ECON 3350/7350: Applied Econometrics for Macroeconomics and Finance

Tutorial 3: Univariate Time Series - II

The point of this question is to suggest a general "road map" for analyzing univariate time series with ARMA models.

1. The file `Merck.csv` contains daily data of stock prices of Merck & Co., Inc. (MRK) during 2001-2013. In what follows, we use y_t to denote the adjusted closing prices (*adjclose* in the data) in time t.

   (a) Load the data to Stata, generate a date variable, declare the data as time series, and keep only observations during January 1, 2011 - January 31, 2012.

   (b) Construct the following variables:

   - Changes in prices: $\Delta y_t = y_t - y_{t-1}$
   - Log returns: $r_t = \log(y_t / y_{t-1})$

   (c) Draw time series plots of y_t and Δy_t and comment on their stationarity[1].

   (d) Compute and plot (using either *ac*/*pac* or *corrgram*) ACF and PACF of y_t and Δy_t. Comment on your findings.

   (e) Based on the ACF and PACF of Δy_t you obtained in (d), propose and estimate at least three ARMA(p, q) models for Δy_t.

   (f) Use AIC and BIC to select an ARMA(p, q) model. Estimate the AR and MA parameters of this model and report estimation results.

   (g) Draw a time series plot of the residuals you obtain via estimating the ARMA model selected in (f). Comment on your findings. Run the Ljung-Box test (at significance level $\alpha = 5\%$) for the white noises hypothesis and report test results. Note that you will need to adjust the degree(s) of freedom as you are analyzing estimation residuals.

   (h) Forecast MRK stock prices in January, 2012. Compare your predicted prices with real prices in the data.

   (i) Repeat (c) - (h) for the log returns r_t. Note that here you forecast the daily returns $(y_t - y_{t-1})/y_{t-1}$ in January, 2012. Hint: Recall that $(y_t - y_{t-1})/y_{t-1} \approx r_t$.

Solution: See the do-file `tutorial3.do`.

(a) Just take a look at the do-file.

[1] You should use only 2011 data for parts (c)-(g).

(b) The same as Part (a).

(c) It seems from the time series plot that $\{y_t\}$ is not likely to be stationary as its mean is time-varying. For $\{\Delta y_t\}$, the time series plot provides evidence for a constant (≈ 0) mean. However, we can observe the phenomenon called *volatility clustering*, i.e., large price changes (absolute returns) often occur in clusters, which is one of the stylized features common in financial data. This phenomenon provides evidence that the variance of Δy_t may depend on t. We will study models capturing stochastic volatility in the future. For now, we just ignore this issue for simplicity.

(d) For $\{y_t\}$, $\rho_1 \approx 1$ and ρ_k decays very slowly as k increases, and the PACF has only 1 peak whose value is approximately 1. This pattern of ACF and PACF implies that $\{y_t\}$ may have *unit root*, i.e., $y_t \sim \mathrm{AR}(1)$ with $a_1 \approx 1$. We know that the sufficient and necessary condition for an AR(1) model to be stationary is $|a_1| < 1$. Thus, we doubt that $\{y_t\}$ is not a stationary process, which is consistent with the intuition we got from the time series plot in Part (a). We will learn how to handle unit root in the future, too. Before that, enjoy stationarity for a while. For $\{\Delta y_t\}$, we can observe a couple of significant (but small) ACF and PACF for comparatively large k (order of lags). There are no clear cut-off points for either ACF or PACF, which suggests that an ARMA process rather than either AR or MA is more suitable for modeling $\{\Delta y_t\}$.

(e) Following the *parsimony principle*, we try ARMA models with order $(1,1)$, $(1,2)$, $(2,1)$, $(2,2)$ and compute their AIC/BIC. See and run the do-file.

(f) We pick ARMA$(1,1)$ as it has the smallest BIC and very similar AIC as ARMA$(1,2)$. The estimated model is

$$\widehat{\Delta y_t} = \underset{(0.0233)}{0.0104} - \underset{(0.0887)}{0.9083}\Delta y_{t-1} + \underset{(0.1059)}{0.8413}\epsilon_{t-1}$$

(g) Though we can detect volatility clustering (not surprising at all as we ignore this intentionally), the process $\{\widehat{\epsilon}_t\}$ of residuals seems to have zero mean. We run Ljung-Box tests for $k = 3, \ldots, 10$. For each k, the Q-statistic (degree of freedom = $k - 2$ here) turns out to be smaller than the corresponding critical value. Hence, we cannot reject the H_0 that $\{\widehat{\epsilon}_t\}$ is a white noise process.

(h) See and run the do-file. For forecasting purpose, the ARMA$(1,1)$ model we fit in Part (f) performs adequately well.

(i) All results here are very similar to what we have for $\{\Delta y_t\}$ unless $\widehat{r}_{t+h}$ has weak predictive ability for r_{t+h}.