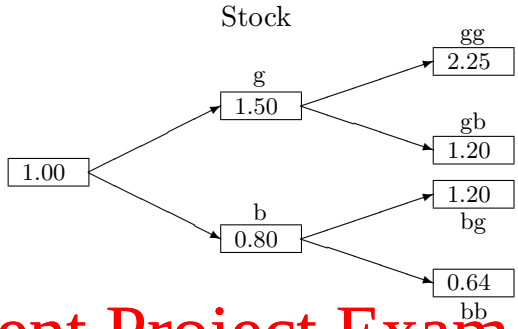
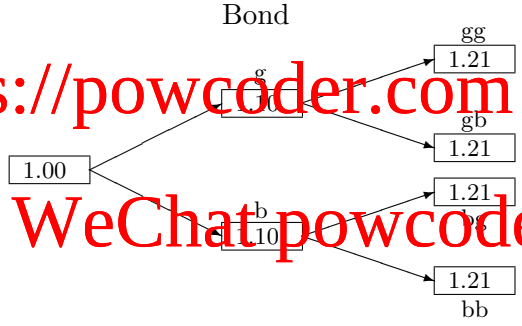
Economics of Finance

Tutorial 4

1. Consider a three period binomial time-state model in which there are two securities, a bond and a stock. The payments made by these securities in each state are shown in the trees below:





(i) Write down the payment matrix, $\mathbf{Q}$, and corresponding price vector, $\mathbf{p_S}$, derived from the following elemental payment combinations:

B0: Buy a Bond at period 0, sell it at the end of the next period;

S0: Buy a Stock at period 0, sell it at the end of the next period;

Bg: At period 1, if the state is **g**, buy a Bond, sell it at the end of the next period;

Sg: At period 1, if the state is **g**, buy a Stock, sell it at the end of the next period;

Bb: At period 1, if the state is **b**, buy a Bond, sell it at the end of the next period;

Sb: At period 1, if the state is **b**, buy a Stock, sell it at the end of the next period.

(ii) Compute the atomic security prices (i.e., the price of one dollar in each of the six future time-states: g, b, gg, gb, bg, bb). Write down the formula you used to derive the atomic security price vector.

(iii) Write down the payment matrix, $\mathbf{Q}$, and corresponding price vector, $\mathbf{p_S}$, derived from the following elemental payment combinations:

B0: Buy a Bond at period 0, sell it at the end of the next period;

S0: Buy a Stock at period 0, sell it at the end of the next period;

Bb: At period 1, if the state is **b**, buy a Bond, sell it at the end of the next period;

Sb: At period 1, if the state is **b**, buy a Stock, sell it at the end of the next period.

B02: Buy a Bond at period 0, sell it at the end of period 2;

S02: Buy a Stock at period 0, sell it at the end of period 2;

(iv) Verify the atomic security prices computed using the payment matrix $\mathbf{Q}$ in part (iii) is the same as the one found using the payment matrix $\mathbf{Q}$ in part (i).

(v) Suppose an investor wants to obtain the following time-state payments:

$$\mathbf{c} = \begin{pmatrix} 0 & 10 & 20 & 20 & 30 & 40 \end{pmatrix}'.$$

The vector of payment combination holdings, $\mathbf{n}$, is calculated as follows: $\mathbf{n} = \mathbf{Q}^{-1}\mathbf{c}$. Calculate $\mathbf{n}$ for both the $\mathbf{Q}$ matrices considered in (i) and (iii) above.

(vi) Take each vector $\mathbf{n}$ from part (v) and calculate how much of the bond and stock the investor must buy or sell in aggregate in each state in period 1 to implement this dynamic strategy? Show your workings, and verify that both $\mathbf{n}$ vectors are describing the same overall strategy.

(vii) Compute the arbitrage-free price of the time-state payment vector $\mathbf{c}$. Explain how you arrived at your answer.

2. Consider a three period binomial time-state model in which there are two securities, a bond and a stock. The payments made by the stock in each state are shown in the tree below. The bond pays 10 percent interest each period.

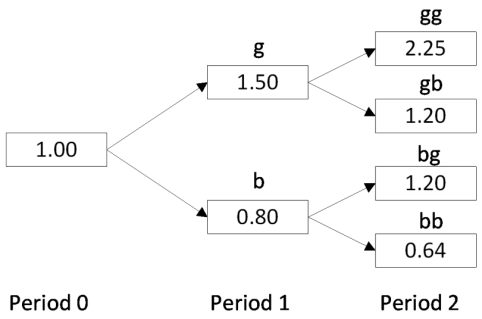


Figure 1: Lattice for the Stock

(i) Calculate the price in Period 0 of an American Call (buy) option that expires at the end of Period 2 with an exercise price of 1.60. Explain how you arrived at your answer.

(ii) Calculate the price in Period 0 of an American Put (sell) option that expires at the end of Period 2 with an exercise price of 1.60. Explain how you arrived at your answer.

3. An investor owns 2,000 shares of Walmart Inc, now selling at $110. The investor has a one-year investment horizon and is concerned of a downside risk. She considers buying a call or put option. A one-year call with a strike price of $110 is now selling at $1.50 whilst the put option with similar strike price and expiry date is selling at $0.41.

i) Given the above data, devise the protective strategy.

ii) Explain why in this situation a call option is more expensive than a put option.

iii) Under which conditions a call option price will be exactly equal to a put option price.