ECOS3010: Tutorial 4 (Answer Key)

Question 1-5. Answer True, False or Uncertain. Briefly explain your answer.

1. The correlation between inflation and unemployment in the original Phillips curve is consistent with cross country evidence on the correlation between inflation and output growth.

False. The original Phillips curve finds that there is a negative correlation between inflation and unemployment, which means that higher inflation is associated with more employment and output. However, cross country evidence suggests that on average, countries with high inflation rates are associated with lower output growth. This is inconsistent with the prediction from the original Phillips curve.

2. In the Lucas model, the current price of the good is higher on the island with less young individuals.

True. On the island with less young individuals, there are less young individuals supplying labor to produce the goods. The relative less supply of the goods leads to a higher price of the goods compared with the island with more young individuals.

3. In the Lucas model, when monetary policy is not random (i.e., the growth rate of money supply is constant), the young cannot infer the true state of the world (i.e., the island where they live and the current money supply) from the prices.

False. It is true that the young individuals in the Lucas model do not know the current money supply and the distribution of the young population across the two islands. However, they are rational, which means that they will make the most correct inference from the information they have. When money supply grows at a constant rate, individuals can infer the current money supply from last period's money supply and the growth rate of money supply. Individuals can also infer whether they live on the island with less young individuals or more young individuals through the prices. They know that there should exist two prices. In addition, they know that the high price should occur on the island with less young individuals and the low price should occur on the island with more young individuals. Overall, the young can infer the true state of the world from the information that they have when monetary policy is not random.

4. Money is superneutral in the Lucas model.

False. In the Lucas model, when the growth rate of money supply increases, the rate of return to labor decreases. As a result, labor supply will decrease and output will decrease. The increase in the growth rate of money supply affects real variables such as labor supply and output in the model. When the change in the growth rate of money supply affects real economic variables in a model, we say that money is not superneutral.

5. In the Lucas model, when monetary policy is random, the young cannot infer the true state of the world from the prices.

True. When the growth rate of money supply is random, the young can no longer infer the current money supply and the distribution of young individuals from the prices. In the model we showed in class, there are potentially four prices. Two of the prices are unique and individuals can infer the true state from these two prices. However, the other two prices are the same. It means that individuals cannot tell whether they are on the island with less young individuals and a lower growth rate of money supply or they are on the island with more young individuals and a higher growth rate of money supply. Overall, the young cannot infer the true state of the world from the information they have when monetary policy is random.

6. Consider the following version of the Lucas model. The number of young individuals

born on island i in period t, N_t^i is random according to the following specification:

$$\begin{aligned} N_t^i &= \frac{4}{5}N \quad \text{with probability 0.5} \\ &= \frac{1}{5}N \quad \text{with probability 0.5.} \end{aligned}$$

Assume that the money supply grows at the constant rate $z_t = z$ in all periods.

(a) Set up the budget constraints of the individuals when young and when old. Also set up the government budget constraint and money market clearing condition. Find the lifetime budget constraint (combine the budget constraints of the young and old).

The first-period budget constraint for an individual who is born in period t and on island i is

$$c_{1,t}^i + l_t^i \leq y \quad \text{or} \quad c_{1,t}^i + v_t^i m_t^i \leq y.$$

The second-period budget constraint for the same individual who is born in period t and on island j in period $t+1$ is

$$c_{2,t+1}^{i,j} \leq v_{t+1}^j m_t^i + a_{t+1}.$$

Combine the first- and second-period budget constraints to get the lifetime budget constraint:

$$c_{1,t}^i + \frac{v_t^i}{v_{t+1}^j} c_{2,t+1}^{i,j} \leq y + \frac{v_t^i}{v_{t+1}^j} a_{t+1}$$

or

$$c_{1,t}^i + \frac{p_{t+1}^j}{p_t^i} c_{2,t+1}^{i,j} \leq y + \frac{p_{t+1}^j}{p_t^i} a_{t+1}.$$

The government budget constraint is

$$M_{t-1} a_t = v_t (M_t - M_{t-1}) = \left(1 - \frac{1}{z_t}\right) v_t M_t$$

Money market clearing condition on island i is

$$N_t^i \left(y - c_{1,t}^i\right) = v_t^i \frac{M_t}{2} \quad \text{or} \quad N_t^i l_t^i = v_t^i \frac{M_t}{2}.$$

(b) On which island would you prefer to be born? Explain with reference to the rate of return to labor.

You would want to be born on the island with the greatest rate of return to labor. In this model, the rate of return to labor is

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j}.$$

Since the price of goods in the next period p_{t+1}^j is independent of the price of goods this period, the greater the price this period, the greater the rate of return to labor. If we label the island with less young individuals as island A and the island with more young individuals as island B. We now need to compare p_t^A with p_t^B. From the money market clearing condition,

$$p_t^i = \frac{1}{v_t^i} = \frac{M_t/2}{N_t^i l_t^i}.$$

It follows that

$$p_t^A = \frac{M_t/2}{\frac{1}{5}Nl_t^A},$$
$$p_t^B = \frac{M_t/2}{\frac{4}{5}Nl_t^i}.$$

Given the assumption that the substitution effect dominates the income effect, we can show that $p_t^A > p_t^B$. (Following similar steps as we did in class, you can prove this by contradiction.) This implies that the rate of return to labor will be higher on island A. In general, individuals would want to be born on the island with the smaller population.

(c) Show how the rate of return to labor and the individual's labor supply depend on the value of z.

Here the growth rate of money supply is constant. The rate of return to labor is

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j} = \frac{\frac{M_t/2}{N_t^i l_t^i}}{\frac{M_{t+1}/2}{N_{t+1}^j l_{t+1}^j}} = \frac{N_{t+1}^j l_{t+1}^j}{N_t^i l_t^i}\frac{M_t}{M_{t+1}} = \frac{N_{t+1}^j l_{t+1}^j}{N_t^i l_t^i}\frac{1}{z}.$$

We can clearly see from this equation that as the value of z increases, the rate of return to labor falls. Such a change will generate a decline in labor supply and hence lower output.

For the following parts, assume that the growth rate of money supply z_t is random according to

$$z_t = 1 \quad \text{with probability } \theta$$
$$= 4 \quad \text{with probability } 1-\theta.$$

The realization of z_t is kept secret from the young until all purchases of goods have occurred (i.e., individuals do not learn M_t until period t is over). Given these changes in assumption, answer the following questions:

(d) How many states of the world would individuals be able to observe if information about every variable were perfectly available? Describe those possible states.

If agents could perfectly observe information about every variable, there would be four states of the world, corresponding to four combinations of N_t^i and z_t. See Table 1 for the prices corresponding to the four states.

	$\frac{4}{5}N$	$\frac{1}{5}N$
$z_t = 1$	$p_t^a = \frac{M_{t-1}/2}{\frac{4}{5}Nl_t^a}$	$p_t^b = \frac{M_{t-1}/2}{\frac{1}{5}Nl_t^b}$
$z_t = 4$	$p_t^c = \frac{4M_{t-1}/2}{\frac{4}{5}Nl_t^c}$	$p_t^d = \frac{4M_{t-1}/2}{\frac{1}{5}Nl_t^d}$

Table 1: Four Possible Prices

(e) How many states of the world are the individuals able to distinguish when there is limited information (i.e., they do not know the value of z_t)?

If individuals do not observe z_t and N^i, two out of four prices would be unique so that individuals can still distinguish. The rest two prices will look the same to individuals. In Table 1, $p_t^a < p_t^b = p_t^c < p_t^d$. If individuals observe p_t^a or p_t^d, they will be able to infer the current z_t and N^i. However, p_t^b and p_t^c are the same. Individuals will not be able to

distinguish between state b where $N^i = N/5$ and $z_t = 1$ versus state c where $N^i = 4N/5$ and $z_t = 4$.

(f) Draw a graph of labor supply and the growth rate of money supply in each possible state of the world when there is limited information. What is the correlation observed between money creation and output?

Following part (e), we find that there will be three possible values for labor supply. If the price p_t^a is observed, individuals know that they are on the island with more young individuals and a lower growth rate of money. The low current price leads to a lower level of labor supply l_t^a. If the price p_t^d is observed, individuals know that they are on the island with less young individuals and a higher growth rate of money supply. The high current price leads to a higher level of labor supply l_t^d. Individuals cannot distinguish between state b and state c. When they observe p_t^b or p_t^c, they are not sure whether they are on the island with less young individuals and a lower growth rate of money supply or they are on the island with more young individuals and a higher growth rate of money supply. Each individual will supply an intermediate level of labor l^*. Overall, $l_t^a < l^* < l_t^d$. See Figure 1 for an illustration. There appears to be a positive correlation between money creation and output.

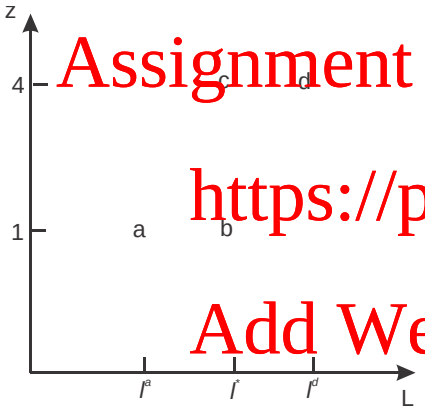

Figure 1: Labor Supply and Growth Rate of Money Supply

(g) Suppose the government wanted to take advantage of the relation between money creation and output. If it always inflate ($\theta = 0$), will the graph you derived in part (f) remain the same. Explain your answer.

If the government always inflate, there will be no randomness to inflation. Individuals will no longer be confused about the world. Only state c and d will exist. They know that $z = 4$. When individuals observe a high price, they know they are on an island with less young individuals. When individuals observe a low price, they know they are on an island with more young individuals. Compared with the level of output at $z = 1$, output produced at $z = 4$ is lower because the higher growth rate of money supply lowers the rate of return to labor and hence labor supply and output fall. The positive correlation between money creation and output disappears when the government tries to exploit it.