

Lecture 5

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Impulse and Step Responses

DC motor example

Impulse response

⇔ Transfer function

If we apply a unit impulse $\delta(t)$ as input, output is called the *impulse response*.

Laplace transform of $\delta(t)$ is 1. So,

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→ Transfer function G = Laplace transform of impulse response g !

Step responses

- Impulses are infinitely large for an infinitely short time. Not often encountered in control.
- In control, for time-domain analysis often of more interest to study the *step response*, i.e. the output when the input is a unit step <https://powcoder.com>
- Reveals how system behaves when reference suddenly changes from one constant to another

$$u(t)=1, t > 0 \Leftrightarrow U(s) = 1/s$$

$$\rightarrow Y(s) = G(s)/s \Leftrightarrow y(t) = \int_0^t g(\tau) d\tau$$

- I.e., impulse response = derivative of step response

Example (DC motor)

- Determine the impulse response of the motor for the input being the supply voltage and the output being the angle of the shaft.

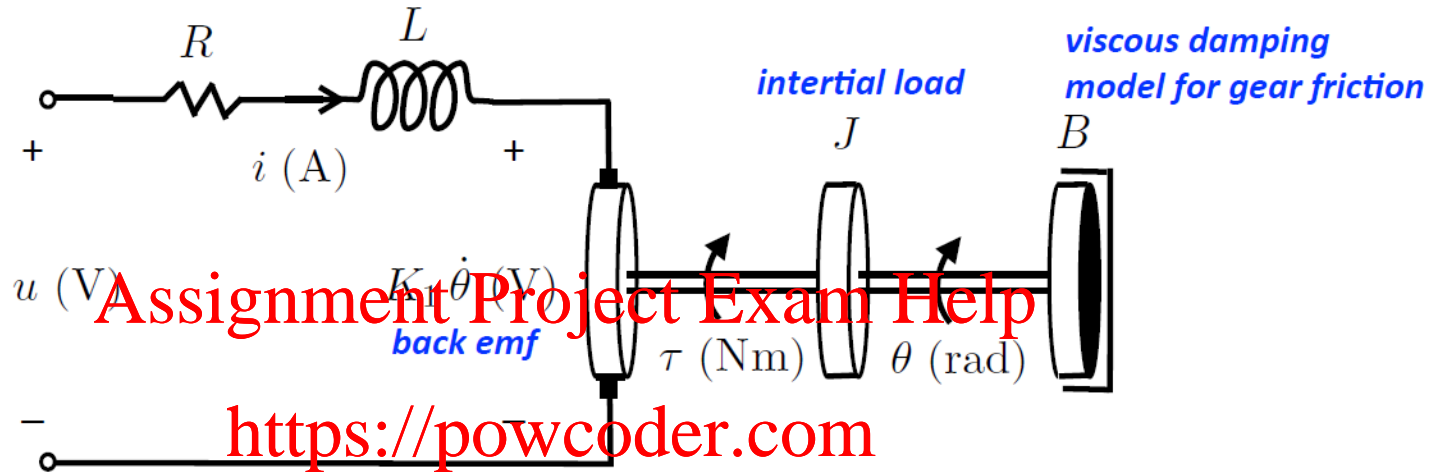
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- Determine the step response for the case when the output is the angular velocity of the shaft and the input is the supply voltage.

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Example (DC motor)



linearised motor equation: $\tau(t) = K_2 i(t) \xleftrightarrow{\mathcal{L}} T(s) = K_2 I(s)$

Newton's second law: $J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) \xleftrightarrow{\mathcal{L}} J(s^2\Theta(s) - s\theta(0) - \dot{\theta}(0)) = T(s) - B(s\Theta(s) - \theta(0))$

Kirchoff's voltage law: $u(t) = Ri(t) + L\frac{di}{dt}(t) + K_1\dot{\theta}(t)$

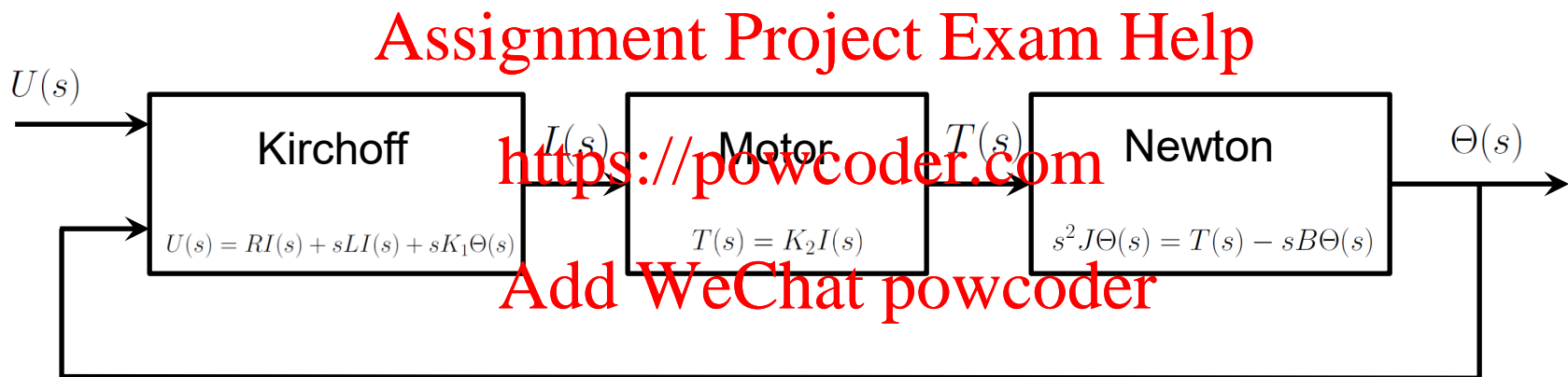
$\xleftrightarrow{\mathcal{L}} U(s) = RI(s) + L(sI(s) - i(0)) + K_1(s\Theta(s) - \theta(0))$

Example

- Assume zero initial conditions:

	Time	Laplace
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Motor	$\tau(t) = K_2 i(t)$	$T(s) = K_2 I(s)$
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Newton	$J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t)$	$s^2 J\Theta(s) = T(s) - sB\Theta(s)$
Kirchoff	$u(t) = Ri(t) + L\frac{di}{dt}(t) + K_1\dot{\theta}(t)$	$U(s) = RI(s) + sLI(s) + sK_1\Theta(s)$

Graphical interpretation



Exercise: redraw this diagram so that it represents the block diagram where each block is given by its transfer function.

Example

- Direct calculations yield

$$(s^2 J + sB)\Theta(s) = T(s) = K_2 I(s)$$

$$I(s)(R + sL) = U(s) - sK_1\Theta(s)$$

$$(sJ^2 + sB)\Theta(s) = K_2 \frac{U(s) - sK_1\Theta(s)}{R + sL}$$

$$(s^2 J + sB)(R + sL)\Theta(s) + sK_1 K_2 \Theta(s) = K_2 U(s)$$

$$\frac{\Theta(s)}{U(s)} = \frac{K_2}{(s^2 J + sB)(R + sL) + sK_1 K_2}$$

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s} \cdot \frac{1}{(T_1 s + 1)(T_2 s + 1)}$$

T_1 and T_2
are real valued for
 $(RJ + BL)^2 >$
 $4(K_1 K_2 + RB)LJ$

Example

- We want to rewrite the transfer function as

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s} \cdot \frac{1}{(T_1s + 1)(T_2s + 1)} = \frac{A_1}{s} + \frac{A_2}{T_1s + 1} + \frac{A_3}{T_2s + 1}$$

- Let us calculate A_1 . We multiply by s :

$$K \cdot \frac{1}{(T_1s + 1)(T_2s + 1)} = \frac{A_1}{1} + s \frac{A_2}{T_1s + 1} + s \frac{A_3}{T_2s + 1}$$

and obtain A_1 by setting $s = 0$, which yields

$$K \frac{1}{1 \cdot 1} = A_1 \Rightarrow A_1 = K$$

Compute A_2 and A_3 in the same way.

Example (summary)

- Transfer function from u (V) to θ (rad)

$$\frac{\Theta(s)}{U(s)} = \frac{K_2}{s((sL + R)(sJ + B) + K_1 K_2)} = \frac{K}{s(T_1 s + 1)(T_2 s + 1)}$$

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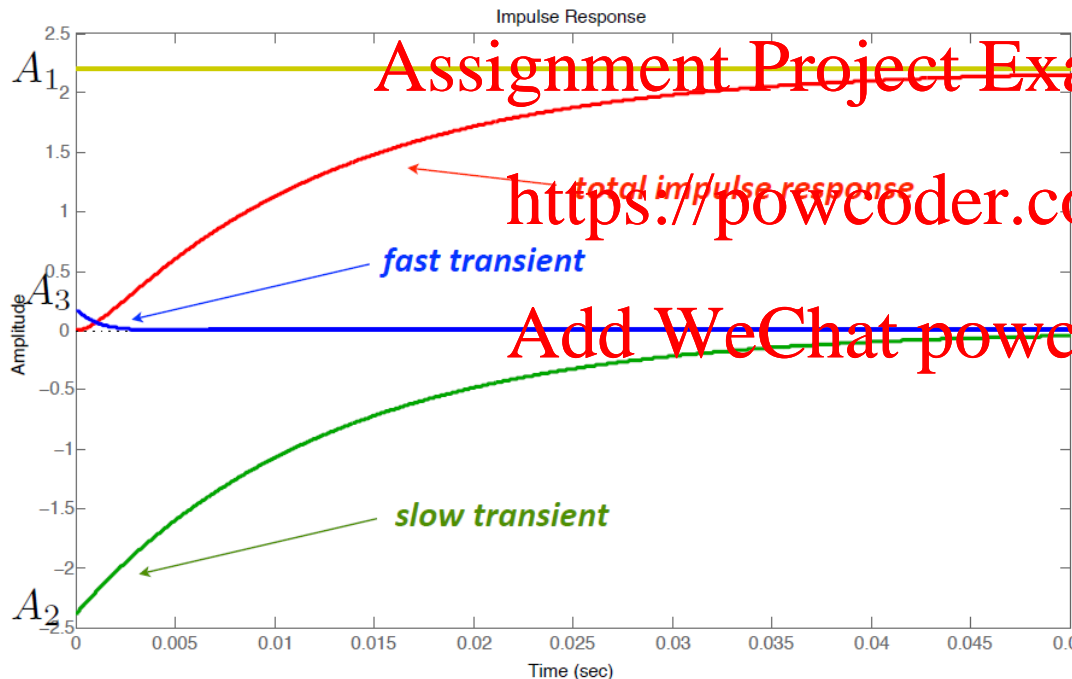
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$$\begin{aligned} \frac{\Theta(s)}{U(s)} &= \frac{K}{s} \cdot \frac{1}{(T_1 s + 1)(T_2 s + 1)} = \frac{A_1}{s} + \frac{A_2}{T_1 s + 1} + \frac{A_3}{T_2 s + 1} \\ &\Leftrightarrow (A_1 + \frac{A_2}{T_1} e^{-\frac{t}{T_1}} + \frac{A_3}{T_2} e^{-\frac{t}{T_2}}) \zeta(t) \end{aligned}$$

$$A_1 = K, A_2 = \frac{K}{-1/T_1} \frac{1}{(-T_2/T_1 + 1)}, A_3 = \frac{K}{-1/T_2} \frac{1}{(-T_1/T_2 + 1)}$$

Example (impulse response)

Suppose: $K \approx 2.2$; $T_1 \approx 12.5$ (ms); $T_2 = 1$ (ms)



```
K = 2.2;  
T1 = 12.5e-3;  
T2 = 1e-3;  
T = T1*4;  
  
A1 = K;  
A2 = (-K*T1)*1/(-T2/T1 + 1);  
A3 = (-K*T2)*1/(-T1/T2 + 1);  
  
sys1 = A1*tf(1,[1 0]);  
sys2 = A2*tf(1,[T1,1]);  
sys3 = A3*tf(1,[T2,1]);  
  
sys = sys1 + sys2 + sys3;  
  
impulse(sys,'r',sys1,'y',sys2,'g', ...  
        sys3,'b', [0:0.0001:T])
```

Example (angular velocity step response)

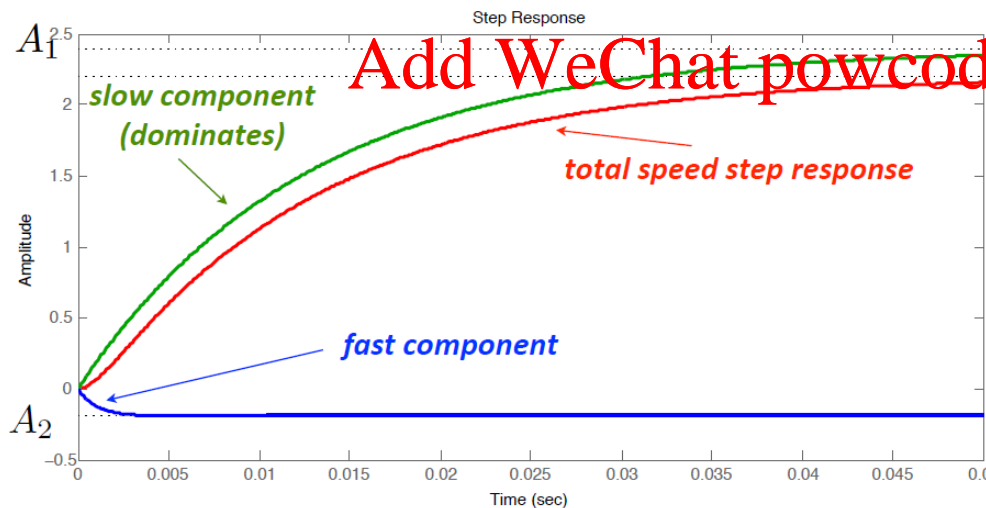
$$\Omega(s) = s\Theta(s) - \theta(0) \xleftrightarrow{\mathcal{L}} \omega(t) = \dot{\theta}(t) \quad \frac{\Omega(s)}{U(s)} = \frac{K}{(T_1s + 1)(T_2s + 1)}$$

For a unit step input: $\Omega(s) = \frac{K}{(T_1s + 1)(T_2s + 1)} \cdot \frac{1}{s}$ unit step input

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where $A_1 = \frac{K}{(-T_1/T_2 + 1)}$ and $A_2 = \frac{K}{(-T_2/T_1 + 1)}$

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```
K = 2.2;
T1 = 12.5e-3;
T2 = 1e-3;
T = T1*4;

A1 = K/(-T2/T1 + 1);
A2 = K/(-T1/T2 + 1);

sys1 = A1*tf(1,[T1,1]);
sys2 = A2*tf(1,[T2,1]);

sys = sys1 + sys2;

step(sys,'r',sys1,'g', ...
      sys2,'b',[0:0.0001:T])
```