Topic 1: A Simple Model of Money

Monetary Economics

ECOS3010

Introduction

- Money has always been important to people and to the economy.
- Money has a long history.
  - Commodity money: shells, beads, cigarettes, silver, gold, ...
  - Fiat money: paper currency $\longrightarrow$ intrinsically useless
  - Emoney: debit card, smart card, ecash,.....
- Are we headed for a cashless society?

Introduction

- Why do we need money?
  - Lack of double coincidence of wants.
  - Lack of record-keeping or credit: money is MEMORY!
- Functions of money:
  - a medium of exchange: primary function;
  - a unit of account;
  - a store of value.
- A suitable framework to study issues related to money: the overlapping generations (OLG) model.
  - highly tractable;
  - an elegant way to introduce money;
  - dynamic.

Environment of the Model

- The economy begins in period 1 and runs forever: $t = 1, 2, ..., \infty$.
- Individuals live for two periods: *young* and *old*.
- In period t, N_t individuals are born. In the first period, N_0 initial old.
- In each period t, N_t young individuals and N_{t-1} old individuals.
- One non-storable good. A very important assumption! Why?
- Each individual receives y units of goods when young and nothing when old.

Environment of the Model

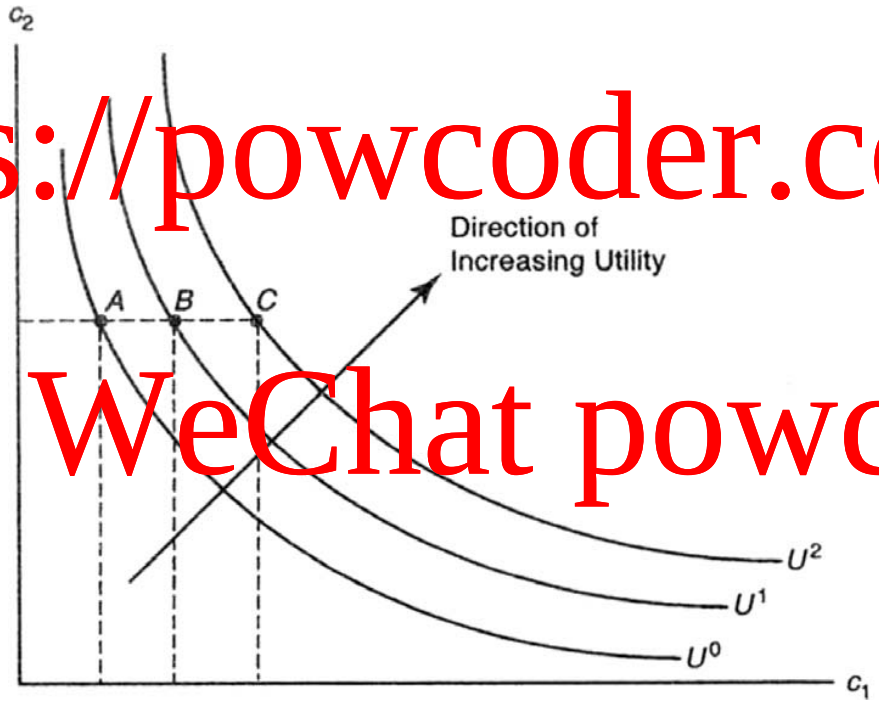
- Here is a summary of the model so far:

Generation	$t=1$	$t=2$	$t=3$	$t=4$	$\rightarrow$
0	0				
1	y	0			
2		y	0		
3			y	0	
$\downarrow$				...	...

- Preferences:
  - Individuals of all future generations value consumption both when young and when old.
  - Initial old value consumption only when old.

Environment of the Model

- Three assumptions about an individual's utility.
  1. Utility is increasing with the consumption.



Environment of the Model

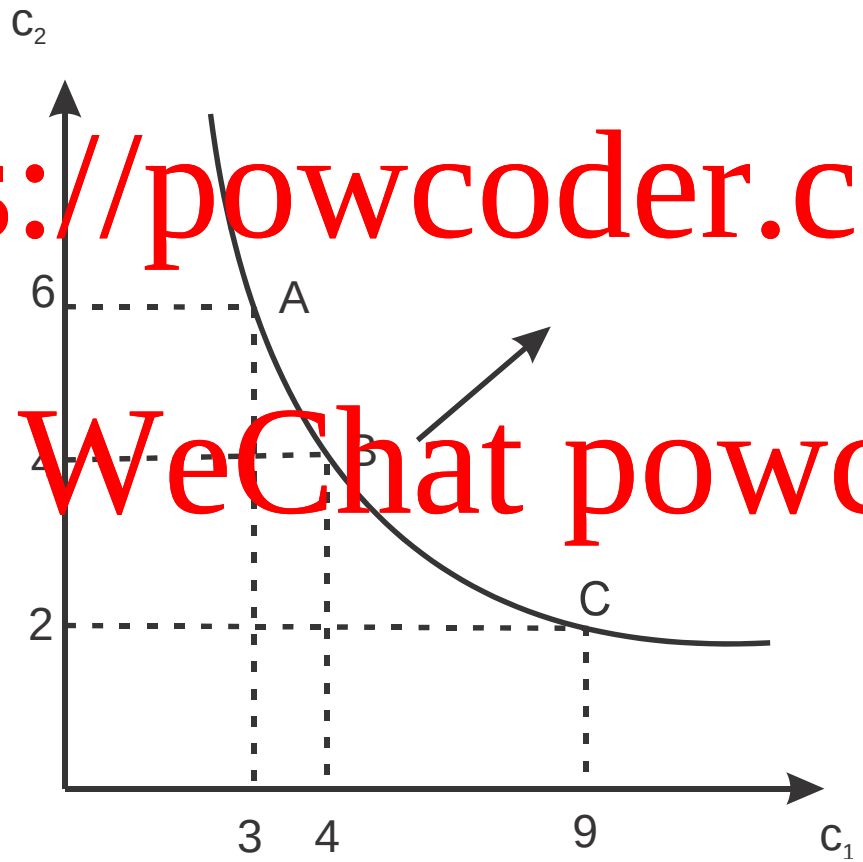
- Three assumptions about an individual's utility.

2. Individuals value some consumption in both periods of life: the indifference curves never cross either axis.

Environment of the Model

- Three assumptions about an individual's utility.

3. Diminishing marginal rate of substitution.
   → a typical indifference curve



Centralized Solution: the Golden Rule Allocation

- Suppose there is a central planner who can allocate the available goods among the young and the old in each period.
- Let $(c_{1,t}, c_{2,t+1})$ denote the consumption bundle by individuals born in period t.

- Resource constraint in period t is

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y.$$

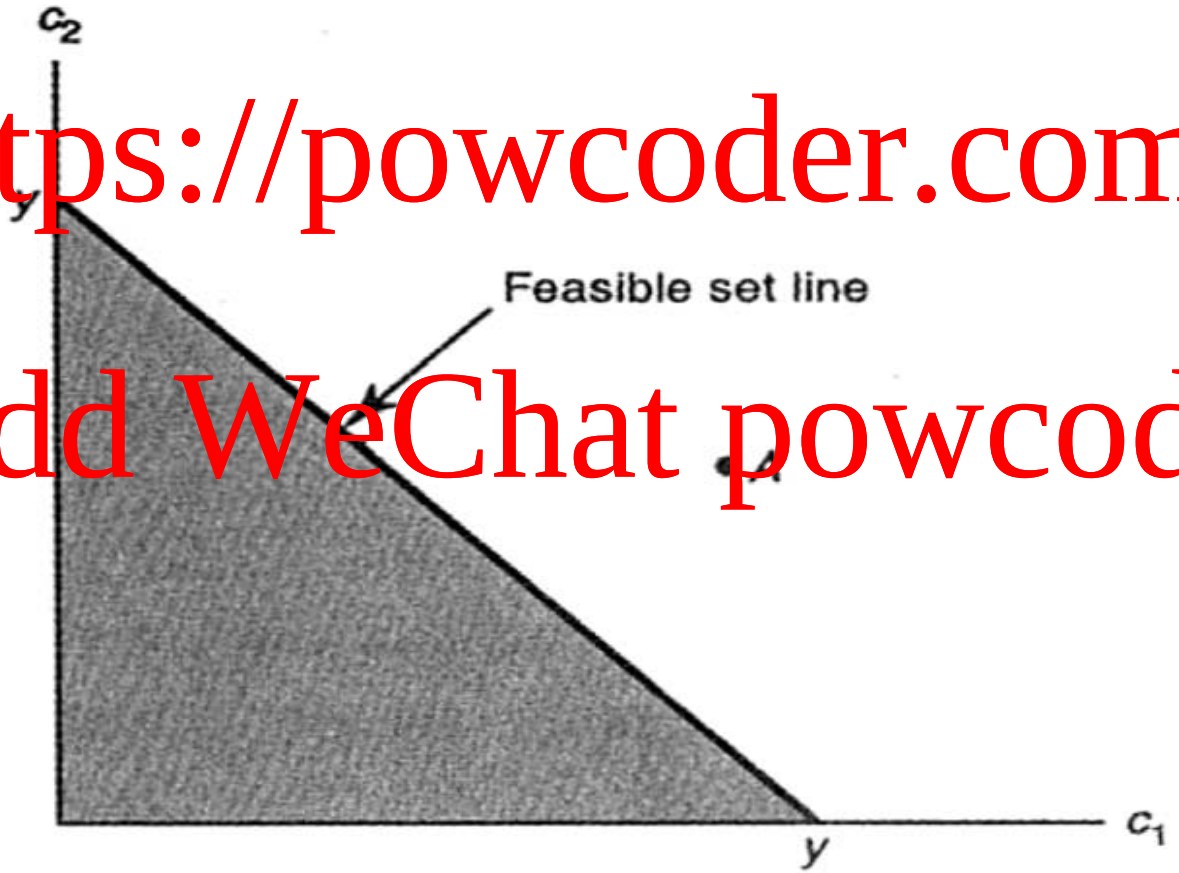
- Suppose for now that for all t,
  - the population is constant: $N_t = N$;
  - we focus on stationary allocations where $c_{1,t} = c_1$ and $c_{2,t} = c_2$. Note: does not necessarily imply $c_1 = c_2$.

Centralized Solution: the Golden Rule Allocation

- Resource constraint simplifies to

$$c_1 + c_2 \leq y.$$

- Graphically, the feasible set is



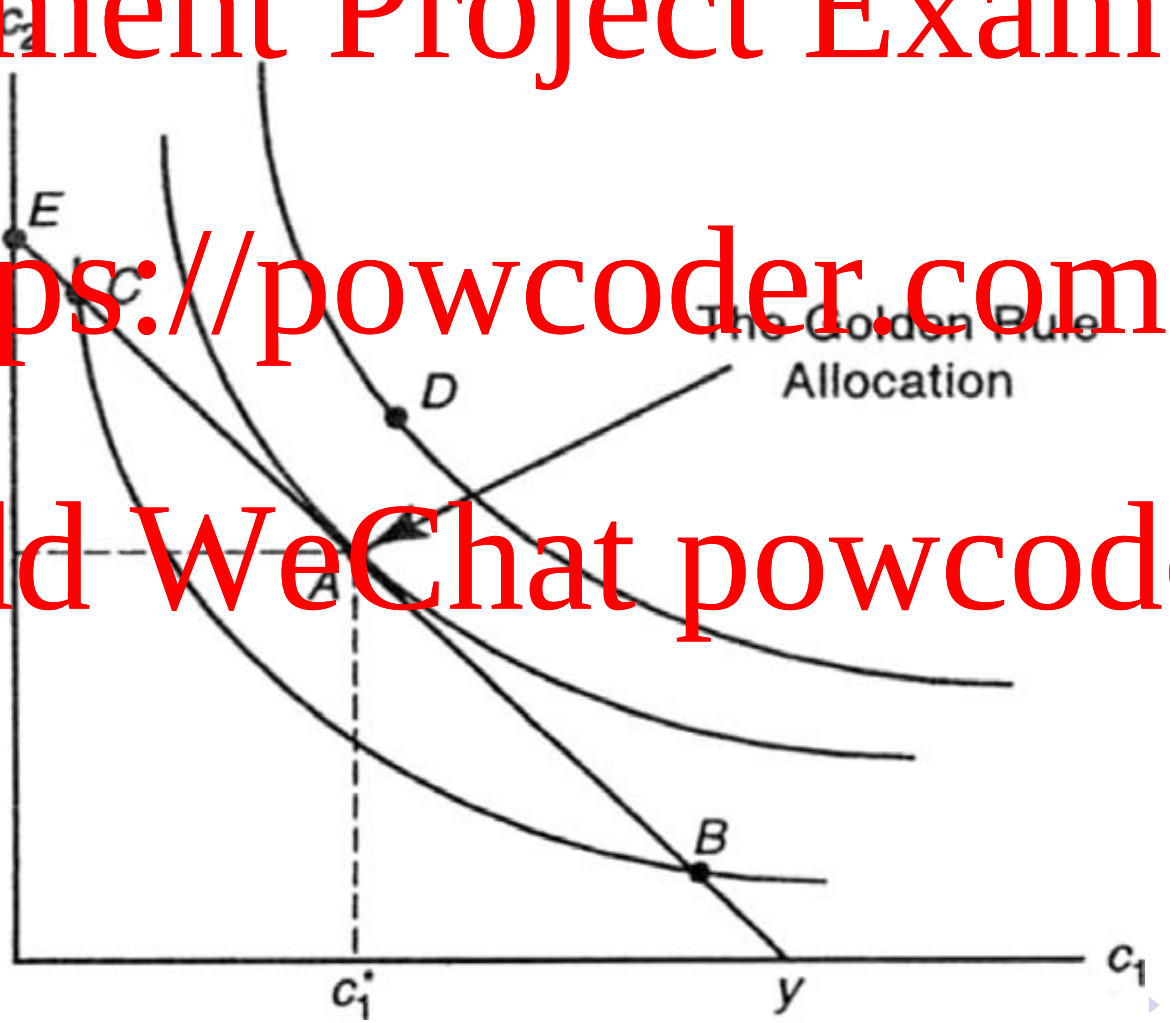
Centralized Solution: the Golden Rule Allocation

- Within the feasible set, which allocation would the planner choose? The combination of (c_1, c_2) that maximizes an individual's utility.
- The golden rule allocation is the allocation within the feasible set that maximizes the utility of future generations. It occurs at the unique point of tangency between the feasible set line and an indifference curve

- Does the golden rule allocation maximize the utility of the initial old?

Centralized Solution: the Golden Rule Allocation

- Point A: the golden rule allocation; Point E: max utility of the initial old.



Decentralized Solutions: a Competitive Equilibrium without Money

- To achieve the golden rule allocation, the planner needs to redistribute c_2^* units of goods from each young to each old in every period.
- Strong assumptions about the power of central planners.

- Can we achieve the golden rule allocation without a planner?
- When individuals trade among themselves $\longrightarrow$ a competitive equilibrium.

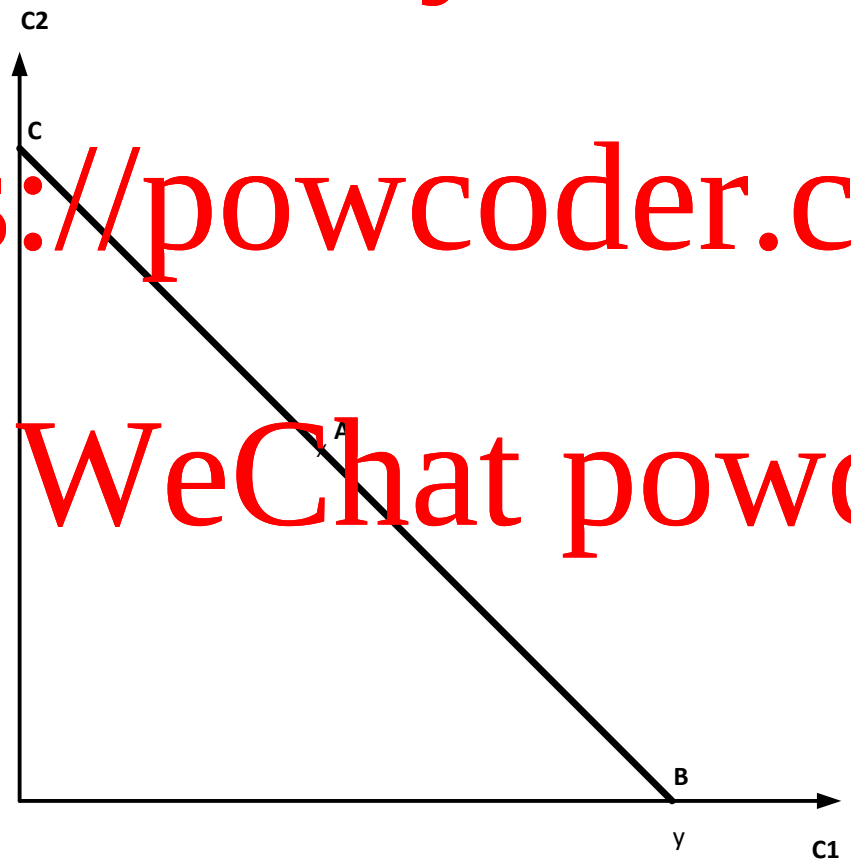
  - Individuals maximize their own utilities.
  - Individuals are price takers.
  - Markets clear.

Decentralized Solutions: a Competitive Equilibrium without Money

- What is the competitive equilibrium allocation?
- No trade can occur in this economy $\longrightarrow$ autarkic allocation: individuals have no economic interaction with others.
  - Lack of double coincidence of wants: the old would like to have some goods from the young, but they have nothing that the young want.
  - No record-keeping or credit.
- Each individual's consumption: $c_1 = y_1$, $c_2 = 0$. (Goods are non-storable!)
- Utility is low: both the future generations and the initial old are **worse off** than **almost** any other feasible consumption bundle.

Decentralized Solutions: a Competitive Equilibrium without Money

- How can we improve the economy?



Decentralized Solutions: a Monetary Equilibrium

- How can the economy achieve a better allocation than the autarkic allocation?
- One way to allow some trading opportunities is to introduce **money**.
- Fiat money:
  - produced by the government (almost) costlessly;
  - cannot be counterfeited;
  - portable;
  - storable.

Decentralized Solutions: a Monetary Equilibrium

- **A monetary equilibrium** is a competitive equilibrium in which there is a **valued** supply of fiat money. That is, the fiat money can be traded for consumption good.

- For fiat money to have value, 2 conditions must be satisfied:
  - supply of money must be limited.
  - impossible (or very costly) to counterfeit.

Decentralized Solutions: a Monetary Equilibrium

Demand for Money

- There is a fixed stock of money: M units.
- Each of the initial old is endowed with M/N_0 units money.
- Are there potential trade opportunities?
  - At $t = 1$, the initial old have money and the young (newborn) have goods. Would they trade? Yes.
  - At $t = 2$, the old (who were young at $t = 1$) have money and the young (newborn) have goods. Would they trade? Yes.
  - At $t = 3, 4, \ldots\ldots$, the old in each period always have some money and the young always have goods.
  - Now each individual can consume in both periods of life.

Decentralized Solutions: a Monetary Equilibrium

Demand for Money

- Consider an individual who is born at time t.
  - $c_{1,t}$: consumption when young;
  - $c_{2,t+1}$: consumption when old;
  - m_t : the number of dollars acquired when young (by giving up some of the endowed consumption good);
  - v_t : the value of money, which implies the price level $p_t = 1/v_t$.

- The individual's budget constraint in the first period of life

$$c_{1,t} + v_t m_t \leq y$$

- The individual's budget constraint in the second period of life

$$c_{2,t+1} \leq v_{t+1} m_t$$

- The individual's life-time budget constraint

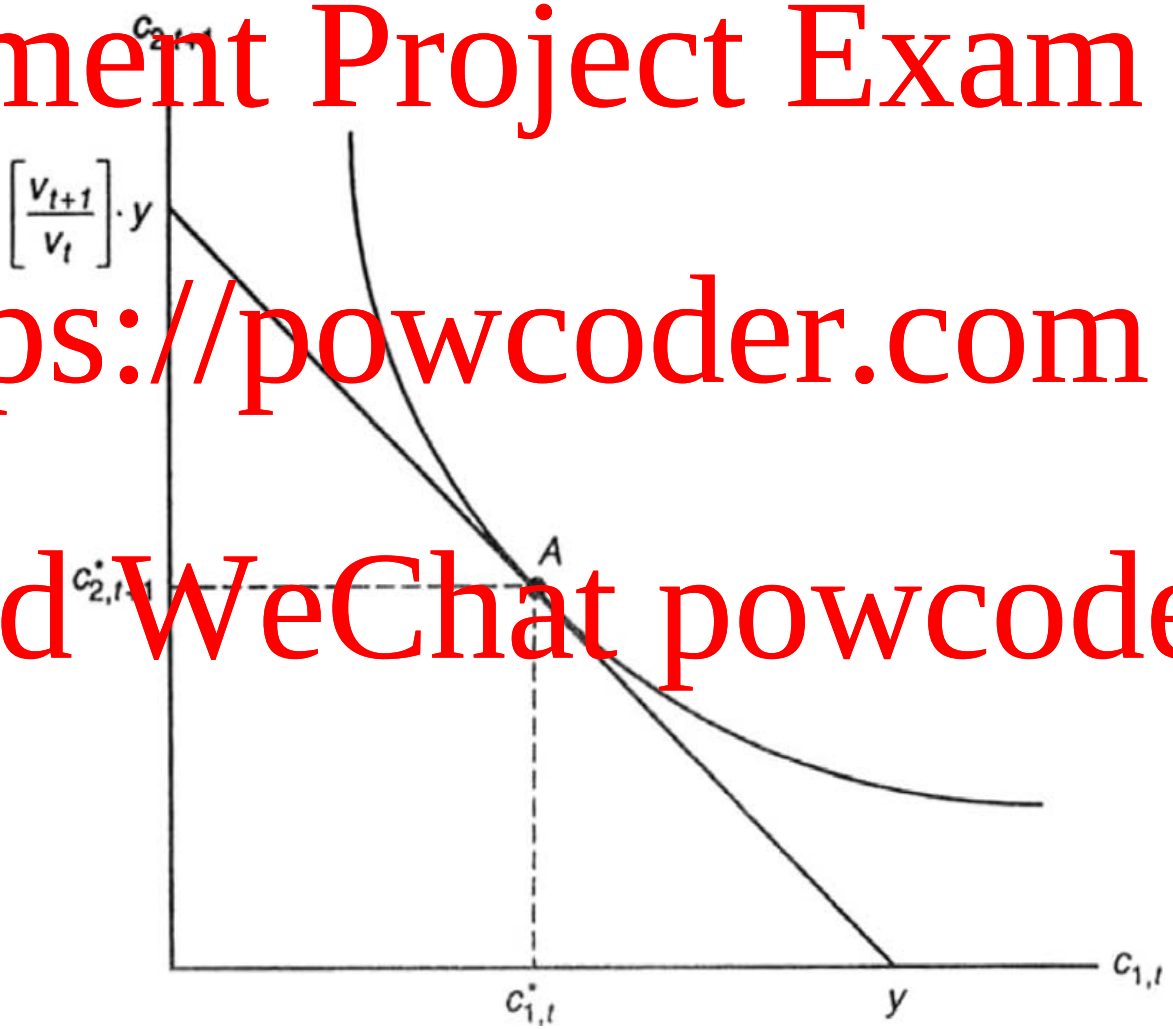
$$c_{1,t} + \left(\frac{v_t}{v_{t+1}}\right) c_{2,t+1} \leq y.$$

  - v_{t+1}/v_t : the real return of money

Decentralized Solutions: a Monetary Equilibrium

Demand for Money

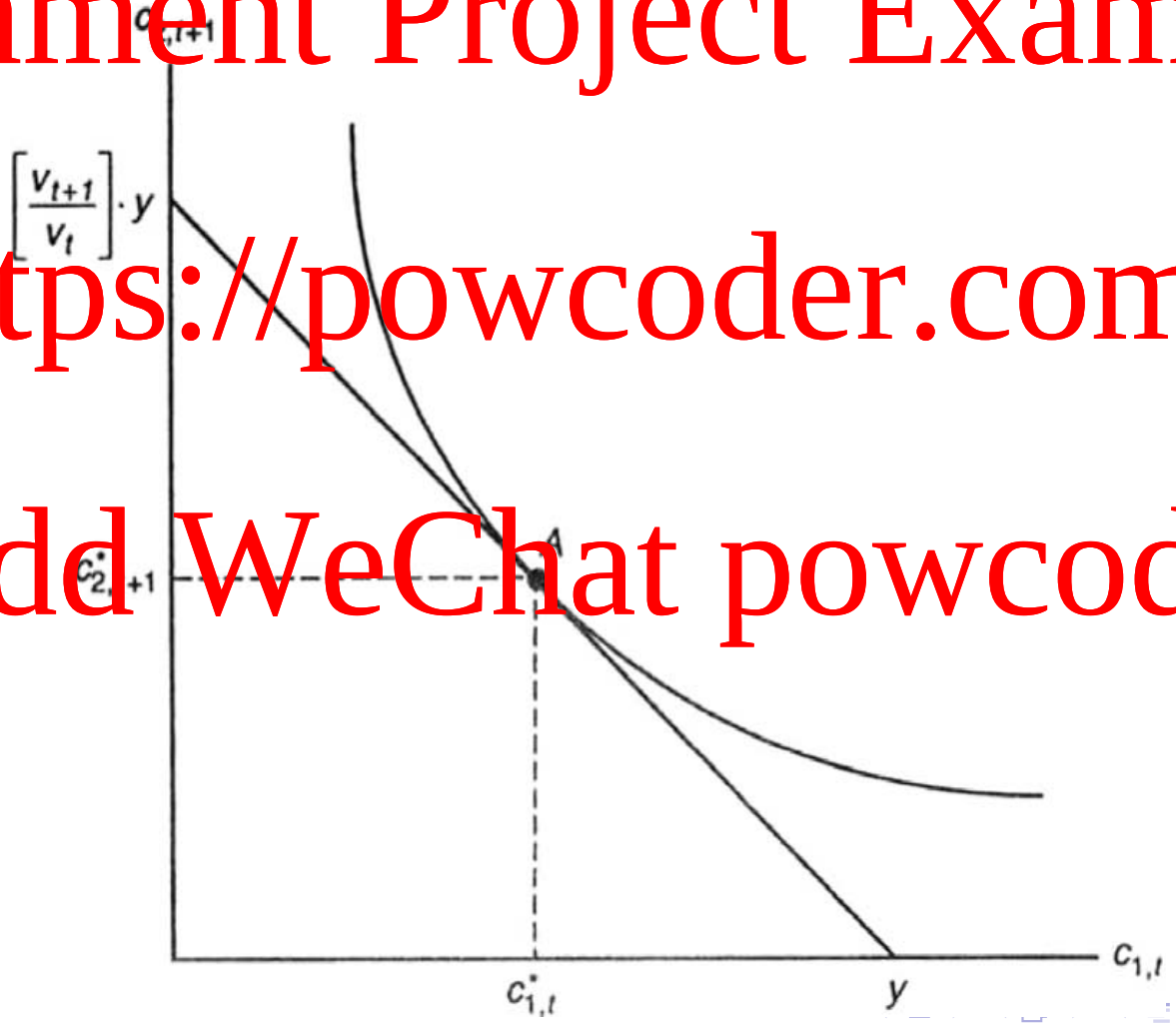
- Graphically, we depict the budget set:



Decentralized Solutions: a Monetary Equilibrium

Demand for Money

- Within the budget set, point A maximizes an individual's utility.



Decentralized Solutions: a Monetary Equilibrium

A Monetary Equilibrium

- It remains to find v_{t+1}/v_t. Recall that in any competitive market, the price (or value) of an object is determined as the price at which the supply of the object equals its demand.
  - demand for money at time t

$$N_t (y - c_{1,t});$$

  - supply of money at time t

$$v_t M_t;$$

  - v_t is determined through

$$N_t (y - c_{1,t}) = v_t M_t \;\rightarrow\; v_t = \frac{N_t (y - c_{1,t})}{M_t}.$$

Decentralized Solutions: a Monetary Equilibrium

A Monetary Equilibrium

- From v_t and v_{t+1}, we can find

$$\frac{v_{t+1}}{v_t} = \quad .$$

- Let's further simplify our economy: suppose we focus on
  - stationary allocations where $c_{1,t} = c_1$ and $c_{2,t+1} = c_2$;
  - a constant population where $N_t = N$;
  - a constant money supply where $M_t = M$.

  for all t.

- Now, we have

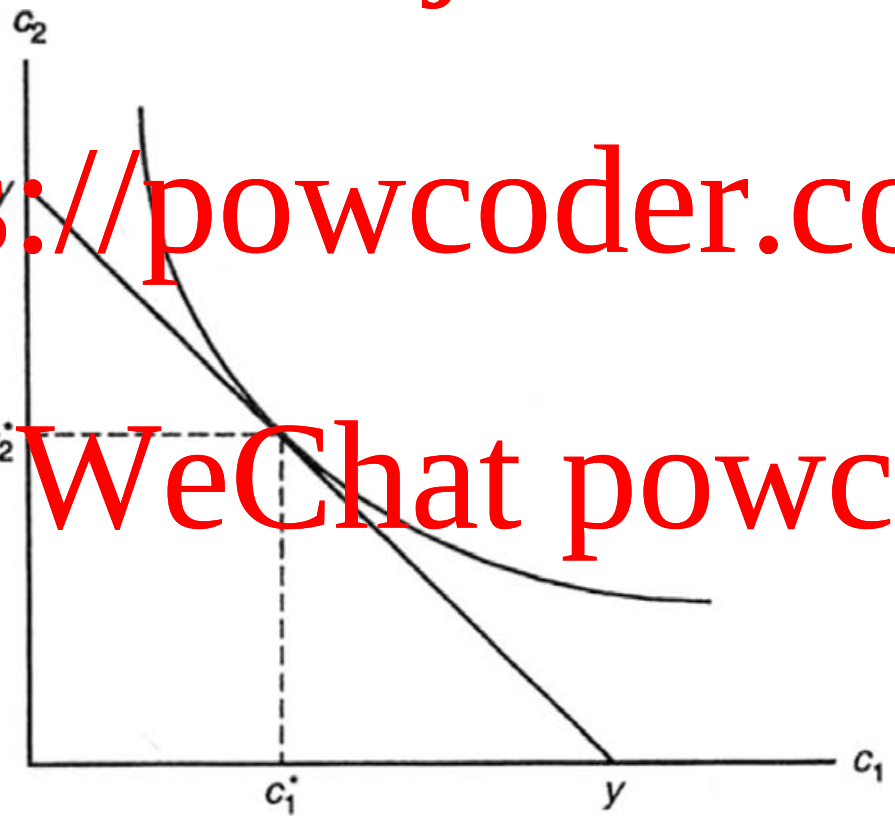
$$\frac{v_{t+1}}{v_t} = 1 \text{ or } v_{t+1} = v_t.$$

  The value of money is constant.

Decentralized Solutions: a Monetary Equilibrium

A Monetary Equilibrium

- We update the graph that depicts monetary equilibrium.



- Quantity theory of money: the price level is proportional to the quantity of money in the economy. In our economy, the price level is

$$p = \frac{1}{v} = \frac{M}{N(y - c_1)}.$$

- Neutrality of money: the nominal size (measured in dollars) of the stock of money M has no effect on the real (measured in goods) values of consumption (c_1, c_2) and real money demand $y - c_1$.

Decentralized Solutions: a Monetary Equilibrium

A Monetary Equilibrium

- Is monetary equilibrium the golden rule?

golden rule	monetary equilibrium
max utility	max utility
subject to the *resource constraint*	subject to the *budget constraint*
resource constraint: $c_1 + c_2 \leq y$	budget constraint: $c_1 + c_2 \leq y$
$\rightarrow$ golden rule = monetary equilibrium	

- Compared to competitive equilibrium without money, the introduction of money allows all future generations to achieve the golden rule allocation. It also benefits the initial old, whose consumption increases from 0 to c_2^*.

An Example

- Suppose that $u(c_1, c_2) = c_1 c_2$. Consider the economy with a constant population and a constant money supply.
- Golden rule allocation

$$\max_{c_1, c_2} \; c_1 c_2 \quad \text{subject to} \quad c_1 + c_2 = y.$$

Substitute c_2 with $c_2 = y - c_1$, it is equivalent to

$$\max_{c_1} \; c_1 (y - c_1).$$

$$\rightarrow$$

$$\text{FOC: } \; y - c_1 + (-c_1) = 0$$

$$\rightarrow$$

$$c_1 = \frac{y}{2} \text{ and } c_2 = \frac{y}{2}$$

An Example

- A Monetary equilibrium

$$\max_{c_1, c_2} \; c_1 c_2 \quad \text{subject to} \quad c_1 + \frac{v_t}{v_{t+1}} c_2 = y.$$

We know that $v_t = v_{t+1}$ so that $v_t / v_{t+1} = 1$. Substitute c_2 with $c_2 = y - c_1$, it is equivalent to

$$\max_{c_1} \; c_1 (y - c_1).$$

$$\rightarrow$$

$$\text{FOC: } \; y - c_1 + (-c_1) = 0$$

$$\rightarrow$$

$$c_1 = \frac{y}{2} \text{ and } c_2 = \frac{y}{2}$$

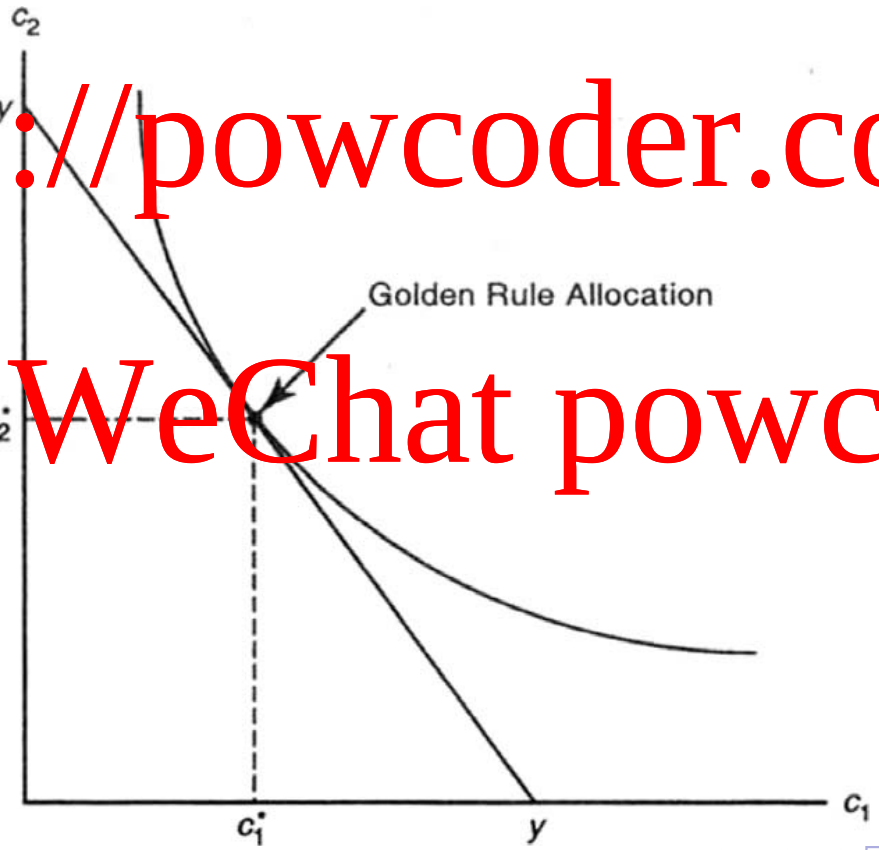
A Growing Economy

- So far we have learned that the introduction of money opens up trade opportunities and monetary equilibrium coincides with the golden rule allocation. We have assumed **a constant money supply** and **a constant population**.
- What if we have a growing population? Suppose that $N_t = nN_{t-1}$ where $n > 1$. How does a growing population affect the golden rule allocation and the monetary equilibrium?

A Growing Economy

- Golden rule allocation: the planner maximizes an individual's utility subject to the resource constraint

$$N_t c_1 + N_{t-1} c_2 \leq N_t y \rightarrow c_1 + \frac{1}{n} c_2 \leq y.$$



A Growing Economy

- A Monetary equilibrium: the individual maximizes his own utility subject to the life-time budget constraint

$$c_1 + \left(\frac{i_t}{v_{t+1}}\right) c_2 \leq y.$$

Recall that

$$v_t = \frac{M}{N_t(y - c_1)} \rightarrow \frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = n$$

When the population is growing, the value of money is also growing at the same speed. Therefore, the budget constraint simplifies to

$$c_1 + \frac{1}{n} c_2 \leq y.$$

- Notice that the budget constraint is identical to the resource constraint. So the monetary equilibrium coincides with the golden rule allocation. Again, the introduction of money helps the economy achieve the best possible allocation!