Test 2 Solution

Question 2

(i) Degraded image (ii) Result of inverse filter

(a) (8 marks) Fig. (i) shows an image degraded by noise. Inverse filter is used to recover the image, producing the result shown in Fig. (ii). Explain mathematically why the degraded image cannot be recovered by the inverse filter.

(b) (9 marks) What can be done to the inverse filter to improve the result? Express your modification mathematically.

(c) (8 marks) Given that the power spectrums of the noise and the undegraded image are not known, can the Wiener filter still be used to recover the degraded image in Fig. 1(i)? If so, describe the workflow of the image recovery process. If not, provide your justification.

Solution:

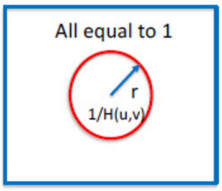
(a) $\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$

The noise is enhanced when $H(u,v)$ is close to zero.

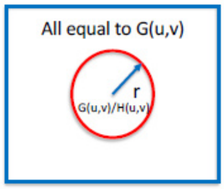
(b) Inverse filter can be improved by filtering only at low frequencies:

$$\hat{F}(u,v) = \begin{cases} \dfrac{G(u,v)}{H(u,v)} & \sqrt{u^2+v^2} \le r \\ G(u,v) & \sqrt{u^2+v^2} > r \end{cases}$$

Inverse Filter with high frequency components



$\hat{F}(u,v)$ Reconstructed Image Spectrum



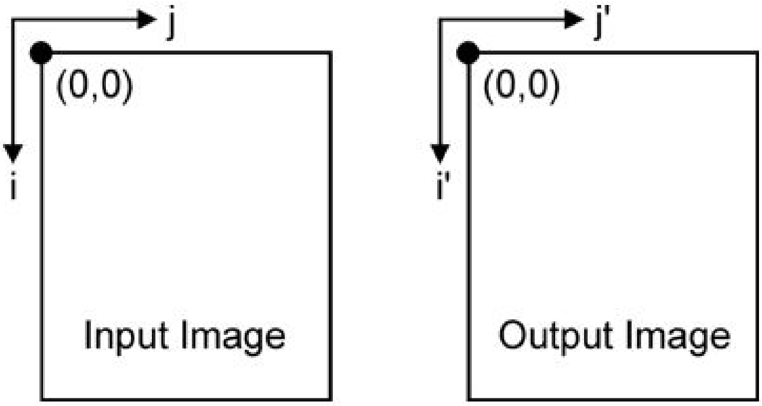
(c) Yes, the ratio between the spectrums of the undegraded image and noise can be approximated by a constant denoted by SNR. The Wiener filter becomes:

$$W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{1}{\mathrm{SNR}}}$$

SNR is tuned empirically for the optimum result.

Question 3

An input image $I(i, j)$ is mapped into the output image $I_0(i', j')$ by (i) first rotating the input image by 60° clockwise about the origin (0, 0) located at the upper left, (ii) then scale the image by a factor of $s_x = s_y = 0.7$ about the origin and (iii) finally translating the image 20 pixels to the right and 15 pixels up. The (i, j) and (i', j') coordinate systems are shown below.



(a) (9 marks) Express the output coordinates (i', j') in terms of the input coordinates (i, j).

(b) (9 marks) The Python built-in tool is used to perform the transformation obtained in Part a. A user is required to specify an affine transformation matrix T to generate an affine transformation matrix that is passed to the `warpaffine` function:


```
a = cv2.warpaffine(im, T, (width, height))
```


Determine `T` and show your steps.

(c) (12 marks) Find the coordinates of the input pixel (i, j), which would be mapped to the output pixel (i', j') = (200, 50). Find the gray level of this output pixel using nearest neighbour interpolation. Express your answer in terms of the gray level of the input image, I, at a location with integer indices.

(d) (8 marks) Find the gray level at (i, j) = (45.6, 100.7) by using bilinear interpolation. A table of gray level values in the input image is given below. Please note that this question does not have a relationship with Parts a-c.

$I(i, j)$
$I(45, 100) = 18$
$I(45, 101) = 45$
$I(46, 100) = 52$
$I(46, 101) = 36$
$I(47, 100) = 21$
$I(47, 101) = 47$
$I(44, 100) = 15$
$I(44, 101) = 40$

Question 3

(a)

$$\begin{bmatrix} i' \\ j' \end{bmatrix} = 0.7 \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} -15 \\ 20 \end{bmatrix}$$

$$= 0.7 \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} -15 \\ 20 \end{bmatrix}$$

(b)

$$x' = -0.7\frac{\sqrt{3}}{2}y + 0.35x + 20$$

$$y' = 0.35y + 0.7\frac{\sqrt{3}}{2}x - 15$$

$$x' = 0.35x - 0.7\frac{\sqrt{3}}{2}y + 20$$

$$y' = 0.7\frac{\sqrt{3}}{2}x + 0.35y - 15$$

The affine matrix

$$= \begin{bmatrix} 0.35 & -0.7\frac{\sqrt{3}}{2} & 20 \\ 0.7\frac{\sqrt{3}}{2} & 0.35 & -15 \end{bmatrix}$$

(c) $(i', j') = (200, 50)$

$$\frac{1}{0.7}\begin{bmatrix} i'+15 \\ j'-20 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i \\ j \end{bmatrix}$$

$$\frac{1}{0.7}\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i'+15 \\ j'-20 \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix}$$

$$\begin{bmatrix} i \\ j \end{bmatrix} = \frac{1}{0.7}\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 200+15 \\ 50-20 \end{bmatrix}$$

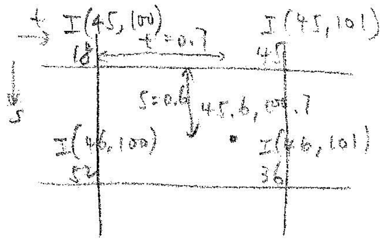
$$\begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} 116.5 \\ 287.4 \end{bmatrix}$$

The gray level of the output pixel using the nearest neighbourhood is $I(117, 287)$

(d) $I(45.6, 100.7)$

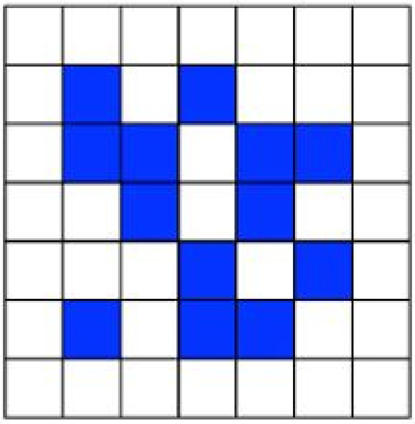
$$= (1-s)(1-t)\, I(45,100) + (1-s)(t)\, I(45,101) + (s)(1-t)\, I(46,100) + (s)(t)\, I(46,101)$$

$$= (0.4)(0.3)\, 18 + (0.4)(0.7)\, 45 + (0.6)(0.3)\, 52 + (0.6)(0.7)\, 36$$

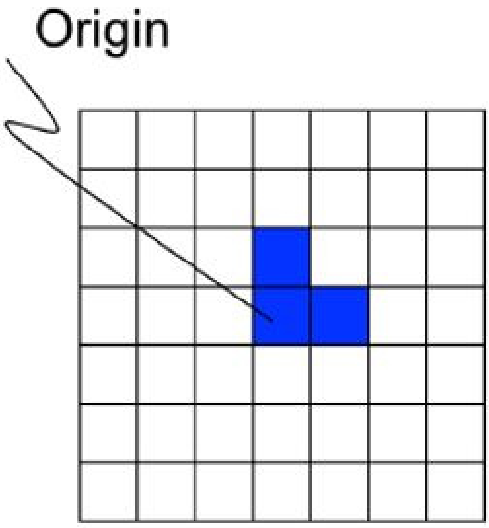


$$= 0.12(18) + 0.28(45) + 0.18(52) + 0.42(36) = \underline{39.24}$$

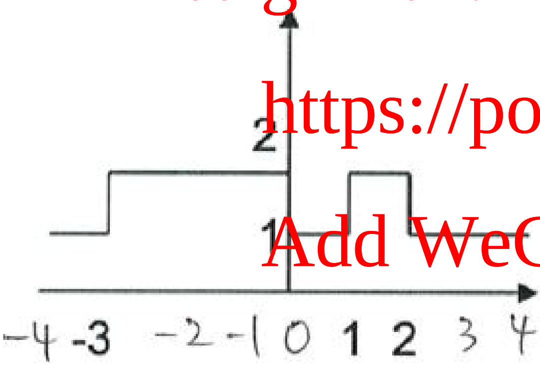
Question 4



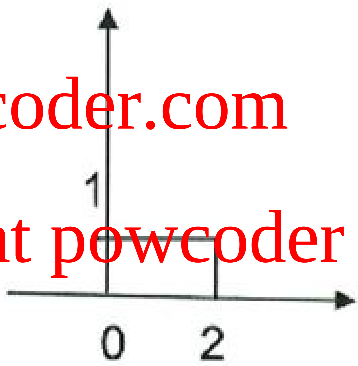
(i) F



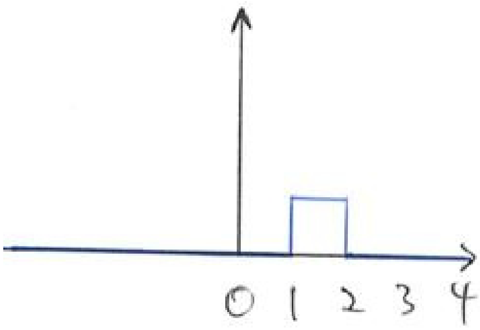
(ii) H



(iii) f



(iv) h



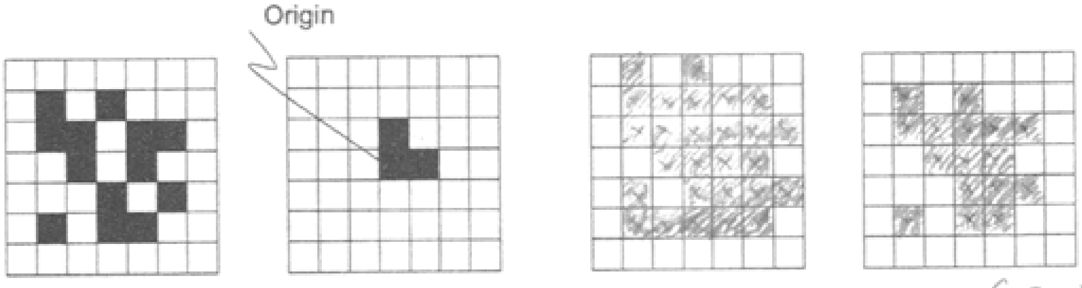
(v)

(a) (11 marks) Find the closing of the binary image F, shown in Fig. (i), by the structural element H, shown in Fig. (ii). Draw the intermediate and final results.

(b) (13 marks) Find the grayscale opening of the function f sketched in Fig. (iii) with the structural element h shown in Fig. (iv). Sketch the immediate and final results.

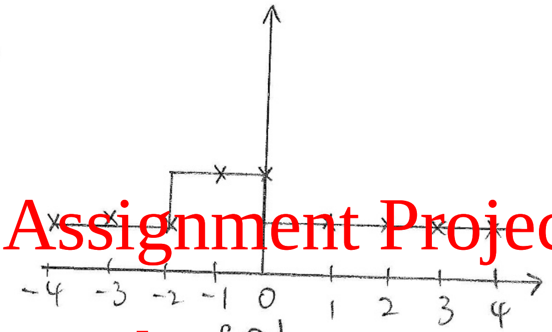
(c) (13 marks) Given the function f and the structural element h in Part b, identify an algorithm that outputs the function shown in Fig. (v). What is this algorithm called? List an application of this algorithm.

(a)

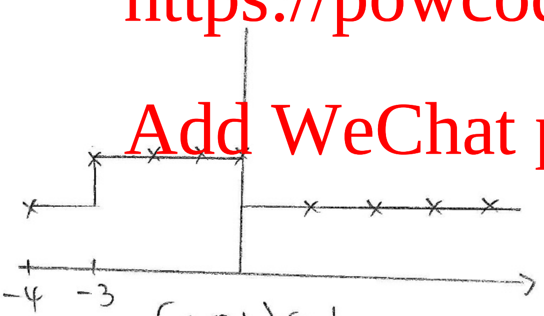


(a) F (b) H (c) $F \oplus H$ (d) $(F \oplus H) \ominus H$

(b)



$f \ominus h$



$f \circ h = (f \ominus h) \oplus h$

(c) Fig (v) is generated by $f - (f \circ h)$

This is the top-hat transformation.

One application of the top-hat transformation is to extract objects of interest from non-uniform background.