Complex Networks

Lecture 1: Basic Concepts

EE 6605

Instructor: G Ron Chen

Some Background and Motivation

- Between two randomly selected persons in the world, how many friends are there connecting them together?
- When searching from one webpage to another through the World Wide Web (WWW), how many clicks are needed on average?
- Where to locate data centres on the Internet to be more efficient and effective for data traffic?
- How could computer viruses propagate so fast and so wide through the Internet?
- Why were people being infected so easily by diseases such as COVID-19, SARS and Flu, all over the world?
- How did rumors spread out in human societies?
- How did electric power blackout emerge from a small local system failure through the huge power grid?
- How did financial crisis spread out over the world?

We are living in a networked world today

- The influence of various complex and dynamical networks is currently pervading all kinds of sciences, ranging from physical to biological, even to social sciences. Its impact on modern engineering and technology is prominent and will be far-reaching.
- On the one hand, networks bring us with benefits and convenience, improve our efficiency of work and quality of life, and create tremendous opportunities that we never had before.
- On the other hand, however, networks also generate harms and damages to humans and societies, typically through epidemic spreading, computer virus propagation, power blackout, and the like.

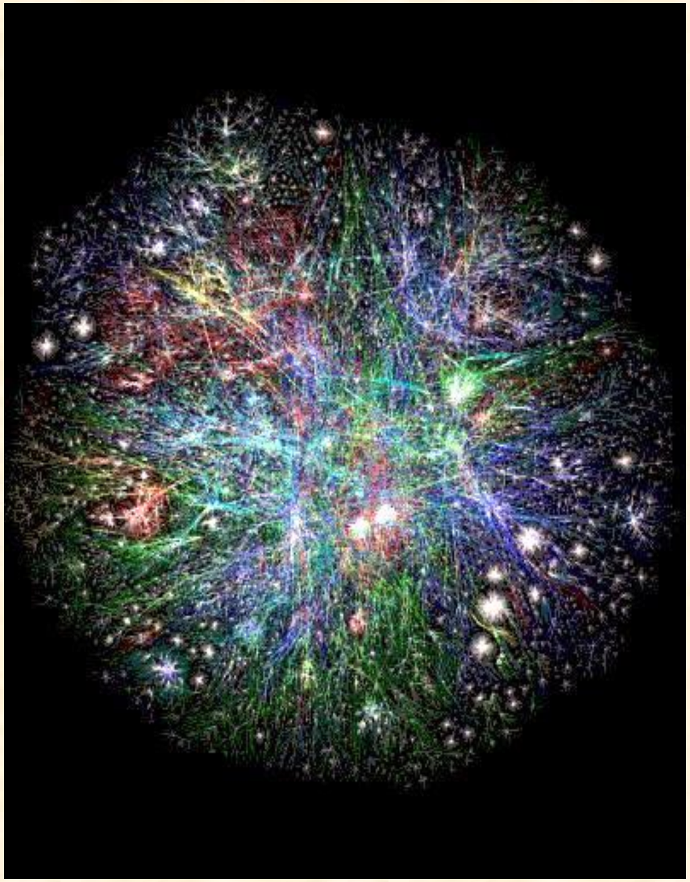
Development of network science studies

- For a long time in the history, studies of communication networks, power networks, transportation networks, biological networks, economic networks, social networks, etc., were carried out separately and independently.
- Recently, there were some rethinking of the general theory of complex dynamical networks towards a better understanding of the intrinsic relations, common properties and shared features of different kinds of networks in the real world.
- The new intention of studying fundamental properties and dynamical behaviors of complex networks, both qualitatively and quantitatively, is important and timely, although very challenging technically.
- The current research along this line has been recognized as a whole as “network science and engineering”, and has indeed become overwhelming.

Network Complexity (I)

***Structural complexity*:**

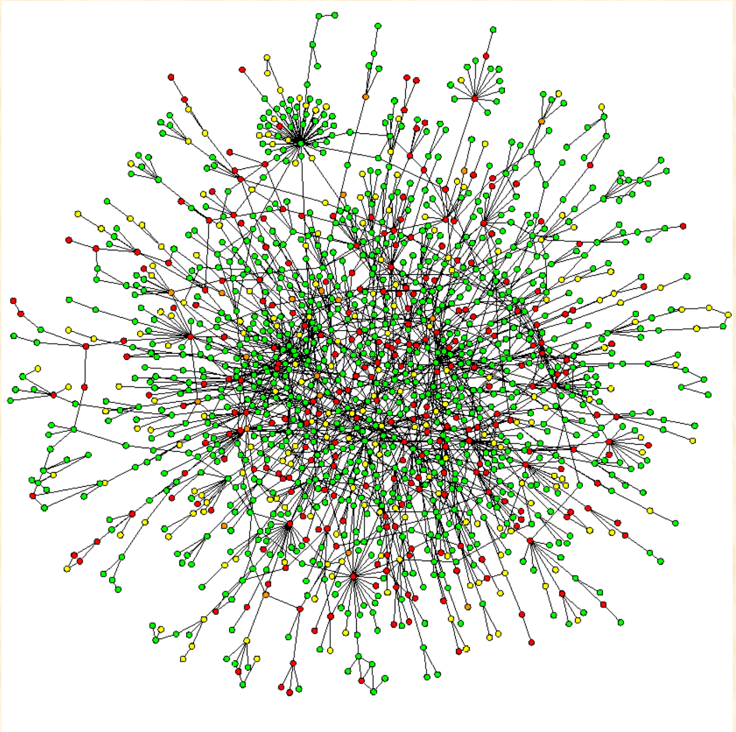
- A network appears to be complicated in structure, which are seemingly messy and disordered.
- The network topology (i.e., structure) may vary in time.
- The connections (i.e., edges) among systems (i.e., nodes) may be weighted, directed, time-varying, and even with noise and uncertainties.



Network Complexity (II)

***Node-dynamical complexity*:**

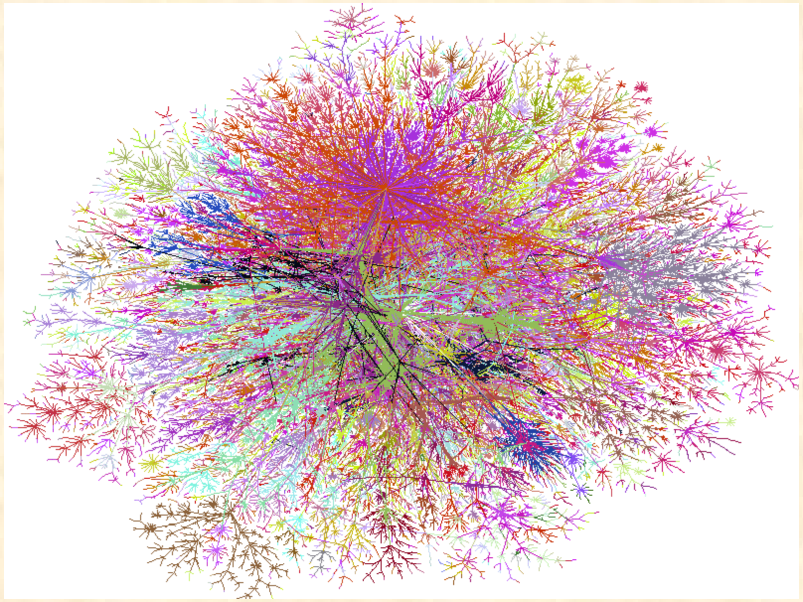
- A network may have different kinds of nodes.
- A node in the network can be a dynamical system, which may have complex dynamics such as bifurcating and even chaotic behaviors.



Network Complexity (III)

***Mutual interactions among various complex factors*:**

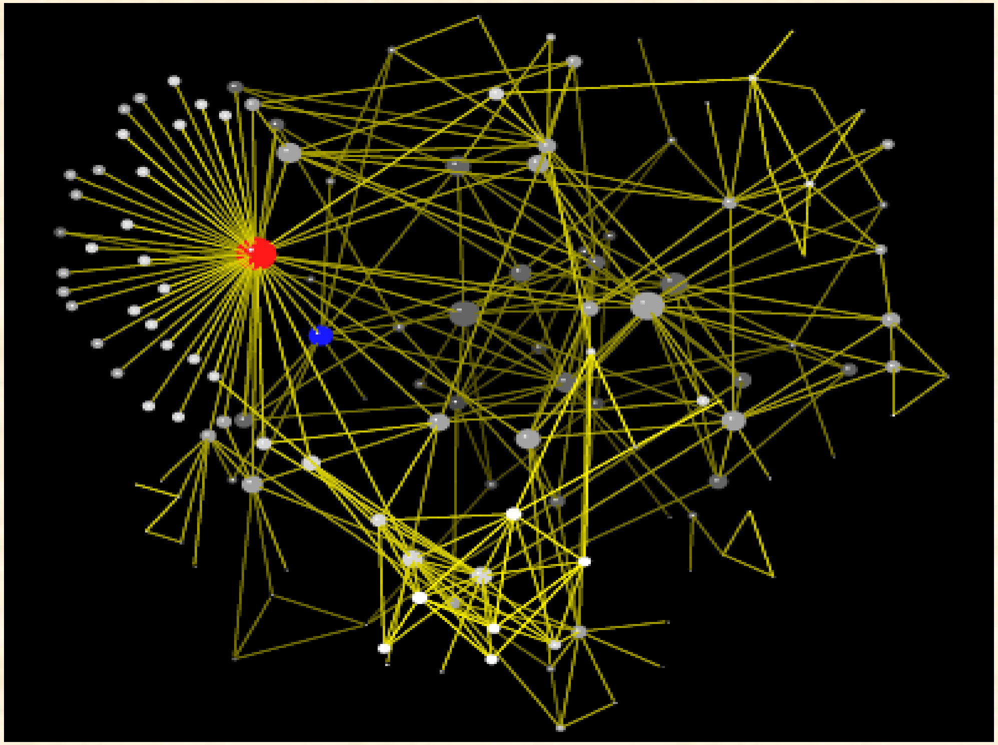
- A real-world network is affected by many internal and external factors.
- The close relations between sub-networks make the already-complicated behaviors of each of them become even more complex.



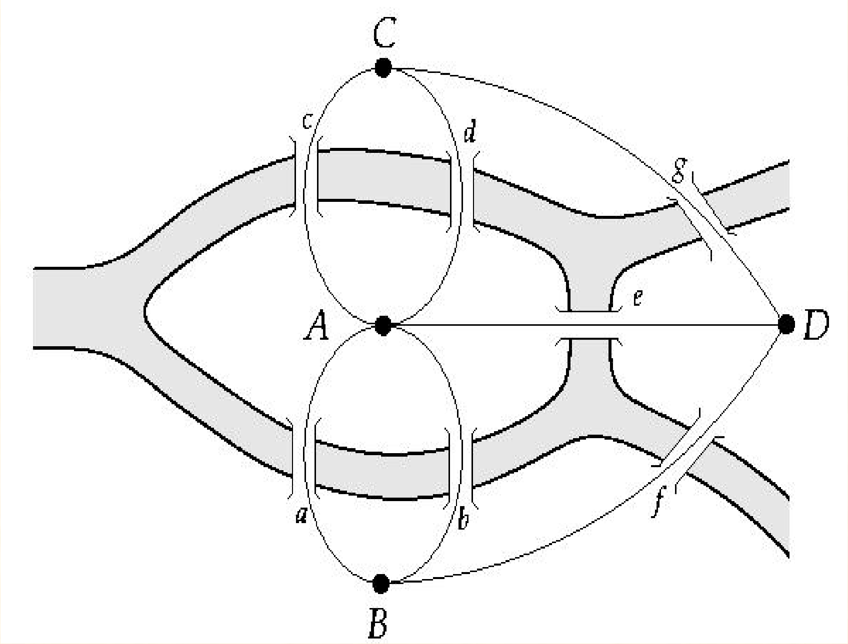
Network of Networks

Internet of Things

Brief History of Network Research



Leonhard Euler (1707-1783)



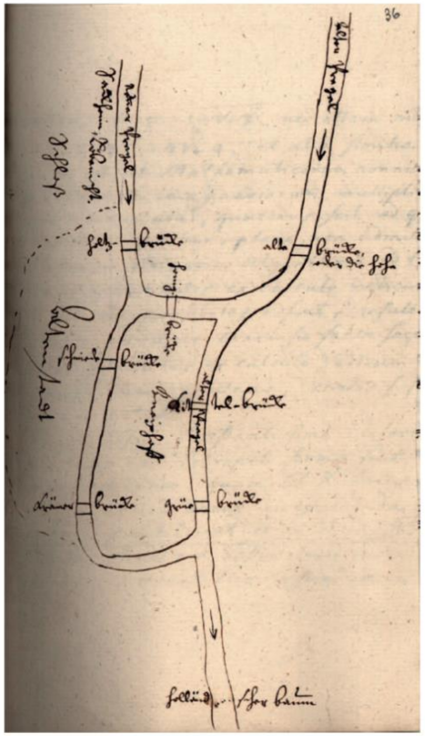
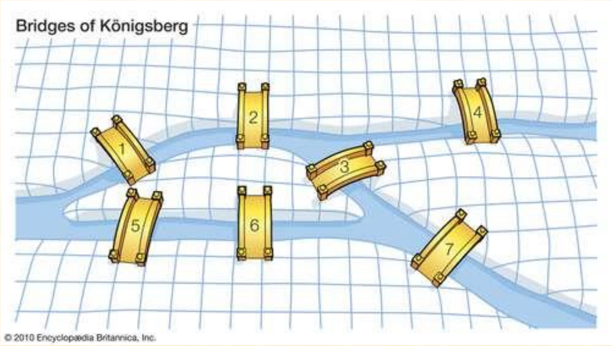


Figure 1: Ehler's drawing of Königsberg, 1736

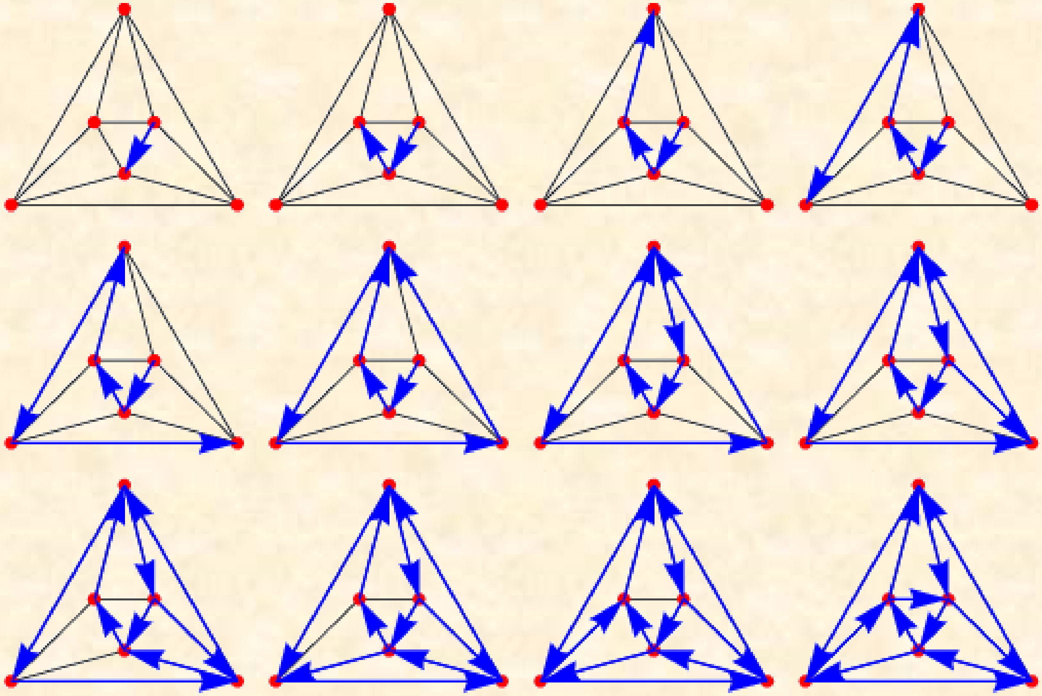
Viewed the problem as a graph
with 4 nodes and 7 edges

From 7 to N



Solvable **if and only if**

Every node has an even degree

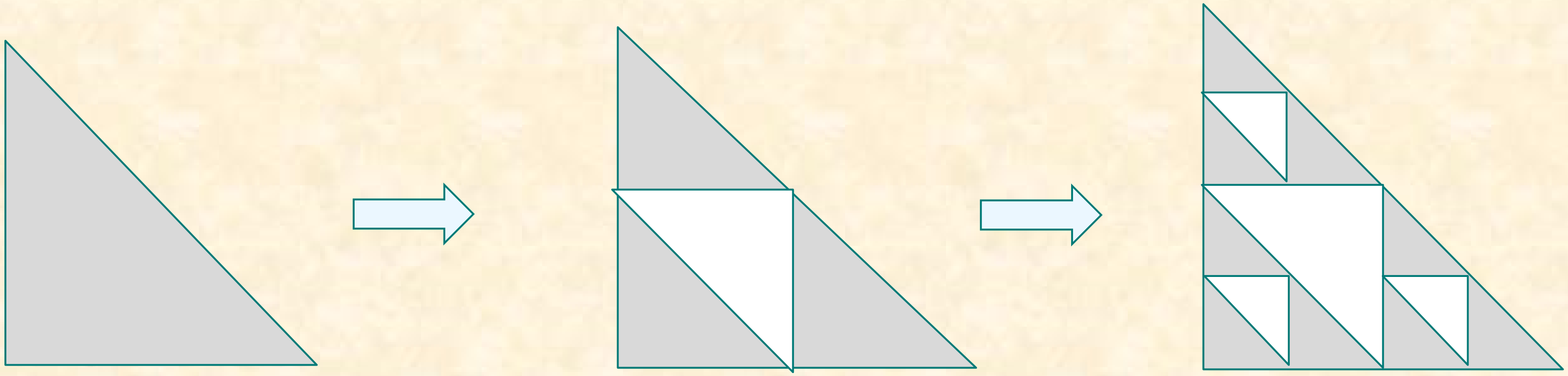


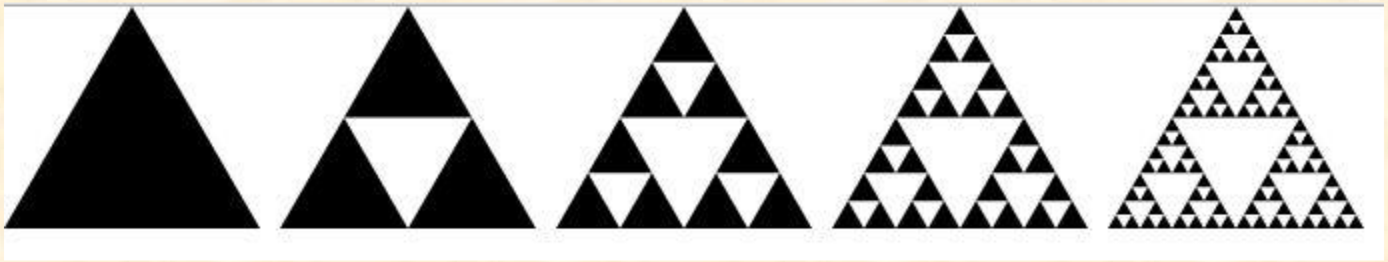
"Eulerian Graph"

Fractals

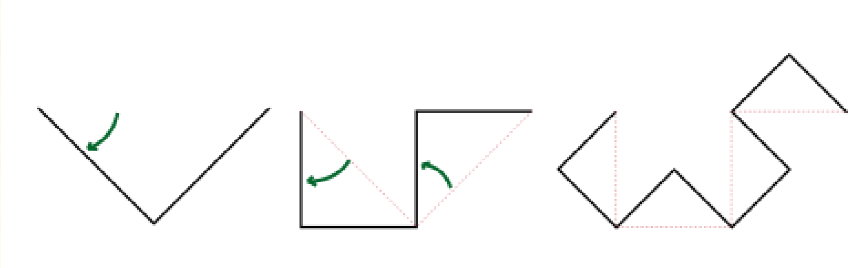
Sierpinski Triangle

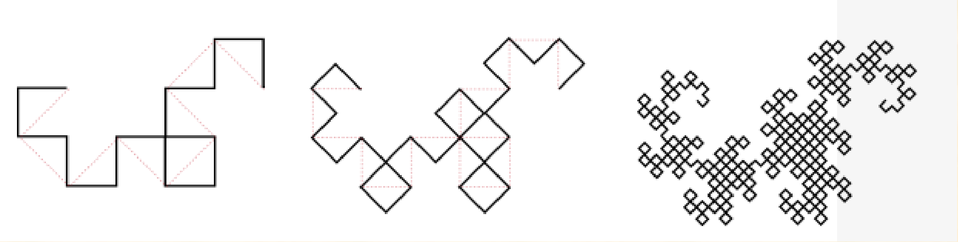
Wacław Sierpińsk
(1882 – 1969)

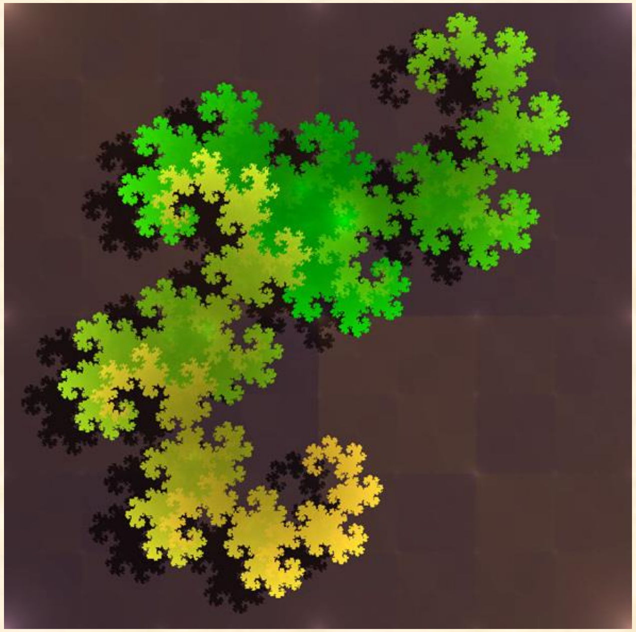




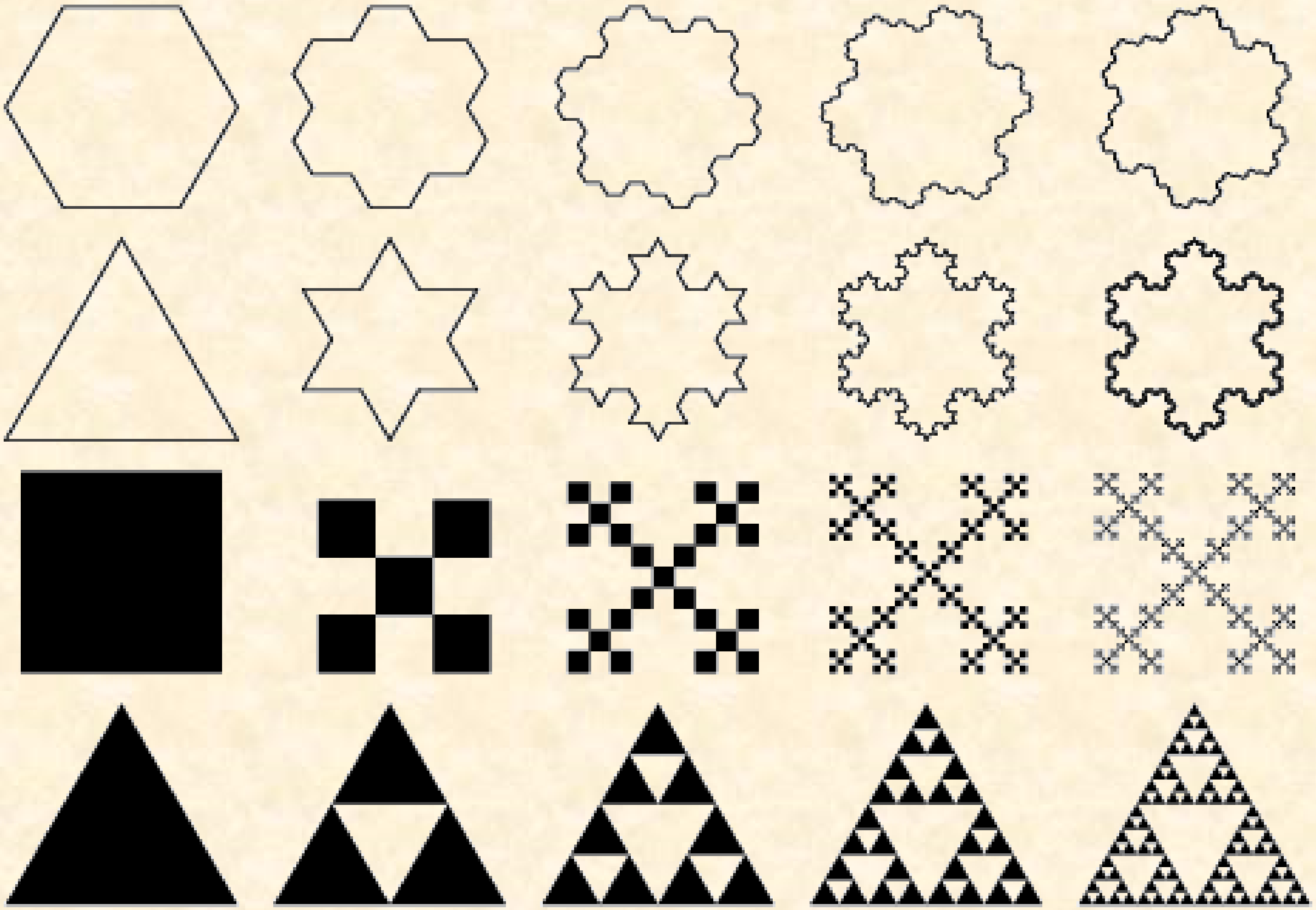
Dragon Curve



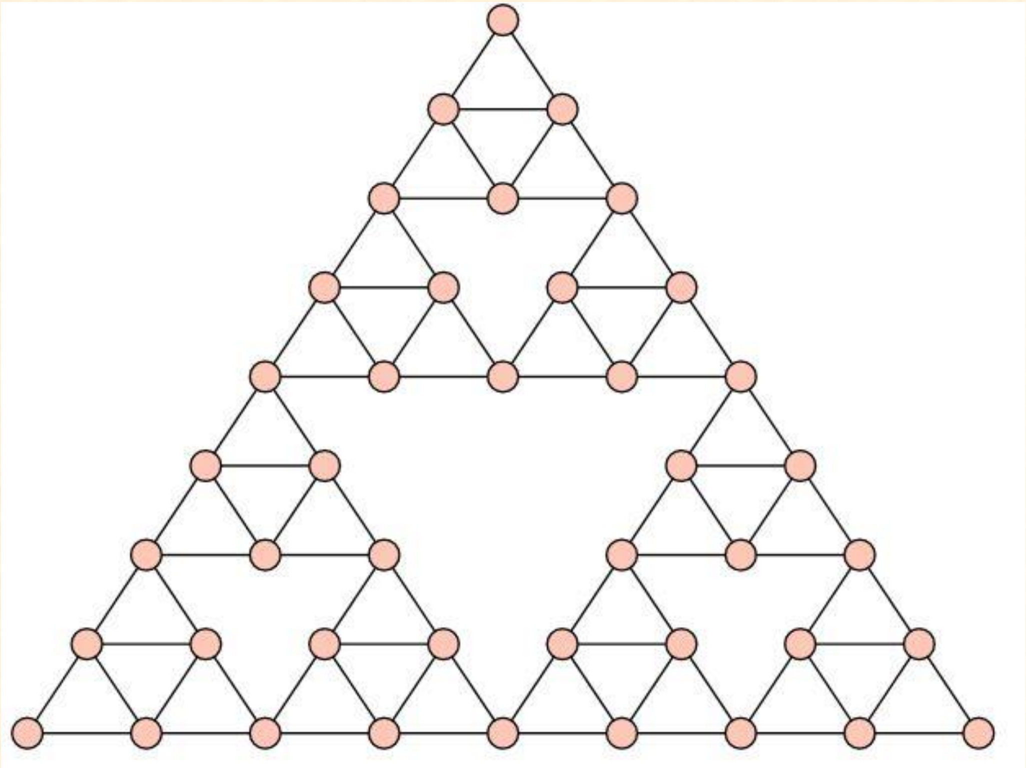




More Examples of **Fractals**



Sierpinski Triangle (Fractal Network)



You can always start from any node, go through every edge once and once only, and finally return to the starting node, no matter how big the Sierpinski triangle is.

Four Color Problem

Q: Given any map, can the regions be colored by using at most **four** different colors, so that no two adjacent regions have the same color?

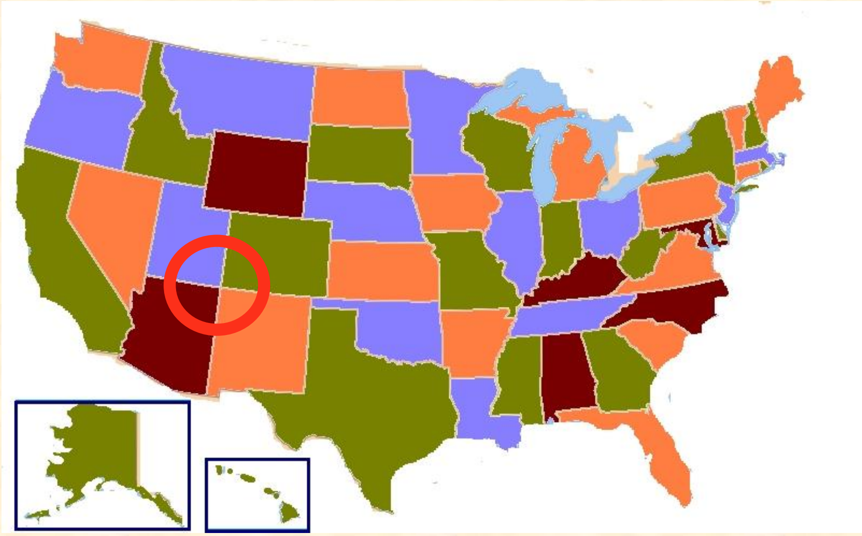
Terminology:

Two regions are called adjacent if they share a common boundary that is not a corner, where a corner is a point shared by more than two regions.

Convention:

All corners, and points that belong to more than two regions, are ignored. A region has to be a simply connected (i.e., contiguous).

Bizarre maps (e.g. using regions of finite area but infinite perimeter) are ignored (which may require many different colors)



Map Coloring as a Network Problem

Q: Given any map, can the regions be colored by using at most four colors, so that no two adjacent regions have the same color?

Four Color Conjecture (Theorem):

Yes, four colors are sufficient

Brief History:

Heawood (1890): Five colors are sufficient (proved mathematically)

Appel and Haken (1976): Four colors are sufficient (proved by computer programming: 1200 machine hours; 10 billion decisions)

Robertson, Sanders, Seymour and Thomas (1997): Simplified the above computer-aided proof

Gonthier (2005): proved by using a general purpose theorem-proving software

Graph Theory

provides a basic tool for studying complex networks

Some inferential historical markers in network science

Year	People	Event
1736	Euler	Seven-bridge problem
1936	König	First graph theory book
1959	Erdös and Rényi	Random graph theory
1967	Milgram	Small-world experiment
1973	Granovetter	Strength of weak ties
1998	Watts and Strogatz	Small-world network model
1999	Barabási and Albert	Scale-free network model

Inter-Personal Ties

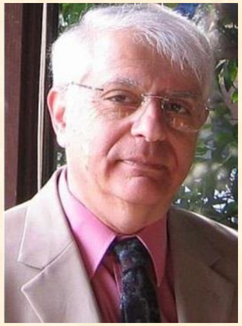
Johann Wolfgang von Goethe (歌德) (Germany, 1809) discussed the "marriage tie"

Anatol Rapoport (Russia, 1954): "the likely contacts of two individuals who are closely acquainted tend to be more overlapping than those of two arbitrarily selected individuals" (became a cornerstone of social network theory)

…

Mark Granovetter (USA, 1973):

"The Strength of Weak Ties"



(1943 --)
Stanford University

(one of the most influential sociology papers, cited by > 57,600 times)

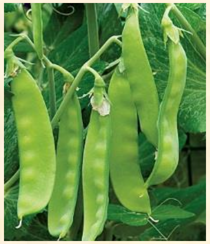
Granovetter observed that most jobs were found through "weak ties" (distant acquaintances)

80-20 Rule

Vilfredo Pareto (Italy, 1906): Observed that about 80% of the land in Italy was owned by about 20% of the population

He then carried out some surveys on a number of other countries and found that all the distributions are quite similar

He further observed that 20% of the pea pods contained 80% of the peas in his garden



Microsoft: By fixing the top 20% most reported bugs, 80% of the errors and crashes would be eliminated

Business: Roughly 80% of profits come from 20% of buyers; 80% of complaints come from 20% of customers; 80% of sales come from 20% of products; 80% of the sales come from 20% of the clients, ...

1992 United Nations Development Program Report:

The richest 20% of population control 82.7% of the world’s global income

According to a report in the 2012 BBC online briefing, about 83% of the population could properly be classified as lurkers, while 17% of the population could be classified as intense contributors

李嘉誠:

一件衣服被我穿上了，80%的人都說好看，那我一定會買；一個生意機會被我遇上了，80%的人都說可以做，那我絕對不會去做。

我深信世界上的 2/8定律，爲什麼世界上80%是窮人，20%是富人？因爲20%的人做了別人看不懂的事，而80%的人不會堅持正確的選擇。

Generalized 80-20 Rule and **Pareto Distribution**

20% people has 80% wealth

20% of the 20% people have 80% of the 80% wealth

and so on ……

p and $(1-p)$

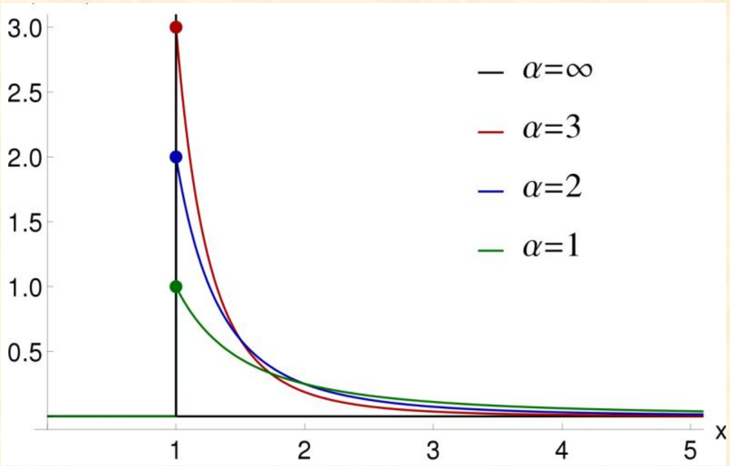
p^2 and $(1-p)^2$

p^3 and $(1-p)^3$

……

$\Longleftrightarrow$

Pareto distribution: $\begin{cases} P(X > x) \sim x^{-\alpha} \\ \alpha = log_{p/(1-p)}\, p \end{cases}$

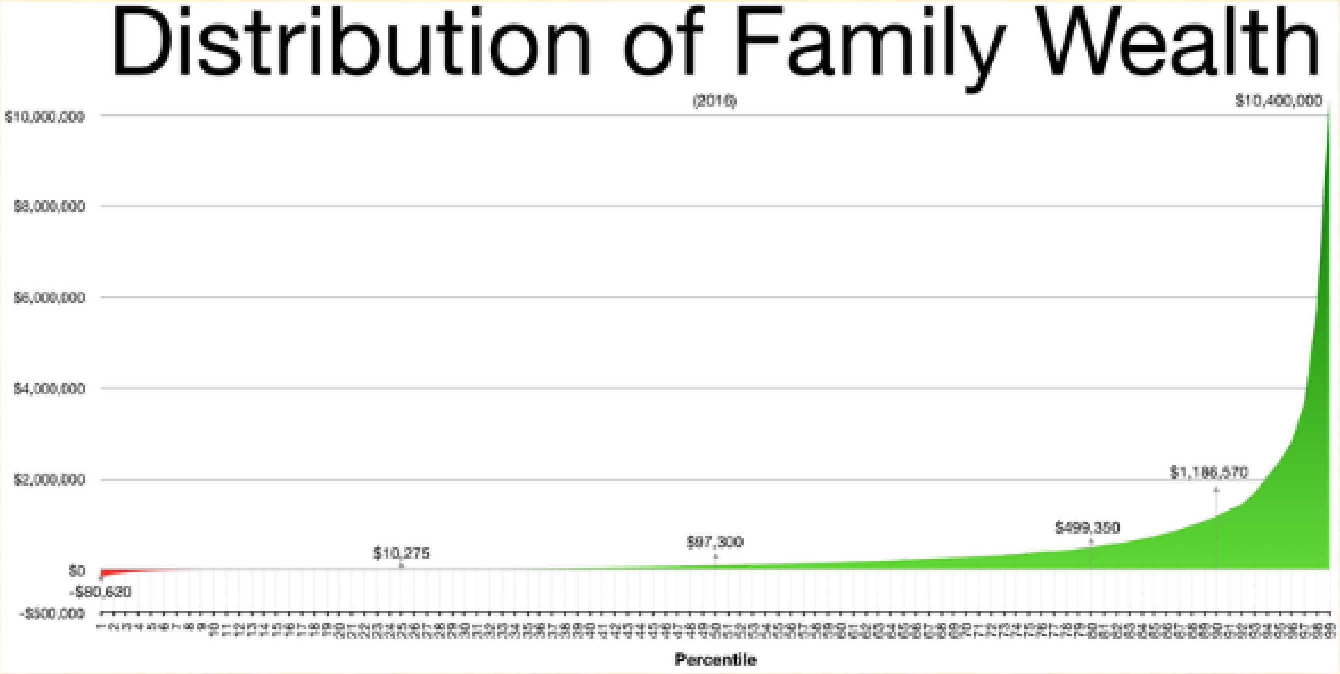


One Report

USA 2014:

The top 1% of people own 40% of nation's wealth

The bottom 80% own only 7%



Another Report (4 December 2015)

CNBC = Consumer News and Business Channel

One More (13 December 2016)

特朗普内阁17人财富超美国1/3家庭总和

EMBARRASSMENT OF RICHES

Trump's first 17 cabinet-level picks have more money than a third of American households combined

What about China?

sina 新闻中心 国内新闻

新闻 宋佳

中国社会不平等趋势扩大：1%家庭占全国1/3财产

微博 微信 QQ空间 + 3.9W 评论

2016年01月13日18:29 第一财经日报

中国目前的收入和财产不平等状况正在日趋严重。顶端1%的家庭占有全国约三分之一的财产，底端25%的家庭拥有的财产总量仅在1%左右。此外，从教 育机会到医疗保障，中国社会的不平等现象整体呈现扩大趋势。日前公布的《中国民生发展报告2015》在深入调研的基础上作出了上述判断。

《中国民生发展报告》丛书是基于北京大学中国家庭追踪调查（China Family Panel Studies， CFPS）撰写的系列专题报告，以全国25个省市160个区县的14960个家庭为基线样本，探讨民生问题状况、差异、原因和社会机制。

Top 1% Families worth as much as 1/3 of China

1% Rule (90-9-1 Rule)

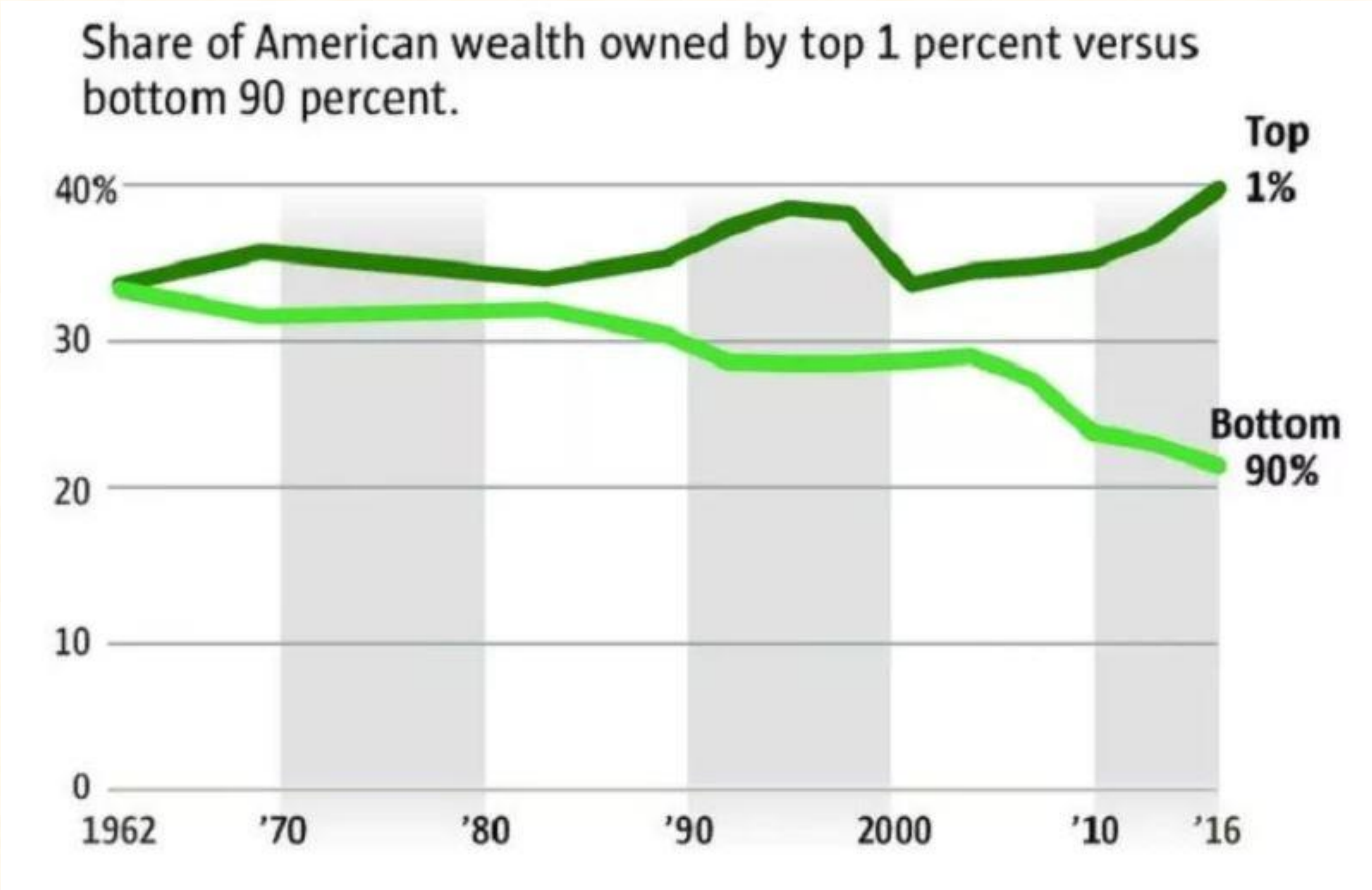
1% rule (known also as 90-9-1 rule): hypothesizing that more people will lurk (沉默) than will participate in a virtual community (WeChat, Facebook, etc.)

Internet: 1% of people create content; 9% edit/modify the content; 90% only view the content without contributing anything to it

Blog: For every blog posted by 1 blogger, 9 people make some comments to follow; 99 other people only view the blog without posting comments

The first 90 percent of the code accounts for the first 10 percent of the development time. The remaining 10 percent of the code accounts for the other 90 percent of the development time.
— Tom Cargill, Bell Labs

Still yet, **one more report** (1% vs 90%)



(2017 年初)

来自国际慈善组织乐施会最新发布的调研报告显示，目前全球最富有的 8 个人分别是：

- 微软始创人比尔·盖茨（美国，净资产 750 亿美元）
- Zara 创始人奥尔特加（西班牙，670 亿美元）
- “股神”巴菲特（美国，608 亿美元）
- 电讯大亨卡洛斯·斯利姆（墨西哥，500 亿美元）
- 亚马逊始创人贝索斯（美国，452 亿美元）
- Facebook 联合创始人扎克伯格（美国，446 亿美元）
- 甲骨文主席埃利森（美国，436 亿美元）
- 彭博社始创人布隆伯格（美国，400 亿美元）。

如果从财富总量来看，全球金字塔顶尖 1%的人所掌握的资源比其他 99% 的财富总和还要多，而且世界过半数人口拥有的资产总数比不上这全球前 8 名富豪财富的总和。

Brain Science

The weight of the brain accounts for only 2% of the human body but it consumes 20% of the total energy

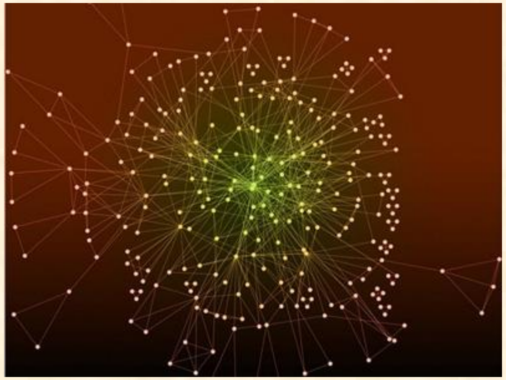
90% of the energy consumption (dark energy) in the brain has not been well understood

M. D. Fox, M. E. Raichle, Spontaneous fluctuations in brain activity observed with functional magnetic resonance imaging, *Nature*, 8: 710-711, 2007

BREAK

10 minutes

Network Topology



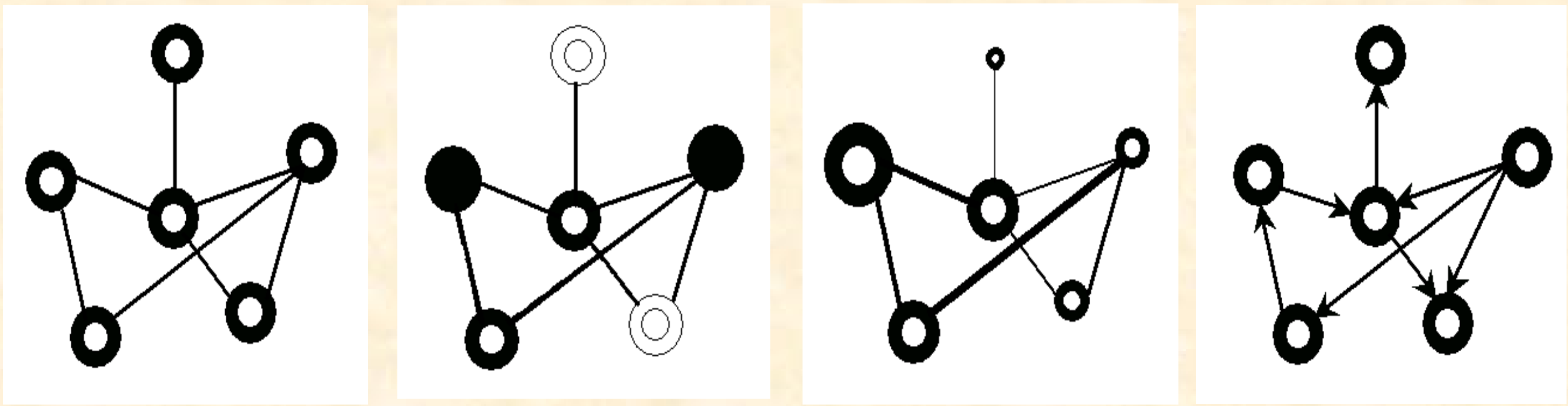
- A **network** is a set of **nodes** interconnected via **edges**

Examples:

- Internet: Nodes – routers Edges – optical fibers
- WWW: Nodes – document files Edges – hyperlinks
- Scientific Citation Network:
  Nodes – papers Edges – citation
- Social Networks:
  Nodes – individuals Edges – relations

An Brief Introduction to Graphs Theory

Some typical graphs:



(a) (b) (c) (d)

(a) undirected and unweighted network with same type of nodes

(b) undirected and unweighted network with different types of nodes and edges

(c) undirected but weighted network with weights on both nodes and edges

(d) directed but unweighted network with same type of nodes

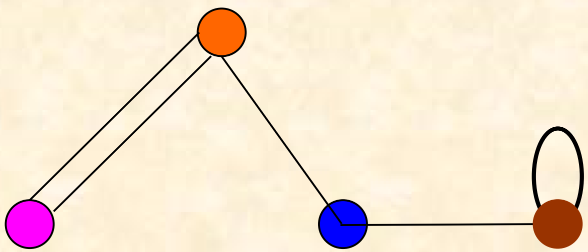
(e) … …

In this course: Nodes: all identical

Edges: undirected or directed, with no multi-connections and self-loops

Some Basics

Only connected（連通）graphs ae considered



graph with self-loop
or multiple edges
are not allowed

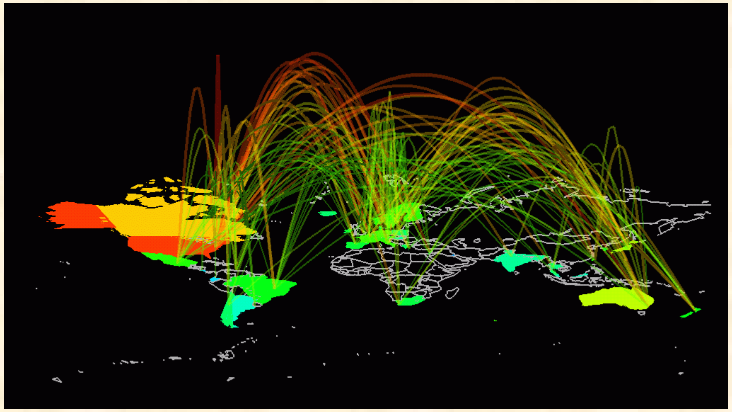
Convention:  not allowed

When a node is removed, all its edges will also be removed

A graph of N nodes has $N(N\text{-}1)/2$ edges

Some Basic Concepts

- **Degree** and **Degree Distribution**
- **Distance** and **Average Path Length**
- **Clustering Coefficient**
- **Coreness**
- **Betweenness**
- **Assortativeness**



Degree and Degree Distribution

- **Degree** k_i **of node** i
  = total number of its edges
- **Average Degree** $\langle k \rangle$ **over the network**

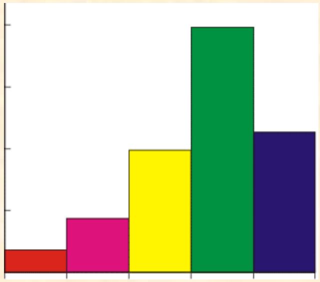
$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i$$

- **The spread of node degrees over a network is characterized by a distribution function:**

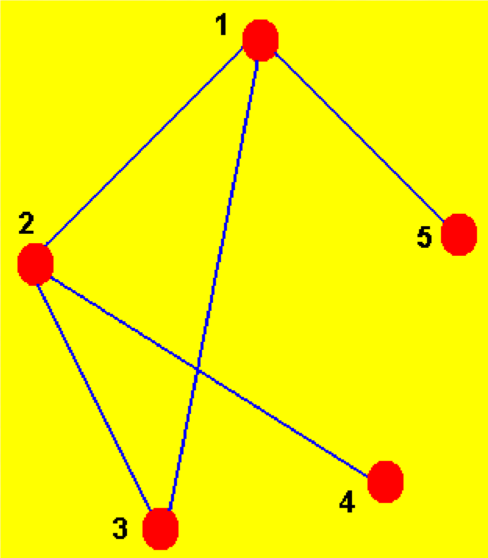
$P(k)$ **= probability that a randomly selected node has exactly degree** k

$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k)$$

histogram



Example:



- **Degree:**

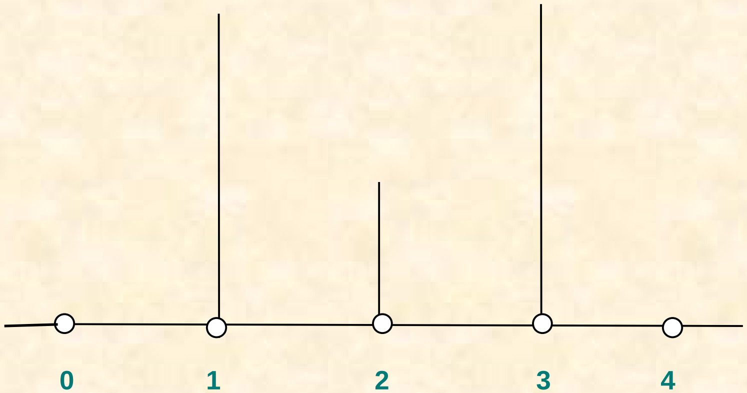
  node1 = 3, node2 = 3, node3 = 2,

  node4 = 1, node5 = 1

- **Average Degree:**

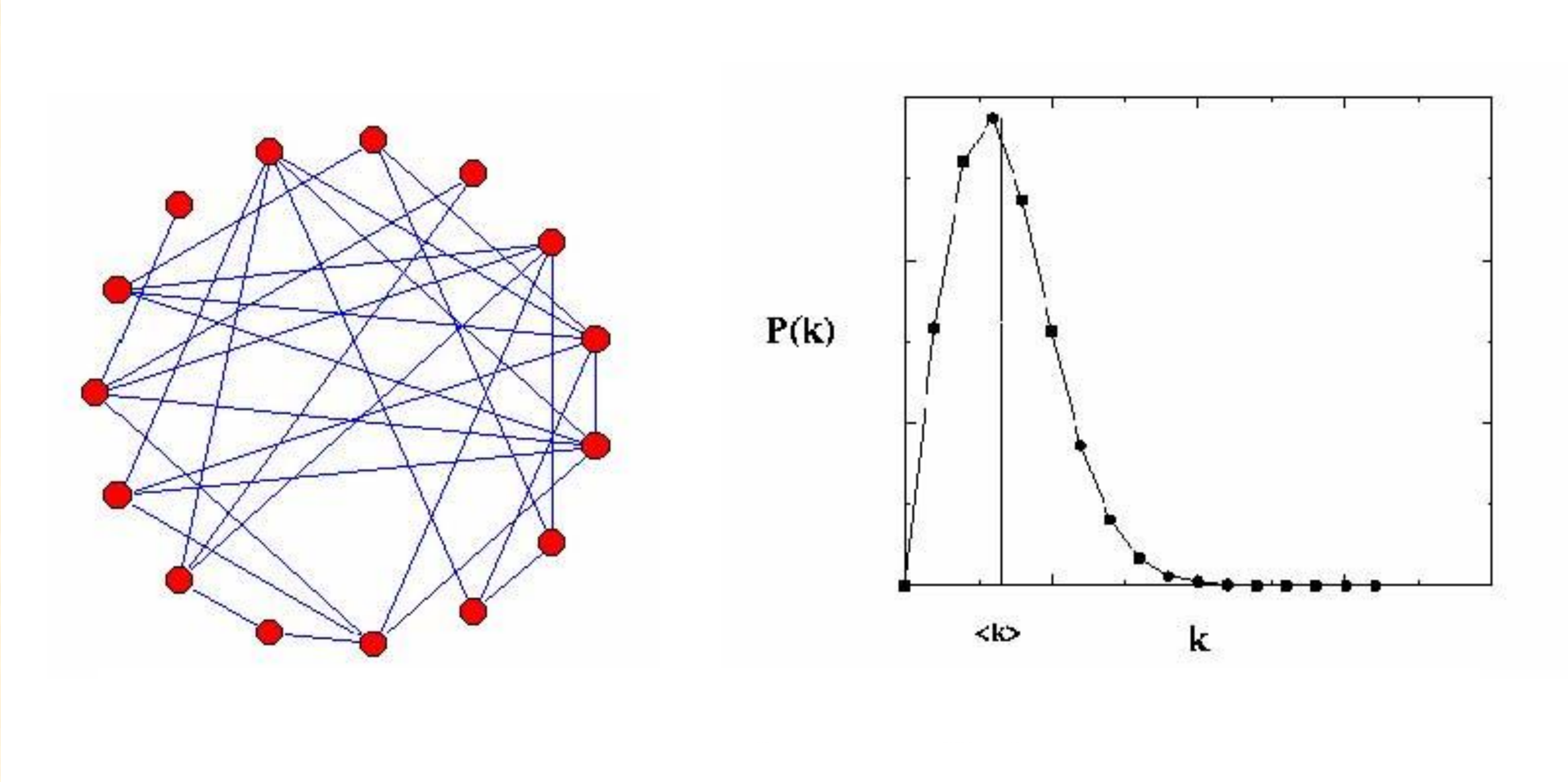
  $\langle k \rangle = (3 + 3 + 2 + 1 + 1) / 5 = 2$

- **Degree Distribution:**

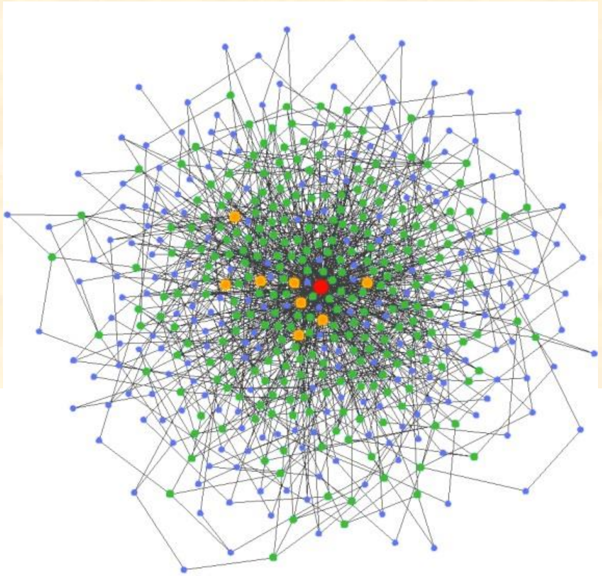


(probability mass function)

Example: Poisson Degree Distribution



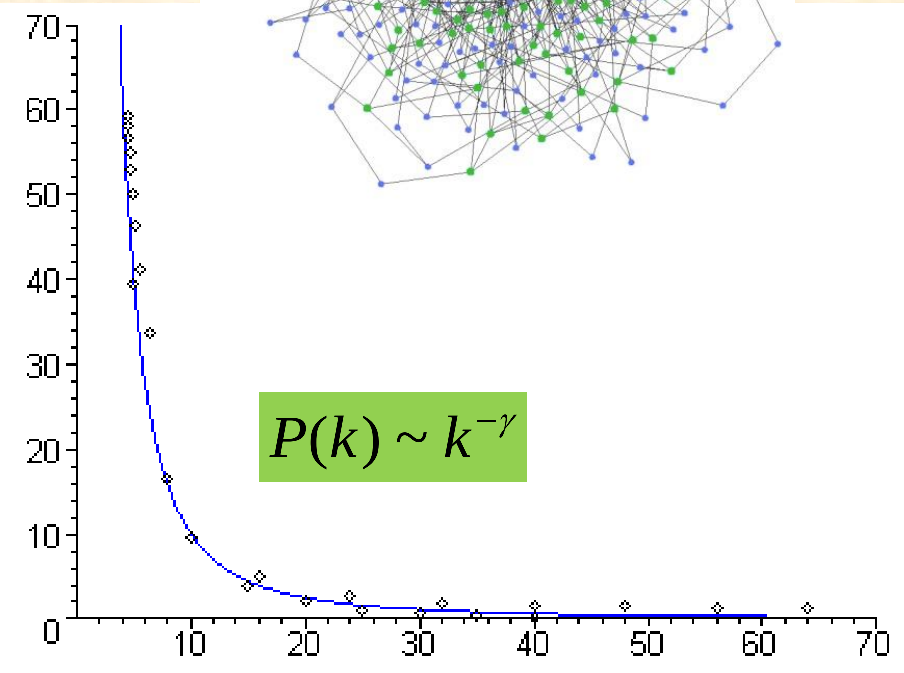
Power-Law Degree Distribution

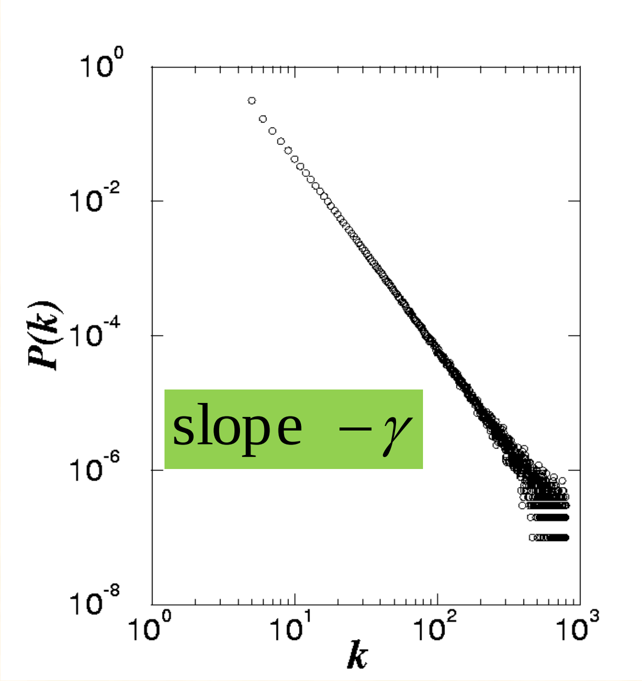


$$P(k) \sim k^{-\gamma}$$

$$\Rightarrow \ln[P(k)] = -\gamma \ln[k]$$

$$\Rightarrow Y = -\gamma X$$





Distance and Average Path Length

- **Distance** $d_{i,j}$ between two nodes i and j

  = the number of edges along the shortest path connecting them
- **Diameter** $D = \max\{d_{i,j}\}$
- **Average path length** L = average over all $d_{i,j}$

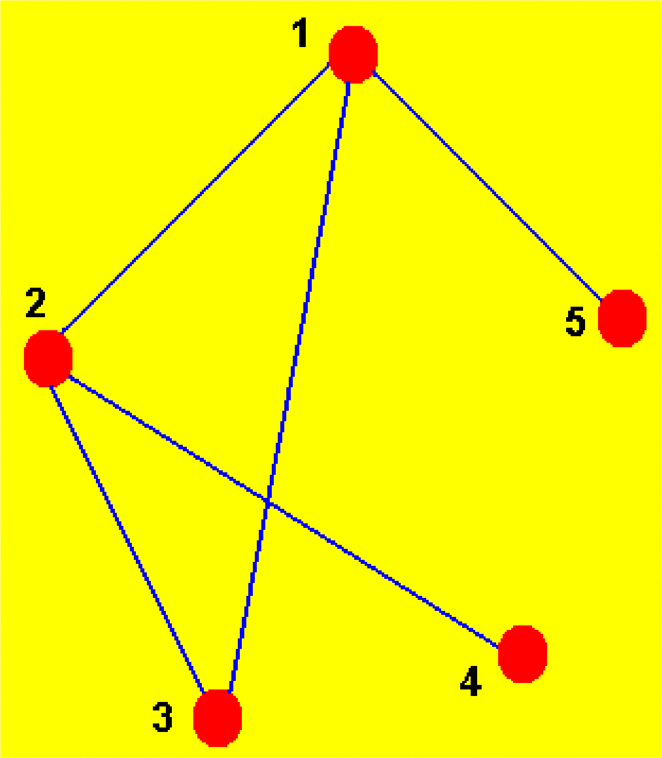
- **For many large and complex networks:**

  small L is a small-world feature

  (For example, most social networks)

Example:

A network having $N = 5$ nodes and 5 edges:



$$L = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i<j} d_{ij}$$

$d_{12} = 1$ $d_{13} = 1$ $d_{14} = 2$ $d_{15} = 1$

$d_{23} = 1$ $d_{24} = 1$ $d_{25} = 2$

$d_{34} = 2$ $d_{35} = 2$

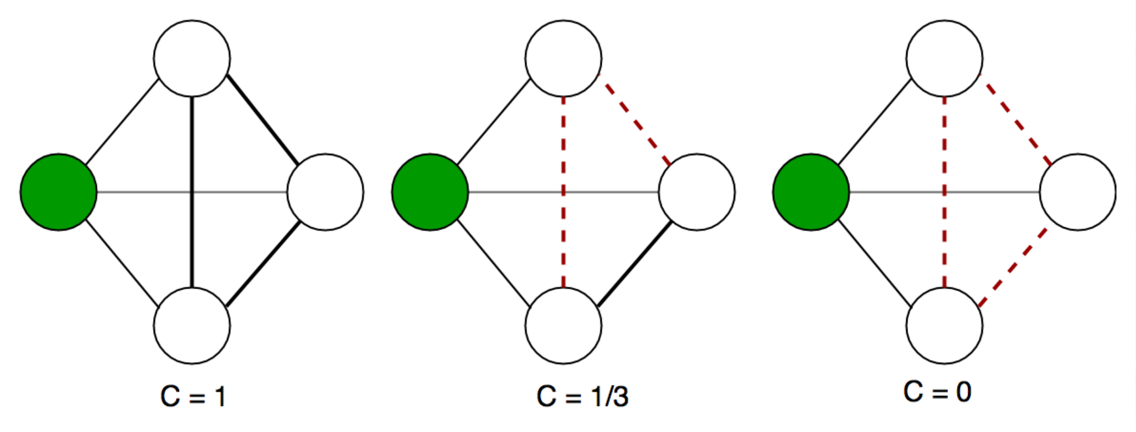
$d_{45} = 3$

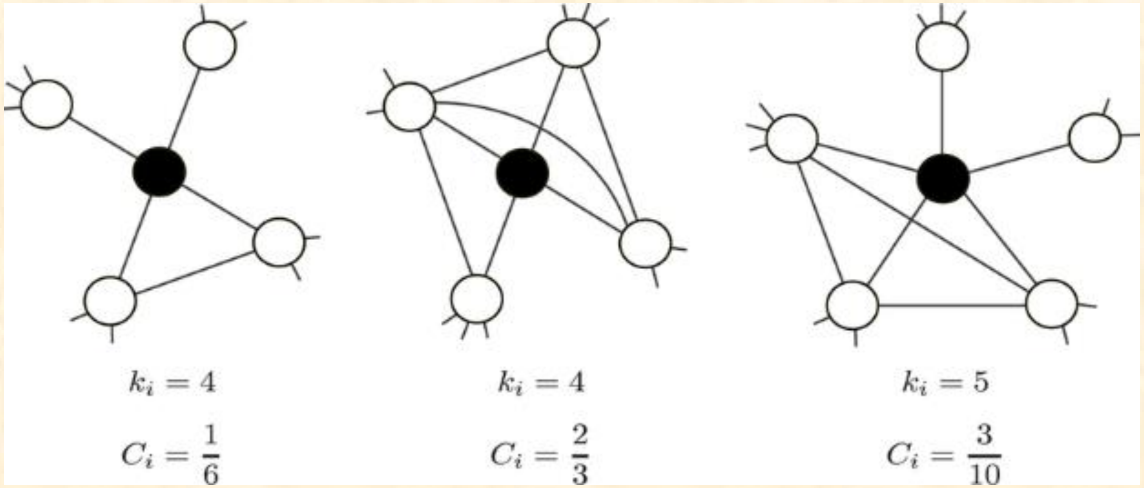
Total = 16

Average: L = 16 / 10 = 1.6

Clustering Coefficient

Among all your friends, how many of them are also friends?





Clustering Coefficient

- **Clustering Coefficient** C of a network:
- **How many of your friends are also friends each other?**
  - **Consider a node, *i***
  - **Suppose that node *i* has k_i edges (so it has k_i neighbors)**
  - **$E(i)$ = number of existing edges of these k_i neighbors actually have**
  - **$T(i)$ = number of possible edges of these k_i neighbors can have**
    $= k_i(k_i - 1)/2$
  - **$C(i) = E(i) / T(i)$ -- Clustering coefficient of node *i***
  - **C = average over all $C(i)$ -- Clustering coefficient of the network**

- **Usually, $0 < C < 1$**
  - **$C = 1$ if all neighbors of a node are connected pair-wise**
  - **$C = 0$ if no neighbors of a node are connected pair-wise**

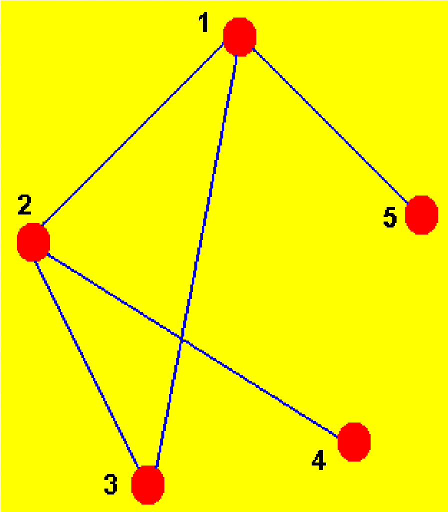
- **For many large and complex networks:**

  large C is a small-world feature

Example:

How to compute the clustering coefficient ?

- Node ***i*** has k_i edges (so it has k_i neighbors)
- $\boldsymbol{E(i)}$ = number of edges of these neighbors actually have
- $\boldsymbol{T(i)}$ = number of edges of these neighbors can possibly have $= k_i(k_i - 1)/2$
- $\boldsymbol{C(i)} = \boldsymbol{E(i)} / \boldsymbol{T(i)} = 2\boldsymbol{E(i)} / k_i(k_i - 1)$
- $\boldsymbol{C}$ = average over all $\boldsymbol{C(i)}$

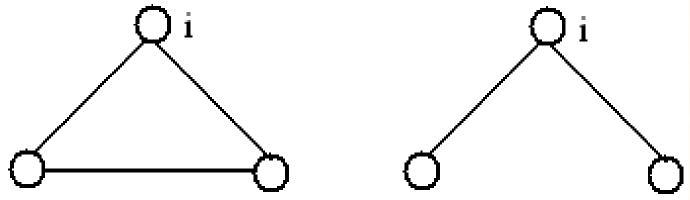


Node-1 has 3 neighbors, $E(1) = 1$, so $C(1) = 2\text{x}1/(3\text{x}2) = 1/3$
Node-2 has 3 neighbors, $E(2) = 1$, so $C(2) = 2\text{x}1/(3\text{x}2) = 1/3$
Node-3 has 2 neighbors, $E(3) = 1$, so $C(3) = 2\text{x}1/(2\text{x}1) = 1$
Node-4 has 1 neighbor, $E(4) = 0$, so $C(4) = 0$
Node-5 has 1 neighbor, $E(5) = 0$, so $C(5) = 0$

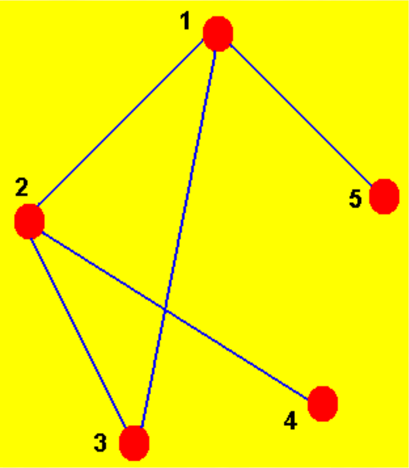
Average $C = (1/3+1/3+1+0+0) / 5 = 1/3$

Another definition from the geometric viewpoint:

$$C(i) = \frac{number\ of\ complete\ triangles\ with\ corner\ i}{number\ of\ all\ triangular\ graphs\ with\ corner\ i}$$



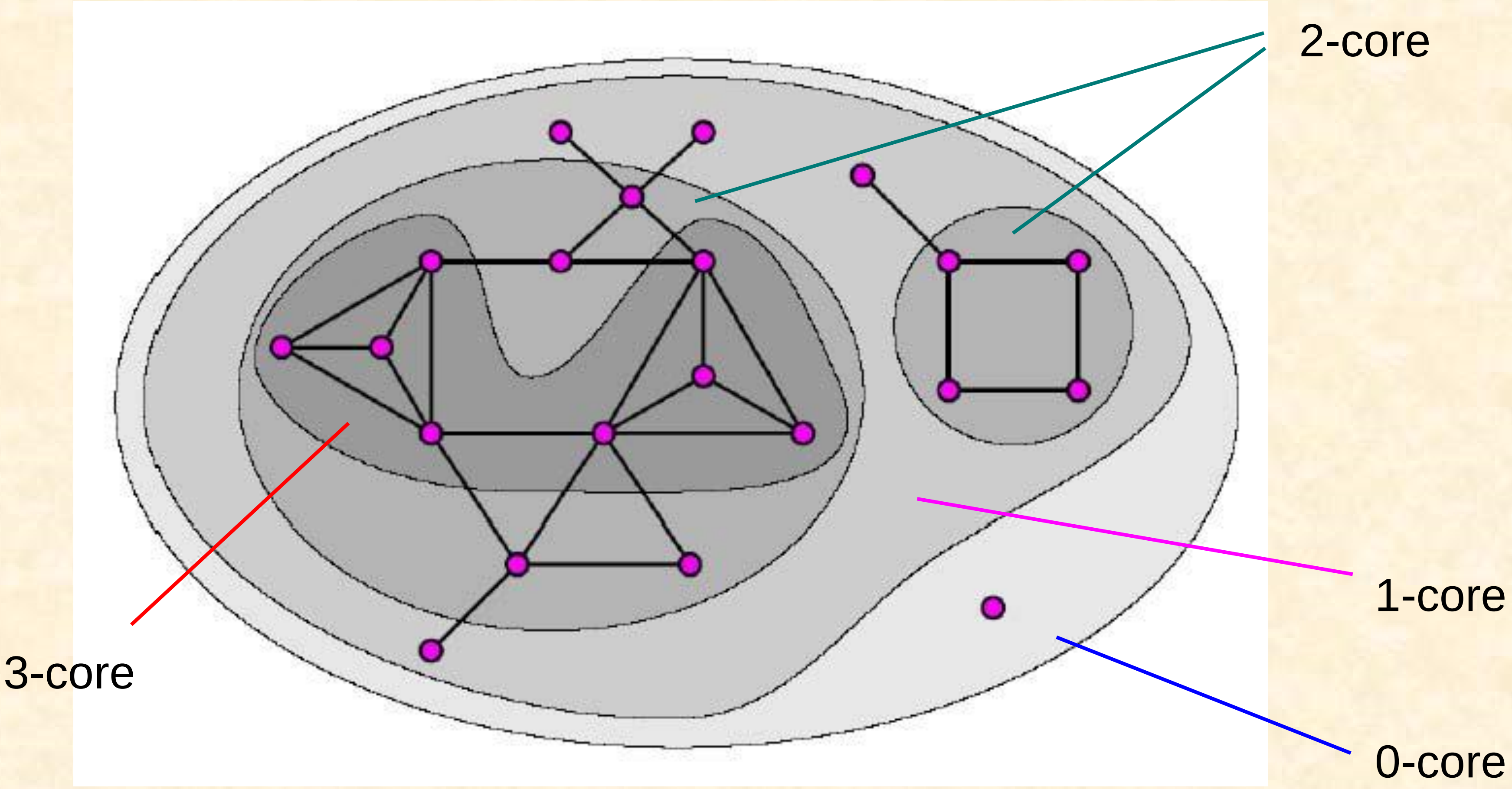
Left – complete triangle is counted in both numerator and denominator
Right – incomplete triangular graph is counted only in denominator



Node-1 has 1 **complete triangle and** 3 **triangular graphs, so** $C(1) = 1/3$
Node-2 has 1 **complete triangle and** 3 **triangular graphs, so** $C(2) = 1/3$
Node-3 has 1 **complete triangle and** 1 **triangular graph, so** $C(3) = 1$
Node-4 has 0 **complete triangles, so** $C(4) = 0$
Node-5 has 0 **complete triangles, so** $C(5) = 0$

Average $C = (1/3+1/3+1+0+0) / 5 = 1/3$

Coreness



Inside a *k* core, every node has degree at least *k*

Coreness

- The ***k*-*core*** in a graph is defined to be the remaining sub-graph after all the nodes with degrees $\leq k-1$ have been removed successively, during which:

  (i) when a node is removed, all its adjacent edges will also be removed;

  (ii) after a node of degree $\leq k-1$ is removed, in the remaining graph all the remaining nodes with a new degree $\leq k-1$ also need to be removed.

- If a node belongs to a k-core of a graph, but it will be removed from the $(k+1)$-core, then this node is said to have *coreness* (core value) k

- The largest coreness in a graph is called the *coreness of the graph*.

Examples:

An isolated node has coreness $k = 0$

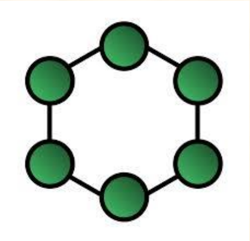
A fully-connected network of size $\boldsymbol{N}$ has coreness $k = N - 1$

Coreness

Star-shaped networks

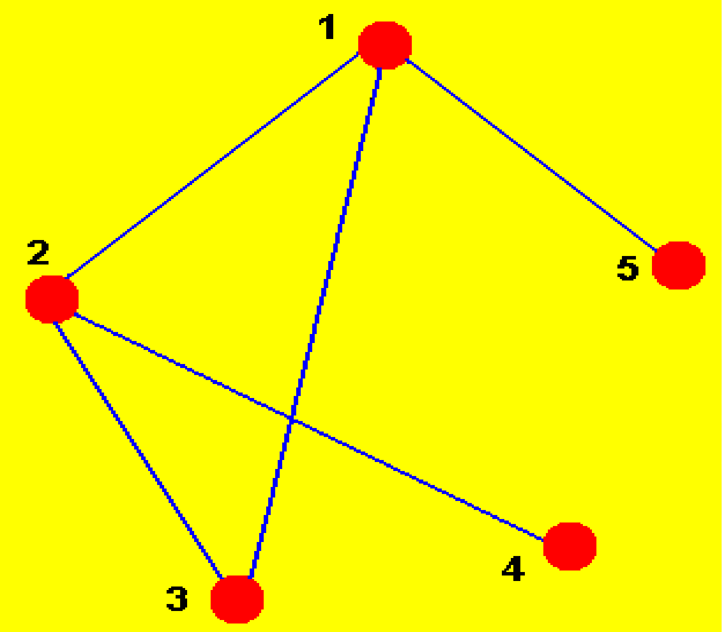
- the 1-core of the network is the network itself
- all nodes, including the centre, have coreness 1
- the coreness of the network is 1

Ring networks



- the 2-core of the network is the network itself
- all nodes have coreness 2
- the coreness of the network is 2

Coreness



Example:

- 1-core is the whole graph
- 2-core is Triangle 1-2-3
- Node 1, Node 2, Node 3 have coreness 2
- Node 4 and Node 5 have coreness 1
- Coreness of the graph is 2

Betweenness

Definition 1 (Node-betweennes) In a network of size N, the ***node-betweenness*** of node i is defined by

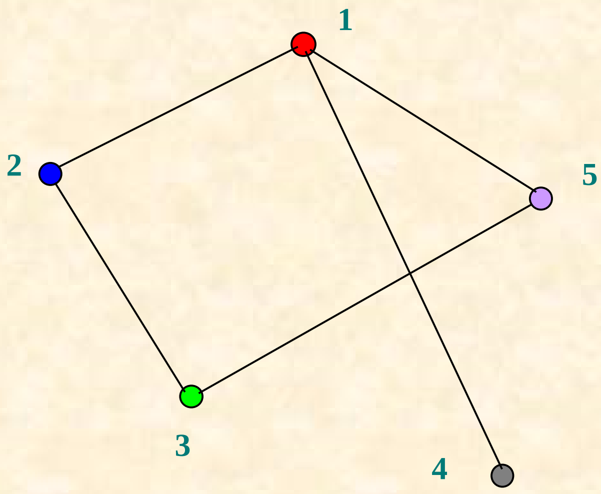
$$B(i) = \sum_{i \neq j \neq l} \frac{L_{jl}(i)}{L_{jl}}$$

where L_{jl} is the number of all existing shortest paths from node j to node l, and $L_{jl}(i)$ is the number of all shortest paths from node j to node l that pass through node i.

(since $i \neq j \neq l$ the node i itself is excluded)

Normalization: divided by total number of node-pairs not including node i as an end node: $(N-1)(N-2)/2$

Node-Betweenness



Example:

betweenness of node 1 is

$$B(1) = \frac{(5,1,4)}{(5,1,4)} + \frac{0}{(5,3)} + \frac{(5,1,2)}{(5,1,2) + (5,3,2)} + \frac{0}{(5,1)} + \frac{(4,1,2,3) + (4,1,5,3)}{(4,1,2,3) + (4,1,5,3)} + \frac{(4,1,2)}{(4,1,2)} + \frac{0}{(4,1)} + \frac{0}{(3,2)} + \frac{0}{(3,1)} + \frac{0}{(2,1)}$$

$$= \frac{1}{1} + \frac{0}{1} + \frac{1}{2} + \frac{0}{1} + \frac{2}{2} + \frac{1}{1} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1} + \frac{0}{1}$$

$$= \frac{7}{2}$$

Normalized: $$B_N(1) = \frac{7/2}{(5-1)(5-2)/2} = \frac{7}{12}$$

Betweenness

Definition 2 (Edge-betweennes)

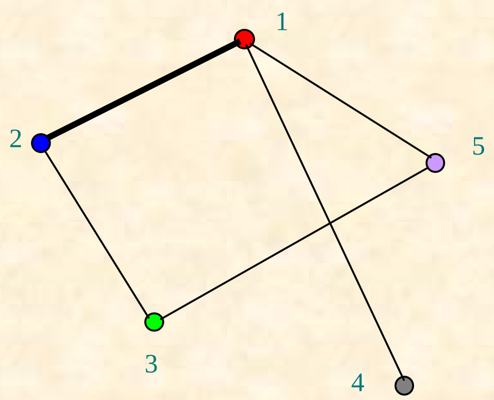
In a network of size N, the ***edge-betweenness*** of edge e_{ij} is defined by

$$B(e_{ij}) = \sum_{(l,q)\neq(i,j)} \frac{\tilde{L}_{lq}(e_{ij})}{\tilde{L}_{lq}}$$

where $\tilde{L}_{lq}$ is the number of all exisitng shortest paths from node l to node q, and $\tilde{L}_{lq}(e_{ij})$ is the number of all shortest paths from node l to node q that pass through edge e_{ij}

Normalization: divided by total number of edges not including e_{ij} namely, divided by $N(N\text{-}1)/2 - 1$

Edge-Betweenness



Example:

betweenness of edge e_{12} is

$$B(e_{12}) = \frac{0}{(5,1,4)} + \frac{0}{(5,3)} + \frac{(5,1,2)}{(5,1,2)+(5,3,2)} + \frac{0}{(5,1)} + \frac{(4,1,2,3)}{(4,1,2,3)+(4,1,5,3)} + \frac{(4,1,2)}{(4,1,2)} + \frac{0}{(4,1)} + \frac{0}{(3,2)} + \frac{(3,2,1)}{(3,2,1)+(3,5,1)}$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} = \frac{5}{2}$$

Here, by convention, $\frac{(2,1)}{(2,1)}$ is not counted

So, the normalization below needs to -1

Normalized: $$B_N(e_{12}) = \frac{5/2}{\frac{5(5-1)}{2} - 1} = \frac{5}{18}$$

(Dis)Assortativity

Assortativity Coefficient of a network is defined by

$$r = \frac{M^{-1}\sum_i j_i k_i - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}{M^{-1}\sum_i \frac{1}{2}(j_i^2 + k_i^2) - \left[M^{-1}\sum_i \frac{1}{2}(j_i + k_i)\right]^2}$$

where k_i and j_i are the degrees of the end nodes of edge i, and M is the total number of edges in the network.

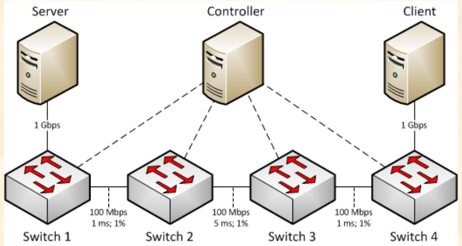
If $r > 0$ then the network is *assortative* (big-big nodes);

if $r < 0$ then the network is *disassortative* (big-small nodes).

(Dis)Assortativity

In a network, if nodes with large degrees tend to connect to nodes also with large degrees on average, the network is said to be assortative

but if nodes with large degrees tend to connect to nodes with small degrees on average, the network is said to be disassortative



Note: networks having the same degree sequence (hence, the same assortativity) may have different topologies

Statistical Properties of Some Real-World Complex Networks

N – number of nodes; ***M*** – number of edges; **<*k*>** - average degree;

L – average path length; ***r*** – power-law exponent; ***C*** – clustering coefficient

	Network	Type	N	M	$<k>$	L	γ	C
Social science	Film actors	undirected	449913	25516482	113	3.48	2.3	0.78
	Company directors	undirected	7673	55392	14.4	4.6	–	0.88
	Math coauthorship	undirected	253339	496489	3.92	7.57	–	0.34
	Physics coauthouship	undirected	52909	245300	9.27	6.19	–	0.56
	Biology coauthorship	undirected	1520251	11803064	15.5	4.92	–	0.6
	Telephone call graph	undirected	47000000	80000000	3.16			
	E-mail messages	undirected	59912	86300	1.44	4.95	1.5/2.0	0.16
	E-mail addresses books	undirected	16881	57029	3.38	5.22	–	0.13
	Student relationships	undirected	573	477	1.66	16	–	0
	Sexual contacts	undirected	2810				3.2	

Mostly assortative

Statistical Properties of Some Real-World Complex Networks

	Network	Type	N	M	$<k>$	L	γ	C
Information Science	WWW nd.edu	directed	269504	1497135	5.55	11.3	2.1/2.4	0.29
	WWW Altavista	directed	203549046	2.13E+09	10.5	16.2	2.1/2.7	
	Citation network	directed	783339	6716198	8.57		3.0/–	
	Roget's Thesaurus	directed	1022	5103	4.99	4.87	–	0.15
	Word co-occurrence	undirected	460902	1.7E+07	70.1		2.7	0.44
Technology	Internet (AS-level)	undirected	10697	31992	5.98	3.31	2.5	0.39
	Power grid	undirected	4941	6594	2.67	19	–	0.08
	Train routes	undirected	587	19603	66.8	2.16	–	0.69
	Software packages	directed	1439	1723	1.2	2.42	1.6/1.4	0.08
	Software classes	directed	1377	2213	1.61	1.51	–	0.01
	Electric circuits	undirected	24097	53248	4.34	11.1	3	0.03
	Peer-to-peer network	undirected	880	1296	1.47	4.28	2.1	0.01

Mostly disassortative

Statistical Properties of Some Real-World Complex Networks

	Network	Type	N	M	$< k >$	L	γ	C
Biology	Metabolic network	undirected	765	3686	9.64	2.56	2.2	0.67
	Protein network	undirected	2115	2240	2.12	6.8	2.4	0.07
	Marine food web	directed	135	598	4.43	2.05	–	0.23
	Freshwater food web	directed	92	997	10.8	1.9	–	0.09
	Neural network	directed	307	2359	7.68	3.97	–	0.28

Various types

Common Features:

Small L and Small C → Random-Graph Networks

Small L but Large C → Small-World Networks

Power-Law $\sim k^{-\gamma}$ → Scale-Free Networks

Present studies of complex networks

- ***Discovering*: Trying to reveal the global statistical properties of a network and to develop measures for these properties.**
- ***Modeling*: Trying to establish a mathematical model of a given network, enabling better understanding of the network statistical properties and the causes of their appearance.**
- ***Analysis*: Trying to find out the basic characteristics and essential features of nodes, edges, and the whole network in a certain topology, to develop fundamental mathematical theories that can describe and predict the network dynamical behaviors.**
- ***Control*: Trying to develop effective methods and techniques that can be used to modify and improve network properties and performances, suggesting new and possibly optimal network designs and utilizations, particularly in the regards of network stability, synchronizability, controllability and data-traffic management.**
- ***Applications*: Trying to apply and utilize some special and fundamental properties and characteristics of complex networks to facilitate the design and applications of network-related problems, such as data-flow congestion control on the Internet and traffic control for city transportations, optimal integrated circuit design for chip fabrication, better decision-making of policy and strategy for commercial trading and financial management, etc.**

End