

$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\begin{aligned} A_1 &= \int_{-\infty}^{a_1} f_x(x) \cdot dx \\ &= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{a_1} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\ &= \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{(a_1-\mu)/\sigma} e^{-\frac{t^2}{2}} dt \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(a_1-\mu)/\sigma} e^{-\frac{t^2}{2}} dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{(\mu-a_1)/\sigma}^{\infty} e^{-\frac{t^2}{2}} (-dy) \end{aligned}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{(\mu-a_1)/\sigma}^{\infty} e^{-\frac{t^2}{2}} dy$$

$$= \Phi\left(\frac{a_1-\mu}{\sigma}\right)$$

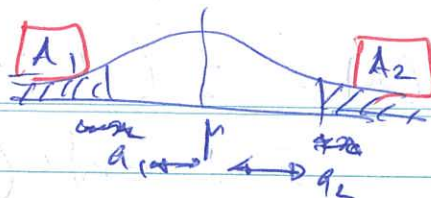
$$\begin{aligned} A_2 &= \int_{a_2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\ &= \int_{(a_2-\mu)/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} dt \end{aligned}$$

$$= \Phi\left(\frac{a_2-\mu}{\sigma}\right) \quad \text{--- (2) upper tail,}$$

$$A_1 = \Phi\left(\frac{\mu-a_1}{\sigma}\right)$$

$$A_2 = \Phi\left(\frac{a_2-\mu}{\sigma}\right)$$

$$\Phi(x) = 1 - \Phi(-x)$$



$$\frac{x-\mu}{\sigma} = t$$

$$dx = \sigma dt$$

$$x = -\infty \Rightarrow t = -\infty$$

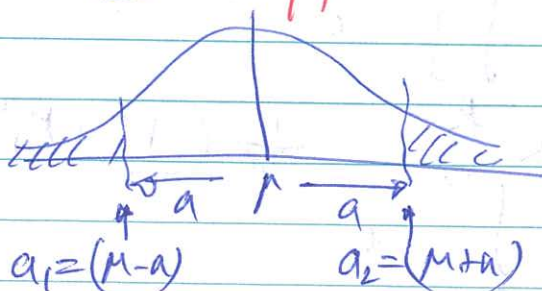
$$x = a_1 \Rightarrow t = \frac{a_1-\mu}{\sigma}$$

$$t = -y$$

$$dt = -dy$$

$$t = -\infty \Rightarrow y = \infty$$

$$t = \frac{a_1-\mu}{\sigma} \Rightarrow y = \frac{\mu-a_1}{\sigma}$$



$$\begin{aligned} A_1 &= \Phi\left(\frac{\mu-\mu+a}{\sigma}\right) = \Phi\left(\frac{a}{\sigma}\right) \\ A_2 &= \Phi\left(\frac{\mu+\mu-a}{\sigma}\right) = \Phi\left(\frac{a}{\sigma}\right) \end{aligned} \quad \Bigg| \quad \equiv$$