

Economics of Finance

**Tutorial 1**

Note: Use Matlab when necessary.

1. Find the inverse of the following matrix:

$$X = \begin{pmatrix} 0.1 & 0.1 & 0 & 0.9 \\ 0.8 & 0.2 & 0.1 & 0 \\ 0 & 0.4 & 0.5 & 0 \\ 0.1 & 0.3 & 0.4 & 0.1 \end{pmatrix}.$$

2. Show that the three columns in the matrix **A** below are linearly dependent

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -3 & 9 \\ 4 & 1 & 11 \end{pmatrix}.$$

3. Compute the determinant of the matrix in question 2. Use `det()` function. Explain what you find.

4. Suppose there are two investment funds, Fund A and Fund B. The exposures of Fund A are 50% in bonds and 50% in stocks while the exposures of Fund B are 30% in bonds and 70% in stocks. An investor desires an asset allocation of 40% in bonds and 60% in stocks using the two funds.

This problem can be solved by setting up the system of simultaneous equations:

$$\begin{aligned} 0.5x_A + 0.3x_B &= 0.4 \\ 0.5x_A + 0.7x_B &= 0.6, \end{aligned}$$

where  $x_A$  and  $x_B$  represent allocations of money invested in Fund A and Fund B respectively.

4.1 Solve the system by repeated substitution and interpret the results.

4.2 Write it in matrix form as  $\mathbf{Ax} = \mathbf{b}$ .

4.3 Solve the system using the inverse of **A**. Check that your solution is correct by computing  $\mathbf{A}(\mathbf{A}^{-1}\mathbf{b})$ .

Note: indeed you could perform this by hand, but let's say, the columns in the matrix in question 1 describe the allocations over four asset classes [domestic bonds (DB), foreign bonds (FB), domestic stocks (DS), foreign stocks (FS)] of four mutual funds denoted by MF1, MF2, MF3, MF4. For example, MF1 holds 10 percent of its assets in DB, 80 percent in FB, zero in DS, and 10 percent in FS. If an investor wants to allocate her wealth between the four funds to obtain, say, the following asset mix: DB=1, FB=0, DS=0, FS=0, it would be really hard to do it by hand. That is why computing skills are very important. By the way this asset mix is referred to as pure asset play (100% in DB in this case).

Questions 5 to 7 refer to a world in which there are only two periods and two possible states of the world in the second period (a good weather state and a bad weather state). Also, apples are the only product produced in this world, and they cannot be stored from one period to the next. The following abbreviations will be used: PA = apple in first period (i.e., present apple), GA = good weather apple, BA = bad weather apple.

5. Dealer I is willing to trade  $0.3PA$  for  $1GA$  (or vice versa), and dealer II is willing to trade  $1GA$  for  $0.6BA$  (or vice versa). What is the arbitrage-free price of a BA in terms of PA?

6. In addition dealer III is willing to trade  $0.6PA$  for  $1BA$  (or vice versa). Are there arbitrage opportunities? If so, design a profitable arbitrage strategy.

7. Suppose now there are transaction costs. The three dealers have the following bid-ask spreads:

Dealer I:    sell  $1PA$  for  $2.5GA$   
              buy    $1PA$  for  $2GA$

Dealer II:   sell  $1GA$  for  $1BA$   
              buy    $1GA$  for  $0.75BA$

Dealer III: sell  $1PA$  for  $2BA$   
              buy    $1PA$  for  $1.5BA$ .

## Assignment Project Exam Help

Are there arbitrage opportunities now? Explain.

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