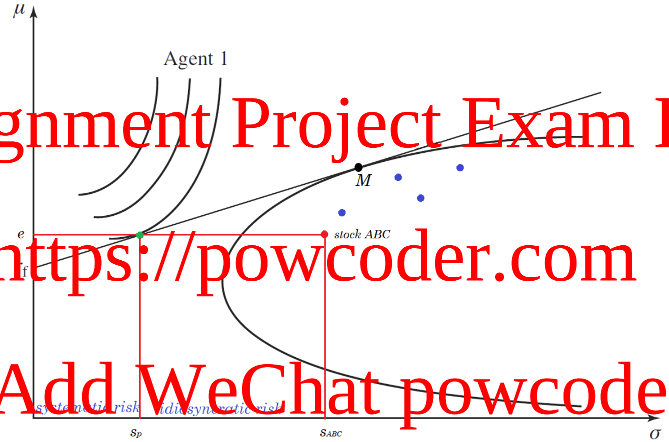
Lecture 8: Capital Asset Pricing Model

Economics of Finance

School of Economics, UNSW

Systematic vs Idiosyncratic Risk



$$s_{ABC} = s_p + s_i$$

- $s_p = \beta s_M$: systematic risk – non-diversifiable
- s_i: idiosyncratic risk – diversifiable
- $\beta \equiv x$: share invested in the market portfolio to replicate e

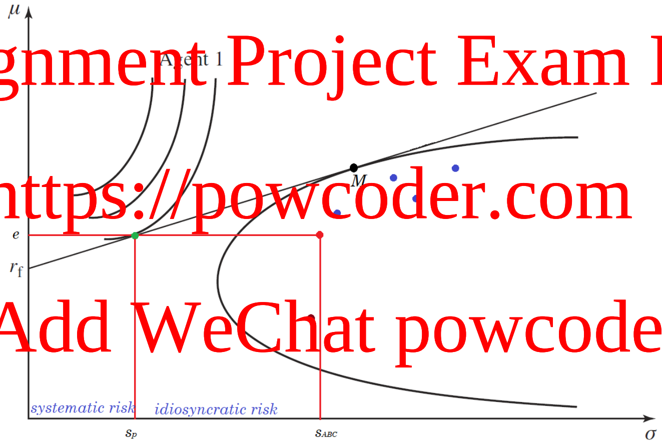
Capital Asset Pricing Model

Capital asset pricing model (CAPM) is a model used to determine an appropriate expected return of any asset

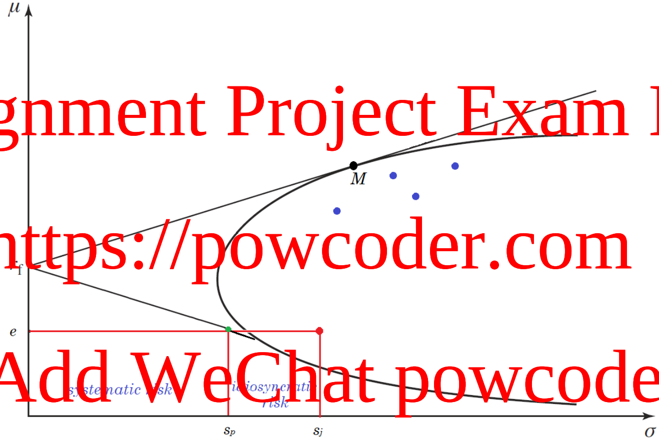
- only systematic risk is valued
- replicate any desired expected asset return e_j using the market portfolio (fraction β_j) and the risk-free asset (fraction $1-\beta_j$)

$$e_j = \beta_j e_M + (1-\beta_j) r_f = r_f + \beta_j (e_M - r_f)$$

What if $e_j < r_f$



Negative β



Still the same

$$e_j = \beta_j e_M + (1 - \beta_j) r_f = r_f + \beta_j (e_M - r_f)$$

Alternative interpretation of β

To infer β_j, regress the actual (historical) excess asset return, $R_j - r_f$, on excess market return $R_M - r_f$:

$$R_j - r_f = \alpha_j + \beta_j(R_M - r_f)$$

From econometrics, we remember that regression coefficient

$$\beta_j = \frac{cov(R_j, R_M)}{var(R_M)}$$

Therefore, β_j indicates how the specific asset co-moves with the market:

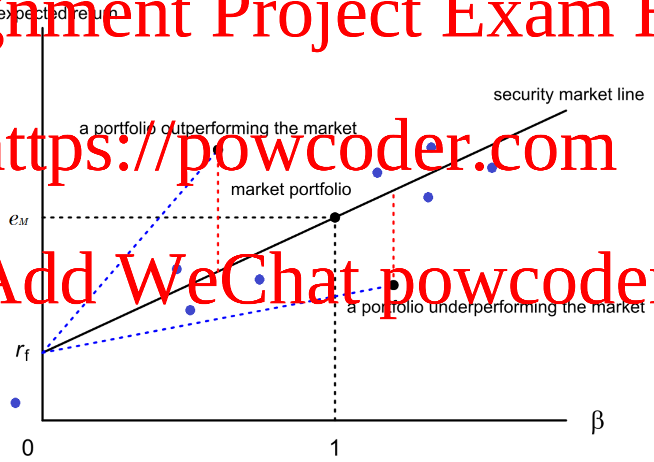
- $\beta > 1$ asset is more volatile than the market
- $0 < \beta < 1$ asset is less volatile than the market
- $\beta < 0$ asset moves in opposite direction – rare and useful

What about α_j? It should be 0 in theory. “Chasing” α.

Security market line

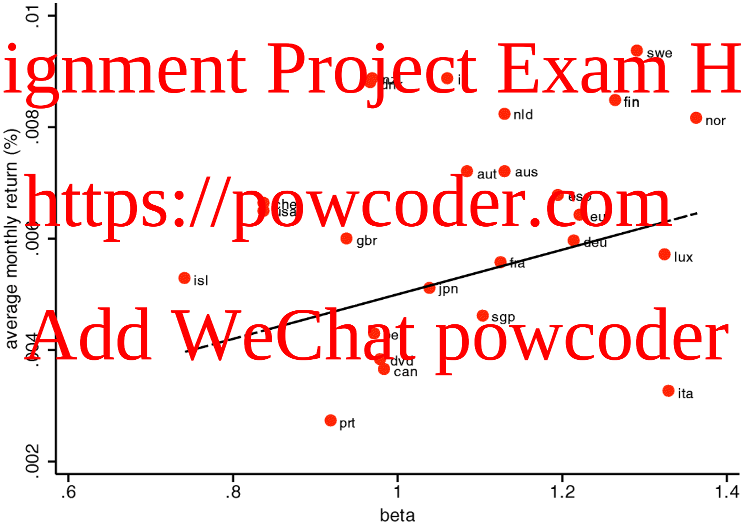
With different β value, the required return for any asset is

$$e = r_f + \beta(e_M - r_f)$$



Example: security market line

Country stock market indices averaged over 1988-2017



Example: pricing with CAPM

Gordon's stock price model

$$P_j = \frac{D_j}{e_j - g_j},$$

D_j current dividend e_j expected rate of return g_j expected rate of dividend growth

Can use $e_j = r_f + \beta_j(e_M - r_f)$ to infer the fair stock price.

The standard deviation

CAPM gives us guidance about the expected return, what about standard deviation (or risk)?

The standard deviation of R_j: $s_j = \beta_j s_m + s_i$

- reflects an *idiosyncratic risk* in addition to *systematic risk*
- recall that market portfolio and CML reach the highest Sharpe ratio
- any other securities are stochastically dominated by M and r_f combinations
- this does not necessarily mean other securities will cease from the market, as they will be traded to construct M.

Arbitrage Pricing Theory (APT)

CAPM provides good benchmark, but reality is more complicated: market risk is just one factor, but there are others

$$R_j = r_f + \beta_{j,1} f_1 +, \ldots, +\beta_{j,K} f_K + \varepsilon_j,$$

- R_j is the expected return of the asset (or portfolio) j
- ε_j idiosyncratic, unexplained part of return
  $E(\varepsilon_j) = 0$, $E(R_j) = e_j$

- r_f is the risk-free rate
- f_k is the factor risk premium
- $\beta_{j,k}$ is the sensitivity of portfolio j to factor k

- K is the number of factors.

Assumptions (similar to standard OLS):

- exogeneity: ε_j and factors f_k are independent
- ε_j for different assets are independent

This is not *pure* arbitrage, but *statistical* arbitrage

Example: applying the model to sector and region

$$R_j = r_f + \beta_{j,m} f_m + \beta_{j,s} f_s + \beta_{j,r} f_r + \varepsilon_j,$$

- f_m is the market risk premium $(e_m - r_f)$;
- f_s is the risk premium for a particular sector, e.g., ASX health care sector index
- f_r is the risk premium for a particular region, e.g., MSCI Asia Pacific index
- βs are sensitivities to the factors

Example: Fama and French Model – 3 Factor Model

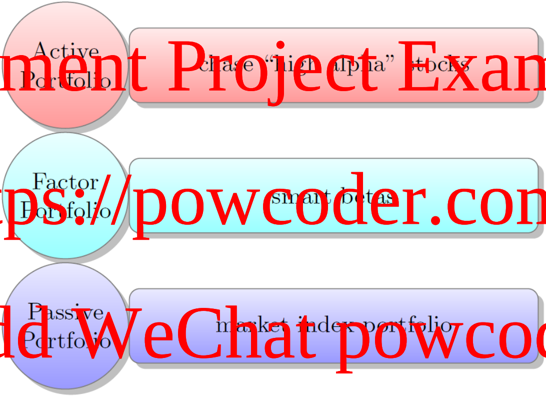
$$R_j = r_f + \beta_{j,m} f_m + \beta_{j,SMB} f_{SMB} + \beta_{j,HML} f_{HML} + \varepsilon_j,$$

- f_m is the excess return on the market portfolio
- f_{SMB} is the size factor attributable to the company's market capitalisation, Small minus Big
- f_{HML} is the value factor driven by the difference between High minus Low book-to-market stocks

Extensions:

- momentum
- operating profitability
- firm investment factor (agressive vs conservative)
- time-varying factors – dynamic factor models

Smart Betas: Bridge between Active and Passive Portfolios



Benefits of smart beta products

- Customisable products
- Lower transaction costs relatively to active portfolio

Wrap up

- Sharpe ratio is maximised by market portfolio
- Optimal risky is independent from individual preferences – the same market portfolio for all

- Rational individual only hold portfolios on CAL
- CAPM is based on replicating portfolio
- takes into account *systematic risk*

- Factor models offer valuable extension to a simple CAPM