Economics of Finance

Tutorial 6

1. (Representative Agent A-D Economy) Consider a world in which there are only two dates: 0 and 1. At date 1 there are two possible states of nature: a bad weather state (B) and a good weather state (G). The B state occurs with probability $\pi_B = 0.3$, while the G state occurs with probability $\pi_G = 0.7$.

There is one non-storable consumption good – say apples. The endowment of apples at time 0, e_0, is 1. At time 1 the endowment of apples is state dependent. In the G state, the endowment, e_G, is 2. In the B state, the endowment, e_B, equals 0.7.

There is one representative consumer in this economy. The consumer's preferences over apples are given by

$$u\left(c_0\right) + \beta \sum_{s_1 \in \{B,G\}} \pi_{s_1} u\left(c_{s_1}\right),$$

where the instantaneous utility function is $u\left(c\right) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 2$. The consumer's time discount factor, β, is 0.98.

In this economy the only traded securities are atomic (Arrow-Debreu) securities. One unit of 'G security' sells at time 0 at a price q_G and pays one unit of consumption at time 1 if state 'G' occurs and nothing otherwise. One unit of 'B security' sells at time 0 at a price q_B and pays one unit of consumption in state 'B' only.

(a) Write down the consumer's set of budget constraints.

(b) Define a *Market Equilibrium* in this economy. Is there any trade of atomic (Arrow-Debreu) securities possible in this economy?

(c) Write down the Lagrangian for the consumer's optimisation problem and find the first order necessary conditions.

(d) Find the prices of the atomic (Arrow-Debreu) securities. What is the one-period discount factor and risk-free interest rate in this economy?

(e) How is the financial market discount factor, *df* different from consumer's time preferences discount factor, β?

2. (General Equilibrium of A-D Economy) Consider a world in which there are only two dates: 0 and 1. At date 1 there are two possible states of nature: a bad weather state (B) and a good weather state (G). That is: $s_1 \in S_1 = \{G, B\}$, while the state at date zero is known: call it s_0 if you will. The B state occurs with probability $\pi_B = 1/3$, while the G state occurs with probability $\pi_G = 2/3$.

There is one non-storable consumption good - say apples. There are two consumers in this economy. Their preferences over apples are exactly the same and are given by the following expected utility function

$$\frac{1}{2}c_0^k + \beta \sum_{s_1 \in S_1} \pi_{s_1} \ln\left(c_{s_1}^k\right),$$

where subscript $k = 1, 2$ denotes consumers. The consumer's time discount factor, β, is 0.9.

The consumers differ in their endowments which are given in the table below:

Consumers	Endowments		
	$t = 0$	t=1	t=1
		B	G
Consumer 1	10	1	2
Consumer 2	5	4	6

In this economy the only traded securities are basic Arrow securities. One unit of 'G security' sells at time 0 at a price q_G and pays one unit of consumption at time 1 if state 'G' occurs and nothing otherwise. One unit of 'B security' sells at time 0 at a price q_B and pays one unit of consumption in state 'B' only.

i) Write down the consumer's budget constraint including amounts of Arrow securities purchased a_{s_1}, where $s_1 \in S_1$ for all times and states.

ii) Define a *Market Equilibrium* in this economy. Is there any trade of Arrow securities possible in this economy?

iii) Write down the Lagrangian for the consumer's optimisation problem and find the first order necessary conditions.

iv) Find the prices of the atomic (Arrow-Debreu) securities and quantities traded.

v) Find the risk neutral probabilities, and the stochastic discount factors of this economy. What does your results suggest?

vi) Suppose that trade is not allowed in the economy. Compare the utilities of consumer without trade and with trade. Is there a Pareto improvement due to the trade?

vii) Suppose that instead of the atomic securities there are two securities, e.g. a stock and a bond, available for trade in this economy. Their payments are given by the following matrix.

$$\begin{array}{cc} & \begin{array}{cc} B & S \end{array} \\ \begin{array}{c} G \\ B \end{array} & \begin{pmatrix} 1 & 1.5 \\ 1 & 0.8 \end{pmatrix} \end{array}$$

Find their arbitrage-free prices at time 0?