Question 2

Consider the following 3-bit image.

2	3	0	4	0
4	2	4	5	7
3	3	6	5	5
1	3	6	1	7
1	2	3	1	5

Fig. 1 Original image

(a) (18 marks) Find a point operation that equalizes this image. Tabulate the point operation. Note that the output image can only have integer values. Draw the histogram of the input and the output image. Show detailed steps.

(b) (12 marks) Determine a point operation that will match the input image shown in Fig. 1 to the following histogram. Show detailed steps.

Gray-level	H
0	0
1	3
2	2
3	3
4	3
5	6
6	5
7	3

Fig. 2 Desired histogram

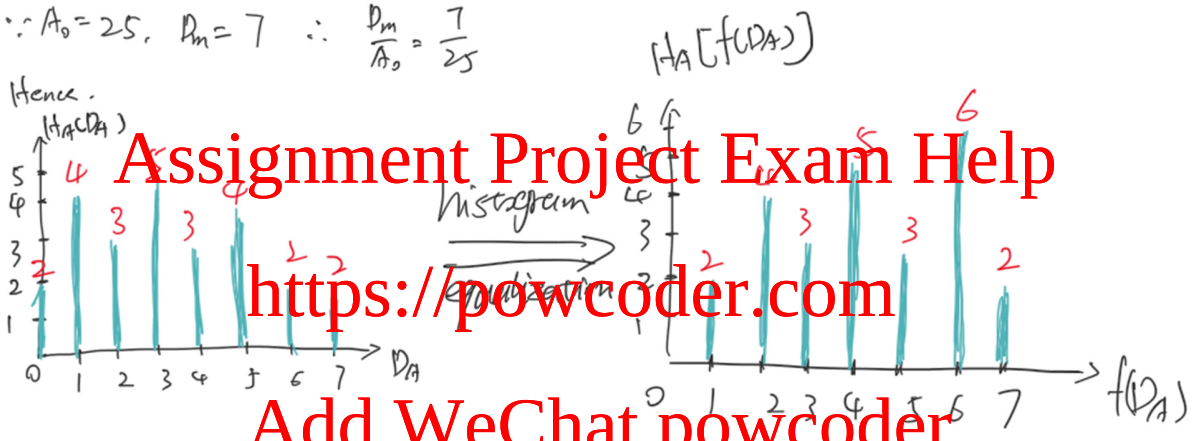
Q_2:

(a)

D_A	$H_A(D_A)$	$\sum_{i=0}^{D_A} H_A(D_A)$	$f(D_A)$	Discretized $f(D_A)$
0	2	2	0.56	1
1	4	6	1.68	2
2	3	9	2.52	3
3	5	14	3.92	4
4	3	17	4.76	5
5	4	21	5.88	6
6	2	23	6.44	6
7	2	25	7	7

$\because A_0 = 25,\ D_m = 7 \quad \therefore \frac{D_m}{A_0} = \frac{7}{25}$

Hence,



(b) we denote the pixel value in $Q(b)$ as D_C.
denote an intermediate equalized histogram as D_B.
Hence. $f_1(D_A) = D_B$. $f_2(D_C) = D_B \Rightarrow D_C = f_2^{-1} f_1(D_A)$

D_A	$H_A(D_A)$	D_B
0	2	1
1	4	2
2	3	3
3	5	4
4	3	5
5	4	6
6	2	6
7	2	7

D_C	$H_C(D_C)$	$\sum_{i=0}^{D_C} H_C(D_C)$
0	0	0
1	3	3
2	2	5
3	3	8
4	3	11
5	6	17
6	5	22
7	3	25

$f_2(D_C)$	D_B
0	0
0.84	1
1.4	1
2.24	2
3.08	3
4.76	5
6.16	6
7	7

D_B	$D_C = f_2^{-1}(D_B)$
0	0
1	1
2	3
3	4
4	4
5	5
6	6
7	7

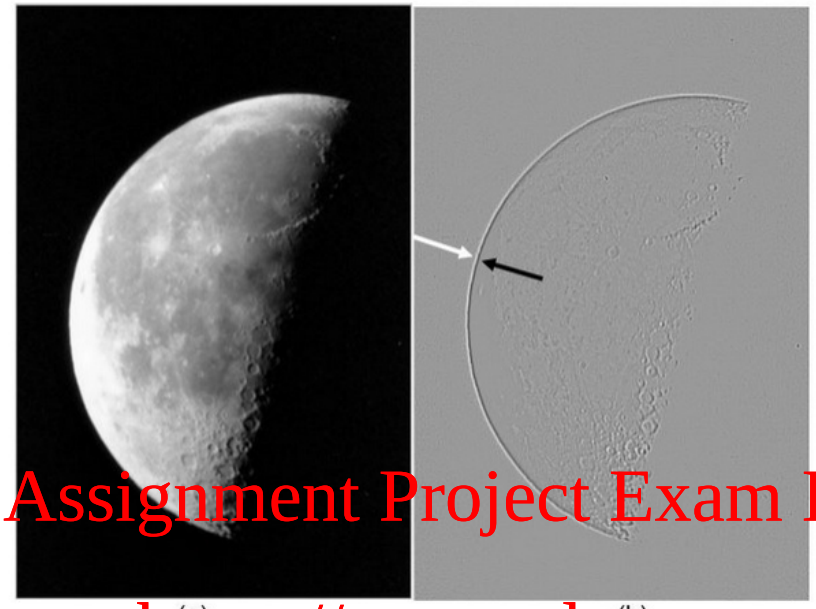
Consequently,

D_A	$f_1(D_A) = D_B$	$D_C = f_2^{-1} f_1(D_A)$
0	1	1
1	2	3
2	3	4
3	4	4
4	5	5
5	6	6
6	6	6
7	7	7

point operation that will match the D_A to D_C.

Question 3

(a) (5 marks) In Fig. 3(b), the edge detected by the Laplacian filter is white on the left side (white arrow) and black on the right side (black arrow). Explain this effect by drawing an intensity profile and the corresponding output by the Laplacian filter.



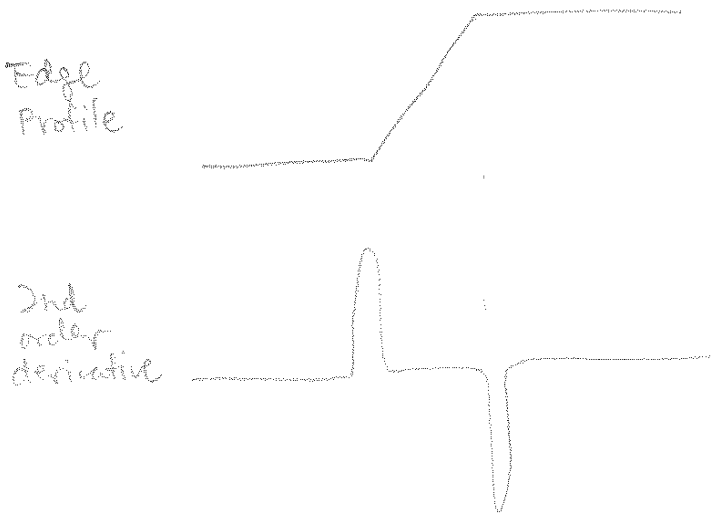
(a) (b)

Fig. 3 (a) The original image; (b) The image obtained using the Laplacian filter.

The Laplacian operation is expressed in the following equation:

$$\nabla^2 I(x,y) = I(x-1,y-1) + I(x,y-1) + I(x+1,y-1) + I(x-1,y) + I(x+1,y) + I(x-1,y+1) + I(x,y+1) + I(x+1,y+1) - 8I(x,y)$$

(a) Laplacian is a second-order derivative filter



the left of the edge is associated with a positive 2nd order derivative, therefore shown as white in the Laplacian-filtered image. The right of the edge is associated with a negative 2nd order derivative, therefore shown as black in the Laplacian-filtered image.

(b) (15 marks)

Determine the discrete-time Fourier transform of the Laplacian filter L(u,v). x and u are horizontal position and frequency, respectively, whereas y and v are vertical position and frequency, respectively. Plot the magnitude of the 1D profiles L(u,0) and L(0,v). Is L(u,v) a highpass or a lowpass filter? Explain your answer

(c)

(c) (15 marks)

The unsharp masking and highboost filtering operations are defined by the following equations:

$$g_{mask}(x,y) = f(x,y) - \overline{f}(x,y)$$

$$g(x,y) = f(x,y) + k g_{mask}(x,y)$$

where k is a constant.

The blurred image $\overline{f}(x,y)$ is generated by a correlation operation with the following weighted average kernel:

$$\frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$g(x,y)$ can be obtained by performing ONE correlation operation between a 3x3 computational molecule $h(x,y)$ and $f(x,y)$ (i.e., $g(x,y) = h \star f(x,y)$). Determine $h(x,y)$ and express your answer in terms of k. Illustrate your steps in determining $h(x,y)$. State all properties of spatial filtering that you used to arrive at your answer.

(b)

$$L(u,v) = \sum_y \sum_x l(x,y)\, e^{-j2\pi ux}\, e^{-j2\pi vy}$$

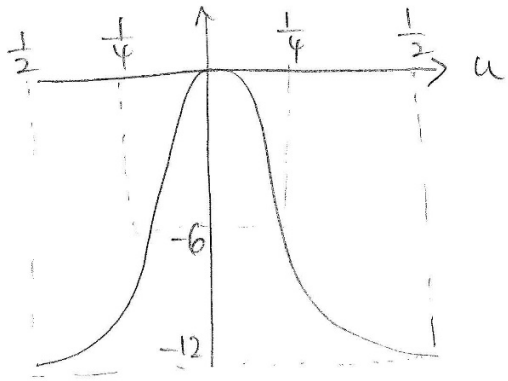
y \ x	-1	0	1
1	1	1	1
0	1	-8	1
1	1	1	1

$$L(u,v) = -8 + e^{j2\pi u} e^{j2\pi v} + e^{j2\pi v} + e^{-j2\pi u} e^{j2\pi v} + e^{j2\pi u} + e^{-j2\pi v} + e^{j2\pi u} e^{-j2\pi v} + e^{-j2\pi u} e^{-j2\pi v}$$

$$L(u,0) = -8 + e^{j2\pi u} + e^{-j2\pi u} + e^{j2\pi u} + e^{-j2\pi u} + e^{j2\pi u} + e^{-j2\pi u}$$

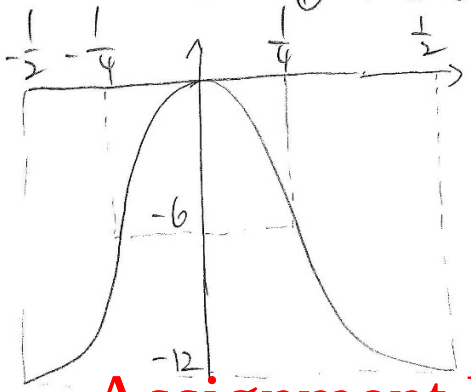
$$= -6 + 3e^{j2\pi u} + 3e^{-j2\pi u}$$

$$= -6 + 6\cos(2\pi u)$$



$$L(0,\nu) = -8 + e^{j2\pi\nu} + e^{j2\pi\nu} + e^{j2\pi\nu} + 1 + 1 + e^{-j2\pi\nu} + e^{-j2\pi\nu} + e^{-j2\pi\nu}$$

$$= -6 + 6\cos(2\pi\nu)$$



The Laplacian filter is a high-pass filter. The magnitude of the DTFT spectrum is high at high freq whereas the magnitude is low at low frequencies.

(c) $g_{mask} =$ (0 0 0 / 0 1 0 / 0 0 0) $-\frac{1}{16}$ (1 2 1 / 2 4 2 / 1 2 1) $* f$

$$g = \left[\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} + k \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{16} \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \right) \right] * f$$

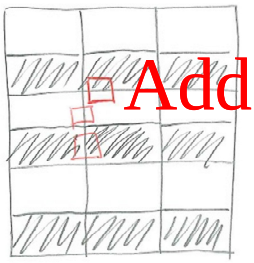
$$h = \begin{array}{|c|c|c|} \hline -\frac{k}{16} & -\frac{k}{8} & -\frac{k}{16} \\ \hline -\frac{k}{8} & k - \frac{k}{4} & -\frac{k}{8} \\ \hline -\frac{k}{16} & -\frac{k}{8} & -\frac{k}{16} \\ \hline \end{array}$$

Question 4

(a) (b)

The original image shown in Fig. 4(a) was filtered by a Gaussian lowpass filter, resulting in the output image shown in Fig. 4(b).

Explain why there is a black streak on the top edge and a white streak at the bottom edge of Fig. 4(b). Also explain why the streak only appears on the top and bottom edges but not the left and right edges.



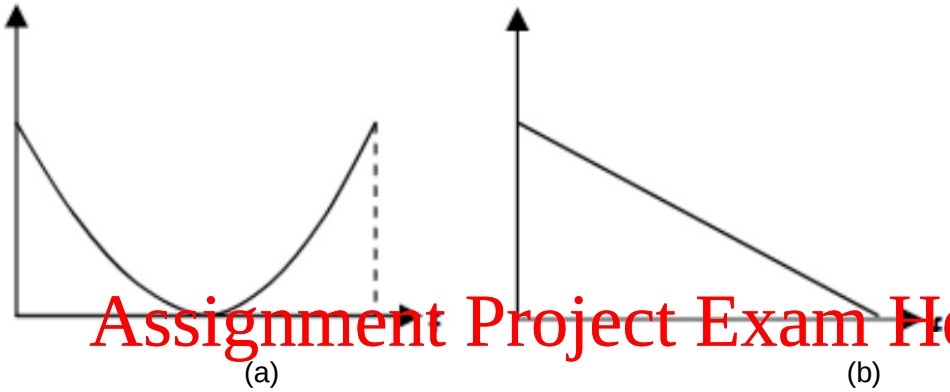
Question 4

Filtering in DFT domain is circular convolution in spatial domain

- Circular convolution with the Gaussian kernel averages black and white regions in the top and bottom (see figure). This explains the black and white streaks on top and bottom border
- Circular convolution averages regions of the same intensity in the right and left borders.

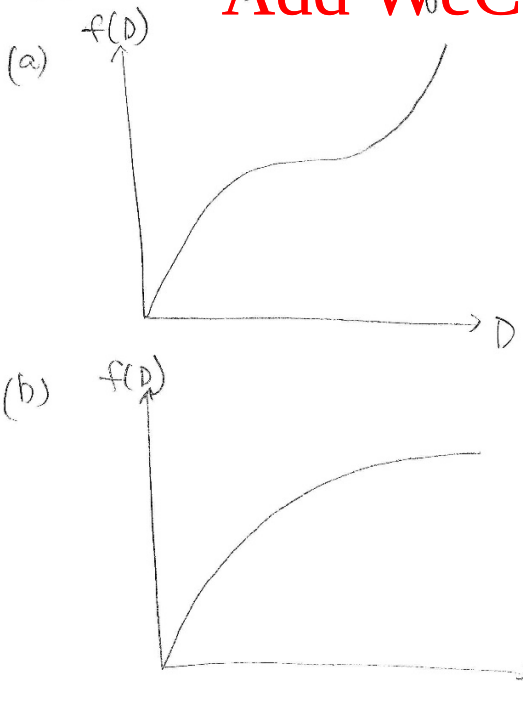
Question 5

The histograms of two example images are shown in fig. 5(a) and (b). Sketch the point operations required to flatten the histogram. Provide qualitative explanations of your plots. Note that a full mathematical derivation is not required as the purpose of this question is to assess your intuitive understanding of histogram equalization.



Question 5

Histogram equalization outputs the cummulative distribution of the histogram.



Question 6

Image A is an 8-bit image shown in Fig. 5a and has a histogram denoted by H_1. Image B shown in Fig. 5b is formed by swapping the top and the bottom halves of Image A and has a histogram denoted by H_2. What is the mathematical relationship between H_1 and H_2? Justify your answer.

Fig. 5a

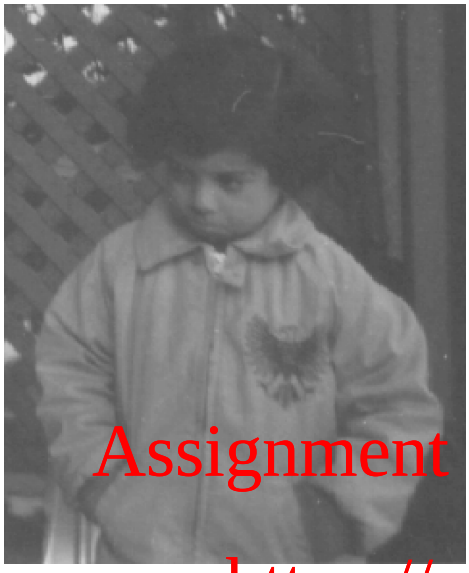


Fig. 5b

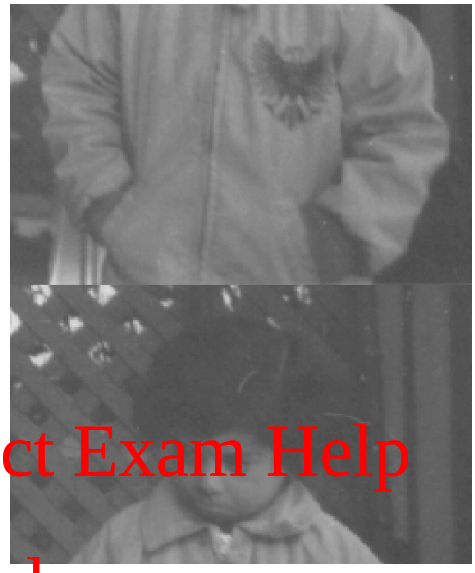


Fig. 5 Histograms of two example images.

Question 6

$H_1 = H_2$

Changing the spatial location of pixels would not change the histogram