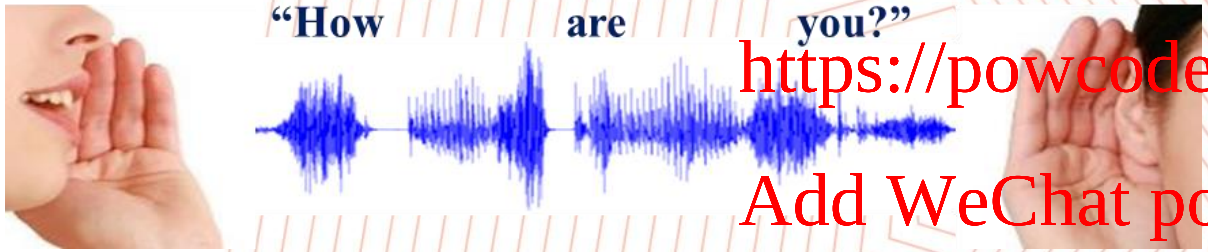
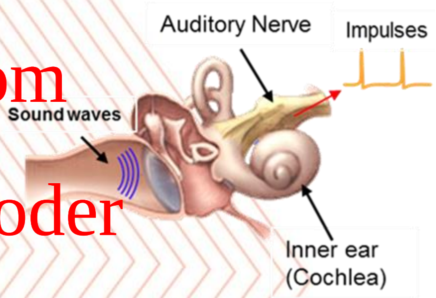
ELEC3104: Mini-Project – Cochlear Signal Processing





Prof. Eliathamby Ambikairajah, School of EE&T

Term 3, 2022

ELEC3104: Mini-Project – Cochlear Signal Processing

TLT – Level 2 (Pass Level): Implementation of a cascaded filter bank model of the cochlea for analysis purposes.

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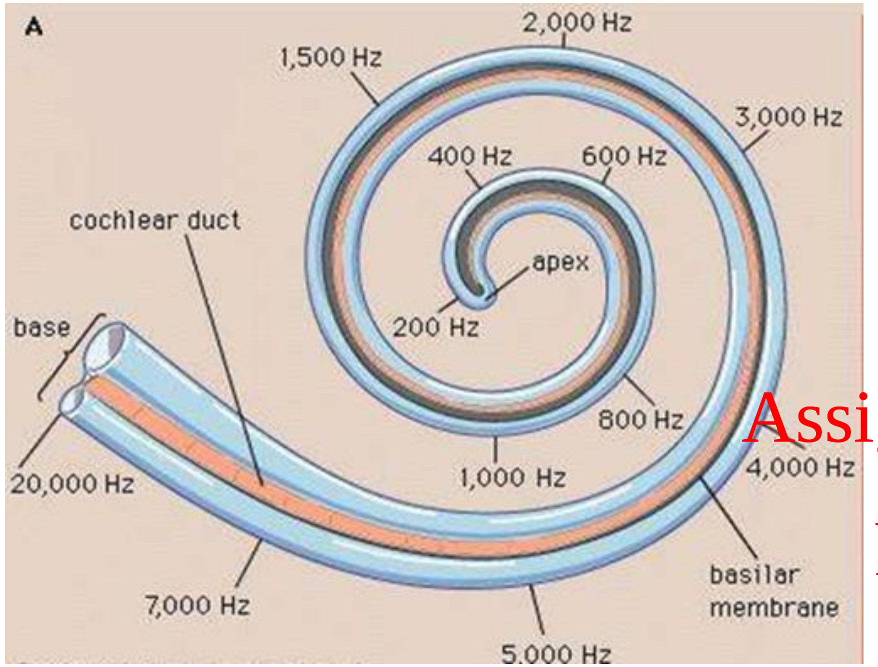
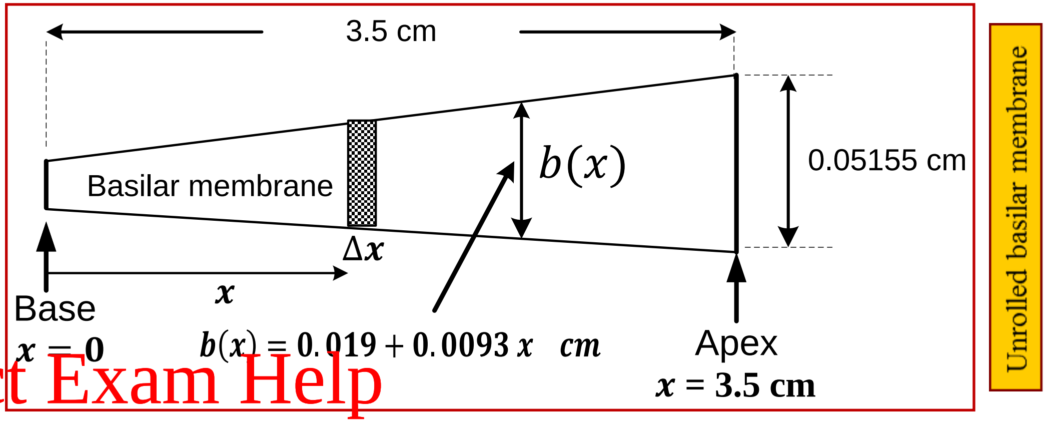
TLT-Level 2 Project Implementation

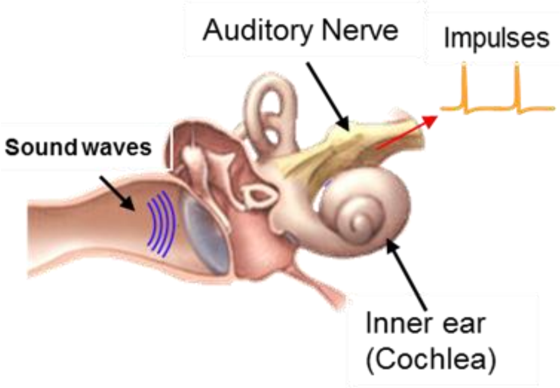
- ✓ For this project, you should implement a digital model of the peripheral auditory system comprising of a model of the outer ear and the middle ear (from TLT Level 1) and the transmission line mode of the cochlea (TLT Level 2).
- ✓ The transmission line model of the cochlea can be implemented as a cascade of many band-pass filters.
- ✓ You should understand how the characteristics of the model are related to the functioning of the cochlea explained in TLT Level 1
- ✓ Validate that all parts of your model operate as desired in terms of impulse responses, frequency responses for a variety of input signals.

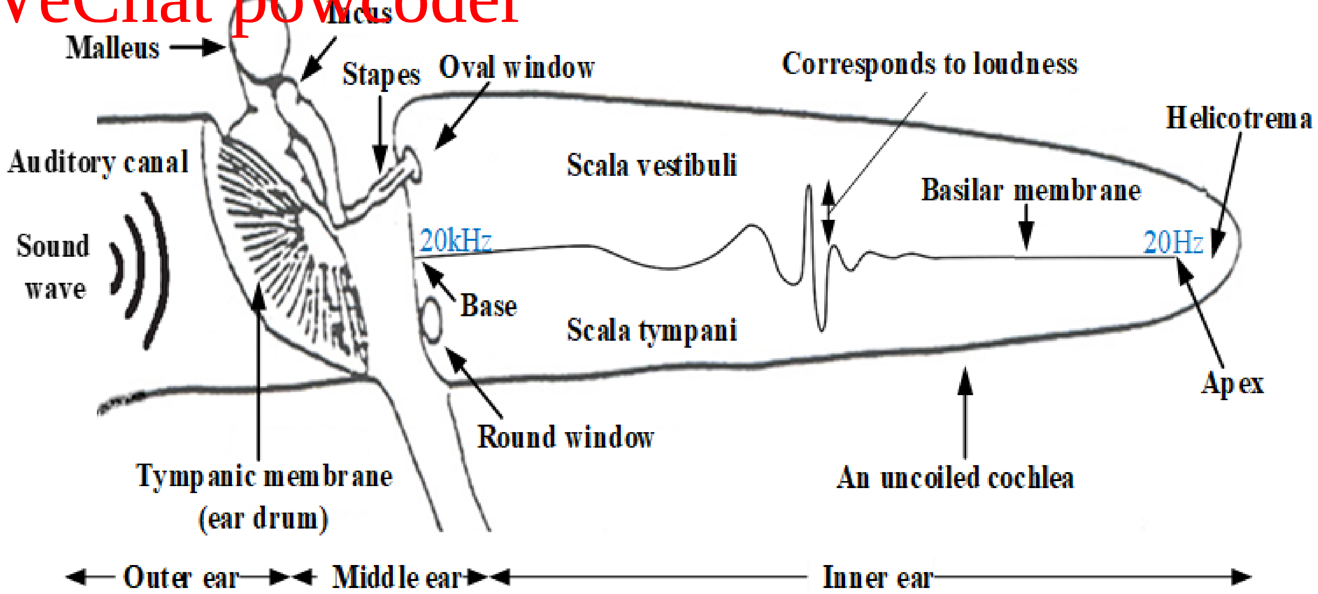
Inner ear (Cochlea)



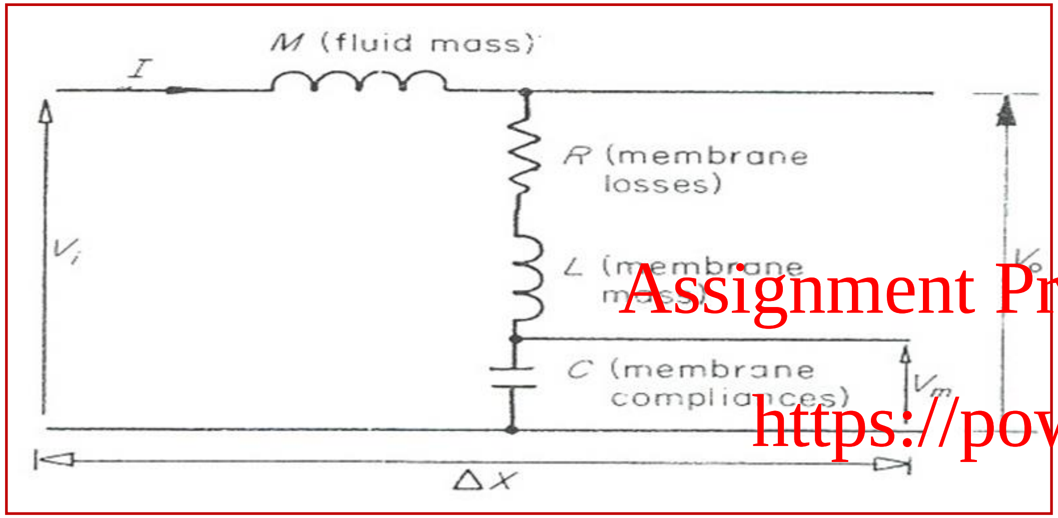


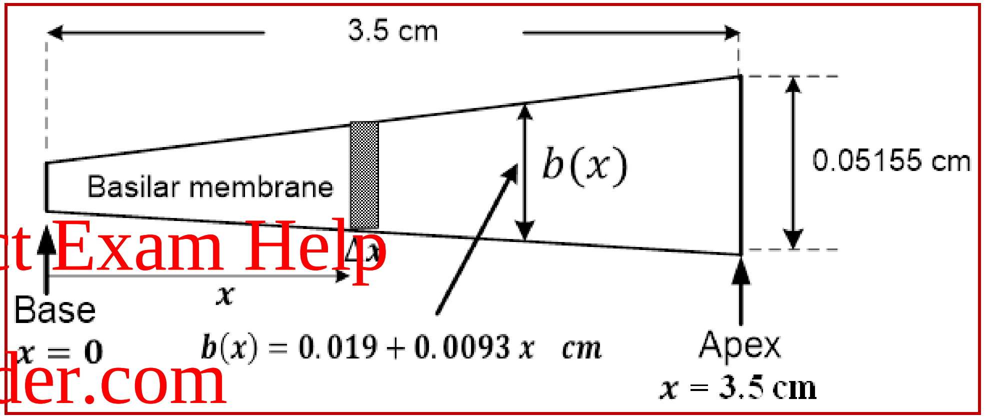
A longitudinal section of an uncoiled cochlea

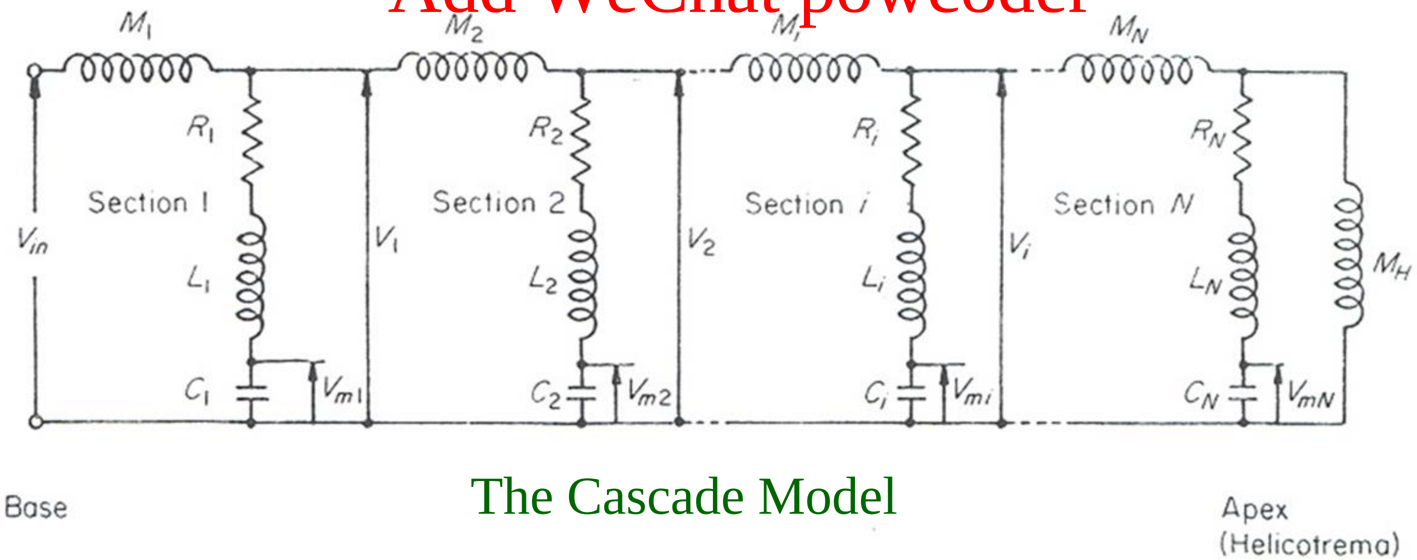




- A simple electrical model of a section of the BM is shown below figure below.







The Cascade Model

Pressure and Displacement Transfer Functions

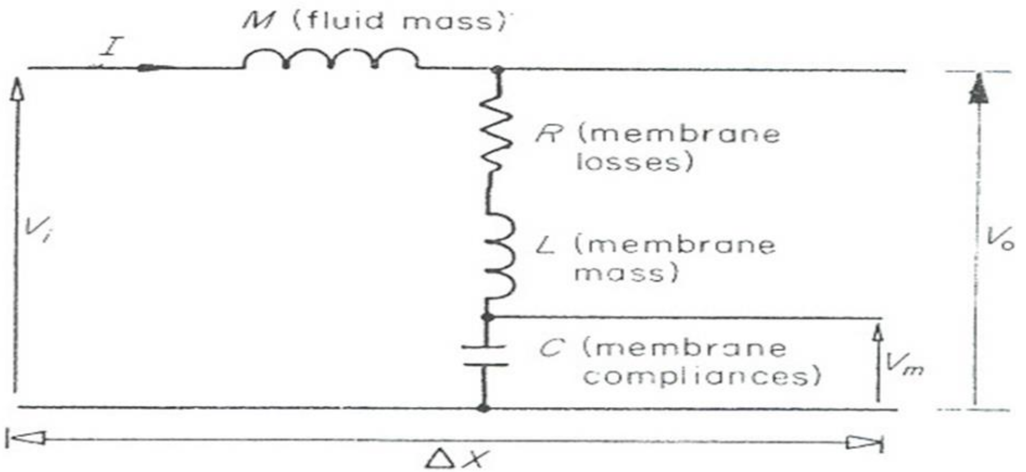
- ✓ The Voltage or **Pressure transfer function** of the isolated section can be obtained as follows:

$$\frac{v_o(s)}{v_i(s)} = K\,\frac{a}{s+a}\cdot\frac{\omega_p^2}{s^2+B_p s+\omega_p^2}\cdot\frac{s^2+B_z s+\omega_z^2}{\omega_z^2}$$

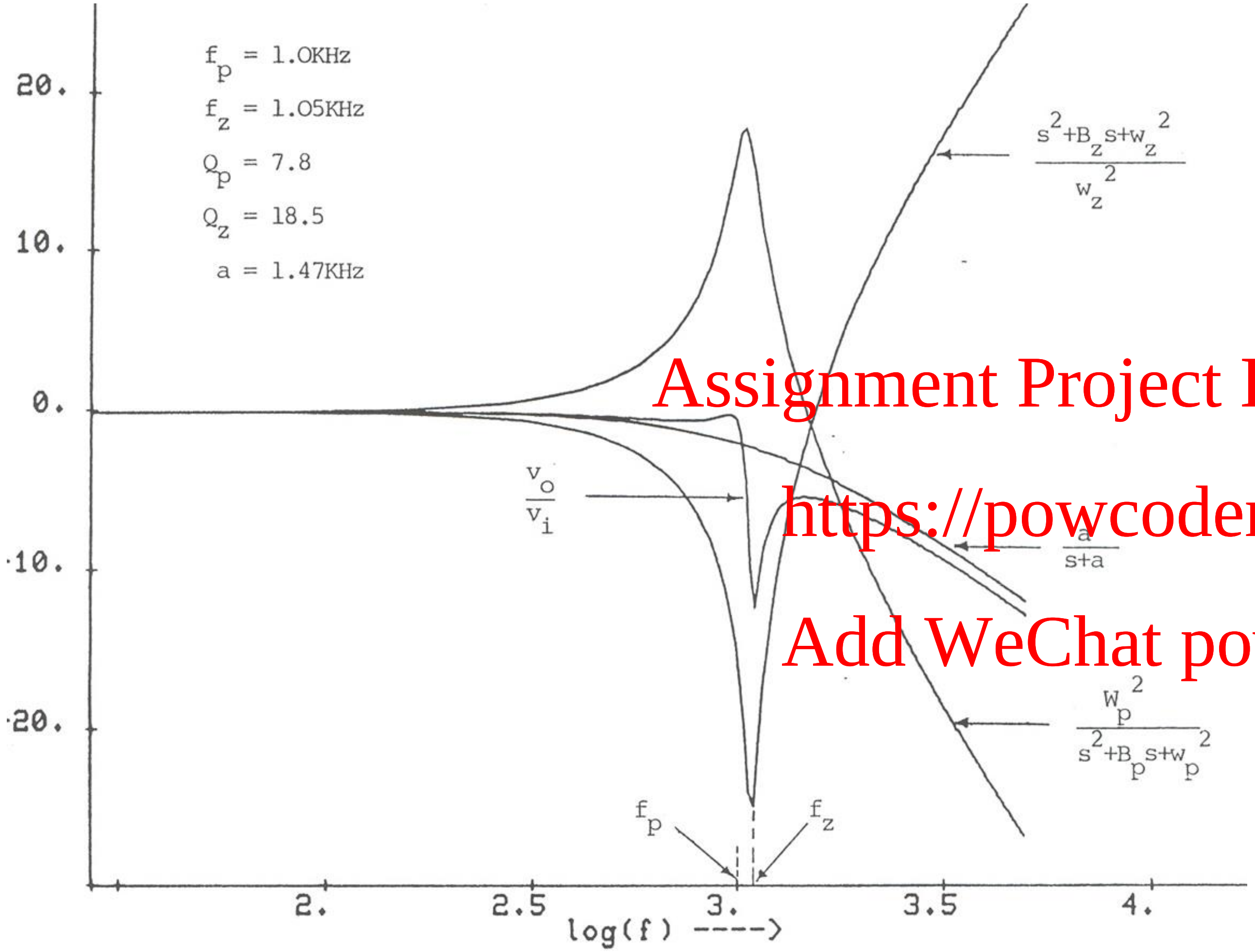
Low pass filter **Resonant pole** **Resonant zero**

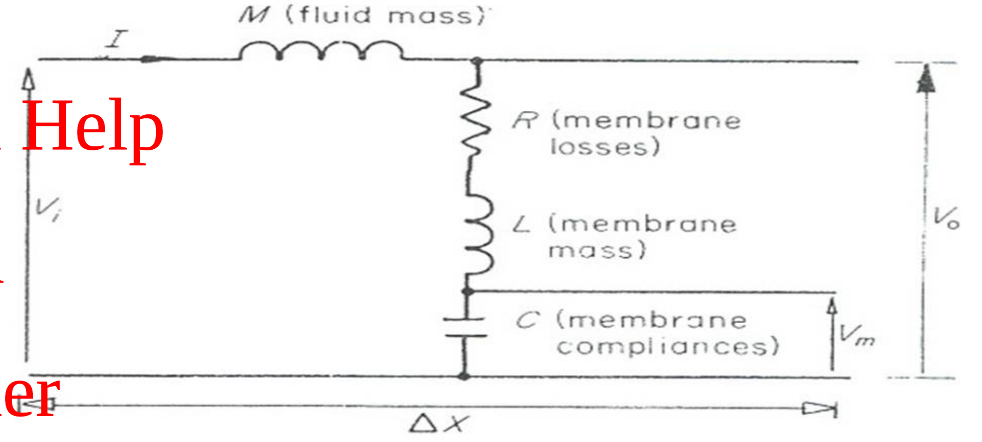
$$\frac{v_m(s)}{v_i(s)} = K\,\frac{a}{s+a}\,\frac{\omega_p^2}{s^2+B_p s+\omega_p^2}$$

- ✓ One can see that the displacement transfer function is contained in the pressure transfer function and therefore a simple cascade arrangement is possible.



Frequency Response - one section of the membrane



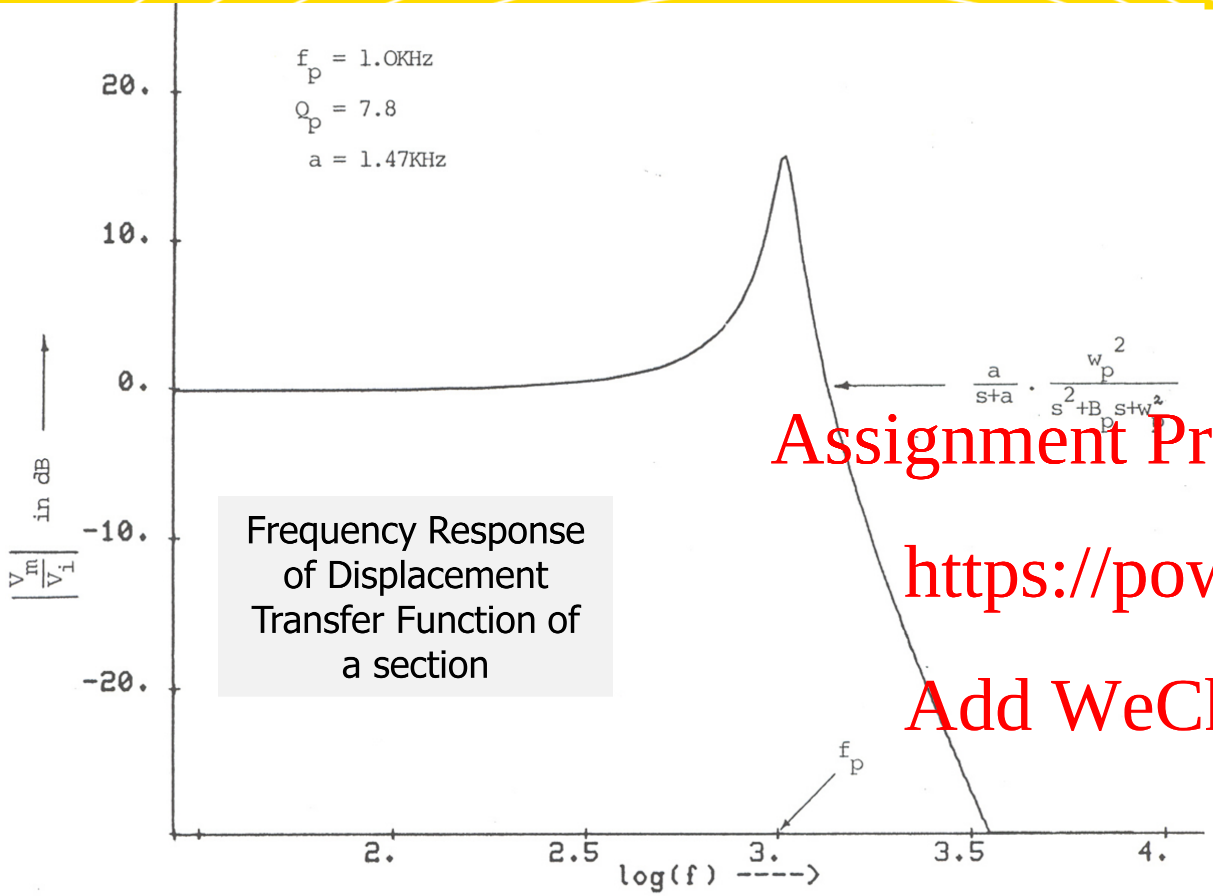


$$\frac{v_o(s)}{v_i(s)} = K \frac{a}{s+a} \frac{\omega_p^2}{s^2+B_p s+\omega_p^2} \frac{s^2+B_z s+\omega_z^2}{\omega_z^2}$$

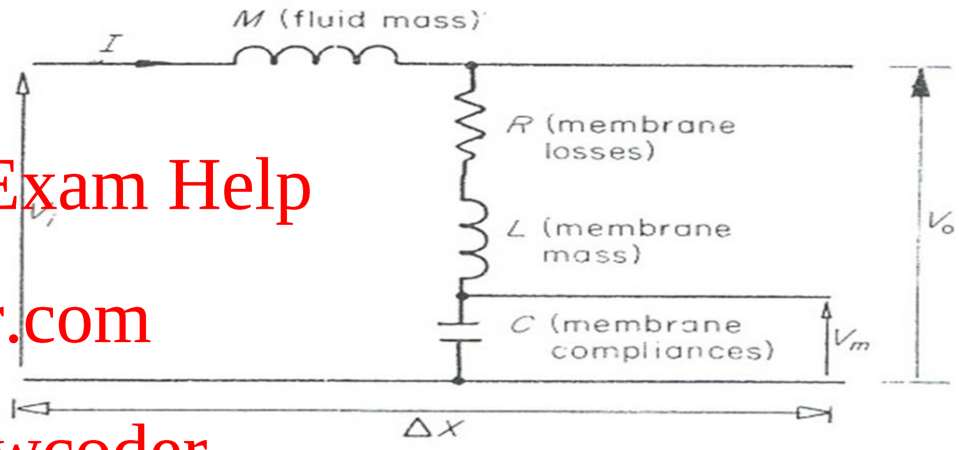
Pressure Transfer Function

Low pass filter | **Resonant pole** | **Resonant zero**

Frequency Response - one section of the membrane



Frequency Response of Displacement Transfer Function of a section



Low pass filter **Resonant pole**

$$\frac{v_m(s)}{v_i(s)} = K \frac{a}{s+a} \frac{\omega_p^2}{s^2 + B_p s + \omega_p^2}$$

Displacement Transfer Function

Digital filter model of the basilar membrane

- ✓ A digital filter model of the basilar membrane can be obtained by transforming the analogue filter equation (given below) to an equivalent digital filter equation.
- ✓ The impulse response of the Basilar Membrane (BM) is an important property and it should be preserved in the digital filter model of the BM. So use **impulse invariant transformation** and is given by:

$$\frac{1}{s+a} \rightarrow \frac{1}{1-e^{-aT}z^{-1}}; \quad \text{T – sampling period}$$

- ✓ On applying the impulse invariant transformation, the pressure transfer function in digital domain can be obtained:

$$\frac{v_o(s)}{v_i(s)} = K \frac{a}{s+a} \cdot \frac{\omega_p^2}{s^2+B_p s+\omega_p^2} \cdot \frac{s^2+B_z s+\omega_z^2}{\omega_z^2}$$

impulse invariant transformation

$$\frac{v_o(z)}{v_i(z)} = K \frac{1-a_0}{1-a_0 z^{-1}} \frac{1-b_1+b_2}{1-b_1 z^{-1}+b_2 z^{-2}} \frac{1-a_1 z^{-1}+a_2 z^{-2}}{1-a_1+a_2}$$

$$\frac{v_m(s)}{v_i(s)} = K \frac{a}{s+a} \frac{\omega_p^2}{s^2+B_p s+\omega_p^2}$$

impulse invariant transformation

$$\frac{v_m(z)}{v_i(z)} = K \frac{1-a_0}{1-a_0 z^{-1}} \frac{(1-b_1+b_2)z^{-1}}{1-b_1 z^{-1}+b_2 z^{-2}}$$

- ✓ Where $a_0, a_1, a_2, b_1, and\ b_2$ are the digital filter coefficients;

Digital filter coefficients

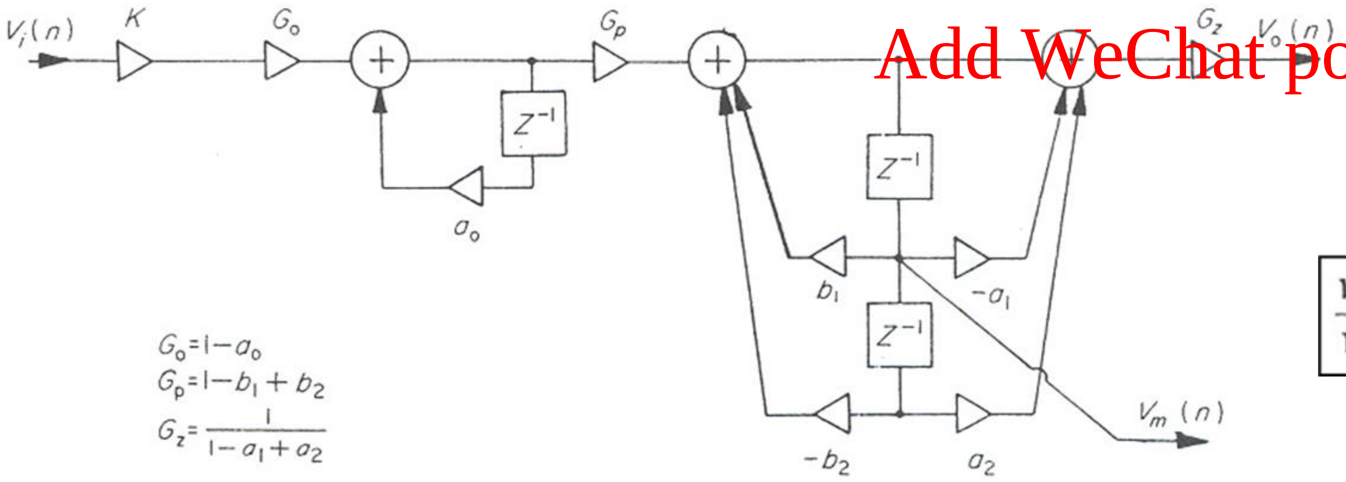
$\boldsymbol{a_1} = 2e^{-p_1 T}\cos(q_1 T)$; $\boldsymbol{a_2} = e^{-2p_1 T}$; $\boldsymbol{p_1} = \frac{\omega_z}{2Q_z}$; $\boldsymbol{q_1} = p_1\sqrt{4Q_z^2 - 1}$;

$\boldsymbol{b_1} = 2e^{-p_2 T}\cos(q_2 T)$; $\boldsymbol{b_2} = e^{-2p_2 T}$; $\boldsymbol{p_2} = \frac{\omega_p}{2Q_p}$; $\boldsymbol{q_2} = p_2\sqrt{4Q_p^2 - 1}$;

Sampling frequency (f_s) = 48 kHz, T = 1/ f_s

$\boldsymbol{a_0} = (2 - \cos\boldsymbol{\theta_c}) - \sqrt{(2 - \cos\boldsymbol{\theta_c})^2 - 1}$;
$\boldsymbol{\theta_c}$ *is the* 3dB cut-off frequency of the low pass filter (choose $\boldsymbol{\theta_c}$ = 1.4 * $\boldsymbol{\omega_z}$)

- ✓ In each section, pressure is converted into displacement of the basilar membrane and transmitted to the following section. This leads to two transfer functions, one relating the output pressure, $V_o(n)$, and the input pressure, $V_i(n)$, and the other relating the output displacement, $V_m(n)$ to the input pressure.
- ✓ Each section of the basilar membrane can be realised as a digital filter as follows:



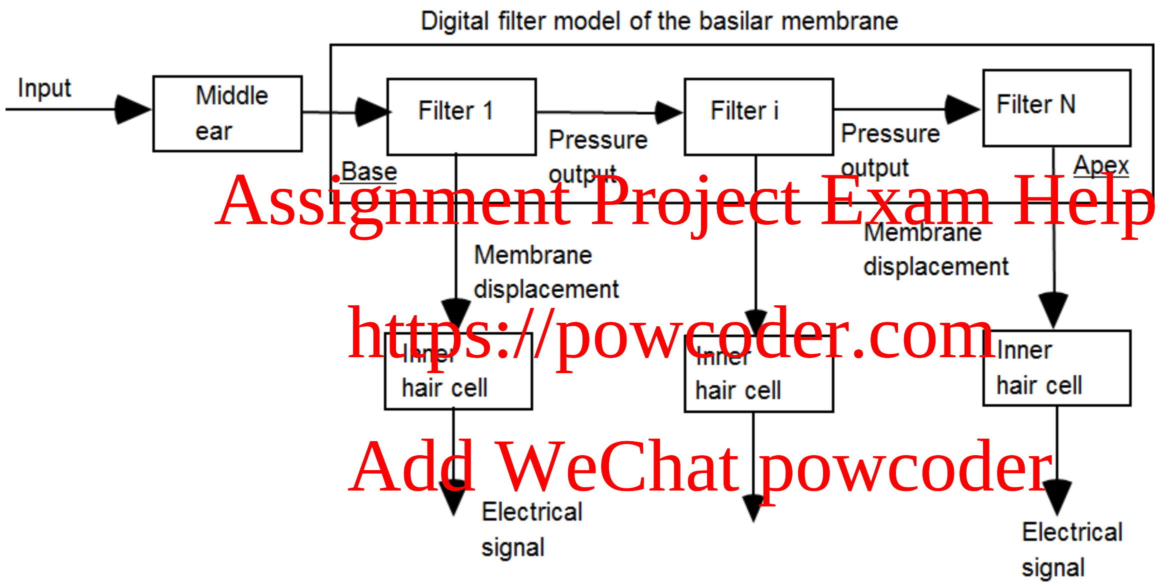
$$\frac{v_o(z)}{v_i(z)} = K \frac{1 - a_0}{1 - a_0 z^{-1}} \frac{1 - b_1 + b_2}{1 - b_1 z^{-1} + b_2 z^{-2}} \frac{1 - a_1 z^{-1} + a_2 z^{-2}}{1 - a_1 + a_2}$$

$$\frac{v_m(z)}{v_i(z)} = K \frac{1 - a_0}{1 - a_0 z^{-1}} \frac{(1 - b_1 + b_2) z^{-1}}{1 - b_1 z^{-1} + b_2 z^{-2}}$$

One can see that the displacement transfer function is contained in the pressure transfer function and hence a simple cascade arrangement is sufficient to represent the cochlear model

Transmission Line Model of the Cochlea

- ✓ The basic model of the cochlea is a transmission line model in which the basilar membrane is modelled as a cascade of 128 low pass filters, notch filters and resonators as shown below. Assume a sampling frequency of 48 kHz.

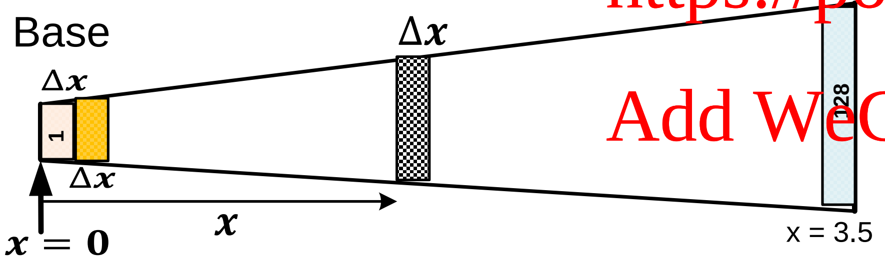


Transmission Line Model

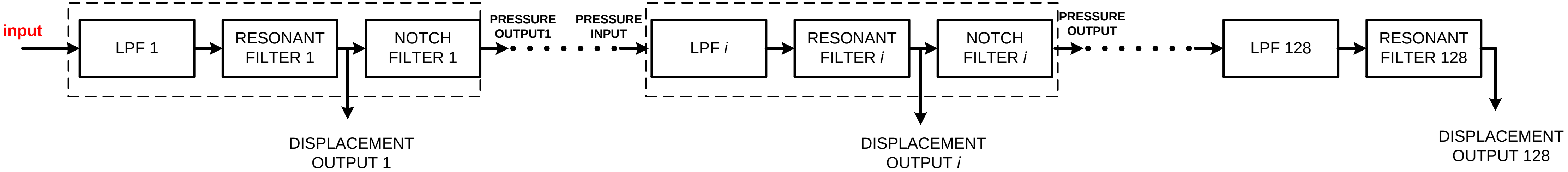
- ✓ Each digital filter section in the model above represents a section of the basilar membrane (tuned to a specific frequency) with 128 sections representing the entire basilar membrane.
- ✓ A stimuls representing pressure at the ear drum (after the outer ear model) is the input to the model shown in the figure above. This stimulus then moves along the transmission line as a travelling wave corresponding to the pressure in the cochlear fluid.

Selection of Frequency Scale

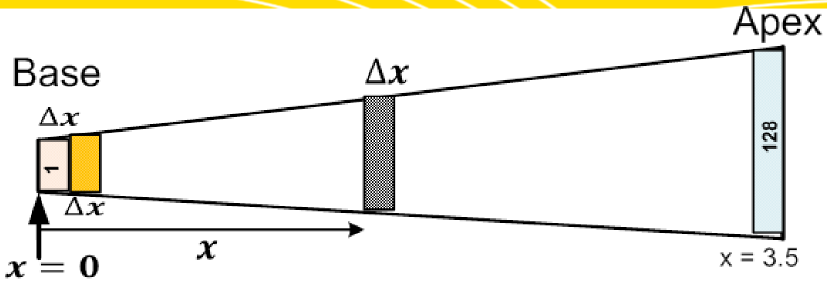
- ✓ The model considers the BM length to be 3.5 cm.
- ✓ If the membrane is simulated using 128 digital filters connected in cascade then the section length (Δx) and the frequency ratio between adjacent sections are constant throughout. That is Δx = 3.5/128 = 0.0275 cm;
- ✓ $$\frac{[f_p]_i}{[f_p]_{i+1}} = \frac{(20000)\,10^{-0.667x}}{(20000)\,10^{-0.667(x+\Delta x)}} = 10^{0.667\Delta x} = 1.0429$$



Filter number	Distance (cm)	Frequency(fp)
1	0	20 kHz
66	1.777	1304.79 Hz
128	3.4727	96.553 Hz



Implementation – Step 1



- ✓ Number of filters N = 128; Length of the BM = 3.5 cm. $\Delta x = \frac{3.5}{128} = 0.0273\ cm$
- ✓ $x = 0, \Delta x, 2\Delta x, 3\Delta x, \ldots \ldots \ldots, 127\Delta x$; $\boldsymbol{Resonant\ frequency}\ \boldsymbol{f_p(n) = (20000)\ 10^{-0.667 n\Delta x}}$

Filter No ($\boldsymbol{n}$)	Distance (x) cm	$\boldsymbol{f_p(n)}$: Resonant Frequency (Hz)	$\boldsymbol{\frac{f_p(n)}{f_p(n+1)}}$	$\boldsymbol{f_z(n)}$: Resonant zero (Hz) (Notch filer)
1	0	$f_p(1)$ = 20000	$\frac{f_p(1)}{f_p(2)}$ = 1.0429	$f_z(1)$ = 1.0429× $f_p(1)$ = 20858
2	Δx	$f_p(2)$ = 19177	$\frac{f_p(2)}{f_p(3)}$ = 1.0429	$f_z(2)$ = 1.0429× $f_p(2)$ = 20000
3	$2\Delta x$	$f_p(3)$ = 18389	$\frac{f_p(3)}{f_p(4)}$ = 1.0429	$f_z(3)$ = 1.0429× $f_p(3)$ = 19178
4	$3\Delta x$	$f_p(4)$ = 17633	.	.
.		.	.	.
128	$127\Delta x$	$f_p(128)$ = 96.55	-	$f_z(128)$ = 1.0429× $f_p(128)$=100.70

Implementation – Step 2

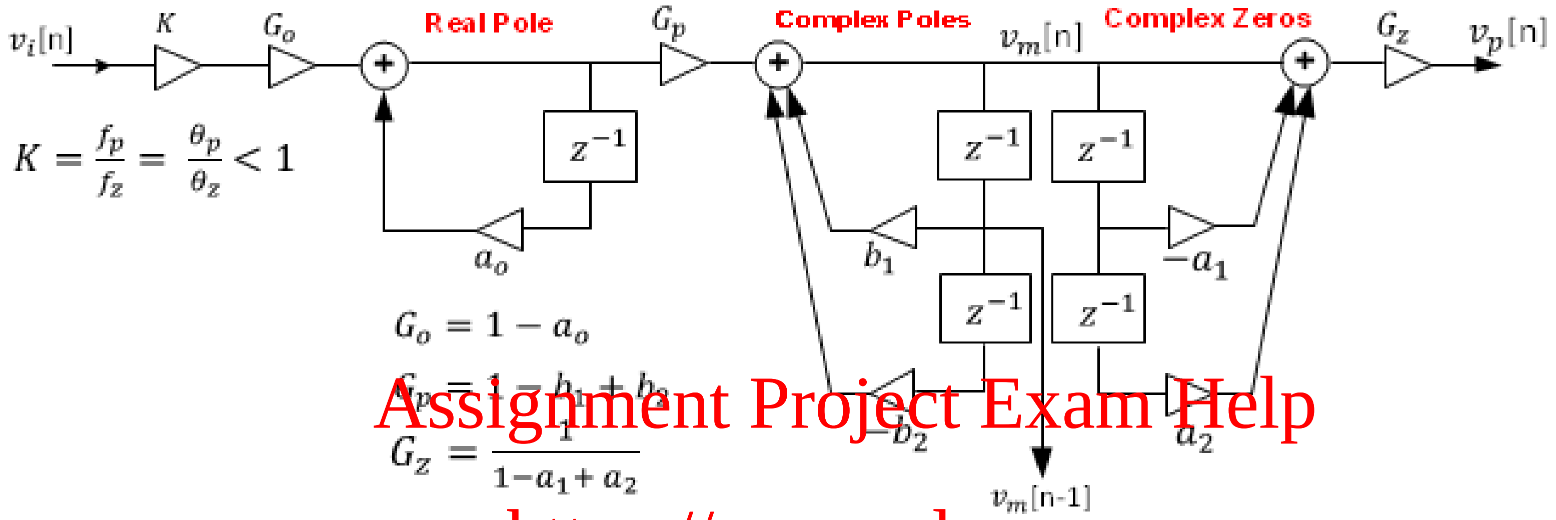
- ✓ Calculate the quality factor values, Q_p and Q_z; and bandwidths, BW_p and BW_z.
- ✓ Q_p varies linearly from 10 (first filter) to 5.5 (128th filter)
- ✓ Q_z varies linearly from 22 (first filter) to 12 (128th filter)
- ✓ You can change these around and observe what happens but ensure $Q_z > Q_p$.
- ✓ $BW_p(n) = \frac{f_p(n)}{Q_p(n)}$ and $BW_z(n) = \frac{f_z(n)}{Q_z(n)}$

Filter No (n):	$f_p(n)$ **in Hz**	$Q_p(n)$	$BW_p(n)$ **in Hz**	$f_z(n)$ **in Hz**	$Q_z(n)$	$BW_z(n)$ **in Hz**
1	20000	10	2000	20858	22	948.1
2	19177	9.96	1925	20000	21.92	912.4
3	18389	9.93	1852	19178	21.84	878.0
.	.	.		.	.	
128	96.55	5.5	17.56	100.70	12	8.4

Design Criteria

- ✓ In order to simulate the basilar membrane accurately with the transmission line model, it is critical that the complex zero frequency is slightly higher than the resonant frequency of the preceding resonator (complex pole filter).
- ✓ Selection of frequency scale: The ratios of the resonant frequencies of two adjacent sections are always constant and equal to 1.0429. i.e., the resonant frequency of the i^{th} section is 1.0429 times the resonant frequency of the $i+1^{th}$ section.
- ✓ From experiments it is known that the Q values (quality factor) for the complex pole section, Q_p, go from 10 (1st filter) down to 5.5 (128th filter). You can interpolate linearly for the intermediate filters.
- ✓ The Q values (quality factor) for the complex zeros section, Q_z, go from 22 (1st filter) down to 12 (128th filter). You can interpolate linearly for intermediate filters.
- ✓ For each section, $Q_z > Q_p$
- ✓ K is chosen as the ratio of complex pole frequency to the complex zero frequency ($\mathrm{K} = \frac{f_p}{f_z}, K < 1$).
- ✓ The cut-off frequency of the low pass filter can be chosen as follows: $\boldsymbol{f_c} = 1.4 * \boldsymbol{f_z}$
- ✓ Note that this model gives the basilar membrane displacement without taking into account fluid coupling. In order to take fluid coupling into account you must apply the spatial differentiation (TLT Level1 slide 12).
- ✓ Use impulse invariant transformations (as outlined earlier) to design the digital filters.

Implementation – Step 3



$$K = \frac{f_p}{f_z} = \frac{\theta_p}{\theta_z} < 1$$

$$G_o = 1 - a_o$$

$$G_p = 1 - b_1 + b_2$$

$$G_z = \frac{1}{1 - a_1 + a_2}$$

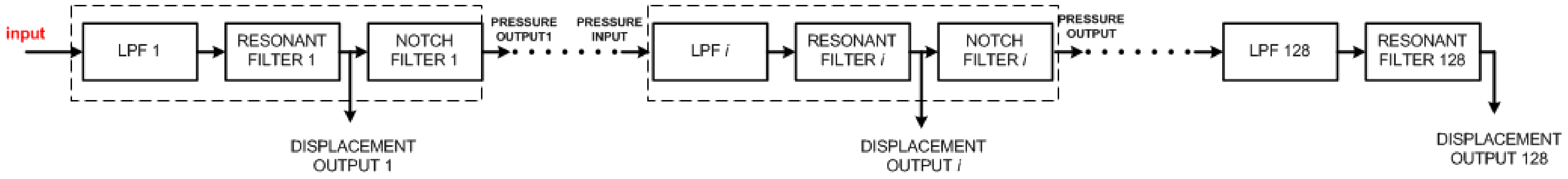
Filter Number	f_p (Hz)	f_z (Hz)	Q_p	Q_z	BW_p(Hz)	BW_z (Hz)
1	20000	20858	10	22	2000	948.1
2	19177	20000	9.96	21.92	1925	912.4
3	18389	19178	9.93	21.84	1852	878.0
.	.	.	.	.		
128	96.55	100.70	5.5	12	17.56	8.4

Filter Number	K	G_0	a_0	G_p	b_1	b_2	G_z	a_1	a_2
1	0.9589	0.8137	0.1863	3.2863	-1.5167	0.7697	0.2729	-1.7514	0.9130
2	0.9589	0.8199	0.1801	3.1975	-1.4202	0.7773	0.2798	-1.6574	0.9160
3	0.9589	0.8242	0.1758	3.0960	-1.3113	0.7847	0.2885	-1.5473	0.9189
.									
.									
128	0.9589	0.0183	0.9817	0.0002	1.9975	0.9977	5759	1.9985	0.9987

If your implementation is right, you should get these parameter values for the selected filters if you started with the same assumptions

Implementation – Step 4

- ✓ Digital filtering in the time domain for following inputs (sampled at 48kHz). You can use *filter*() in MATLAB to implement filtering.
  - ✓ Impulse
  - ✓ single sinusoid – Try initially with 1kHz, then others of your choice.
- ✓ Make sure you obtain all the 128 displacement outputs and 128 pressure outputs for each input signal. You can store these as 2 matrices with 128 columns and as many rows as there are samples in the input signal.
- ✓ From the impulse responses of each section, obtain the magnitude response (take FFT of the impulse response) and make sure it is what you expect.
- ✓ For the sinusoidal input, plot a row of the output displacement matrix to observe basilar membrane displacement at that time.
- ✓ Note that spatial differentiation is carried out across columns (within each row).



Spatial Differentiation

Spatial Differentiation

- ✓ Spatial differentiation of the membrane displacement represents coupling between the cilia of the inner hair cells, through the fluid in the subtectorial space.
- ✓ Spatial differentiation refers to taking the derivative with respect to the position (along the basilar membrane). A discrete model is given by:

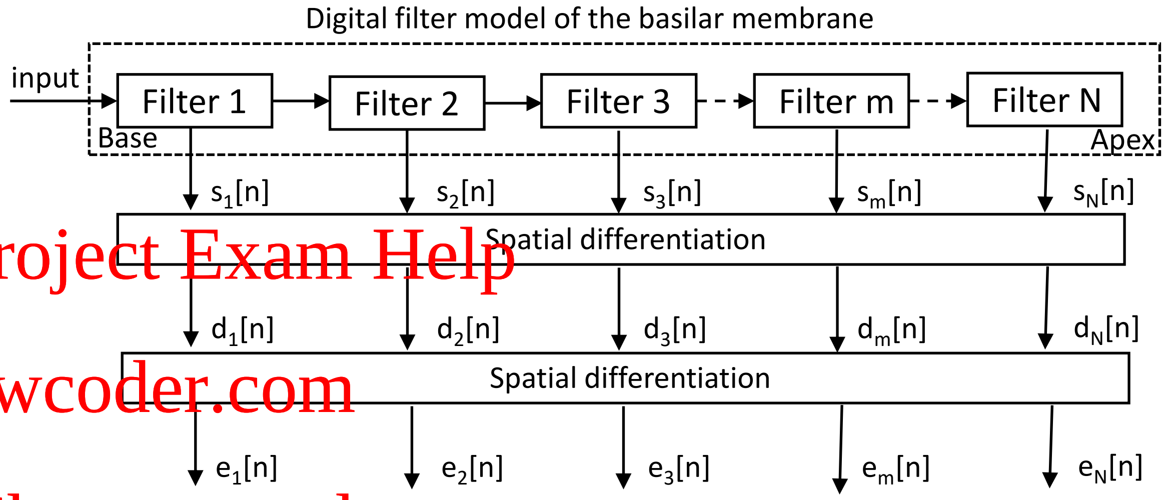
  d_m[n]=s_m[n]- s_{m+1}[n]

  {$e.g.$ d_1[n]=s_1[n]- s_2[n]}

- ✓ The second spatial differentiation is given by:

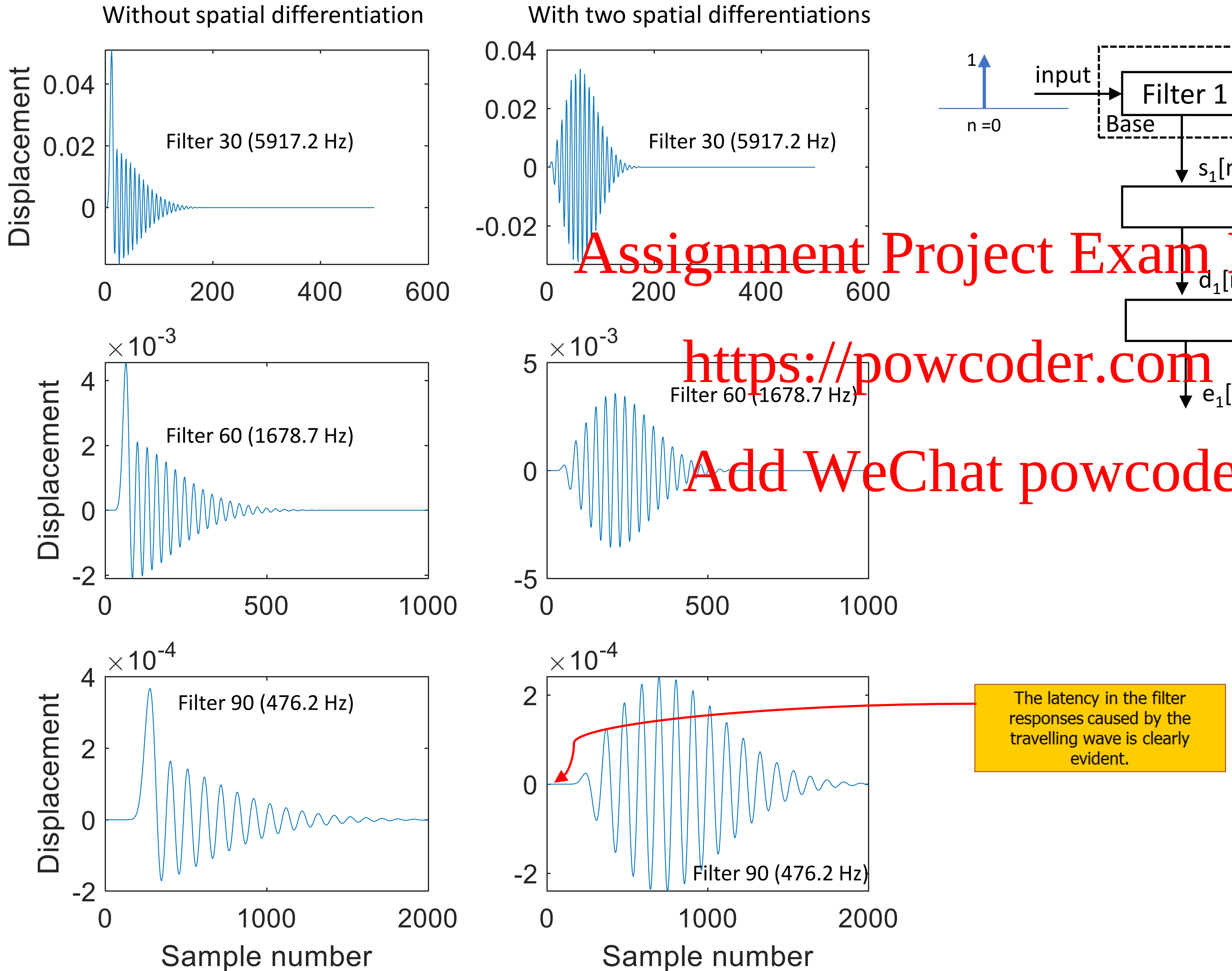
  e_m[n]=d_m[n]- d_{m+1}[n]

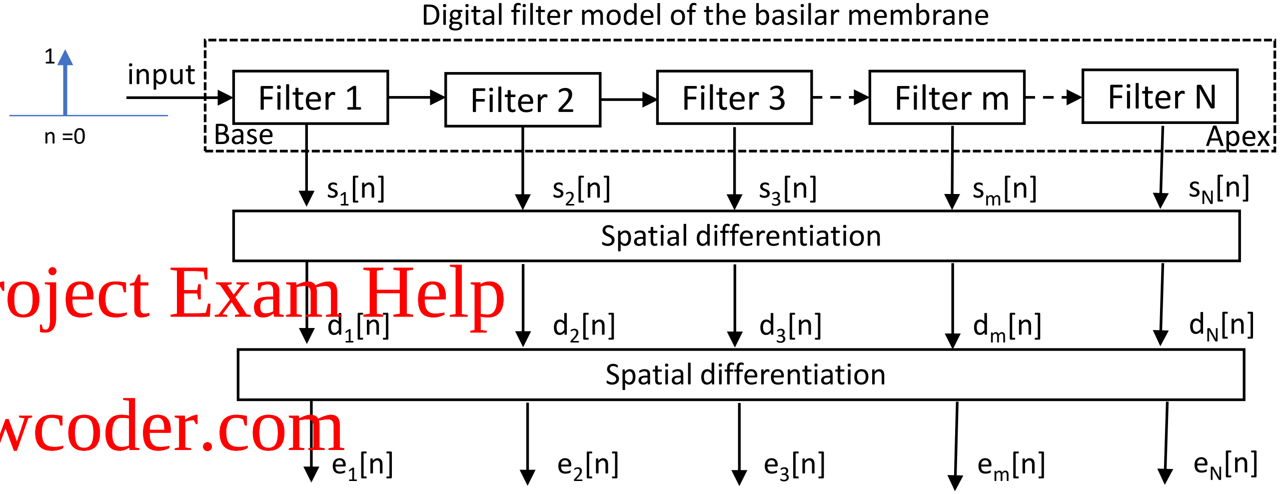
  {$e.g.$ e_1[n]=d_1[n]- d_2[n]}

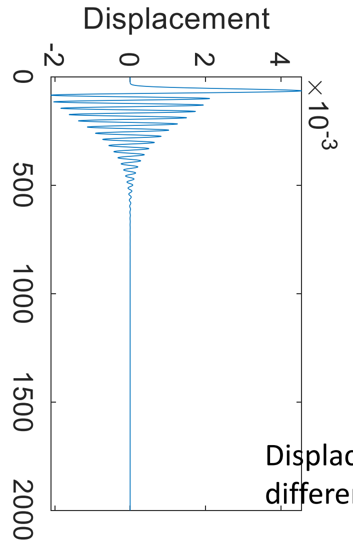


Are you on the right track?

✓ If your implementation is on the right track, you should observe the following impulse responses (roughly) at different sections of the membrane. Look at the membrane displacements when giving an impulse input.



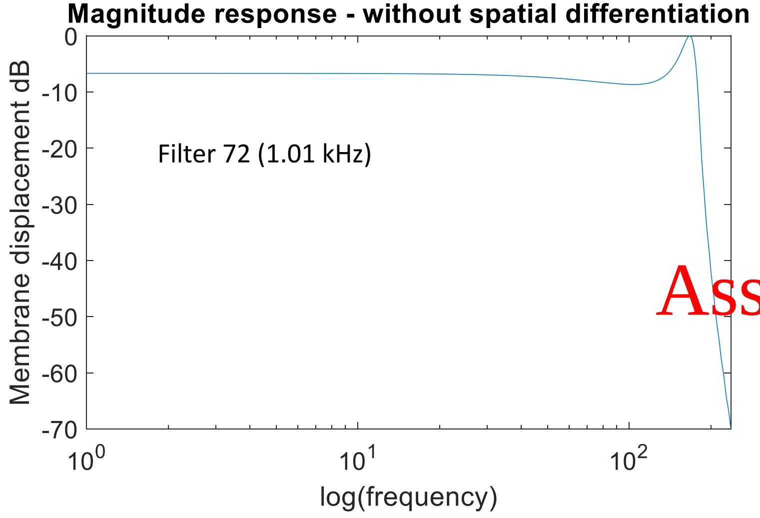


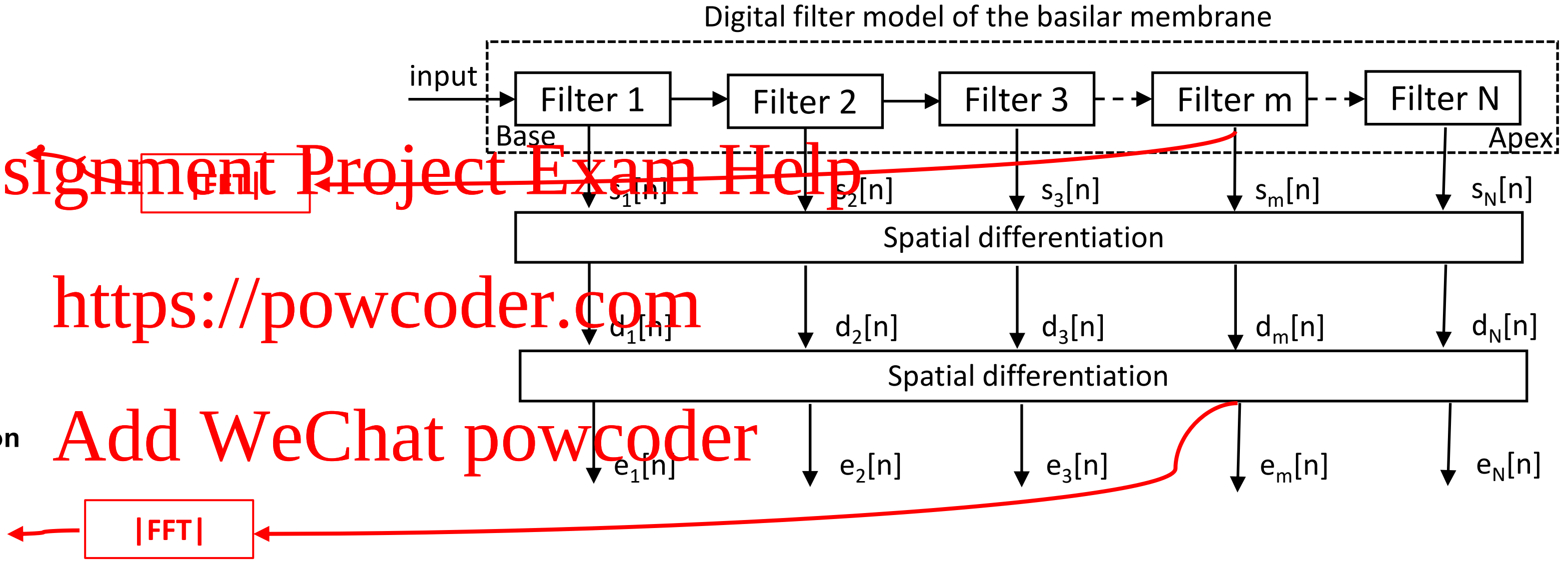


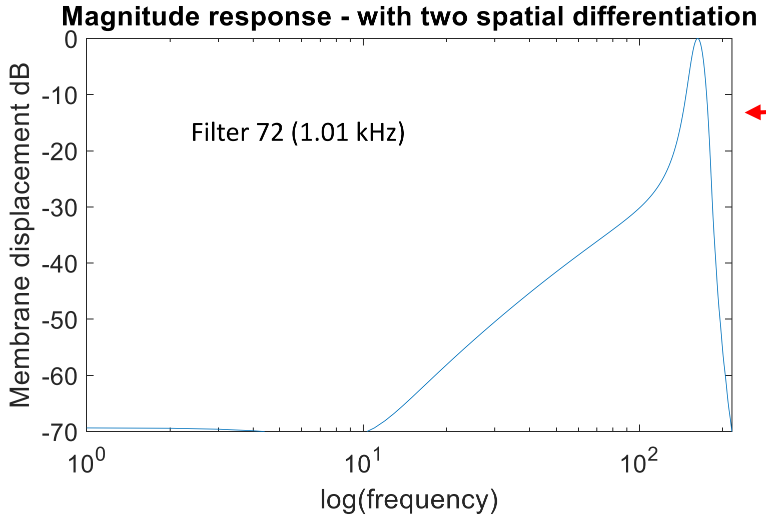
Displacement without spatial differentiation

Are you on the right track?

✓ If your implementation is on the right track, you should observe the following magnitude response (roughly) at the appropriate section of the basilar membrane with and without spatial differentiation.





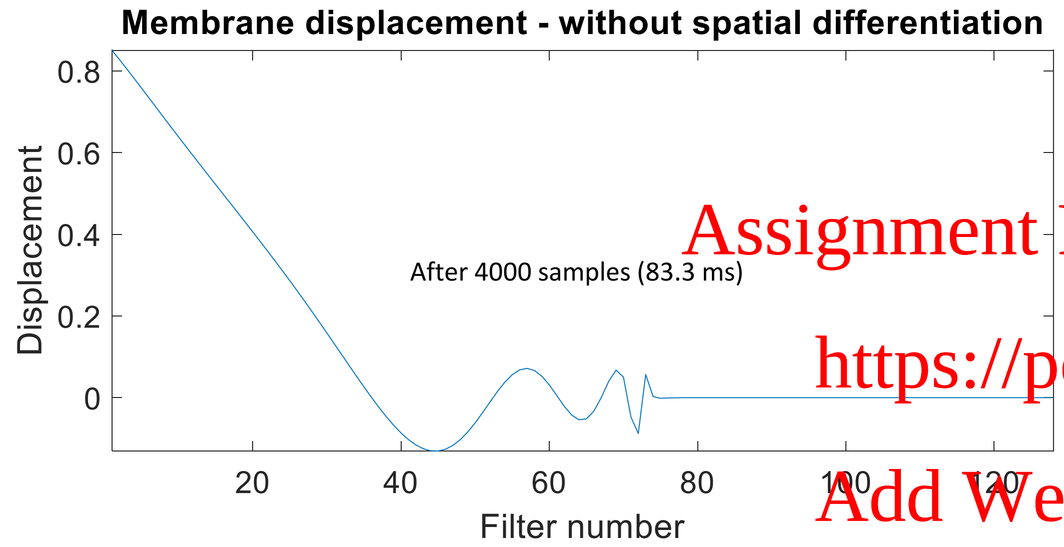


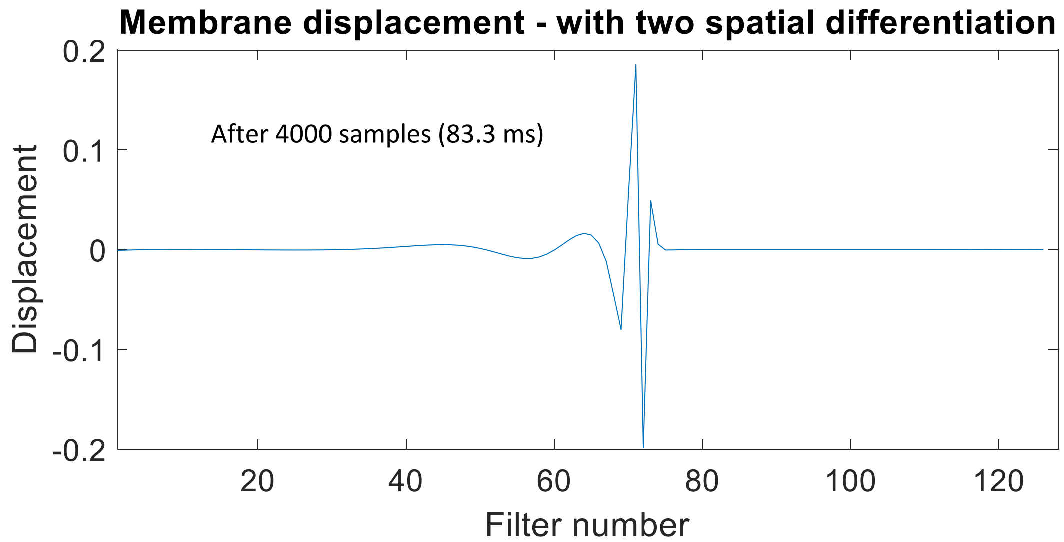
Frequency response of a particular filter can be obtained by taking the FFT of the impulse response of that filter.

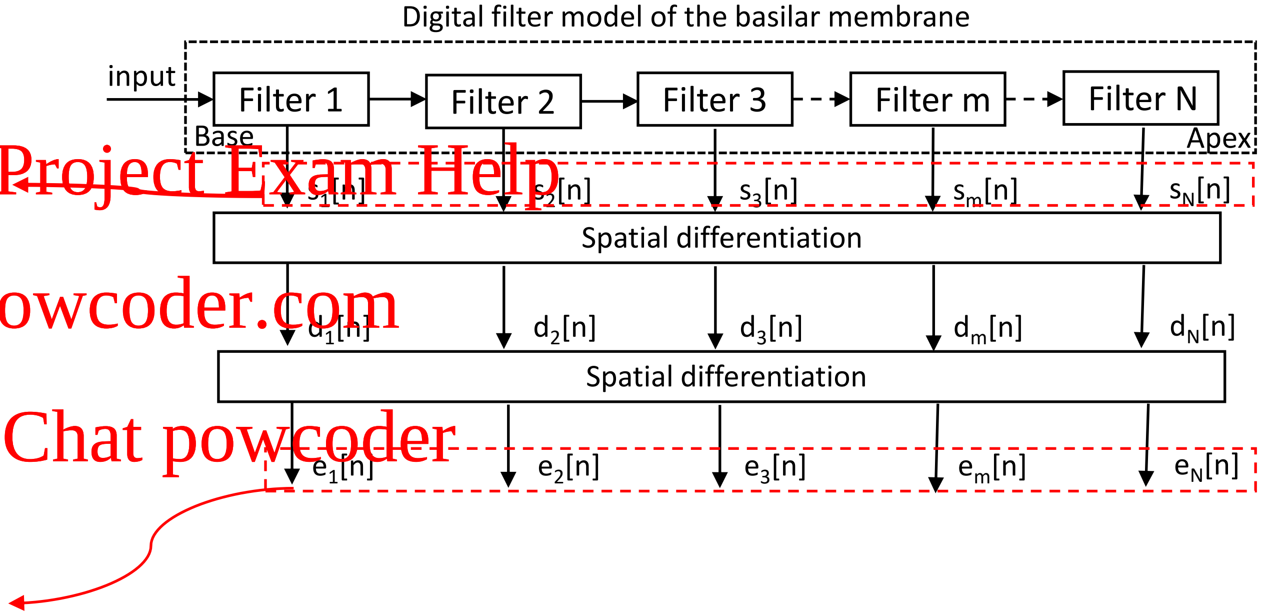
Frequency response of the section of the 128-filter auditory model corresponding to a frequency of 1 kHz.

Are you on the right track?

- ✓ If your implementation is on the right track, given a sinusoidal input, you should observe the following displacement (after two spatial differentiation) plotted against section number. As expected the membrane should exhibit activities up to the appropriate resonant section (where it shows maximum displacement) and no appreciable activity thereafter.



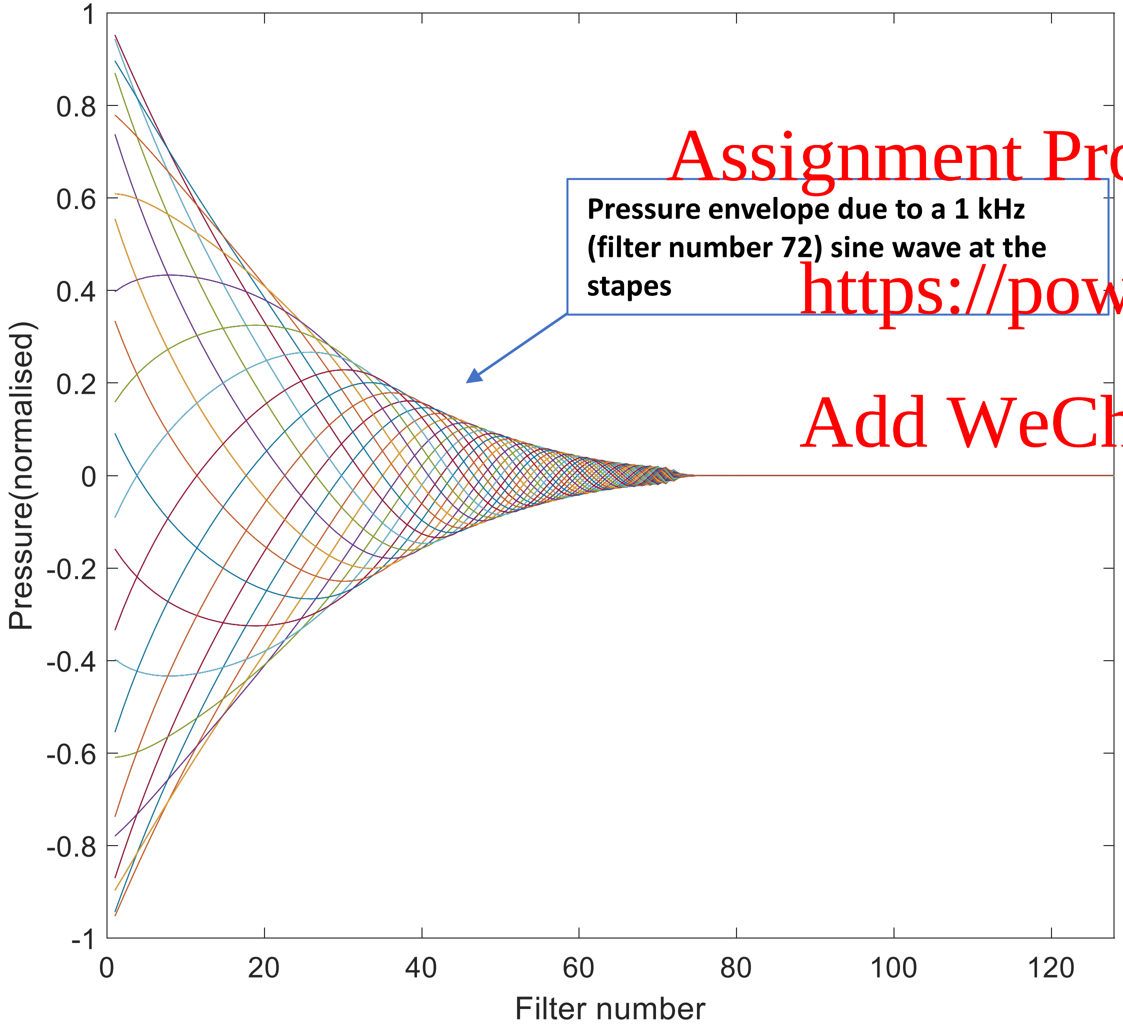




✓If your implementation of spatial differentiation is right, you should observe the following (roughly) for a 1kHz sinusoidal input.

Are you on the right track?

✓ If you plot the pressure outputs of different sections at a series of regular time steps (superimpose them on the same plot), you should observe something similar to the following. Observing the envelope, the pressure is high at the basal end and decays down to zero at the resonant position.



TLT level 2 Final Validation:

1. Apply a signal which is a sum of three sinusoids (Try a sum of a low frequency, a mid frequency and a high frequency sinusoid), 1000-2000 samples, of equal amplitude and frequencies of your choice, to the input of the cochlear model.

2. Plot membrane displacement (in one figure):
   i. Without spatial differentiation
   ii. With one spatial differentiation
   iii. With two spatial differentiation

3. Explain your results to your lab demonstrator.

Reflection

You should reflect on your project to see the following:

- ✓ What is the function of the basilar membrane and how does it respond to various input stimuli?
- ✓ What will happen if you include the outer and middle ear models at the input of the transmission line model of the cochlea in terms of hair cell output?
- ✓ What is the effect of spatial differentiation on the basilar membrane displacement?
- ✓ As a low frequency wave travels along the basilar membrane, the fluid pressure decreases and becomes zero. Can you explain this in terms of the travelling wave?
- ✓ When the input has multiple frequencies and one of the tones is removed after a period of time, how and when would the hair cell response change?
- ✓ What is the effect of the Q factors of your filters on the overall model? And what is the function of the complex zeros in your model?