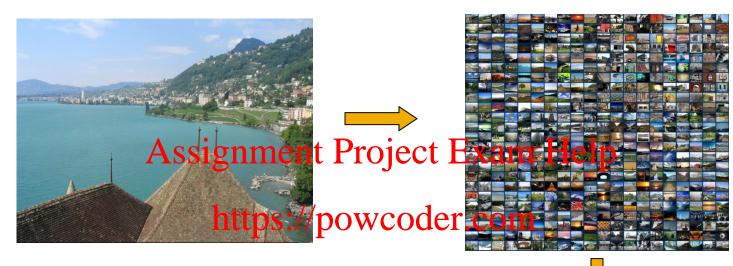
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Finding Similar Items: Locality & Semsitive Hashing

Adapted from slides of: Jure Leskovec, Anand Rajaraman, Jeff Ullman Stanford University





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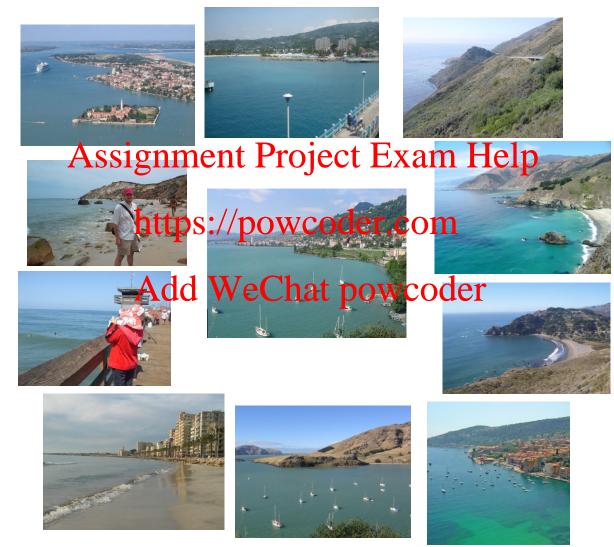








10 nearest neighbors from a collection of 20,000



10 nearest neighbors from a collection of 2 million

A Common Metaphor

- Many problems can be expressed as finding "similar" sets:
 - Find near-neighbors in high-dimensional space
- Examples:
 - Pages with sith ar words der.com
 - For duplicate detection classification by topic
 - Customers who purchased similar products
 - Products with similar customer sets
 - Images with similar features
 - Users who visited similar websites
 - DNA sequences with high similarity
 - For clustering, classification of functional entities

Problem formuation

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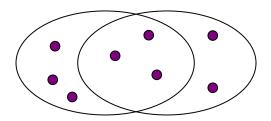
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Finding Similar Items

Distance Measures

- Goal: Find near-neighbors in high-dim. space
- We formally define "near neighbors" as points that are a "small distance" apart
- For each apparent on Pweet Framed to define what "distance" means coder.com
- Today: Jaccard distance/similarity
- The Jaccard similarity of two pets is the size of their intersection divided by the size of their union: $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$
- Jaccard distance: $d(C_1, C_2) = 1 |C_1 \cap C_2| / |C_1 \cup C_2|$



3 in intersection 8 in union Jaccard similarity= 3/8 Jaccard distance = 5/8

Task: Finding Similar

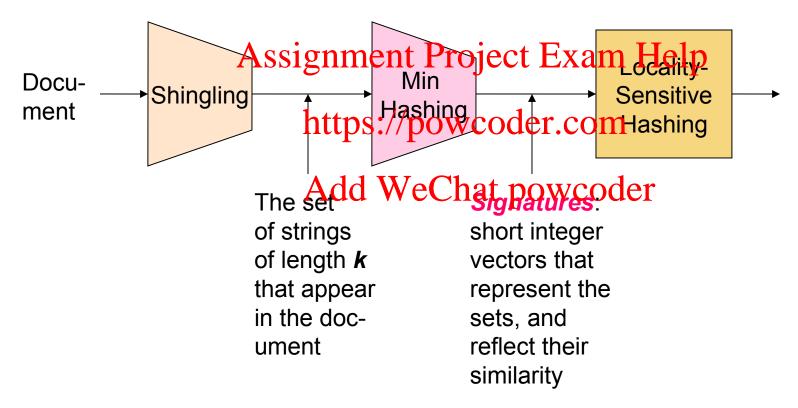
cuments

- Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs
- Applications:
 Assignment Project Exam Help
 Mirror websites, or approximate mirrors
 - Don't want the thosy betwinced en center
 - Similar news articles at many news sites
 Cluster articles by "same story
- Problems:
 - Many small pieces of one document can appear out of order in another
 - Too many documents to compare all pairs
 - Documents are so large or so many that they cannot fit in main memory

3 Essential Steps for Similar Docs

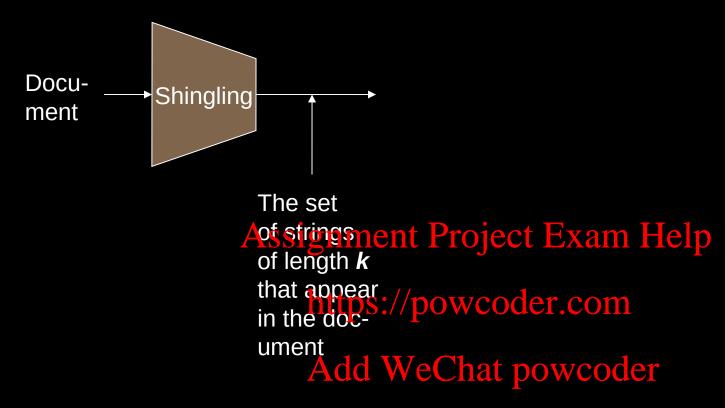
- 1. Shingling: Convert documents to sets
- 2. Min-Hashingin Convertelange sets to short signatures, while preserving similarity https://powcoder.com
- 3. Locality-Sensitive Floatings To Eus on pairs of signatures likely to be from similar documents
 - Candidate pairs!

The Big Picture



Candidate pairs

those pairs of signatures that we need to test for similarity



Shingling

Step 1: Shingling: Convert documents to sets

Documents as High-Dim Data

- Step 1: Shingling: Convert documents to sets
- Simple approaches Project Exam Help
 - Document = set of words appearing in document https://powcoder.com

 Document = set of "important" words
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- A different way: Shingles!

Define: Shingles

- A k-shingle (or k-gram) for a document is a sequence of k tokens that appears in the doc
 - Tokens easige releasta Ptoject Next and I show thing else, depending on the application https://powcoder.com
 Assume tokens = characters for examples
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- Example: k=2; document D₁ = abcab Set of 2-shingles: $S(D_1) = \{ab, bc, ca\}$
 - Option: Shingles as a bag (multiset), count ab twice: $S'(D_1) = \{ab, bc, ca, ab\}$

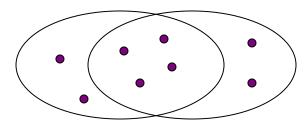
Compressing Shingles

- To compress long shingles, we can hash them to (say) 4 bytes
- Representsagdocumente by Ethan set pf hash values of its k-shingles https://powcoder.com
- **Example:** k=2; document D_1 = abcab Add WeChat powcoder Set of 2-shingles: $S(D_1)$ = {ab, bc, ca} Hash the singles: $h(D_1)$ = {1, 5, 7}

Similarity Metric for Shingles

- Document D_1 is a set of its k-shingles $C_1=S(D_1)$
- Equivalently, each document is a 0/1 vectorsignthenspace of Examinates
 - Each unique shingle is a dimension
- Vectors are very sparse
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 A natural similarity measure is the **Jaccard similarity:**

$$sim(D_1, D_2) = |C_1 \cap C_2|/|C_1 \cup C_2|$$

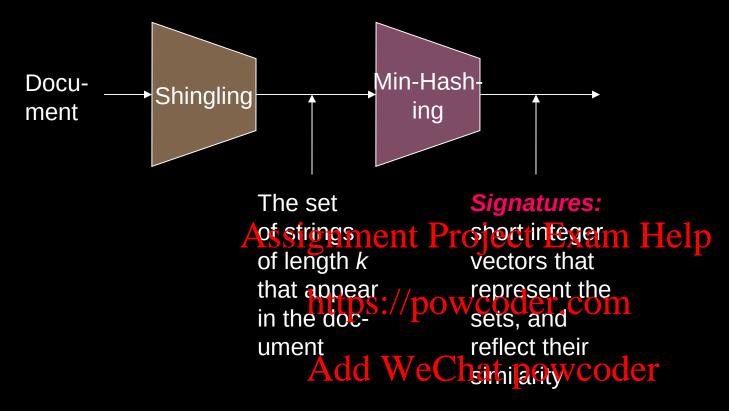


Working Assumption

- Documents that have lots of shingles in common have similar text, even if the text appears instifferent of the Exam Help
- Caveat: You must pick k large enough, or most documents will have hat our stringles
 - k = 5 is OK for short documents
 - k = 10 is better for long documents

Motivation for Minhash/LSH

- Suppose we need to find near-duplicate documents among N=1 million documents
- Assignment Project Exam Help
 Naïvely, we would have to compute pairwise
 Jaccard similarities for every pair of docs
 - N(N-1)/2Add **Wo**Chat powersoles
 - At 10⁵ secs/day and 10⁶ comparisons/sec, it would take 5 days
- For N = 10 million, it takes more than a year...

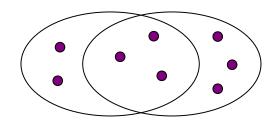


MinHashing

Step 2: Minhashing: Convert large sets to short signatures, while preserving similarity

Encoding Sets as Bit Vectors

Many similarity problems can be formalized as finding subsets that have significant intersection Assignment Project Exam Help Encode sets using 0/1 (bit, boolean) vectors



- - One dimension the selephent in the conversal set
- Interpret set intersection as bitwise AND, and Add WeChat powcoder set union as bitwise **OR**
- **Example:** $C_1 = 10111$; $C_2 = 10011$
 - Size of intersection = 3; size of union = 4,
 - **Jaccard similarity** (not distance) = 3/4
 - Distance: $d(C_1,C_2) = 1 (Jaccard similarity) = 1/4$

An example of the matrix

$$S_1 = \{a, d\}, S_2 = \{A\}dA = WeChatapowcodef\}.$$

It saves space to represent a sparse matrix of 0's and 1's by the positions in which the 1's appear

From Sets to Boolean

Matrices

- Rows = elements (shingles)
- Columns = sets (documents)
 - 1 in row e and column s if and only if e is a member of s
 - Column simulative in the corresponding sets (rows with value 1)
 - Typical matrix is sparse!
- Each document is a column:
 - **Example:** $sim(C_1, C_2) = ?$
 - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
 - $d(C_1,C_2) = 1 (Jaccard similarity) = 3/6$

	Documents								
1 j	<u>P</u> 1	1	1	0					
0	1	1	0	1					
	0	1	0	1					
ပါ။။မျှင်ခ	0	0	0	1					
Ö	1	0	0	1					
	1	1	1	0					
	1	0	1	0					

Outline: Finding Similar

- So far:
 - Documents → Sets of shingles
 - Representsigtmasnboodejantvextonslielanmatrix
- Next goal: Find similar columns while https://powcoder.com computing small signatures
 Add WeChat powcoder
 Similarity of columns == similarity of signatures

Outline: Finding Similar

- **Next Goal: Find similar columns, Small signatures**
- Naïve approach:
 - 1) Signatures of columns
 - 2) Examine pairs of signatures to find similar columns

 ttps://powcoder.com

 Essential: Similarities of signatures and columns are related
 - 3) Optional: Abethythethallymms with similar signatures are really similar
- Warnings:
 - Comparing all pairs may take too much time: Job for LSH
 - These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures)

- Key idea: "hash" each column C to a small signature h(C), such that:
 - (1) h(C) is small enough that the signature fits in RAM Assignment Project Exam Help
 - (2) $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$ is the same as the "similarity" of signatures

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- Goal: Find a hash function h(·) such that:
 - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
 - If sim(C₁,C₂) is low, then with high prob. h(C₁) / h(C₂)
- Hash docs into buckets. Expect that "most" pairs of near duplicate docs hash into the same bucket!

Min-Hashing

- Goal: Find a hash function h(·) such that:
 - if $sim(C_1, C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
 - if sim(C^A₁,ε₂)gs to ω, t Repi with Frigh Hebb. h(C₁) / h(C₂)

https://powcoder.com

- Clearly, the hash function depends on Add WeChat powcoder the similarity metric:
 - Not all similarity metrics have a suitable hash function
- There is a suitable hash function for the Jaccard similarity: It is called Min-Hashing

Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation π
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 Define a "hash" function $h_{\pi}(C)$ = the index of the https://powcoder.com first (in the permuted order π) row in which column C has Availue (Chat powcoder $h_{\pi}(C) = min_{\pi} \pi(C)$
- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Example

Element	S_1	S_2	S_3	S_4	_	Eleme	ent	S_1	S_2	S_3	S_4
a	1	0	0	1	_	b		0	0	1	0
\boldsymbol{b}	0 🛕	Ssig	nim	efit F	Proje	ect Ex	kam	Не	18	1	0
\boldsymbol{c}	0	1	0	1		a		1	76	0	1
d	1	0 1	iting	·Anc	WCC	oder de	าดท	1	0	1	1
e	0	0	1	.′ ₀ PC		C		0	1	0	1

Apply one permutation. Add WeChat powcoder

h(S1) = a; h(S2)=c; h(S3)=b; h(S4)=a)

Element	S1	S2	S3	S4
h	a	С	b	a

permutate

Another example

Element	S_1	S_2	S_3	S_4		Element	S_1	S_2	S_3	S_4
1	1	0	0	1]	2	1	0	1	0
2	1 /	Assi	gnn	nent	Proj	ect Exar	nH	elp	0	0
3	0	1	1	0		6	0	Ō	1	1
4	0	0	http	sy/j	owc	oder.com	n_1	0	0	1
5	1	0	0	0		4	0	0	1	1
6	0	0	Add	1 W	eC ha	it powco	der	1	1	0

$$m(S_1) = 2$$

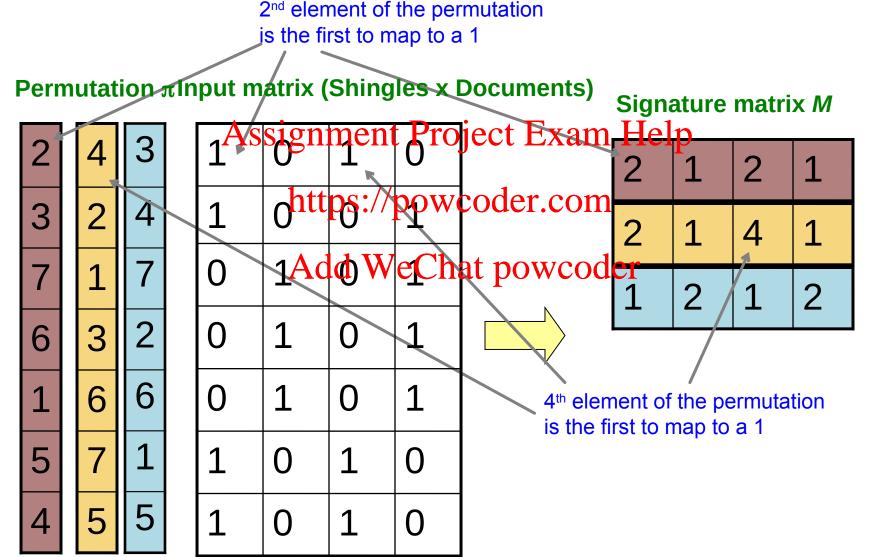
 $m(S_2) = 3$
 $m(S_3) = 2$
 $m(S_4) = 6$

Min-Hashing Example

Note: Another (equivalent) way is

to

store row indexes: soft permutation



The Min-Hash Property

- Choose a random permutation π
- Claim: $Pr[h_{\pi}(C_1) = h_{\pi}(C_2)] = sim(C_1, C_2)$
- Why? Assignment Project Exam Help
 - Let X be a doc (set of shingles), y∈X is a shingle https://powcoder.com
 Then: Pr[π(y) = min(π(X))] = 1/[X]
 - - It is equally like Adoat Wre Colorate the min element
 - Let y be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
 - Then either: $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$

One of the two cols had to have 1 at position y

()

- So the prob. that **both** are true is the prob. $\mathbf{y} \in C_1 \cap C_2$
- $\Pr[\min(\pi(C_1)) = \min(\pi(C_2))] = |C_1 \cap C_2| / |C_1 \cup C_2| = sim(C_1, C_2)$

 \cap

N

Four Types of Rows

• Given cols C₁ and C₂, rows may be classified as:

```
    C<sub>1</sub>
    A
    Assignment Project Exam Help
    B
    C
    https://powcoder.com
    D
    Add WeChat powcoder
    a = # rows of type A, etc.
```

- Note: $sim(C_1, C_2) = a/(a + b + c)$
- Then: $Pr[h(C_1) = h(C_2)] = Sim(C_1, C_2)$
 - Look down the cols C₁ and C₂ until we see a 1
 - If it's a type-A row, then $h(C_1) = h(C_2)$ If a type-B or type-C row, then not

Similarity for Signatures

- We know: $Pr[h_{\pi}(C_1) = h_{\pi}(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions
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- The similarity of two signatures is the fraction of the hash functions in which they agree

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- Note: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hashing Example

Permutation π Input matrix (Shingles x Documents) Signature matrix *M* Project Exam Hel 3 4 powcoder.com that powcoder 6 ()**Similarities:** 6 0 2-4 1-2 1-3 5 ()0.75 0.75 Col/Col

\$ig/Sig

1.00

Min-Hash Signatures

- Pick K=100 random permutations of the rows
- Think of sig(C) as a column vector
- sig(C)[i] Assigned in Pto the kth permutation, the index of the first row that has a 1 in column C

- $sig(C)[i] = min_{Add}(C)$ Note: The sketch (signature) of document C is small ~100 bytes!
- We achieved our goal! We "compressed" long bit vectors into short signatures

Implementation Trick

- Permuting rows even once is prohibitive
- Row hashing!
 - Pick K = 100 hash functions k Exam Help
- Ordering under k, gives a random row permutation!
 https://powcoder.com
 One-pass implementation
- - For each column Care hashpawe. Reep a "slot" for the min-hash value
 - Initialize all sig(C)[i] = ∞
 - Scan rows looking for 1s
 - Suppose row j has 1 in column C
 - Then for each k;:
 - If $k_i(j) < sig(C)[i]$, then $sig(C)[i] \leftarrow k_i(j)$

How to pick a random hash function h(x)? **Universal hashing:**

 $h_{a,b}(x) = ((a \cdot x + b) \mod p) \mod N$ where:

a,b ... random integers $p \dots prime number (p > N)$

Implementation

The hash function takes the row index as input, and outputs another "row index" (simulating row permutation). There are n hash functions For each row r, do the following:

- 1. Compute $h_1(r), h_2(r), \dots, h_n(r)$ Project Exam Help
- For each column c do the following:
 - (a) If c has 0 in relations in relations were con-
 - (b) However, if c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to the smaller of the current while of SIG(i, c) and $h_i(r)$.

Row	S_1	S_2	S_3	S_4	$x+1 \mod 5$	$3x + 1 \mod 5$
0	1	0	0	1	1	1
1	0	0	1	0	2	4
2	0	1	0	1	3	2
3	1	0	1	1	4	0
4	0	0	1	0	0	3

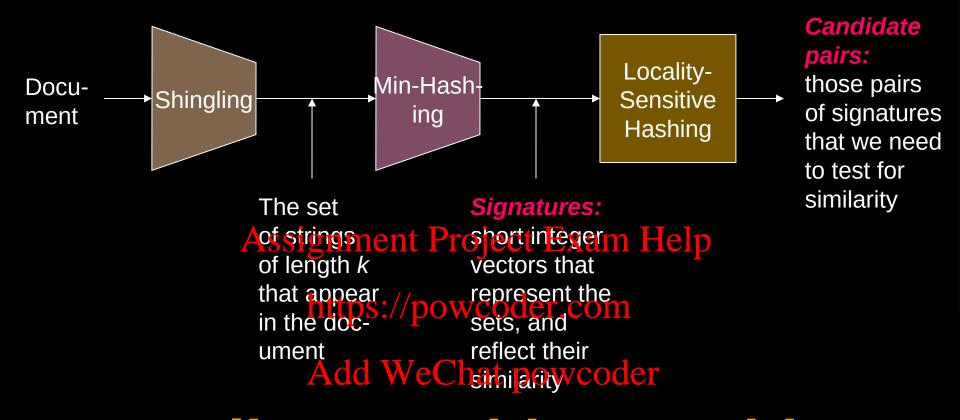
- 1. Compute $h_1(r), h_2(r), ..., h_n(r)$.
- For each column c do the following:
 - (a) If c has 0 in row r, do nothing.
 - (b) However, if c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to the singular part of the projective for the projective c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r, then for each i = 1, 2, ..., n set SIG(i, c) to c has 1 in row r.

	Row	S	ttos	:#/r	ÓW	codemetin	3x+1 me	od 5
I	0	1	0	0	1	1	1	
	1	0	0	117	0	2	4	
	2	₀ A	laa	W 6	e G n	at powcoo	ner ₂	
	3	1	0	1	1	4	0	
	4	0	0	1	0	0	3	

	S_1	S_2	S_3	S_4
h_1	∞	∞	∞	∞
h_2	∞	∞	∞	∞

	S_1	S_2	S_3	S_4
h_1	1	∞	2	1
h_2	1	∞	4	1

	S_1	S_2	S_3	S_4
h_1	1	3	2	1
h_2	1	2	4	1



Locality Sensitive Hashing

Step 3: Locality-Sensitive Hashing:

Focus on pairs of signatures likely to be from similar documents

Original references

LSH was developed by Indyk and Motwani (Indyk and Motwani, 1998) and later refined by Gionis anghoworkgest (Sionis Jetpal., 1999) for solving high-dimensional computational https://powcoder.com/geometry problems such as finding nearest neighbors
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LSH: First Cut

2 1 4 1
1 2 1 2
2 1 2 1

- Goal: Find documents with Jaccard similarity at least s (for some similarity threshold, e.g., s=0.8) Assignment Project Exam Help
- LSH General idea: Use a function f(x,y) that tells https://powcoder.com/whether x and y is a candidate pair: a pair of elements whose small arttpowest be evaluated
- For Min-Hash matrices:
 - Hash columns of signature matrix M to many buckets
 - Each pair of documents that hashes into the same bucket is a candidate pair

Candidates from Min-Ha

```
2 1 4 1
1 2 1 2
2 1 2 1
```

- Pick a similarity threshold s (0 < s < 1)</p>
- Columns x and y of M are a candidate pair if their signatures a greecode at least fraction s of their rows:

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 M (i, x) = M (i, y) for at least frac. s values of i
 - We expect documents x and y to have the same (Jaccard) similarity as their signatures

LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

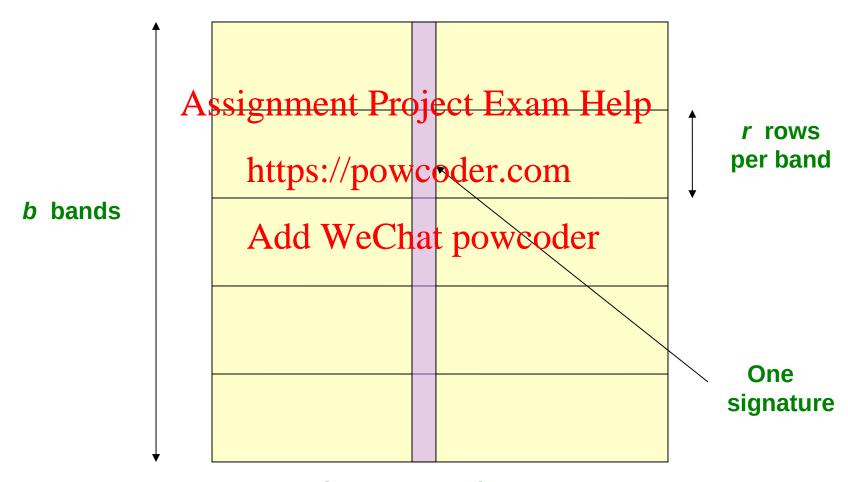
Big idea: Hash columns of signature matrix M several times

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- Arrange that (only) similar columns are https://powcoder.com likely to hash to the same bucket, with high probability WeChat powcoder
- Candidate pairs are those that hash to the same bucket

Partition M into b Band

2 1 4 1
1 2 1 2
2 1 2 1



Signature matrix *M*

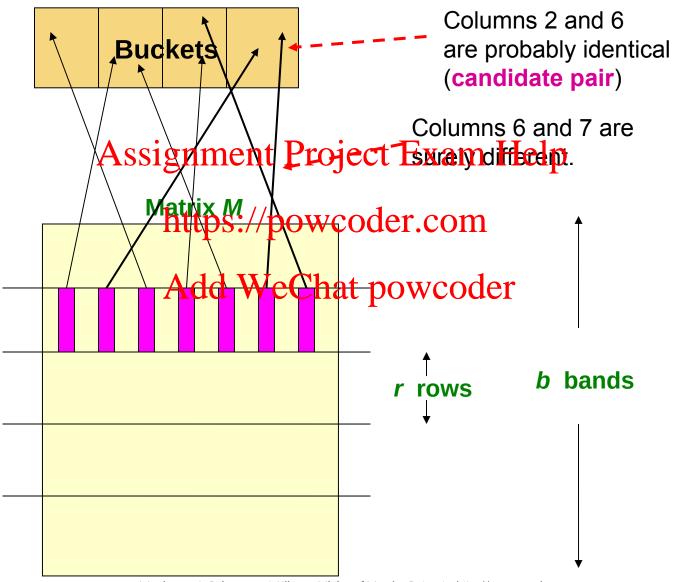
Partition M into Bands

- Divide matrix M into b bands of r rows
- For each band hash its portion of each column to a hash table with k buckets https://powcoder.com
 Make **k** as large as possible

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- Candidate column pairs are those that hash to the same bucket for ≥ 1 band
- Tune b and r to catch most similar pairs, but few non-similar pairs

Hashing Bands



Simplifying Assumption

- There are **enough buckets** that columns are unlikely to hash to the same bucket unless they are **identical** image articular band
- https://powcoder.com
 Hereafter, we assume that "same bucket"
 means "identical in Grant Page der
- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

2	1	4	1
1	2	1	2
2	1	2	1

Assume the following case:

- Suppose 100,000 columns of M (100k docs)
- Signatures of nhohintegers (xows)elp
- Therefore, signatures take 40Mb https://powcoder.com
- Choose b = 20 bands of r = 5 integers/band Add WeChat powcoder
- **Goal:** Find pairs of documents that are at least *s* = 0.8 similar

C₁, C₂ are 80% Similar

```
2 1 4 1
1 2 1 2
2 1 2 1
```

- Find pairs of s=0.8 similarity, set b=20, r=5
- **Assume:** $sim(C_1, C_2) = 0.8$
 - Since sim(C, C) s we want C C to be a candidate pair: We want them to hash to at least 1 common bucket (at least one bandlissidentical der.com
- Probability C., C. identical in one particular band: (0.8)⁵ = 0.328 Chat powcoder
- Probability C_1 , C_2 are **not** similar in all of the 20 bands: $(1-0.328)^{20} = 0.00035$
 - i.e., about 1/3000th of the 80%-similar column pairs are false negatives (we miss them)
 - We would find 99.965% pairs of truly similar documents

C, are 30% Similar

- **Find pairs of** *s***=**0.8 similarity, set **b=**20, **r=**5
- **Assume:** $sim(C_1, C_2) = 0.3$
 - Since sim(C, C) < s we want C, C to hash to NO common buckets (all bands should be different)
- Probability Chttpsidenticalin come particular band: $(0.3)^5 = 0.00243$ Probability C_1 , C_2 identical in at least 1 of 20
- bands: $1 (1 0.00243)^{20} = 0.0474$
 - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming candidate pairs
 - They are false positives since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold s

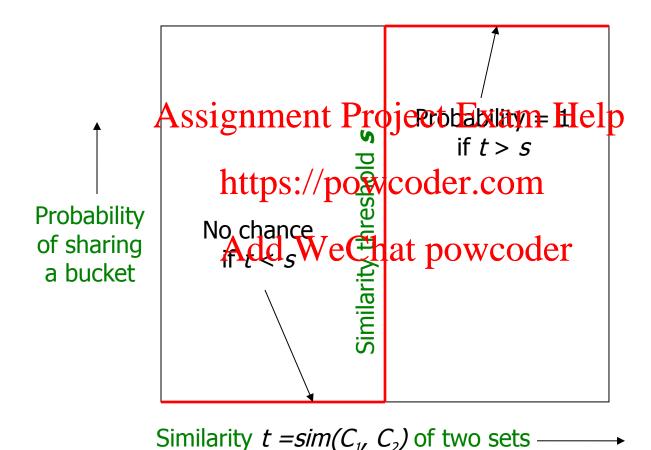
LSH Involves a Tradeof

2 1 4 1
1 2 1 2
2 1 2 1

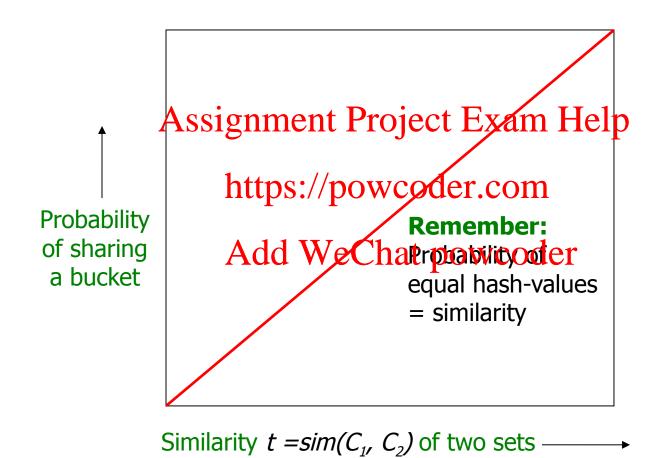
Pick:

- The number of Min-Hashes (rows of M)
- The number of th
- The numbernatory The nu
- Example: If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Analysis of LSH – What We Want



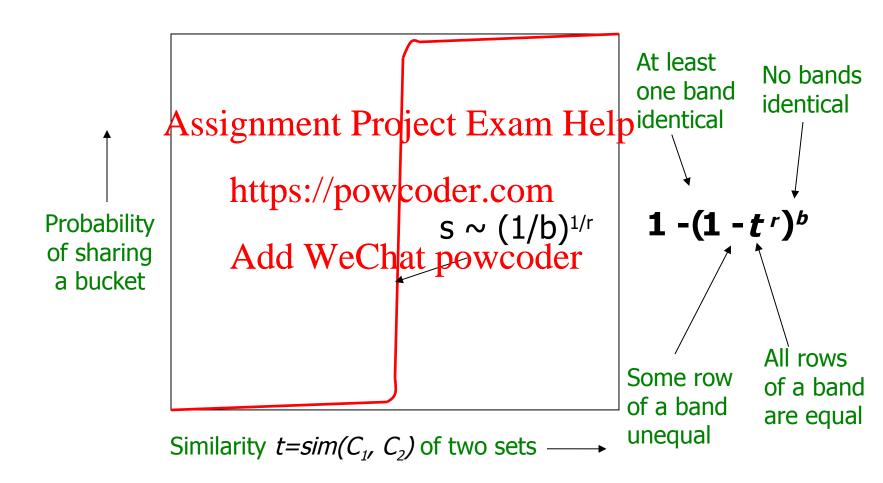
What 1 Band of 1 Row Gives



b bands, r rows/band

- Columns C₁ and C₂ have similarity t
- Pick any band (r rows)
 - Prob. that signment Project Exam Help
 - Prob. that same row in band unequal = 1 tr
- Prob. that no band identical = $(1 t^r)^b$
- Prob. that at least 1 band identical = 1 (1 t^r)^b

What b Bands of r Rows Gives You



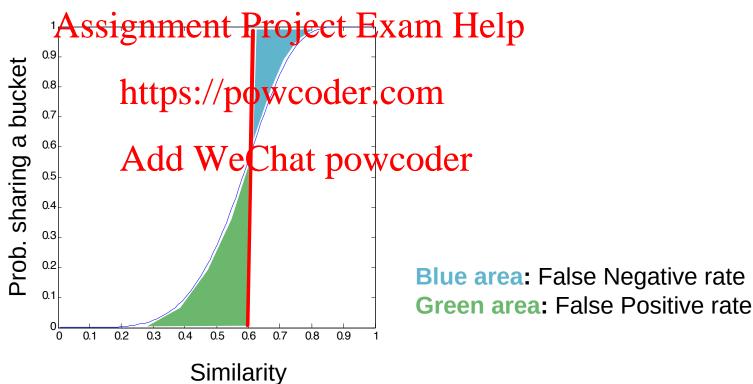
Example: b = 20; r = 5

- Similarity threshold s
- Prob. that at least 1 band is identical:

Assig	n m en	t Project) Exam F	lelp
h	tt <mark>3</mark> s://	powedder.com	
A	3 dd W .4	047 <mark>eChat powcoder</mark> .186	
	.5	.470	
	.6	.802	
	.7	.975	
	.8	.9996	

Picking r and b: The S-curve

- Picking r and b to get the best S-curve
 - 50 hash-functions (r=5, b=10)



LSH Summary

- Tune M, b, r to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures Assignment Project Exam Help
- Check in mahtime movy that candidate pairs really do have similar signatures
- Optional: In another pass through data, check that the remaining candidate pairs really represent similar documents

Summary: 3 Steps

- Shingling: Convert documents to sets
 - We used hashing to assign each shingle an ID
- Min-Hashing: Convert large sets to short signatures, while preserving similarity https://powcoder.com.
 We used similarity preserving hashing to generate
 - We used similarity preserving hashing to generate signatures with dr_{T} by C_{T} by C_{T} $\text{C}_{\text{T$
 - We used hashing to get around generating random permutations
- Locality-Sensitive Hashing: Focus on pairs of signatures likely to be from similar documents
 - We used hashing to find candidate pairs of similarity s



日本語要約

Assembling large genomes with single-molecule sequencing and locality sensitive haldling

Konstantin Berlin, Sergey katep Style pobwedin emen prake, Jane M Landolin & Adam M Phillippy

Affiliations | Contributions Add We Chath powcoder

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