City University of Hong Kong
Department of Economics and Finance
Course EF 5213 Assignment #4 **(due April 8, 2018)**

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1. Under the Black-Schole regime, it is possible to generate the implied volatility surface, with respect to option strike price K and maturity term T, based on the market prices of plain vanilla call options written on the same asset. As discussed in Example 5.2, the plain vanilla call option prices $c(K, T)$ can be converted into a set of implied volatilities $\nu(K, T)$ utilizing the Black-Schole pricing formula with current asset price S_0 and risk-free interest rate r. In practice, the implied volatility surface is parameterized as

$$\nu(K,T) = b_0(T) + b_1(T)\left(\frac{X}{\sqrt{T}}\right) + b_2(T)\left(\frac{X}{\sqrt{T}}\right)^2 + b_3(T)\left(\frac{X}{\sqrt{T}}\right)^3 \quad , \quad X = tan^{-1}\left(ln\left(\frac{K}{S_0 e^{rT}}\right)\right) \quad \text{called moneyness}$$

with coefficients $b_0(T)$, $b_1(T)$, $b_2(T)$, and $b_3(T)$ depending on the maturity term. For each of the maturity term $\{T_1, T_2, \ldots , T_n\}$ in the market data, the volatility skew (ν *versus* K) can be obtained by least-square fitting of the coefficients $b_0(T)$, $b_1(T)$, $b_2(T)$, and $b_3(T)$ to the implied volatilities in the data. Using then the contours of volatility skew, the volatility term structure (ν *versus* T) for arbitrary strike K can be obtained through cubic spline interpolation. The volatility term structure can also be extended to $T = 0$ and $T \to \infty$ by linearly extrapolating the left-end and right-end cubic polynomials, respectively. In this way, it is possible to estimate the implied volatility $\nu(K , T)$ for any strike price and maturity within the regions $0 < K < \infty$ and $0 \le T < \infty$.

(a) Using the market prices of the European call options in the attached comma separated values file, develope a VBA routine that generates the implied volatility surface for the underlying asset. Your solution should be able to evaluate the interpolated value of implied volatility $\nu(K , T)$ for chosen strike price and maturity within the regions $0 < K < \infty$ and $0 \le T < \infty$, respectively.

Note : Enclosed VBA subroutine *LeastSquareFit*() that is capable to perform a least-square fitting of the coefficients $\{c_1, \ldots , c_M\}$ in $y = c_1\phi_1(x) + \ldots + c_M\,\phi_M(x)$ given points $\{(x_1, y_1), \ldots , (x_N, y_N)\}$.

There are mispriced call options in the attached file that violate $call \ge S_0 - Ke^{-rT}$.

(50 points)

(b) In the stochastic model, the local volatility $\sigma(S_t, t)$, with asset price S_t at time t, can be calibrated from the implied volatility surface $\nu(K, T)$ using the Dupire formula given by

$$[\sigma^2(S_t, t)]_{S_t = K\,,\,t = T} = \frac{\nu^2 + 2T\nu\dfrac{\partial\nu}{\partial T} + 2rKT\nu\dfrac{\partial\nu}{\partial K}}{\left(1 + \beta K\dfrac{\partial\nu}{\partial K}\right)^2 + K^2 T\nu\left(\dfrac{\partial^2\nu}{\partial K^2} - \beta\left(\dfrac{\partial\nu}{\partial K}\right)^2\right)} \quad , \quad \beta = \frac{ln(S_0/K) + (\,r + \tfrac{1}{2}\nu^2)T}{\nu}$$

Use implicit finite difference method to price the accumulator contract in assignment 2 based on the local volatility as calibrated above. You can estimate the first and second derivatives of a function to the second order of Δx as

$$g'(x) = \frac{g(x + \Delta x) - g(x - \Delta x)}{2\Delta x} \quad , \quad g''(x) = \frac{g(x + \Delta x) - 2g(x) + g(x - \Delta x)}{(\Delta x)^2}$$

Practically, you can choose $\Delta x = 10^{-8}\,x$. **(30 points)**