Homework

Exercise $\max_{c_1, c_2, n_1, n_2} \ln(c_1) + \alpha \ln(1-n_1) + \beta[\ln(c_2) + \alpha \ln(1-n_2)]$

$$c_1 + c_2 = n_1 w_1 + n_2 w_2$$

$$\mathcal{L} = \ln(c_1) + \alpha \ln(1-n_1) + \beta[\ln(c_2) + \alpha \ln(1-n_2)] + \lambda[n_1 w_1 + n_2 w_2 - c_1 - c_2]$$

For c_1: $\frac{1}{c_1} - \lambda = 0$ ①

c_2: $\frac{\beta}{c_2} - \lambda = 0$ ②

n_1: $\frac{\alpha}{1-n_1} = \lambda w_1$ ③

n_2: $\frac{\beta\alpha}{1-n_2} = \lambda w_2$ ④

λ: $n_1 w_1 + n_2 w_2 - c_1 - c_2 = 0$ ⑤

① → $\lambda = \frac{1}{c_1}$

② → $c_2 = \frac{\beta}{\lambda} = \beta c_1$

all sub into ⑤

③ → $n_1 = 1 - \frac{\alpha}{\lambda w_1} = 1 - \frac{\alpha}{w_1} \cdot c_1$

④ → $n_2 = 1 - \frac{\beta\alpha}{\lambda w_2} = 1 - \frac{\beta\alpha}{w_2} \cdot c_1$

intertemporal

$$\Rightarrow c_1 = \frac{w_1 + w_2}{(1+\alpha)(1+\beta)} \qquad c_2 = \beta c_1 = \frac{\beta(w_1+w_2)}{(1+\alpha)(1+\beta)}$$

↑ Life cycle consumption theory

$$n_1 = 1 - \frac{\alpha}{w_1} \cdot \frac{w_1+w_2}{(1+\alpha)(1+\beta)} \qquad n_2 = 1 - \frac{\beta\alpha}{w_2} \cdot \frac{w_1+w_2}{(1+\alpha)(1+\beta)}$$

$w_1 \uparrow \rightarrow n_1 \uparrow$, $n_2 \downarrow$ reduce leisure when relatively expensive

$w_2 \uparrow \rightarrow n_1 \downarrow$, $n_2 \uparrow$ enjoy more leisure when relatively cheaper