Lesson 8: Capital Asset Pricing Model

Economics of Finance

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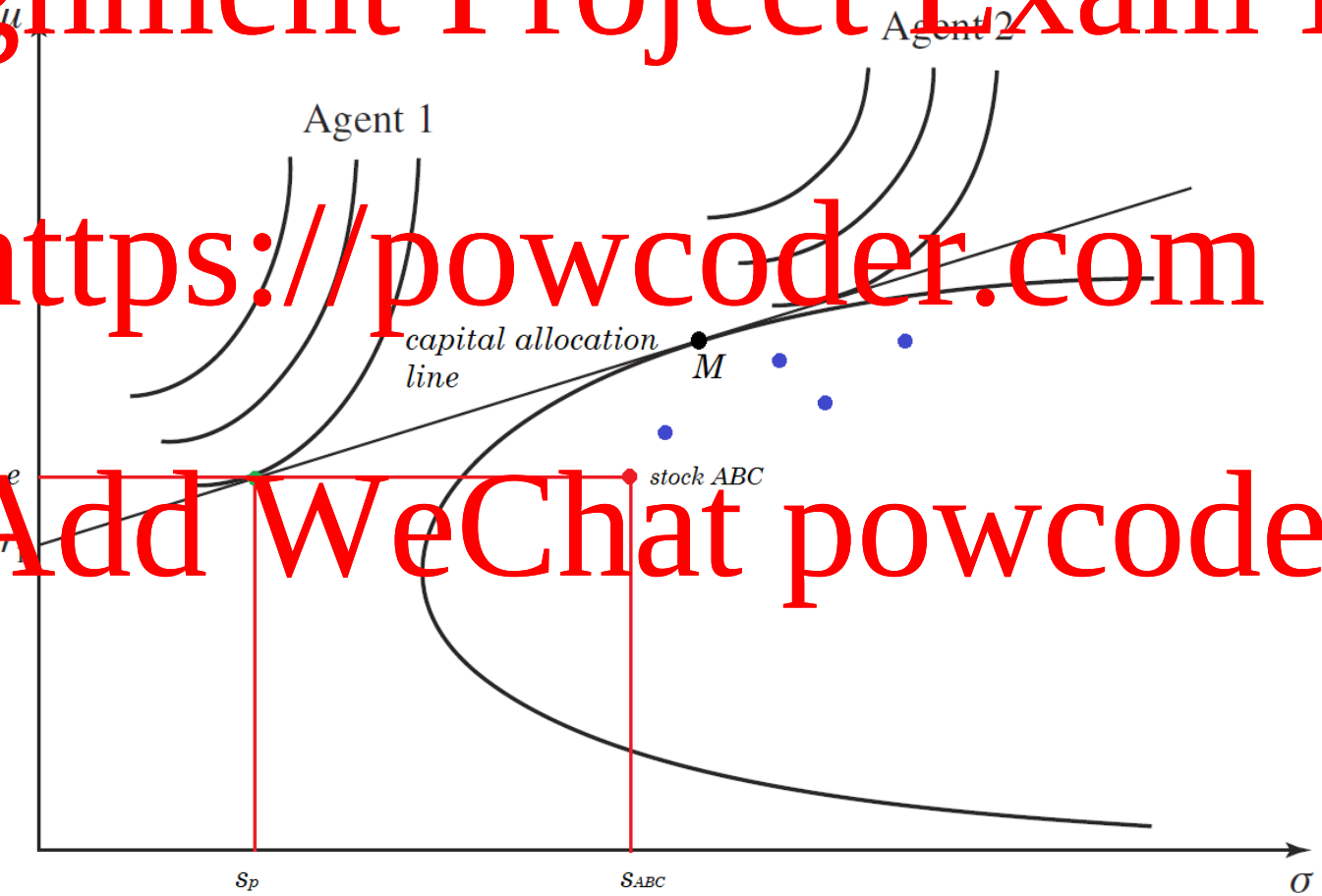
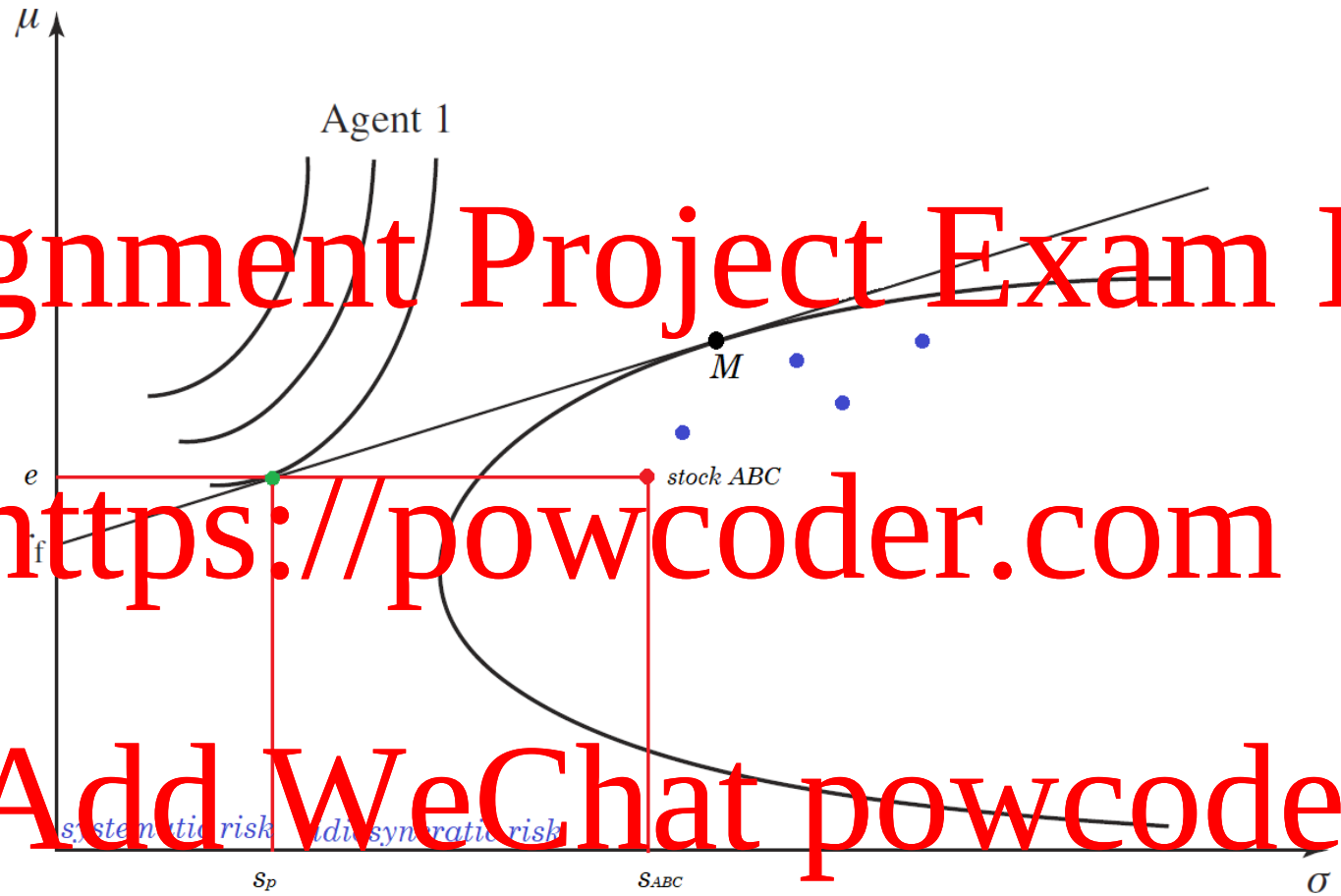
Capital allocation line

A maximum Sharpe ratio is obtained for any portfolio on the straight line from r_f tangent with the efficient frontier at M. This line is called *capital allocation line* (CAL).



Systematic vs Idiosyncratic Risk



$$s_{ABC} = s_p + s_i$$

- $s_p = \beta s_M$: systematic risk – non-diversifiable
- s_i: idiosyncratic risk – diversifiable
- $\beta \equiv x$: share invested in the market portfolio to replicate e

Capital Asset Pricing Model

Capital asset pricing model (CAPM) is a model used to determine an appropriate expected return of any asset

- only systematic risk is valued
- replicate any desired expected asset return e_j using the market portfolio (fraction β_j) and the risk-free asset (fraction $1-\beta_j$)

$$e_j = \beta_j e_M + (1-\beta_j) r_f = r_f + \beta_j (e_M - r_f)$$

Alternative interpretation of β

To infer β_j, regress the actual (historical) excess asset return, $R_j - r_f$, on excess market return $R_M - r_f$:

$$R_j - r_f = \alpha_j + \beta_j (R_M - r_f)$$

From econometrics, we remember that regression coefficient

$$\beta_j = \frac{cov(R_j, R_M)}{var(R_M)}$$

Therefore, β_j indicates how the specific asset co-moves with the market.

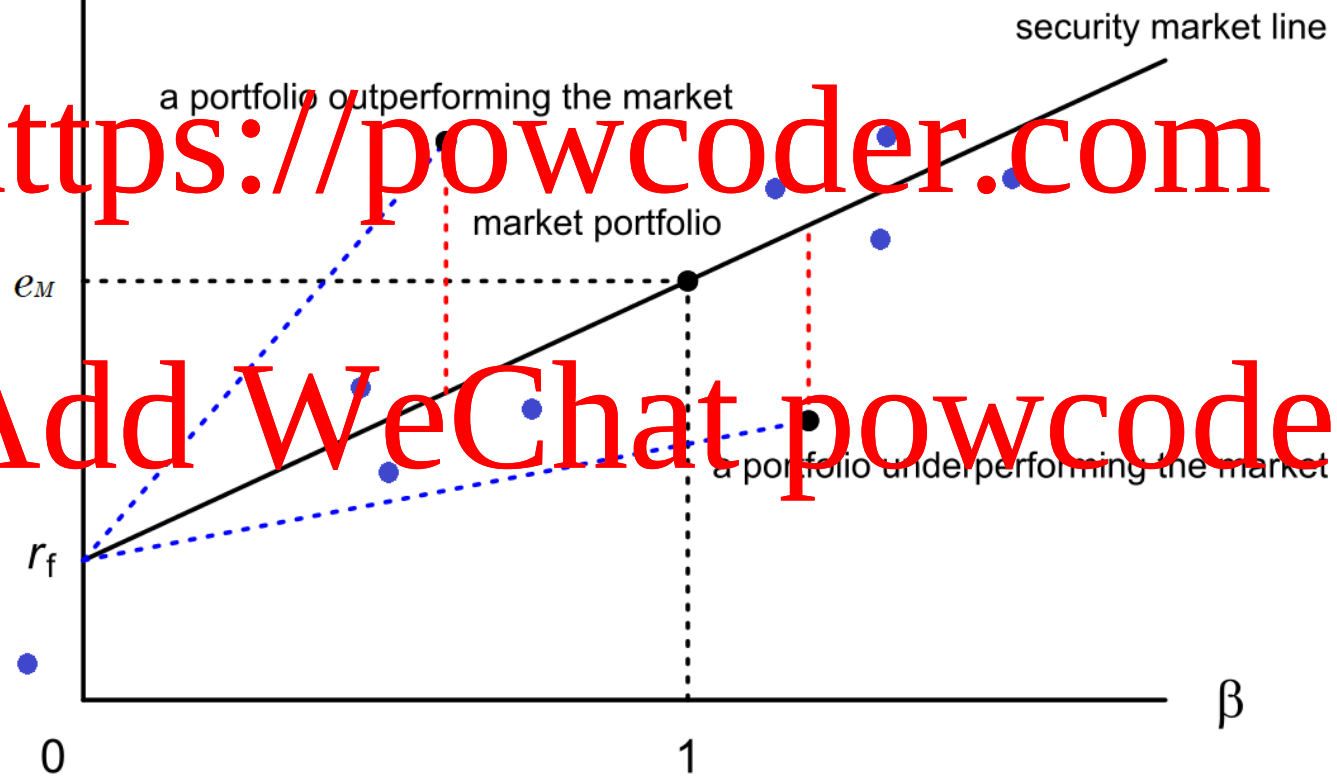
- $\beta > 1$ asset is more volatile than the market
- $0 < \beta < 1$ asset is less volatile than the market
- $\beta < 0$ asset moves in opposite direction – rare and useful

What about α_j? It should be 0 in theory. "Chasing" α.

Security market line

With different β value, the required return for any asset is

$$e = r_f + \beta(e_M - r_f)$$



Arbitrage Pricing Theory (APT)

CAPM provides good benchmark, but reality is more complicated: market risk is just one factor, but there are others

$$R_j = r_f + \beta_{j,1} f_1 + \ldots + \beta_{j,K} f_K + \varepsilon_j$$

- R_j is the expected return of the asset (or portfolio) j
- ε_j idiosyncratic, unexplained part of return
  $E(\varepsilon_j) = 0$, $E(R_j) = \theta_j$

- r_f is the risk-free rate
- f_k is the factor risk premium
- $\beta_{j,k}$ is the sensitivity of portfolio j to factor k

- K is the number of factors.

Assumptions (similar to standard OLS):

- exogeneity: ε_j and factors f_k are independent
- ε_j for different assets are independent

This is not *pure* arbitrage, but *statistical* arbitrage