Alastair Hall

ECON61001:
Econometric Methods

Autumn 2020

Problem Set for Tutorial 3

The first question considers the construction of one-sided tests.

1. Consider the regression model

$$y = X\beta_0 + u$$

and let $\hat{\beta}_T$ denote the OLS estimator of β_0. Assume Assumptions CA1-CA6 (see Lecture 1) hold. Suppose it is desired to test $H_0 : \beta_{0,i} \leq \beta_{*,i}$ versus $H_1 : \beta_{0,i} > \beta_{*,i}$.

   (a) Suggest an appropriate decision rule for performing this hypothesis test at the $100\alpha\%$ significance level.

   (b) Consider the following regression model from Lectures 1 & 2:

$$y_t = \text{controls} + \beta_{belt,0} * belt_t + \beta_{mph,0} * mph_t + u,$$

   with the set of controls being those used in (2.18) in the Lecture Notes. Use the coefficient estimator in (2.18) in Example 2.2 and the standard error reported in Example 2.3 of the Lecture Notes to test: $H_0 : \beta_{mph,0} \leq 0$ versus $H_1 : \beta_{mph,0} > 0$ using a 5% significance level. Interpret the outcome of the test in terms of the impact of this law.

The next question relates to testing hypotheses involving linear restrictions on β_0.

2. Consider the linear regression model

$$y = X\beta_0 + u$$

with $k = 5$. In lectures, we discussed procedures for testing the null hypothesis $R\beta_0 = r$. Show that the following null hypotheses fit within this framework.

   (a) $H_0 : \beta_{0,2} - \beta_{0,4} = 0$.

   (b) $H_0 : \beta_{0,1} + \beta_{0,2} = \beta_{0,3}$ and $1 + \beta_{0,4}/3 = \beta_{0,5}$.

Verify that both cases satisfy the restriction $rank(R) = n_r$.

In the lecture, we discussed testing $H_0 : \beta_{0,i} = \beta_{*,i}$ using a t-statistic. In this question, we consider testing this hypothesis using a F-statistic and show that the results of the two approaches are identical.

3. Consider the linear regression model

$$y = X\beta_0 + u$$

(a) Verify that the restriction $\beta_{0,i} = \beta_{*,i}$ can be written equivalently as $R\beta_0 = r$ where R is the $1 \times k$ vector equal to the the i^{th} row of I_k and $r = \beta_{*,i}$.

(b) Define

$$\text{t-stat} \;=\; \frac{\hat{\beta}_{T,i} - \beta_{*,i}}{\hat{\sigma}_T \sqrt{m_{i,i}}}$$

where $m_{i,i}$ is the $(i,i)^{th}$ element of $(X'X)^{-1}$, and recall this is the t-statistic used to test $H_0 : \beta_{0,i} = \beta_{*,i}$. Show that

$$(\text{t-stat})^2 \;=\; F$$

where F is the F- statistic for testing $R\beta_0 = r$ using the definitions of R, r in part (a).

In this question, you verify the properties of the RLS estimator discussed in lectures.

4. Consider the linear regression model

$$y \;=\; X\beta_0 + u$$

where y is the $T \times 1$ vector containing the observable dependent variable, X is a $T \times k$ matrix of observable explanatory variables and u is the $T \times 1$ vector of unobservable errors, and let Assumptions *CA1-CA6* from the lectures hold. Let $\hat{\beta}_{R,T}$ denote the RLS estimator based on the linear restrictions $R\beta = r$ where where R is a $n_r \times k$ matrix of pre-specified constants with rank equal to n_r and r is a $n_r \times 1$ vector of pre-specified constants that is,

$$\hat{\beta}_{R,T} \;=\; \hat{\beta}_T - (X'X)^{-1}R'\{R(X'X)^{-1}R'\}^{-1}(R\hat{\beta}_T - r),$$

where $\hat{\beta}_T$ is the OLS estimator of β_0.

(a) Assuming that $R\beta_0 = r$, show that $E[\hat{\beta}_{R,T}] = \beta_0$ and $Var[\hat{\beta}_{R,T}] = \sigma_0^2 D$ where

$$D \;=\; (X'X)^{-1} - (X'X)^{-1}R'\{R(X'X)^{-1}R'\}^{-1}R(X'X)^{-1}.$$

(b) Show that if $R\beta_0 \neq r$ then $E[\hat{\beta}_{R,T}] \neq \beta_0$.

Hint: you can take advantage of the properties of the OLS estimator in answering these questions.