# Lesson 7: Modern Portfolio Theory

**Economics of Finance**



School of Economics, UNSW

# Expected utility



The *mean-variance* expected utility takes the form:

$$Eu = e - \frac{c}{t} = e - \frac{s^2}{t},$$

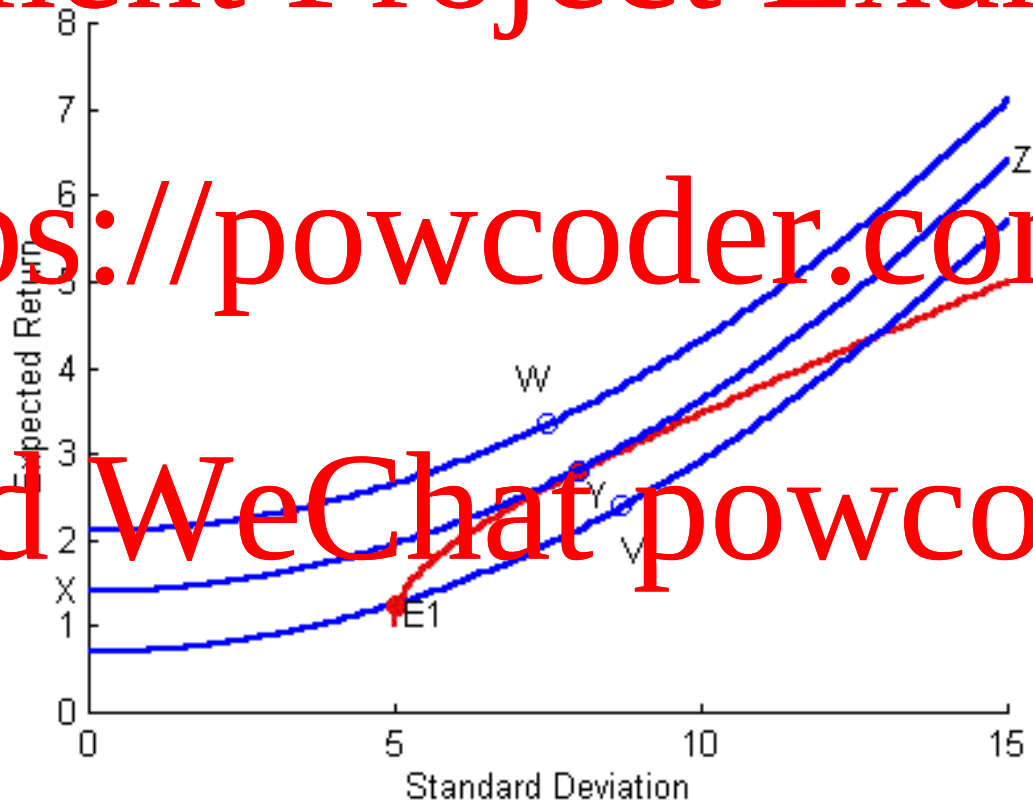


- $e$ is the expected return
- $s$ is the standard deviation of the expected return,
- $t = 2/c$ is the investor's *risk tolerance* and $c$ is risk aversion.
- $t$ or $c$ can be time-varying and wealth-dependent, but for simplicity we assume they are constant

# Optimal Portfolio Choice

- Investor will maximize the utility (blue indifference curves)
- Given the $e - s$ opportunities (red) available on the market – efficient frontier of a portfolio



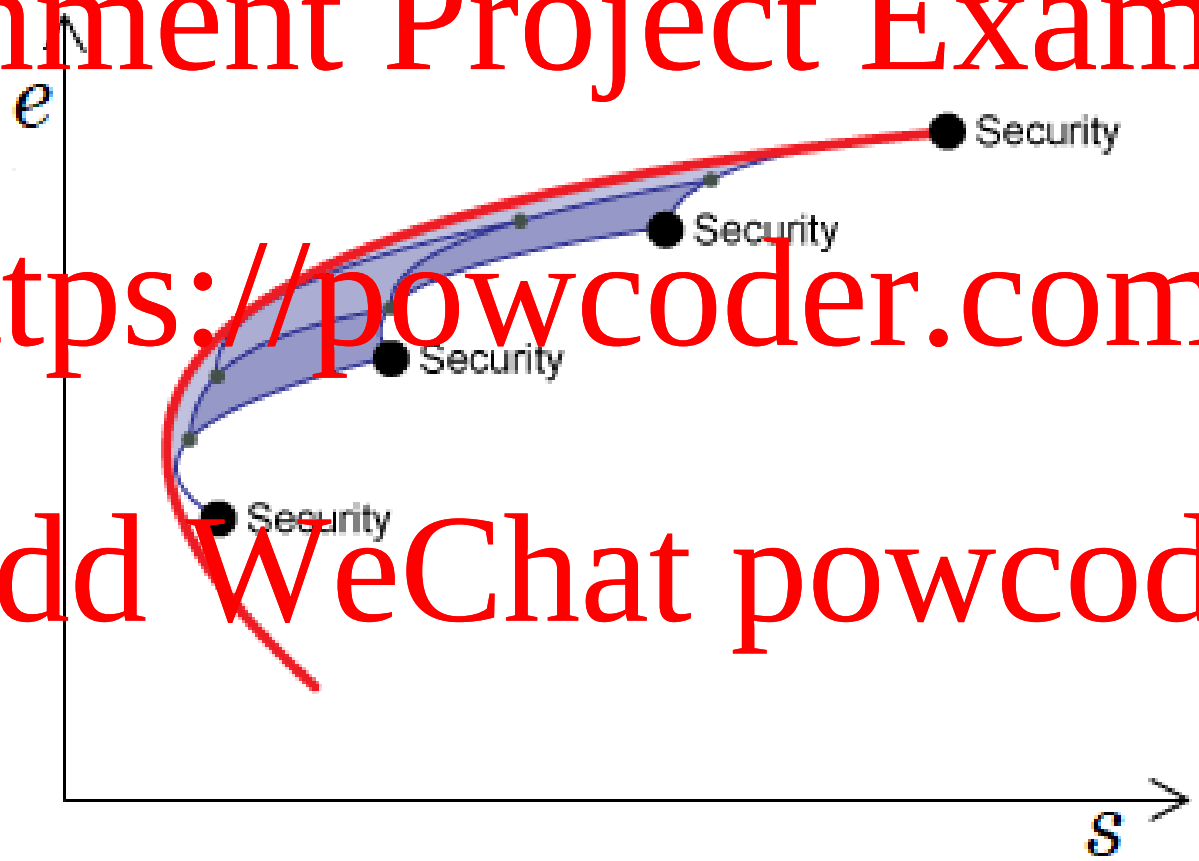



Why is efficient frontier concave?

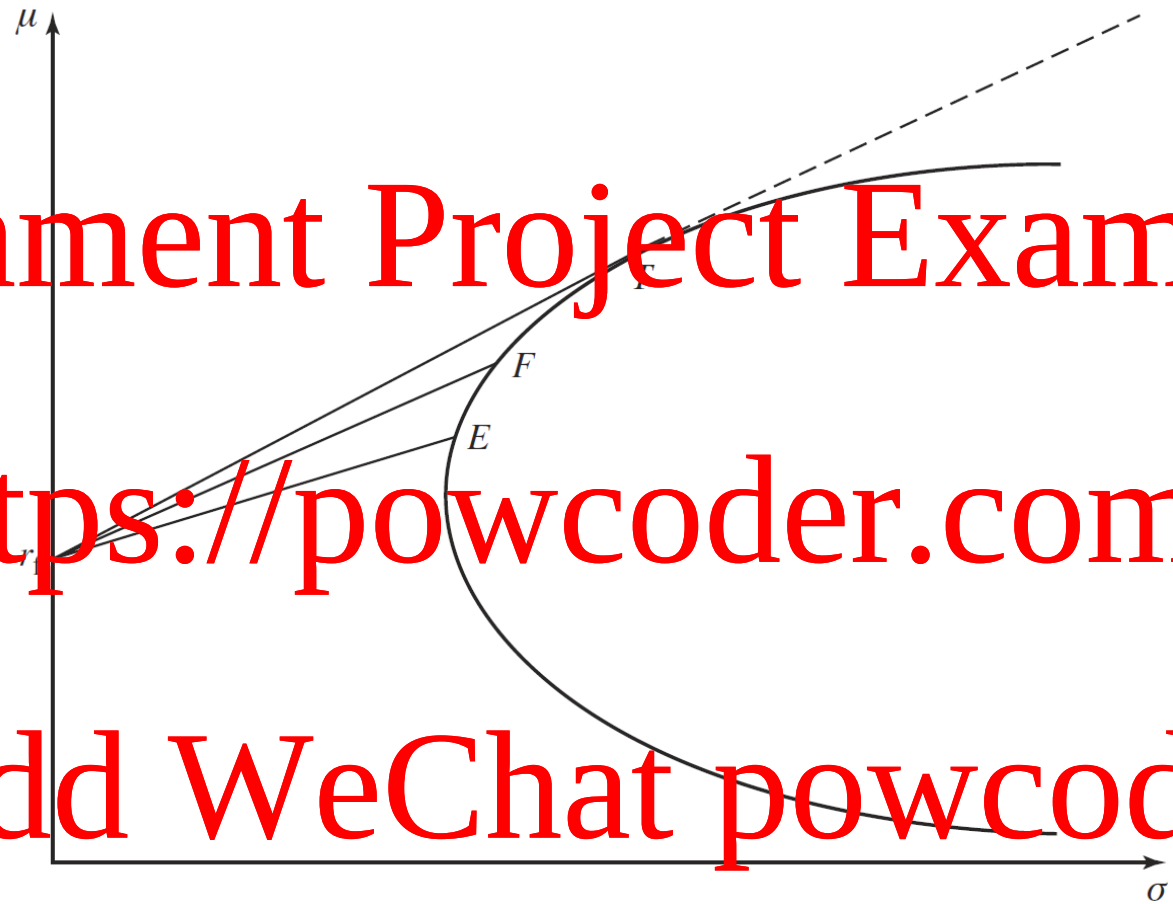
# Efficient Frontier: many securities

Efficient frontier is as the most "optimal" portfolio of portfolios of all risky securities





# Combine portfolios of risky assets with a risk-free asset






The point of tangency $T$ is called "Market portfolio", the best portfolio of risky assets on the market you can use to combine with the risk-free asset.

What about combinations with $F$ and $E$ portfolios?

# Sharpe ratio



William Sharpe proposed the following *Sharpe ratio*

$$S = \frac{e - e_f}{s}$$

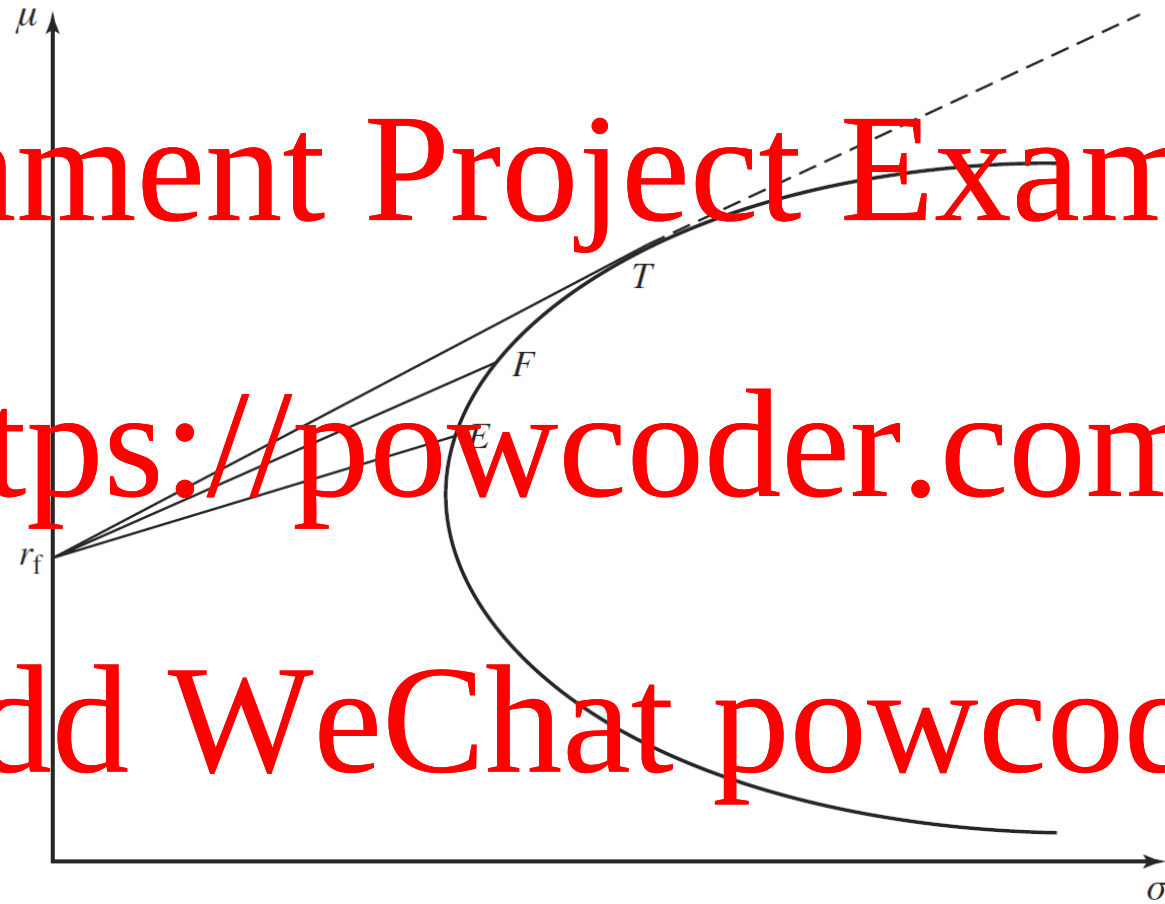


where $e - e_f$ is excess return and $s$ is standard deviation (risk) of a risky asset/portfolio.

- reflect the tradeoff between excess return and risk
- simple way to compare different stocks/portfolios
- usually $S > 1$ is acceptable, $S > 2$ is very good

# Sharpe ratio of the Market portfolio






$S_M = \frac{e_M - e_f}{s_M}$ is the slope to the tangent line and therefore the *best Sharpe* ratio available on the market