Economics of Finance

Tutorial 5

1. Two securities X and Y make the following payments (for each dollar invested) in the good and bad weather states:

	X	Y
B	1.10	0.80
G	1.00	1.50

Suppose the good outcome occurs with probability 0.6 and the bad outcome with probability 0.4.

(i) Compute the expected rate of return on securities X and Y.

(ii) Compute the atomic security prices.

(iii) Compute the risk-free rate of return. Construct a portfolio of securities X and Y that pays the same amount in the good and bad states.

(iv) Compute the expected rate of return and risk premia of the *atomic* securities.

(v) Compute the forward prices of the atomic securities (risk neutral probabilities).

(vi) Compare the forward price of each atomic security with the probability of that state being observed. Why are the forward prices and associated probabilities not equal?

2. Consider the portfolio Z that makes the following payments in four different states (VB, B, G, VG). You are also given (physical) probabilities and forward prices (risk neutral probabilities) of each state

	c	prob	f
VB	00	0.3	0.5
B	10	0.2	0.3
G	10	0.1	0.1
VG	50	0.4	0.1

Suppose the risk-free rate of return is 5 percent.

(i) Compute the risk premium of portfolio Z.

(ii) Compute the risk premia of the four atomic securities.

(iii) Will the *market* portfolio pay a risk premium in this case? Explain.

3. (Expected utility) Suppose you are faced with the following gamble scenario:

- Consume 6000 with probability 0.4
- Consume 3000 with probability 0.6.

Suppose further that your utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma = 1/2$. (Later will learn that γ is the coefficient of relative risk aversion. The higher γ, the more you dislike risk)

(a) What is your expected utility?

(b) What is your expected consumption?

(c) What is your attitude towards risk?

(d) Certainty equivalent, CE, refers to the guaranteed amount of consumption that an individual would view as equally desirable as a risky gamble, that is, $EU_{\text{gamble}} = u(CE)$. Compute the certainty equivalent of the gamble.

(e) Suppose now that the coefficient of relative risk aversion is $\gamma = 2$. Answer to the questions (a)–(d) above for with $\gamma = 2$. Are agents more or less tolerant to risk than before?

4. (The Role of Finance) Consider an economy in which a representative agent lives for two periods, year 0 and year 1. The representative agent derives utility from consumption and their time discount rate is β. Suppose there is *no uncertainty*. The agent life-time utility is given by:

$$U(c_0, c_1) = \ln(c_0) + \beta \ln(c_1),$$

The agent receives an initial endowment, e, at time zero and recieves income (say from labor) in period zero and one, y_0 and y_1, respectively. The agent can save, s, or borrow (negative s) money at interest rate i.

(a) Write down the maximization problem in detail.

(b) Write down the Lagrangian that represents the maximization problem.

(c) Derive the first order conditions.

(d) Interpret the trade-offs you find.

(e) Solve for equilibrium consumption, and saving/borrowing.

(f) Suppose $y_0 = 0.4$, $e = 0.6$ and $y_1 = 3$, $\beta = 0.98$ while $i = 0.05$. Compare the welfare (utility) of equilibrium with financing options (saving/borrowing available) and without. Comment on your result.