Complex Dynamical Networks:

Lecture 4b: Internet

-- Topology and Modeling

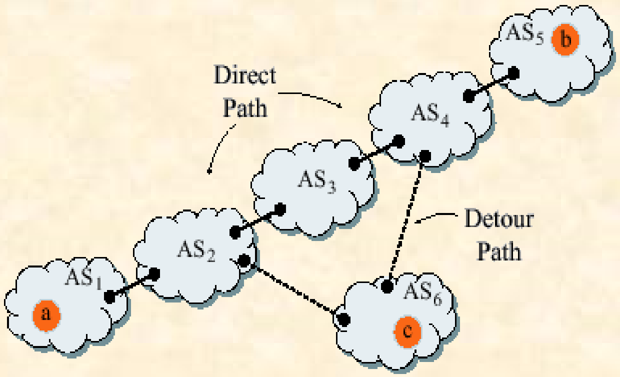
EE 6605

Instructor: G Ron Chen

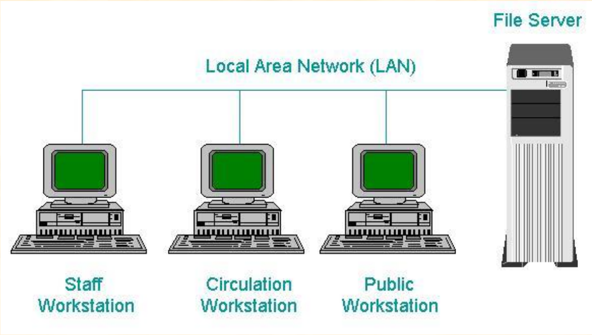
Network Topology Modeling

- Graph representations

- **AS**-level:
  - nodes are domains (AS)
  - edges are peering relationships



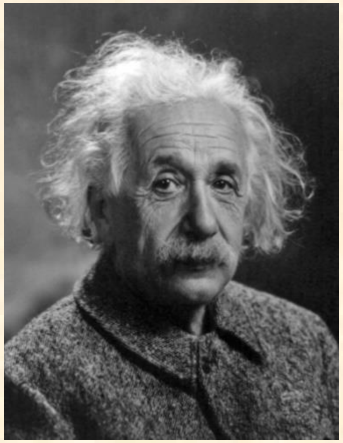
- **Router**-level:
  - nodes are routers
  - edges are one-hop IP connections



- **PC**-level: not manageable today
  - nodes are PCs, hand-held sets
  - edges are optical fibers

Internet Modeling

A model should be made "as simple as possible, but not simpler." – Albert Einstein



Recall simple generic models:

Random-graph model

Small-world model

Scale-free model

Actually, there are better ones

Representative Models

--- --- --- (1980s)

- **Waxman** (Waxman 1988)
  Router-level model capturing locality

--- --- --- (1990s)

- **Transit-Stub** (Zegura 1997), Tiers (Doar 1997)
  Router level model capturing hierarchy and clustering

--- --- --- (2000s)

- **Inet** (Jin 2000)
  AS-level model based on degree sequence
- **BRITE** (Medina 2000)
  AS-level model based on evolution
- **BA-Model** (Barabasi-Albert 1999-2000)
  AS-level model based on degree sequence and evolution

  - **HOT** (CalTech 2004-2005)
    Heuristic Optimized Tradeoffs
  - **MLW** (Fan-Chen, 2006-2010)
    Multi-Local-Worlds

Router-Level Internet Topology

- A common software tool to represent the router-level Internet topology by a graph is *traceroute*
- The traceroute uses hop-limited probe, which consists of a hop-limited IP (Internet Protocol) packets and the corresponding ICMP (Internet Control Message Protocol) responses, to probe every possible IP address and record every reached router and the corresponding edges
- Commercial: *traceroute* software or its IPv6 version *traceroute6(8)*

Router-Level Internet Topology

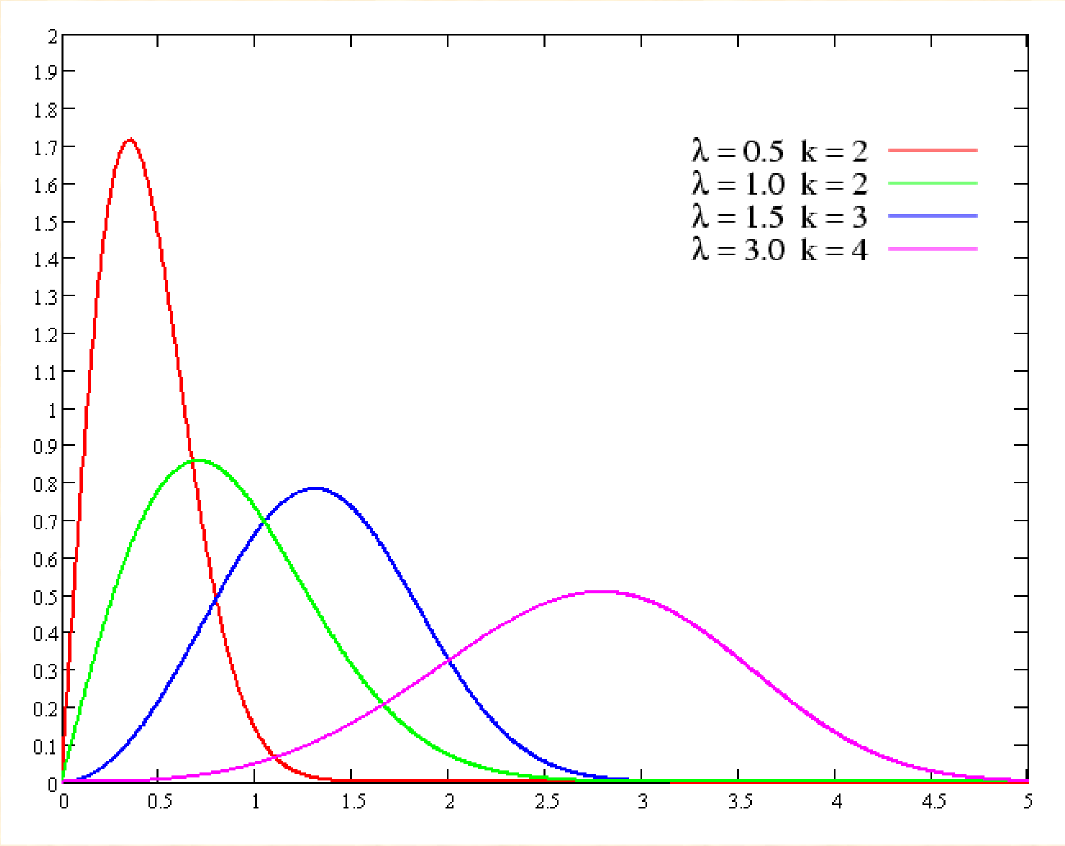
- Some analysis on the real data collected during October-November of 1999 shows that in the router-level of the Internet topology:
  - basically, it does not have hierarchical structure
  - power-law node-distribution is not prominent but Weibull distribution seems better, yet the latter can only reflect Transit but not Stub subnets

- Some analytical results on the real data collected during December 2001 -- January 2002 show that the Weibull distribution can better fit the complementary cumulative distribution function of router out-degree than the Pareto and Power-law distributions

- Today, data size is too big to manage for modeling

Weibull distribution

$$f_k(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$



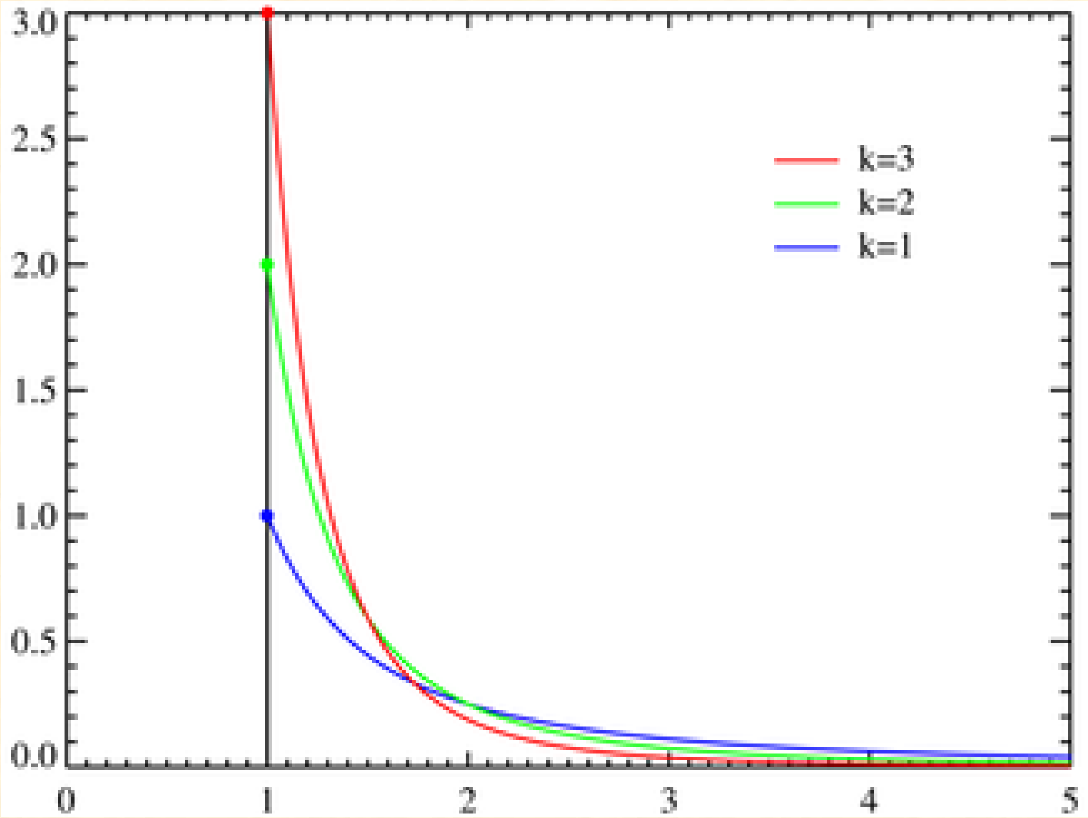
x

k and λ are constant parameters

Pareto distribution

$$f_k(x) = k\frac{\lambda^k}{x^{k+1}}, \quad x \ge \lambda > 0$$

k and λ are constant parameters



x

First Generation of Internet Topology Models

1980s

Waxman Model

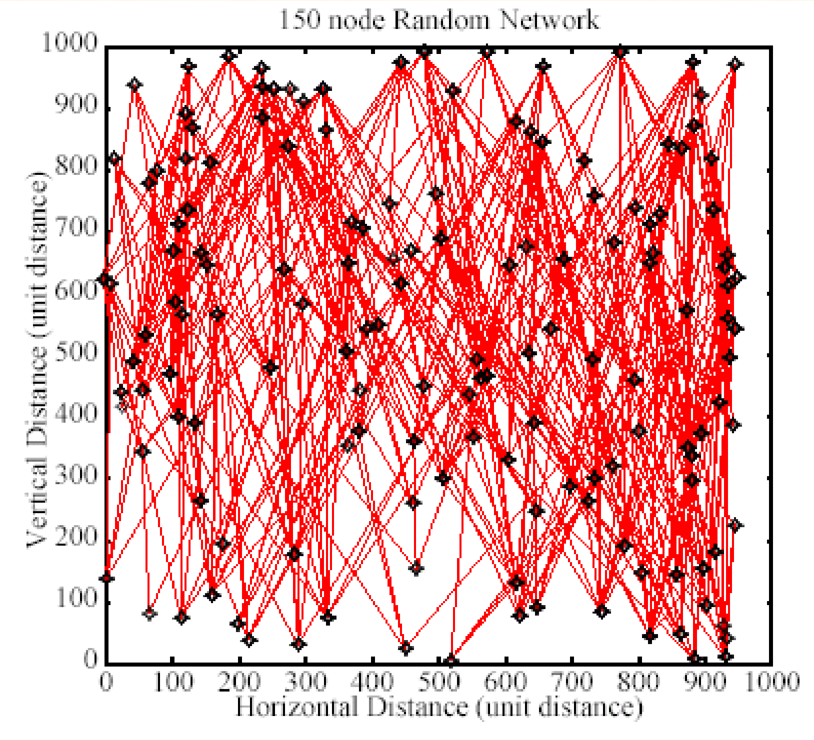
Waxman model:

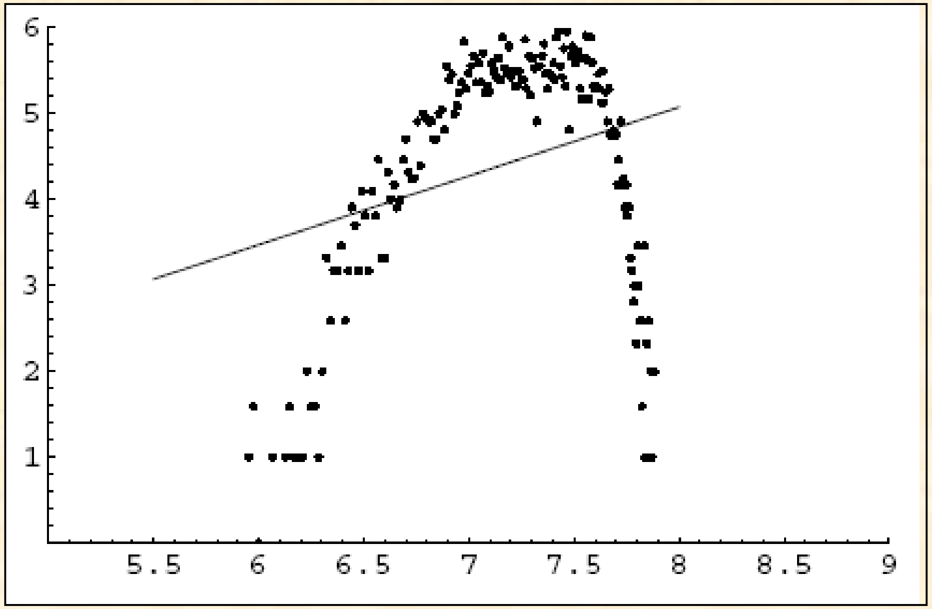
- Start with N nodes, randomly placed on a lattice, at most one in each small square
- Each step, for every pair of two nodes, u and v, connect them by an edge according to the following probability (called Waxman probability):

$$P(u,v) = \alpha\, e^{-d(u,v)\,/\,(\beta\, L_{\max})}$$

where $d(u,v)$ is the distance, $L_{\max}$ is the longest distance, and α is the (normalized) average number of edges and β is a parameter determined by the average path length, satisfying $0 < \alpha, \beta \leq 1$, that together make the formula a probability distribution function

Waxman Model





$N = 150, \quad \alpha = 0.25, \beta = 0.3$

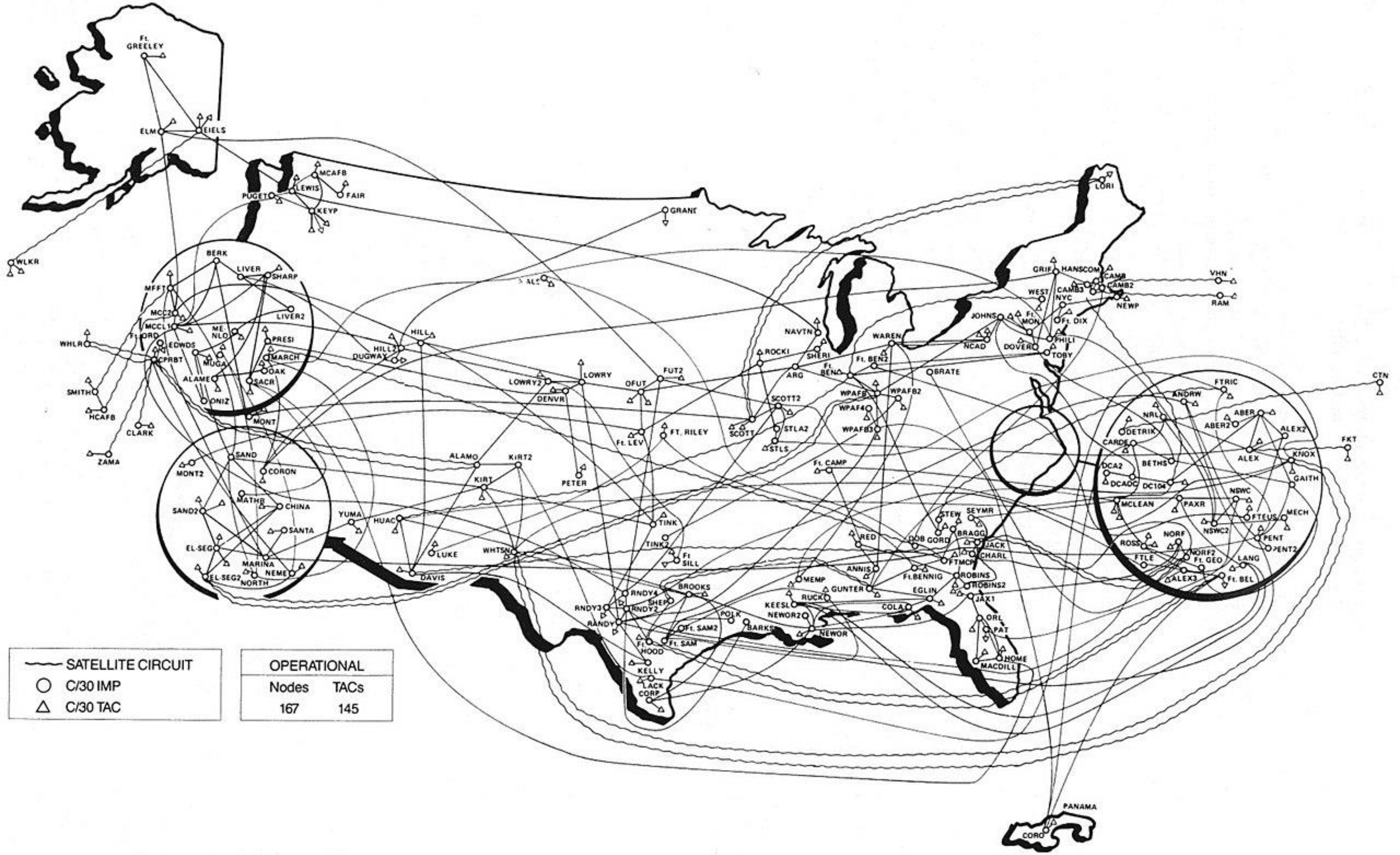
Degree distribution (~ Weibull)

(Waxman, 1988)

(Medina *et al*., 2000)

Second Generation of Internet Topology Models

1990s



MILNET (Military Network), part of the ARPANET,
unclassified by US Department of Defense (1989)

Transit-Stub Topology

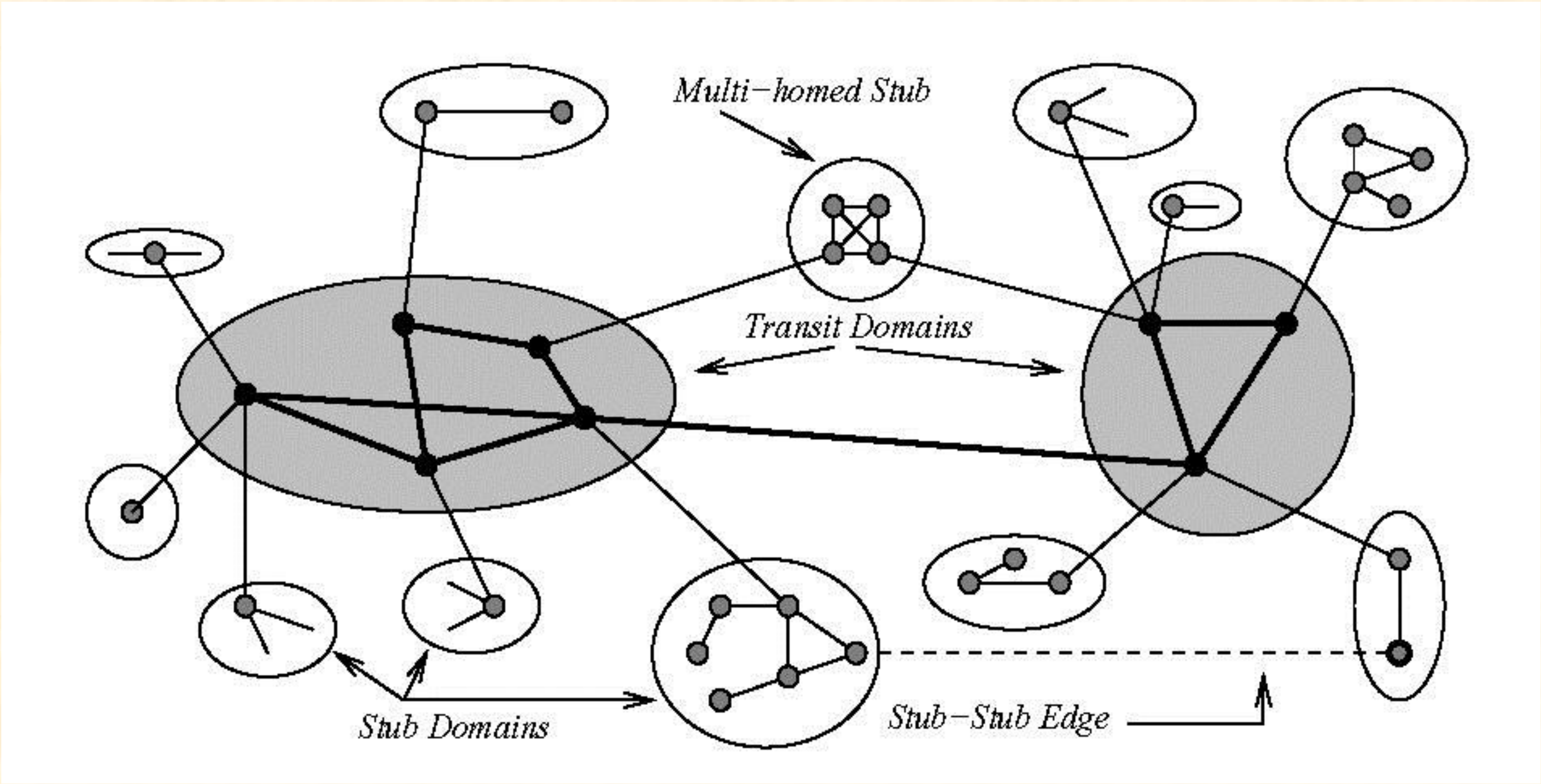


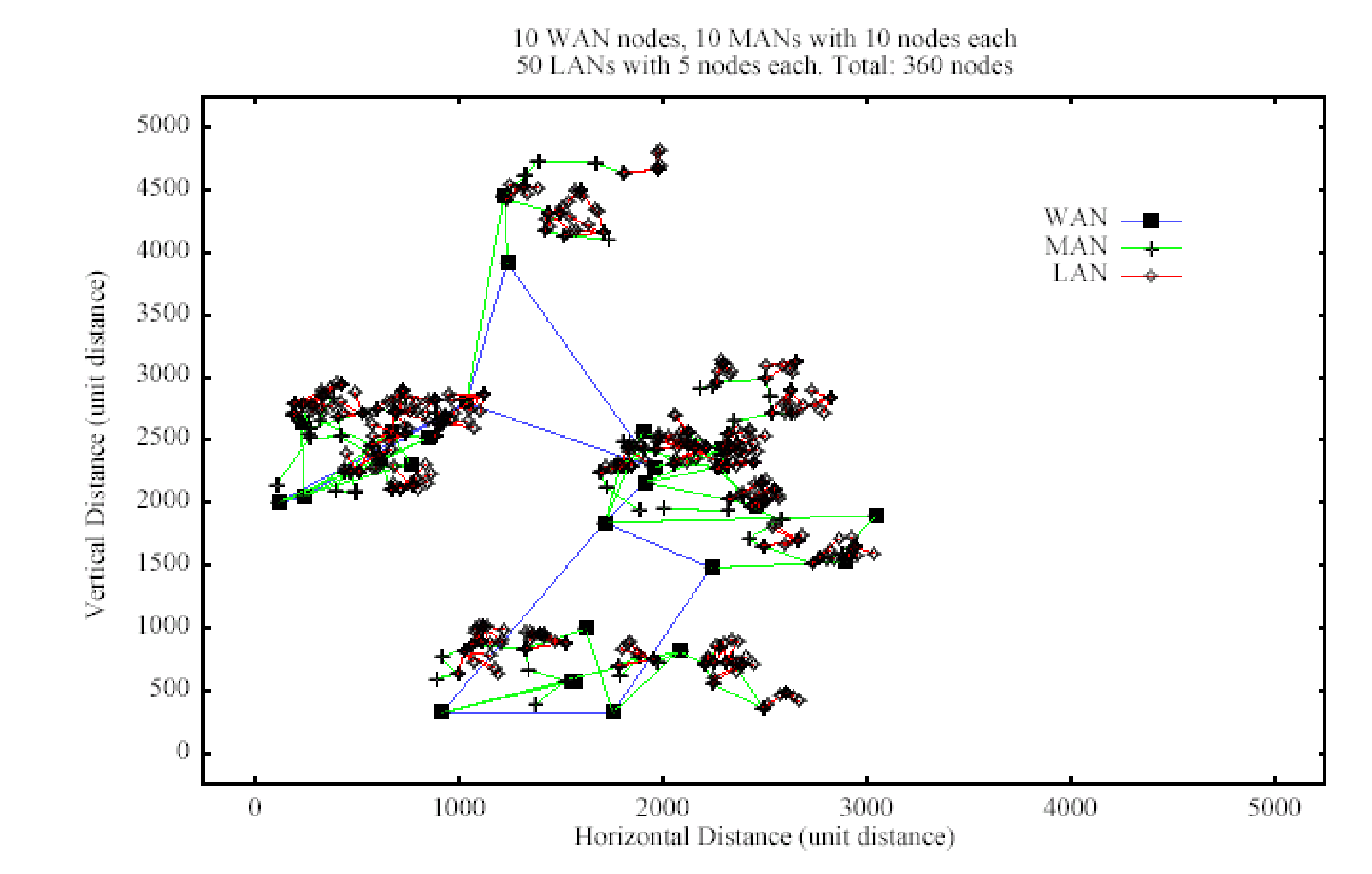
Illustration of network structure from Transit-Stub topology generator

(Hierarchy with Communities)

Transit-Stub Topology Generator

- Modeling Data
- Generate all Transit domains:
  - Use a random-graph generation method (e.g., the Waxman algorithm), where each node represents a Transit domain.
  - Generate nodes in each Transit domain by adding some nodes around the Transit point, and then connect these nodes with edges at random.
- Generate Stubs for each Transit:
  - This is similar to the above Transit-domain generation, but at a lower level.
  - Connect every Stub domain to a Transit domain: Randomly select one node from a Stub domain and then connect this node to the Transit domain by an edge.
- Generate LANs for each Stub:
  - This is similar to the above Transit-Stub generation, but at the lowest level.
  - They all have star-shaped structures.
  - Connect each LAN to a Stub domain.

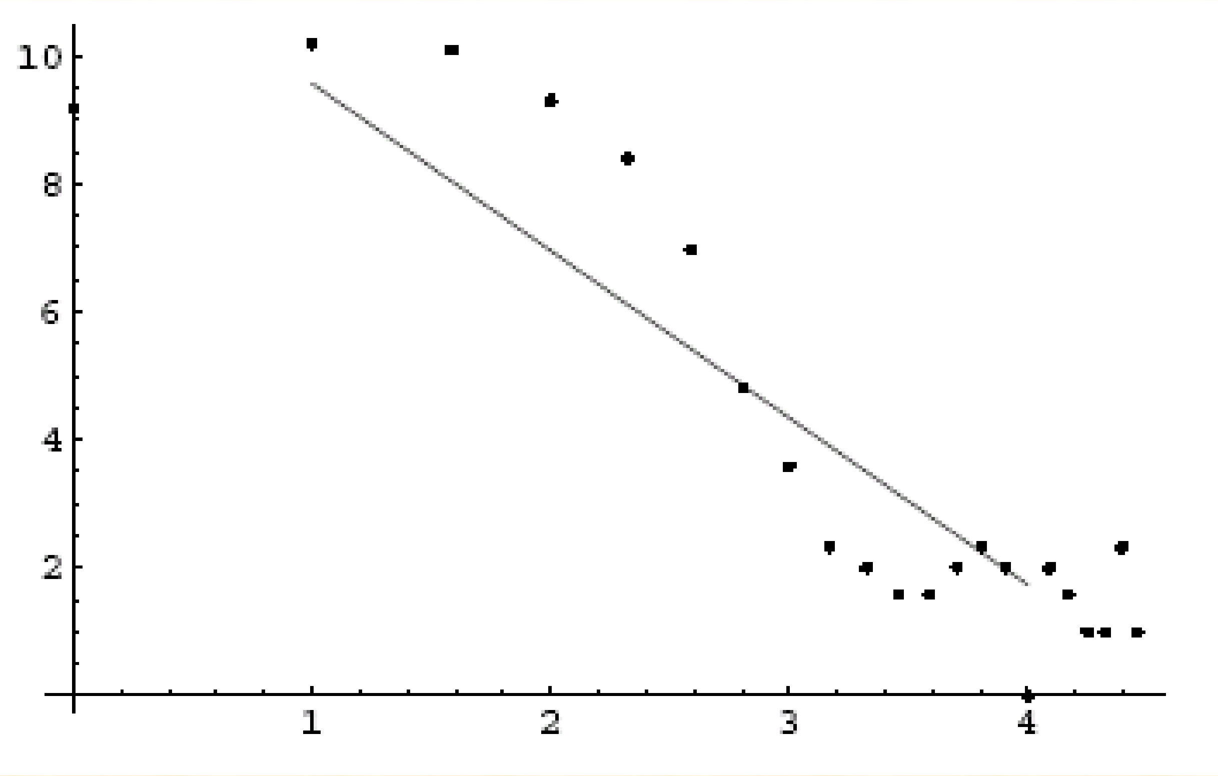
Transit-Stub Topology Generator



A typical Transit-Stub topology
(Hierarchy with Communities)

(Calvert, 1997)

Transit-Stub Topology Generator



Out-degree distribution of a Transit-Stub network with 6660 nodes

~ Weibull

(Medina et al., 2000)

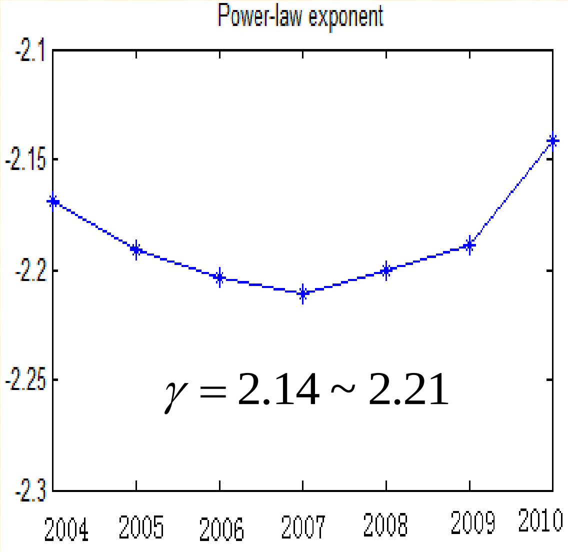
Third Generation of
Internet Topology Models

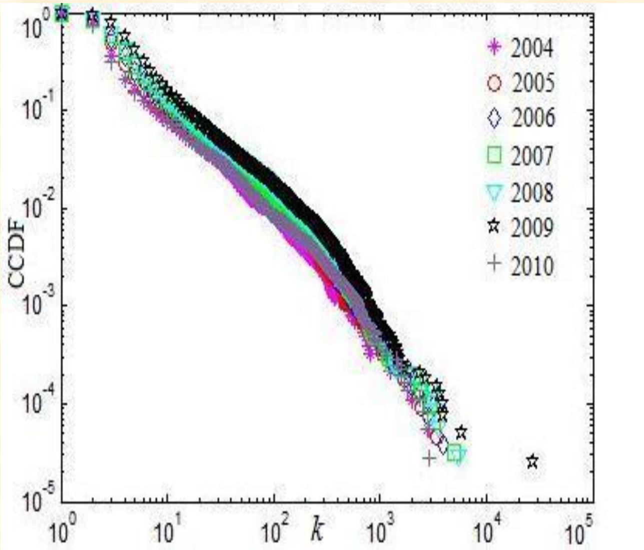
2000s

Inet

- Router-level model and AS-level model
- Input:
  - Total number of nodes
  - Percentage of degree-one nodes
- Degree sequence: power-law

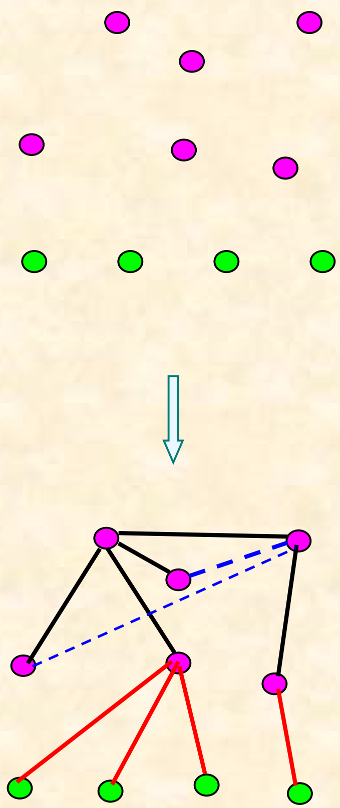
$$P(k) \propto k^{-\gamma}$$





Inet

- From the given data set, let V_1 be the set of all degree-1 nodes, typically has about 30% of the total (green nodes). Let the rest be V_2 (pink nodes).
- Generate a spanning tree consisting of nodes from V_2



To generate the spanning tree in a network G, start from isolated initial conditions, and then a node i is connected to a node j, both in V_2, according to the following (preferential attachment) probability:

$$\Pi(i,j) = \frac{w_i^{\,j}}{\sum_{k \in G} w_i^{\,k}}$$

$w_i^{\,j}$ -- weight (reverse distance) from i to j

- Connect the degree-1 nodes from V_1 to the spanning tree, according to the above same probability.
- Connect high-degree nodes to those available nodes which did not connect to V_1, also according to the above same probability (blue dashed lines).

(Jin 2000)

Recall: Analysis on the Internet data of April 2002 shows that nodes with degrees of 1, 2, and 3 were 26%, 38% and 14%, respectively, which sums up to about 80% of the whole network (Zhou and Mondragon, 2004)

Inet

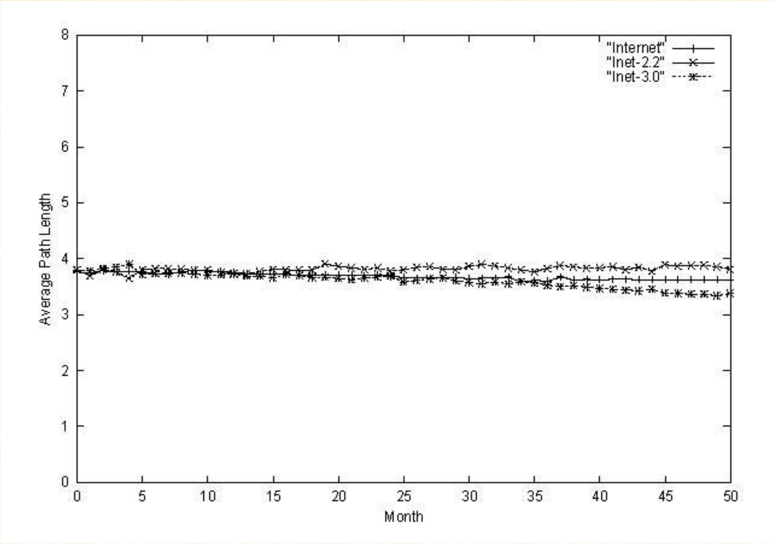
Started by University of Michigan (2002)

Inet Framework

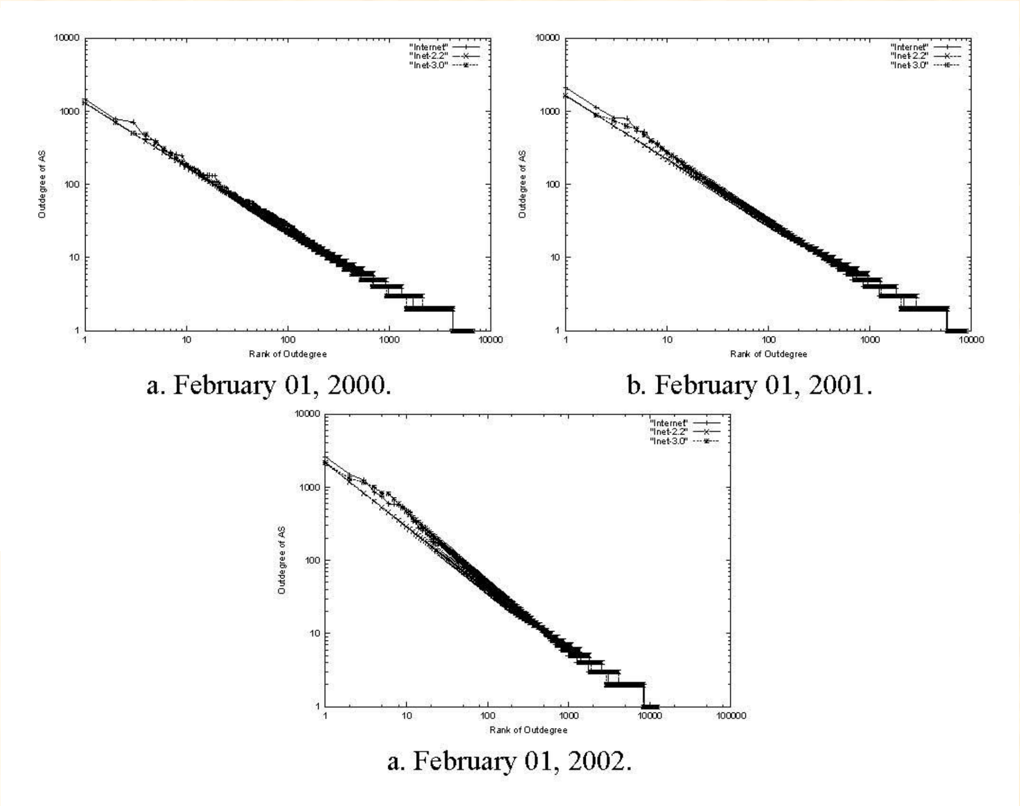
Inet-3.6.6 Program

Simulations based on real data:

6700 nodes (Feb. 2000), 8880 nodes (Feb. 2001) and 12700 nodes (Feb. 2002)



↑ Average path lengths



a. February 01, 2000.

b. February 01, 2001.

a. February 01, 2002.

Out-degree power-law distributions →

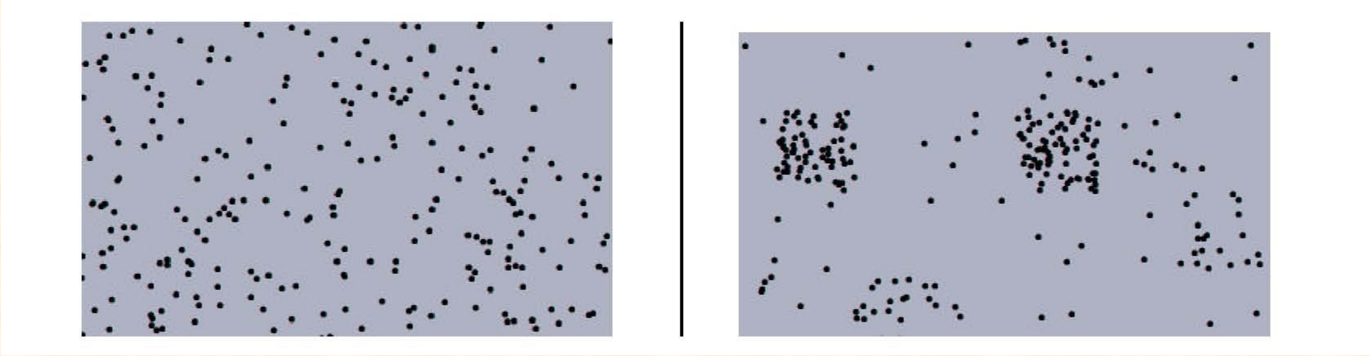
BRITE

(Boston university Representative Internet Topology gEnerator)

Modeling   NS-3 codes

Framework –

- Set a lattice on the plane, divide the lattice into some large squares, and then further divide all large squares into small squares.
- According to a certain (e.g., uniform or Pareto) distribution, determine how many nodes will be assigned into each large square.
- Then, in each large square, randomly pick a small square and assign at most one future node to it (next page shows how to add future nodes).



Assign nodes: (a) Uniform distribution (b) Pareto distribution

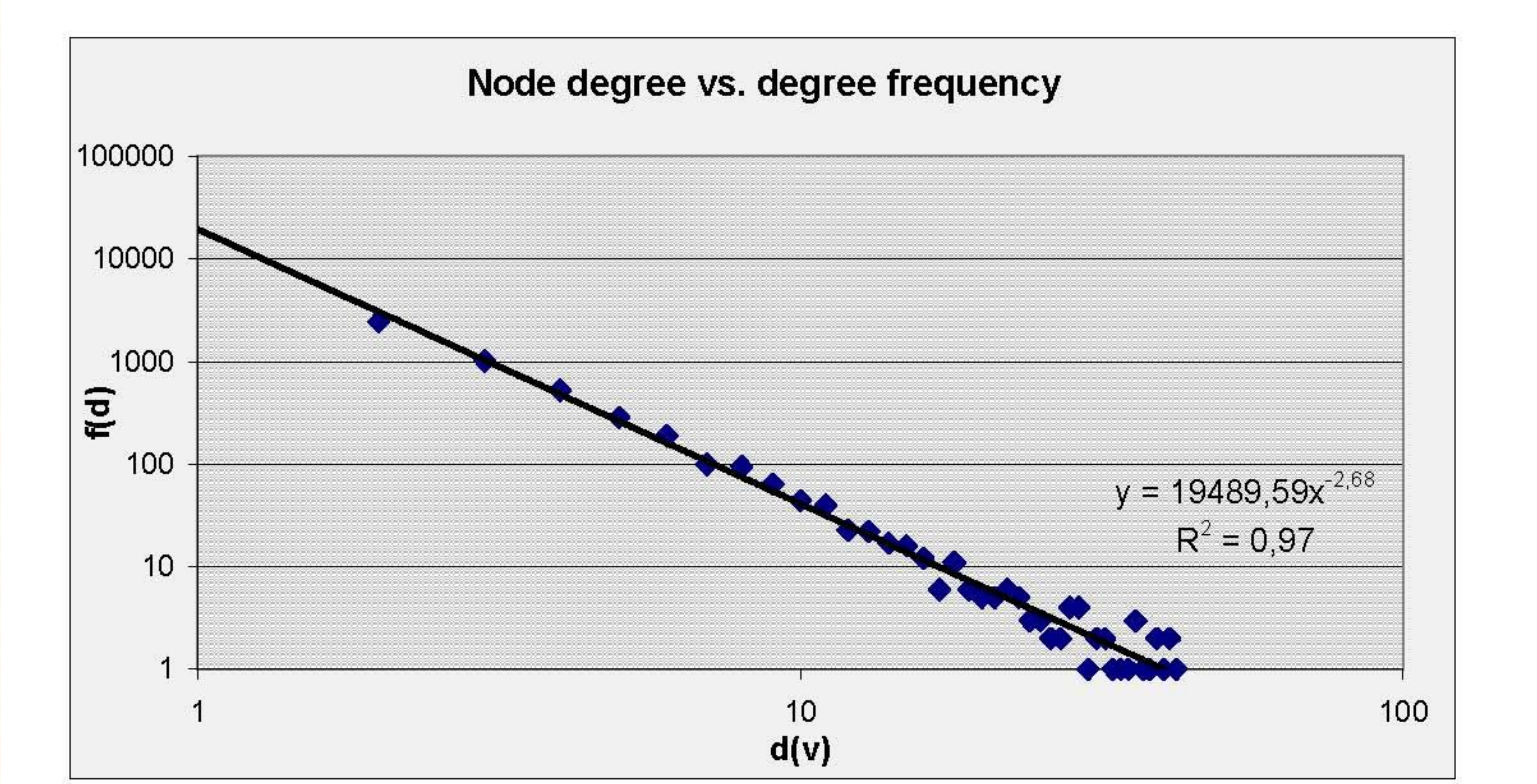
Now, start to add nodes:

- Initially, generate a random graph with m_0 nodes
- Then, add more nodes to the graph gradually.
- The way to connect nodes is determined by two parameters: Incremental Growth (IG) and Preferential Connectivity (PC):
  - if $IG = 0$ then put m nodes onto the plane simultaneously, and randomly pick one node among them and then connect it to the other nodes;
  - if $IG = 1$ then put one node onto the plane each time, and connect this new node to m existing nodes in the network.
- The way to establish connections is based on the PC parameter value:
  - if $PC = 0$ then follow the Waxman probability to connect the new node to the existing nodes;
  - if $PC = 1$ then follow the BA linear preferential attachment probability;
  - if $PC = 2$ then use the following weighted preferential attachment probability:

    $$\Pi(k_i) = \frac{w_i k_i}{\sum_{j \in C} w_j k_j}$$

    where k_i is the degree of node i, w_i is the Waxman probability, and C is the set of all m nodes being connected to node i.

BRITE

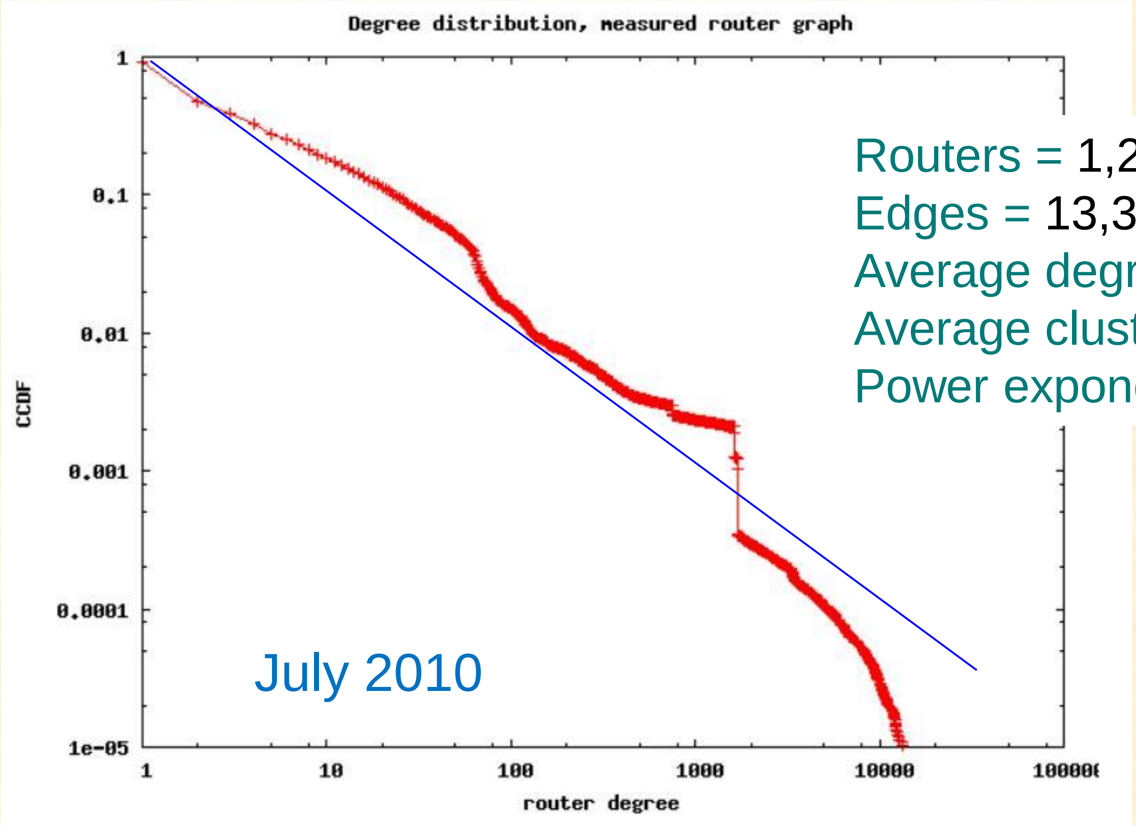


Node-degree distribution - 5000 router nodes

(Di Fatta et al., 2001)

Internet Topology

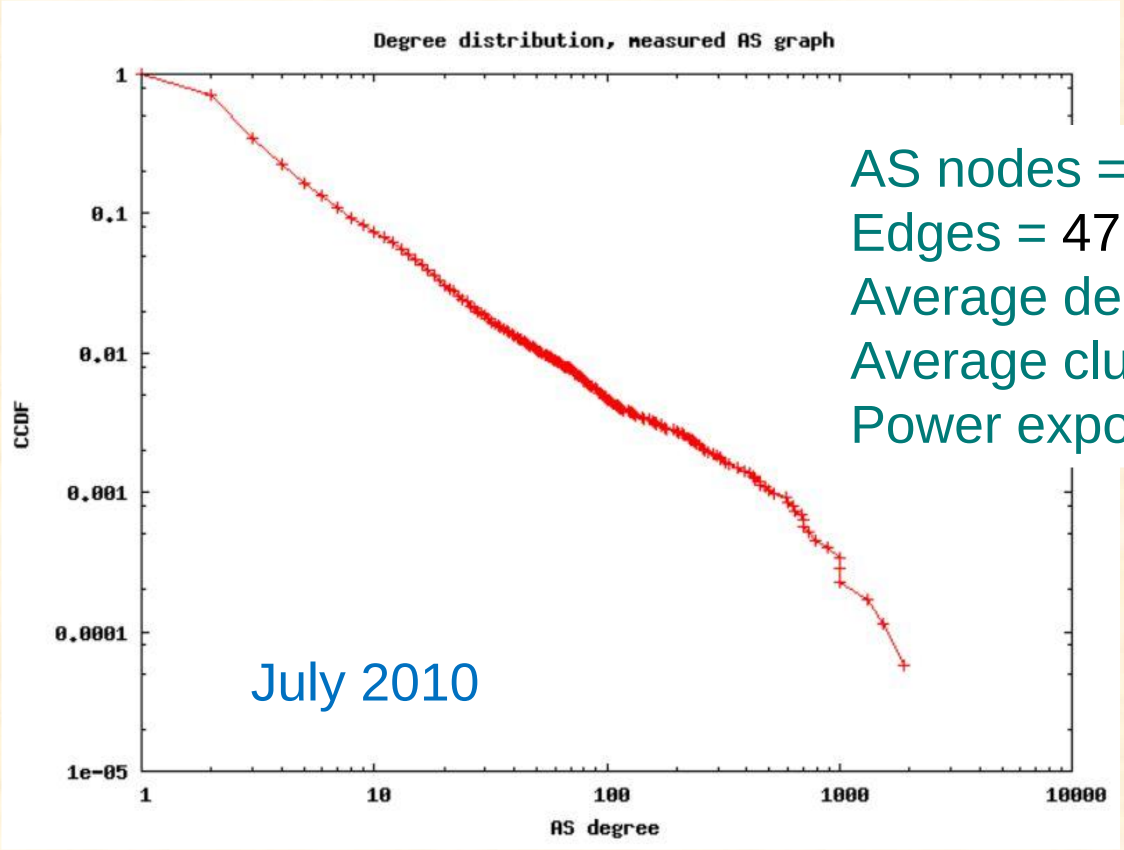
at Router Level



Routers = 1,204,038
Edges = 13,349,748
Average degree = 22.17
Average clustering = 0.73
Power exponent = 2.34

CAIDA

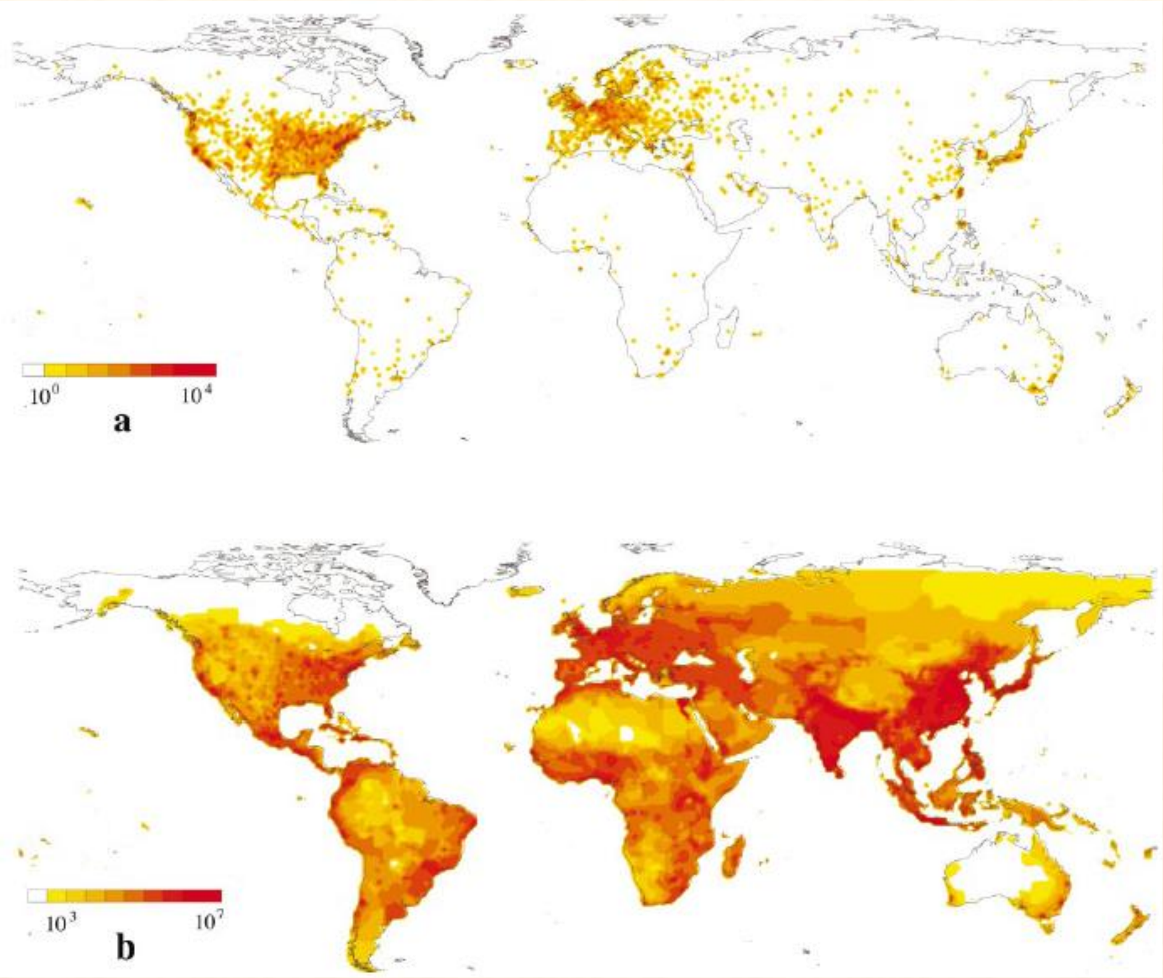
Internet Topology
at **AS level**



AS nodes = 17,492
Edges = 47,804
Average degree = 5.4
Average clustering = 0.25
Power exponent = 2.13

CAIDA

Geographic Layout of the Internet



(a) Router density (b) Human population density

(Yook et al., 2002)

Geographic Layout of the Internet

Correlation between router interfaces and human population

	Population (Millions)	Interface	People per interface
Australia	18	18,277	975
Japan	136	37,649	3,631
Mexico	154	4,361	35,534
USA	299	282,048	1,061
South America	341	10,131	33,752
W. Europe	366	95,993	3,817
Africa	837	8,379	100,011

Yook *et al.* (2002)

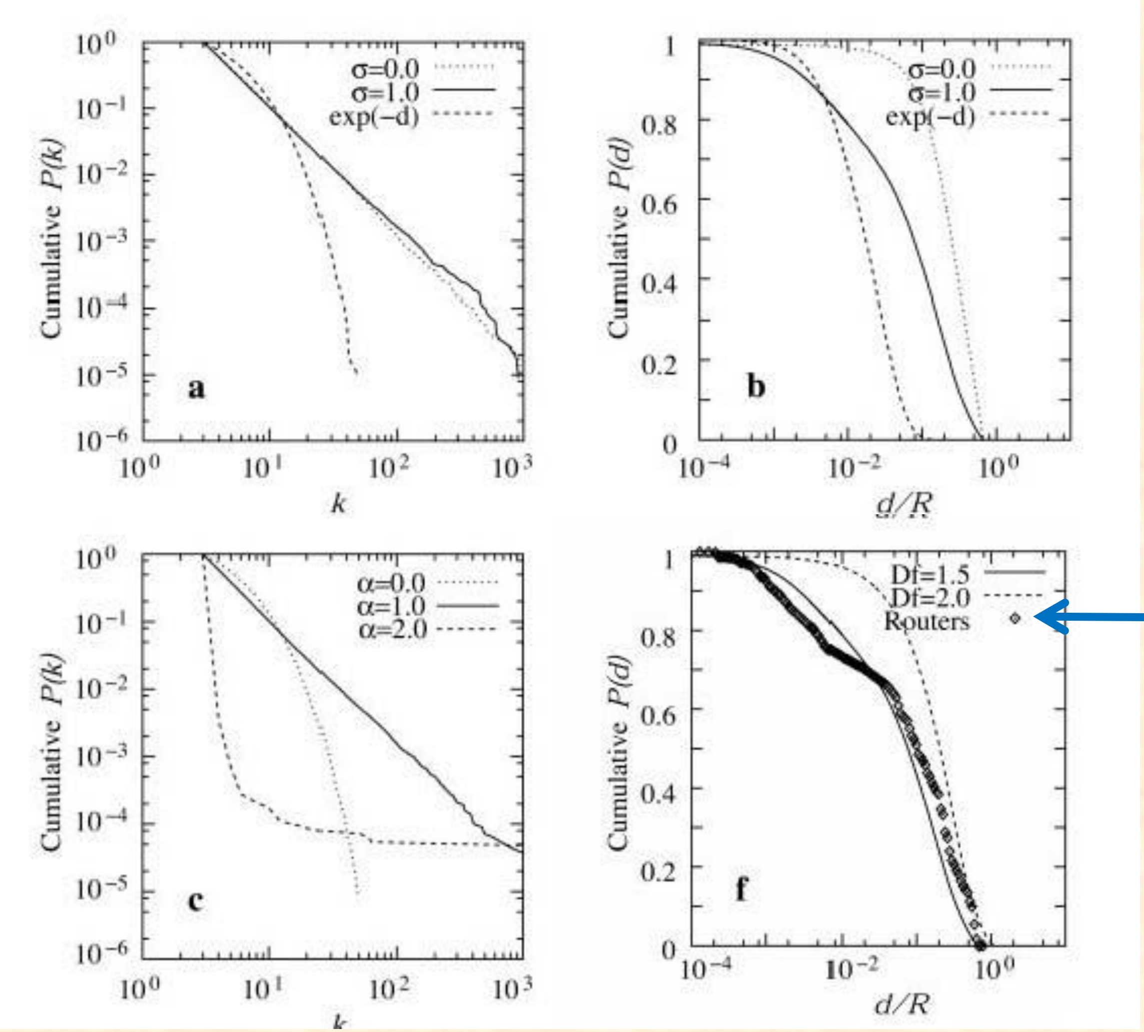
GeoBA model (Yook *et al.*, PNAS 2002)

- Start with a lattice consisting of many small squares
- Assign to the squares a user population density $\rho(x, y)$
- At each step, add a node i into a square centered at (x,y) its user population density $\rho(x, y)$
- This new node i will bring in m new edges, and each edge connects to an existing node j of degree k_j, with geographic distance d_{ij} to node i, according to the probability (nonlinear preferential attachment)

$$\Pi(k_j, d_{ij}) \sim \frac{k_j^{\alpha}}{d_{ij}^{\sigma}}$$

where α and σ are constant parameters.

GeoBA Model: Simulation/Data



α =1 gives power-law

(Yook *et al*., PNAS 2002)

Facebook Modeling

Facebook: The largest social network today

Existing Models:

- Scale-Free (BA) model
- Distance Social Network (DSN) model
- Asymmetric Weights Dynamic (AWD) model
- Exponential Random Graph models
    - Markov Random (MR) model
    - Random Triangle (RT) model

Existing Models

BA model

- Growth: starting from a connected network with $m_0 \geq \mathbf{1}$ introduce 1 new node to the network each time, and this new node is connected to m existing nodes, where $1 \leq m \leq m_0$.

- Preferential attachment: the new node is connected to each of the m existing nodes, according to the following probability:

$$\prod_i = \frac{k_i}{\sum_{j=1}^{N} k_{\boldsymbol{j}}}$$

where for node *i* of degree k_i

DSN model

- A set of N isolated individuals as nodes which are randomly placed within a social space, H, where vector $h_i^n = (h_i^1, \ldots, h_i^{d_H})$ defines the position of the i-th node and d_H is the dimension of H

- For each pair of nodes, their connection probability is

$$\Pr\left(h_i^n, h_j^n\right) = \frac{1}{1 + [b_n^{-1} d_n(h_i^n, h_j^n)]^{\alpha_n}}$$

where $d_n(h_i^n, h_j^n)$ is the Euclidean distance, b_n^{-1} controls the average degree and α_n > 1 is a measure of homophily, i.e., the tendency of people to connect to similar people. So, as α_n increases, the cluster coefficient becomes larger.

Existing Models

AWD model

In a network of size N, each node goes out to visit other nodes. The choice of whom to visit is determined by the relative weights each node has assigned to the others. So, the i-th node has a vector of weights $(w_{i1}, \ldots, w_{in})$. The connection probability that this node i visits node j is

$$\Pr(i,j) = \frac{w_{ij}}{\sum_k w_{ik}}$$

MR model

Its probability distribution:

$$\Pr(\mathbf{Y}=\mathbf{y}) = \left(\frac{1}{k}\right)\exp(\theta L(\mathbf{y}) + \sigma_2 S_2(\mathbf{y}) + \sigma_3 S_3(\mathbf{y}) + \tau T(\mathbf{y}))$$

where θ is a parameter; $L(\mathbf{y})$ is the number of edges; $S_2(\mathbf{y})$ and $S_3(\mathbf{y})$ are the numbers of two-stars and three-stars in the network, y, and $T(\mathbf{y})$ is the number of triangles in $\mathbf{y}$.

RT model

It is a Markov random model which considers only the triangle effects and sets all other parameters to zero

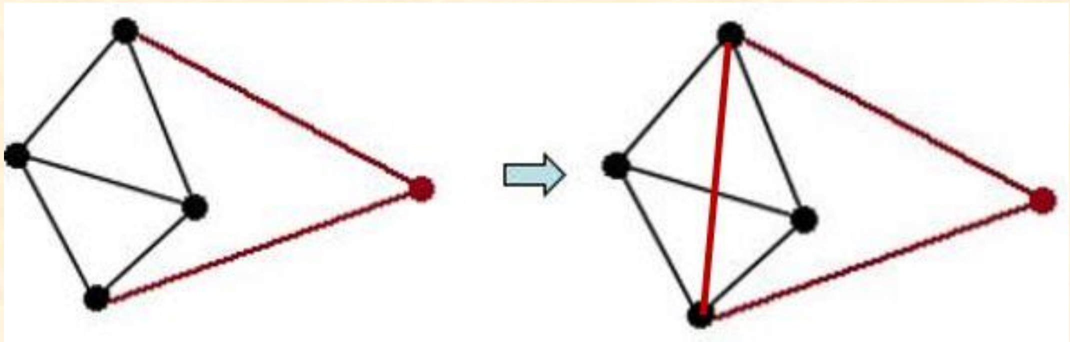
Henneberg Network

Step 1. Start with a small (triangularly) connected network.

Step 2. Node addition: Add a new node to the existing network.
Connect it to a randomly-picked pair of existing nodes.

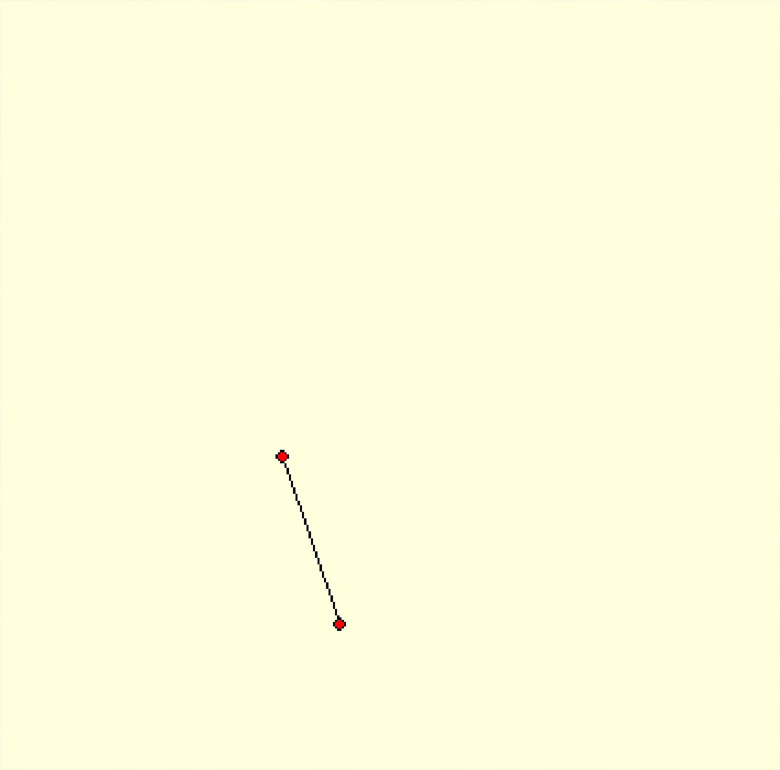
Step 3. Edge addition: If the pair of old nodes were connected,
do nothing; otherwise, connect them with a new edge.

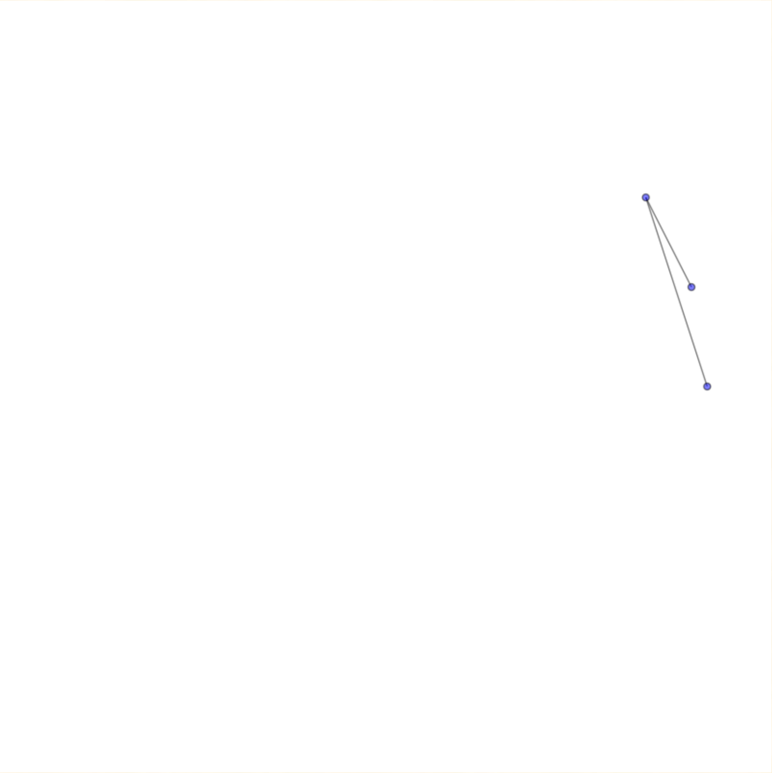
Step 4. Repeat Steps 2 and 3 till the network has the desired size.



Original Model L. Henneberg (1911)

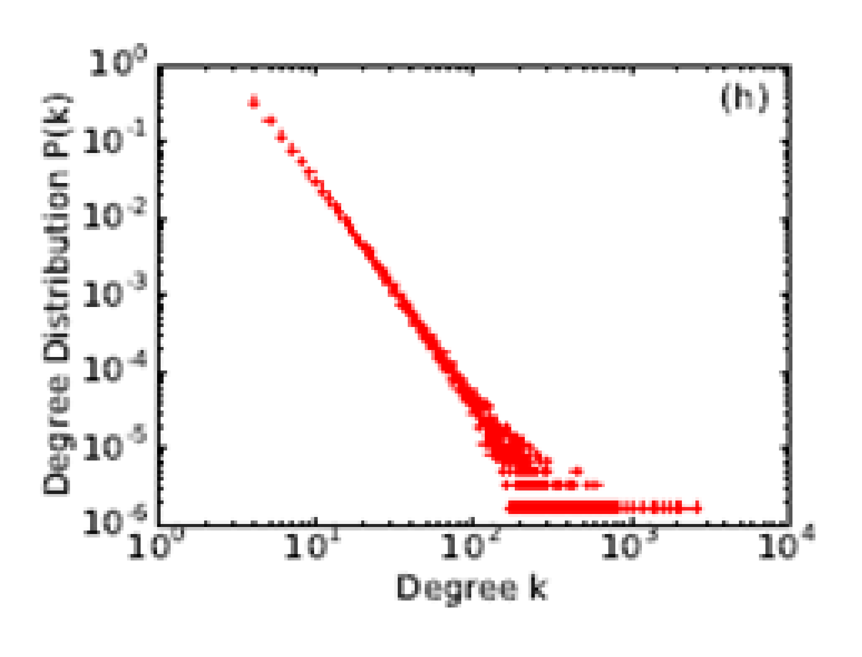
Facebook is Scale-Free Network





Different ways to displace

Modified Henneberg Model



Modified Henneberg Model

with preferential attachment connections

BREAK

10 minutes

Modeling the Internet

Starting from scale-free models …

Limitation
of most scale-free Internet models

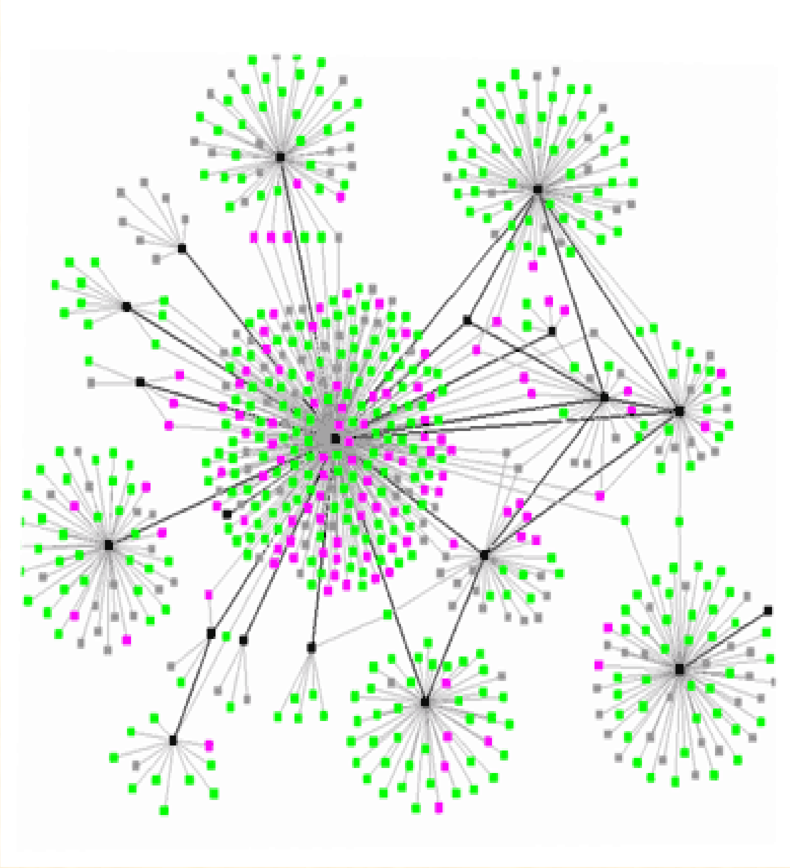
Preferential Attachment (or, its variants):

$$\Pi_i = \frac{k_i}{\sum_j k_j}$$

They all use global preferential attachment -

- Every newly added node requires the connectivity information of **all nodes** in the network
- A real network only uses **local** preferential attachment with information of only **some nodes** in the network

Question: How to describe a topology of the AS-level Internet with localization property?

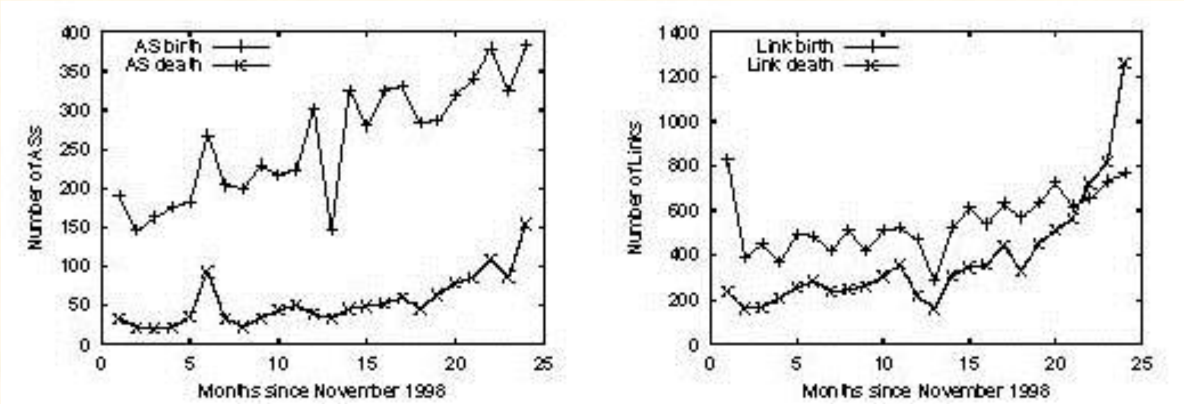


- The Internet consists of several sub-networks: each sub-network is called a "**local-world**"
- The newly added node only needs connectivity information of those nodes in a local-world
- The connections among different local-worlds are sparse
- The connections of nodes within the same local-world are dense

A Scale-Free Network Model for Internet
--- Multi-Local-World (MLW) Model

This model includes 4 events:

- Addition of local-worlds
- Addition of new nodes to local-worlds with preferential attachments
- Addition and deletion of edges of new nodes to local-worlds
- Addition of edges among local-worlds

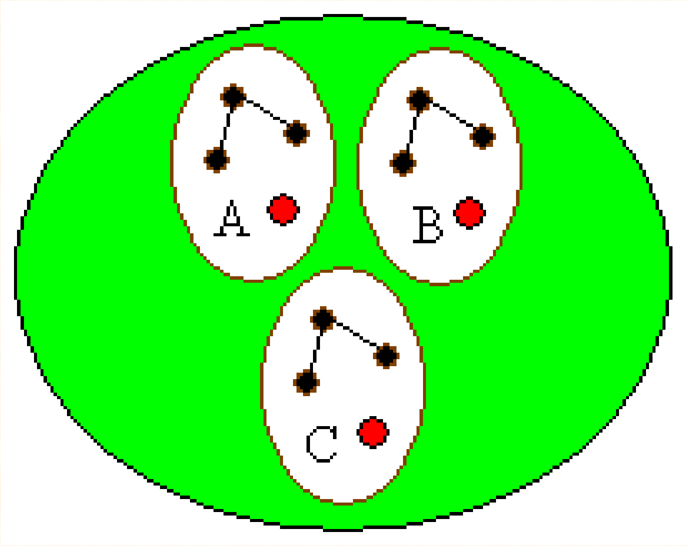


Oregon data (1998)

G. Chen, Z. Fan and X. Li (2005), Z. Fan and G. Chen (2006, 2009)

MLW Model

Start with m isolated local-worlds, with m_0 nodes and e_0 edges in each local-world



Example:

Start with $m = 3$ local-worlds (A, B, C), with $m_0 = 3$ nodes (black circles) and $e_0 = 2$ edges in each local-world

Each local-world has a unique identifier (red circle)

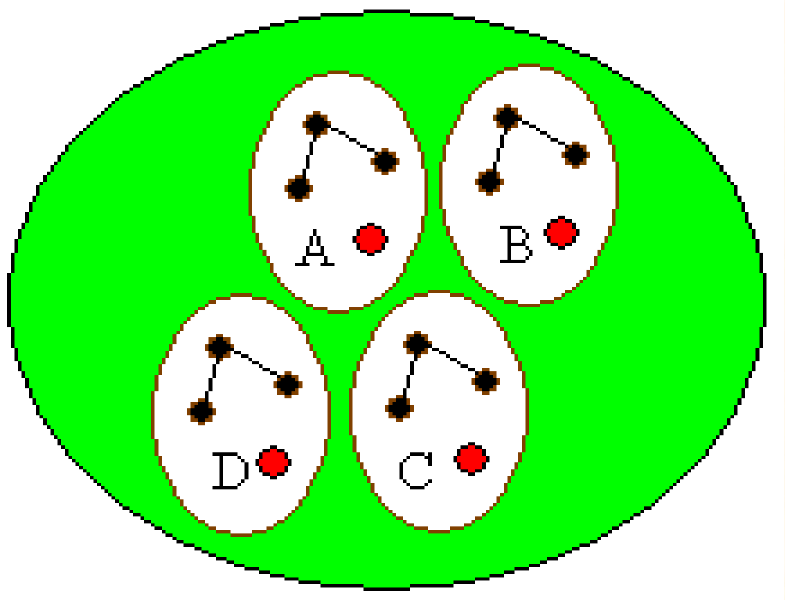
MLW Model

At each step, perform one of the five operations:

(i) **With probability p a new local-world is created, which contains m_0 nodes and e_0 edges.** Meanwhile, a unique identifier is generated for this new local-world.

Local world D is created with probability p

(with $m_0 = 3$ nodes (black circles) and $e_0 = 2$ edges)



MLW Model

(ii) With probability *q*, a new node is added to an existing local-world, which has m_1 edges with the nodes within the same local-world:

First, a local-world Ω is selected at random. Then, a node to which the new node connects in the local-world Ω is chosen with probability

$$\Pi(k_i) = \frac{k_i + \alpha}{\sum_{j \in \Omega} (k_j + \alpha)} \qquad (1)$$

MLW Model

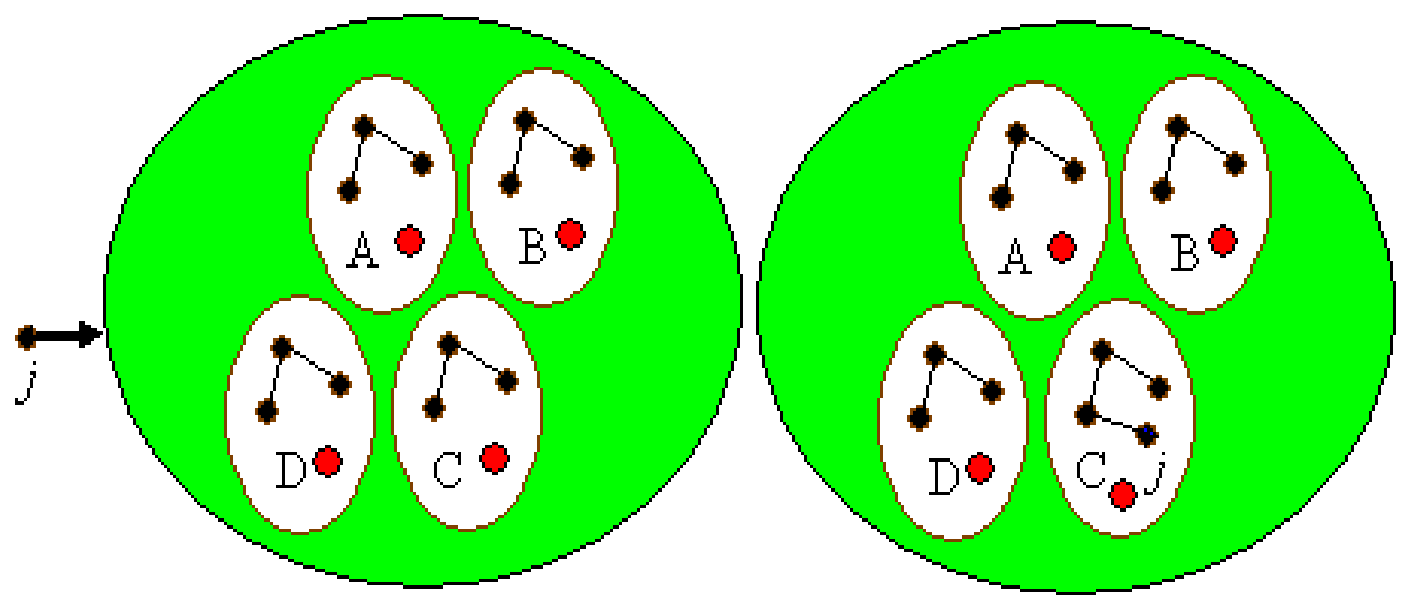
(Step (ii) continued)

In (1), Ω is the Ω-th local-world in which node i locates, and the parameter $\alpha > 0$ represents the "attractiveness" of node i which is used to govern the probability for "young" nodes to get new edges.

This process is repeated m_1 times.

MLW Model

(Step **(ii)** continued)



Example (continued) **A new node j joins the network. First, it selects the local-world C where it will locate, and then connects an existing node ($m_1 = 1$) in this local-world with preferential attachment probability given by (1)**

MLW Model

(iii) With probability *r*, m_2 edges are added to a chosen local-world:

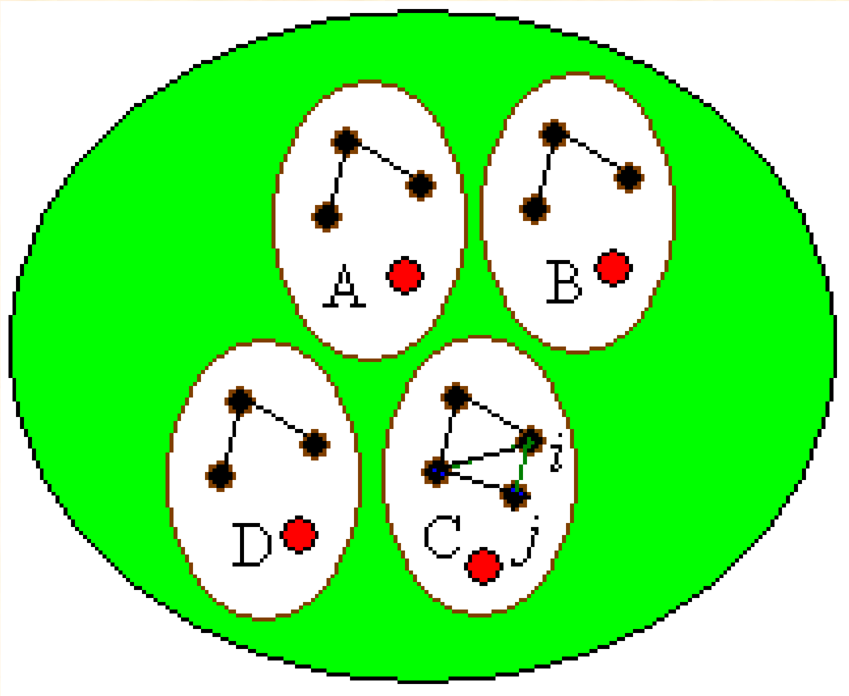
First, a local-world Ω is selected at random.

Then, one end of an edge is chosen at random, while the other end of the edge is selected by (1)

This process is repeated m_2 times

MLW Model

(Step **(iii)** continued)



Example:

First, **local-world** ***C*** is chosen at random.
Then, $m_2 = 2$ edges are added to this local-world.

One end of an edge is selected at random, while the other end of the edge is chosen with a probability given by (1)

MLW Model

(iv) With probability *s*, m_3 edges are deleted within a chosen local-world:

First, a local-world Ω is selected at random.

Then, one end of an edge is chosen at random,
while the other end of the edge is selected with probability

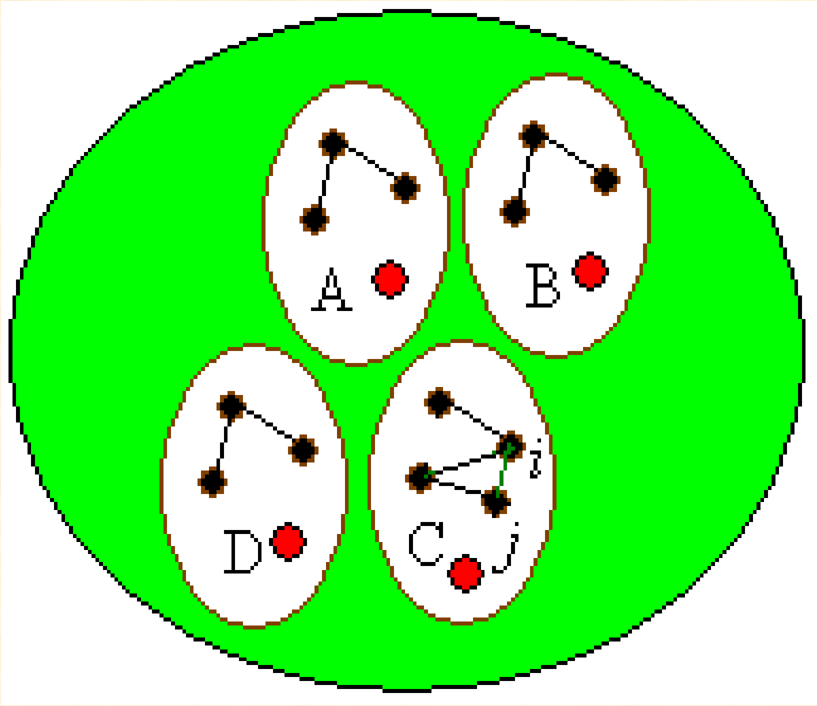
$$\Pi'(k_i) = \frac{1}{N_\Omega(t)-1}(1-\Pi(k_i)) \qquad (2)$$

where $N_\Omega(t)$ represents the number of nodes within the Ω -*th* local-world, and $\Pi(k_i)$ is determined by (1)

This process is repeated m_3 times.

MLW Model

(Step **(iv)** continued)



Example:

First, **local-world** *C* is chosen at random.

Then, $m_3 = 1$ edge is deleted within this chosen local-world.

An end of the edge is selected at random, while the other end of the edge is chosen with probability given by (2)

MLW Model

(v) With probability *u*, a selected local-world has m_4 edges with the other existing local-worlds:

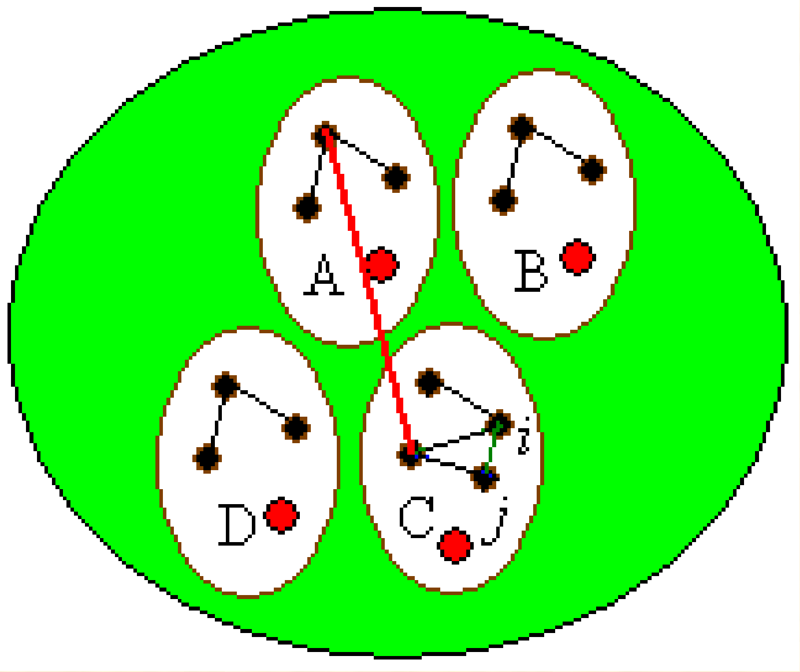
First, randomly select a local-world and a node in this local-world with probability given by (1).

Then, the selected node is acted as one end of an edge, while the other node of the edge, which is in another local-world chosen at random, is selected with probability given by (1).

This process is repeated m_4 times.

MLW Model

(Step **(iv)** continued)

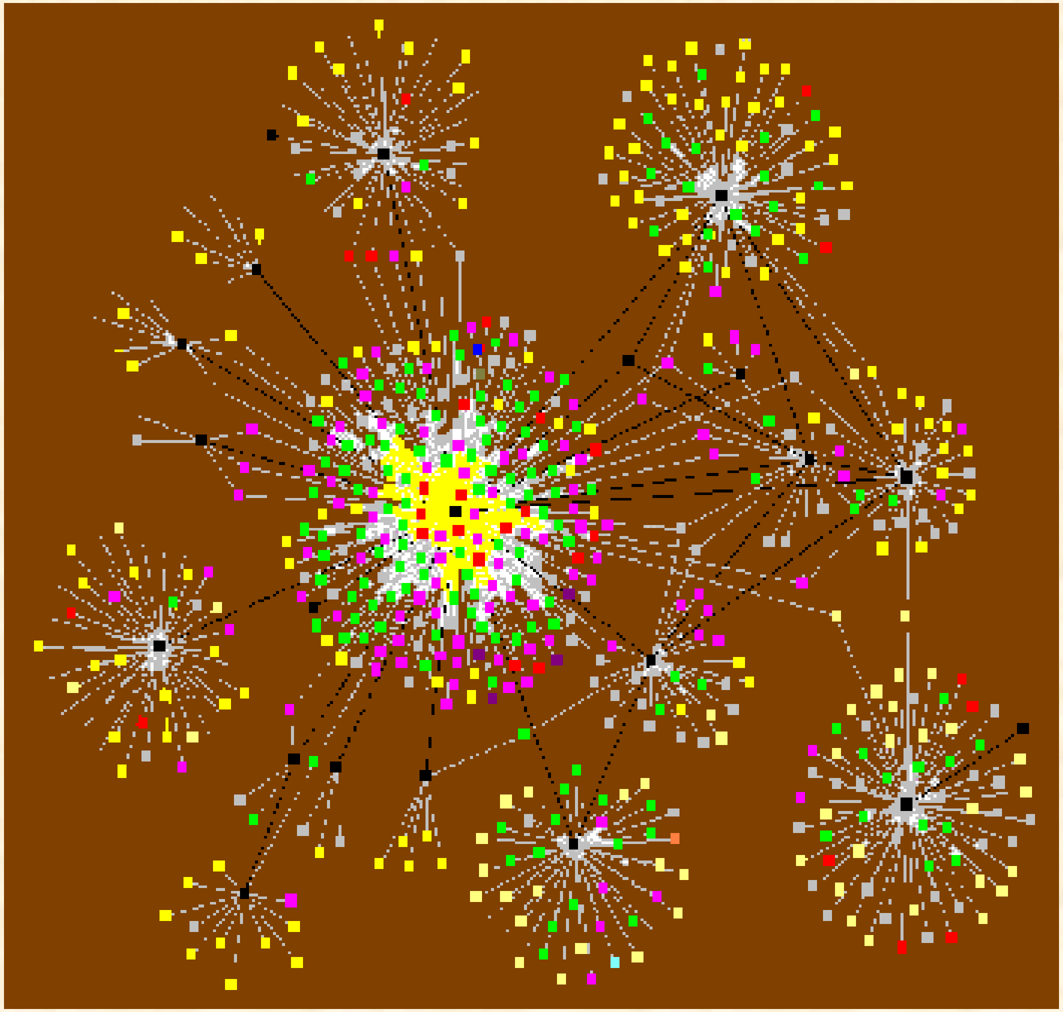


Example:

Depending on the probability u, $m_4 = 1$ link is added between two nodes in two different local-worlds.

Both ends of the link are chosen with preferential attachment according to a probability given by (1)

Illustration of a Resulting Network



MLW Model

Degree Distribution has a power-law form:

$$P(k)=\frac{t}{a(3m+t(1+2p))}\left(m_1+b/a\right)^{1/a}\left(k+b/a\right)^{-\gamma}$$

Here: $0\le p,r,s,u\le 1,\ \ 0<q\le 1,\quad p+q+r+s+u=1$

$$\gamma=1+1/a$$

$$a=\frac{qm_1}{c}+\frac{rm_2(q+m_0p-p)}{(q+m_0p)c}+\frac{sm_3p}{(q+m_0p)c}+\frac{2um_4}{c}$$

$$b=\frac{q\alpha m_1}{c}+\frac{rm_2}{(q+m_0p)}+\frac{rm_2(q+m_0p-p)\alpha}{(q+m_0p)c}+\frac{sm_3p\alpha}{(q+m_0p)c}-\frac{2sm_3}{(q+m_0p)}+\frac{2um_4\alpha}{c}$$

Evaluating the Internet models

Internet Models at the AS-level:

- Waxman model
- Transit-stub model

~Weibull distribution

- Fluctuation-driven model
- BA model
- Generalized BA (GBA) model
- Fitness model
- HOT model

MLW Model

Power-law distribution

GBA is also called EBA (Extended BA)

Evaluating the Internet models (cont.)

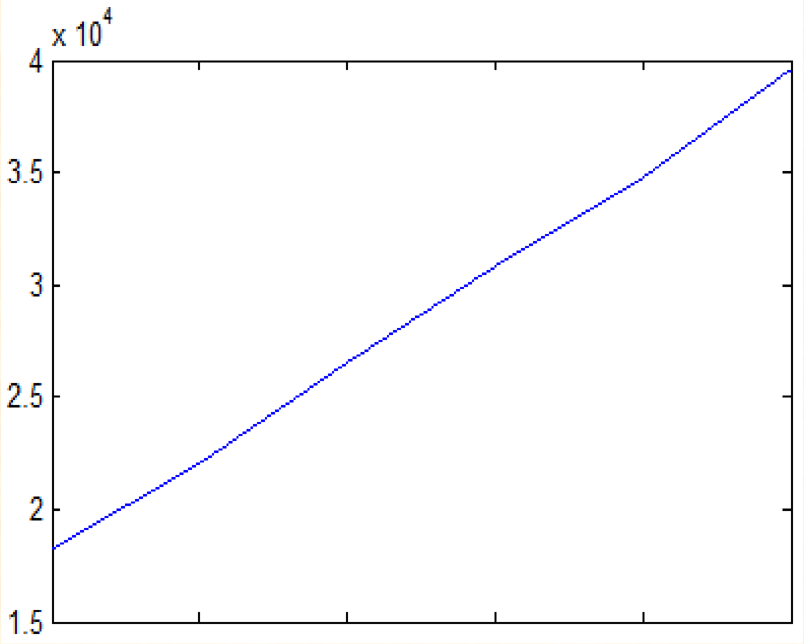
- Fluctuation-driven model - **Exponentially growing network**

- BA model
- Generalized BA (GBA) model
- Fitness model
- HOT model
- MLW model

Linearly growing networks

GBA = EBA

Evaluating the Internet models (cont.)



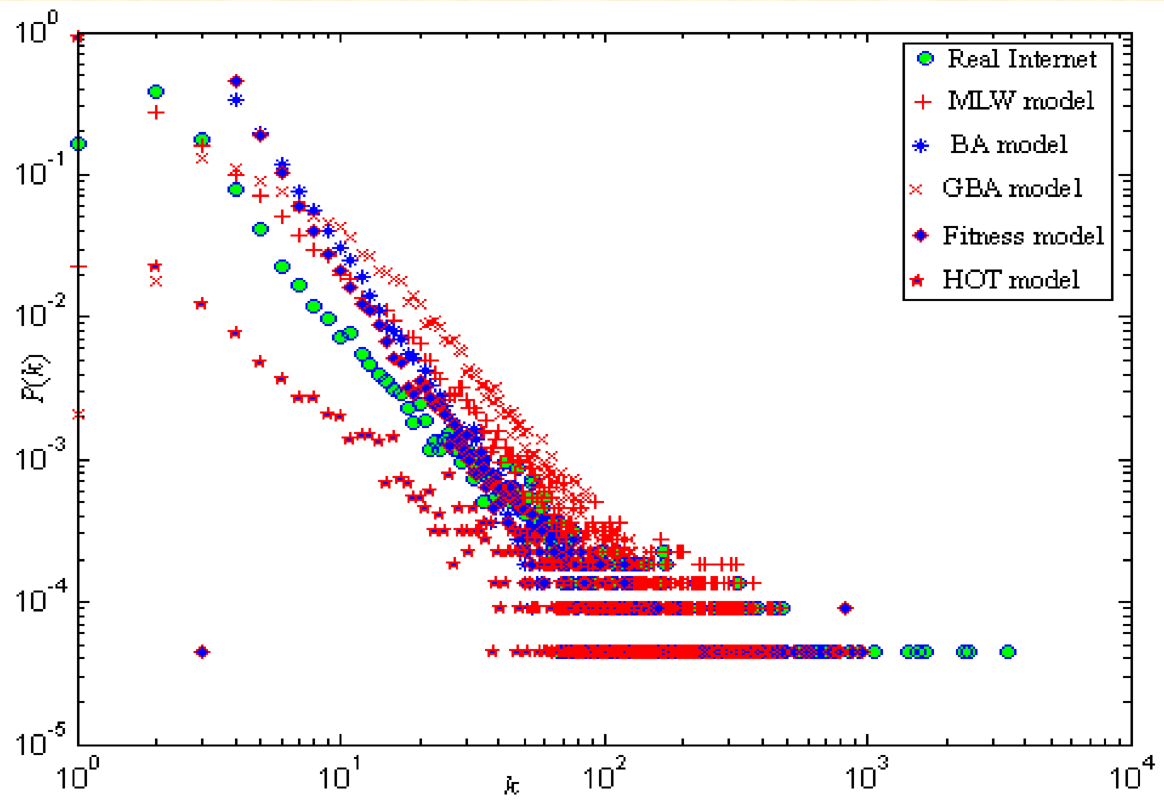
Number of Nodes

(from 2004 to 2010)

Linearly growing network

Fluctuation-driven model is NOT suitable for the AS-level Internet

Evaluating the Internet models (cont.)



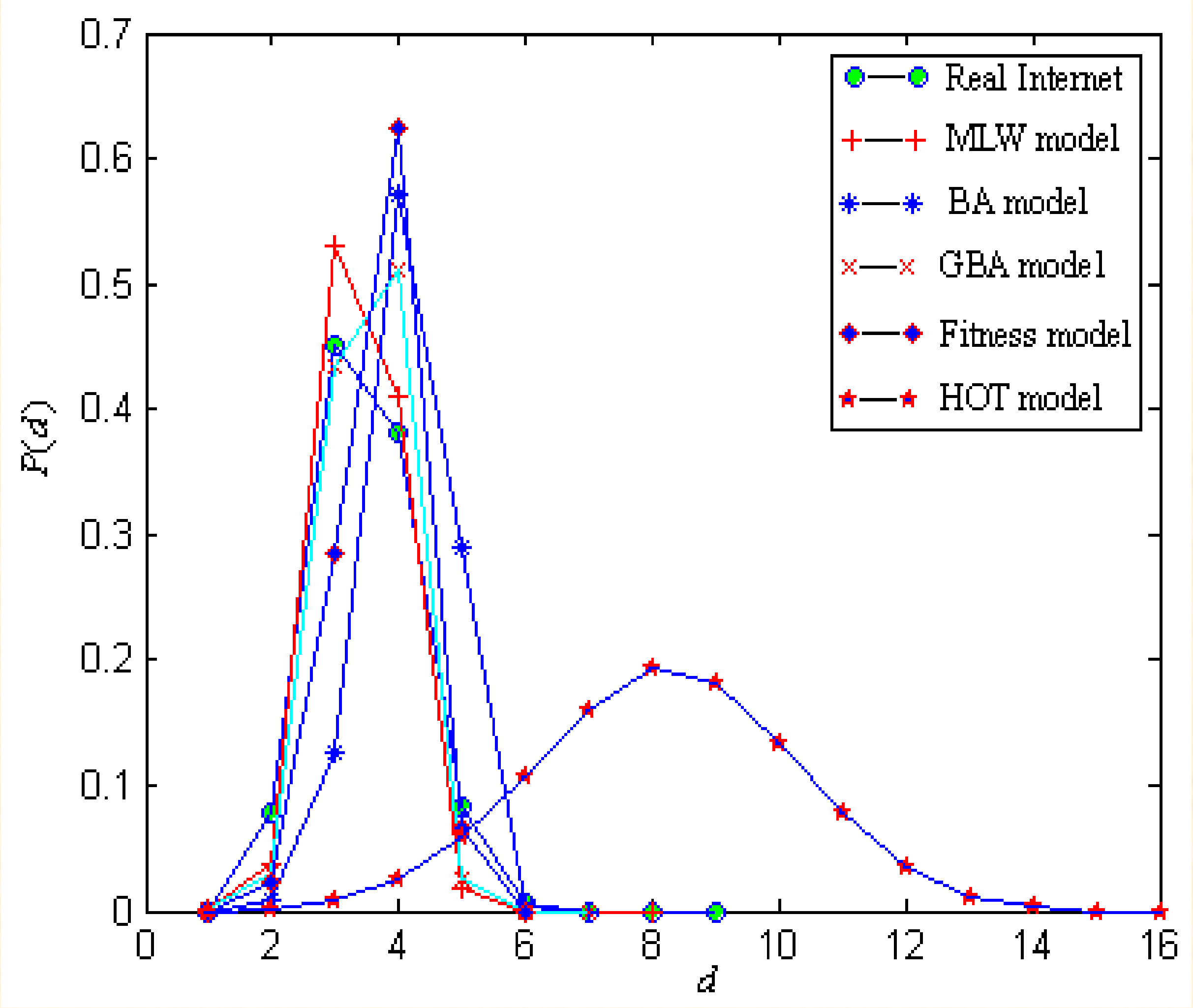
Internet snapshot on May 15, 2005

Power Exponent:

r = 2.2 for real Internet
r = 3.0 for BA model
r = 1.5 for HOT model

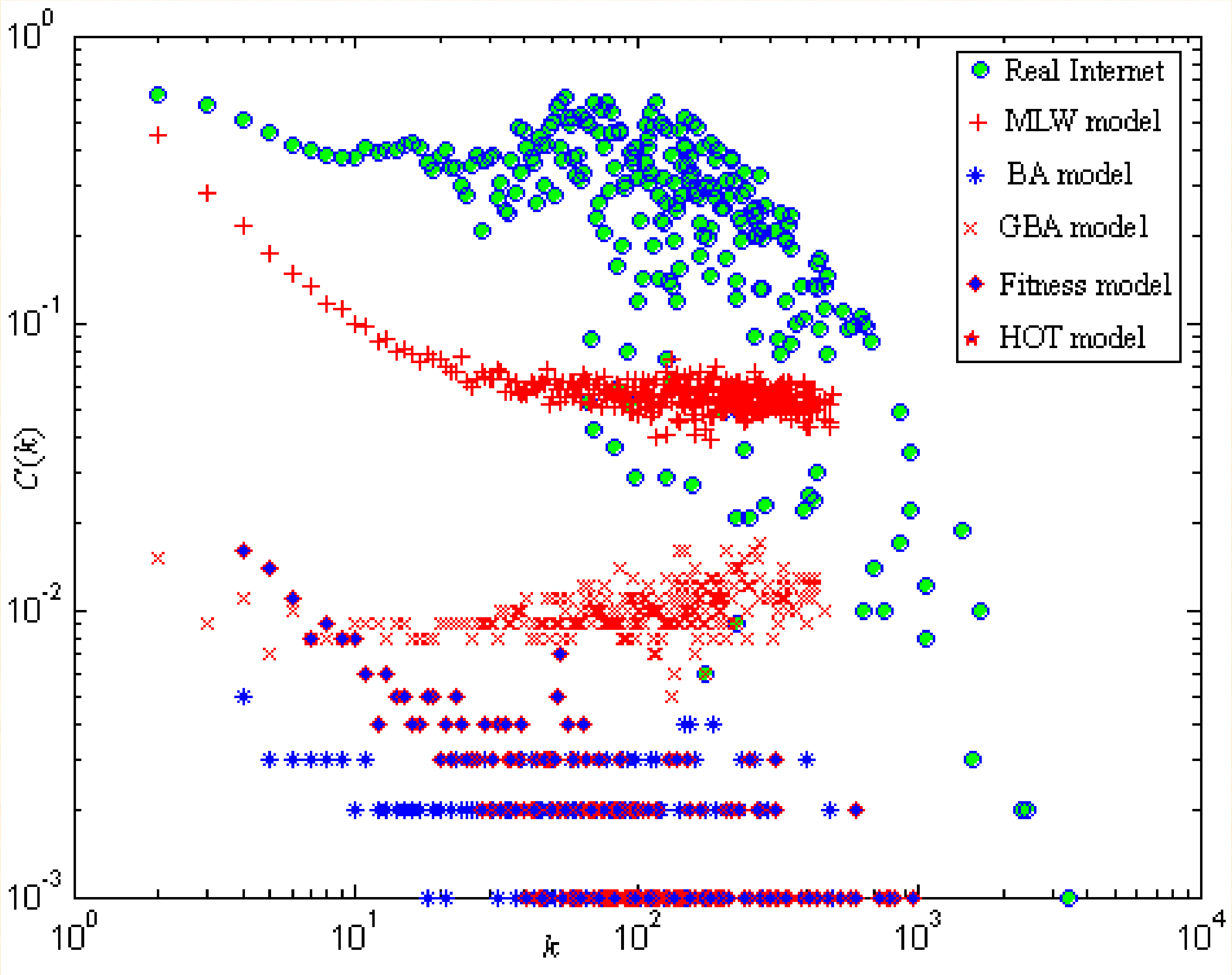
BA and HOT models can NOT capture the scale-free feature of the Internet

Evaluating the Internet models (cont.)



Distance distributions of the Internet and of different scale-free models

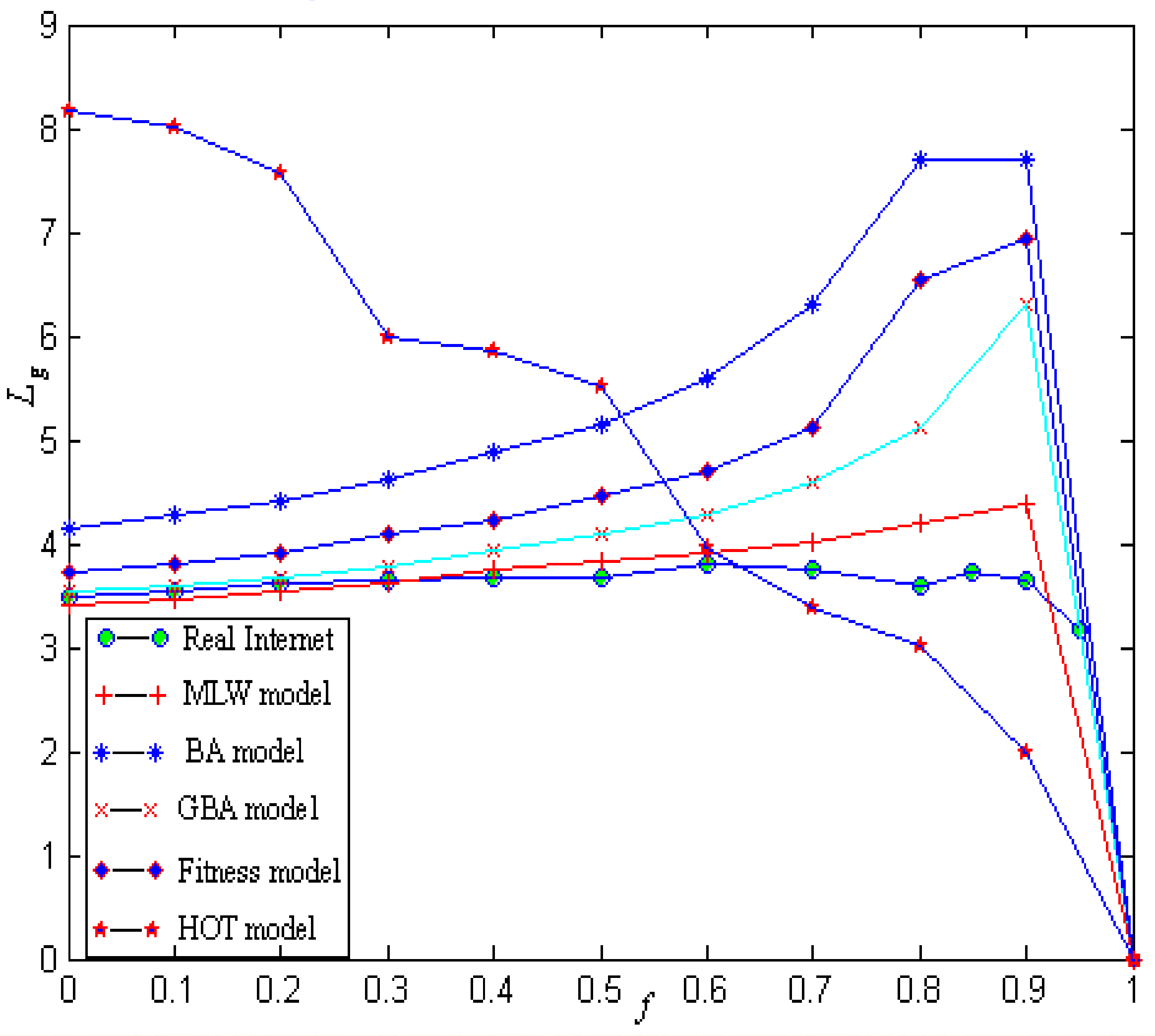
Evaluating the Internet models (cont.)



Clustering coefficients as functions of the degree for the real Internet and the BA, GBA, Fitness, HOT, and MLW models.

BA, GBA, Fitness, and HOT models can NOT capture the small-world feature of the real Internet

Evaluating the Internet models (cont.)



f - Fraction of removed edges

Comparison of the **average shortest path-lengths** in the giant component for the real Internet and the five models studied.

Evaluating the Internet models (cont.)

Comparison: Four Models vs Real Internet

	BA	GBA	Fitness	MLW	**Real Internet** (May 15,2005)
N	21999	21999	21999	21999	21999
$\overline{C}$	0.003	0.01	0.01	**0.24**	**0.46**
$\overline{d}$	4.14	**3.49**	3.71	3.45	**3.49**
γ	3	2.69	2.45	**2.36**	**2.18**
λ_{max}	27.82	62.83	39.16	**111.87**	**141.12**

Comparison of MLW Model with other models

Internet Modeling	Scale-free feature	Small-world feature	Community structure
BA model	Yes	No	No
EBA model	Yes	Yes	No
Fitness model	Yes	No	No
HOT model	Yes	No	No
MLW model	YES	YES	YES

MLW model is better than the BA, EBA, Fitness and HOT models in capturing the scale-free and small-world features of the Internet

Further Evaluating the Internet models

Summary -

MLW model is better than BA, EBA, Fitness, and HOT models in capturing the scale-free and small-world features of the Internet

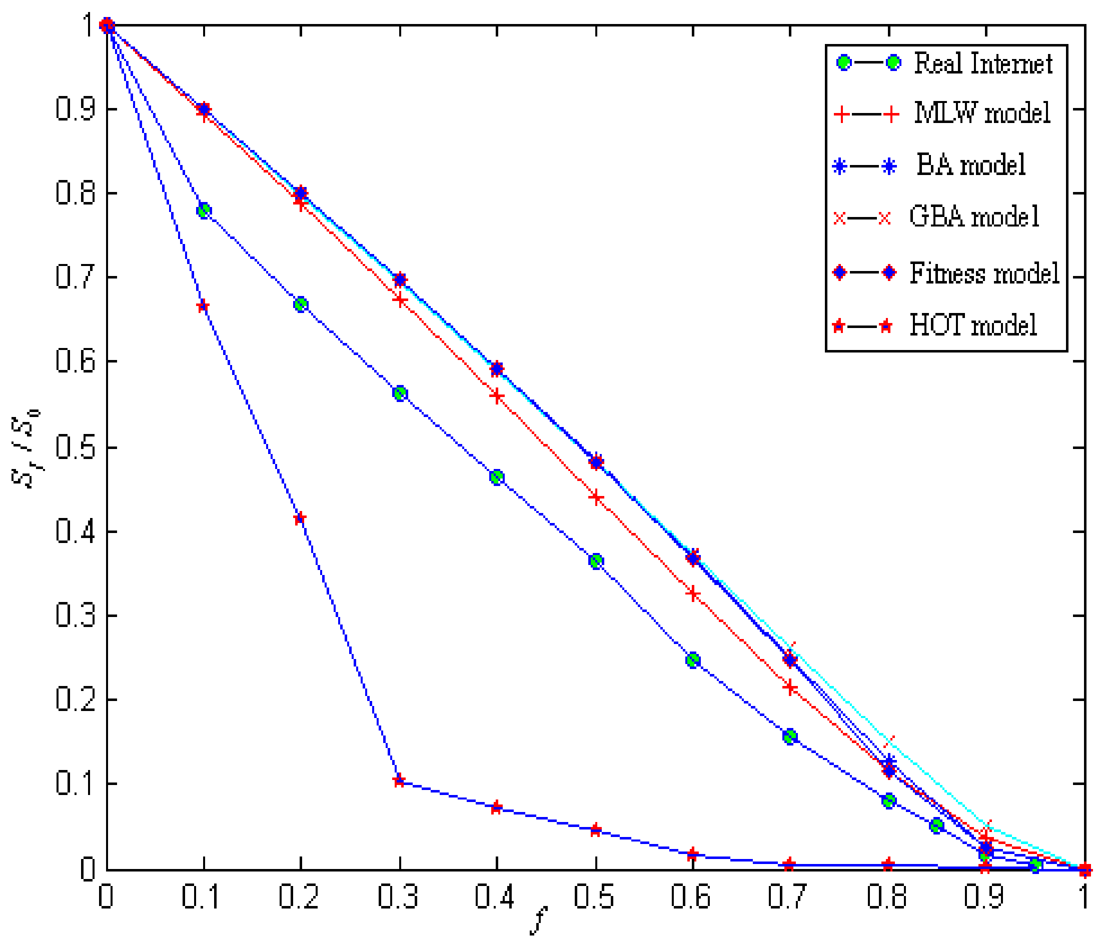
Topological Statistics -

degree distribution, power-law exponent,
distance distribution, clustering coefficient,
average shortest path-length, hierarchical clustering

But what about performances?

Comparison: What about performances?

Robustness against random attacks

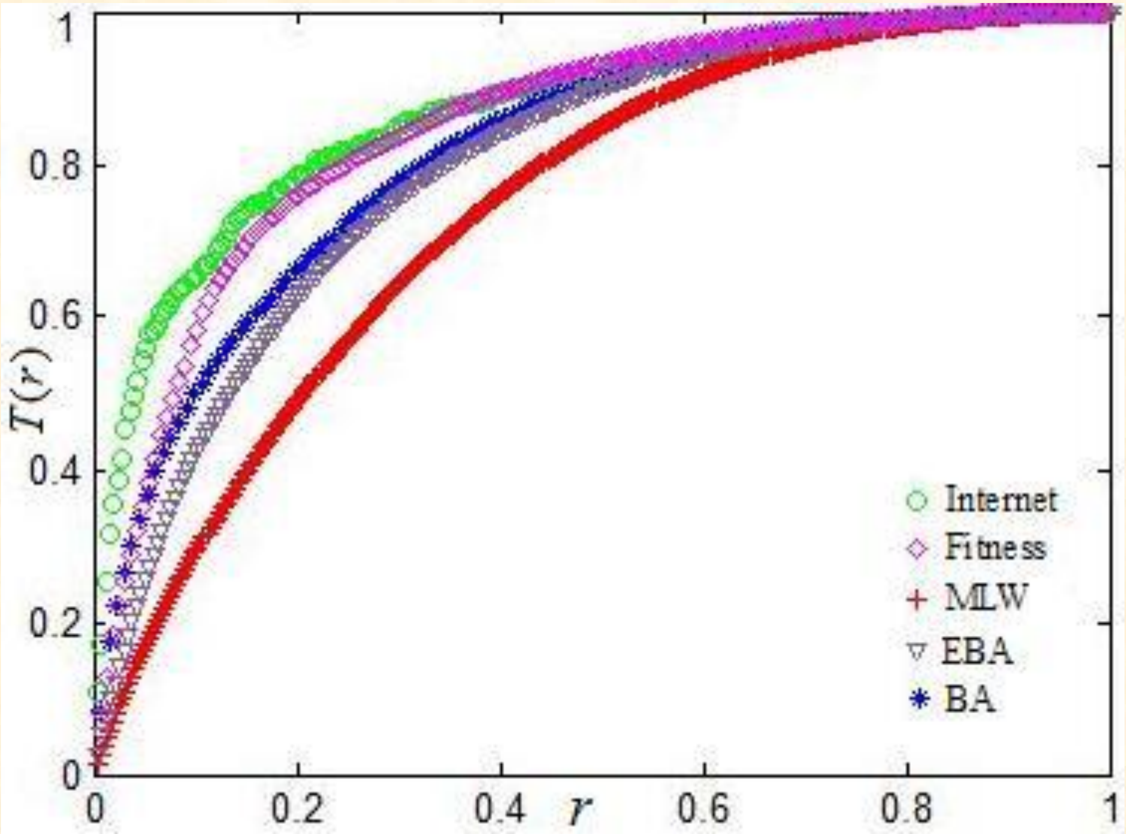


S_f: the size of the largest component after a fraction of nodes, f, in the network are randomly removed.

S_f / S_0 measures the capability of the network in which nodes still can communicate each other after the f portion of nodes has been randomly removed.

Performance comparisons

Traffic load distribution



MLW model does not perform well

$T(r)$ - the ratio of the traffic load of the first r largest nodes over the total traffic load of the whole network

Verification of the methods used

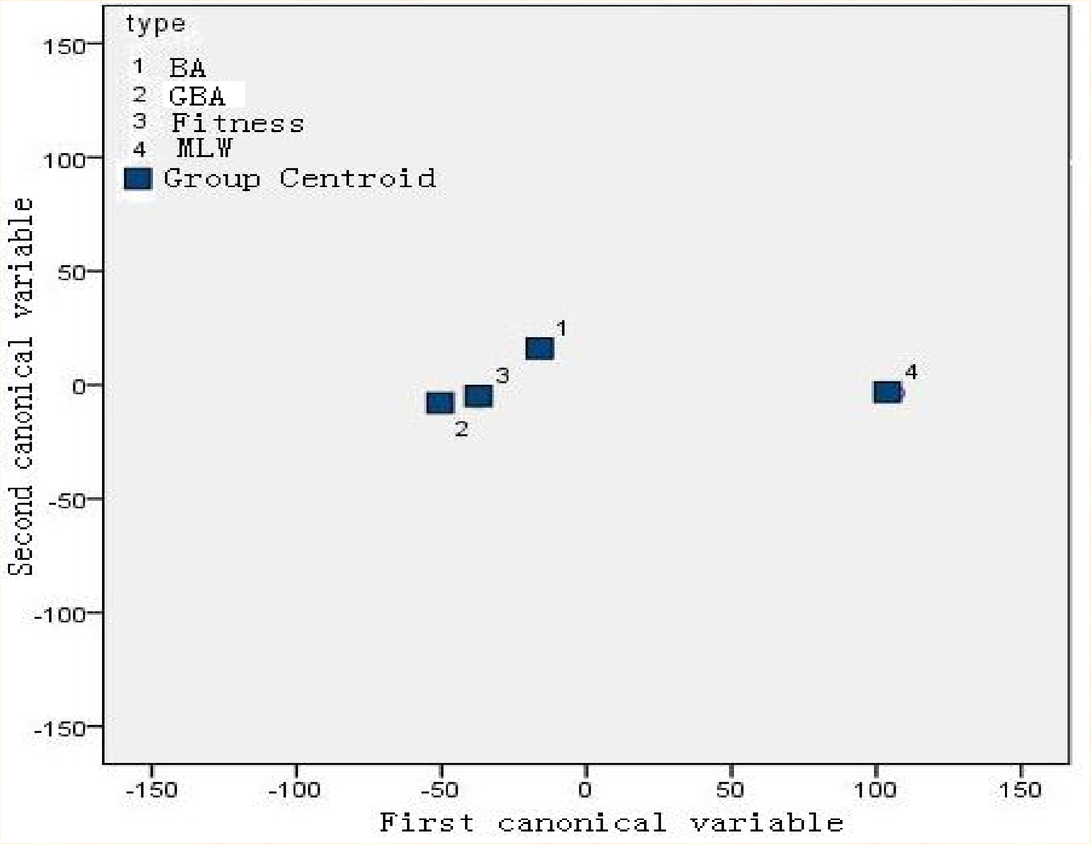
- Canonical variable analysis
- Bayesian decision theory

Topological measurements are projected into a reduced dimensional feature space by using canonical analysis, so that the Bayesian decision method can be applied onto a more representative feature space in a lower dimension

L. F. Costa, F. A. Rodrigues, G. Travieso, P. R. V. Boas, Advances in Physics 56(2007): 167

Comparison: Bayesian Test

Consider: average clustering coefficient, average distance, and largest nonzero eigenvalue of adjacency matrix



Bayesian decision method

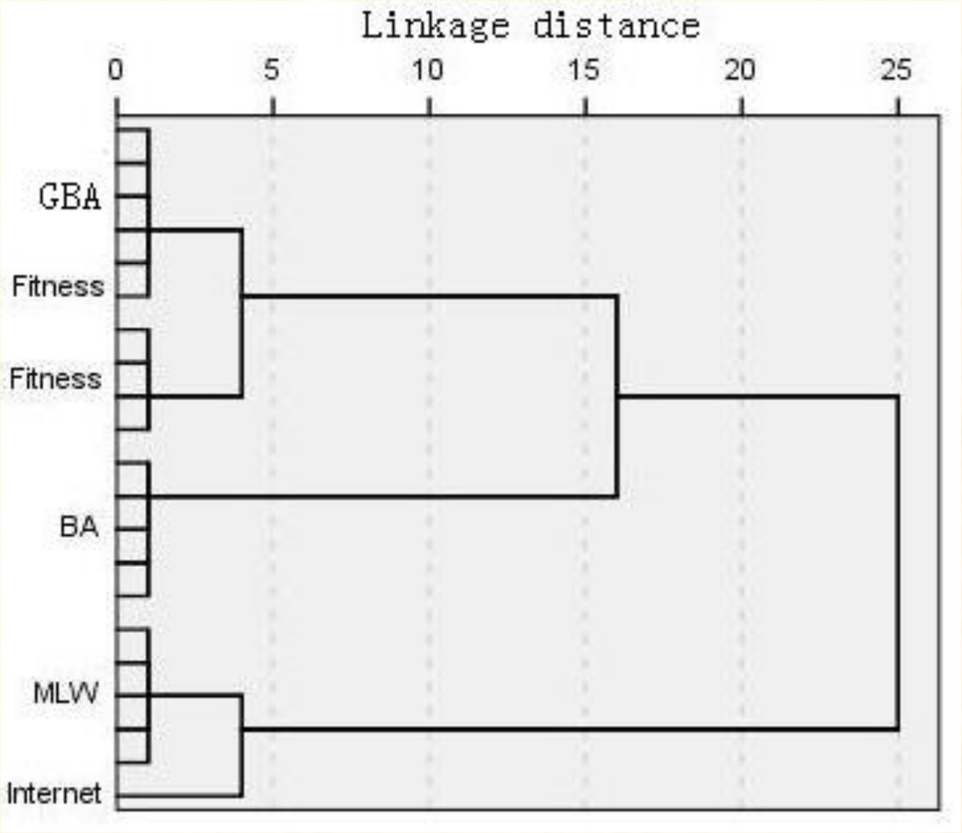
→

MLW model is most compatible with the Internet

Comparison: **Hierarchical Clustering Algorithm**

Applying the **hierarchical clustering algorithm** to evaluate different Internet models

Principle:
The sooner two networks are merged, the more similar they are



L. F. Costa, F. N. Silva,
J. Stat. Phys. 125(2006): 841

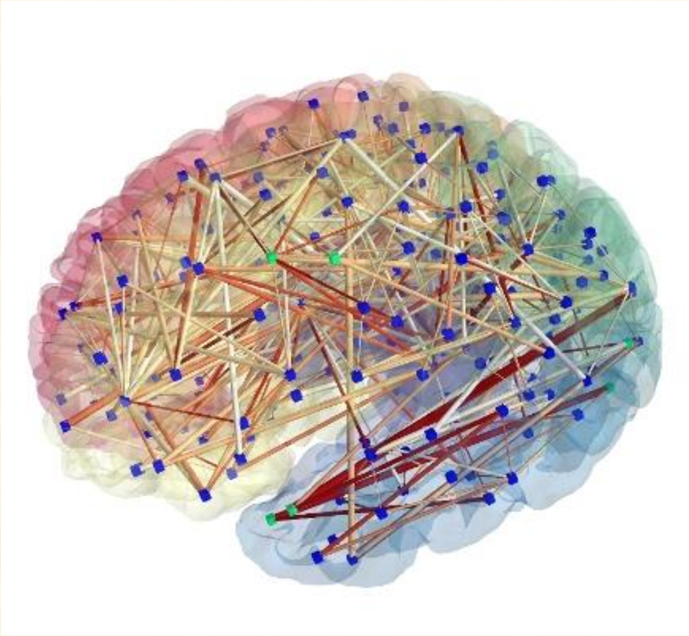
MLW model is closest to the Internet

Remarks

- **MLW** is the best model for the AS-level Internet as compared to Fluctuation-Driven, BA, EBA, and Fitness models
- These comparisons were performed only based on part of the Internet features:
  - degree distribution
  - distance distribution
  - average path-length
  - clustering coefficient
  - robustness against random attack
- The MLW model is rather complicated
- More comparisons are needed
- Internet is too complex to comprehend. As of today, there is no commonly-agreed model of the Internet

→ Good models are badly needed

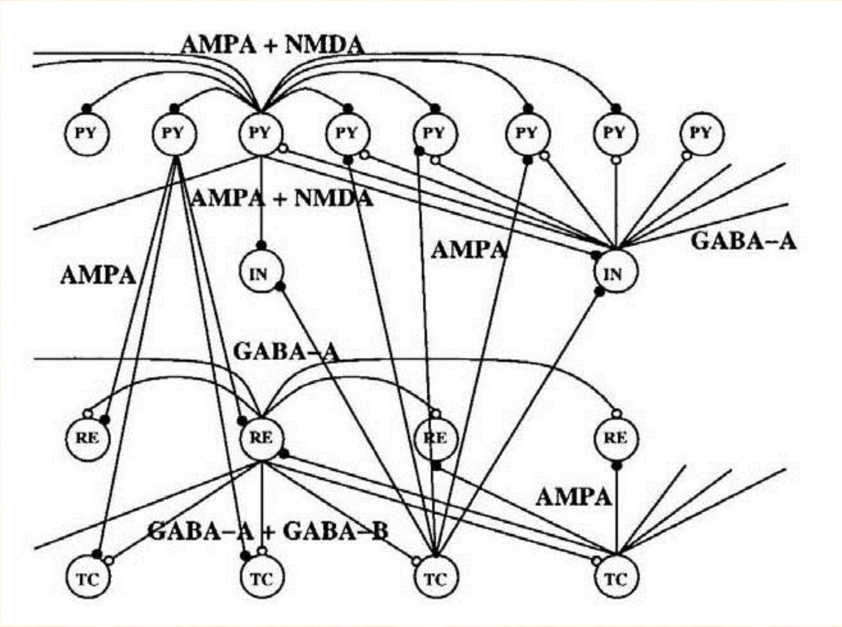
Brain and Internet

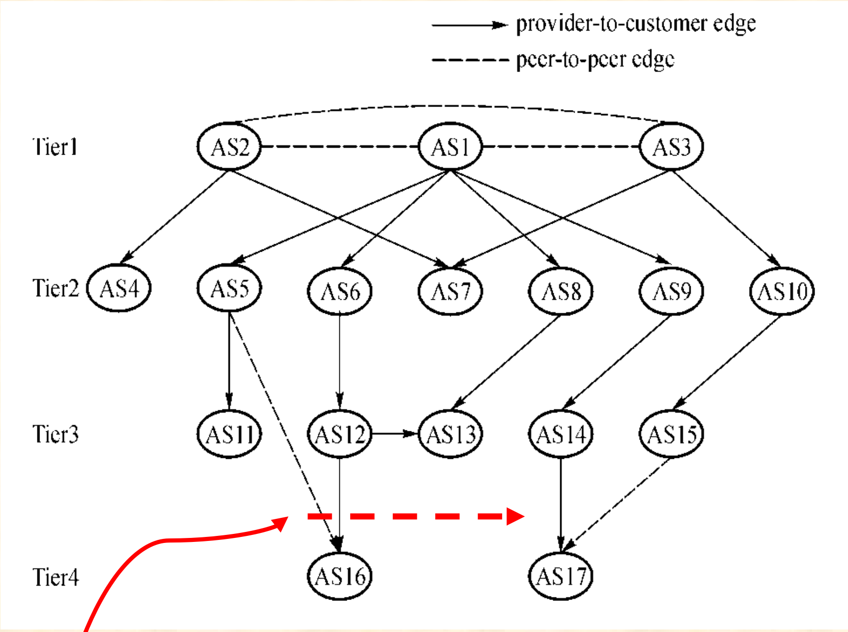


Any Inspiration ?

Topology

Brain ↔ Internet





M. Bazhenov et al., *J. Neuroscience*, 2002

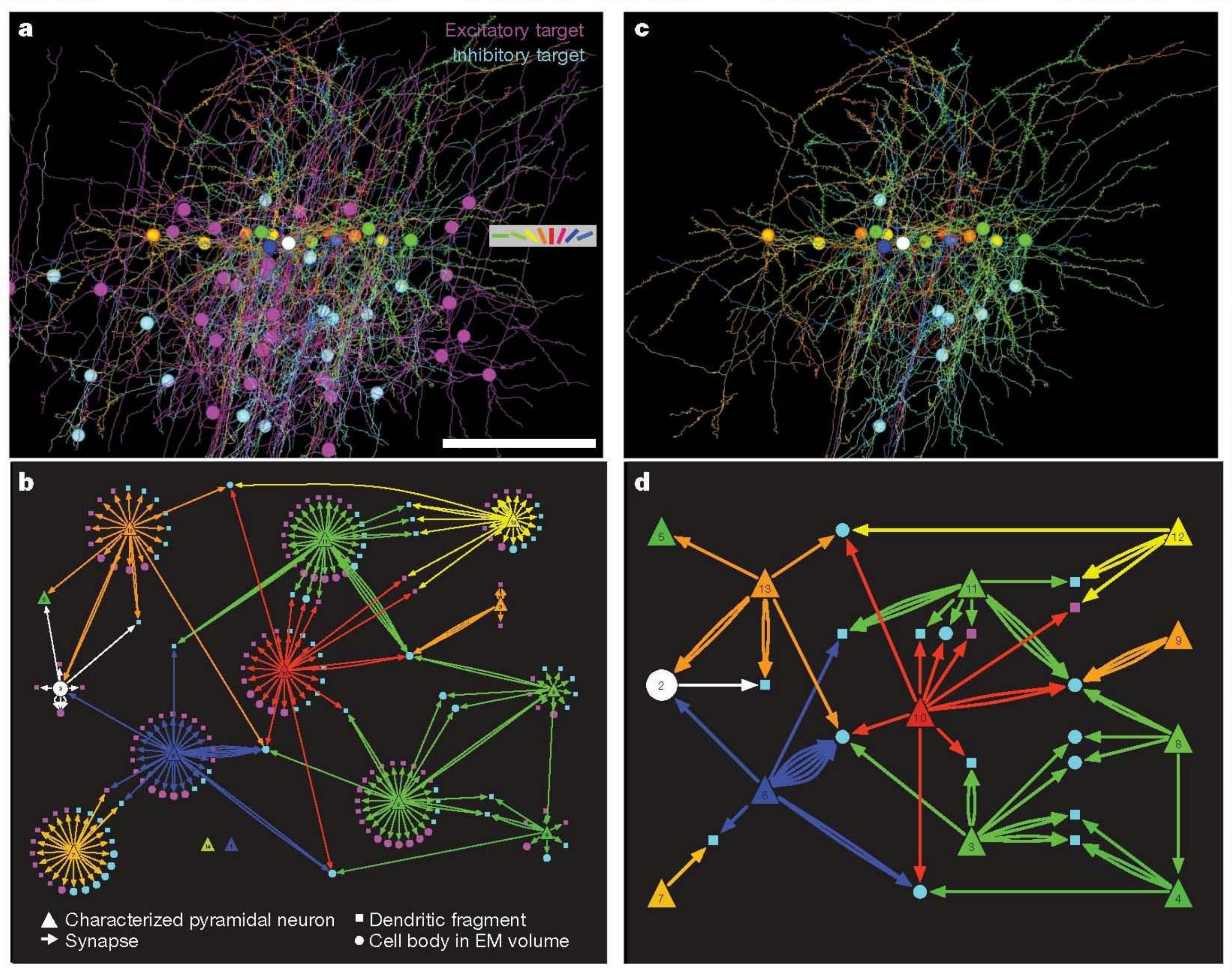
S. I. Cai et al., *Proc. IEEE CDC*, 2004

IXP (Internet Exchange Point)

Brain Topology

Small-world Topology

Community Structure



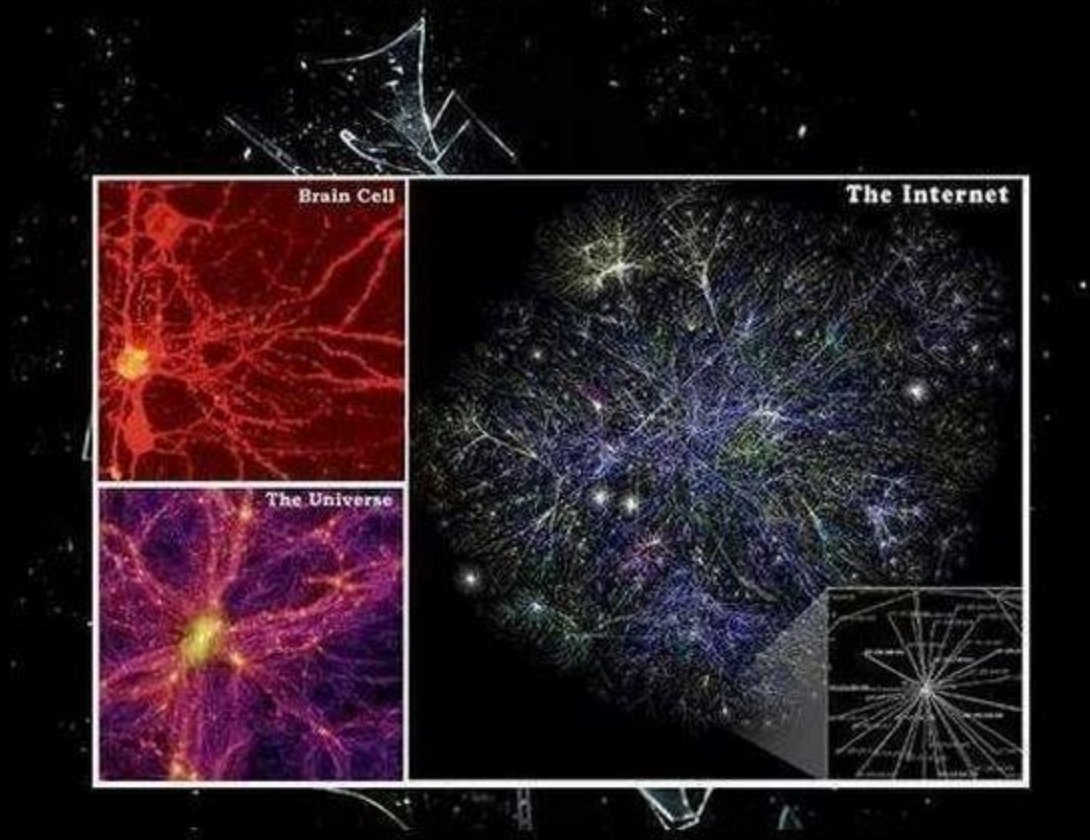
D. D. Bock et al., *Nature*, 2011

HUMAN BRAIN CELLS, OUR UNIVERSE AND THE INTERNET HAVE VERY SIMILAR STRUCTURES!!

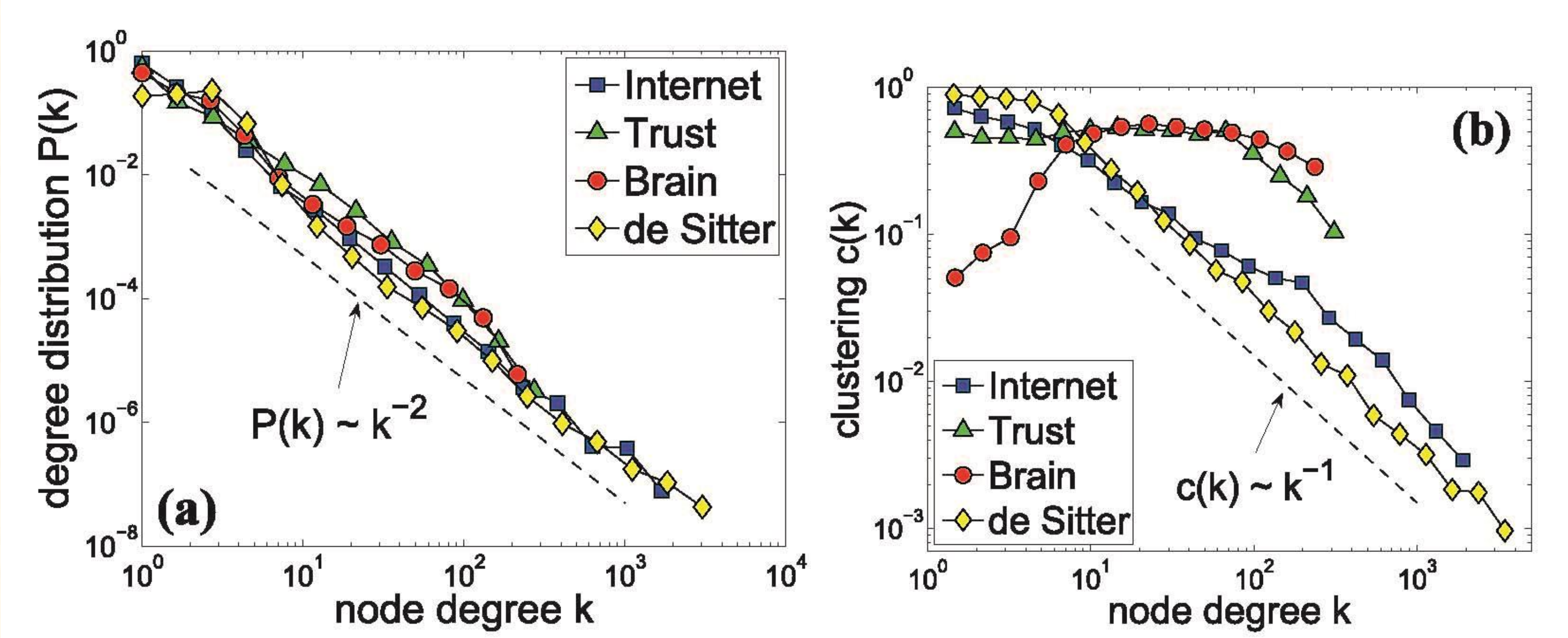
December 13, 2013 **by** marcanthony213 **in** Uncategorized **and tagged** brain cells, cellular structures, energy, frequencies, internet, structures, universe, vibrations

D. Krioukov et al., *Scientific Report*, 2012

www.unbelievable-facts.tumblr.com



Brain Topology



Internet (CAIDA Data); Trust (Social Network); de Sitter (Universe Space-time Data)

N = 24,000

D. Krioukov et al., *Scientific Report*, 2012

Brain Topology

Brain Neural Networks: functioning

Hierachical Structure

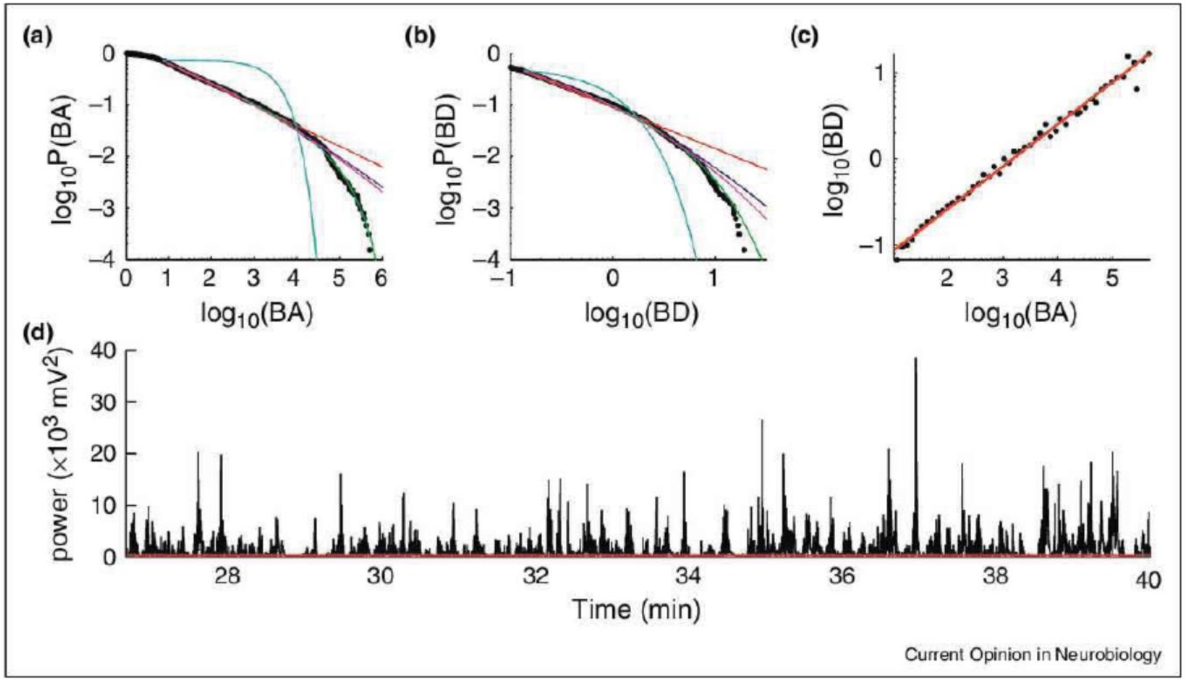
Upper level: High-frequency neuronal bursting

Middle level: … …

Lower level: Diurnal neuroendocrine rhythm

J. A. Robert, et al, The heavy tail of the human brain, *Neuobiology*, 2014

Brain Topology



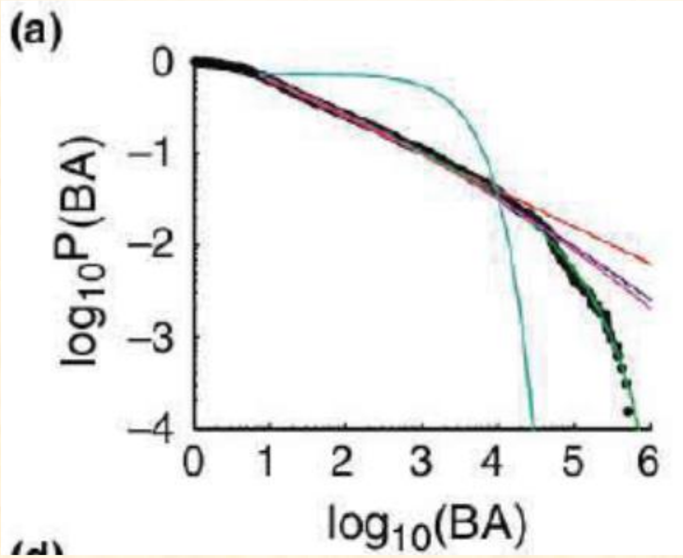
Scale-free dynamics in post-hypoxic neonatal EEG. **(a)** Upper CDF of burst area (BA) with fits to candidate distributions: power law (red), exponentially-truncated power law (green; exponent 1.36), lognormal (blue), stretched exponential (magenta), and exponential (light blue). **(b)** Upper CDF of burst duration (BD) with fits (truncated power law, green, has exponent 1.56). **(c)** Relationship between BD and BA with least-squares fit (red; slope 0.51). **(d)** Example burst suppression instantaneous power with threshold overlaid (red). Figure adapted from Ref. [44•].

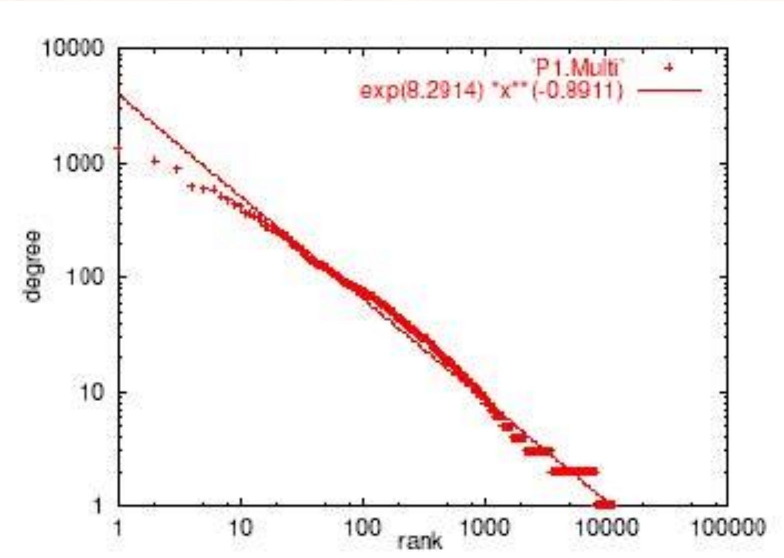
J. A. Robert, et al, The heavy tail of the human brain, *Neuobiology*, 2014

Brain Topology

Brain

Internet

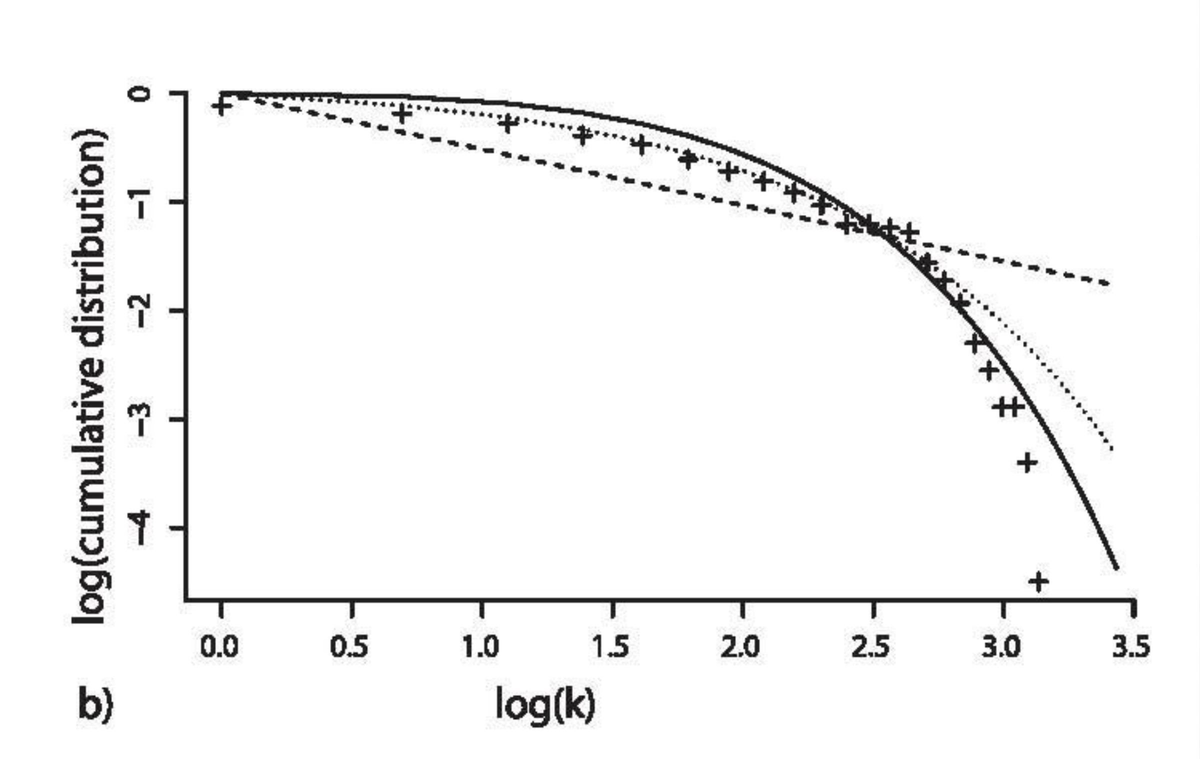




J. A. Robert, et al, *Neuobiology*, 2014

Faloutsos (3 brothers), *ACM SIGCOMM*, 1999

Brain Topology



Degree distribution of **small-world** brain **functional** network

S. Achard, et al, *Neurobiology*, 2006

Brain Topology

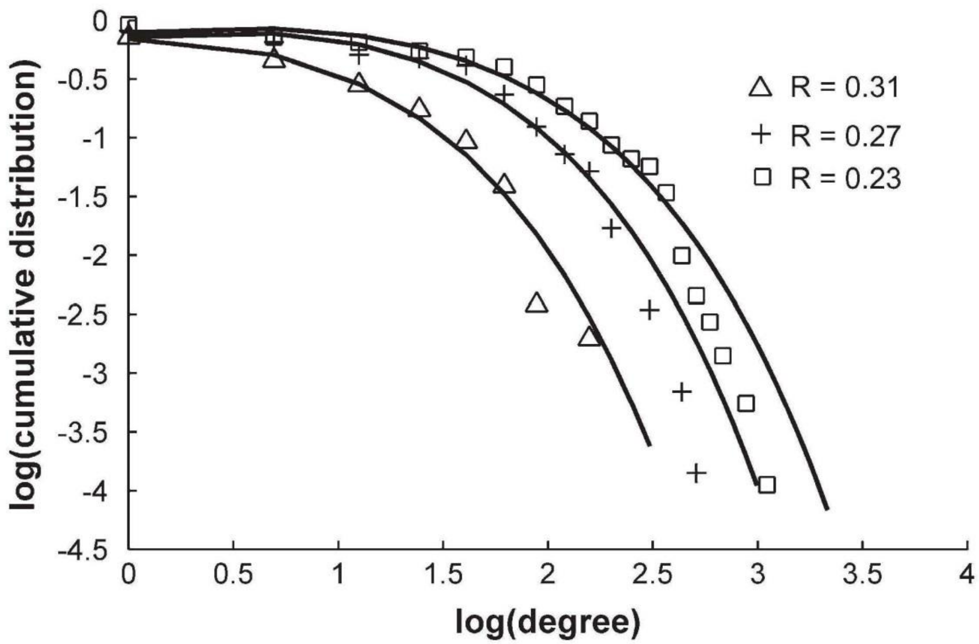


Figure 5. The degree distributions of human brain anatomical networks for 3 different correlation threshold values. The black solid lines indicate exponentially truncated power-law fits in the log–log plot of the cumulative probability degree versus the degree for the 3 correlation threshold values. Here a cumulative distribution was used to reduce the effects of noise on this smaller data set (Strogatz 2001). Upright

Degree distribution of **small-world** brain **anatomical** network

Yong He, et al, *Cerebral Cortex*, 2007; *PLoS One*, 2011

Inspiration

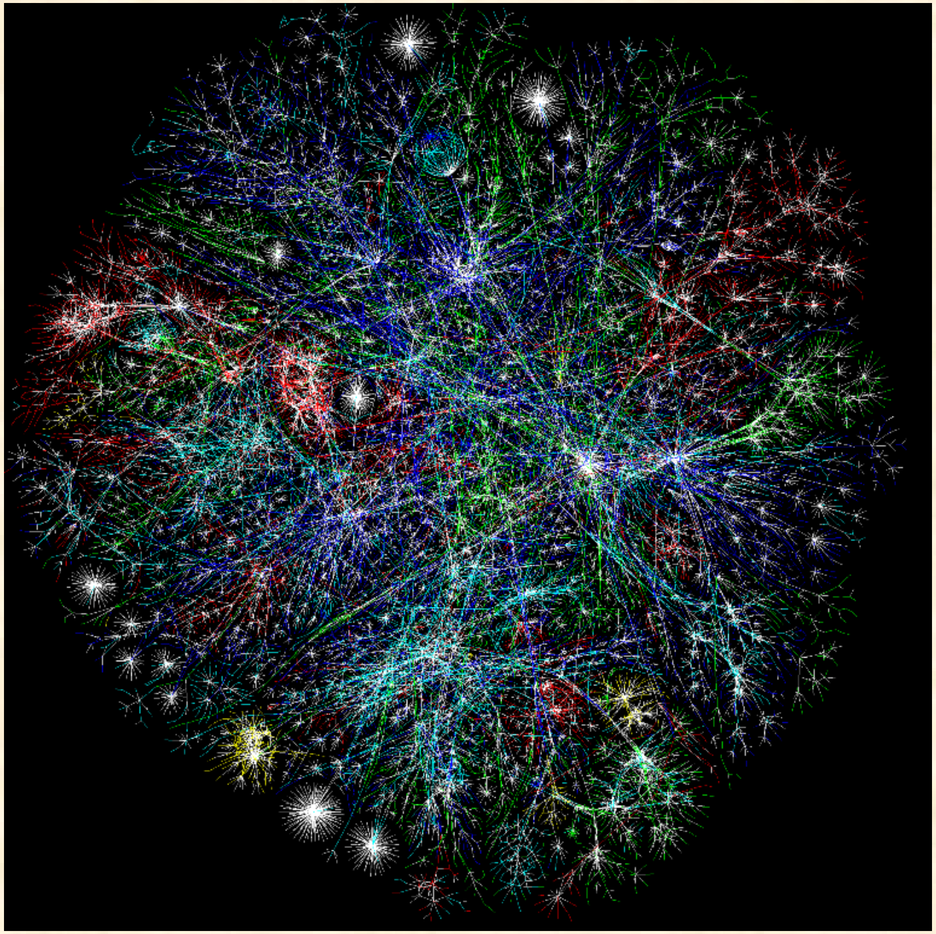
Structure:

Layers + Communities

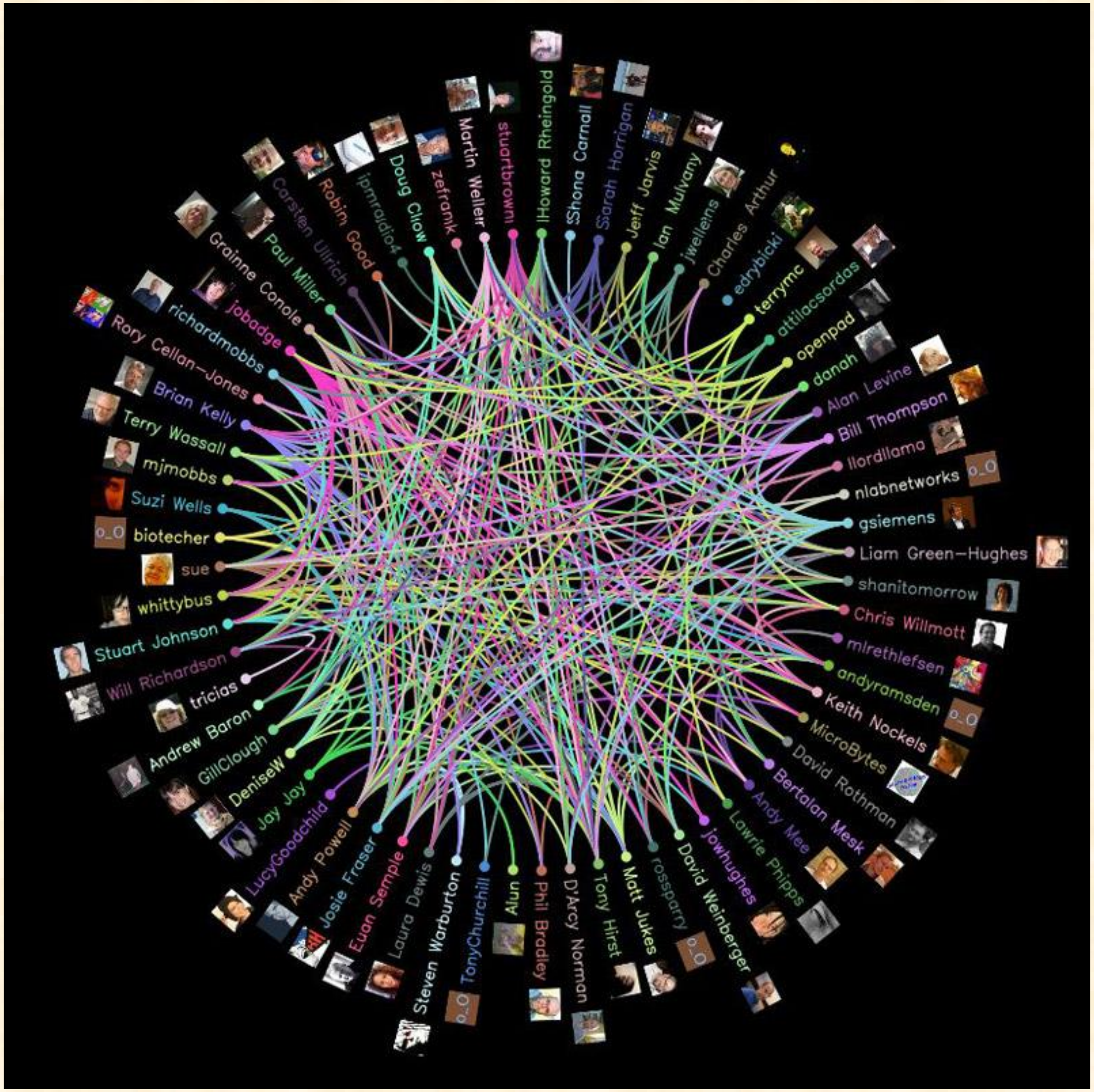
Feature: Small-world

Distribution:

Power-law with flat head and heavy tail



Internet



Social Networks are typical Small-World Networks

Human mobility and **behaviors** both **affect Internet topological evolution**

ScienceDaily®
Your source for the latest research news

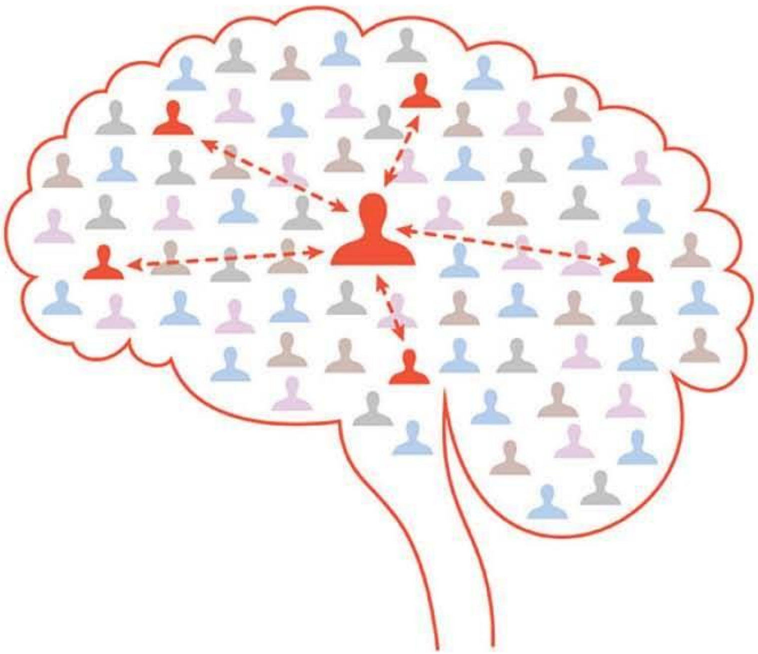
Brain Facebook Structure

The brain's social network: Nerve cells interact like friends on Facebook

Date: February 4, 2015

Source: University of Basel

Neurons in the brain are wired like a social network, report researchers from Biozentrum, University of Basel. Each nerve cell has links with many others, but the strongest bonds form between the few cells most similar to each other. The results are published in the journal Nature.



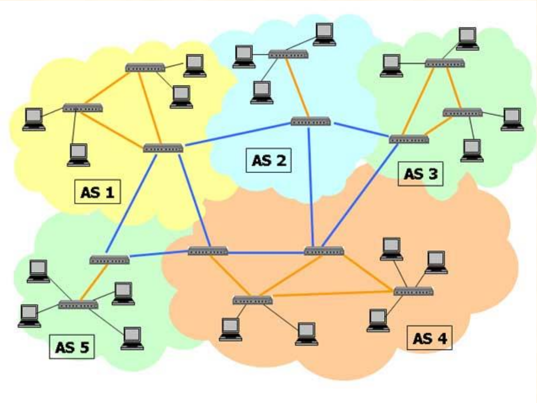
A neural network is like a social network: The strongest bonds exist between like-minded partners.

Credit: Biozentrum, University of Basel

Nerve cells form a bewildering meshwork of connections called synapses -- up to several thousand per cell. Yet not all synaptic connections are equal. The overwhelming majority of connections are weak, and cells make only very few strong links. "We wanted to see if there are rules that explain how neurons connect in complex networks comprising millions of neurons," says Professor

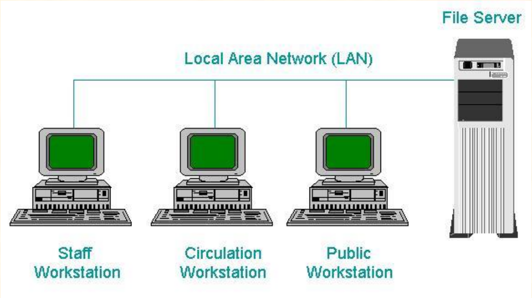
Internet Modeling

Graph Theory



❖ AS-level:

> nodes are domains (AS)
> edges are peering relationships
> modeling manageable



❖ Router-level:

> nodes are routers
> edges are one-hop IP connections
> partially manageable

❖ PC-level:

> nodes are PCs, hand-held sets
> edges are optical fibers
> modeling not manageable

End