

**ECOS3010: Assignment 2 (Total: 50 marks) Due 11:59 pm, Friday, October 28, 2022**

1. Homework must be turned in on the day it is due. Work not submitted on or before the due date is subject to a penalty of 5% per calendar day late. If work is submitted more than 10 days after the due date, or is submitted after the return date, the mark will be zero. Each assignment is worth 10% of total weight.

2. **TYPE your work (including all mathematical equations).** Homework is submitted as a typed .pdf file, no exceptions. Untyped work will not be marked and will receive a mark of zero. You can draw a graph by hand, scan it, and include it as a figure in the PDF. Please don't forget to include your name and student number.

3. Carefully explain your work.

Question 1-5. Answer True, False or Uncertain. Briefly explain your answer. (each question 4 marks)

1. In an international economy of perfectly substitutable currencies, an increase in the stock of one country's money reduces real value of all monies.

2. The negative correlation between inflation and the real interest rate can be explained by the Fisher effect.

3. The rate of return equality holds in the model of illiquidity.

4. The rate of return equality is inconsistent with the observations found in the Equity Premium Puzzle.

5. Cooperative stabilization can help countries have a fixed exchange rate regime and avoid high inflation.

6. (10 marks) Consider an economy of three-period-lived people in overlapping generations. Each individual is endowed with  $y$  goods when young and old and nothing when middle-aged. The population of each generation born in period  $t$  is  $N_t$  where  $N_t = nN_{t-1}$ . There are no assets other than loans.

(a) Explain how private debt can be used to provide for consumption when middle-aged. Point out who lends to whom and write the condition for the equality of supply and demand for loans in period  $t$ . (5 marks)

(b) Write the budget constraints for the young, the middle-aged, and the old. Be sure to define any notation you introduce. (5 marks)

7. (10 marks) Consider the model of illiquidity where individuals live for three periods. Each individual is endowed with  $y$  units of the consumption good when young and with nothing in the other two periods of life. Let  $N_t$  be the number of individuals in the generation born at  $t$  with  $N_t = nN_{t-1}$ . There are two types of assets in the economy. (No inside money or private IOUs are available.) One asset is fiat money, with a fixed supply  $M$ . The other asset is capital. A unit of capital can be created from a unit of the consumption good in any period  $t$  and capital can be created in any amount. Two periods after it is created, a unit of capital produces  $X$  units of the consumption good and then disintegrates. Let  $X > n^2$ .

Assume that the stock of money is distributed equally to the initial middle-aged and each initial old can produce  $Xk_0$  units of the consumption good in the first period. Now suppose that an individual's preference is given by

$$U(c_1, c_2, c_3) = c_1 c_2 c_3.$$

- (a) Find the rate of return on fiat money. (2 marks)
- (b) Describe and explain how an individual finances his consumption in the second-period and third-period of life. (2 marks)
- (c) Write down the budget constraints faced by an individual when young, middle-aged and old. (2 marks)
- (d) Solve for the optimal stationary allocation of  $(c_1^*, c_2^*, c_3^*)$  for all future generations. (4 marks)

8. (10 marks) Consider the OLG model with capital. Each individual is endowed with  $y$  units of the consumption good when young and with nothing when old. Let  $N$  be the number of individuals in each generation. Suppose there is one asset available in the economy – capital. A unit of capital can be created from a unit of the consumption good in any period  $t$  and capital can be created in any amount. One period after it is created, a unit of capital produces  $X$  units of the consumption good and then disintegrates. Assume that each initial old can produce  $Xk_0$  units of the consumption good in the first period. Now suppose that an individual's preference is given by

$$U(c_1, c_2) = (c_1)^{\frac{1}{2}}(c_2)^{\frac{1}{2}}.$$

We focus on stationary allocations.

- (a) Write down the budget constraints faced by an individual when young and old. Combine the budget constraints to find the lifetime budget constraint for an individual. (1 mark)
- (b) Solve for the optimal allocation of  $(c_1^*, c_2^*)$  for all future generations. What is the optimal  $k^*$ ? (2 marks)

Now suppose an additional asset is available in the economy – fiat money. Money supply grows at a constant rate  $z$ ,  $M_t = zM_{t-1}$ . The government imposes a legal restriction such that individuals must carry a real money balance of at least  $q$  units of the consumption good from the young to old. That is, the amount of money held by a young individual should be at least worth  $q$  units of the consumption good. Assume that  $q$  is a small number and exogenously given.

- (c) Describe how an individual finances his second-period consumption if  $X > 1/z$ . What if  $X \leq 1/z$ ? (2 marks)
- (d) From now on, let's assume  $X > 1/z$ . Write down the budget constraints faced by an individual when young and old. (1 mark)
- (e) Solve for the optimal allocation of  $(c_1^*, c_2^*)$  for all future generations. What is the optimal  $k^*$ ? (2 marks)
- (f) How does  $z$  affect the choices of  $(k^*, c_1^*, c_2^*)$ ? Does the Tobin effect exist? (1 mark)
- (g) Explain how the choices of  $(k^*, c_1^*, c_2^*)$  would change if  $q$  is very big? (Hint: you can think about the extreme case that  $q$  is approaching  $y$ .) (1 mark)