Alastair Hall | **ECON61001: Econometric Methods** | Semester 1, 2020-21

Problem Set for Tutorial 4

In this question you explore further the different scalings of the sample mean in the WLLN and CLT. In the Lecture Notes Section 3.1 Example 3.2, it is remarked that the CLT is an exact result for Case (i) in which $\{v_t\}_{t=1}^{T}$ *are independently and identically distributed normal random variables. In this question you verify this statement and consider its implications for the large sample behaviour of the sample mean scaled by different functions of the sample size.*

1. Let $\{v_t\}_{t=1}^{T}$ be a sequence of independently and identically distributed standard normal random variables (often written using the mathematical shorthand $v_t \sim IN(0,1)$, $t = 1, 2, \ldots T$) and set $\bar{v}_T = T^{-1}\sum_{t=1}^{T} v_t$.

   (a) Show that $T^{1/2}\bar{v}_T \sim N(0,1)$. *Hint: Write* $\sum_{t=1}^{T} v_t = \iota_T' v$ *where* $v = (v_1, v_2, \ldots, v_T)'$ *and* ι_T *is a* $T \times 1$ *vector of ones, and use Lemma 2.1 in the Lecture Notes, noting that* $v_t \sim IN(0,1)$, $t = 1, 2, \ldots T$ *implies* $v \sim N(0, I_T)$.

   (b) Let n be a finite positive constant and $z \sim N(0,1)$. Using part (a), show that $P(|\bar{v}_T| < n) = P(|z| < T^{1/2}n)$ and use this result to deduce $lim_{T\to\infty} P(|\bar{v}_T| < n)$. Relate this limiting behaviour to the WLLN.

   (c) Now consider $T\bar{v}_T$. Show that $P(|T\bar{v}_T| < n) = P(|z| < T^{-1/2}n)$ and so deduce $lim_{T\to\infty} P(|T\bar{v}_T| < n)$.

In this question, you consider the probability distribution of the errors in a type of regression model known as the linear probability model (LPM).

2. Consider the linear regression model

$$y_i = x_i'\beta_0 + u_i$$

   in which y_i is an indicator variable that takes the value one if an event occurs (such as an individual is employed) and zero otherwise. Assess: (i) whether x_i and u_i are independent; (ii) whether conditional on x_i, u_i can have a normal distribution.

In lectures, we stated that the the OLS estimator of the error variance is consistent. In this question, you establish this result.

3. Consider the linear regression model

$$y_i \; = \; x_i'\beta_0 \; + \; u_i$$

   where $x_i' = (1, x_{2,i}')$, the data are cross-sectional and the model satisfies Assumptions *CS1* - *CS5* so that: $\{\ (u_i, x_i'),\ i = 1, 2, \ldots N\}$ forms an independent and identically distributed sequence; $E[x_i x_i'] = Q$, finite, p.d.; $E[u_i|x_i] = 0$; $Var[u_i|x_i] = \sigma_0^2$, a positive, finite constant. Let $\hat{\sigma}_N^2$ denote the OLS estimator of σ_0^2. Show that $\hat{\sigma}_N^2 \xrightarrow{p} \sigma_0^2$.

In Tutorial 2, Question 2, you derived the bias of the OLS estimator when relevant regressors have been omitted from the model. In this question, you establish the large sample analogue to this result.

4. Consider the linear regression model

$$y_i \;=\; x_{i,1}'\beta_{0,1} \;+\; x_{i,2}'\beta_{0,2} \;+\; u_i$$

where $x_i' = (x_{i,1}', x_{i,2}')$, $x_{i,\ell}$ is $k_\ell \times 1$ for $\ell = 1, 2$, $k = k_1 + k_2$, and Assumptions *CS1* - *CS5* are satisfied so that: $\{\, (u_i, x_i'),\ i = 1, 2, \ldots N\}$ forms an independent and identically distributed sequence; $E[x_i x_i'] = Q$, finite, p.d.; $E[u_i|x_i] = 0$; $Var[u_i|x_i] = \sigma_0^2$, a positive, finite constant. Suppose that a researcher estimates the following model by OLS,

$$y_i \;=\; x_{i,1}'\gamma_* \;+\; \text{error}.$$

Let $\hat{\gamma}_N$ be the OLS estimator of γ_*. Show that $\hat{\gamma}_N \xrightarrow{p} \beta_{0,1} + Q_{1,1}^{-1}Q_{1,2}\beta_{0,2}$, where $E[x_{i,j}x_{i,\ell}'] = Q_{j,\ell}$ for $j, \ell = 1, 2$.

In Lecture 4, we discussed methods for testing nonlinear restrictions on β_0. In this question, you must use these methods to propose an appropriate statistic for testing a particular hypothesis.

5. Consider the linear regression model

$$y_i \;=\; x_i'\beta_0 \;+\; u_i$$

where: $\{\, (u_i, x_{2,i}'),\ i = 1, 2, \ldots N\}$ forms an independent and identically distributed sequence; $E[x_i x_i'] = Q$, finite, p.d.; $E[u_i|x_i] = 0$; $Var[u_i|x_i] = \sigma_0^2$, a positive, finite constant. Assume $k = 5$ that is, β_0 is 5×1 and that $\beta_{0,i} \neq 0$ for $i = 2, 3$. Suppose it is desired to $H_0 : \beta_{0,2}\beta_{0,3} - 1 = 0$ versus $H_1 : \beta_{0,2}\beta_{0,3} - 1 \neq 0$. Propose a test statistic and state the appropriate decision rule.