CITY UNIVERSITY OF HONG KONG

Course code & title: **EE6605**

Complex Networks: Modeling, Dynamics and Control

Session: Semester B 2015/16

Time allowed: **2** Hours

This paper has 6 pages including this cover page.

- This exam consists of **7** questions.
- Answer **all** questions.

- You may directly draw on figures in the exam papers and return them with the blue answer book.

This is a **closed-book** examination.

Candidates are allowed to use the following materials/aids:

Regular Calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

* **Return** These Exam Question Papers with the Blue Answer Book

* **Student ID**: _____

Question 1 [40 Marks] (Graph Theory and Applications)

Q-1.1 [20 Marks]

Answer all the **T**rue or **F**alse questions below, and show a brief reason for T or a specific counterexample for F, where every network has $N\ (\geq 3)$ nodes to avoid trivial cases.

(1) **[4 Marks]** Undirected graphs

1.1) If a connected graph with N nodes has m edges and its complementary graph has n edges, then $m+n=N(N-1)/2$.

1.2) If a connected graph with N nodes has m edges and its isomorphic graph has n edges, then $m+n \leq N(N-1)$.

(2) **[6 Marks]** Directed graphs (digraphs)

2.1) If a digraph is directed Eulerian then its underlying graph is also Eulerian.

2.2) If a digraph is directed Hamiltonian then its underlying graph is also Hamiltonian.

2.3) If a digraph is strongly connected then it has a directed spanning tree.

(3) **[6 Marks]** In a digraph, if there is a directed path from node u to node v, then node v is said to be *accessible* from node u. Moreover, if every node is accessible by a node w, then node w is called a *root* of the digraph.

3.1) In a directed tree, every node is accessible from the root.

3.2) If a digraph has a directed spanning tree, then every node is accessible from its root.

3.3) In a directed Hamiltonian graph, every node is a root of a tree.

(4) **[4 Marks]**

4.1) A perfect matching in a digraph is always a perfect matching in its underlying graph.

4.4) A directed tree always has a perfect matching and it is unique.

Q-1.2 [20 Marks]

(1) **[10 Marks]**

1.1) **[5 marks]** For the graph shown in Figure Q1-1, write out three adjacency matrices:

(a) the adjacency matrix A_1 of the component with nodes 1, 2 and 3;

(b) the adjacency matrix A_2 of the component with nodes 4 and 5;

(c) the adjacency matrix A_3 of the whole graph with nodes 1, 2, 3, 4 and 5.

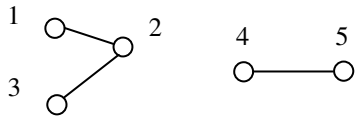


Figure Q1-1

1.2) **[5 Marks]** By comparing A_3 to A_1 and A_2, how can you tell that A_3 represents a disconnected graph with 2 components? In general, if you are given a large-sized

adjacency matrix A, how can you tell whether or not it represents a graph with 1 component, a graph with 2 components, or a graph with 3 components, and so on?

(2) **[10 Marks]**

2.1) **[5 Marks]** Consider the Chinese postman problem on a lattice with $3\times3=9$ nodes as shown in Figure Q1-2a, where every edge has length 1. To find an optimal solution, at least how many extra edges do you need to add? Why?

2.2) **[5 Marks]** Consider the Chinese postman problem on the lattice with $n\times n=n^2$ nodes as shown in Figure Q1-2b, where every edge has length 1. To find an optimal solution, at least how many extra edges do you need to add? Why?

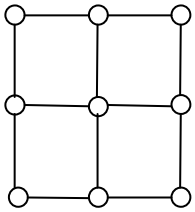


Figure Q1-2a

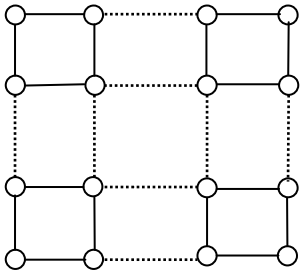


Figure Q1-2b

Question 2 [10 Marks] (Internet Modelling)

Figure Q2 illustrates the basic structure of the Internet at the highest level.

(1) **[2 Marks]** What does AS mean? What are those ?

(2) **[3 Marks]** What type of network does this Internet topology look like? Why do you think so?

(3) **[5 Marks]** Try to represent this network by a simpler but equivalent three-level network, and then further to represent this network by a two-level network. (You can describe how to do it by words, no need to draw pictures.)

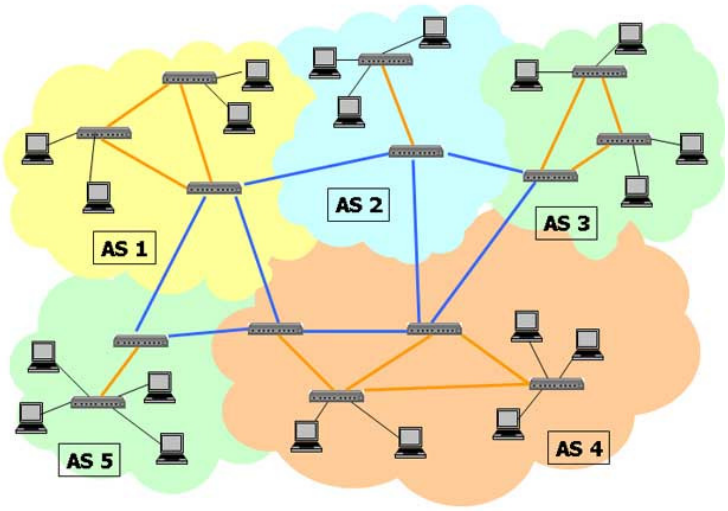


Figure Q2

Question 3 [10 Marks] (Epidemic Spreading)

Consider a wireless network of many identical sensor nodes distributed over a lattice, where every node has the same maximum signal-transmission range $r(t)$ at each time t, as shown in Figure Q3. Initially, all nodes are *susceptible* (S). Assume that the central node in the figure is now infected by a virus, which spreads to all its neighbors within the range $r(t)$ through signal broadcasting at time t. Then, these neighbors become *infective* (I), and every infected sensor will relay the virus to their neighbors in the next time step. At the next time step, the original central node and its nearby neighbors within range $r(t)-r_0$ cannot affect any new nodes outside their ranges, so they become *unharmful* (which is denoted as U), or called invalid. This process continues to spread out, so more and more nodes become infected while more and more nodes from the center propogating outwards become unharmful.

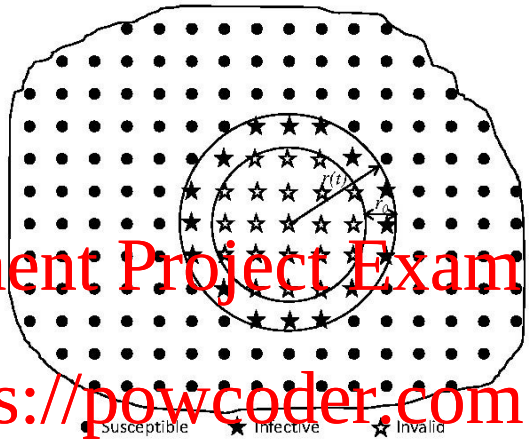


Figure Q3

(1) **[2 Marks]** To describe the above virus spreading, will you use SI, SIS, or SIR model? Why?

(2) **[4 Marks]** Recall the SI model: $\frac{dI(t)}{dt}=\lambda S(t)I(t)$, where λ is the effective spreading rate. If you want to modify this SI model to describe the above spreading process by adding one more term to the right-hand side, as $\frac{dI(t)}{dt}=\lambda S(t)I(t)+(?)$, what term $(?)$ will you use (in terms of S, I, and/or U)?

(3) **[4 Marks]** How will you include the parameters $r(t)$ and r_0 into your modifed model above?

Questions 4 [10 Marks] (Community Structures)

Given a network as shown in Figuire Q4.

(1) **[6 Marks]** Based on visual estimation without actually computing the edge betweenness, apply the Girvan–Newman Algorithm to detect the community structure of the network.

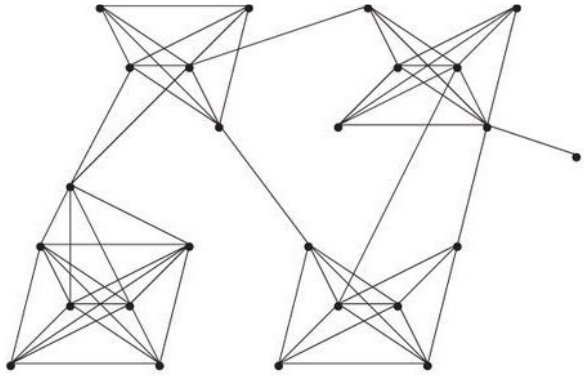


Figure Q4

(2) **[4 Marks]**

2.1) If the Planted L-Partition Algorithm is used, the number $L = ?$

2.2) If the Clique-Based Algorithm is used, the largest clique size $k = ?$

Questions 5 [10 Marks] (Network Games)

Consider the road map shown in Figure Q5a, with 4 cars to go from A to B.
For some cars to go through road AD, or CB, each car needs to pay 5 dollars.
For x cars to go through road AC, or DB, each car needs to pay x dollars.
(Here, x can be 1, 2, 3, or 4.)

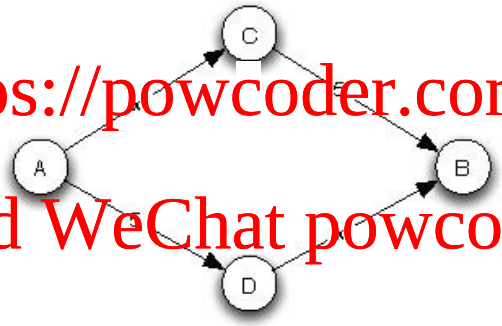


Figure Q5a

(1) **[2 Marks]** If 2 cars go through ACB and 2 other cars go through ADB, as shown in Figure Q5b, how much each car needs to pay? Totally how much for the 4 cars to pay altogether?

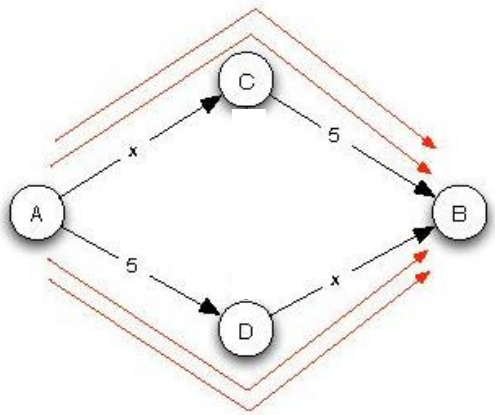


Figure Q5b

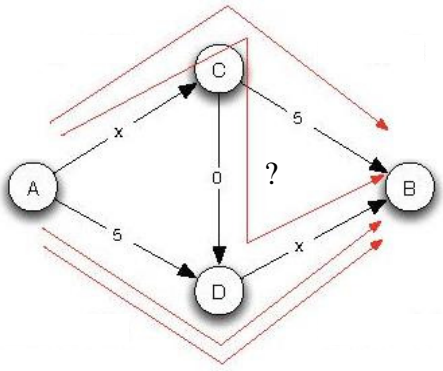


Figure Q5c

(2) **[2 Marks]** Now, suppose that one road CD is open for cars, free of charge, as shown in Figure Q5c. If you are the driver of one car at point C, will you make a turn to go CDB? Assuming that all other cars do not change their roads.

(3) **[6 Marks]** Explain one advantage and one disadvantage of adding the free road CD.

Questions 6 [10 Marks] (Network Synchronization)

Are the following pairs of signals phase-synchronizing? If so, for each pair, what kind of synchronization (in-phase, anti-phase, shifted-phase) they perform?

[2 Marks Each]

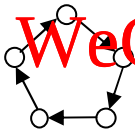
(1) $\sin(t)$ and $5\sin(t)$
(2) $\sin(t)$ and $\cos(t)$
(3) $\sin(t)$ and $\sin(5t)$
(4) $\sin(t)$ and $5+\sin(t)$
(5) $\sin(t)$ and $\sin(-t)$

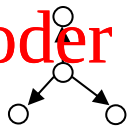
Questions 7 [10 Marks] (Network Control)

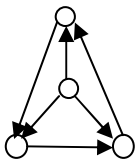
At least how many controllers do you need in order to control the following given network?

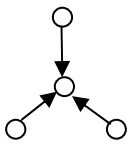
[2 Marks Each]

(1) 

(2) 

(3) 

(4) 

(5) 

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