ECE5884 Wireless Communications

Week 2: Wireless Channel (Path Loss and Shadowing)

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August 1, 2022

Course outline

This week: Ref. Ch. 2 of [Goldsmith, 2005]

- Week 1: Overview of Wireless Communications
- Week 2: Wireless Channel (Path Loss and Shadowing)
- Week 3: Wireless Channel Models
- Week 4: Capacity of Wireless Channels
- Week 5: Digital Modulation and Detection
- Week 6: Performance Analysis
- Week 7: Equalization
- Week 8: Multicarrier Modulation (OFDM)
- Week 9: Diversity Techniques
- Week 10: Multiple-Antenna Systems (MIMO Communications)
- Week 11: Multiuser Systems
- Week 12: Guest Lecture (Emerging 5G/6G Technologies)

Normal/Gaussian distribution: X is continuous probability distribution for a real-valued random variable (RV) with the mean or expectation μ, the standard deviation σ and the variance σ^2.

$$\textit{Notation}: \quad X \sim \mathcal{N}(\mu, \sigma) \text{ or } X \sim \mathcal{N}(\mu, \sigma^2) \tag{1}$$

$$\textit{PDF}: \quad f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\sigma)^2}{2\sigma^2}}; -\infty < x < \infty \tag{2}$$

$$\textit{CDF}: \quad F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right] = 1 - Q\left(\frac{x-\mu}{\sigma}\right) \tag{3}$$

$$\textit{Error fun}: \quad \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt \tag{4}$$

$$\textit{CEF}: \quad \operatorname{erfc}(x) = 1 - \operatorname{erf}(x) \tag{5}$$

$$\textit{Q-fun}: \quad Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}}\,dt = \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \tag{6}$$

$$Q(x) = 1 - Q(-x) \tag{7}$$

Watch YouTube “How are *erf*(.), Q(.), and Gaussian Tails Related?”:
https://www.youtube.com/watch?v=6VBRoJj7aJQ

1. If RV X is log-normally distributed, then $Y = \ln(X)$ has a normal distribution.
2. If RV Y has a normal distribution, then the exponential function of Y, $X = e^Y$, has a log-normal distribution.

$$Y = \ln(X) \sim \mathcal{N}(\mu, \sigma^2) \tag{8}$$

$$\text{then} \quad X \sim \text{Lognormal}(\mu, \sigma^2) \tag{9}$$

$$\text{PDF:} \quad f_X(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}; x > 0 \tag{10}$$

$$\text{CDF:} \quad F_X(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\log(x) - \mu}{\sqrt{2}\sigma}\right)\right] \tag{11}$$

Transmit signal model (recap)

A bandpass signal $s(t)$ at carrier frequency f_c:

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \tag{12}$$

where $s_I(t)$ (in-phase component of $s(t)$) and $s_Q(t)$ (quadrature component of $s(t)$) are real lowpass (baseband) signals of bandwidth $B << f_c$.

A complex lowpass signal:

$$u(t) = s_I(t) + js_Q(t), \text{ so } s_I(t) = \Re\{u(t)\} \text{ and } s_Q(t) = \Im\{u(t)\}. \tag{13}$$

Then,

$$\begin{aligned} s(t) &= \Re\{u(t)\}\cos(2\pi f_c t) - \Im\{u(t)\}\sin(2\pi f_c t) & (14)\\ &= \Re\{u(t)e^{j2\pi f_c t}\} & (15) \end{aligned}$$

This is the complex lowpass representation of the bandpass signal $s(t)$, and the baseband signal $u(t)$ is called the equivalent lowpass signal for $s(t)$ or its complex envelope.

The received signal is the convolution of $s(t)$ with the channel impulse response $h(t)$ plus an additional noise component $n(t)$ introduced by the channel:

$$
\begin{aligned}
r(t) &= s(t) * h(t) + n(t) && (16)\\
&= \Re\{v(t)e^{j2\pi f_c t}\} + n(t); \text{ where } v(t) = u(t) * c(t) && (17)
\end{aligned}
$$

and $c(t)$ is the equivalent lowpass channel impulse response for $h(t)$.

Note: we will neglect the random noise component $n(t)$ in our analysis in Week 2 and Week 3.

Radio wave propagation

We characterize the primary phenomena that affect signal propagation: path loss, shadowing, signal reflection, diffraction and scattering.

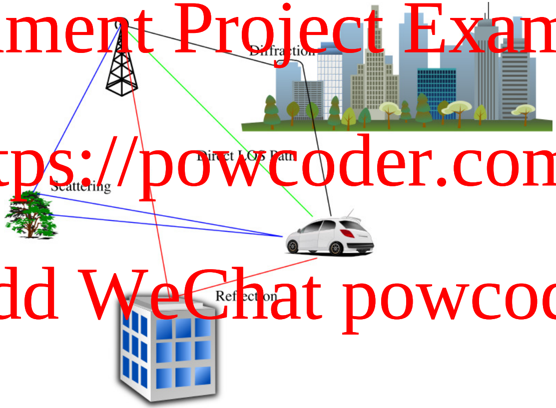


Figure 1: Multipath propagation due to reflection, diffraction and scattering.

https://openclipart.org/detail/194650/multipath-propagation

Path loss

- Path loss characterizes how a signal’s received power decreases with transmit-receive distance; occurs over long distances (100-1000 m).

The linear path loss is the ratio of transmit power to receive power:

$$P_L = \frac{P_t}{P_r} \tag{18}$$

The linear path loss in decibels

$$\begin{aligned} P_L(dB) &= 10 \log_{10}\left(\frac{P_t}{P_r}\right)(dB) \qquad (19) \\ &= 10 \log_{10}(P_t)(dBm) - 10 \log_{10}(P_r)(dBm) \qquad (20) \end{aligned}$$

The dB path gain is defined as the negative of the dB path loss:

$$P_G = -P_L(dB) = 10 \log_{10}\left(\frac{P_r}{P_t}\right)(dB) \tag{21}$$

- 15 dBm (32 mW) Typical wireless LAN transmission power in laptops.
- 27 dBm (500 mW) Typical cellular phone transmission power
- 33 dBm (2 W) Maximal output from a UMTS/3G mobile phone
- 46 dBm (40 W) Maximum allowed output of a single port LTE BS

Line-of-sight (LOS) propagation

Free-space signal propagation: There are no obstructions between the transmitter and receiver. Then, the signal propagates along a straight line between Tx and Rx.

- Line-of-sight (LOS) channel

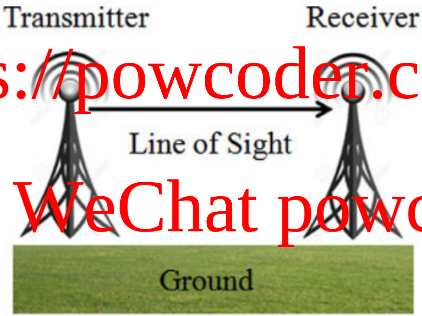


Figure 2: Line-of-sight (LOS) propagation.

Free-space path loss

- The received signal [Parsons, 2000]

$$r(t) = \Re\left[\left(\frac{\lambda\sqrt{G_t G_r}\, u(t-\tau_\ell) e^{-j2\pi d/\lambda}}{4\pi d}\right) e^{j2\pi f_c t}\right] \tag{22}$$

  - G_t – the transmit antenna power gain;
  - G_r – the receive antenna power gain;
  - τ_ℓ – the signal propagation delay;
  - $e^{-j2\pi d/\lambda}$ – the phase shift due to the distance d.

- The received signal power (use $re^{i\theta} = r(\cos\theta + j\sin\theta)$ and $|re^{i\theta}| = r$)

$$P_r = \left(\frac{P_t G_t}{4\pi d^2}\right)\left(\frac{\lambda^2 G_r}{4\pi}\right) \quad \text{The Friis formula [Friis, 1946]} \tag{23}$$

- Free-space path loss

$$P_L = -10\log_{10}\left[G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2\right] \text{ in dB} \tag{24}$$

$$P_L \propto d^{-2} \tag{25}$$

Two-Ray multipath model

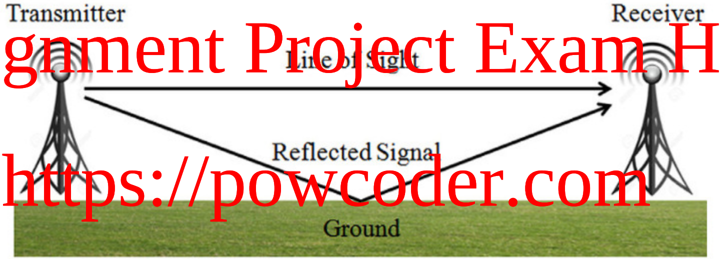


Figure 3: A simple point-to-point wireless communications system.

- The received signal consists of two components:
  1. The LOS component
  2. A reflected component which is the signal reflected off the ground.

Two-Ray multipath (mathematical) model

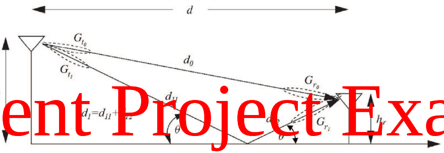


Figure 4: Two ray model.

The received signal ($\sqrt{G_{t_0}G_{r_0}} = \sqrt{G_0}$ and $\sqrt{G_{t_1}G_{r_1}} = \sqrt{G_1}$):

$$r(t) = \frac{\lambda}{4\pi}\Re\left[\left(\frac{\sqrt{G_0}u(t-\tau_0)e^{-j2\pi d_0/\lambda}}{d_0} + \frac{\sqrt{G_1}u(t-\tau_1)e^{-j2\pi d_1/\lambda}}{d_1}\right)e^{j2\pi f_c t}\right] \quad (26)$$

The received power ($d \gg 0$, $u(t-\tau_0) \approx u(t-\tau_1)$, $\sqrt{G_0} = \sqrt{G_1} = \sqrt{G}$, etc.):

$$P_r \approx P_t\left(\frac{\sqrt{G}h_t h_r}{d^2}\right)^2 \propto d^{-4} \quad (27)$$

The received power is inversely proportion to d^4.

Simplified path-loss model

A general received signal due to multiple rays:

$$r(t) = \Re\left[\left(\text{LoS} + \sum_{i=1}^{N_r} \text{reflected}_i + \sum_{j=1}^{N_d} \text{diffracted}_j + \sum_{k=1}^{N_s} \text{scattered}_k\right) e^{j2\pi f_c t}\right] \quad (28)$$

- The complexity of signal propagation makes it difficult to obtain a single model that characterizes path loss accurately across a range of different environments and frequencies.

- Best to use a simple model that captures the essence of signal propagation, e.g., single-slope model:

$$P_r = P_t K \left(\frac{d_r}{d}\right)^{\alpha} \quad (29)$$

- d_r is a reference distance for the antenna far field, α is the path-loss exponent, and K is a unit-less constant.
- K, d_r, and α can be obtained to approximate either an analytical or empirical model (Lab 2: Path loss measurement using NI-USRP).

Shadowing

- Shadowing is the attenuation caused by obstacles between the transmitter and receiver that absorb the transmitted signal. shadowing occurs over distances that are proportional to the length of the obstructing object (10-100 m).

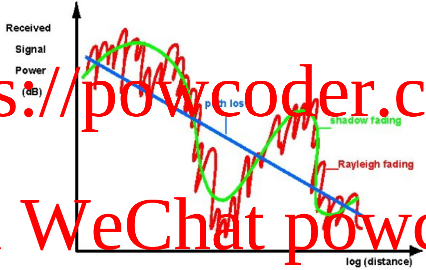


Figure 5: Path loss, shadowing and fading vs distance.

- Random blockages due to location, size, and dielectric properties of the blocking objects cause the random attenuation, and are unknown.
- Statistical models must be used to characterize this attenuation.

Log-Normal shadowing

- The ratio of transmit-to-receive power $\psi = \frac{P_t}{P_r}$ is a RV with a log-normal distribution:

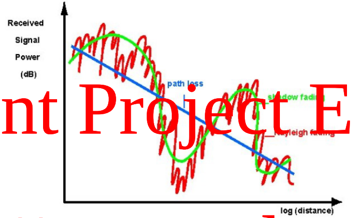
$$\text{PDF:} \quad f_\psi(x) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{\text{dB}}} x} e^{-\frac{\left(10\log_{10} x - \mu_{\psi_{\text{dB}}}\right)^2}{2\sigma^2_{\psi_{\text{dB}}}}}, \; x > 0 \tag{30}$$

$$\text{CDF:} \quad F_\psi(x) = \frac{1}{2}\text{erfc}\left(\frac{\mu - \xi\ln(x)}{\sqrt{2}\sigma}\right) \tag{31}$$

where
  - $\xi = 10/\ln(10)$,
  - $\mu_{\psi_{\text{dB}}}$ is the mean of $\psi_{\text{dB}} = 10\log_{10}(\psi)$ in dB, and
  - $\sigma_{\psi_{\text{dB}}}$ is the standard deviation of ψ_{dB} in dB.
- ψ_{dB} is then a Gaussian distribution with mean $\mu_{\psi_{\text{dB}}}$ and standard deviation $\sigma_{\psi_{\text{dB}}}$.

$$\psi_{\text{dB}} \sim \mathcal{N}\left(\mu_{\psi_{\text{dB}}}, \sigma_{\psi_{\text{dB}}}\right); \quad f_{\psi_{\text{dB}}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{\text{dB}}}} e^{-\frac{\left(x-\mu_{\psi_{\text{dB}}}\right)^2}{2\sigma^2_{\psi_{\text{dB}}}}} \tag{32}$$

Combined path loss and shadowing



$$\frac{P_r}{P_t}(\text{dB}) = \text{Path-loss}\,(\text{dB}) - \text{Shadowing}\,(\text{dB}) \tag{33}$$

$$= 10\log_{10}(K) + 10\alpha\log_{10}\left(\frac{d_r}{d}\right) - \psi_{\text{dB}} \tag{34}$$

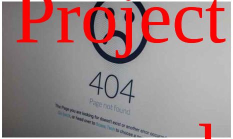
For the simplified path-loss model,

$$P_r\,(\text{dBm}) = \underbrace{P_t\,(\text{dBm}) + 10\log_{10}(K)\,(\text{dB}) - 10\alpha\log_{10}\left(\frac{d}{d_r}\right)(\text{dB})}_{\text{Fixed for a given } d,\ \triangleq\ C(d)} - \underbrace{\psi_{\text{dB}}}_{\text{RV}} \tag{35}$$

$$P_r(d) = C(d) - \psi_{\text{dB}} \tag{36}$$

Outage probability

Wireless systems typically require a target minimum received power level P_{min} (or equivalently a minimum signal-to-noise ratio (SNR)). Performance of a wireless network becomes unacceptable below this threshold P_{min}.



Outage probability $P_{out}(P_{min}, d)$: The probability at which the received power value falls below a power threshold P_{min} at distance d.

Outage probability for the the simplified path-loss model and log-normal shadowing with $\psi_{\mathrm{dB}} \sim \mathcal{N}(0, \sigma_{\psi_{\mathrm{dB}}})$:

$$
\begin{aligned}
P_{out}(P_{min}, d) &= \Pr\left[P_r(d) < P_{min}\right] && (37)\\
&= \Pr\left[C(d) - \psi_{\mathrm{dB}} < P_{min}\right] && (38)\\
&= \Pr\left[\psi_{\mathrm{dB}} > C(d) - P_{min}\right] && (39)\\
&= Q\left(\frac{C(d) - P_{min}}{\sigma_{\psi_{\mathrm{dB}}}}\right) && (40)
\end{aligned}
$$

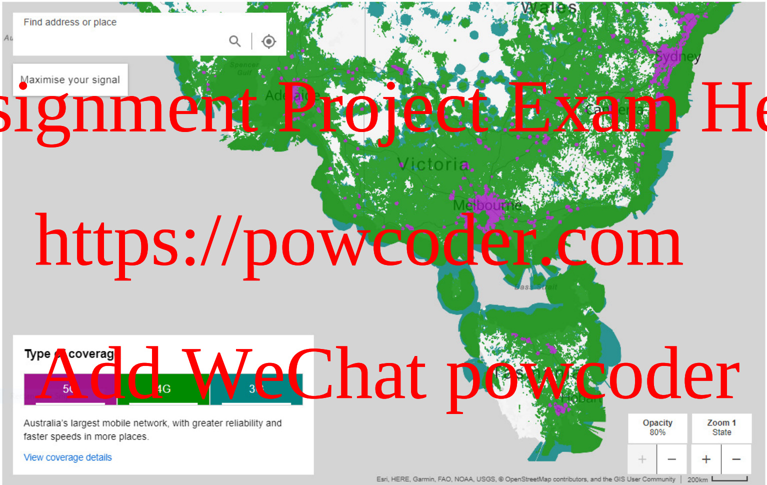
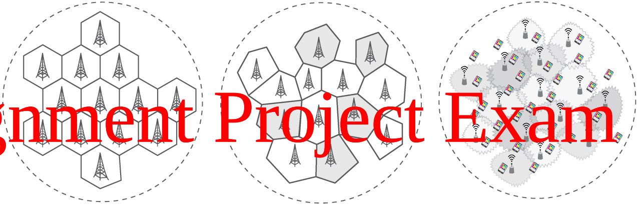


Figure 6: Telstra coverage maps (access on 29/07/2022).

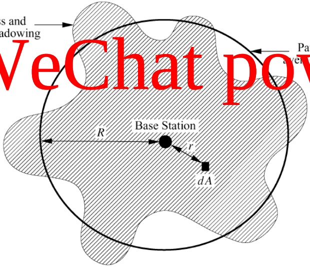
https://www.telstra.com.au/coverage-networks/our-coverage

Coverage area



(a) Regular hexagon (b) Irregular polygon (c) Fractal shape

Figure 7: The evolution of wireless cellular coverage boundaries.



Cell coverage percentage

The expected percentage of locations within a cell where received power exceeds P_{min} (the fraction of useful service area).

$$C = \frac{Coverage\ area}{\pi R^2} = \frac{\int_0^{2\pi} d\theta \int_0^R \hat{P}_{out}(r)\, r\, dr}{\pi R^2} \tag{41}$$

where

$$\hat{P}_{out}(P_{min}, r) = \Pr\left(P_r(r) \geq P_{min}\right) = 1 - P_{out}(P_{min}, r) \tag{42}$$

$$= Q\left(\frac{P_{min} - P_t - 10\log_{10}(K) - 10\alpha\log_{10}\left(\frac{d_r}{r}\right)}{\sigma_{\psi_{\mathrm{dB}}}}\right) \tag{43}$$

$$= Q\left(\frac{P_{min} - \left(P_t + 10\log_{10}(K) + 10\alpha\log_{10}\left(\frac{d_r}{R}\right) + 10\alpha\log_{10}\left(\frac{R}{r}\right)\right)}{\sigma_{\psi_{\mathrm{dB}}}}\right) \tag{44}$$

$$= Q\left(\frac{P_{min} - \left(P_t + 10\log_{10}(K) - 10\alpha\log_{10}\left(\frac{R}{d_r}\right)\right)}{\sigma_{\psi_{\mathrm{dB}}}} + \frac{10\alpha\log_{10}\left(\frac{r}{R}\right)}{\sigma_{\psi_{\mathrm{dB}}}}\right) \tag{45}$$

If

$$a = \frac{P_{min} - \left(P_t + 10\log_{10}(K) - 10\alpha\log_{10}\left(\frac{R}{d_r}\right)\right)}{\sigma_{\psi_{\text{dB}}}} \text{ and } b = \frac{10\alpha\log_{10}(e)}{\sigma_{\psi_{\text{dB}}}}$$

$$\hat{P}_{out}(P_{min}, r) = Q\left(a + b\ln\left(\frac{r}{R}\right)\right) \tag{46}$$

Then,

$$C = \frac{2\pi\int_0^R Q\left(a + b\ln\left(\frac{r}{R}\right)\right) r\,dr}{\pi R^2} = Q(a) + e^{\frac{2-2ab}{b^2}} Q\left(\frac{2-ab}{b}\right) \tag{47}$$

References

- A. Goldsmith, *Wireless Communications*, Cambridge University Press, USA, 2005.
- D. Parsons, *The Mobile Radio Propagation Channel*, 2nd Ed., Wiley, New York, 2000.
- H. T. Friis, *A note on a simple transmission formula*, Proc. IRE, vol. 34, no. 5, pp. 254-256, May 1946.

Thank You!

See you again ☺