ECOS3010: Tutorial 3 (Answer Key)

Question 1-4. Answer True, False or Uncertain. Briefly explain your answer.

1. Suppose that the government creates new money to finance its own purchases. Comparing monetary equilibrium and the golden rule allocation, we find that both c_1 and c_2 are lower in a monetary equilibrium than at the golden rule allocation.

False. In a monetary equilibrium, young individuals would choose to trade less goods for money when money supply is growing. As a result, the old consume less but the young consume more compared to the golden rule allocation. That is, c_1 is higher and c_2 is lower in a monetary equilibrium.

2. To finance the same amount of government purchases, using a lump-sum tax is better than using the inflation tax (money creation).

Uncertain. When using the inflation tax to finance government purchases, we know that the allocation in the monetary equilibrium achieves a lower level utility than the golden rule allocation. If the government uses a lump-sum tax instead of the inflation tax, it is possible to design the lump-sum tax such that the allocation in the monetary equilibrium can coincide with the golden rule allocation. In particular, it requires that the government keeps a constant money supply and sets the per capita lump-sum tax to the per capita government purchases.

3. In most countries, seigniorage contributes significantly to total government revenue.

False. In most industrialized countries during normal times, seigniorage contributes to total government revenue, but it accounts for only a small fraction of total government revenue. However, for countries that experience high inflation episodes, seigniorage could contribute significantly to total government revenue.

4. The government can finance any amount of government purchases by creating new money.

False. When the government creates new money to finance its own purchases, a higher growth rate of money supply does not always lead to a higher level of seigniorage in real terms. A higher growth rate of money supply means that a higher fraction of real value of money stock will be collected by the government. However, the real value of the total money stock decreases when the growth rate of money supply increases. Overall, when the growth rate of money supply is low, a higher growth rate leads to more seigniorage. When the growth rate of money supply is high, a higher growth rate leads to less seigniorage.

5. Assume that people face a lump-sum tax of τ goods when old and a growth rate of money supply of $z > 1$. The tax and the newly created money are used to finance government purchases of g goods per young individual in every period. There are N individuals in every generation.

(a) Find the individual's budget constraints when young and when old. Combine them to form the individual's lifetime budget constraint and graph this constraint.

An individual's first-period budget constraint is

$$c_1 + v_t m_t \leq y.$$

The second-period budget constraint is

$$c_2 \leq v_{t+1} m_t - \tau.$$

Combine the two period budget constraints to get the lifetime budget constraint:

$$c_1 + \frac{v_t}{v_{t+1}} c_2 \leq y - \frac{v_t}{v_{t+1}} \tau.$$

To find the value of v_t/v_{t+1}, we need to find the value of money v_t first. From the money market clearing condition, we have

$$N(y - c_1) = v_t M_t.$$

It follows that

$$v_t = \frac{N(y - c_1)}{M_t}.$$

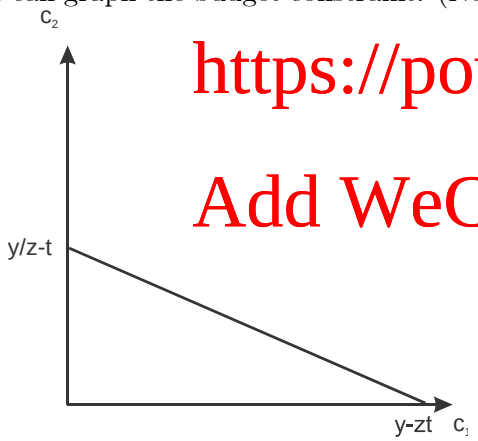
We can then derive money's rate of return as

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N(y-c_1)}{M_{t+1}}}{\frac{N(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{z}.$$

The individual's budget constraint is updated as

$$c_1 + \frac{1}{z} c_2 \leq y - \tau.$$

We can graph the budget constraint. (Note that I use t to represent τ in the figures below.)



(b) Find the government's budget constraint.

The government's budget constraint is

$$g = \tau + \frac{(M_t - M_{t-1}) v_t}{N} = \tau + \frac{\left(1 - \frac{1}{z}\right) M_t v_t}{N}.$$

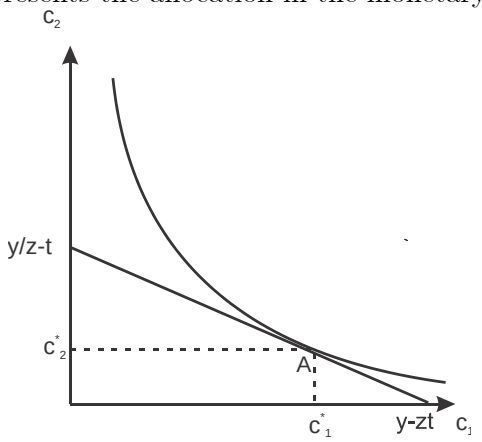
Using the money market clearing condition, we have

$$\begin{aligned} g &= \tau + \frac{\left(1 - \frac{1}{z}\right) N (y - c_1)}{N} \\ &= \tau + \left(1 - \frac{1}{z}\right)(y - c_1). \end{aligned}$$

(c) Graph the stationary monetary equilibrium.

The stationary monetary equilibrium can be found at the tangency point between the

indifference curve and the individual's budget constraint. Point A in the figure below represents the allocation in the monetary equilibrium



(d) Find the stationary monetary equilibrium when $z = 1$ and add it to the graph in (c).

When $z = 1$, the individual's budget constraint is

$$c_1 + c_2 \leq y - \tau.$$

We graph the budget constraint and find the allocation in a monetary equilibrium at point B when $z = 1$. Please see the attached PDF file.

(e) Compare the real balances of fiat money when $z > 1$ to the value when $z = 1$.

Comparing c_1 when $z > 1$ and when $z = 1$, we find that the c_1 is higher when $z > 1$ than when $z = 1$. Since the real money balances held by any individual is

$$v_t m_t = y - c_1,$$

the real money balances of money is lower when $z > 1$ than when $z = 1$. That is, a higher growth rate of money supply reduces the real money balances held by any individual. As a result, the aggregate real money balances in the economy also decline as z increases.

(f) What is the optimal choice of (τ, z) so that individuals achieve the highest level of utility?

To achieve the highest level of utility, the allocation of (c_1, c_2) should be the golden rule allocation. The golden rule allocation can be found when the planner maximizes an individual's utility subject to the economy's resource constraint. In this economy, the resource constraint is

$$c_1 + c_2 \leq y - g,$$

where g is the per capita government purchases. For the optimal policy, the government chooses (τ, z) so that the individual's budget constraint can coincide with the resource constraint. The optimal policy is to let $\tau = g$ and $z = 1$.

6. Consider an economy with a constant population of $N = 1000$. Individuals are endowed with $y = 20$ units of the consumption good when young and nothing when old. All seigniorage revenue is used to finance government expenditures. There are no subsidies and no taxes other than seigniorage. Suppose that preferences are such that each individual

wishes to hold real balances of fiat money worth

$$\frac{y}{1+\frac{v_t}{v_{t+1}}}$$

goods.

(a) Use the equality of supply and demand in the money market to find the total real balances of fiat money in a stationary equilibrium as a function of the rate of fiat money creation z.

The total real balances of money is

$$v_t M_t.$$

The money market clearing condition implies that

$$v_t M_t = N\left(y - c_1\right).$$

The value of money in period t is then

$$v_t = \frac{N\left(y - c_1\right)}{M_t}.$$

Money's rate of return is

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N(y-c_1)}{M_{t+1}}}{\frac{N(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{z}.$$

The amount of money in real terms that each young individual chooses to hold is

$$v_t m_t = y - c_1 = \frac{y}{1+\frac{v_t}{v_{t+1}}}.$$

Using $v_{t+1}/v_t = 1/z$, we have

$$y - c_1 = \frac{y}{1+z}.$$

Therefore, the total real balances of money as a function of z is

$$\begin{aligned} v_t M_t &= N\left(y - c_1\right) \\ &= N\frac{y}{1+z} \\ &= \frac{1000 \times 20}{1+z} \\ &= \frac{20,000}{1+z}. \end{aligned}$$

We can see that a higher z leads to a lower $v_t M_t$.

(b) Use your answer in part (a) to find total seigniorage revenue as a function of z. Graph this function and explain its shape.

The total seigniorage revenue in real terms is

$$G = \left(M_t - M_{t-1}\right) v_t = \left(1 - \frac{1}{z}\right) M_t v_t.$$

Using the solution of $v_t M_t$ that we found in part (a), we have

$$\begin{aligned} G &= \left(1 - \frac{1}{z}\right) \frac{20,000}{1+z} \\ &= 20,000 \frac{1 - \frac{1}{z}}{1+z}. \end{aligned}$$

We can graph G as a function of z. (Note that I use excel to graph the example below. You can use your preferred software to do this.)