ECON 3350/7350: Applied Econometrics for Macroeconomics and Finance

Tutorial 4: Single Equation Models of Multiple Time Series

ARDL and ECM.

1. Derive the ECM representation of the following ARDL(1,1,2) model:

$$c_t = \delta + \theta_1 c_{t-1} + \gamma_0 a_t + \gamma_1 a_{t-1} + \lambda_0 y_t + \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \epsilon_t$$

   Which parameter(s) in the resulting ECM are long-run multiplier(s) and adjustment parameter(s)?

Solution:

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Error correction presentation of ARDL(1,1,2)

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$$\Delta c_t = \delta + \alpha(c_{t-1} - \beta_1 a_{t-1} - \beta_2 y_{t-1})$$
$$+\gamma_0 \Delta a_t + \lambda_0 \Delta y_t - \lambda_2 \Delta y_{t-1} + \epsilon_t$$

Where,

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The long run multipliers are $\beta_1 = \frac{\gamma_0+\gamma_1}{1-\theta_1}$ and $\beta_2 = \frac{\lambda_0+\lambda_1+\lambda_2}{1-\theta_1}$

The adjustment parameter is $\alpha = -(1-\theta_1)$

To convert ARDL(1,1,2) to error correction presentation:

Add and Subtract following terms to ARDL(1,1,2):

$$c_{t-1}$$

$$\gamma_0 a_{t-1}$$

$$\lambda_0 y_{t-1}$$

$$\lambda_2 y_{t-1}$$

$$c_t = \delta + \theta_1 c_{t-1} - c_{t-1} + c_{t-1} + \gamma_0 a_t - \gamma_0 a_{t-1} + \gamma_0 a_{t-1} + \gamma_1 a_{t-1} + \lambda_0 y_t - \lambda_0 y_{t-1} + \lambda_0 y_{t-1}$$
$$+ \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \lambda_2 y_{t-1} - \lambda_2 y_{t-1} + \epsilon_t$$

$$c_t - c_{t-1} = \delta + (\theta_1 - 1)c_{t-1} + \gamma_0(a_t - a_{t-1}) + (\gamma_0 + \gamma_1)a_{t-1} + \lambda_0(y_t - y_{t-1}) + (\lambda_0 + \lambda_1$$
$$+ \lambda_2)y_{t-1} - \lambda_2 y_{t-1} + \lambda_2 y_{t-2} + \epsilon_t$$

$$c_t - c_{t-1} = \delta + (\theta_1 - 1)c_{t-1} + (\gamma_0 + \gamma_1)a_{t-1} + (\lambda_0 + \lambda_1 + \lambda_2)y_{t-1}$$
$$+\gamma_0(a_t - a_{t-1}) + \lambda_0(y_t - y_{t-1}) - \lambda_2(y_{t-1} - y_{t-2}) + \epsilon_t$$

$$c_t - c_{t-1} = \delta + (\theta_1 - 1)c_{t-1} + (\gamma_0 + \gamma_1)a_{t-1} + (\lambda_0 + \lambda_1 + \lambda_2)y_{t-1}$$
$$+\gamma_0 \Delta a_t + \lambda_0 \Delta y_t - \lambda_2 \Delta y_{t-1} + \epsilon_t$$

$$\Delta c_t = \delta + \alpha(c_{t-1} - \beta_1 a_{t-1} - \beta_2 y_{t-1})$$
$$+\gamma_0 \Delta a_t + \lambda_0 \Delta y_t - \lambda_2 \Delta y_{t-1} + \epsilon_t$$

2. The file `wealth.csv` contains observations on:

- c_t = the log of total real per capita expenditures on durables, nondurables and services;
- a_t = the log of a measure of real per capita household net worth(including all financial and household wealth); and
- y_t = the log of after-tax labour income.

The data are from 1952Q2 through 2006Q2 (see Koop, G., S. Potter and R. W. Strachan (2008) "Re-examining the consumption-wealth relationship: The role of uncertainty" *Journal of Money, Credit and Banking*, Vol. 40, No. 2.3, 341-367.

(a) Draw time series plots of c_t, a_t, and y_t. Compute and plot the ACF and PACF of c_t, a_t, and y_t. Comment on your findings.

(b) Fit ARDL(p, q, m) models to the data with each component order of (p, q, m) up to 2. Use BIC for model selection. Report the best model. Hint: Install the `ardl` package and use its `ardl` command.

(c) Estimate the ECM representation of the ARDL model selected in Part (b) and report the estimated model. Hint: Use the `ec1` option with the `ardl` command.

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Solution: See the do-file `tutorial4.do`.

(a) For all these three processes, we can see that

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(1) There is an obvious (upward) trend.

(2) ACF decays very slowly.

(3) PACF has one peak very close to 1.

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These results imply that none of these three processes is stationary and probably the most suitable model for them is an ARMA model with AR coefficient ≈ 1.

(b) Based on BIC, we choose the ARDL(1,2,2) model. The estimated model can be represented as

$$\begin{aligned}\widehat{c}_t =& \underset{(0.0347)}{0.0725} + \underset{(0.0001)}{0.0002}t + \underset{(0.0284)}{0.9145}c_{t-1} + \underset{(0.0187)}{0.0558}a_t + \underset{(0.0258)}{0.0009}a_{t-1} - \underset{(0.0180)}{0.0511}a_{t-2} \\ &+ \underset{(0.0424)}{0.3589}y_t - \underset{(0.0595)}{0.1204}y_{t-1} - \underset{(0.0435)}{0.1972}y_{t-2} \\ R^2 =& 0.9998\end{aligned}$$

(c) The estimated ECM can be represented as

$$\begin{aligned}\widehat{\Delta c_t} =& \underset{(0.0347)}{0.0725} + \underset{(0.0001)}{0.0002}t - \underset{(0.0284)}{0.0855}(c_{t-1} - \underset{(0.0865)}{0.0651}a_{t-1} - \underset{(0.0943)}{0.4836}y_{t-1} \\ &+ \underset{(0.0187)}{0.0558}\Delta a_t + \underset{(0.0180)}{0.0511}\Delta a_{t-1} + \underset{(0.0424)}{0.3589}\Delta y_t - \underset{(0.0436)}{0.1972}\Delta y_{t-1} \\ R^2 =& 0.4142\end{aligned}$$