ECE5884 Wireless Communications – Assignment 1 22 Aug. 2022

1. (5 points) Assume your mobile phone is located 50 m from a base station transmitting at 10 W. The mobile phone network is operating at 900 MHz. Free space propagation is assumed and transmitter and receiver directivity is given by 1 each. You may use the Friis formula.

   (a) Find the received power at your mobile phone.

   (b) How does the received power change if the system frequency is 1800 MHz?

2. (5 points) Spectral efficiency indicates how effectively the bandwidth is used for data transmission. Assume that a wireless system has a spectral efficiency of 2 bps/Hz. If the coherence bandwidth of the channel is found out to be 500 kHz, then calculate which of the following data rates will experience flat fading: 8 kbps, 40 kbps, 100 kbps, 6 Mbps. Justify your answer!

3. (10 points) A wireless system experiences both path loss and shadowing effects where path loss follows the single-slope simplified model (eq. (29) Week 2 slides), and shadowing follows log-normal distribution (eq. (32) Week 2 slides).

   (a) Derive an outage probability expression at distance d for the minimum received power P_{min}.

   (b) Calculate the outage probability with $\alpha = 3, d_r = 1\,m, d = 100\,m, K = -31.54\,dB, P_t = 5\,mW$, $P_{min} = -100$ dBm, $\mu_{\psi_{dB}} = 0$ dB and $\sigma_{\psi_{dB}} = 3.65$ dB.

4. (10 points MATLAB) A multipath fast-fading channel of wireless communications system is given as $h = h_r + jh_i$ which is a complex number. We assume $h_r \sim \mathcal{N}(m_r, \sigma^2)$ and $h_i \sim \mathcal{N}(m_i, \sigma^2)$ which are Gaussian distributions with non-zero means and variance σ^2. Then, the envelop of h, i.e., $|h|$, follows a Rician distribution. The PDF of $|h|$ is given in eq. (13) Week 3 slides. Using MATLAB simulations, verify the analytical expression of PDF when $m_r = 1.5$, $m_i = 1.2$ and $\sigma^2 = 0.5$. You need to submit a MATLAB code!

5. (18 points) We have a simple wireless system with a transmitter and receiver pair which are implemented with a single antenna. Then, the received signal at the receiver at time t may be written as

$$r(t) = h\, s(t) + n(t). \tag{1}$$

Here, h is the flat-fading wireless channel; $s(t)$ is the transmitted signal which has the average transmit power P_s (W); and $n(t)$ is the additive white Gaussian noise (AWGN) which follows a circularly symmetric complex Gaussian distribution, i.e., $n(t) \sim \mathcal{CN}(0, N_0)$, where N_0 (W/Hz) is the average noise power. The channel bandwidth of the wireless system is B (Hz). We neglect path-loss and shadowing effects. We assume that the envelop of h, i.e., $|h|$, follows Nakagami-m distribution (m is a positive integer) as given in eq. (16) Week 3 slides.

(a) Write an expression for the instantaneous received signal-to-noise ratio (SNR) γ in terms of h, P_s, N_0 and B.

(b) Derive an expression for the SNR outage probability, when the received SNR falls below a threshold γ_{th}. You may give the outage probability in terms of m, γ_{th} and $\bar{\gamma}$ where $\bar{\gamma}$ is the average SNR which is defined as $\bar{\gamma} = \frac{\Omega_p P_s}{N_0 B}$. Please provide details of your derivation!

(c) Plot SNR outage probability vs $\bar{\gamma}$ for $\gamma_{th} = 5$ dB and $m = 1, 2, 3$ when $\bar{\gamma}$ varies from -5 dB to 20 dB. Use semi-log plot, i.e., y-axis in log scale and x-axis in linear scale where $\bar{\gamma}$ is in dB.

(d) What do you observe in the above plot when m increases? Explain why?

(e) (**Optional**) Derive an asymptotic expression for the SNR outage probability when the average SNR is large, i,e, $\bar{\gamma} \to \infty$. You may plot asymptotic results for $m = 1, 2, 3$ on the same graph in (c).

(f) For the same wireless system, we assume that only the receiver knows channel state information (CSI) perfectly. Then, derive an expression for the capacity outage probability if the predetermined capacity threshold is C_{th}.

Hint#1: You may use the following integral solution:

$$\int_0^d x^{a-1} e^{-bx}\, dx = \frac{\Gamma(a) - \Gamma(a, b\, d)}{b^a}; a, b, d > 0$$

where $\Gamma(x)$ id the Gamma function and $\Gamma[a, x]$ is the the incomplete Gamma function. You may implement this function in following software as:

$$\Gamma(n,x) = \begin{cases} \text{Gamma[n, x]}, & \text{Mathematica.} \\ \text{gammainc(x,n,'upper')*gamma(n)}, & \text{MATLAB.} \end{cases}$$

Hint#2: You may use the following series expansion:

$$\lim_{x \to 0} \Gamma[n,x] \approx \Gamma[n] - \frac{x^n}{n}.$$

6. (12 points) Consider a flat fading channel of bandwidth 20 MHz and where, for a fixed transmit power $\bar{P}$, the received SNR is one of six values: $\gamma_1 = 20\,\text{dB}$, $\gamma_2 = 15\,\text{dB}$, $\gamma_3 = 10\,\text{dB}$, $\gamma_4 = 5\,\text{dB}$, $\gamma_1 = 0\,\text{dB}$, and $\gamma_6 = -5\,\text{dB}$. The probabilities associated with each state are $p_1 = p_6 = 0.1$, $p_2 = p_4 = 0.15$, and $p_3 = p_5 = 0.25$.

(a) Assume that only the receiver has CSI. Find the Shannon (ergodic) capacity of this channel.

(b) Assume that both the receiver and transmitter have CSI. Give the optimal power adaptation policy for this channel, and the corresponding Shannon capacity.