ECON 61001: *Hypothesis Testing: Power*

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Hypothesis testing

- A statistical hypothesis is a conjecture about the distribution of one or more random variables.
- The classical theory of hypothesis testing provides a framework for deciding whether a particular hypothesis is correct.
- Within this framework, there are only two possible decisions: the hypothesis is true or it is not. A decision procedure for such a problem is called a test.

Assume: our hypothesis involves θ, the parameter vector indexing distribution of V, and Θ denote the parameter space with $\Theta \subset \mathbb{R}^p$.

Divide Θ into two mutually exclusive and exhaustive parts:

$$\begin{aligned} \Theta_0 &= \{\theta : \text{ such that the hypothesis is true}\}, \\ \Theta_1 &= \{\theta : \text{ such that the hypothesis is false}\} \end{aligned}$$

Using this partition, we can state the object as being to test the null hypothesis,

$$H_0 : \theta \in \Theta_0$$

against the alternative hypothesis,

$$H_1 : \theta \in \Theta_1.$$

Base inference on some test statistic; denoted by S_T.

Decision rule:

$$S_T \in R_0 \quad \Rightarrow \quad H_0 \text{ is accepted or rather not rejected}$$

$$S_T \in R_1 \quad \Rightarrow \quad H_0 \text{ is rejected in favour of } H_1$$

In the companion podcast discussed how R_0 and R_1 are chosen to control the probability of a Type I error.

Now consider the properties of the test under H_1.

Power of a test

Let $P_\theta(\,\cdot\,)$ denote the probability of the event in parentheses if the parameter vector takes the value θ.

Define $\beta(\theta) = P_\theta(R_0) = 1 - P_\theta(R_1)$; that is, $\beta(\theta)$ describes the probability of a type II error for values of θ that satisfy H_1.

The power function of the test is:

$$\pi(\,\cdot\,) = 1 - \beta(\,\cdot\,).$$

$\rightarrow$ for $\theta_* \in \Theta_1$, $\pi(\theta_*)$ is:

- the probability of correctly rejecting H_0 when $\theta = \theta_*$.
- the power of the test against the alternative $\theta = \theta_*$.

Suppose that:

- $y_t \sim IN(\theta, \sigma^2)$, $t = 1, 2, \ldots T$; assume σ^2 known.
- wish to test $H_0 : \theta = 0$ versus $H_1 : \theta \neq 0$.

- Decision rule: reject $H_0 : \theta = 0$ in favour of $H_1 : \theta \neq 0$ at the 5% significance level if $|\tau_T| > 1.96$.

So power of the test is given by

$$\pi(\theta) \ = \ P(|\tau_T| \ > \ 1.96 \,|\, \theta, \theta \in \Theta_1).$$

To evaluate $\pi(\theta)$ we need the distribution of τ_T if $\theta \neq 0$.

We have

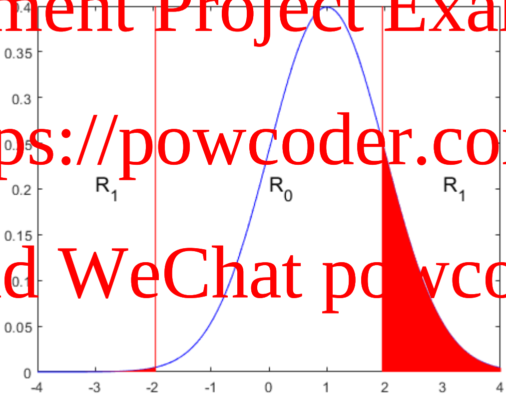
$$\tau_T = \frac{\bar{v}_T}{\sqrt{\sigma^2/T}} = \frac{\bar{v}_T - \theta}{\sqrt{\sigma^2/T}} + \frac{\theta}{\sqrt{\sigma^2/T}} \sim N(\mu, 1),$$

where $\mu = \theta/(\sqrt{\sigma^2/T})$.

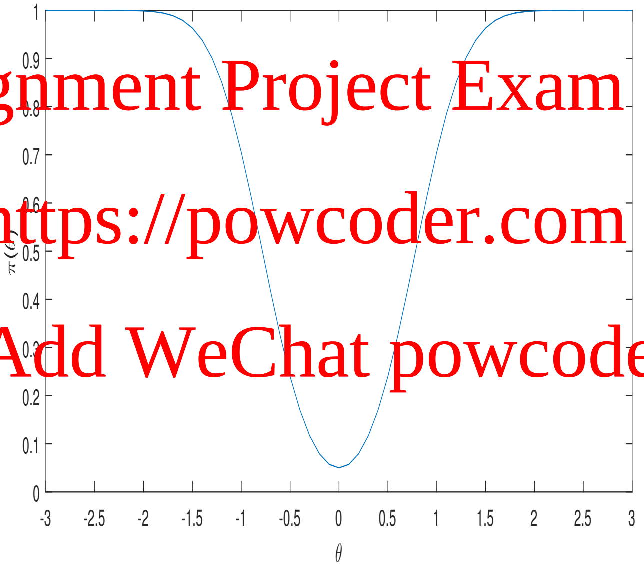
Power is a function of μ.

Next slide shows power for $\theta = \sigma/\sqrt{T}$, *i.e.* $\mu = 1$.

Example 2.8 in Lecture Notes: power of test when $\theta = \sigma/\sqrt{T}$



Example 2.8 in Lecture Notes: power curve



From plot of power function for this two-sided test we see that:

- power depends on $|\theta|$ in this case
- $P_\theta(R_1) > \alpha$ for all $\theta \in \Theta_1 \rightarrow$ test is said to be unbiased.

Further discussion of the topics in this podcast can be found in Section 2.8.1 of the Lecture Notes.