EECS 340 Algorithms

2018 Spring Semester

Homework 4

Due on March 28, 2018, 12:45pm

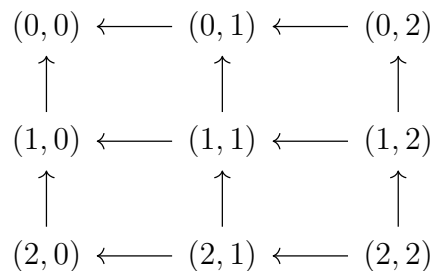This assignment covers graph algorithms.

1. **Warm-up: BFS**

   Given the graph:

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   A ——— B

   C ——— D     E

   F     G
   ```
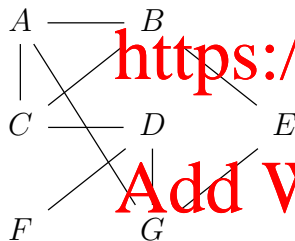
   (a) Draw the shortest path tree found by a BFS. ($\star$)

   (b) Annotate the tree in part (a) with the order vertices are visited in your BFS. ($\star$)

   (c) List the cross-edges of the BFS. ($\star$)

2. **Warm-up : Topological Sorting**

   (a) Find four distinct topological orderings of the nodes in the following directed acyclic graph, presented here as a 3×3 grid. ($\star$)

$$(0,0) \longleftarrow (0,1) \longleftarrow (0,2)$$
$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$
$$(1,0) \longleftarrow (1,1) \longleftarrow (1,2)$$
$$\uparrow \qquad\qquad \uparrow \qquad\qquad \uparrow$$
$$(2,0) \longleftarrow (2,1) \longleftarrow (2,2)$$

(b) Describe how to generalize your orderings to the case of a graph with the same connectivity pattern on an $n \times n$ grid. $(\star\star)$

3. **A Unicycle Problem**

   Prove that a cycle exists in an undirected graph if and only if a BFS of that graph has a cross-edge. $(\star\star)$

   Your proof may use the following facts from graph theory:

   - There exists a unique path between any two vertices of a tree.
   - Adding any edge to a tree creates a unique cycle.

4. **A Bicycle Problem**

   Solve C-13.15 in the text. You may assume that the graph is connected. $(\star\star)$

5. **Maximal Independent Set** [1] An *independent set* I of an undirected graph $G = (V, E)$ is a subset $I \subseteq V$ such that if $u, v \in I$, then $(u, v) \notin E$. A *maximal independent set* M is an independent set which cannot have any vertices added to it without losing the independent set property. Describe an efficient algorithm to compute the maximal independent set of any such G. $(\star\star)$

6. **Application: Procrastinator's Sort**

   The year is 430BC. Socrastes the Philosopher has a collection of important ethical questions to ponder over. Some of these questions have dependencies, i.e: "Is it ethical to consume meat?" should be answered before the question "What should I eat for lunch?" is resolved. Socrastes may only consider one question at a time, but nevertheless, Socrastes is a masterful philosopher. Whenever Socrastes begins thinking about a question, Socrastes is guaranteed to answer it in a few days. Socrastes will *always* respect the dependencies between questions in choosing which questions to think about. However, whenever Socrastes has a *choice* between two questions [2] to ponder, Socrastes will always choose the less important question to think about. [3] Suppose that Socrastes ranks the importance of questions on an integer scale from 1 to 10. If the number of questions is n and the number of dependencies is m, describe an algorithm which returns a possible order that Socrastes might think about the questions in $O(n + m)$ time. $(\star \star \star)$

7. **Application: Zombie Apocalypse**

   The year is 2050AD. CWRU's HvZ got a little out of hand this year, primarily due to one BME student's discovery of a zombifying virus. Luckily, Babs has constructed an elaborate labyrinth [4] where all students are currently in hiding. Suppose that the labyrinth is represented by an $m \times n$ matrix of values in $\{H, Z, W\}$, where H, Z,

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   [1]Reproduced from A-13.3, only with 100% fewer lawyers involved.

   [2]To reiterate, for Socrastes to have a choice beween two or more questions, the questions must have no remaining prerequisites for Socrastes to handle.

   [3]This fatal flaw is widely regarded by philosophers to have set back the field several thousand years.

   [4]Known as KSL

and W correspond to humans, zombies, and walls, respectively. Each minute, all zombies infect all humans which are directly adjacent to them in the labyrinth, e.g, if the state of the labyrinth at zero minutes is:

$$
\begin{array}{ccc}
H & W & H \\
 & & \uparrow \\
H \longleftarrow Z & Z \\
 & & \downarrow \\
H & W & H
\end{array}
$$
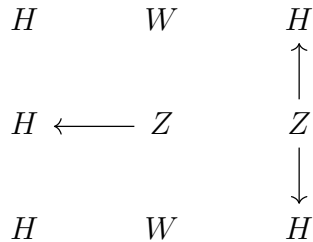
then the state of the labyrinth at one minute is:

$$
\begin{array}{ccc}
H & W & Z \\
Z & Z & Z \\
H & W & Z
\end{array}
$$

Describe an algorithm to determine how long (in minutes) it will take for all humans in Babs' labyrinth to become zombies. Assume that no human is safe. $(\star \, \star \, \star)$

8. **The following is for EECS 454 students only**. Read Section 17.3 in the textbook and then solve the following problems:

   (a) R-17.2

   (b) C-17.7

   (c) C-17.8

Submission

Hand in your paper in-class by the beginning of class on the due date. Always check Canvas for updates and corrections.