ECOS3010: Tutorial 1 (Answer Key)

Question 1-5. Answer True, False or Uncertain. Briefly explain your answer.

1. The main difference between fiat money and commodity money is that fiat money is intrinsically useless.

True. Historically, commodity money takes many forms including shell, bead, silver, gold and etc.. Commodity money has its own value. Unlike commodity money, fiat money is intrinsically useless. Fiat money is often issued by the government.

2. The golden rule allocation maximizes the utilities of both the future generations and the initial old.

False. The golden rule maximizes the utilities of the future generations, but does not maximize the utilities of the initial old. In our model, the initial old consume only when old. So they would like allocate all consumption to old. In contrast, all future generations prefer to consume both when young and when old.

3. In a monetary equilibrium, individuals maximize their utilities subject to the resource constraints.

False. In a monetary equilibrium, individuals maximize their utilities subject to the lifetime budget constraints.

4. Our model of money is consistent with the quantity theory of money.

True. Quantity theory of money says that the price level is proportional to the quantity of money in the economy. In our model, the equality of money supply and money demand determines the equilibrium price level, which is proportional to the quantity of money.

5. When money supply is constant and population is growing at a constant rate $N_t = nN_{t-1}$, the allocation from a monetary equilibrium is not the golden rule allocation.

False. When the population is growing and money supply is constant, the allocation in a monetary equilibrium still achieves the golden rule allocation. One can find that the resource constraint coincides with an individual's budget constraint. So individuals in a monetary equilibrium consume the same consumption bundle as the golden rule allocation chosen by the planner.

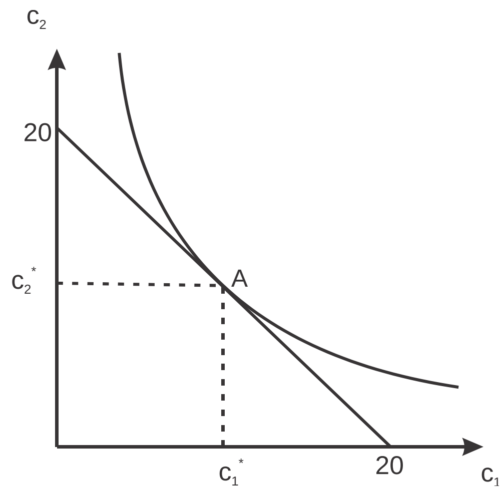
6. Consider an economy with a constant population of $N = 100$. Individuals are endowed with $y = 20$ units of the consumption good when young and nothing when old.

(a) What is the equation for the feasible set of this economy? Portray the feasible set on a graph. With arbitrarily drawn indifference curves, illustrate the stationary combination of c_1 and c_2 that maximizes the utility of future generations.

Feasible set:

$$100c_1 + 100c_2 \leq 100 \times 20 \ \rightarrow \ c_1 + c_2 \leq 20.$$

Point A in on the graph maximizes the utility of future generations.



(b) Now look at a monetary equilibrium. Write down equations that represent the constraints on first- and second-period consumption for a typical individual. Combine these constraints into a lifetime budget constraint.

First-period budget constraint:

$$c_1 + v_t m_t \leq 20.$$

Second-period budget constraint:

$$c_2 \leq v_{t+1} m_t.$$

Lifetime budget constraint: using the first- and second-period budget constraints

$$\frac{c_2}{v_{t+1}} \leq m_t \leq \frac{20 - c_1}{v_t} \;\rightarrow\; c_1 + \frac{v_t}{v_{t+1}} c_2 \leq 20.$$

(c) Suppose the initial old are endowed with a total of $M = 400$ units of fiat money. What condition represents the clearing of the money market in an arbitrary period t? Use this condition to find the real rate of return of fiat money.

Aggregate real demand for money in period t:

$$N\,(y - c_1) = 100 \times (20 - c_1).$$

Aggregate real supply of money in period t:

$$v_t M = 400 v_t.$$

The value of money is determined by the equality of money supply and money demand. Therefore, we have

$$400 v_t = 100 \times (20 - c_1) \text{ and } v_t = \frac{100 \times (20 - c_1)}{400}.$$

Similarly,

$$v_{t+1} = \frac{100 \times (20 - c_1)}{400}.$$

We can now find that the real rate of return of fiat money is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{100\times(20-c_1)}{400}}{\frac{100\times(20-c_1)}{400}} = 1.$$

The value of money is constant.

Now suppose that preferences are such that $u(c_1, c_2) = c_1^{1/2} + c_2^{1/2}$.

(d) Find an individual's real demand for money. Use the assumption about preferences and your answer in part (c) to find an exact numerical value.

In a monetary equilibrium, an individual maximizes his utility subject to the budget constraint. Mathematically,

$$\max_{c_1, c_2} \quad c_1^{1/2} + c_2^{1/2} \ \text{ subject to } c_1 + c_2 = 20,$$

where we have substituted v_t/v_{t+1} in the budget constraint by 1. From the budget constraint, $c_2 = 20 - c_1$. We substitute the expression of c_2 into the utility function to have the unconstrained maximization problem:

$$\max_{c_1, c_2} \quad c_1^{1/2} + (20 - c_1)^{1/2}.$$

The first-order condition is

$$\begin{aligned} \frac{1}{2} c_1^{-1/2} + \frac{1}{2}(20 - c_1)^{-1/2} \times (-1) &= 0 \\ \rightarrow \ c_1^{-1/2} &= (20 - c_1)^{-1/2} \\ \rightarrow \ c_1 &= 20 - c_1 \\ \rightarrow \ c_1 &= 10. \end{aligned}$$

It follows that an individual's real demand for money is

$$y - c_1 = 20 - 10 = 10.$$

(e) What is the value of money in period t, v_t? What is the price of the consumption good p_t?

The value of money in period t is

$$v_t = \frac{100 \times (20 - c_1)}{400} = \frac{100 \times (20 - 10)}{400} = 2.5.$$

Therefore, the price level is

$$p_t = \frac{1}{v_t} = \frac{1}{2.5} = 0.4.$$

(f) Suppose instead that the initial old were endowed with a total of 800 units of fiat money. How do your answers to part (e) change? Are the initial old better off with more units of money?

If the initial old is endowed with 800 units of money, it won't affect the choice of (c_1, c_2).

(You can try to verify it.) The value of money in period t is thus

$$v_t = \frac{100 \times (20 - c_1)}{800} = \frac{100 \times (20 - 10)}{800} = 1.25.$$

The price level is

$$p_t = \frac{1}{v_t} = \frac{1}{1.25} = 0.8.$$

The initial old consume $c_2 = 10$, which is the same as before. So they are not better off with more units of money. In this economy, money is neutral. The change in the stock of money does not affect any real variables such as c_1 and c_2. Only nominal variables such as v_t and p_t are affected.

7. In this chapter, we modeled growth in an economy by a growing population. We could also achieve a growing economy by having an endowment that increases over time. To see this, consider the following economy: Let the number of young people born in each period be constant at N. There is a constant stock of fiat money, M. Each young person born in period t is endowed with y_t units of the consumption good when young and nothing when old. The individual endowment grows over time so that $y_t = \alpha y_{t-1}$ where $\alpha > 1$. For simplicity, assume that in each period t, individuals desire to hold real money balances equal to one-half of their endowment, so that $v_t m_t = y_t/2$.

(a) Write down equations that represent the constraints on first- and second-period consumption for a typical individual. Combine these constraints into a lifetime budget constraint.

First-period budget constraint:

$$c_1 + v_t m_t \le y_t \ .$$

Second-period budget constraint:

$$c_2 \le v_{t+1} m_t.$$

As before, we combine the previous two budget constraints to get an individual's lifetime budget constraint:

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \le y_t.$$

(b) Write down the condition that represents the clearing of the money market in an arbitrary period t. Use this condition to find the real rate of return of fiat money in a monetary equilibrium. Explain the path over time of the value of fiat money.

Aggregate real demand for money in period t:

$$N\left(y_t - c_1\right).$$

Aggregate real supply of money in period t:

$$v_t M.$$

When money market clears, we have

$$N\left(y - c_1\right) = v_t M \ \rightarrow \ v_t = \frac{N\left(y_t - c_1\right)}{M}.$$

We know that the preferences are such that $v_t m_t = y_t/2$. It implies that $c_1 = y_t - v_t m_t =$

$y_t/2$. The value of money in period t is

$$v_t = \frac{N\left(y_t - \frac{y_t}{2}\right)}{M} = \frac{Ny_t}{2M}.$$

Similarly,

$$v_{t+1} = \frac{Ny_{t+1}}{2M}.$$

It follows that the real rate of return of fiat money is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{Ny_{t+1}}{2M}}{\frac{Ny_t}{2M}} = \frac{y_{t+1}}{y_t} = \alpha.$$

The value of fiat money grows at a constant rate α. In our lecture, we modeled growth in the economy by growth in the number of young people born each period. We found that in that case, the rate of return of fiat money equal to n, the growth rate of the economy. In this example, α is the growth rate of the economy (it is the gross rate of change of the total endowment.). We discover that even in this more complicated setup, the rate of return of fiat money is equal to the growth rate of the economy when the money supply is fixed.