ECOS3010: Tutorial 2 (Answer Key)

Question 1-5. Answer True, False or Uncertain. Briefly explain your answer.

1. According to RBA's definition of monetary aggregates, M1 includes more assets than M3 does.

False. According to RBA's definition of monetary aggregates, M3 includes M1 plus all other deposits in the economy. So M3 includes more assets than M1 does.

2. Suppose that the government increases money supply and gives new money to the old in every period. Comparing monetary equilibrium with the golden rule allocation, we find that all future generations achieve a lower level of utility but the initial old enjoys a higher level of utility in the monetary equilibrium.

False. When the government increases money supply and gives new money to the old in every period, the allocation in the monetary equilibrium is not the golden rule allocation. In particular, all future generations in the monetary equilibrium achieve a lower level of utility. The initial old also achieve a lower level of utility because c_2 in the monetary equilibrium is lower than c_2 at the golden rule allocation.

3. Suppose that money supply grows at a constant rate z ($z > 1$). Comparing monetary equilibrium and the golden rule allocation, we find that individuals consume too much when young and too little when old in the monetary equilibrium.

True. In a monetary equilibrium, inflation makes young individuals trade less goods for money. As a result, the old have less money to purchase goods. Comparing monetary equilibrium with the golden rule allocation, we find that individuals consume too much when young and too little when old.

4. Suppose that the population grows at a constant rate n ($n > 1$) and money supply grows at a constant rate z ($z > 1$), the value of money falls over time.

Uncertain. When the population grows at a constant rate n and money supply grows at a constant rate z, we can derive money's rate of return as $v_{t+1}/v_t = n/z$. If $n < z$, the value of money falls over time. If $n > z$, the value of money increases over time. And if $n = z$, the value of money stays constant.

5. When the population is growing, fixing the price level is the optimal policy.

False. When the population is growing, money's rate of return is given by $v_{t+1}/v_t = n/z$. The allocation in the monetary equilibrium is generally not the golden rule allocation. The optimal policy requires that an individual's budget constraint is identical to the planner's resource constraint. It implies that we need $z = 1$ for a monetary equilibrium to achieve the golden rule allocation. In this case, the value of money increases over time and the price level actually falls over time.

(Note that for question 4 and question 5, my explanation uses the expression of v_{t+1}/v_t. If you cannot memorize the exact expression of v_{t+1}/v_t, it is fine to explain your answer in words as long as the intuition is correct.)

6. Let $N_t = nN_{t-1}$ and $M_t = zM_{t-1}$ for every period t, where z and n are both greater than 1. The money created each period is used to finance a lump-sum subsidy of a_t^* goods to each young individual.

(a) Find the equation for the budget constraint of an individual in the monetary equilibrium. Graph it. Show an arbitrary indifference curve tangent to the budget constraint and indicate the levels of c_1 and c_2 that would be chosen by an individual in this equilibrium.

An individual's first- and second-period budget constraints are

$$c_1 + v_t m_t \leq y + a_t^* \quad \text{and} \quad c_2 \leq v_{t+1} m_t.$$

Combining these two period budget constraints together, we derive the lifetime budget constraint

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leq y + a_t^*.$$

The value of money is derived from the money market clearing condition (when aggregate money supply equals aggregate money demand),

$$N_t \left(y + a^* - c_1 \right) = v_t M_t \quad \rightarrow \quad v_t = \frac{N_t \left(y + a^* - c_1 \right)}{M_t}.$$

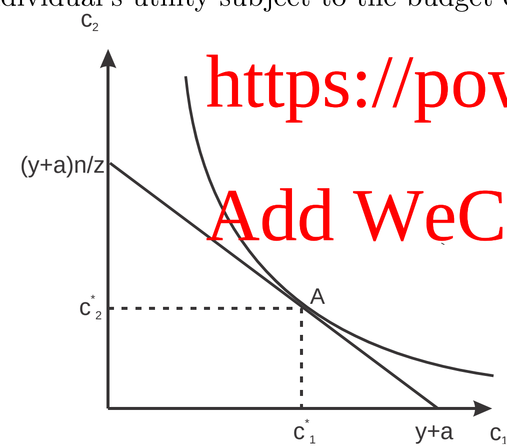
It follows that money's rate of return is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y+a^*-c_1)}{M_{t+1}}}{\frac{N_t(y+a^*-c_1)}{M_t}} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} = \frac{n}{z}.$$

We can update our budget constraint as

$$c_1 + \frac{z}{n} c_2 \leq y + a^*.$$

Graphically, we can draw the budget constraint and label point A as the allocation in a monetary equilibrium. The consumption bundle (c_1^*, c_2^*) at point A maximizes an individual's utility subject to the budget constraint.

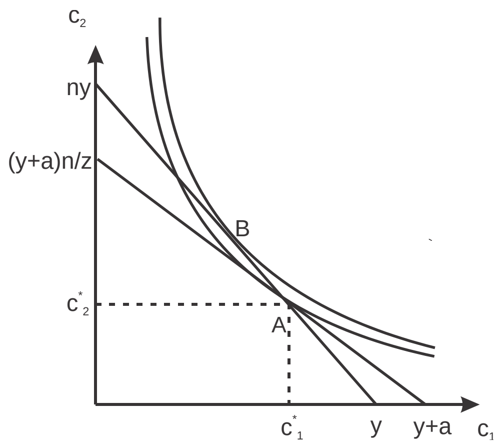


(b) On the graph you drew in part (a), draw the resource constraint. Take advantage of the fact that the resource constraint goes through the monetary equilibrium (c_1^*, c_2^*). Label your graph carefully, distinguishing between the budget and resource constraints.

We first derive the resource constraint faced by the planner,

$$N_t c_1 + N_{t-1} c_2 \leq N_t y \quad \rightarrow \quad c_1 + \frac{1}{n} c_2 \leq y.$$

We add the resource constraint to the graph and label the golden rule allocation as point B.



(c) Show that the monetary equilibrium does not maximize the utility of the future generations. Support your assertion with references to the graph you drew of the budget and feasible constraints.

We can clearly see from the graph that there are feasible points that are preferred by the future generations to (c_1^*, c_2^*). One such point is point B which is the golden rule allocation. Point B lies on a higher indifference curve than (c_1^*, c_2^*). This shows that given the resource constraint (or feasible constraint), the stationary monetary equilibrium (c_1^*, c_2^*) does not maximize the utility of future generations. Furthermore, the initial old also prefer a point like B since it gives them higher second-period consumption than c_2^*.

(Note that we use the term "feasible constraint" and the term "resource constraint" interchangeably.)

7. Consider an overlapping generations model with the following characteristics: Each generation is composed of $1,000$ individuals. The money supply changes according to $M_t = 2M_{t-1}$. The initial old own a total of $10,000$ units of money ($M_0 = \$10,000$). Each period, the newly printed money is given to the old of that period as a lump-sum transfer. Each individual is endowed with 20 units of the consumption good when born and nothing when old. Preferences are such that individuals wish to save 10 units when young at the equilibrium rate of return on money.

(a) What is the gross real rate of return on money in this economy (v_{t+1}/v_t)?

To derive the rate return on money, we first need to find the value of money. The money market clearing condition is

$$N\left(y - c_1\right) = v_t M_t \quad \rightarrow \quad v_t = \frac{N\left(y - c_1\right)}{M_t}.$$

It follows that

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N(y-c_1)}{M_{t+1}}}{\frac{N(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{2}.$$

The rate of return on money is $1/2$. The value of money falls by a half every period.

(b) How many goods does an individual consume when young (c_1)?

Individuals allocate their endowments between consumption and saving when young. The first-period budget constraint is

$$c_1 + v_t m_t \leq y.$$

When individuals save 10 units when young, it means that $v_t m_t = 10$. Given that $y = 20$, we have $c_1 = y - v_t m_t = 10$. The consumption when young is 10 units of good.

(c) How many goods does an individual receive as a transfer (a)?

From the government budget constraint, the transfer is from the newly printed money. In aggregate,

$$Na = v_t \left(M_t - M_{t-1} \right) = v_t \left(M_t - \frac{M_t}{2} \right) = \frac{1}{2} v_t M_t.$$

To find the value of $v_t M_t$, we use the money market clearing condition

$$v_t M_t = N \left(y - c_1 \right) = 1000 \times (20 - 10) = 10,000.$$

It follows that the amount of the transfer in real terms is given by

$$a = \frac{\frac{1}{2} v_t M_t}{N} = \frac{\frac{1}{2} \times 10,000}{1000} = 5.$$

(d) How many goods does an individual consume when old (c_2)?

From an individual's second-period budget constraint,

$$c_2 \leq v_{t+1} m_t + a.$$

Recall that $v_t m_t = 10$. We can derive the value of $v_{t+1} m_t$ as

$$v_{t+1} m_t = \frac{v_{t+1}}{v_t} v_t m_t = \frac{1}{2} \times 10 = 5.$$

Therefore, we have

$$c_2 = v_{t+1} m_t + a = 5 + 5 = 10.$$

The second-period consumption is 10 units of good.

(e) What is the price of the consumption good in period 1 in dollars (p_1)?

The price is the inverse of the value of money,

$$p_1 = \frac{1}{v_1}.$$

We will find the value of v_1. Given that

$$v_t = \frac{N \left(y - c_1 \right)}{M_t},$$

we have

$$v_1 = \frac{N \left(y - c_1 \right)}{M_1} = \frac{1000 \times (20 - 10)}{2 \times M_0} = \frac{1000 \times (20 - 10)}{2 \times 10000} = \frac{1}{2}.$$

Finally, the price level in period 1 is

$$p_1 = \frac{1}{v_1} = 2.$$