{P_s}\right] = F_{|h|^2}\left(\frac{N_0\gamma_{th}}{P_s}\right) && (26)
\end{aligned}
$$

- For Rayleigh fading (use (12))

$$
P_{out} = 1 - e^{-\frac{\left(\frac{N_0\gamma_{th}}{P_s}\right)}{2\sigma^2}} = 1 - e^{-\left(\frac{\gamma_{th}}{2\sigma^2}\frac{N_0}{P_s}\right)} \qquad (27)
$$

- Similarly, you can evaluate the SNR outage probabilities for Rician and Nakagami-*m* fading channels!

References

A. Goldsmith, *Wireless Communications*, Cambridge University Press, USA, 2005.

Thank You!

See you again ☺