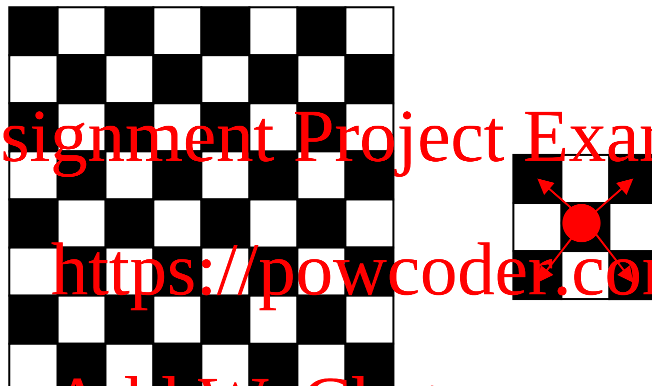
HOMEWORK PROBLEMS #4

4-1 A standard chessboard is shown in the following Figure (a). Define a new "bishop-pawn" marked by a red disc as shown in Figure (b), which can move only "one step by one step" along the diagonal direction, as indicated in Figure (b).

Can this "bishop-pawn" start from a black block, somewhere on the chessboard, then move to visit every black block, once and once only, and
(1) needs not to return to the starting block;
(2) finally returns to the starting block.

If so, how? If not, why?



(a) (b)

4-2 Answer the following **T**rue or **F**alse questions, with a few words or (counter)examples to support your answers (to avoid trivial cases, assume all network sizes $N \geq 3$):

(4-2.1) Undirected graphs

1.1) In a Hamiltonian graph, every node has an even degree.

1.2) If a graph is both Eulerian and Hamiltonian, then it has an even number of edges.

1.3) A circuit with an even number of edges is bipartite.

1.4) If two graphs are isomorphic then they are homeomorphic.

1.5) If two graphs are homeomorphic then they are isomorphic.

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1.6) For any even number r, the r-regular graph has an even number of edges.

(4-2.2) Digraphs

2.1) If a digraph is directed Eulerian then its underlying graph is also Eulerian.

2.2) If a digraph is directed Hamiltonian then its underlying graph is also Hamiltonian.

2.3) If a digraph is strongly connected then it has a directed spanning tree.

2.4) A directed tree always has a perfect matching and it is unique.

2.5) A matching in a digraph is also a matching in its underlying graph.

2.6) A perfect matching in a digraph is always a perfect matching in its underlying graph.

(4-2.3) In a digraph, denote the node's in-degree by $d_{in} > 0$ and out-degree by $d_{out} < 0$.

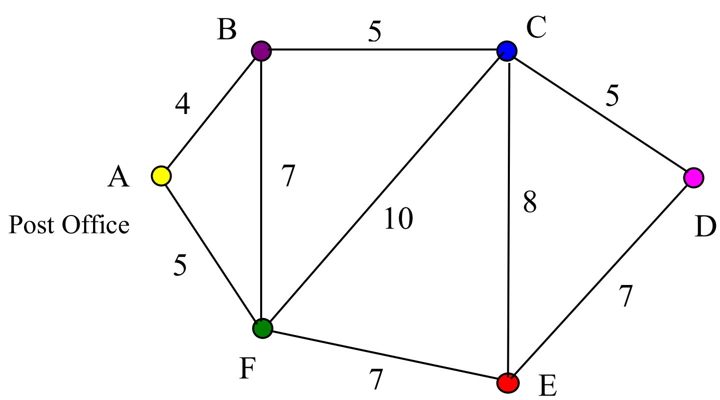
3.1) A connected digraph is directed Eulerian *if and only if* every node satisfies $d_{in} + d_{out} = 0$.

3.2) A connected digraph is directed Eulerian *if and only if* $\sum_{\text{all nodes}} d_{in} + \sum_{\text{all nodes}} d_{out} = 0$.

3.3) A connected digraph is directed bipartite *if and only if* every node satisfies $d_{in} + d_{out} = 0$.

3.4) A connected digraph is directed bipartite *if and only if* $\sum_{\text{all nodes}} d_{in} + \sum_{\text{all nodes}} d_{out} = 0$.

4-3 Solve the Chinese postman problem shown below, where the time for the postman to walk through each street has been indicated.



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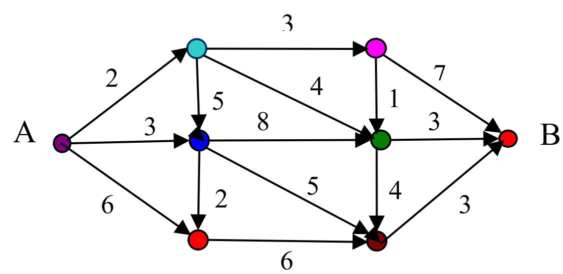
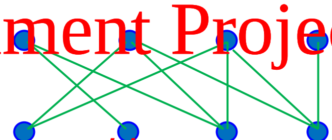
4-4 Solve the maximum flow problem on the following graph, where numbers are rewards (show your steps):



4-5 Use the augmenting path algorithm to find a maximum matching of the following bipartite graph (show your steps):



Is your result a perfect matching?

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