Week 1: Overview and Math Review

Monetary Economics

ECOS3010

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  - Consultation Hour: Mondays 1:45-2:45 pm via Zoom.

- Textbook: Bruce Champ, Scott Freeman and Joseph Haslag, Modeling Monetary Economics, 5th edition, Cambridge University Press.

- Assessment
    - Assignment 1 (10%): Available Friday August 26, 5 pm. Due Friday September 16, 11:59 pm.
    - Assignment 2 (10%): Available Friday October 7, 5 pm. Due Friday October 28, 11:59 pm.
    - Midterm exam (30%): Monday, Sep 19, 11 am.
    - Final exam (50%): during exam period.

Overview

- Lecture topics
  - Money
  - Inflation
  - Price surprises and the Philips curve
  - International monetary system
  - Money and capital
  - Liquidity and financial intermediation
  - Bank risks, liquidity risks and bank panics
  - Crisis

Important Mathematical Concepts

- **Differentiation and chain rule**:

Consider three functions $F(x, y), x(a)$, and $y(a)$.

Q: what's the derivative of F w.r.t to a

A: using chain rule,

$$\frac{\partial F(x,y)}{\partial a} = \frac{\partial F(x,y)}{\partial x}\frac{\partial x(a)}{\partial a} + \frac{\partial F(x,y)}{\partial y}\frac{\partial y(a)}{\partial a}$$

- Example:
  - *problem*: suppose firm's profit function is $\Pi(y) = -y^4 + 6y^2 - 5$ where Π is the firm's profit and y is the amount of output. Assume firm's production function is $y = 5L^{2/3}$, where L is the amount of labour input. Apply the Chain Rule to compute the derivative of Π with respect to L.

  - *solution*:

$$\begin{aligned}
\frac{d}{dL}(\Pi(y(L))) &= \Pi'(y) \cdot y'(L) \\
&= [-4(y)^3 + 12y] \cdot (\frac{10}{3} L^{-1/3}) \\
&= [-4(5L^{2/3})^3 + 12(5L^{2/3})] \cdot (\frac{10}{3} L^{-1/3})
\end{aligned}$$

Chain rule, log and exponential functions III

which after simplifying equals

$$(-4 \cdot 125L^2 + 60L^{2/3}) \cdot (\frac{10}{3}L^{-1/3}) = -\frac{5000}{3}L^{5/3} + 200L^{1/3}.$$

- **Properties of log functions**:

$$\begin{aligned} \ln(ab) &= \ln(a) + \ln(b) \\ \ln(\frac{a}{b}) &= \ln(a) - \ln(b) \\ \ln(a^b) &= b\ln(a) \\ \frac{d\ln(x)}{dx} &= \frac{1}{x} \\ \ln(1+x) &\approx x \quad \text{, if } x \text{ is small} \end{aligned}$$

- **Exponential function**:

$$e^{a-b} = \frac{e^a}{e^b}$$
$$e^{\ln(a)} = a$$

Unconstrained optimization I

- consider a function $y = f(x)$. Necessary (first-order) condition for optimization of this function:

$$\frac{dy}{dx} = \frac{df(x)}{dx} = 0$$

- it's a maximum if $\frac{d^2 f(x)}{dx^2} \leq 0$ (a **concave** function), a minimum if $\frac{d^2 f(x)}{dx^2} \geq 0$ (a **convex** function).

  - example 1 (concave):

$$\begin{aligned} y &= f(x) \\ &= x^{\alpha} - x + 1 \quad \text{, where } 0 < \alpha < 1 \end{aligned}$$

figure: ....

- example 2 (convex):

$$\begin{aligned} y &= f(x) \\ &= x^{\beta} - x + 1 \quad \text{, where } \beta > 1 \end{aligned}$$

figure: ....

- the value of x that maximizes y is called

$$x^* = \arg\max_x f(x)$$

$y^* = f(x^*)$ is the maximum value of the function.

- **Example 1**:

  - *problem*

$$\begin{aligned} \max_{x,y} \quad f(x,y) &= xy \\ s.t. \quad 3x + 4y &= 16 \end{aligned}$$

  - *solution*:

- **Example 2:**

  - consider the following utility maximization problem of a consumer/household

$$\max_{x,y} U = U(x, y)$$
$$s.t. \quad p_x x + p_y y \leq M$$

  - *solution:*

Constrained optimization (Lagrangian method) I

- It is not always possible to express x as a function of y explicitly. In that case, we can apply the Lagrangian method.

- **Example 1**:
  - *problem*:

$$\begin{aligned} \max_{x,y} \quad f(x,y) &= xy \\ s.t. \qquad 3\,x + 4y &= 16 \end{aligned}$$

  - *solution*:
    - first, form a Lagrangian with Lagrange multiplier λ

$$L = xy + \lambda\left[16 - 3x - 4y\right]$$

Constrained optimization (Lagrangian method) II

- next, take the derivatives w.r.t. x, y, and λ yielding

$$\begin{aligned} 0 &= y - 3\lambda \\ 0 &= x - 4\lambda \\ 0 &= 16 - 3x - 4y \end{aligned}$$

- last, solve for three values using the system of equations above, yielding

$$\begin{aligned} x &= \frac{8}{3} \\ y &= 2 \\ \lambda &= \frac{2}{3} \end{aligned}$$

- **Example 2:**

  - consider the following utility maximization problem of a consumer/household

$$\max_{x,y} U = U(x,y)$$
$$s.t. \quad p_x x + p_y y \leq M$$

  - we can solve this problem using the Lagrangian method

  - solving the problem:
    - form the Lagrangian function (**note the way the constraint is written!**)

$$L = U(x,y) + \lambda[M - p_x x - p_y y]$$

Constrained optimization (Lagrangian method) IV

- first-order conditions (FOC)

$$
\begin{aligned}
\frac{\partial L}{\partial x} &= U_x(x, y) - \lambda p_x = 0 \\
\frac{\partial L}{\partial y} &= U_y(x, y) - \lambda p_y = 0 \\
\frac{\partial L}{\partial \lambda} &= M - p_x x - p_y y = 0
\end{aligned}
$$

- **the solution**: combining the first two FOCs to eliminate λ, we have (the maximum utility, subject to the budget constraint, must satisfy these conditions)

$$\frac{U_x}{U_y} = \frac{p_x}{p_y}$$

and

$$p_x x + p_y y = M$$

$$\begin{aligned} \max \quad f(x, y, z) &= (1+x)yz \\ s.t. \quad x + y + z &\leq 1 \end{aligned}$$