Lecture 1: Time-State Claims

Economics of Finance

School of Economics, UNSW

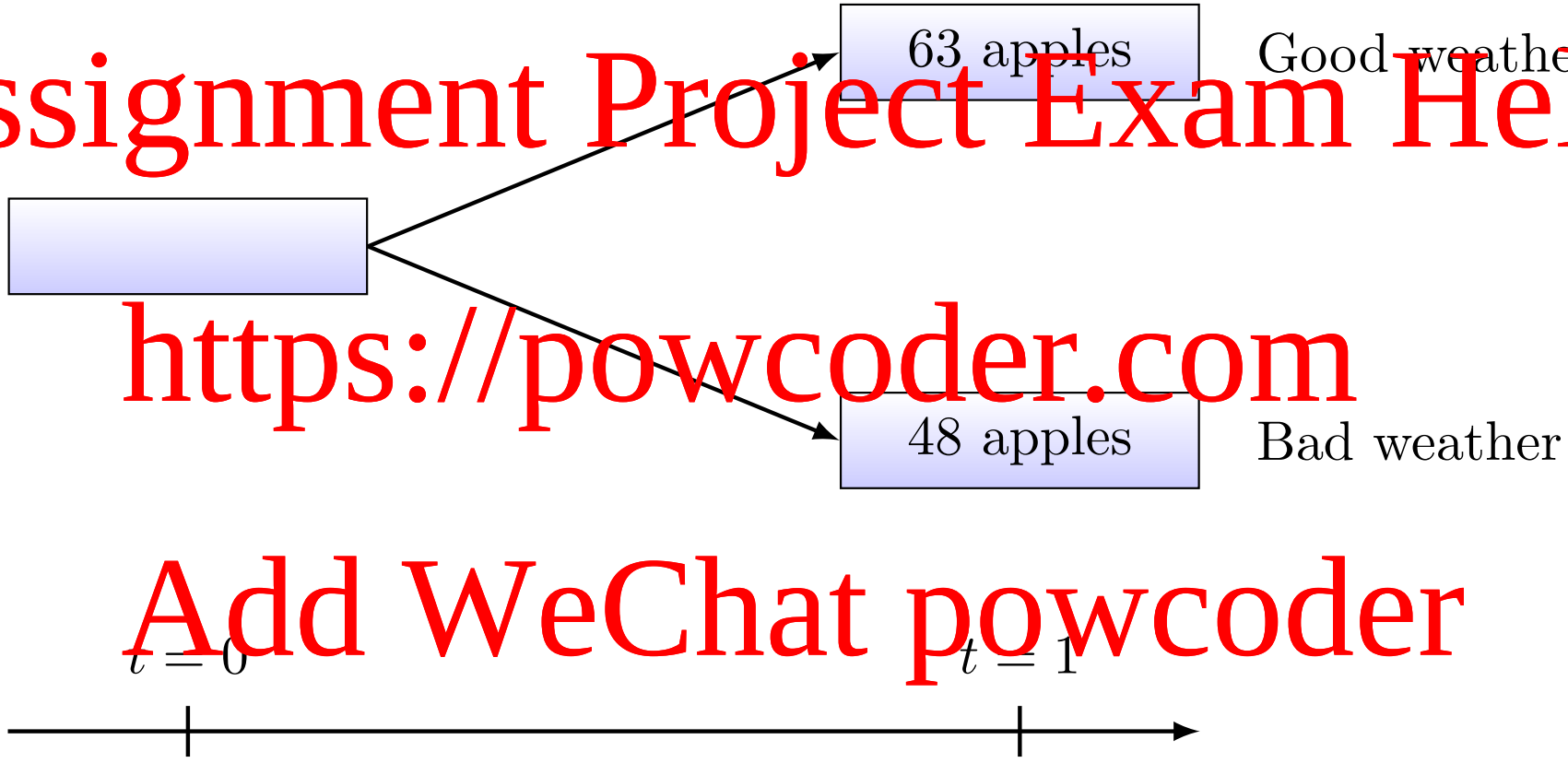
What is finance?

> *'Finance concentrates on exchanges in which money of one type or another is likely to appear on both sides of a trade.'*
>
> — *William F. Sharpe*

Finance deals with payment now, payment in the future and uncertainty.

- Key factors: *Time & Uncertainty.*

Example: an apple tree



Two time periods: $t = 0$, $t = 1$, spring- no apples and fall-uncertain apples.

Why do we care?

Types of questions relevant for finance:

- How much apple does an apple tree worth?→ *Pricing Problem*
- How do we optimise our future apple stream? → *Portfolio Problem*

Why is it important?

Catastrophic financial consequences when we get it wrong!

"All the News That's Fit to Print"

The New York Times

Late Edition

NEW YORK, TUESDAY, OCTOBER 20, 1987

30 CENTS

STOCKS PLUNGE 508 POINTS, A DROP OF 22.6%;
604 MILLION VOLUME NEARLY DOUBLES RECORD

U.S. Ships Shell Iran Installation In Gulf Reprisal

Offshore Target Termed a Base for Gunboats

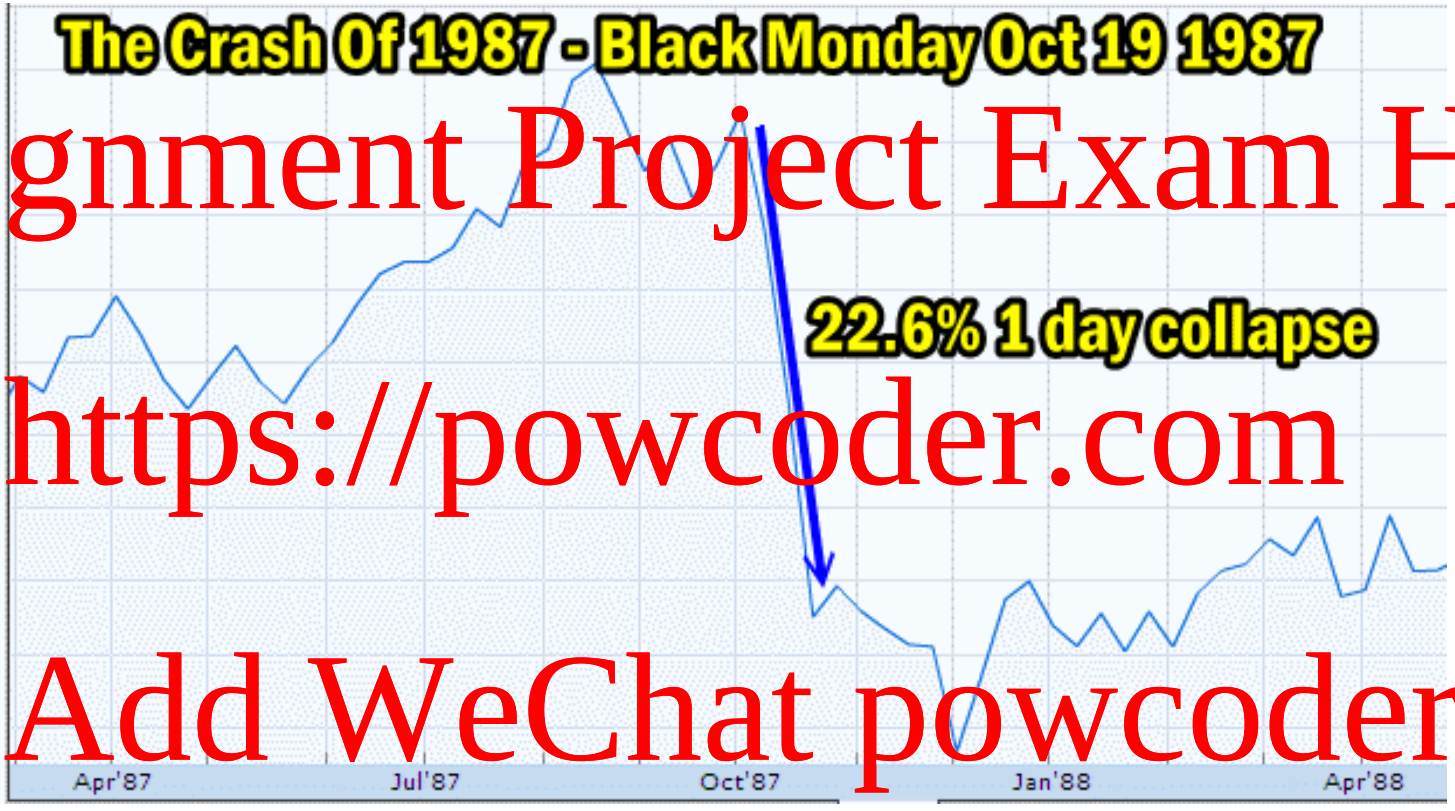
By STEVEN V. ROBERTS

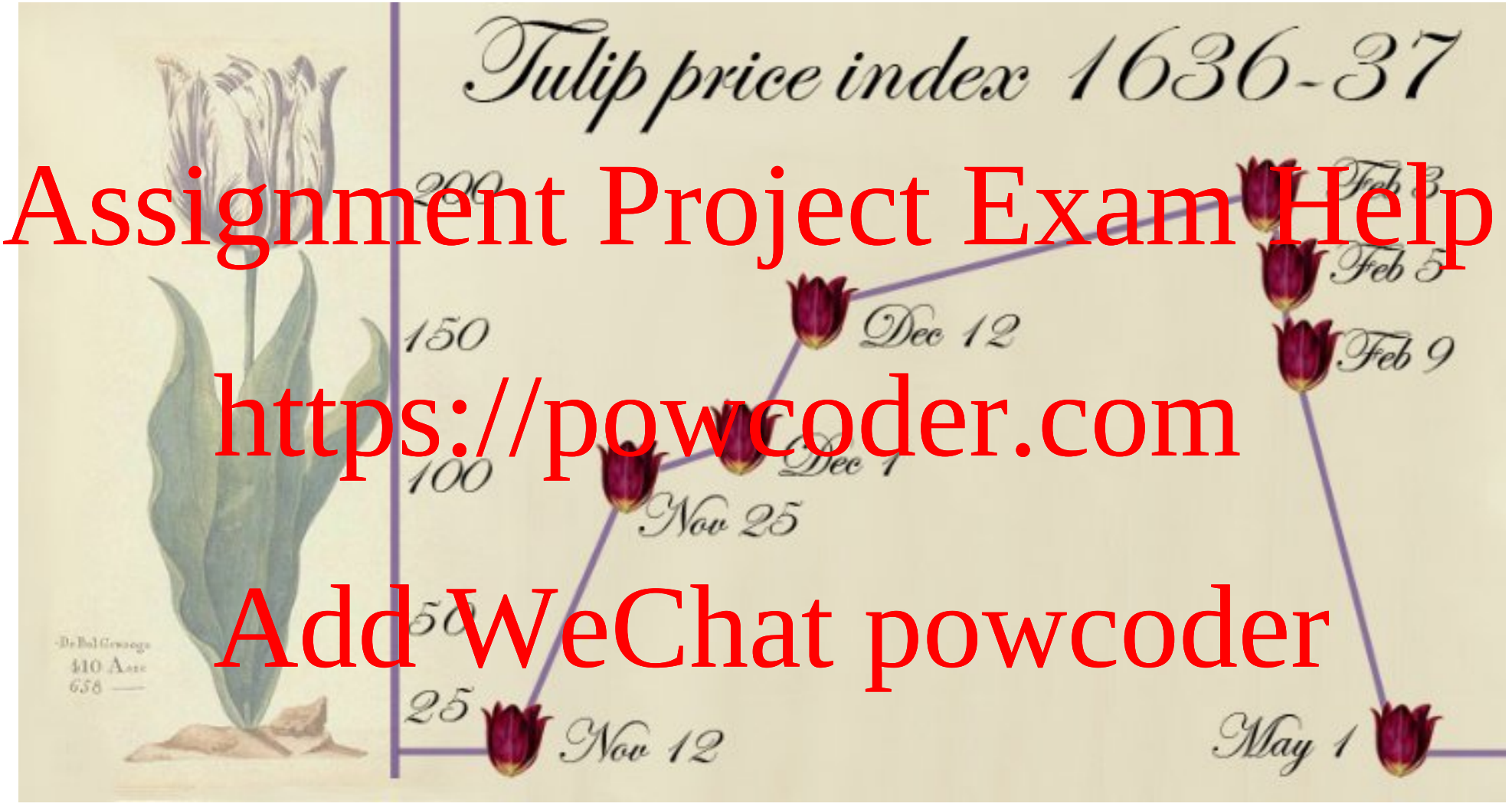
A Huge Blow to the Five-Year Bull Market

WORLDWIDE IMPACT

Trading Raises ... Recession — Tape 2 Hours Late

By LAWRENCE J. DeMARIA





Why economics of finance?

Economics is a science of people and market.

Some questions:

- Why do people behave differently in financial market?
- Does the asset price reflects efficient allocation?
- How does financial market implement the risk sharing role?
- How does trading financial asset improves social welfare?
- Does completing the financial market improves efficiency?

Financial contract: an Arrow-Debreu paradigm

Kenneth Arrow and Gerard Debreu introduced the concept of (state-) contingent contract

> *'A contract for the transfer of a commodity [specifying], in addition to its physical properties, its location and date, an event on the occurrence of which the transfer is conditional.'*

Gerard Debreu, *Theory of Value*, The Cowles Foundation Monograph, 1959

In short, a financial contract is a *Time-state claim*.

A simple environment

Key elements: Discrete time & discrete states

Two time periods:

- Time 0 - today
- Time 1 - a year from now

Two possible states of the world:

- G: good weather
- B: bad weather

These states of the world are:

- mutually exclusive (no state that is both good and bad)
- exhaustive (one and only one of the states will occur)

An all-apple economy

Suppose the only commodity traded in this economy is apple

- No money *per se*;
- Apple is the unit of account (*numeraire*)

Why apples?

- Consumable (it's good);
- Countable, and perfectly divisible (it is measurable);
- Non-storable (timing matters!).

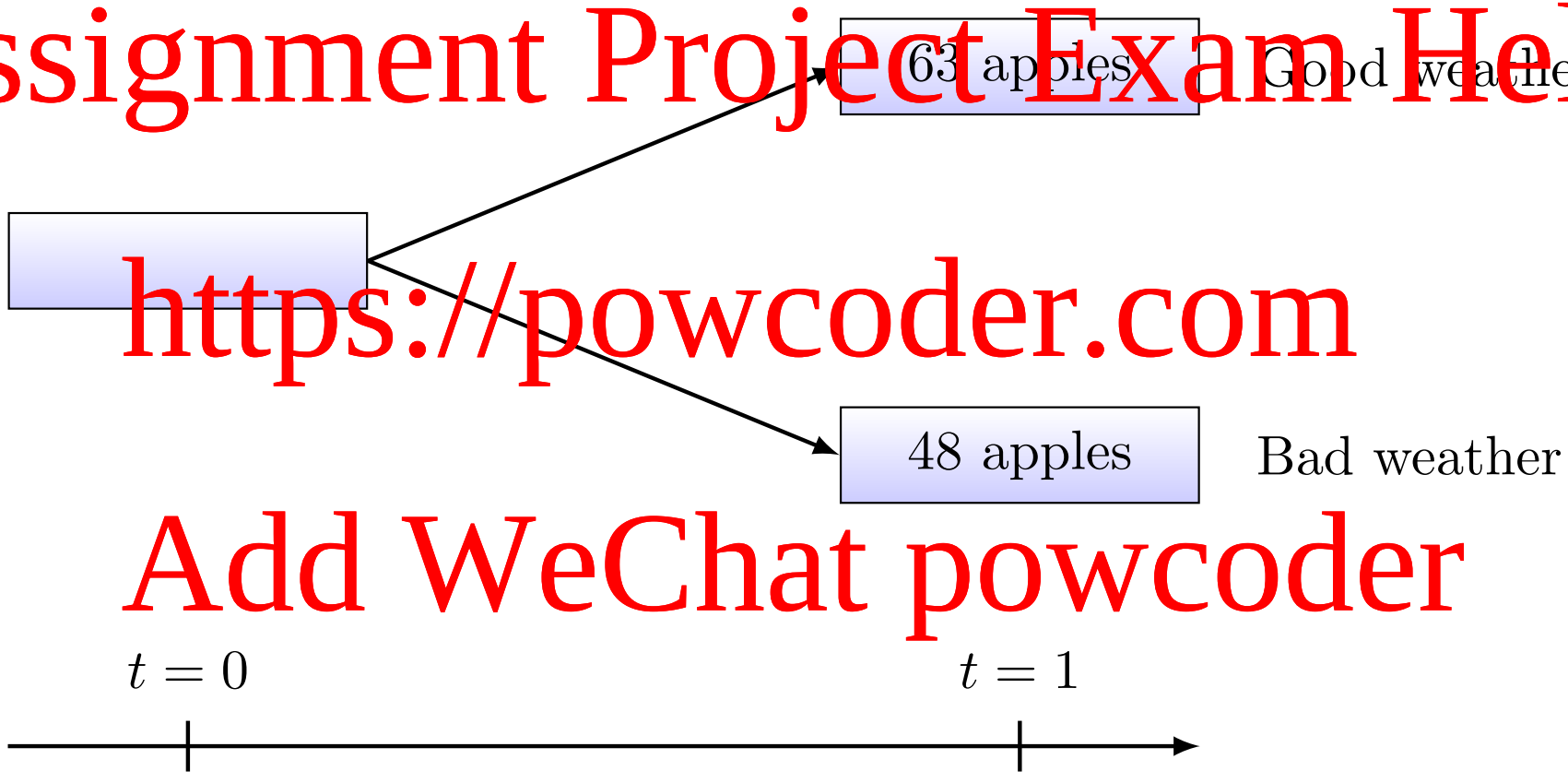
State-contingent production

The only type of productive investment is: APPLE TREE

The tree will produce:

- 63 apples if the weather is good
- 48 apples if the weather is bad

An apple tree



Elementary claims

There are two elementary *time-state* claims:

- One apple at time 1 if the weather is GOOD
- One apple at time 1 if the weather is BAD

We will refer to these claims as:

- GA - 'Good weather apples',
- BA - 'Bad weather apples'

Similarly, we will refer to a present apple as 'PA'.

Atomic security

We interchangeably refer to a claim as a *security*. A security is a certificate of the following form:

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I, Jane Smith, promise to deliver to the bearer of
this certificate one apple at the end of year 1 if
and only if the weather during the year has been
good.
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- Implicitly we assume that a credit agency has established that the security is AAA, i.e. default free.
- Atomic security is an atomic time-state claim (also known as basic Arrow-Debreu security, 'primitive' security)

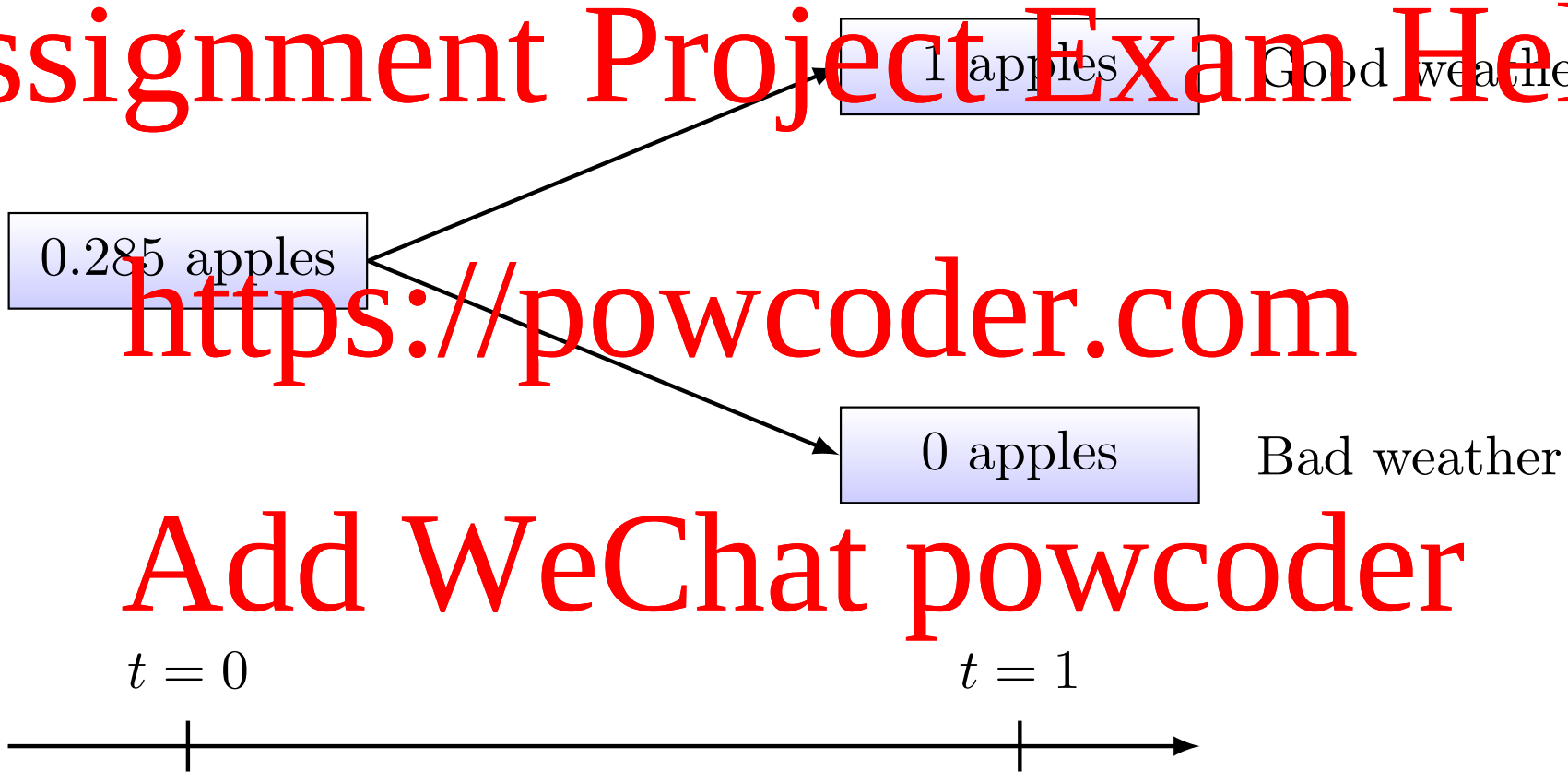
Dealers

There exists a group of dealers stand ready to trade atomic claims.

G Dealer is willing to trade:

- 0.285 PA for 1.0 GA or
- 1.0 GA for 0.285 PA or
- any multiple of those.

Good weather apple

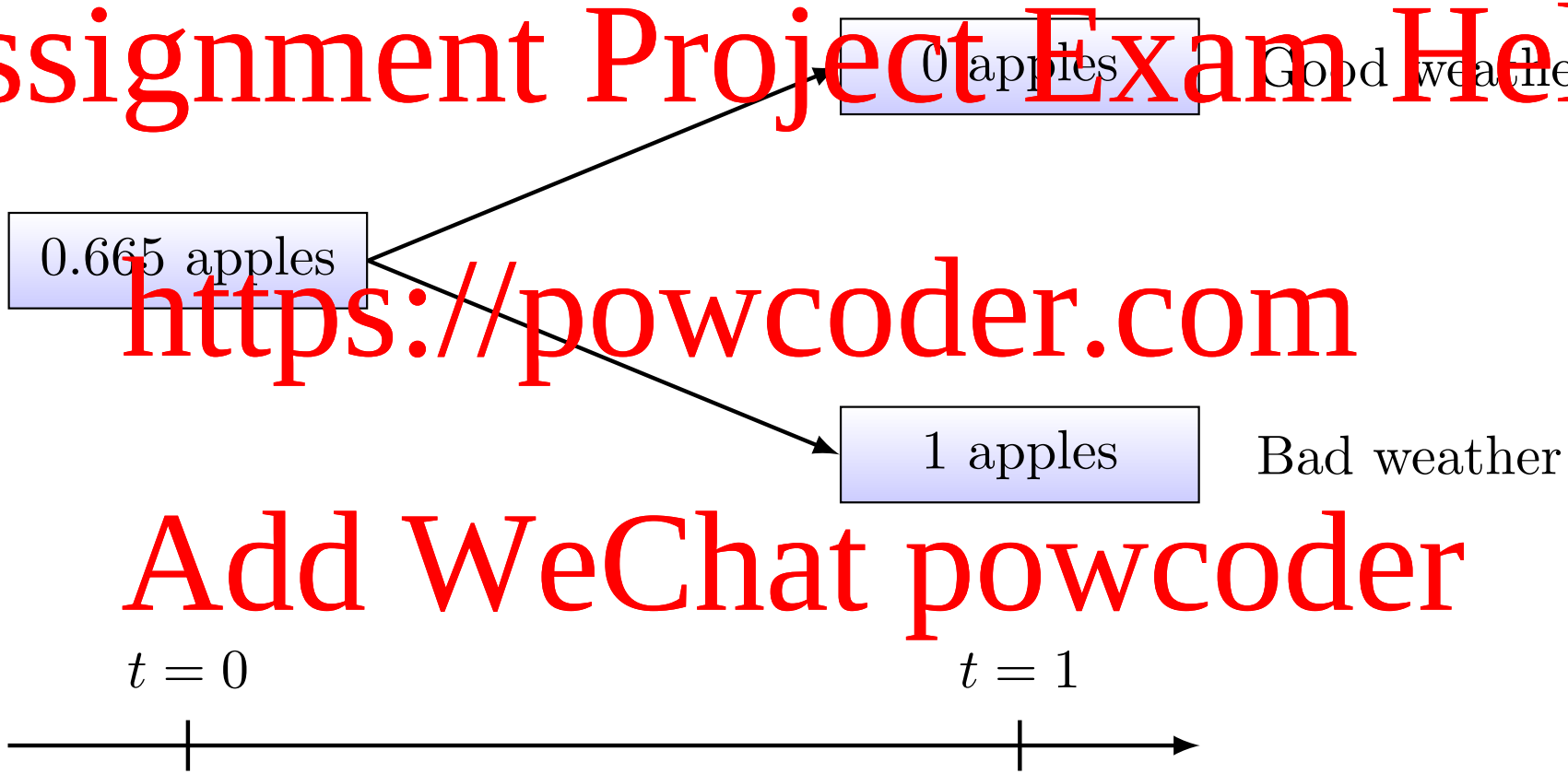


B Dealer

Further to Dealer G, Dealer B is willing to trade:

- 0.665 PA for 1.0 BA or
- 1.0 BA for 0.665 PA or
- any multiple of these

Atomic security: Bad weather apple



Complete Market

We have two explicit markets: $PA \leftrightarrows GA$ and $PA \leftrightarrows BA$.

- Notice we can make combinations of any arbitrary number GA and BA to construct *any* portfolio we desire
- We call this *complete market*
- So far all trades involves PA payment. What about other possible trades?

Other Types of Trade

Consider the following contract:

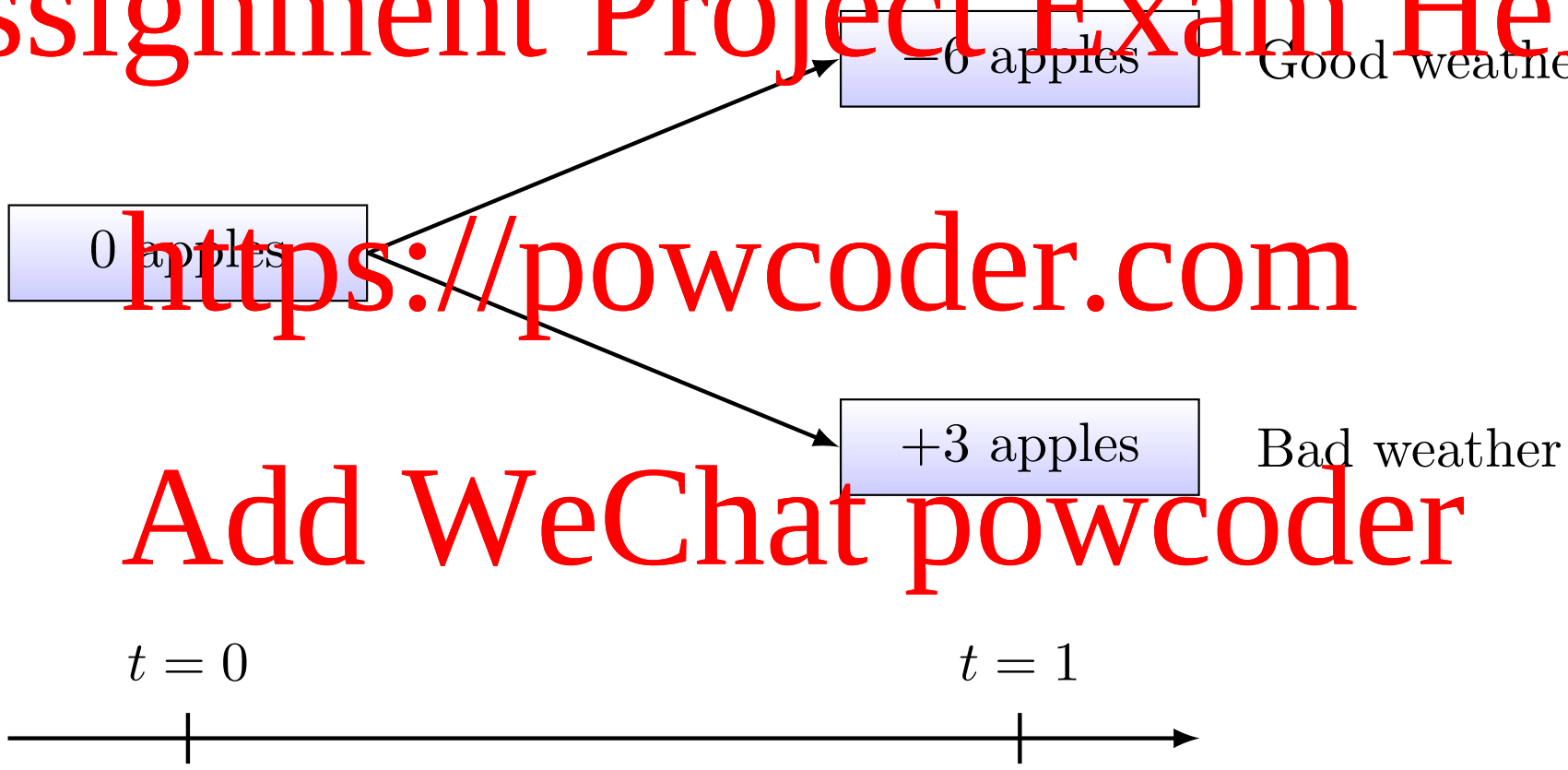
- Party A promises to pay Party B: 6 apples if the weather is good
- Party B promises to pay Party A: 3 apples if the weather is bad
- Neither party pays the other anything today (on signing)

Such a contract is called a *swap*.

Swap is the third possible type of trade in our world:
$GA \leftrightarrows BA$.

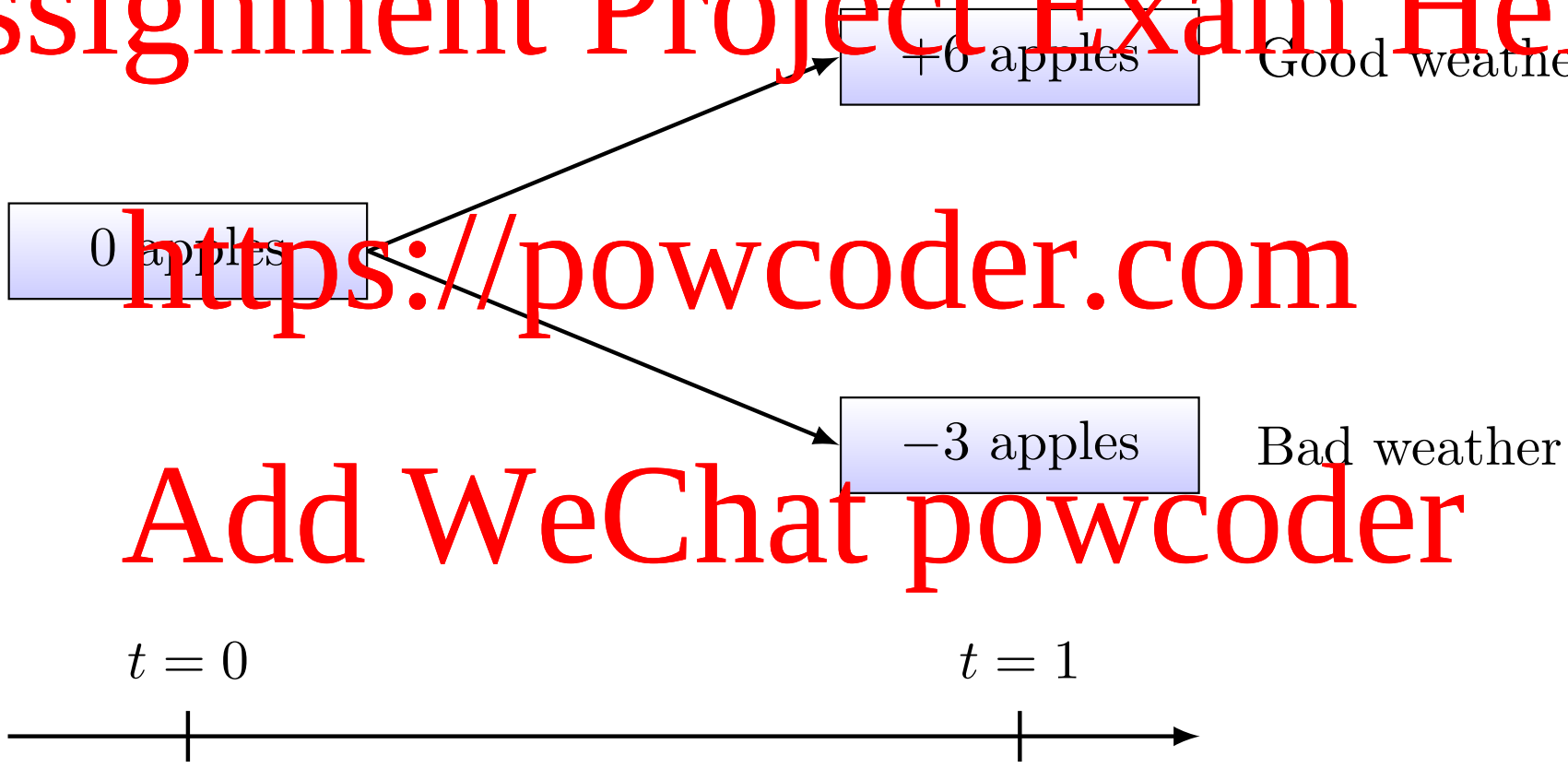
Swap

Perspective of Party A



Swap

Perspective of Party B



Put it all together

- Dealer G trades 0.285 PA for 1.0 GA or vice versa;
- Dealer B trades 0.665 PA for 1.0 BA or vice versa;
- Party A trades 6 GA for 3 BA or vice versa;

Looks like we can make some profit out there. How?

Arbitrage

An *arbitrage* provides a positive net payoff in at least one time and state and no negative net payoff in any time and state.

The payment matrix

- each row represents a transaction;
- each column represents a time-state combination;

	PA	GA	BA
Party A			
Dealer B			
Dealer G			
Net			

Arbitrage strategy

We now construct a set of transactions which implements an arbitrage.

Step 1: Go to Party A, and sign a contract swapping 6GA with 3BA;

	PA	BA	GA
Party A	0	3	–6
Dealer B			
Dealer G			
Net			

This creates a position of $-6GA$ and $3BA$ on your balance sheet.

Dealer B

Step 2: Go to Dealer B, and sell $3BA$ to her. In return, receive a credit of

$$3 \times 0.665 = 1.995PA.$$

	PA	BA	GA
Party A	0	3	-6
Dealer B	$3 \times 0.665 = 1.995$	-3	0
Dealer G			
Net			

This transaction close out the position of BA.

Dealer G

Step 3: Go to Dealer G, and buy $6GA$ from her. Pay

$$6 \times 0.285 = 1.710PA.$$

	PA	BA	GA
Party A	0	3	-6
Dealer B	$3 \times 0.665 = 1.995$	-3	0
Dealer G	$-6 \times 0.285 = -1.710$	0	6
Net			

This transaction close out the position of GA.

Summary

To finish off, summarize the transactions:

	PA	BA	GA
Party A	0	3	-6
Dealer B	$3 \times 0.666 = 1.998$	-3	0
Dealer G	$-6 \times 0.285 = -1.710$	0	6
Net	0.285	0	0

Our strategy is creating a profit without any lost in any state. By definition, this is an *arbitrage*.

Several ways to arbitrage

Step 1: Go to Party A, sign a contract swapping 6GA with 3BA

Step 2: Go to Dealer B, and sell $3BA$ to her. In return, receive a credit of

$$3 \times 0.665 = 1.995PA.$$

Step 3: Go to Dealer G, and use all the of $1.995PA$ you received to buy GAs from her

$$1.995PA/0.285 = 7GA$$

	PA	BA	GA
Party A	0	3	−6
Dealer B	$3 \times 0.665 = 1.995$	3	0
Dealer G	$-7 \times 0.285 = -1.995$	0	7
Net	0	0	1

This is still an *arbitrage* as there is a net profit in GA.
It will be realised only if GA happens.

Arbitrage free environment

- When an arbitrage opportunity arises, traders will exploit it and cause the terms of trade to adjust until no arbitrage is possible.

- We call this arbitrage free environment.

The Law of One Price (LOP)

Definition: (LOP) *In an arbitrage-free economy with no transactions costs, any given time-state claim will sell for the same price, no matter how obtained. This holds for any 'package' of time-state claims.*

- In the 'real world' transactions costs are usually present;
- The lack of arbitrage opportunities only insures that prices for a given set of time-state claims will fall within a band narrow enough to preclude generating a positive profit *net of transactions costs* out of trading.