Assignment 4

Deadline: **December 13th 11:30 pm**

Notice

- Please note your all members name in the top of the answer.
- Written answers to assignments must be typed; Graphs and numerical work need not be typed, but should be legible.
- Include the process of your calculations.
- There is no late submission. Please keep the deadline in your mind!

Q1. Price discrimination - 1st degree (35 points)

Consider the market of some good with a unique firm in it. In this market there are only two consumers, one of low valuation and one with high valuation. The firm is able to perfectly distinguish between them. Each consumer's gross consumer surplus is given by the following expression:

$$\mathcal{U}(\theta_i, q_i) = \theta_i \left(aq_i - \frac{q_i^2}{w} \right)$$

for $i = 1, 2$.

The monopolist's cost function is given by $C(q_i) = 3q_i^2 + 100$, for $i = 1,\ 2$.

a) (3 points) Set up the firm's maximization problem, detailing the objective function and the participants constraints.

b) (3 points) Briefly explain why, at a solution, both constraints must hold with equality.

c) (6 points) Assume $a = 5$, $w = 2$, $\theta_1 = 14$ and $\theta_2 = 24$. Solve for the firm's optimal packages: (T_1^*, q_1^*) and (T_2^*, q_2^*).

d) (3 points) Find the firm's profit under this solution.

e) (3 point) What is the market's total surplus? (**Hint**: you do not need to do any calculation to answer this question.)

f) (17 points) Solve the monopolist's problem but suppose now that the monopolist uses two-part tariffs: $T_1 = A_1 + p_1 q_1$ and $T_2 = A_2 + p_2 q_2$. Follow these steps:

(a) (6 points) Set up the firm's problem. Be very careful when defining the objective function (profits should be a function of prices and parameters).

(b) (5 points) Take FOCs with respect to prices to find optimal prices, and then replace these values in the demand functions to find equilibrium quantities.

(c) (3 points) Calculate the fixed part of each tariff.

(d) (3 points) Finally, conclude this exercise by showing that the solution to this problem (under two-part tariffs) is exactly the same as the solution obtained before when the firm was using packages.

Q2. Price discrimination - 2nd degree (35 points)

Consider the market of some good with a unique firm in it. In this market there are only two types of consumers, one of low valuation and one with high valuation. A fraction λ of the consumers correspond to the low valuation type and the remaining $1 - \lambda$ are high valuation type. The firm cannot distinguish between types. Each consumer's utility function (or gross consumer surplus) is given by the following expression:

$$\mathcal{U}(\theta, q) = \theta q - \frac{q^2}{2}$$

The firm faces a unitary cost c for producing the good.
Assume $\theta_1 = 75$, $\theta_2 = 80$, $\lambda = \frac{1}{2}$ and $c = 25$.

a) (10 points) Set up the firm's problem and solve for the optimal packages (T_1^*, q_1^*) and (T_2^*, q_2^*).

(**Hint**: *When setting up the firm's problem, you do not need to write down all four constraints and show which ones hold and which ones do not. You can just write down the "relaxed version" as is done on the lecture notes).*

b) (4 points) With these results, compute the firm's profits and consumer surplus for each type (verify that it's zero for the low type). Finally, compute total surplus.

c) (15 points) Suppose now that the monopolist is restricted to using a two-part tariff system, namely $T_1 = A_1 + p_1 q_1$ and $T_2 = A_2 + p_2 q_2$. Set up the firm's problem and find the optimal tariff schemes (A_1^*, p_1^*) and (A_2^*, p_2^*).

(**Hint**: *For this kind of preferences, indirect utility function (net consumer surplus) is given by* $V(\theta_i, p_i) = \frac{(\theta_i - p_i)^2}{2}$ *and demand function is* $q_i = \theta_i - p_i$. *for* $i = 1,\ 2$).

d) (4 points) Compute firm's profits, consumer surplus for each type and total surplus under this pricing system.

e) (2 points) Provide a brief conceptual explanation as to why the firm is making less profits under two-part tariffs than under packages.

Q3. Price discrimination: 3rd degree (45 points)

Suppose that there are N_1 consumers of type θ_1 and N_2 consumers of type θ_2. Preferences are $U(\theta_i, q_i) = \theta_i \left(aq_i - \frac{q_i^2}{2}\right)$ for $i = 1,\ 2$. The parameters are given as $\theta_1 = 12$, $\theta_2 = 15$, and $a = 4$.

The monopolist can distinguish between types but it cannot distinguish between consumers that belong to the same type. Therefore, the monopolist has to care for two separate groups of consumers. Let's assume that the monopolist has a cost function $C(Q) = cQ$ where $Q = q_1 + q_2$.

a) (3 points) Derive the demand function for each group.

b) (6 points) Once you have each group's demand function, set up the firm's maximization problem and find the optimal choices of output for each group, q_1^* and q_2^*.

c) (3 points) Assume $c = 6$, $N_1 = 1$, and $N_2 = 2$ what are the optimal prices? What are the firm's profits?

d) (3 points) Calculate (net) consumer surplus for groups one and two. Conclude this exercise by computing aggregate welfare (total surplus)

Q4. Entry Deterrence: Stackelberg Revisited (15 points)

Consider a market with only one firm operating in it, which we identify as the "leader" firm. There is however, a second firm evaluating whether or not to enter to market. We shall identify this firm as the "entrant" one. Cost functions are given by: $C_L(q_L) = 2q_L$ and $C_E = 2q_E + F$ where F is the fixed entry cost that the entrant must pay in order to start producing.

Market demand is given by $P(Q) = 20 - Q$ where $Q = q_L + q_E$ (of course, if the entrant is not in the market, then $q_E = 0$).

As in the Stackelberg environment, competition takes place in two stages: in the first stage, the leader chooses its quantity and in the second stage, the entrant observes the leader's quantity and decides to enter or not. If it does, the entrant chooses at the same time how much to produce.

a) (7 points) Find an expression for the entrant's reaction function.

b) (5 points) Assuming $F = 16$, what will the leader choose to do: accommodate to entry or deter it?

c) (3 points) What if $F = 72$? What can you say about entry in this market now?