Definition of probability

Probability is a numerical measure of the likelihood that an event will occur.

Notation: The probability of an event E is written P(E) and pronounced "P of E"

Scale: $0 \le P(E) \le 1$

If P(E) = 0, event E cannot occur

If P(E) = 1, event E is certain to occur

Our focus in this review is to see the fundamental method of computing P(E)

uregina.ca/~gingrich/e224s1708.ppt

Terms for probability

- Probability
- Likelihood
- Odds
- Chances
- Random selection
- Random or stochastic processes

Experiments and outcomes

An experiment is a process that generates well-defined outcomes

- Tossing a coin
- Rolling a single die or a pair of dice
- Applying for a job
- Salary expected after completing a degree
- Drilling an oil well
- Federal election

these are sometimes termed *random experiments*.

Classical method

Experiment (eg. Flip of a coin – outcome of H or T)

- Exact outcome is unknown before conducting experiment
- All possible outcomes of experiment are known
- Each outcome has a probability
- Experiment can be repeated under uniform conditions

Together these conditions produce regularities or patterns in outcomes

Sample space

- An **experiment** is a process that generates well-defined outcomes.
- The **sample space**, S, for an experiment is the set of all experimental outcomes.
- Each of these outcomes is referred to as a **sample point**. For an experiment with n sample points, label these E_1, E_2, E_3, … , E_n.
- The sample space of an experiment is the set of all sample points, ie. S = {E_1, E_2, E_3, … , E_n}.
  - E.g. toss a coin, S={head, tail}
  - Roll a single die, S={1,2,3,4,5,6}
  - Federal election, S={candidate 1, candidate 2}
  - Roll a pair of dice, what is the sample space?

Two requirements for assigning probabilities

- If E_i is the ith outcome of an experiment, then its probability is no less than zero and no greater than 1. That is, for all sample points or outcomes, E_i, $0 \leq P(E_i) \leq 1$, for $i = 1, 2, \ldots n$.
- The sum of the probabilities of all the outcomes equals 1, that is,

  $P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n) = 1$ or

$$\sum_{i=1}^{i=n} P(E_i) = 1$$

Examples of sample spaces

- Flip a coin (n = 2). S = {H, T}, where H = head, T= tail.

  P(H) = 1/2 = 0.50 and P(T) = 1/2 = 0.50.

  P(H) + P(T) = 0.50 + 0.50 = 1.

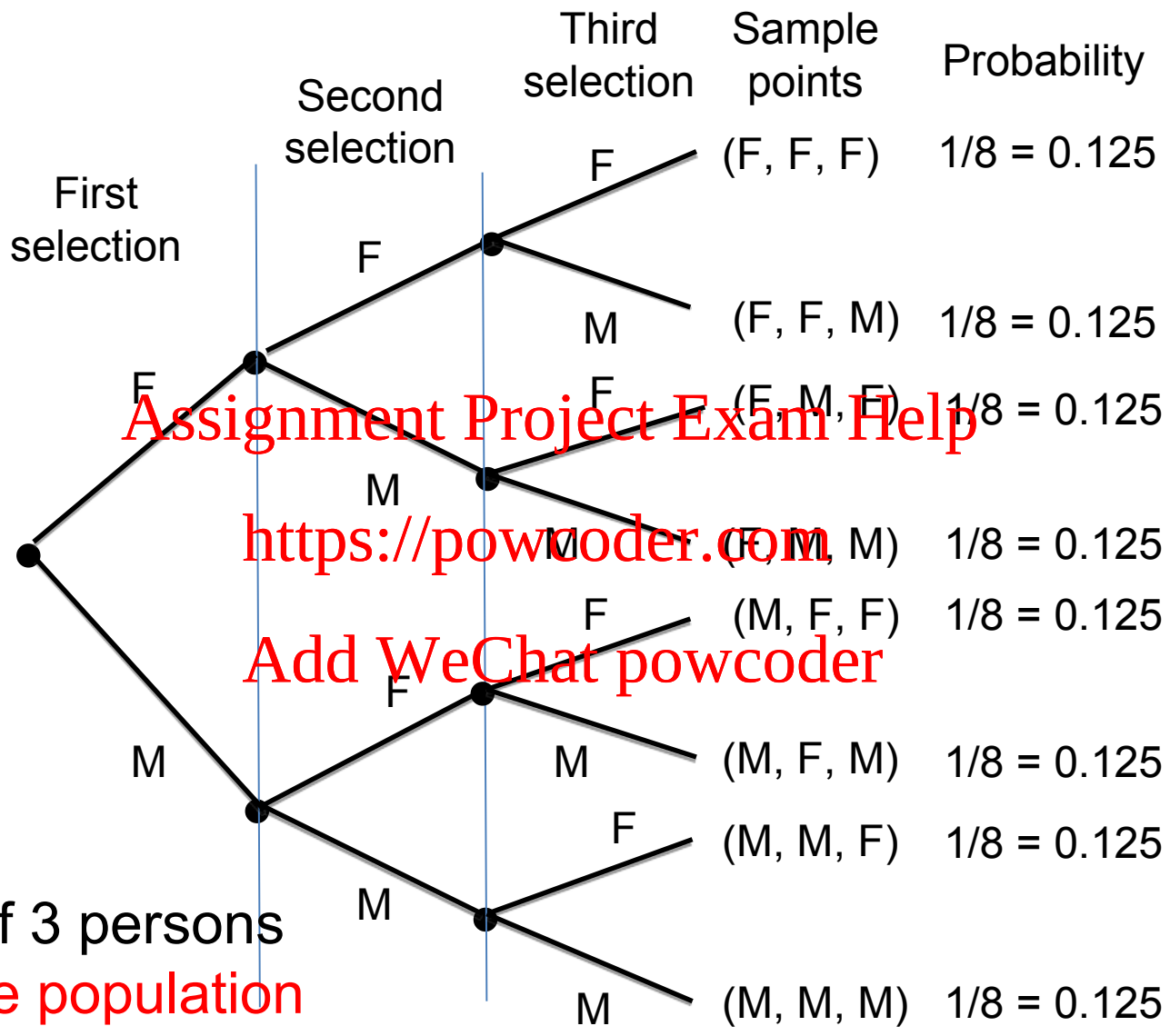
- One roll of a ten-sided die (n = 10). S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

  Probability of each side occurring is 1/10.

  P(0) + P(1) + P(2) + … + P(9) =1.

Multiple-step experiments

- If an experiment with k steps has n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, etc., then the sample space has $n_1 \times n_2 \times n_3 \times \ldots \times n_k$ sample points
- E.g. If 3 persons are randomly selected from a large population with half females and half males, there are 2 × 2 × 2 = 8 outcomes for the gender of the persons selected. Each outcome is equally likely.



Selection of 3 persons from a large population of half females and half males.

Event

- An **event** is a collection of sample points
- The probability of any event is equal to the sum of the probabilities of the sample points in the event
- Event of selecting exactly two females (2F).

  P (2F) = 1/8 + 1/8 + 1/8 = 3/8 = 0.375.

  This is the sum of the probabilities of the events FFM, FMF, and MFF.

Exercises

- P (2 heads in two tosses of a coin) = ?
- P (obtain 4 in roll of one die (6 sides by default)) = ?
- P (total of 4 when rolling two dice (6 sides)) = ?

If one randomly selects individuals from a large population that is ½ male and ½ female:

- P (one male and one female if 2 individuals selected) = ?

- P (two females and one male if 3 individuals selected) = ?

Please submit your solution to the following question via Canvas

If one randomly selects individuals from 20 individuals (10 females and 10 males):

- P (one male and one female if 2 individuals selected) = ?
- P (two females and one male if 3 individuals selected) = ?

Relationships of Probability

- Complement of an event
- Addition law – intersection and union
- Mutually exclusive
- Conditional probability
- Independence
- Multiplication law

Number of students by major and Excel skill level

Major of student	Excel skill level				Total
	None (N)	Low (L)	Medium (M)	High (H)	
Math (MA)	0	2	4	0	6
Business (B)	1	3	6	3	13
Economics (E)	2	12	8	2	24
Other (O)	0	1	1	1	3
Total	3	18	19	6	46

Q1: The probability that a student in this class is not an Economics major (*E*) is 22/46.

Q2: Probability of a randomly selected student being an Economics major (*E*) or having high Excel skills (*H*)?

$$P(E \cup H) = P(E) + P(H) - P(E \cap H)$$
$$= 24/46 + 6/46 - 2/46$$
$$= 28/46 = 0.609$$

Complement of an event

- The **complement** A_c of an event A is the set of all sample points that are not in event A.
- $P(A) + P(A_c) = 1$, $P(A_c) = 1 - P(A)$
- Examples: The probability that a student in this class is not an Economics major (E) is 22/46.

  $P(E) = 24/46 = 0.522$

  $P(E_c) = 1 - 0.522 = 0.478$

Combining events

What is the probability that more than one event has occurred? If there are two events:

Both events occur – this is referred to as the **intersection** of the two events.

At least one of the events occur – this is referred to as the **union** of the two events.

Neither event occurs.

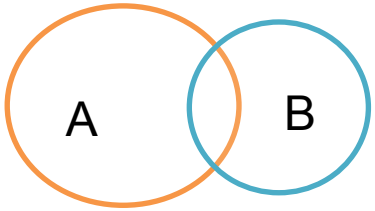
Union of two events

- The **union** of events *A* and *B* is the event containing all the sample points of either *A* or *B*, or both.
- The notation for the union is P(*A*∪*B*).
- Read this as "probability of *A* union *B*" or the "probability of *A* or *B*".
- The probability of *A* or *B* is the sum of the probabilities of all the sample points that are in either *A* or *B*, making sure that none are counted twice.

Intersection of two events

- The **intersection** of events *A* and *B* is the event containing only the sample points belonging to both *A* and *B*.
- The notation for the intersection is P(*A*∩*B*).
- Read this as "probability of *A* intersection *B*" or the "probability of *A* and *B*."
- The probability of *A* and *B* is the sum of the probabilities of all the sample points common to both *A* and *B*.

Addition law



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The probability that at least one event occurs is the probability of one event plus the probability of the other. But to avoid double counting, the probability of the intersection of the two events is subtracted.

Examples

- Probability of a randomly selected student being an Economics major (*E*) or having high Excel skills (*H*)?

  $P(E \cup H) = P(E) + P(H) - P(E \cap H)$

  $= 24/46 + 6/46 - 2/46$

  $= 28/46 = 0.609$

- Probability of Business (*B*) or low Excel skills (*L*)?

  $P(B \cup L) = P(B) + P(L) - P(B \cap L)$

Mutually exclusive events

- Two events are **mutually exclusive** if the events have no sample points in common.
- If two events *A* and *B* are mutually exclusive, the probability of *A* and *B* is zero.
- In this case, the probability of *A* or *B* is the sum of the probability of *A* and the probability of *B*. That is,

  P(*A*∩*B*) = 0 if *A* and *B* are mutually exclusive.

  Then P(*A*∪*B*) = P(*A*) + P(*B*)

Example of mutually exclusive events

If 3 persons are randomly selected from a large population of half females and half males, what is the probability of selecting 3 females (*A*) and at least 2 males (*B*)?

Event *A* has sample point (F, F, F) .

Event *B* has sample points (F, M, F), (M, F, M), (M, M, F) and (M, M, M) .

Events A and B have no sample points in common, so P($A \cap B$) = 0.

Conditional probabilities

- A **conditional probability** refers to the probability of an event *A* occurring, given that another event *B* has occurred.
- Notation: $P(A \mid B)$
- Read this as the "conditional probability of *A* given *B*" or the "probability of *A* given *B*."

Formulae for conditional probabilities

- The probability of *A* given *B* is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- The probability of *B* given *A* is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Number of students by major and Excel skill level

Major of student	Excel skill level				Total
	None (N)	Low (L)	Medium (M)	High (H)	
Math (MA)	0	2	4	0	6
Business (B)	1	3	6	3	13
Economics (E)	2	12	8	2	24
Other (O)	0	1	1	1	3
Total	3	18	19	6	46

$$P(L \mid MA) = P(L \cap MA) / P(MA) = (2/46) / (6/46) = 2/6 = 0.333$$

Examples of conditional probabilities from student survey

- Probability that each major has low skill level?

  $P(L \mid MA) = P(L \cap MA) / P(MA) = (2/46) / (6/46) = 2/6 = 0.333$

  $P(L \mid B) = 3 / 13 = 0.231$

  $P(L \mid E) = 0.500$

  $P(L \mid O) = 0.333$

Number of students by major and Excel skill level

Major of student	Excel skill level				Total
	None (N)	Low (L)	Medium (M)	High (H)	
Math (MA)	0	2	4	0	6
Business (B)	1	3	6	3	13
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Other (O)	0	1	1	1	3
Total	3	18	19	6	46

If a student has a high skill level in Excel, 1) what is the probability his or her major is Business? 2) what is the probability his or her major is Other (O)?

Exercises - solutions

- P (2 heads in two tosses of a coin) = 1/4 = 0.250
- P (obtain 4 in roll of one die) = 1/6 = 0.167
- P (total of 4 when rolling two dice) = 3/36 = 1/12 = 0.083

If one randomly selects individuals from a large population that is ½ male and ½ female:

- P (one male and one female if 2 individuals selected) = 2/4 = 1/2 = 0.500

- P (two females and one male if 3 individuals selected) = 3/8 = 0.375

If a student has a high skill level in Excel, what is the probability his or her major is Business? Other?

$P(B \mid H) = P(B \cap H) / P(H) = (3/46) / (6/46) = 3/6 = 0.500$

$P(O \mid H) = 0.167$