Topic 5: Capital and Private Debt

Monetary Economics

ECOS3010

Introduction

- So far, the only asset in our model is fiat money. In the real world, there are many other assets.
  - capital: goods that are saved for production;
  - private debt: loans that facilitate borrowing and lending;
  - and more.

- In this chapter, we will examine the interaction between money and other assets (capital and private debt).
  - A model with capital.
  - A model with private debt.
  - How can money and other assets coexist?
  - Inflation rate and interest rate.

A Model with Capital

- Consider the following production technology:
  - at time t, k_t units of consumption goods are saved as capital goods at time t;
  - at time $t+1$, xk_t units of consumption goods can be produced with k_t units of capital, x is a constant;
  - capital fully depreciates after production.

- Back to our basic OLG model, **suppose that there is no money**. Instead, technology allows the young to save in the form of capital. The old can use capital to produce consumption goods. In addition,
  - each initial old is endowed with k_0 units of capital;
  - population is growing at a constant rate n, $N_t = nN_{t-1}$.

A Model with Capital

- Suppose we focus on stationary allocations. An individual faces
  - the first-period budget constraint

$$c_1 + k \leq y;$$

  - the second-period budget constraint

$$c_2 \leq xk.$$

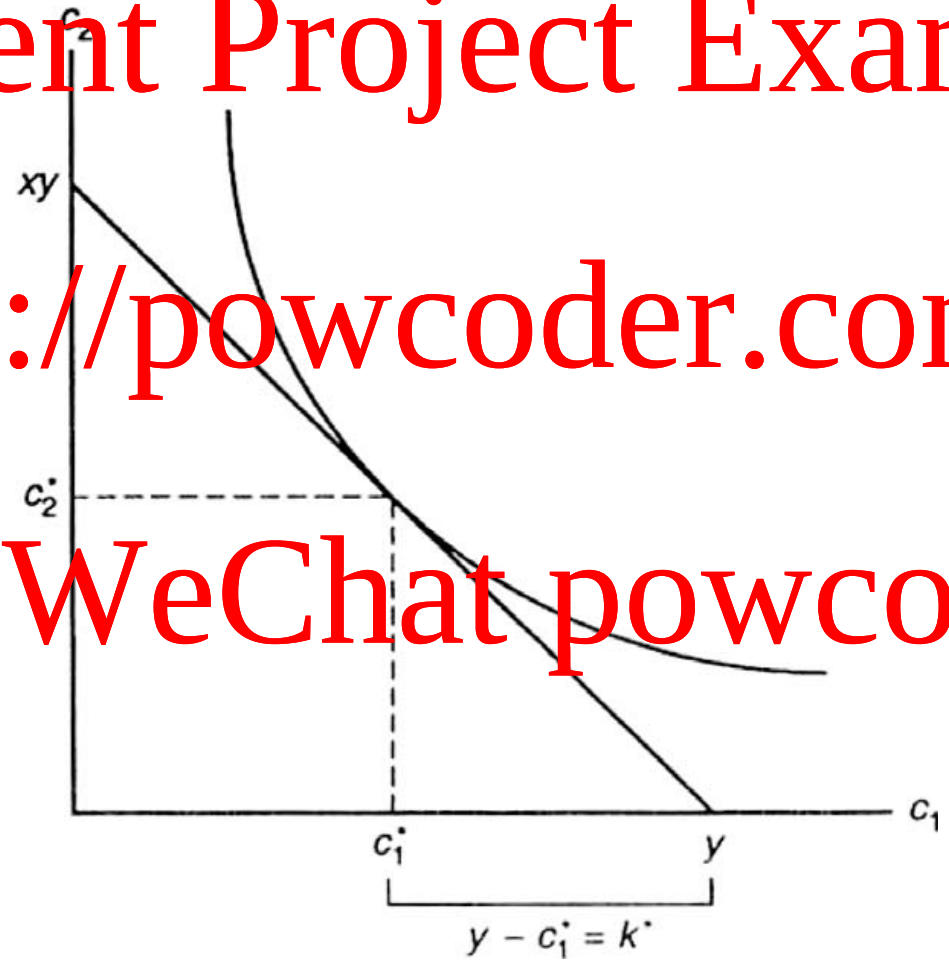
  - the lifetime budget constraint

$$c_1 + \frac{c_2}{x} \leq y,$$

combining the two period budget constraints.

A Model with Capital

- For any given x, we can depict the lifetime budget constraint.



A Model with Capital

- How much capital will an individual desire? The individual chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint. The optimal choice of k^* is

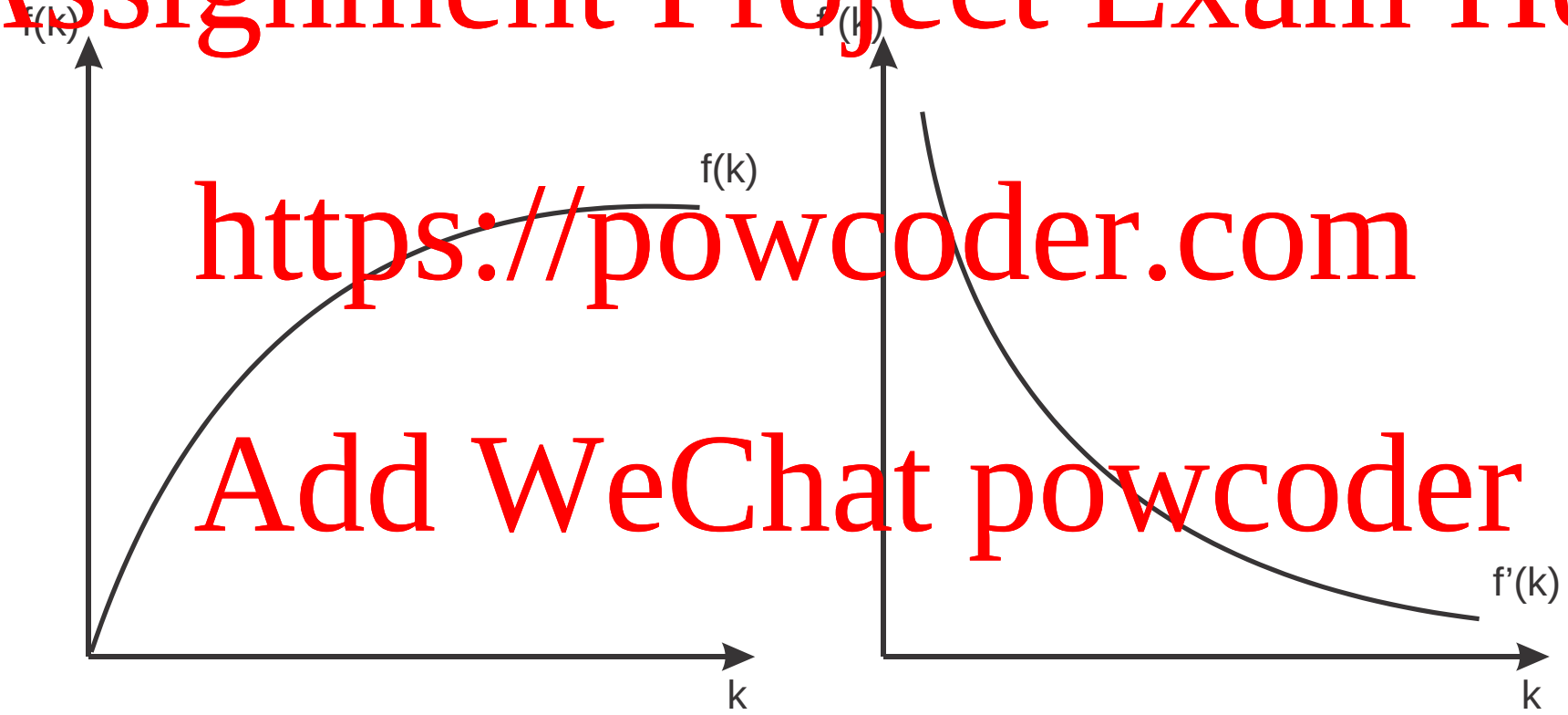
$$k^* = y - c_1^*.$$

- This simple model assumes that the marginal product of capital is a constant x. A more realistic assumption is that capital exhibits a "diminishing marginal product". Let $f(k)$ denote a general production function. In general, $f'(k) > 0$. A diminishing marginal product of capital means that

$$f''(k) < 0.$$

A Model with Capital

- An illustration of $f(k)$ with diminishing marginal product.



A Model with Private Debt

- Consider private debt as IOUs issued by individuals —private loans.
  Let there be two types of individuals:
  - borrowers: endowed with nothing when young and y units of goods when old;
  - lenders: endowed with y units of goods when young and nothing when old.

- In each generation, half of the people are borrowers and the rest half are lenders.

- To begin with, suppose that private debt is the only asset in the economy. **There is neither money nor capital.**

A Model with Private Debt

- For a lender,
  - the first period budget constraint is

$$c_{1,L} + l \leq y,$$

    where l denotes the amount of loans;
  - the second period budget constraint is

$$c_{2,L} \leq rl,$$

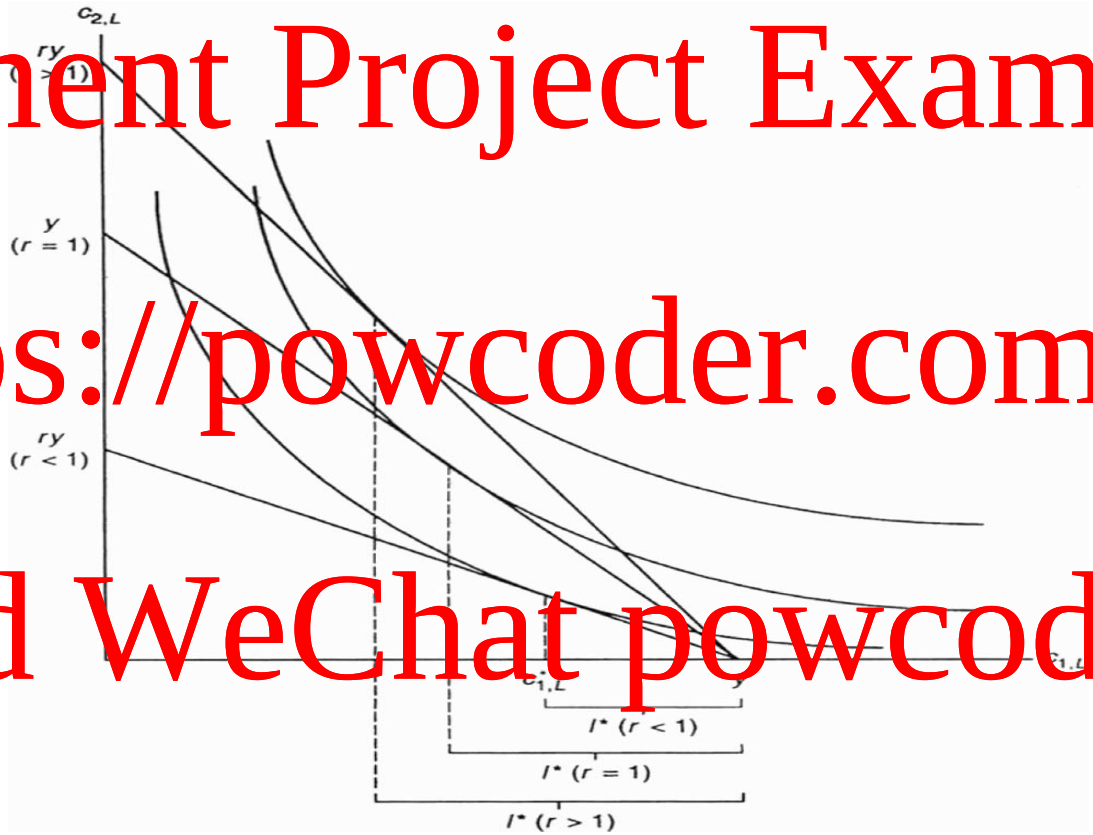
    where r is the gross real interest rate on loans;
  - the lifetime budget constraint is

$$c_{1,L} + \frac{c_{2,L}}{r} \leq y,$$

    by combining the two period budget constraints.

A Model with Private Debt

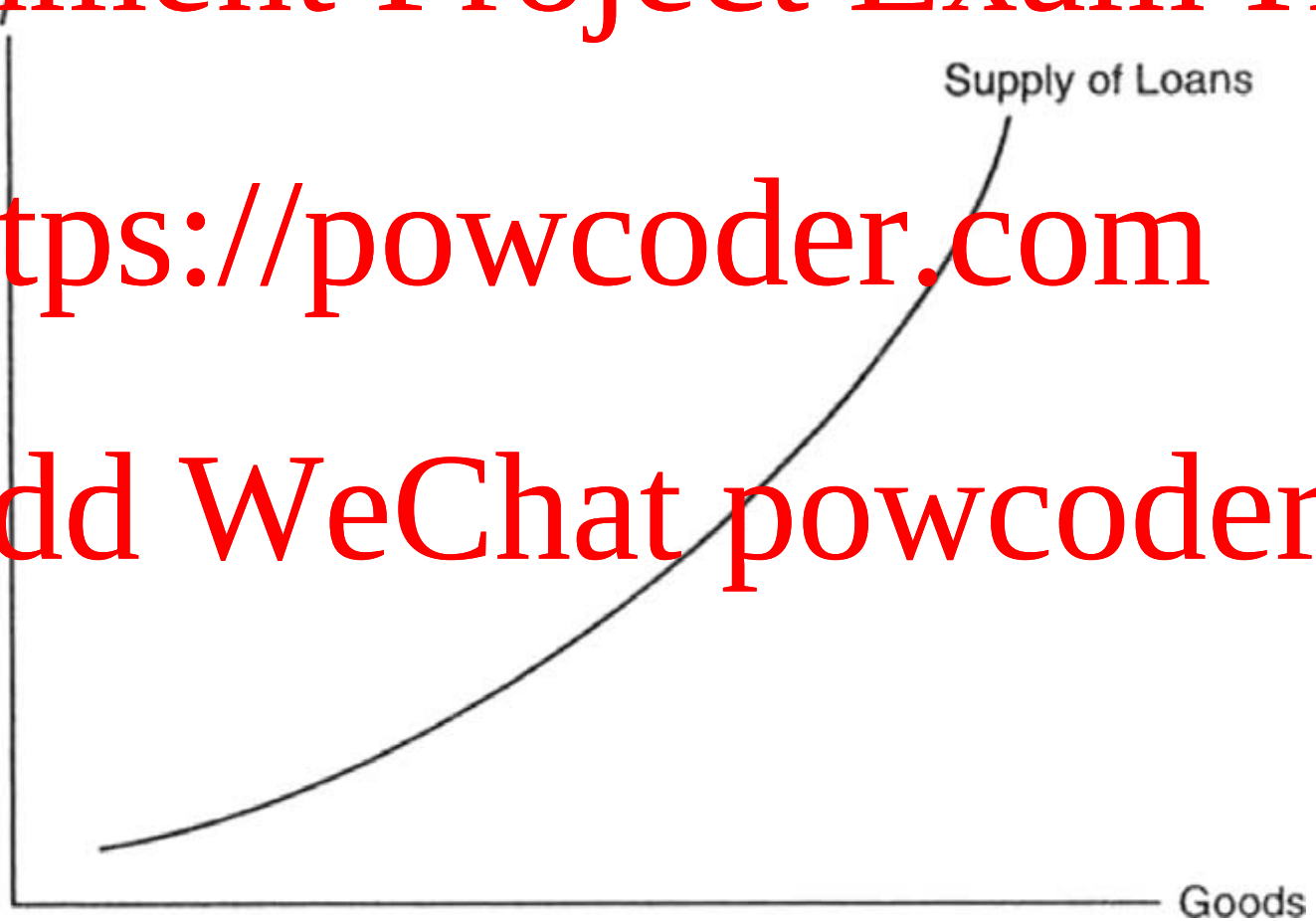
- We can depict the lender's lifetime budget constraint.



- The lender chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint.

A Model with Private Debt

- We assume that preferences are such that as r increases, $c_{1,L}$ decreases so that l increases. Let L denote the aggregate supply of loans.



A Model with Private Debt

- For a borrower,
  - the first-period budget constraint is

$$c_{1,B} \leq b,$$

  where b denotes the amount of debt (loans);
  - the second-period budget constraint is

$$c_{2,B} \leq y - rb,$$

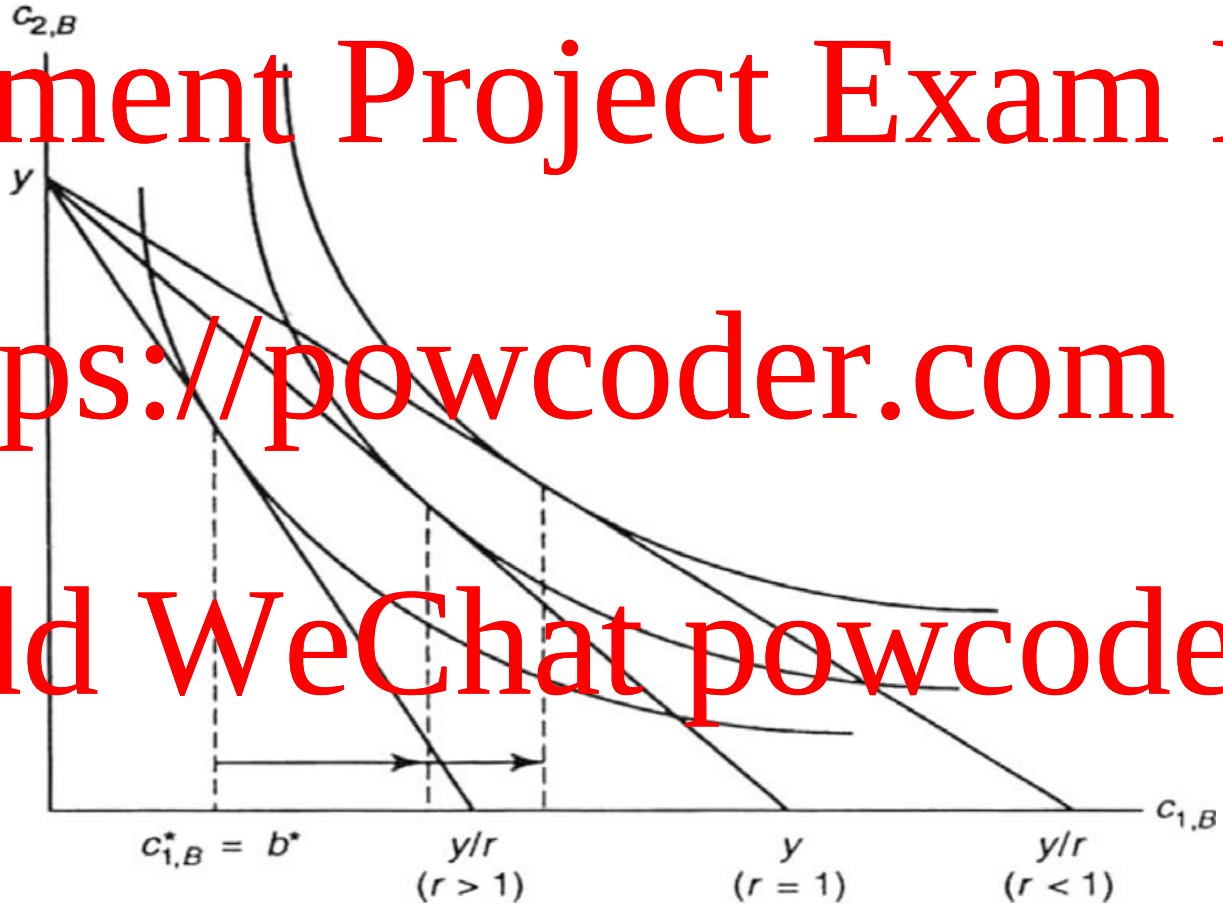
  - the lifetime budget constraint is

$$c_{1,B} + \frac{c_{2,B}}{r} \leq \frac{y}{r},$$

  by combining the two period budget constraints.

A Model with Private Debt

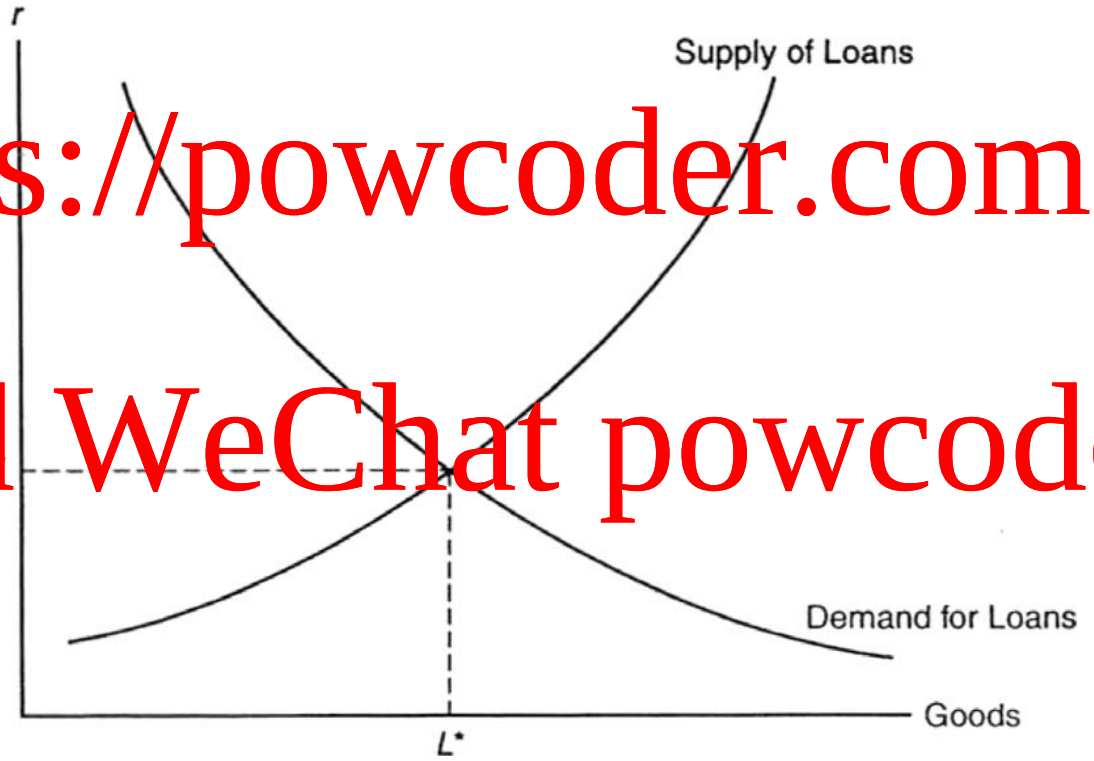
- We can depict the borrower's lifetime budget constraint.



- The borrower chooses (c_1, c_2) such that the indifference curve is tangent to the budget constraint.

A Model with Private Debt

- We assume that preferences are such that as r increases, $c_{1,B}$ decreases so that b decreases. Let B denote the aggregate demand for loans.



Rate of Return Equality

- Suppose that we introduce capital to our model with private debt.
  - The marginal product of capital is x.
  - The rate of return on loans is r^*.
- How should **lenders** choose between capital and private loans?
  - What would happen if $x > r^*$?
  - What would happen if $x < r^*$?
  - What would happen if $x = r^*$?

Rate of Return Equality

- For people to be willing to hold both capital and loans as assets, we must have

$$x = r.$$

or more generally

$$f'(k) = r$$

- Suppose that there are many assets available to individuals. Without any uncertainty about returns and any government restrictions (**perfect substitutes**) the rate of return on these assets must be identical for people to hold all available assets simultaneously.

- We refer to this as the principle of "**rate-of-return equality**".

Coexistence of Money and Other Assets

- If we introduce money into our model with capital and private debt, the rate-of-return equality requires that for all assets to be held by individuals,

$$\frac{n}{z} = r = x.$$

Here n is the growth rate of population and z is the growth rate of money supply; so n/z is the rate of return on money.

- **If all assets are perfect substitutes**, then rate-of-return equality holds for all assets to coexist. In this case, how does money interact with other assets?

  - Interaction between money and capital: the Tobin effect.

Coexistence of Money and Other Assets

The Tobin Effect

- Consider a standard OLG model with money and capital. Each young is endowed with y units of consumption goods when young and nothing when old.
- Suppose that capital displays a diminishing marginal product. That is, $f''(k) < 0$.

- For both money and capital to be valued, we must have

$$f'(k) = \frac{n}{z}$$

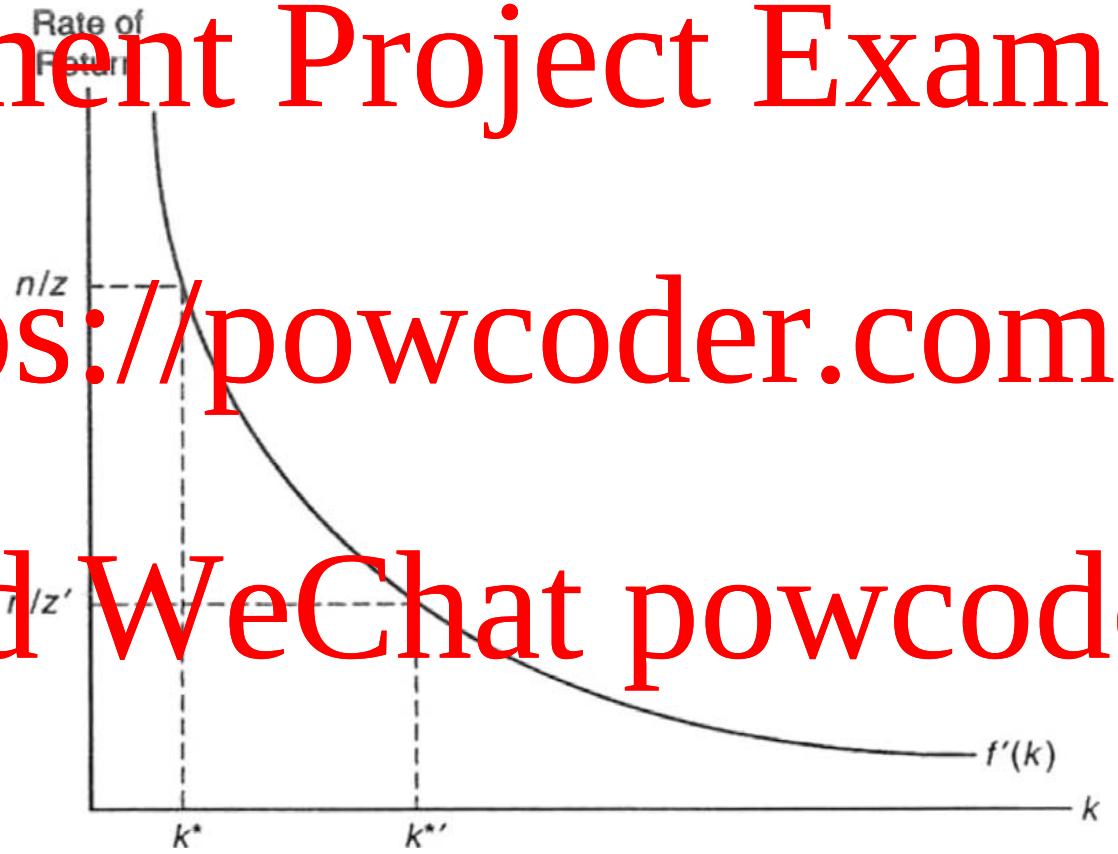
For any given (n, z), we can find a desired level of capital stock.

- What if there is a permanent increase in z?

Coexistence of Money and Other Assets

The Tobin Effect

- Graphically, we show the determination of k.



- When z increases, n/z decreases and k^* increases. (In the figure, $z' > z$.)

- The substitution of capital for money in reaction to an increase in inflation, described by Tobin (1965), is called the "**Tobin effect**".
- Does the Tobin effect suggest that an increase in z could help to increase output? In our model, the output in period t is

$$GDP_t = N_t y + N_{t-1} f\left(k_{t-1}\right).$$

If we live in a world **where money and capital are perfect substitutes**, **an increase in z would raise k and hence output**. Should the policy maker use z as a tool to raise output?

- Output $\neq$ Welfare.
- The Tobin effect is not large in the real world.

- In the real world, fiat money and other assets **are not perfect substitutes.** In particular, the rate of return on money is generally lower than the rate of return on other assets. What are the effects of anticipated inflation on interest rate, capital and output?

- This raises an obvious question: why would money still be valued? We postpone this important question for later chapters. For now, consider a simple argument: legal restriction. Each young is required by law to acquire money worth a fixed number of goods q^*.

- Nominal interest rate and real interest rate.
  - **Nominal** interest rates: the number of **dollars** paid in interest for each dollar lent.
  - **Real** interest rates: the number of **goods** paid in interest for each good lent.
  - Nominal interest rates are the ones cited by financial intermediaries and the press.
  - In times of inflation, nominal interest rates do not reflect the real rate of return.
  - Let R_t and r_t denote the nominal interest rate and the real interest rate. Let p_t denote the price of a good.

Coexistence of Money and Other Assets

When Money and Other Assets are not Perfect Substitutes

- We can express the gross real interest rate r_t as

$$r_t = \frac{R_t p_t}{p_{t+1}} = R_t \frac{p_t}{p_{t+1}}. \quad (1)$$

If we rearrange (1), we can obtain

$$\underbrace{R_t - 1}_{\text{net nominal interest rate}} = \underbrace{(r_t - 1)}_{\text{net real interest rate}} + \underbrace{\left(\frac{p_{t+}}{p_t} - 1\right)}_{\text{net inflation rate}}$$

$$+ (r_t - 1)\left(\frac{p_{t+1}}{p_t} - 1\right).$$

- For low values of the real interest rate and the inflation rate, we have approximately:

net nominal interest rate = net real interest rate + net inflation rate.

Coexistence of Money and Other Assets

When Money and Other Assets are not Perfect Substitutes

- **Anticipated inflation and the nominal interest rate**: the predicted full adjustment of the nominal interest rate to anticipated inflation is called the "*Fisher effect*" – named after Irving Fisher.
- Consider a model with money, capital and private debt. Suppose that the marginal product of capital is x. The rate-of-return equality implies that

$$x = r_t = R_t \frac{p_t}{p_{t+1}}.$$

Note that here money is held by people **because of the legal restriction**. **The rate-of-return equality applies to capital and private debt**. As the rate of return on money is $p_t / p_{t+1} = n/z$, we have

$$x = R_t \frac{n}{z} \quad \text{or} \quad R_t = x \frac{z}{n}.$$

- The real interest rate is a constant x. The nominal interest rate rises with anticipated inflation to keep the real interest rate constant x.

- Inflation and nominal interest rate in the U.S.

insert US data

- Inflation rate and nominal interest rate in Australia.

insert AU data

Coexistence of Money and Other Assets

When Money and Other Assets are not Perfect Substitutes

- There is a tendency for nominal interest rates and inflation rates to move together in accordance with the Fisher effect. However, the gap between the nominal interest rate and the inflation rate is not constant due to changes in the real interest rate.
- Will inflation affect the real interest rate? If so, how?

  - In our previous example,

$$x = r_t = R_t \frac{n}{z}.$$

    The real interest rate is always a constant x by the rate-of-return equality. An increase in inflation will only affect the nominal interest rate, but not the real interest rate. The key assumption is a constant marginal product of capital.
  - What if capital exhibits a diminishing marginal product?

Coexistence of Money and Other Assets

When Money and Other Assets are not Perfect Substitutes

- **Anticipated inflation and the real interest rate**: an exception to the Fisher effect could occur **if money, private debt, and capital are substitutes and capital has a diminishing marginal product**. In this case,
  - an increase in the inflation rate leads to an increase in capital – the Tobin effect;
  - an increase in capital leads to a decline in the real interest rate because of diminishing marginal product of capital;
  - overall, an increase in inflation will still lead to a rise in the nominal interest rate, however, because of the simultaneous decrease in the real interest rate, the nominal interest rate will not rise by the full amount of the rise in inflation.

- Inflation rate and real interest rate in the U.S.

insert US scatterplot

- Inflation rate and real interest rate in Australia.

insert AU scatterplot

Risk

- So far, we have assumed that all assets pay a rate of return that is known with complete certainty. What would happen to rate-of-return equality if instead one asset has a random rate of return?
  - For example: there is some positive probability that a loan will not be repaid by the borrower. If the marginal product of capital is a constant x, then capital and private debt are **not** perfect substitutes.
- If people do not care about risk (are "risk neutral"), then the rate-of-return equality still holds on average. Suppose that an asset pays return $r_1, r_2, \ldots, r_n$ with probabilities $\pi_1, \pi_2, \ldots, \pi_n$, respectively. The expected rate of return on this asset can be calculated as

$$E(r) = \pi_1 r_1 + \pi_2 r_2 + \cdots + \pi_n r_n.$$

The rate-of-return equality is modified to

$$E(r) = x.$$

Risk

- Are people risk neutral? Probability not.
- If people are risk averse, people may still accept a risky asset. However, the rate-of-return equality will not hold in this case. In fact, the expected rate of return on this risky asset must exceed that of the risk-free asset, compensating for the risk.
- The extra average rate of return that is necessary to entice people to hold a risky asset is called a risk premium,

$$\text{risk premium} = E\left(r_{risky}\right) - r_{safe}.$$

The greater the potential loss and the greater the probability of the loss, the larger the risk premium must be.

- In many real economies, why do people choose to hold fiat money when many alternative assets appear to offer greater rates of return?