ECON 3350/7350

Single Equation Models of Multiple Time Series

Eric Eisenstat

The University of Queensland

Tutorial 4

$ARMA(p,q)$ with deterministic trend

$$y_t = a_0 + a_2 t + a_1 y_{t-1} + \varepsilon_t;\ |a_1| < 1$$

- y_t is **trend stationary** because if we take the trend out the new process is stationary. We return to deterministic and stochastic trends next week.
- De-trending

$$y_t - a_2 t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

$$\widetilde{y}_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

- $\widetilde{y}_t$ is an $ARMA(1,0)$

$ARDL(p, l, s)$ with trend

- For c_t, a_t, and y_t we could have an $ARDL(p, q, m)$:

$$\theta(L)c_t = \delta + \gamma(L)a_t + \lambda(L)y_t + \varepsilon_t$$

Where,

$$\theta(L) = (1 - \theta_1 L - \theta_2 L^2 - ... - \theta_p L^p) = \sum_{i=0}^{p} \theta_i L^i$$

$$\gamma(L) = (\gamma_0 + \gamma_1 L + \gamma_2 L^2 - ... + \gamma_q L^q) = \sum_{j=0}^{q} \gamma_j L^j$$

$$\lambda(L) = (\lambda_0 + \lambda_1 L + \lambda_2 L^2 - ... + \lambda_m L^m) = \sum_{j=0}^{m} \lambda_j L^j$$

- Adding a deterministic trend

$$\theta(L)c_t = \delta_0 + \delta_1 t + \gamma(L)a_t + \lambda(L)y_t + \varepsilon_t$$

The ARDL Family of Models

Using $ARDL(1,1)$

$$y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$$

1. Static Regression:
$y_t = \delta + \theta_0 x_t + \varepsilon_t$;
Restrictions: $a_1 = 0; \quad \theta_1 = 0$

2. First order autoregressive process:
$y_t = \delta + a_1 y_{t-1} + \varepsilon_t$;
Restrictions: $\theta_0 = 0; \quad \theta_1 = 0$

3. Leading indicator equation:
$y_t = \delta + \theta_1 x_{t-1} + \varepsilon_t$;
Restrictions: $a_1 = 0; \quad \theta_0 = 0$

4. Equation in first differences:
$\Delta y_t = \delta + \theta_0 \Delta x_t + \varepsilon_t$;
Restrictions: $a_1 = 1, \theta_0 = -\theta_1$

The ARDL Family of Models-II

$$y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$$

5. First order distributed lag model:

$y_t = \delta + \theta_0 x_t + \theta_1 x_{t-1} + \varepsilon_t$

Restrictions: $a_1 = 0$

6. Partial adjustment model:

$y_t = \delta + a_1 y_{t-1} + \theta_0 x_t + \varepsilon_t$

Restrictions: $\theta_1 = 0$

7. Dead Start model (lagged information only):

$y_t = \delta + a_1 y_{t-1} + \theta_1 x_{t-1} + \varepsilon_t$

Restrictions: $\theta_0 = 0$

8. Proportional Response Model:

$y_t = \delta + a_1 (y_{t-1} - x_{t-1}) + \theta_0 x_t + \varepsilon_t$

Restrictions: $\theta_1 = -a_1$

9. Error Correction Mechanism:

$\Delta y_t = \delta + \alpha(y_{t-1} - \beta x_{t-1}) + \theta_0 \Delta x_t + \varepsilon_t$

where, $\beta = \dfrac{(\theta_1 + \theta_0)}{(1 - a_1)}$; $\alpha = a_1 - 1$

This is a re-arrangement of the ARDL equation.

Multipliers

1. Immediate Response or Impact Multiplier

$$\frac{\partial c_t}{\partial a_t} = \gamma_0$$

2. The Effect after one period, two periods, ...

$$\frac{\partial c_{t+1}}{\partial a_t} = \theta_1 \frac{\partial c_t}{\partial a_t} + \gamma_1 = \theta_1\gamma_0 + \gamma_1$$

$$\frac{\partial c_{t+2}}{\partial a_t} = \theta_1 \frac{\partial c_{t+1}}{\partial a_t} = \theta_1(\theta_1\gamma_0 + \gamma_1)$$

3. Long-run multiplier

$$LRM = \frac{\gamma(1)}{\theta(1)} = \frac{(\gamma_0 + \gamma_1 + \gamma_2 + ... + \gamma_p)}{(1 - \theta_1 - \theta_2 - ... - \theta_p)}$$