ECE5884 Wireless Communications – Quiz 3 **8 August 2022**

1. Write only ONE or TWO sentences for each of the following topics. Just write as you understood ☺.

   (a) Coherence time

   (b) Coherence bandwidth

   (c) Fast fading channel

   (d) Flat fading channel

   (e) Narrowband communications

2. (MATLAB) A multipath fast-fading channel of wireless communications system is given as

$$h = h_r + jh_i$$

which is a complex number. We assume that h follows a circularly symmetric complex Gaussian distribution, i.e., $h \sim \mathcal{CN}(0, \Omega_p)$, where Ω_p is the average envelop power. This implies that h_r, $h_i \sim \mathcal{N}(0, \Omega_p/2)$ which are Gaussian distributions with mean zero and variance $\Omega_p/2$. Then, the envelop of h, i.e., $|h|$, follows a Rayleigh distribution.

PDFs and CDFs of $|h|$ and $|h|^2$ can be given as follows (also see Slide#12 of the Lecture note or Slide#10 of the Workshop note):

$$\text{PDF of } |h|: \quad f_{|h|}(z) = \frac{2z}{\Omega_p} e^{-\frac{z^2}{\Omega_p}}; z \geq 0 \tag{1}$$

$$\text{CDF of } |h|: \quad F_{|h|}(z) = 1 - e^{-\frac{z^2}{\Omega_p}} \tag{2}$$

$$\text{PDF of } |h|^2: \quad f_{|h|^2}(z) = \frac{1}{\Omega_p} e^{-\frac{t}{\Omega_p}} \tag{3}$$

$$\text{CDF of } |h|^2: \quad F_{|h|^2}(z) = 1 - e^{-\frac{t}{\Omega_p}} \tag{4}$$

Using MATLAB simulations, please verify above PDFs and CDFs when $\Omega_p = 5\,\text{dBm}$!

(Hint: For an example, you may generate a random variable h in MATLAB as

$$h = sqrt(omega/2) * randn(1,1) + 1i * sqrt(omega/2) * randn(1,1).$$

Try to get your plots as in Figure 1 ☺)

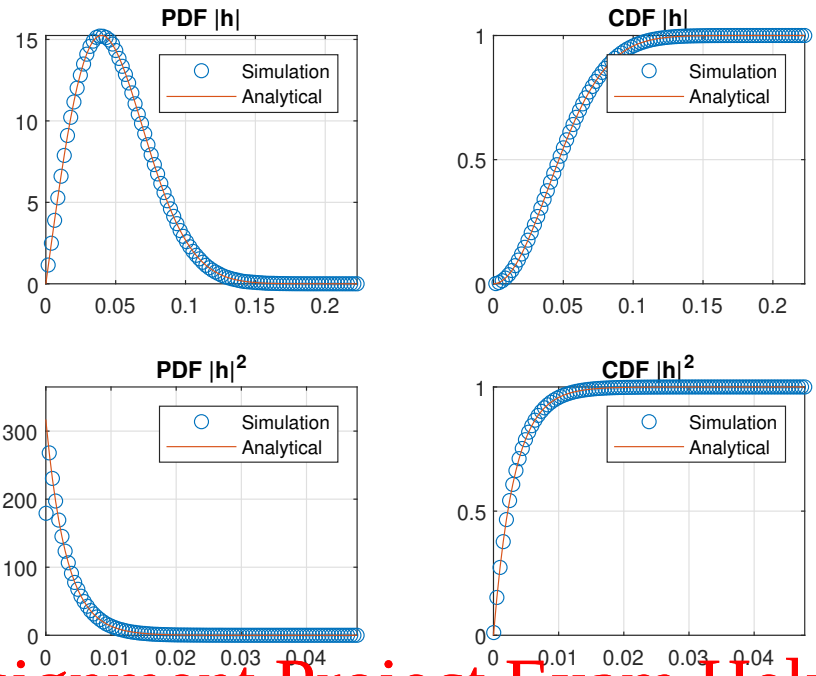

Figure 1: Example.

3. We have a simple wireless system with a transmitter and receiver pair which are implemented with a single antenna. Then, the received signal at the receiver at time t may be written as

$$r(t) = h\, s(t) + n(t). \tag{5}$$

Here, h is the wireless channel as explained in Problem#2; $s(t)$ is the transmitted signal which has the average transmit power P_s (W); and $n(t)$ is the additive white Gaussian noise (AWGN) which follows a circularly symmetric complex Gaussian distribution, i.e., $n(t) \sim \mathcal{CN}(0, N_0)$, where N_0 (W/Hz) is the average noise power. The channel bandwidth of the wireless system is B (Hz). We neglect path-loss and shadowing effects.

(a) Write an expression for the instantaneous received signal-to-noise ratio (SNR) γ in terms of h, P_s, N_0 and B.

(b) Derive an expression for the SNR outage probability, when the received SNR falls below a threshold γ_{th}. Please provide details of your derivation!

(c) The wireless system requires the SNR outage probability less than 10^{-5}. What is the minimum signal power P_s (W) which satisfies this requirement when $N_0 = 10^{-9}$ (W/Hz), $B = 1$ (Hz), $\gamma_{th} = 5$ (dB) and $\Omega_p = 5$ (dBm)?