

**Student to complete:**

Family name  
Other names  
Student number  
Table number


**ECTE962**  
**Telecommunication System Modeling**  
**CCNU Wollongong Joint Institute**

**Assignment Project Exam Help**

**Examination Paper**  
**Summer Session 2020**

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Exam duration 3 hours.

Items permitted by examiner UOW approved calculators, Open book.

Aids supplied Nil.

Directions to students Attempt all questions. Marks are indicated accordingly.

Make sure your answers are CLEAR and READABLE.

The examination paper is printed on BOTH SIDES.

Candidates should note that questions are to be answered as written – no consultation (individual or group) on questions will be given.

Any assumptions made should be recorded with your answer.

Examination papers must be written in ink.

Papers written in pencil will NOT be marked.

**This exam paper and answer booklet(s) must not be removed from the exam venue**

**QUESTION 1 (18 marks)**

In this exercise, we consider a  $M/M/2/3$  queue with servers having identical service rates.

a) Draw the Markov chain associated to this  $M/M/2/3$  queue. Label all the states and transitions clearly. (4 Marks)

b) Let  $X_t$  be the random variable describing the state of the queue at time  $t$ , and assume the queue is in a state  $i$  at time  $t$ . Let us consider a small time step  $\Delta t$ , such that  $\Delta t$  is very small in comparison to the average time between arrivals and  $\Delta t$  is very small in comparison to the mean service times of the queue. Give the transition probability matrix  $\mathbf{P}(\Delta t)$  associated to this  $M/M/2/3$  queue. We recall that we can define the transition probability matrix  $\mathbf{P}(\Delta t)$  such that its coefficients corresponds to  $P_{ij}(\Delta t) = \Pr(X_{t+\Delta t}=i | X_t=j)$ . (4 marks)

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The transition rate matrix  $\mathbf{Q}$  associated to a continuous time Markov chain can be defined as

$$\mathbf{p}'_t = \mathbf{p}_t \mathbf{Q}$$

where  $\mathbf{p}'_t$  is the time derivative of  $\mathbf{p}_t$  and  $\mathbf{p}_t$  corresponds to a row vector containing the state probabilities at time  $t$ . If the chain contains  $n$  states then  $\mathbf{p}_t = (\Pr(X_t=0), \Pr(X_t=1), \dots, \Pr(X_t=n))$  and  $\mathbf{Q}$  is an  $n \times n$  matrix. Using the transition probability matrix computed in the previous question, we can also define  $\mathbf{Q}$  as  $\mathbf{Q} = \lim_{\Delta t \rightarrow 0} (\mathbf{P} - \mathbf{I})/\Delta t$  where  $\mathbf{I}$  is the identity matrix.

c) Give the transition probability matrix  $\mathbf{Q}$  associated to the M/M/2/3 queue. (4 marks)

d) Use  $\mathbf{p}'_t = \mathbf{p}_t \mathbf{Q}$  to find a system of linear equations involving the state probabilities in the steady-state for the M/M/2/3 queue. (4 marks)

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e) Can you solve this system of linear equations without any additional condition? If no, specify which additional condition(s) is needed. (2 marks)

**QUESTION 2 (8 marks)**

ATM switches have a fixed service time duration. A queue in an ATM switch is found to have a Poisson arrival process. Over a long period of time, the average length of the queue is found to be 2.5 packets. Find the fraction of time the queue is empty.

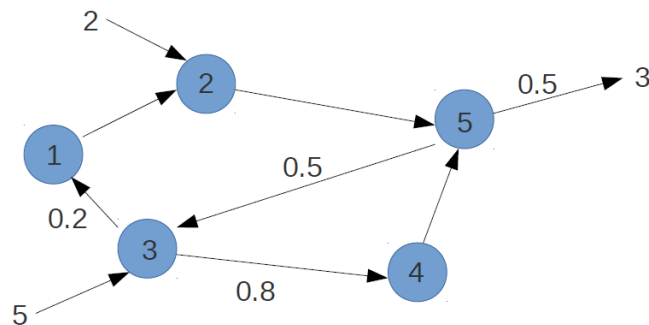
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**QUESTION 3 (14 marks)**

Consider the Jackson network shown below where all nodes are M/M/1. The arrival rate of external traffic to nodes 1, 2, 3 and 4 are shown on the diagram. Also routing information is shown on each link. Assume that the service rates are as follows:  $\mu_1 = 16$ ,  $\mu_2 = 32$ ,  $\mu_3 = 40$ ,  $\mu_4 = 20$ ,  $\mu_5 = 28$ .



(a) Give the routing matrix associated to this network. (3 marks)

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(b) Compute the arrival rates at each node. (4 marks)

(c) Are the conditions of stability satisfied for each node? Justify your answer. (4 marks)

(d) Find the probability that there are 5 customers in the network and all of them are in node 4. (3 marks)

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**QUESTION 4 (8 marks)**

Consider a general birth and death queuing system in which,

$$\begin{aligned}\lambda_k &= \alpha^k \lambda \\ \text{for } k \geq 0, 0 \leq \alpha < 1 \\ \mu_k &= \mu \\ \text{for } k \geq 1 \\ \rho &= \lambda / \mu\end{aligned}$$

(a) Find the steady-state probability  $p(k)$  of having  $k$  customers in the system. Express  $p(k)$  as a function of  $\rho$ ,  $\alpha$  and  $k$ . (4 marks)

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b) Show that this system is always stable (ie stable for any values of  $\lambda$  and  $\mu$ ) if  $0 \leq \alpha < 1$ . (4 marks)

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**QUESTION 5 (16 marks)**

Consider the following function:

$$f(\mathbf{x}) = x^2 - 2x^3 + 2\exp(x) - 4 + y^2$$

with  $\mathbf{x} = (x, y) \in \mathbb{R}^2$ .

(a) Specify the gradient and the Hessian of the function. (2 marks)

(b) The function  $x \rightarrow x^2 - 2x^3 + 2\exp(x)$  admits three critical points at  $x \approx -0.35$ ,  $x \approx 1.27$  and  $x \approx 3.51$ . Deduce how many critical points the function  $f$  admits. For each critical point, specify if it is a minimum, a maximum or a saddle point. (4 marks)

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b) Show that the Hessian at  $\mathbf{x}=(1,0.5)$  is not a positive semi-definite matrix and is not a negative semi-definite matrix. (4 marks)



c) Since at  $\mathbf{x}=(1,0.5)$  the Hessian is indefinite, it cannot be used as a local metric. To circumvent this issue, we propose to use the steepest descent method with the matrix  $2\mathbf{I}$  (i.e. twice the identity matrix) as the local metric. Apply the first minimization step using  $\mathbf{x}_0 = (1, 0.5)$  and  $\mu_0 = 1$ . (3 marks)

d) In view of the point  $\mathbf{x}_1$  found in the previous question, do you think additional steps using the steepest descent method will allow to converge towards the global minimum? If no, towards which critical point do you think it will converge? Justify your answers. (3 marks)

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**QUESTION 6 (18 marks)**

Consider the following system of equations where  $x, y, z$  and  $a$  are all positive integers (i.e. natural numbers):

$$x = 2y, \quad y = z - 1, \quad a = 2x + 1$$

(a) Draw the graph associated to this constraint satisfaction problem with variables  $x, y, z$  and  $a$  as vertices. Give the degree of each vertex. (4 marks)

(b) List the variables in the graph which are associated to cut vertices. (3 marks)

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(d) We provide the following unary constraints :  $0 < x < 10$ ,  $0 < y < 10$ ,  $0 < z < 10$  and  $0 < a < 10$ . Apply node consistency and arc consistency in order to find the resulting domains for each variable. Write down the domains for each variables in the first row of the table in question (e). (5 marks)

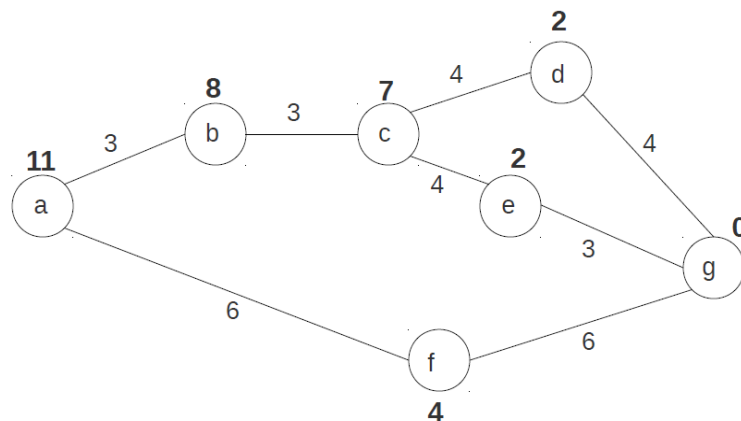
(e) Find a solution of the problem using Forward Checking. Assign variables using the following order a, x, y, z and assign values for each variable in increasing order. Detail the steps in the following table. (4 marks)

	a	x	y	z
domains found in question (d)				
a ← 5	5			

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**QUESTION 7 (10 marks)**


(a) Is the heuristic in this network admissible? Is the heuristic consistent? (4 marks)

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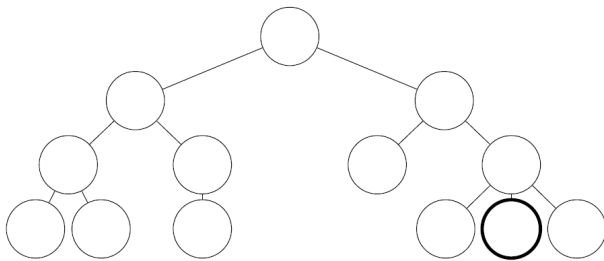
(b) Apply the A\* tree search algorithm to find the shortest path(s) from a to g. In the table below, make the following entry for each step of the algorithm (6 marks):

- When node N is expanded, write: expand N
- When the value of node N is (re)-calculated, write N:  $g(N) + h(N) = f(N)$

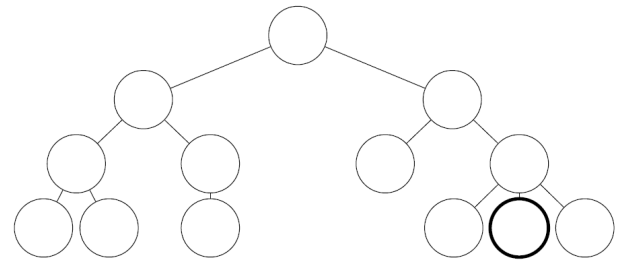
Expand a	

**QUESTION 8 (8 marks)**

(a) Give the order in which the nodes are visited depending on the search algorithm. The bold node is the goal node, the search should stop once the goal node is reached. Use the following conventions: when a direction of exploration is needed, navigate from left to right in the tree (i.e. always start from the left at a given depth). (4 marks)



Depth-first search

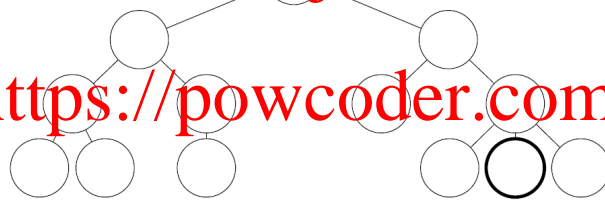


Breadth-first search

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Iterative deepening depth-first search

(b) Is it possible to draw a tree where iterative deepening depth-first search reaches the goal node faster than breadth-first search? Justify your answer. (2 marks)

(c) Is it possible to draw a tree where iterative deepening depth-first search reaches the goal node faster than depth-first search? Justify your answer. (2 marks)

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