

Economics of Finance

Tutorial 1

School of Economics

1. Find the inverse of the following matrix:

$$X = \begin{pmatrix} 0.1 & 0.1 & 0 & 0.9 \\ 0.8 & 0.2 & 0.1 & 0 \\ 0 & 0.4 & 0.5 & 0 \\ 0.1 & 0.3 & 0.4 & 0.1 \end{pmatrix}.$$

Solution

The inverse matrix is:

$$X^{-1} = \begin{pmatrix} -0.375 & 0.875 & -2.875 & 3.375 \\ 2.5 & 2.5 & 17.5 & -22.5 \\ -2.0 & -2.0 & -12.0 & 18.0 \\ 0.875 & -0.375 & -1.625 & 2.125 \end{pmatrix}.$$

The following fragment of Octave(MATLAB) code entered in the command prompt will do the job:

```
>> X = [0.1 0.1 0 0.9; 0.8 0.2 0.1 0; 0 0.4 0.5 0; 0.1 0.3 0.4 0.1];  
>> X_1 = inv(X)
```

2. Show that the three columns in the matrix **A** below are linearly dependent

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -3 & 9 \\ 4 & 1 & 11 \end{pmatrix}.$$

Solution

The three column vectors of the matrix **A** can be summed as follows

$$\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 3 \\ 9 \\ 11 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

where $\lambda_1 = 3, \lambda_2 = -1, \lambda_3 = -1$. Hence the three columns are linearly dependent. This means the matrix **A** is singular.

3. Compute the determinant of the matrix in question 2. Use `det()` function. Explain what you find.

Solution

A square matrix is nonsingular if and only if its determinant is nonzero. Hence this result again implies that the matrix **A** is singular. This can be quickly checked:

```
>> A = [1 0 3; 2 -3 9; 4 1 11];  
>> det(A)
```

```
ans =
```

```
0
```

4. Suppose there are two investment funds, Fund A and Fund B. The exposures of Fund A are 50% in bonds and 50% in stocks while the exposures of Fund B are 30% in bonds and 70% in stocks. An investor desires an asset allocation of 40% in bonds and 60% in stocks using the two funds.

This problem can be solved by setting up the system of simultaneous equations:

$$\begin{aligned}0.5x_A + 0.3x_B &= 0.4 \\0.5x_A + 0.7x_B &= 0.6,\end{aligned}$$

where x_A and x_B represent allocations of money invested in Fund A and Fund B respectively.

4.1 Solve the system by repeated substitution and interpret the results.

4.2 Write it in matrix form as $\mathbf{Ax} = \mathbf{b}$.

4.3 Solve the system using the inverse of \mathbf{A} . Check that your solution is correct by computing $\mathbf{A}(\mathbf{A}^{-1}\mathbf{b})$.

Solution

4.1 Trivial to show that $x_A = 0.5$ and $x_B = 0.5$. This shows that to achieve exposures of 40% in bonds and 60% in stocks, the investor should put 50% of assets in Fund A and 50% of assets in Fund B.

4.2

$$\begin{pmatrix} 0.5 & 0.3 \\ 0.5 & 0.7 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix}$$

4.3 From Octave

```
>> A = [0.5 0.3; 0.5 0.7]
```

```
A =
```

```
0.5000    0.3000
0.5000    0.7000
```

```
>> b = [0.4; 0.6]
```

```
b =
```

```
0.4000
0.6000
```

```
>> inv(A)*b
```

```
ans =
```

```
0.5000
0.5000
```

```
>> A*ans
```

```
ans =
```

```
0.4000
0.6000
```

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Questions 5 to 7 refer to a world in which there are only two periods and two possible states of the world in the second period (a good weather state and a bad weather state). Also, apples are the only product produced in this world, and they cannot be stored from one period to the next. The following abbreviations will be used: PA = apple in first period (i.e., present apple), GA = good weather apple, BA = bad weather apple.

5. Dealer I is willing to trade 0.3PA for 1GA (or vice versa), and dealer II is willing to trade 1GA for 0.6BA (or vice versa). What is the arbitrage-free price of a BA in terms of PA?

Solution

Consider the following trades. First, trade 1PA for $\frac{1}{0.3}$ GA. Then, trade the obtained amount of GA for BA. These trades can be summarized as

$$1PA \rightarrow \frac{1}{0.3}GA \rightarrow \frac{1}{0.3}0.6BA = 2BA.$$

Hence 1BA costs 0.5PA.

6. In addition dealer III is willing to trade 0.6PA for 1BA (or vice versa). Are there arbitrage opportunities? If so, design a profitable arbitrage strategy.

Solution

One way to determine whether there are arbitrage opportunities is the following. Consider the impact of trading in a clockwise and anticlockwise direction:

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Clockwise : $1PA \rightarrow \frac{1}{0.3}GA \rightarrow 2BA \rightarrow 1.2PA$

This trading sequence generates a profit of 0.2PA. Hence, there are arbitrage opportunities if you trade in the clockwise direction.

$$\text{Anticlockwise : } 1PA \rightarrow \frac{1}{0.6}BA \rightarrow \frac{1}{(0.6)^2}GA \rightarrow \frac{0.3}{(0.6)^2}PA = \frac{5}{6}PA$$

If you trade in the anticlockwise direction, you will make a loss. Also, note that if you trade in one direction followed by the other direction you must break even, i.e., $1.2 \times \frac{5}{6} = 1$ (when there are no transaction costs).

7. Suppose now there are transaction costs. The three dealers have the following bid-ask spreads:

Dealer I:

sell 1PA for 2.5GA

buy 1PA for 2GA

Dealer II:

sell 1GA for 1BA

buy 1GA for 0.75BA

Dealer III:

sell 1PA for 2BA

buy 1PA for 1.5BA.

Are there arbitrage opportunities now? Explain.

Solution

In the previous question it would have been sufficient to check the trades in one direction only. If you ended up with anything other than 1PA, then arbitrage opportunities must exist. In this question, if you end up with an answer less than 1PA in one direction, this tells you nothing about what will happen if you trade in the other direction. In such cases, you must check in both directions.

$$\text{Clockwise : } 1PA \longrightarrow 2GA \longrightarrow 1.5BA \longrightarrow 0.75PA$$

This trading sequence generates a loss of 0.25PA. Hence there are no arbitrage opportunities if you trade in the clockwise direction.

$$\text{Anticlockwise : } 1PA \longrightarrow 1.5BA \longrightarrow 1.5GA \longrightarrow 0.6PA$$

Hence if you trade in the anticlockwise direction, you also make a loss. There are no arbitrage opportunities.

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