$$d(\ln x) = \frac{1}{x} \cdot dx$$

$$\boxed{\frac{d(\ln x)}{dx} = \frac{1}{x}}$$

$y = x^\alpha - x + 1$ $\quad 0 < \alpha < 1$

1st order : $\alpha x^{\alpha-1} - 1$

2nd order : $\alpha \cdot (\alpha-1) \cdot x^{\alpha-2}$

< 0

< 0

concave

$\alpha = 0.5$

FOC: $0.5 x^{-0.5} - 1 = 0$

$\rightarrow x^* = \frac{1}{4}$

$y = x^\beta - x + 1, \quad \beta > 1$

1st order: $\beta \cdot x^{\beta-1} - 1$

2nd - - : $\beta \cdot (\beta - 1) \cdot x^{\beta-2}$

> 0

$\beta = 2$

$\rightarrow$ FOC: $2 \cdot \lambda - 1 = 0$

$$\lambda_{min} = \frac{1}{2}$$

$$y_{min} = \frac{3}{4}$$

$$\max_{x,y} f(x,y) = xy$$

s.t. $3x + 4y = 16$

b.c. $\rightarrow$ $x = \frac{16 - 4y}{3}$

→ Max (y) $\frac{16-4y}{3} \cdot y$

For wrt y

$$-\frac{4}{3}y + \frac{16-4y}{3} = 0$$

→ $y^* = 2$

$x^* = \frac{8}{3}$

$$\max_{x,y} \; u(x,y)$$

s.t. $p_x \cdot x + p_y \cdot y \leq M$

b.c. $\rightarrow x = \dfrac{M - p_y \cdot y}{p_x}$

$\rightarrow \max_{y} \; u\left(\dfrac{M - p_y \cdot y}{p_x}, y\right)$

For: $u'(x) \cdot x'(y) + u'(y) = 0$

1 eqn $\rightarrow y^* \xrightarrow{\text{b.c.}} x^*$

$$x+\sqrt{xy}+y \le 1$$

$$\sqrt{xy} = 1-x-y$$

$$xy = (1-x-y)^2$$

$$L = xy - \lambda[3x+4y-16]$$

$$\max_{x,y,z} f(x,y,z) = (1+x)\cdot y \cdot z$$

$$s.t.\ x+y+z \le 1$$

Lagrangian

$$\mathcal{L} = (1+x)yz + \lambda(1-x-y-z)$$

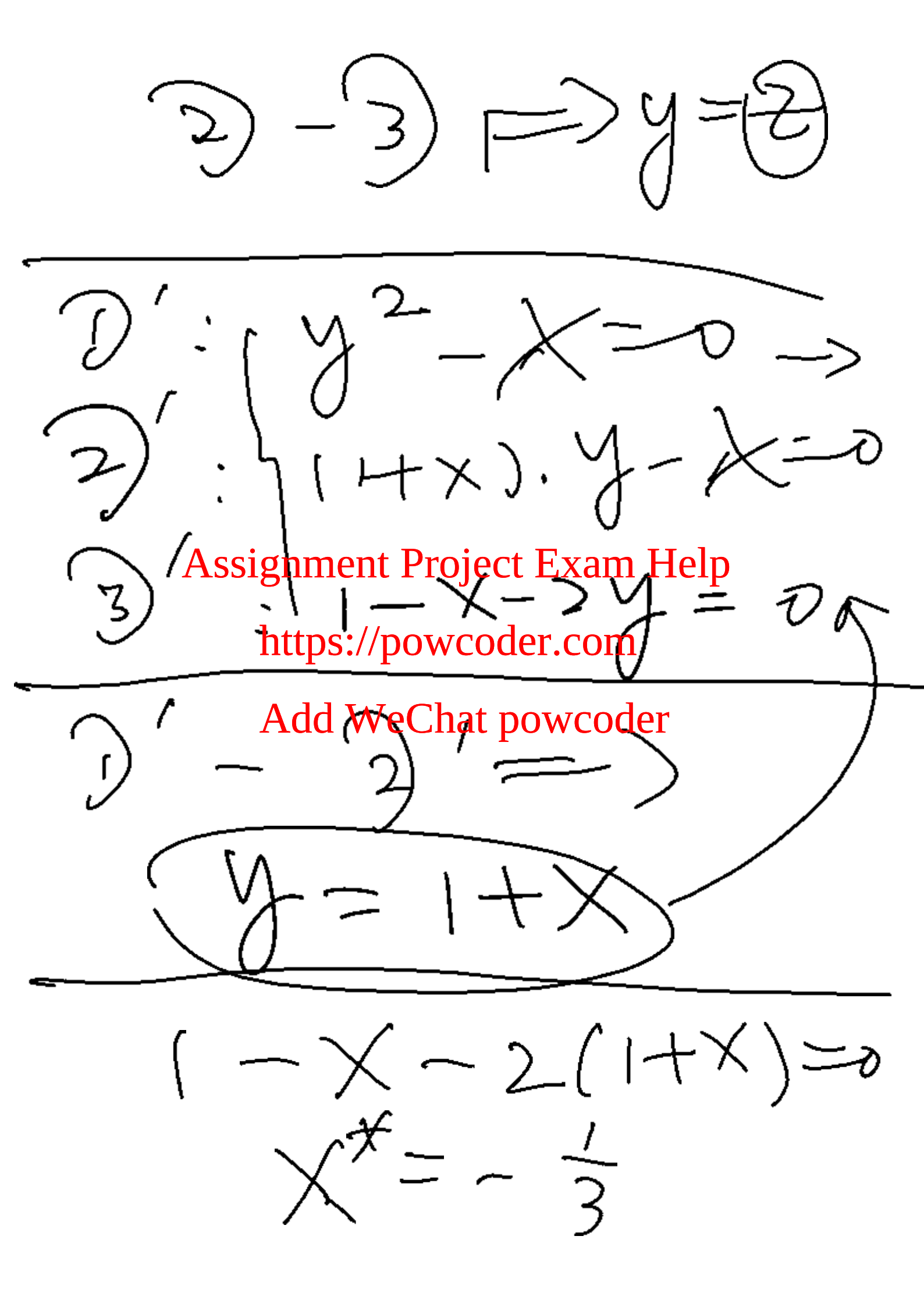
For

wrt x: $\frac{\partial \mathcal{L}}{\partial x} : yz - \lambda = 0$ (1)

y: $\frac{\partial \mathcal{L}}{\partial y} : (1+x)z - \lambda = 0$ (2)

z: $\frac{\partial \mathcal{L}}{\partial z} : (1+x)\cdot y - \lambda = 0$ (3)

λ: $1 - x - y - z = 0$ (4)



$$y^* = \frac{2}{3}$$

$$z^* = \frac{2}{3}$$

$$\lambda = ?$$

$$f^* = \frac{8}{27}$$

max C_1, C_2 n_1, n_2 ← dynamic

$$\ln C_1 + \alpha \ln(1-n_1) + \beta\left[\ln C_2 + \alpha \ln(1-n_2)\right]$$

s.t. $C_1 + C_2 = n_1 w_1 + n_2 w_2$

lifetime b.c.