# ECE5884 Wireless Communications Quiz 7 5 September 2022

1. Given $x(t)$ a continuous time signal, a snapshot of this signal from $[-T/2,\ T/2]$ *window* (a rectangle function) is given by $x_T\,(t) = rect(t/T)x(t)$. Using the Fourier transform tables given below. Show that a finite duration signal cannot be bandlimited.

2. Given a lowpass signal given

$$x(t) = 5 + 3\cos\left(2\pi 10^6 t + \frac{\pi}{4}\right) + \cos\left(\frac{4}{3}\pi 10^6 t + \frac{\pi}{4}\right)$$

   a. What is the Nyquist frequency $f_N$ and the Nyquist sampling rate $f_s$
   b. Given $fs = 5f_N$, write the discrete time equivalent $x[nT_s]$ of the continuous time signal $x(t)$.

3. What is Nyquist criterion for pulseshaping in digital communication systems?
   a. Write the expression for Raised cosine filter response (Week 7 Lecture Slide 15). What is the occupied low-pass bandwidth of RC pulse?
   b. Design a raised cosine (RC) pulse shape for a bandlimited signal with occupied bandwidth of $25kHz$ and can support a minimum data rate of $f_b = 1/T = 45kbps$. Assuming BPSK modulation, two RC pulses per second can be transmitted over the channel. (Hint: Week 7 Lecture Slide 15, write the expression for occupied bandwidth and equate it to 25kHz to determine the roll-off factor).
   
   Note: As Nyquist pulses satisfy the zero ISI criterion, it is possible to overlap these pulses and send two pulses per sec. Rectangular pulses are not Nyquist pulses hence, the pulse rate is 1 pulse/sec.
   
   c. If rectangular pulses were used instead for transmission, what is the required bandwidth to support a data rate of $45kbps$.



| Property | Aperiodic Signal | Fourier Transform |
|---|---|---|
| | $x(t)$ | $\mathsf{x}(f)$ |
| | $y(t)$ | $\mathsf{y}(f)$ |
| Linearity | $ax(t)+by(t)$ | $a\,\mathsf{x}(f)+b\mathsf{y}(f)$ |
| Time shifting | $x(t-t_0)$ | $e^{-j2\pi f_c t_0}\,\mathsf{x}(f)$ |
| Frequency shifting | $e^{j2\pi f_0 t}x(t)$ | $\mathsf{x}(f-f_0)$ |
| Conjugation | $x^*(t)$ | $\mathsf{x}^*(-f)$ |
| Time reversal | $x(-t)$ | $\mathsf{x}(-f)$ |
| Time scaling | $x(at)$ | $\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$ |
| Convolution | $x(t)*y(t)=\int_{-\infty}^{\infty}x(\tau)y(t-\tau)\mathrm{d}\tau$ | $\mathsf{x}(f)\mathsf{y}(f)$ |
| Autocorrelation | $x(t)*x^*(-t)$ | $\lvert\mathsf{x}(f)\rvert^2$ |
| Multiplication | $x(t)y(t)$ | $\mathsf{x}(f)*\mathsf{y}(f)=\int_{-\infty}^{\infty}X(\theta)Y(f-\theta)\mathrm{d}\theta$ |
| Differentiation in time | $\frac{d^n x(t)}{dt^n}$ | $(j2\pi f)^n\,\mathsf{x}(f)$ |
| Integration | $\int_{-\infty}^{t}x(\tau)\mathrm{d}\tau$ | $\frac{1}{j2\pi f}\mathsf{x}(f)+\frac{1}{2}X(0)\delta(f)$ |
| Differentiation in frequency | $t^n x(t)$ | $\left(\frac{j}{2\pi}\right)^n\frac{\mathrm{d}^n X(f)}{\mathrm{d}f^n}$ |
| Modulation (1) | $x(t)e^{j2\pi f_0 t}$ | $\mathsf{x}(f-f_0)$ |
| Modulation (2) | $x(t)\cos(2\pi f_0 t)$ | $\frac{1}{2}[\mathsf{x}(f-f_0)+X(f+f_0)]$ |
| Modulation (3) | $x(t)\sin(2\pi f_0 t)$ | $\frac{1}{j2}[\mathsf{x}(f-f_0)-X(f+f_0)]$ |
| Conjugate symmetry for real signals | $x(t)$ is real | $\begin{cases}\mathsf{x}(f)=\mathsf{x}^*(-f)\\ \mathrm{Re}\{\mathsf{x}(f)\}=\mathrm{Re}\{\mathsf{x}(-f)\}\\ \mathrm{Im}\{\mathsf{x}(f)\}=-\mathrm{Im}\{\mathsf{x}(-f)\}\\ \lvert\mathsf{x}(f)\rvert=\lvert\mathsf{x}(-f)\rvert\\ \measuredangle\mathsf{x}(f)=-\measuredangle\mathsf{x}(-f)\end{cases}$ |
| Symmetry for real and even signals | $x(t)$ real and even | $\mathsf{x}(f)$ real and even |
| Symmetry for real and odd signals | $x(t)$ real and odd | $\mathsf{x}(f)$ purely imaginary and odd |
| Even-odd decomposition for real signals | $x_e(t)=\mathrm{Ev}\{x(t)\}$ [$x(t)$real]<br>$x_o(t)=\mathrm{Od}\{x(t)\}$ [$x(t)$real] | $\mathrm{Re}\{\mathsf{x}(f)\}$<br>$j\mathrm{Im}\{\mathsf{x}(f)\}$ |
| Duality | $x(t)\longleftrightarrow\mathsf{x}(f)$ | $\mathsf{x}(t)\longleftrightarrow x(-f)$ |
| Parseval's theorem | $\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,\mathrm{d}t=\int_{-\infty}^{\infty}\lvert\mathsf{x}(f)\rvert^2\,\mathrm{d}f$ | |
| | $\int_{-\infty}^{\infty}x(t)y^*(t)\mathrm{d}t=\int_{-\infty}^{\infty}\mathsf{x}(f)\mathsf{y}^*(f)\mathrm{d}f$ | |
| ($x(t)$ and $y(t)$ real) | $\int_{-\infty}^{\infty}x(t)y(-t)\mathrm{d}t=\int_{-\infty}^{\infty}\mathsf{x}(f)\mathsf{y}(f)\mathrm{d}f$ | |

| Function Name | Time-Domain Signal $x(t)$ | Frequency-Domain Signal $\mathsf{x}(f)$ |
|---|---|---|
| Impulse | $\delta(t)$ | $1$ |
| DC | $1$ | $\delta(f)$ |
| Complex exponential | $e^{j2\pi f_0 t}$ | $\delta(f-f_0)$ |
| Cosine | $\cos(2\pi f_0 t+\theta)$ | $\frac{1}{2}\left[e^{j\theta}\delta(f-f_0)+e^{-j\theta}\delta(f+f_0)\right]$ |
| Sine | $\sin(2\pi f_0 t+\theta)$ | $\frac{1}{2j}\left[e^{j\theta}\delta(f-f_0)-e^{-j\theta}\delta(f+f_0)\right]$ |
| Unit step | $u(t)=\begin{cases}1 & t\geqslant 0\\ 0 & t<0\end{cases}$ | $\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$ |
| Sign | $\mathrm{sgn}(t)=\begin{cases}1 & t\geqslant 0\\ -1 & t<0\end{cases}$ | $\frac{1}{j\pi f}$ |
| Impulse train | $\mathrm{III}(t/T)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$ | $\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{T})$ |
| Fourier series | $\sum_{k=-\infty}^{\infty}a_k e^{j2\pi f_0 kt}$ where $a_k=\frac{1}{T}\int_T x(t)e^{-j2\pi f_0 kt}\mathrm{d}t$ | $\sum_{n=-\infty}^{\infty}a_n\delta(f-nf_0)$ |
| Rectangle pulse | $\mathrm{rect}\left(\frac{t}{T}\right)=\begin{cases}1 & \lvert t\rvert\leqslant\frac{T}{2}\\ 0 & \text{elsewhere}\end{cases}$ | $T\mathrm{sinc}(fT)=\frac{\sin(\pi fT)}{\pi f}$ |
| Triangle pulse | $\Lambda\left(\frac{t}{W}\right)=\begin{cases}1-\frac{\lvert t\rvert}{W} & \lvert t\rvert\leqslant W\\ 0 & \text{elsewhere}\end{cases}$ | $W\mathrm{sinc}^2(fW)$ |
| Sinc pulse | $\mathrm{sinc}(Wt)=\frac{\sin(\pi Wt)}{\pi Wt}$ | $\frac{1}{W}\mathrm{rect}\left(\frac{f}{W}\right)$ |
| Sinc$^2$ pulse | $\mathrm{sinc}^2(Wt)$ | $\frac{1}{W}\Lambda\left(\frac{f}{W}\right)$ |
| Exponential pulse | $e^{-a\lvert t\rvert}$ with $a>0$ | $\frac{2a}{a^2+(2\pi f)^2}$ |
| Decaying exponential | $e^{-at}u(t)$ with $\mathrm{Re}\{a\}>0$ | $\frac{1}{a+j2\pi f}$ |
| | $te^{-at}u(t)$ with $\mathrm{Re}\{a\}>0$ | $\frac{1}{(a+j2\pi f)^2}$ |
| | $\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$ with $\mathrm{Re}\{a\}>0$ | $\frac{1}{(a+j2\pi f)^n}$ |
| Linear decaying | $\frac{1}{t}$ | $-j\pi\,\mathrm{sgn}(f)$ |

4. Given a baseband continuous time signal $x(t)$ passing through a continuous-time channel $h(t)$ to produce $y(t)$ at the baseband receiver input, explain the steps involved in deriving the discrete-time equivalent system model involving $x[n]$, $h[n]$ and $y[n]$.
    a. Given a bit sequence [1 0 1 1 1 1 0 1 0 1], calculate the 4-QAM symbol sequence s[n] mapped from a normalised 4-QAM constellation map.
    b. Calculate the channel output $y[n]$, if the channel is flat fading and defined by the expression $h = 0.3e^{j\text{pi}/4}$.
    c. A training sequence that is BPSK modulated $t = [1\ 1\ 1\ -1\ -1\ 1\ -1]$ is used in the transmission over the channel defined in (b). Using Least squares approach in Matlab, estimate the channel. Use the following code template and complete the steps inside the for loop.

```
clear all
t = [1 1 1 -1 -1 1 -1 ]';
N_t = length(t);
N_d = 14;
N_f = N_t+N_d;
BPSK_sym_map = [-1,1];
h=0.3*exp(j*pi/4);
SNR = [0:5:30];
SNR_lin = 10.^(SNR/10);
for snr_loop = 1:length(SNR_lin)
snr = SNR_lin(snr_loop)
sigma_n = sqrt(abs(h*h')/snr);
for n = 1:1000

%     Generate complex AWGN sample with standard deviation sigma_n
%     Generate a random index to draw a BPSK symbol from the map, use
randi
%     Generate a the BPSK data symbol using the the random index name
it, data_sym
%     Organize the transmission frame as, Frame = [t;data_sym];
%     calculate the channel output y
%     Calculate the LS channel estimate using the training vector and
%     y(1:N_t) the channel output during training phase
%     calculate the squared error error_sq

end
mse_gamma(snr_loop) =mean(error_sq);

end

semilogy(SNR,mse_gamma)
```