L3_2 Two's Complement

EECS 370 – Introduction to Computer Organization – Fall 2020

Learning Objectives

- Represent signed and unsigned numbers in binary (base 2)
- Negate positive and negative signed values
- Complete arithmetic operations (addition and subtraction) by hand using signed and unsigned binary numbers

Binary Review

Review!

- Before starting this video, get comfortable with representing numbers in binary
- Resources:

  - Video reviews (EECS 370 website): https://www.eecs.umich.edu/courses/eecs370/eecs370.f20/video_reviews/

  - Resource documents (EECS 370 website): https://www.eecs.umich.edu/courses/eecs370/eecs370.f20/resources/

  - Lecture 2 video on Binary numbers: https://drive.google.com/drive/folders/1RDqMynHaAMFW6hRLvky9XJZR2dhBD-bl or Media gallery on Canvas https://umich.instructure.com/courses/394380/external_tools/6329

Review!

Binary Addition

- We can already represent non-negative numbers in binary

$$6 \text{ (base 10)} = 2^2 (4) + 2^1 (2) = 110 \text{ (base 2)}$$

- We can do arithmetic with binary numbers

3 + 2 = 5 (base 10)	3 + 5 = 8 (base 10)

Binary Addition

Review!

- We can already represent non-negative numbers in binary

$$6 \text{ (base 10)} = 2^2\ (4) + 2^1\ (2) = 110 \text{ (base 2)}$$

4 2 1

- We can do arithmetic with binary numbers

3 + 2 = 5 (base 10)

```
  1
 0 0 1 1
 0 0 1 0
--------
 0 1 0 1
```

3 + 5 = 8 (base 10)

```
1 1  1
 0 0 1 1
 0 1 0 1
--------
 1 0 0 0
```

What about Negative Numbers?

- Thoughts: add another bit for sign, use one of the existing bits for sign

1 2

1: 5 = 0101 $\boxed{1}$ 0101 −5 = $\boxed{0}$ 0101

sign bit

2: $\boxed{0}$ 101

sign bit

What about Negative Numbers?

- Design space preferences:
  - Representation of positive and negative values
  - Representation of signed and unsigned values
  - Single way to represent 0
  - Equal magnitude of positive and negative values (roughly)
  - Simple (not complex) to detect sign (positive or negative)
  - Simple negation of a number
  - Simple storage for signed and unsigned
  - Simple, non-redundant hardware for operations
    - E.g., one hardware addition unit for signed and unsigned numbers

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- Thought: use existing bit of binary number for signed values

Two's Complement

Unsigned Binary Representation

- 1011 in binary is 13 in decimal

$$\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 2^3 & 2^2 & 2^1 & 2^0 \end{array} = 8 + 4 + 1 = 13$$

$$\rightarrow 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$8 + 4 + 0 + 1 = 13$$

Two's Complement Binary Representation

- 1011 in binary is 13 in decimal

0 1 1

1 0 0 0

$$\underset{2^3}{1} \quad \underset{2^2}{1} \quad \underset{2^1}{0} \quad \underset{2^0}{1} = 8 + 4 + 1 = 13$$

- Two's complement numbers are very similar to unsigned binary

EXCEPT the first (most significant) digit is negative in two's complement

$$\underset{-(2^3)}{1} \quad \underset{2^2}{1} \quad \underset{2^1}{0} \quad \underset{2^0}{1} = -8 + 4 + 1 = -3$$

Two's Complement – Exercise 1

What is 1010 (binary)

1. Decimal unsigned value?
2. Decimal signed (two's complement) value?

4 BITS

unsigned

1 0 1 0

2^3 2^2 2^1 2^0

Signed – 2's complement

1 0 1 0

$-(2^3)$ 2^2 2^1 2^0

Two's Complement – Exercise 1

What is 1010 (binary)

4 BITS

1. Decimal unsigned value?
2. Decimal signed (two's complement) value?

unsigned

1 0 1 0

2^3 2^2 2^1 2^0

$1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

$8 + 0 + 2 + 0$

$= 10$

Signed – 2's complement

1 0 1 0

$-(2^3)$ 2^2 2^1 2^0

$-8 + 0 + 2 + 0$

-6

Two’s Complement – Exercise 1

What is 1010 (binary)

1. Decimal unsigned value?
2. Decimal signed (two’s complement) value?

unsigned

1 0 1 0

2^3 2^2 2^1 2^0

8 + 2 = 10

Signed – 2’s complement

1 0 1 0

$-(2^3)$ 2^2 2^1 2^0

-8 + 2 = -6

Two's Complement Range

- What is the range of representation of a 4-bit 2's complement number?
  - [-8, 7]

- What is the range of representation of an n-bit 2's complement number?
  - [$-2^{(n-1)}$, $2^{(n-1)} - 1$]

32 bits: - [$-(2^{31})$ to $2^{31} - 1$]

Negating Two's Complement

- Useful trick: You can negate a 2's complement number by inverting all the bits and adding 1.

5 (decimal) in binary is		0	1	0	1
Negate (invert) all bits		1	0	1	0
Add 1		1	0	1	0
	+	0	0	0	1
		1	0	1	1
		$-(2^3)$	2^2	2^1	2^0

$$-8 + 0 + 2 + 1 = -5$$

Two's Complement – Exercise 2

How would you represent -3 (decimal) in 2's complement binary using 4 bits?

Two's Complement – Exercise 2

How would you represent -3 (decimal) in 2's complement binary using 4 bits?

1. Convert 3 (decimal) to binary
2. Negate binary
    1. Invert all bits
    2. Add one

Two’s Complement – Exercise 2

How would you represent -3 (decimal) in 2’s complement binary using 4 bits?

1. Convert 3 (decimal) to binary
2. Negate binary
    1. Invert all bits
    2. Add one

0 0 1 1

1 1 0 0

$\boxed{1\ 1\ 0\ 1}$

Two's Complement – Exercise 2

How would you represent -3 (decimal) in 2's complement binary using 4 bits?

1. Convert 3 (decimal) to binary
2. Negate binary
   1. Invert all bits
   2. Add one

1. Convert 3 to binary
   1. 3 -> 0011
2. Convert to 2's complement
   1. 0011 -> 1100
   2. 1100 + 1 = 1101

Signed – 2's complement

1	1	0	1
-2^3	2^2	2^1	2^0

-8 + 4 + 0 + 1 = -3

Sign Extension

- With two's compliment, it matters how many bits are used!

```
 5 (decimal) in binary (4 bits) is   0101

 5 (decimal) in binary (8 bits) is   0000 0101

-5 (decimal) in binary (4 bits) is   1011

-5 (decimal) in binary (8 bits) is   1111 1011

                                     NOT 0000 1011
```


need to **extend the most significant (sign) bit**

LC-2K: programmer (you) need to do this!

Two’s Complement Arithmetic

Decimal	2’s Complement Binary	Decimal	2’s Complement Binary
0	0000	-1	1111
1	0001	-2	1110
2	0010	-3	1101
3	0011	-4	1100
4	0100	-5	1011
5	0101	-6	1010
6	0110	-7	1001
7	0111	-8	1000

7 – 6 = 7 + (-6) = 1

6 – 7 = 6 + (-7) = -1

Two's Complement Arithmetic

Decimal	2's Complement Binary	Decimal	2's Complement Binary
0	0000	-1	1111
1	0001	-2	1110
2	0010	-3	1101
3	0011	-4	1100
4	0100	-5	1011
5	0101	-6	1010
6	0110	-7	1001
7	0111	-8	1000

7 – 6 = 7 + (-6) = 1

1 0 1 1 1 1
1 0 1 0
$\boxed{1}$ 0 0 0 1

6 – 7 = 6 + (-7) = -1

0 1 1 0
1 0 0 1
$\boxed{1\ 1\ 1\ 1}$

Logistics

- This is the second of 3 videos for lecture 3
  - L3_1 – ISAs – Instructions and Memory
  - L2_2 – Two's Complement
  - L2_3 – LC-2K ISA
- There are two worksheets for lecture 3
  1. Addressing and 2's complement
  2. LC-2K program encoding
- When ready, move on to L3_3