

City University of Hong Kong  
Department of Economics and Finance  
Course EF5210 Assignment #4

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**Question 1 (due April 25, 2018)**

With an accumulator contract, investor agrees to buy a certain amount ( $M$ ) of the underlying asset at a fixed price ( $F$ ) over a period of time at settlement dates  $\{\tau_1, \tau_2, \dots, \tau_n\}$ . There is also a knock out price ( $L > F$ ) that terminates the contract. The risk-neutral valuation of the accumulator contract is given by

$$f_0 = M \hat{E} \left( \sum_{i=1}^m e^{-r\tau_i} (S_{\tau_i} - F) \mid S_0 \right)$$

where  $\tau_m$  is the last settlement date prior to the termination of the contract. There is no need to settle the contract in case when it is being terminated at settlement date.

Use Monte-Carlo simulation to price an accumulator contract with the following parameters :

Current price of the underlying asset  $S_0 = \$100$ ,

Annualized risk-free interest rate  $r = 5\%$

Annualized volatility of the underlying asset  $\sigma = 10\%$

Number of settlement dates in the accumulator contract  $n = 4$ ,

Settlement dates in year of the accumulator contract  $\{\tau_1 = 0.25, \tau_2 = 0.5, \tau_3 = 0.75, \tau_4 = 1.00\}$ ,

Knock out price of the contract  $L = \$120$ ,

Agreed purchase price of the contract  $F = \$96$ ,

Agreed purchase unit of the contract  $M = 100$ ,

Number of samples in the simulation  $N_{sample} = 100000$ , and

Number of simulated steps between two settlement dates  $N = 100$

Use antithetic variate method to reduce the standard error in the simulation.

*Note :* You can use any programming language of your own preference. Please submit your numerical results together with the source code.

( 40 points )

**Question 2 (due May 2, 2018)**

Suppose the risk-neutral process for the short rate is given by

$$dr_t = \theta dt + \sigma(t) dz_t$$

where  $\theta$  is a constant factor and  $\sigma(t)$  is a function of time.

- (a) Show that the forward price  $P_t(T)$  of a zero-coupon bond with unit dollar par value and maturity at  $T$  is given by

$$P_t(T) = (\$1) A(t, T) e^{-B(t, T) r_t}$$

where  $B(t, T) = (T - t)$

$$A(t, T) = e^{-\frac{1}{2}\theta(T-t)^2 + \frac{1}{2}\int_t^T \sigma^2(s)(T-s)^2 ds}$$

**(15 points)**

- (b) Consider the option with maturity at  $T$  written on a zero-coupon bond with unit dollar par value and maturity at a later time  $T^*$ . The maturity payoff of this option is given by

$$f_T = \sqrt{P_\tau(T^*)P_T(T^*)} \quad , \quad \text{where } \tau < T$$

Use Jamshidian formula to determine the current price of this option

**(15 points)**