Economics of Finance

Tutorial 2

1. An apple tree firm offers for sale a bond and stock. An apple tree produces 70GA and 45BA. The bond pays 20GA and 20BA. The stock pays 50GA and 25BA. The price of the bond is 18PA, and the price of the stock is 30PA.

(i) Construct and graph the opportunity set for future apples per present apple. Plot the bond and stock on the opportunity set. Calculate the arbitrage-free atomic prices and also plot them on the opportunity set.

(ii) Holding the payments made by both the bond and stock in each state fixed, and the price of the bond fixed, calculate the range of prices of the stock that generate positive prices for both the atomic securities. (Hint: Observe the relative positions of the bond and stock on the graph, in terms of security payoffs and prices.

(iii) A new security X appears on the market that pays 10GA and 35BA and has a price of 24PA. Plot this security on the opportunity set. Does it provide any arbitrage opportunities? If so, design a profitable arbitrage strategy.

(iv) Another security Y appears on the market that pays 50GA and 30BA and has a price of 40PA. Plot this security on the opportunity set. Does it provide any arbitrage opportunities? If so, design a profitable arbitrage strategy.

2. Suppose there are three possible states of the world in the next period, denoted by good weather (GW), fair weather (FW) and bad weather (BW). Also, three securities are available on the market with payoffs in each state listed below.

	Bond	Stock	Security X
GW	20	43	26
FW	20	35	10
BW	20	30	0

(i) Suppose an investor desires the following portfolio:

$$\mathbf{c}_{(states \times 1)} = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}.$$

Is it possible? Why?

(ii) The prices of the three securities are as follows: $p_{Bond} = 10$, $p_{Stock} = 20$, $p_X = 15$. Are there arbitrage opportunities? If so, design a profitable arbitrage strategy; if not, explain why.

(iii) Suppose security X disappears from the market.

(a) Construct a portfolio that pays 10 in GW and 15 in FW. What is the arbitrage-free price of this portfolio? What is its payoff in the bad weather state?

(b) Construct a portfolio that pays 10 in GW and 20 in BW. What is the arbitrage-free price of this portfolio? What is its payoff in the fair weather state?

(c) Construct a portfolio that pays 15 in FW and 20 in BW. What is the arbitrage-free price of this portfolio? What is its payoff in the good weather state?

(d) Is it possible for an investment firm to provide the time-state claims in (i) to a client without incurring any risk? Explain.

(e) What is the minimum cost to for the investor? (Hint: this part is for lecture 3, and you will need to use optimisation function.)

(f) What is the minimum cost if X becomes available? Comment on your results.