The purpose of this session is to introduce you to the autocorrelated error term in linear regression models and GLS.

Serial Correlation

Consider an autorogressive process of order one AR(1) of the form

$$u_t = c + \rho u_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0,1) \qquad t = 1, ..., T \tag{1}$$

a) Write a function which takes 3 input parameters (c, ρ, T) and simulates an AR(1) process defined in equation (1). Plot the simulated process for $c = 0.1, \rho = 0.8$ and $T = 500$ together with a horizontal line of the unconditional mean.

b) Practice in R how to create a matrix of the form (an autocorrelation matrix for $T = 5$):

$$\Omega_5 = \begin{bmatrix} 1 & 0.8 & 0.8^2 & 0.8^3 & 0.8^4 \\ 0.8 & 1 & 0.8 & 0.8^2 & 0.8^3 \\ 0.8^2 & 0.8 & 1 & 0.8 & 0.8^2 \\ 0.8^3 & 0.8^2 & 0.8 & 1 & 0.8 \\ 0.8^4 & 0.8^3 & 0.8^2 & 0.8 & 1 \end{bmatrix}$$

c) Simulate a bivariate regression model

$$\begin{aligned} y_t &= 1 - 3x_t + u_t, \qquad t = 1, ..., T \\ u_t &= 0.8 \cdot u_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0,1), \end{aligned}$$

where $x_t \sim \mathcal{N}(5,1)$ and $T = 500$. Compute OLS estimates for the intercept and slope parameters. Consider Example 4.3 and compute the GLS solution using $\Sigma = \dfrac{\sigma_\varepsilon^2}{1-0.8^2}\Omega_T$.

d) Use the code from c) to conduct a simulation study to examine the distribution of the OLS and GLS estimators for the slope over $mc = 1000$ simulation draws. Plot the histograms of $\hat{\beta}_1(OLS)$ and $\hat{\beta}_1(GLS)$ next to each other. Use the option `xlim = c(-3.15, -2.85)` when plotting the histograms. Which estimator is more efficient?

Exercises to solve at home

No homework this week. Enjoy the midterm!