University of Manchester

ECON61001: Econometric Methods

Mock Exam

Release date/time:

Submission deadline:

Instructions:

- You must answer **all five questions in Section A** and **two out of the four questions in Section B**. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in your answer submission: answers beyond that number will not be considered.

- Your answers could be typed or hand-written (and scanned to a single pdf file that can be submitted) or a combination of a typed answer with included images of algebra or figures.

- Where relevant, questions include word limits. These are limits, not targets. Excellent answers can be shorter than the word limit. If you go beyond the word limit the additional text will be ignored. Where a question includes a word limit you HAVE to include a word count for your answer (excluding formulae). You could use https://wordcounter.net to obtain word counts.

- Candidates are advised that the examiners attach considerable importance to the clarity with which answers are expressed.

- **You must correctly enter your registration number and the course code on your answer.**

SECTION A

1.(a) Let A denote a $m \times m$ symmetric matrix. If A is positive definite then what does this imply about any quadratic form involving A? **[1 marks]**

1.(b) If A is a $m \times m$ positive definite matrix then what is $rank(A)$, the rank of A. Justify your answer.

[3 marks]

1.(c) Consider

$$A_1 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 7 & 5 \\ 5 & 3 \end{bmatrix}.$$

Verify whether or not A_1 and A_2 are positive definite, being sure to justify your answer. **[4 marks]**

2. Let $\{(y_i, x_i')\}_{i=1}^N$ be a sequence of independently and identically distributed (i.i.d.) random vectors. Suppose that y_i is a dummy variable and so has a sample space of $\{0, 1\}$ with $P(y_i = 1|x_i) = \Lambda(x_i'\beta_0)$ where $\Lambda(\cdot)$ is the cumulative distribution function of the logistic distribution. Derive:

   (a) Derive $E[y_i|x_i]$, $E[y_i]$ and $Var[y_i|x_i]$. **[4 marks]**

   (b) Now suppose that a researcher estimates a logit model based on $\{(y_i, x_i')\}_{i=1}^N$, and let $x_{i,j}$ be the j^{th} element of x_i and a continuous random variable.

      (i) Derive the marginal response of $P(y_i = 1|x_i)$ to a change in $x_{i,j}$ that is, $\partial\Lambda(x_i'\beta_0)/\partial x_{i,j}$. **[2 marks]**

      (ii) What is the limit of the marginal response function in your answer to part(i) as the index $x_i'\beta_0$ tends to infinity? Provide an intuitive explanation for why this must be the case. **[2 marks]**

3. Let $\hat{\theta}_T$ be an estimator of the unknown parameter θ_0. A researcher claims that *"if $\hat{\theta}_T$ is an unbiased estimator of θ_0 then it must also be a consistent estimator of θ_0"*. Evaluate this claim, being sure to justify your argument briefly and define any statistical concepts to which you refer. **[8 marks]**

Continued over

SECTION A continued

4. Consider the following model:

$$y_i \; = \; x_i\beta_0 \, + \, u_i, \qquad i = 1, 2, \ldots, n \tag{1}$$

where x_i is a scalar observable variable, u_i is the unobserved error. Let z_i be a $q \times 1$ vector of observable variables. Assume $\{(x_i, u_i, z_i')'\}$ is a sequence of independently and identically distributed random vectors.

(a) State the conditions under which z_i said to be a *valid instrument* for x_i in this model. Can these conditions be tested? If so then explain how? If not then explain why not? *(Word limit: 150 words)* **[2 marks]**

(b) Suppose now that

$$x_i \; = \; z_i'\gamma_0 \, + \, v_i,$$

where $E[z_i z_i'] = M_{zz}$, a positive definite matrix, and for $w_i = (u_i, v_i)'$, $E[w_i|z_i] = 0$ and $Var[w_i|z_i] = \Omega_0$. Provide a set of conditions involving the parameters of the model, β_0,γ_0, and Ω_0 under which x_i is an endogenous regressor and z_i is a valid instrument for x_i in this model, being sure to justify your answer carefully. **[6 marks]**.

5. Let $\{e_t\}$ be a univariate white noise process.

(a) Assess which of the following three series are *covariance stationary* providing a justification for your answer in each case:

(i) $u_t = e_t$;
(ii) $v_t = (-1)^t + e_t$;
(iii) $w_t = (-1)^t e_t$.

[5 marks]

(b) Assess which of the series in part (a) are *strictly stationary* providing a justification for your answer in each case. **[3 marks]**

Continued over

SECTION B

6. Consider the linear regression model

$$y \;=\; X\beta_0 \;+\; u \qquad (2)$$

where y is $T \times 1$ with t^{th} element y_t, X is $T \times k$ with t^{th} row $x_t' = [1, x_{2,t}']$, u is $T \times 1$ with t^{th} element u_t, β_0 is a $k \times 1$ vector of unknown parameters. Assume that (2) is the true model for y, X is **fixed in repeated samples**, $rank(X) = k$, $u \sim N(0, \sigma_0^2 I_T)$ for some unknown scalar constant σ_0^2. Let Z be a $T \times k$ matrix that is **fixed in repeated samples** with $rank(Z) = k$ and assume $Z'X$ is nonsingular. Define $\tilde{\beta}_T = (Z'X)^{-1}Z'y$, and let $\tilde{\beta}_{T,i}$ be the i^{th} element of $\tilde{\beta}_T$.

(a) Show that $\tilde{\beta}_T$ is an unbiased estimator of β_0. **[4 marks]**

(b) Show that $Var[\tilde{\beta}_T] = \sigma_0^2(X'Z(Z'Z)^{-1}Z'X)^{-1}$. **[9 marks]**

(c) State the formula for $Cov[\tilde{\beta}_{T,i}, \tilde{\beta}_{T,j}]$ in terms of σ_0^2 and $(X'Z(Z'Z)^{-1}Z'X)^{-1}$. **[2 marks]**

(d) Show that $\tilde{\beta}_T \sim N\left(\beta_0, Var[\tilde{\beta}_T]\right)$. **[3 marks]**

(e) A researcher argues that given the results in parts (a), (b) and (d) there is no reason to prefer inferences based on the OLS estimator of β_0 over inferences based on $\tilde{\beta}_T$. Do you agree? Justify your answer. **[4 marks]**

(f) Suppose now that X and Z are stochastic with $E[u|Z] = 0$. Is $\tilde{\beta}_T$ an unbiased and/or a consistent estimator of β_0? Justify your answer but there is no need to provide a formal analysis of the probability limit of $\tilde{\beta}_T$. **[8 marks]**

Continued over

SECTION B continued

7.(a) Consider the model

$$u_t \;=\; w_t \,+\, \phi w_{t-2}, \qquad t = 1, 2, \ldots, T, \tag{3}$$

where $\phi \neq 0$, and $\{w_t\}_{t=-\infty}^{\infty}$ is a sequence of independently and identically distributed random variables with $E[w_t] = 0$ and $Var[w_t] = \sigma^2$. Let u denote the $T \times 1$ vector with t^{th} element u_t. Derive $Var[u] = \Omega$ in terms of ϕ and σ^2. **[14 marks]**

7.(b) Consider the times series regression model

$$y_t \;=\; x_t'\beta_0 \,+\, u_t, \; t = 1, 2, \ldots, T \tag{6}$$

where $x_t = (1, y_{t-1})'$, $\beta_0 = (\beta_{0,1}, \beta_{0,2},)'$ and $\{u_t\}$ is generated as in part (a). You may **assume** that y_t has the following $MA(\infty)$ representation:

$$y_t \;=\; \mu_y \,+\, \sum_{i=0}^{\infty} \psi_{0,i} u_{t-i},$$

where μ_y is a constant and $\psi_{0,1} \neq 0$.

(i) Evaluate whether y_{t-1} is contemporaneously exogenous in (6). **[10 marks]**

(ii) Evaluate whether y_{t-1} is strictly exogenous in (6). **[6 marks]**

Continued over

SECTION B continued

8.(a) Consider the linear regression model

$$y_i \;=\; x_i'\beta_0 \,+\, u_i \tag{4}$$

where $x_i = (1, x_{2,i}')'$ and β_0 are $k \times 1$ vectors. Assume $\{(u_i, x_{2,i}')',\ i = 1, 2, \ldots N\}$ are independently and identically distributed with: (i) $E[x_i x_i'] = Q$, a finite, positive definite $k \times k$ matrix of constants; (ii) $E[u_i|x_i] = 0$; (iii) $Var[u_i|x_i] = h(x_i) > 0$. The OLS estimator of β_0 is $\hat{\beta}_N \;=\; (X'X)^{-1}X'y$ where y is a $N \times 1$ vector with i^{th} element y_i, X is a $N \times k$ matrix with i^{th} row x_i'.

Show that

$$N^{1/2}(\hat{\beta}_N \,-\, \beta_0) \;\xrightarrow{d}\; N\,(\,0,\, V_h\,)\,,$$

where $V_h = Q^{-1}\Omega_h Q^{-1}$, $\Omega_h = E[h(x_i)x_i x_i']$. **[8 marks]**

Hint: you may quote the generic form of the Weak Law of Large Numbers, $N^{-1}\sum_{i=1}^{N} z_i \xrightarrow{p} \mu_z$, *but must verify* μ_z *for the specific choices of* z_t *relevant to your answer. Also you may quote the generic form of the Central Limit Theorem,* $N^{-1/2}\sum_{i=1}^{N}(z_i - \mu_z) \xrightarrow{d} N(0, \Omega)$, *but must verify* μ_z *and* Ω *for the specific choices of* z_i *relevant to your answer.*

8.(b) Consider the following model

$$y_t \;=\; \beta_{0,1} \,+\, \beta_{0,2}x_{2,t} \,+\, \beta_{0,3}x_{3,t} \,+\, u_t, \text{ for } t = 1, 2, \ldots, T.$$

Let u be the $T \times 1$ vector with t^{th} element u_t and X be the $T \times 3$ matrix with t^{th} row $(1, x_{2,t}, x_{3,t})$. Suppose that a researcher estimates the model via OLS based on sample of size $T = 100$, and obtains the fitted equation:

$$\begin{array}{ccccccc} \widehat{y}_t = & 0.389 & + & 0.336x_{2,t} & + & 1.896x_{3,t} & \qquad R^2 = 0.455 \\ & (1.216) & & (0.234) & & (0.927) & \\ & [1.362] & & [0.286] & & [1.602] & \\ & \{1.321\} & & \{0.321\} & & \{1.421\} & \end{array} \tag{5}$$

where OLS Standard errors in parentheses ( ), White Standard Errors in square brackets [ ] and Newey-West Standard Errors in { }. In parts (i) - (iii) specified on the next page:

- Discuss how to test the hypothesis stated with correct asymptotic size α providing the relevant test statistic and its distribution.
- If more than one test may be formed based on the information in equation (5) state so, providing details on how to perform each test.
- Perform the test given the information in equation (5), discussing the choice of test in the case more than one option is available.

Continued over

SECTION B continued

8.(b) (i)

$$H_0 : \beta_{0,1} = 0; \quad H_A : \beta_{0,1} \neq 0$$

for $\alpha = 0.05$ where $Var(u_t|X) = \sigma^2|x_{2,t}|$ and $E[u_t u_{t-j}|X] = 0$ for all t and all $j \neq 0$. **[7 marks]**

(ii)

$$H_0 : \beta_{0,2} \leq 0; \quad H_A : \beta_{0,2} > 0$$

for $\alpha = 0.01$ if $u_t = \epsilon_t x_{1,t}$ where $Corr(\epsilon_t, \epsilon_{t-j}) = \exp(-|j|)$ for all t and j. **[7 marks]**

(iii)

$$H_0 : \beta_{0,2} = \beta_{0,3} = 0; \quad H_A : \beta_{0,2} \neq 0 \text{ and/or } \beta_{0,3} \neq 0$$

for $\alpha = 0.1$ where $u|X \sim N(0, \sigma^2 I_T)$. **[8 marks]**

Continued over

SECTION B continued

9. Let $\{(y_i, x_i')\}_{i=1}^{N}$ be a sequence of independently and identically distributed (i.i.d.) random vectors. Suppose that y_i is a dummy variable and so has a sample space of $\{0, 1\}$ with $P(y_i = 1|x_i) = \Phi(x_i'\beta_0)$ where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

   (a) Assume that $x_i = 1$. Show that the maximum likelihood estimator of β_0 is $\hat{\beta} = \Phi^{-1}(\bar{y})$ where $\bar{y}$ is the sample mean of y and $\Phi^{-1}(\,.\,)$ denotes the inverse of the cumulative distribution function of the standard normal distribution that is, if $\Phi(z) = p$ then $z = \Phi^{-1}(p)$ for any $z \in (-\infty, \infty)$. **[12 marks]**

   (b) A researcher is interested in modeling the probability that a citizen of a US town votes in favour of an increase in the local tax rate to provide additional funding for public schools as a function of certain family and household characteristics. Let $yesvm$ be a dummy variable that takes the value one if the citizen votes in favour of the tax increase and the explanatory variables are: $loginc$, the log of annual household income; $ptcon$, the log of property taxes paid in the year the vote took place; $years$ the number of years the voter has been living in the community; $school$, a dummy variable that takes the value one if the voter works in the public school system; and four other control variables denoted $x1$, $x2$, $x3$ and $x4$ below. Using the *Stata* output on the next page answer the following questions.

      (i) Test whether the amount of property taxes paid by a citizen affects the probability that they vote for the tax increase. Be sure to specify the null and alternative hypothesis, and the decision rule. **[4 marks]**

      (ii) What do the results reveal about how household income affects the probability of voting in favour of the tax increase? Be sure to justify your answer. **[4 marks]**

      (iii) Fifty nine out the ninety five citizens in the sample voted for the increase. Use the Likelihood Ratio statistic to test whether the explanatory variables in the model collectively help to explain the probability that a citizen votes in favour of the tax increase. Be sure to specify the null and alternative hypotheses, and the decision rule, and to explain how you calculate the test statistic. **[10 marks]**

Continued over

9.(b) contd The *Stata* output for the model is as follows in which certain portions have been deleted for the purpose of this question:

```
Probit regression                            Number of obs    =       95

Log likelihood = -52.844
------------------------------------------------------------------
 yesvm    |       Coef.   Std. Err.    z    P>|z|   [95% Conf.Interval]
---------+--------------------------------------------------------
   x1     |     0.2896     0.6962
   x2     |     0.8817     0.7810
   x3     |     0.4000     1.2932
   x4     |    -0.5189     0.7724
 years    |    -0.0241     0.0272    [      output deleted        ]
 school   |     2.7890     1.4859
 loginc   |     2.4341     0.8210
 ptcon    |    -2.4217     1.0982
 _cons    |    -7.2366     7.7340
------------------------------------------------------------------
```


END OF EXAMINATION

1 Table 1: Percentage Points for the t distribution

Student's t Distribution Function for Selected Probabilities The table provides values of $t_{\alpha,v}$ where $\Pr(T \leq t_{\alpha,v}) = \alpha$ and $T \sim t_v$										
α	0.750	0.800	0.900	0.950	0.975	0.990	0.995	0.9975	0.999	0.9995
ν	Values of $t_{\alpha,v}$									
1	1.000	1.376	3.078	6.314	12.706	31.821	63.657			
2	0.816	1.061	1.886	2.920	4.303	6.965	9.925			
3	0.765	0.978	1.638	2.353	3.182	4.541	5.841			
4	0.741	0.941	1.533	2.132	2.776	3.747	4.604			
5	0.727	0.920	1.476	2.015	2.571	3.365	4.032	4.773		
6	0.718	0.906	1.440	1.943	2.447	3.143	3.707	4.317	5.208	
7	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
70	0.678	0.847	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
80	0.678	0.846	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
90	0.677	0.846	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
100	0.677	0.845	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
110	0.677	0.845	1.289	1.659	1.982	2.361	2.621	2.865	3.166	3.381
120	0.677	0.845	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.282	1.645	1.960	2.326	2.576	2.808	3.090	3.297

2 Table 2: Percentage Points for the χ^2 distribution

The χ^2 Distribution Function for Selected Probabilities											
The table provides values of $\chi^2_{\alpha,v}$ where $\Pr(\chi^2 \leq \chi^2_{\alpha,v}) = \alpha$ and $\chi^2 \sim \chi^2_v$											
α	0.005	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99	0.995
v	Values of $\chi^2_{\alpha,v}$										
1	0.000	0.000	0.001	0.004	0.016	0.455	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	1.386	4.605	5.991	7.378	9.210	10.60
3	0.072	0.115	0.216	0.352	0.584	2.366	6.251	7.815	9.348	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	3.357	7.779	9.488	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.610	4.351	9.236	11.07	12.83	15.09	16.75
6	0.676	0.872	1.237	1.635	2.204	5.348	10.64	12.59	14.45	16.81	18.55
7	0.989	1.239	1.690	2.167	2.833	6.346	12.02	14.07	16.01	18.48	20.28
8	1.344	1.646	2.180	2.733	3.490	7.344	13.36	15.51	17.53	20.09	21.95
9	1.735	2.088	2.700	3.325	4.168	8.343	14.68	16.92	19.02	21.67	23.59
10	2.156	2.558	3.247	3.940	4.865	9.342	15.99	18.31	20.48	23.21	25.19
11	2.603	3.053	3.816	4.575	5.578	10.34	17.28	19.68	21.92	24.72	26.76
12	3.074	3.571	4.404	5.226	6.304	11.34	18.55	21.03	23.34	26.22	28.30
13	3.565	4.107	5.009	5.892	7.042	12.34	19.81	22.36	24.74	27.69	29.82
14	4.075	4.660	5.629	6.571	7.790	13.34	21.06	23.68	26.12	29.14	31.32
15	4.601	5.229	6.262	7.261	8.547	14.34	22.31	25.00	27.49	30.58	32.80
16	5.142	5.812	6.908	7.962	9.312	15.34	23.54	26.30	28.85	32.00	34.27
17	5.697	6.408	7.564	8.672	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.265	7.015	8.231	9.390	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	6.844	7.633	8.907	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.434	8.260	9.591	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.034	8.897	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.643	9.542	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.260	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.886	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	34.34	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	44.34	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	89.33	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	99.33	118.5	124.3	129.6	135.8	140.2
150	109.1	112.7	118.0	122.7	128.3	149.3	172.6	179.6	185.8	193.2	198.4
200	152.2	156.4	162.7	168.3	174.8	199.3	226.0	234.0	241.1	249.4	255.3

3 Table 3: Upper 5% percentage points for the F distribution

The F Distribution Function for $\alpha = 0.05$												
The table provides values of F_{α,v_1,v_2} where $\Pr(F \geq F_{\alpha,v_1,v_2}) = 0.05$ and $F \sim F(v_1, v_2)$												
	$v_1 \rightarrow$											
$v_2 \downarrow$	1	2	3	4	5	6	7	8	9	10	12	15
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.96
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	1.97	1.89
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.87
55	4.02	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01	1.93	1.85
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.81
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.79
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.78
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.77
110	3.93	3.08	2.69	2.45	2.30	2.18	2.09	2.02	1.97	1.92	1.84	1.76
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.73

4 Table 4: Upper 1% percentage points for the F distribution

The F Distribution Function for $\alpha = 0.01$

The table provides values of F_{α,v_1,v_2} where $\Pr(F \geq F_{\alpha,v_1,v_2}) = 0.01$ and $F \sim F(v_1, v_2)$

$v_2 \downarrow$ \ $v_1 \rightarrow$	1	2	3	4	5	6	7	8	9	10	12	15
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.60
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.61	2.46
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.42
55	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66	2.53	2.38
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.31
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.27
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.24
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.22
110	6.87	4.80	3.96	3.49	3.19	2.97	2.81	2.68	2.57	2.49	2.35	2.21
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.16