1. **([15 marks] Independent and identical (i.i.d.) random variables).** Say we have independent random variables X and Y, and we know their probability density functions (PFDs) $f_X(t) = 2t\,e^{-t^2}$ and $f_Y(t) = 2t\,e^{-t^2}$ which are Rayleigh distributions. For fixed values $a > 0$, $b > 0$ and $c > 0$, find cumulative distribution functions (CDFs) of following Z:

   (a) [**5 marks**] $Z = \frac{a\,X^2}{b\,Y^2}$

   (b) [**5 marks**] $Z = \frac{a\,X^2}{b\,Y^2+c}$

   (c) [**5 marks**] Verify above CDF expressions by using MATLAB simulations for $a = 0.3; b = 0.5; c = 0.8$.

2. **([5 marks] Co-channel reuse factor).** By using geometric arguments, show that the co-channel reuse factor for cellular deployments based on hexagonal cells is given by $\frac{D}{R} = \sqrt{3N}$, where D is the co-channel reuse distance between cells using the same set of carrier frequencies, R is the radius of the cells (for hexagonal cells, R is the distance from the center to the corner of a cell), and N is the reuse cluster size.

3. **([15 marks] Cellular systems).** Consider a regular hexagonal cell deployment, where the mobile stations (MSs) and base stations (BSs) use omnidirectional antennas. Suppose that we are interested in the uplink (from MS to BS) channel performance and consider only the first tier of co-channel interferers. Ignore the effects of shadowing and multipath fading, and assume that the propagation path loss is described by the simplified path-loss model (refer the class note).

   (a) [**10 marks**] Determine the worst case carrier-to-interference ratio, $\frac{C}{I}$, for cluster sizes $N = 3,\ 4,\ 7$ when the path loss exponent $\beta = 3.5$. You may refer Slide#19 of Week#10 lecture note to identify the worst-case locations of MSs.

   (b) [**5 marks**] What are the minimum cluster sizes that are needed if threshold levels of the carrier-to-interference ratio at radio receivers are 5 dB and 10.5 dB?

4. **([25 marks] Multi-antenna systems).** A single-input multiple-output (SIMO) system consists of a single-antenna transmitter and N antennas receiver. The multipath channel between the transmitter and the ith receiver antenna is denoted as h_i where all h_i, $i = 1, \cdots, N$ are independent and

identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, i.e., $h_i \sim \mathcal{CN}(0,1)$. The complex-valued additive white Gaussian noise (AWGN) of the ith antenna is denoted as n_i which follows $n_i \sim \mathcal{CN}(0, N_0)$. The average transmit power is P. The channel state information (CSI) is only available at the receiver.

(a) [**3 marks**] Write the end-to-end SNR at the receiver with maximal ratio combining (MRC) in terms of h_i and $\bar{\gamma}$ where the average SNR is $\bar{\gamma} = \frac{P}{N_0}$.

(b) [**7 marks**] Derive a closed-form expression for the SNR outage probability, when the received SNR falls below a threshold γ_{th}. All important steps of the derivation should be provided (You may use *Hint#1*).

(c) [**5 marks**] By using asymptotic analysis, i.e., for high average SNR $\bar{\gamma} = \frac{P}{N_0} \to \infty$, derive the achievable diversity order and the array gain of this MRC system. Does this system provide the full-diversity order? Justify your answer (You may use *Hint#2*).

(d) [**7 marks**] Verify analytical outage probability expression in (b) and asymptotic analysis in (c) by using MATLAB simulations. You may plot outage probability vs average SNR $\bar{\gamma}$ where $\bar{\gamma}$ varies from -10 dB to 24 dB when $\gamma_{th} = 5$ dB for $N = 2$, $N = 3$ and $N = 4$.

(e) [**3 marks**] How much power do you save at 10^{-3} outage probability level when N increases from $N = 2$ to $N = 3$ when $\gamma_{th} = 5$ dB.? You may provide the answer as a ratio between two power values.

Hint#1: We have L number of i.i.d., random variables $\{X_1, \cdots, X_i, \cdots, X_L\}$ where each X_i follows an exponential distribution, i.e., $X_i \sim ae^{-ax}$ for $a > 0$. Then, the sum of L i.i.d. exponential random variables $Z = \sum_{i=1}^{L} X_i$ is a Gamma random variable where its distributions can be given as

$$\text{PDF:} \quad f_Z(z) = \frac{a^L}{\Gamma(L)} z^{L-1} e^{-az} \tag{1}$$

$$\text{CDF:} \quad F_Z(z) = \frac{1}{\Gamma(L)} \gamma(L, ax) \tag{2}$$

where $\gamma(u,v)$ is the lower incomplete Gamma function, which can be implemented in MATHEMATICA as $Gamma(u, 0, v)$ and in MATLAB as $gammainc(v, u, 'lower') * gamma(u)$.

Hint#2: You may use the following series expansion:

$$\lim_{x \to 0} \gamma[n, x] \approx \frac{x^n}{n}. \tag{3}$$