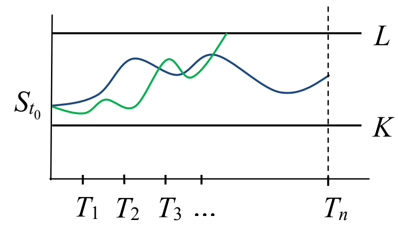
City University of Hong Kong
Department of Economics and Finance
Course EF5213 Assignment #2 **(due March 4, 2018)**

1. Use VBA as programming tool, implement the implicit finite difference method under the Crank-Nicholson scheme to price an accumulator contract written on a stock with Cox-Ingersoll-Ross volatility structure as $\sigma(S_t, t) = \sigma\sqrt{S_t}$, where S_t is the underlying stock price at time t. Consider the following input parameters in your implementation :

On option : r – Risk free interest rate
σ – Volatility factor of the underlying stock
Q – Contract size
$\{ T_1 , T_2 , \dots , T_n \}$ – Settlement dates of the contract
K – Strike price of the contract
L – Knock-out price of the contract



On precision : i_{max} – Number of steps to maturity
j_{max} – Size parameter of the tridiagonal matrix
ΔS – Price increment in the lattice

To improve efficiency, use the enclosed *SolveAxb* routine from *Numerical Recipies* in solving the matrix equation $\boldsymbol{A}_{n\times n}\, \boldsymbol{x}_{n\times 1} = \boldsymbol{b}_{n\times 1}$ based on singular value decomposition.

Note : With accumulator, investor agrees and also entitles to buy a certain amount Q of a stock at a fixed price K and at every settlement dates $\{ T_1 , T_2 , \dots , T_n \}$ over a period of time. There is a knock-out price ($L > K$) that terminates the contract.

- Define time steps t_i with $i_{max}\,\Delta t = T_n$.

- Perform a backward iteration that starts off from T_n with payoff $F(S_j , T_n) = Q \times (S_j - K)$.

- If t_i corresponds to a settlement date, (*i.e.* $t_i - \tfrac{1}{2}\Delta t < T_k \le t_i + \tfrac{1}{2}\Delta t$ for some T_k)

$$F(S_j < L , t_i) = F_{itr}(S_j < L , t_i) + Q \times (S_j - K)$$
$$F(S_j \ge L , t_i) = Q \times (S_j - K)$$

  where $F_{itr}()$ is the option vector immediately after the backward iteration

- If t_i is not a settlement date, $F(S_j < L , t_i) = F_{itr}(S_j < L , t_i)$
  $F(S_j \ge L , t_i) = 0$

Referring to the transition at boundaries $F(S_0, t_{i+1}) = b_0\, F(S_0, t_i)$ and $F(S_{jmax} , t_{i+1}) = b_{jmax}\, F(S_{jmax} , t_i)$. It is always true that $b_{jmax} = 1$ for $S_{jmax} \to \infty$. Also, we must have $b_0 = 1$ whenever the option can be exercised. Otherwise, it is generally true that $b_0 = e^{r\Delta t}$.

(50 points)