

Economics of Finance

Tutorial 7

1. Suppose an investor decides to construct a portfolio consisting of a risk-free asset that pays 6 percent and a stock index fund that has an expected rate of return of 12 percent and a standard deviation of 20 percent. Let x denote the proportion invested in the stock index fund. This implies that the proportion $(1 - x)$ is invested in the risk-free asset. (In your answers use 6 for 6 percent etc).

- (a) Find the equation for the efficient frontier. Graph the efficient frontier in (μ_p, σ_p) space where μ_p is the expected return on the portfolio and σ_p the standard deviation of the return on the portfolio.
- (b) Suppose the expected utility of the investor is given by

$$Eu = \mu_p - 0.025\sigma_p^2. \quad (1)$$

What does expected utility imply in terms of preferences? What is the interpretation of the coefficient in front of σ_p^2 in equation (1)?

- (c) What is the investor's optimal portfolio, assuming that he/she is an expected utility maximiser? (In other words, find the investor's optimal choice for x). Show graphically the optimal portfolio choice in (μ_p, σ_p) space.
- (d) Consider the portfolio with $\mu_p = 8.4$ and $\sigma_p = 8$. Show that this portfolio is efficient. Demonstrate that this portfolio is not optimal in the sense of maximizing the investor's expected utility. (Hint: Calculate the **certainty equivalent** of this portfolio and the one in part (c) and compare the two). Show graphically the certainty equivalent of this portfolio and the one in part (c) on the same graph in (μ_p, σ_p) space.
- (e) Suppose the expected utility of the investor is given by

$$Eu = \mu_p - 0.01\sigma_p^2. \quad (2)$$

What is the investor's optimal portfolio in this case? Compare your answer to the optimal portfolio you found in part (c). Comment

- (f) Is there a relationship between the standard deviation of the optimal portfolio you found in part (c) and (e) and the investors degree of risk tolerance. If so, can that relationship be described precisely in mathematical terms?

2. Consider the following two stocks: stock 1 has expected return $\mu_1 = 6$ and standard deviation $\sigma_1 = 16$; stock 2 has expected return $\mu_2 = 10$ and standard deviation $\sigma_2 = 20$. Assume that the correlation between the returns on the two stocks is 0.75, that is, the correlation coefficient $r_{12} = 0.75$.

- (a) Write down an equation for the expected return (μ_p) and the variance (v_p) of the return on the portfolio as a function of x_2 only, where x_2 is the proportion of invested wealth in stock 2.
- (b) Find the minimum variance portfolio. What is the expected return and variance of this portfolio?
- (c) Is Stock 1 on the efficient frontier? Draw a graph in expected return-standard deviation space to illustrate your answer.