

---

Assignment Project Exam Help  
Lecture 4  
<https://powcoder.com>

~~Add WeChat powcoder~~

---

**Revision of Laplace Transforms**  
**Algebra of Block Diagrams**



# Outline

- Motivation
- Complex numbers
- Definition of Laplace Transform
- Properties of Laplace Transform
- Partial fraction expansion
- Conclusions

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



# Why we love Laplace transforms

- Time derivatives, integrals and convolutions become algebraic operations in s-domain → Much, much simpler to analyse and design systems – and write them down concisely.
- Different functions in time-domain yield different functions in s-domain → No loss of information going from one domain to the other
- Leads to crucial concepts of transfer functions, poles, zeros etc



# Complex numbers

- Complex numbers consist of real and imaginary parts:

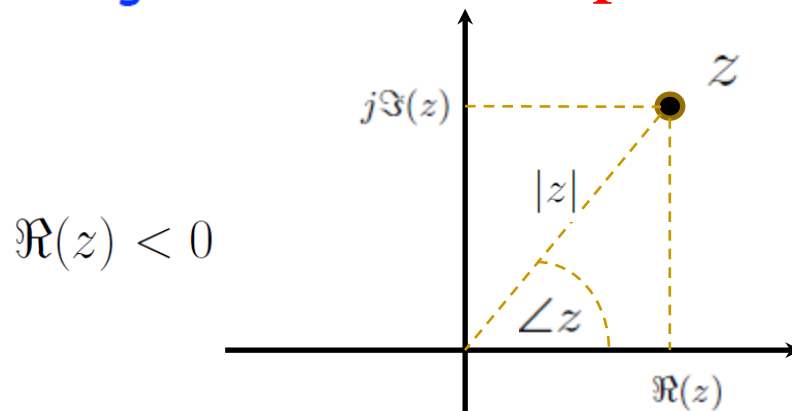
Assignment Project Exam Help

$$z = \Re(z) + j\Im(z)$$

$$= \sqrt{\Re(z)^2 + \Im(z)^2} e^{j \arctan(\Im(z)/\Re(z))}$$

$$= |z| e^{j\angle z} \quad \text{phase (in radians!)}$$

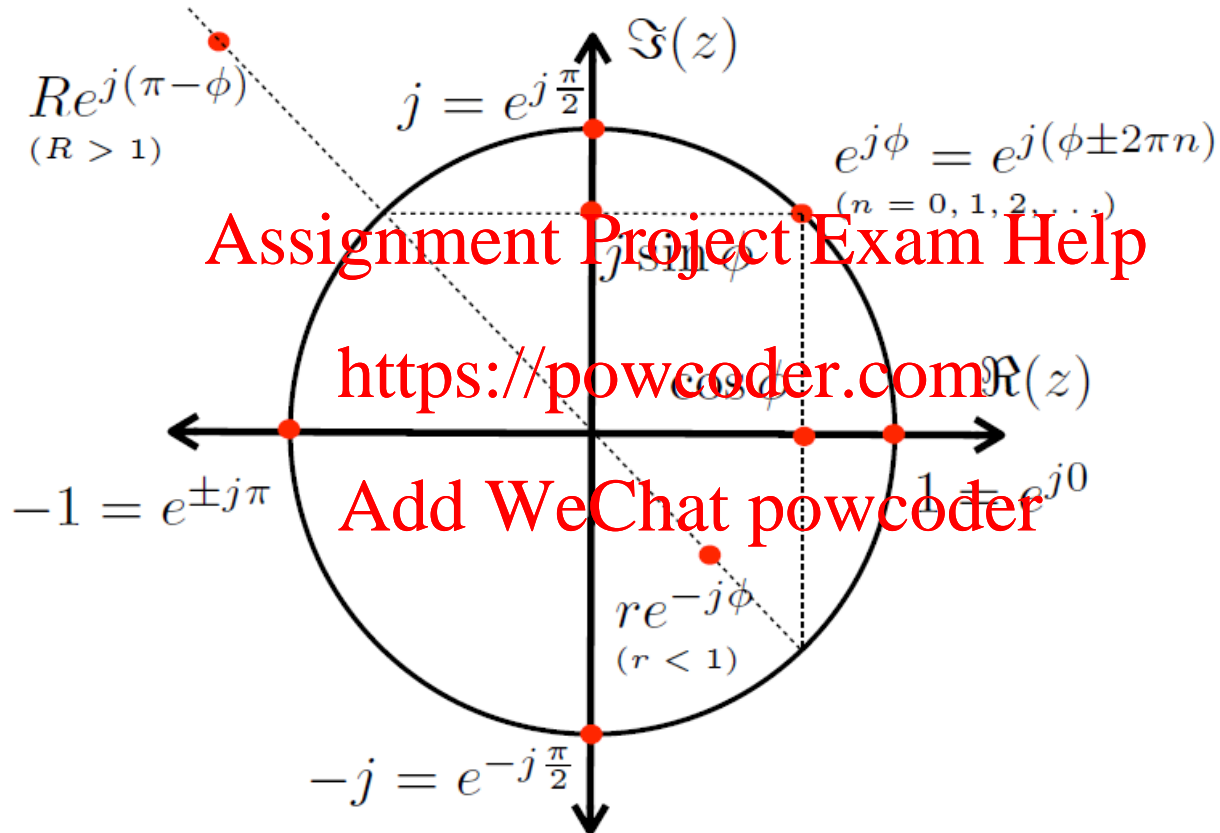
*magnitude* Add WeChat powcoder



$$j := \sqrt{-1}$$



# Complex numbers in complex plane





# Operations with complex numbers

- Addition of complex numbers:

$$z_1 + z_2 = (\Re(z_1) + \Re(z_2)) + j(\Im(z_1) + \Im(z_2))$$

Assignment Project Exam Help

- Multiplication of complex numbers:

<https://powcoder.com>

$$z_1 \cdot z_2 = |z_1| |z_2| e^{j(\angle z_1 + \angle z_2)}$$

Add WeChat powcoder

- Exponential of a complex number:

$$e^z = e^{\Re(z)} e^{j\Im(z)} = e^{\Re(z)} (\cos \Im(z) + j \sin \Im(z))$$



---

Assignment Project Exam Help

Laplace transform  
<https://powcoder.com>

Add WeChat powcoder

---



# Motivation (possible design steps)

## Time domain:

Step 1: Modelling

Step 2: Design specifications

## Complex (Laplace) domain.

Step 4: analysis/design

$y(t)$

Assignment Project Exam Help

Step 3:  $\mathcal{L}[y]$

$Y(s)$

$$\sum_{k=0}^n a_k \frac{d^k y}{dt^k} = \sum_{k=0}^n b_k \frac{d^k u}{dt^k}$$

<https://powcoder.com>

$$Y(s) = G(s)U(s)$$

Overshoot, Undershoot,  
Rise time, Setting time,  
Steady-state error

Add WeChat powcoder

Locations of poles  
and zeroes of transfer function  
in the complex plane

Stability

Step 5:  $\mathcal{L}^{-1}[Y]$

Locations of poles in  
the complex plane

Step 6: Verification, interpretation



# Laplace transform

- Laplace transform for signal  $y(t), 0 \leq t < \infty$  is

$$\mathcal{L}[y(t)] = Y(s) = \int_{0-}^{\infty} e^{-st} y(t) dt$$

- Inverse Laplace transform of  $Y(s)$  is

$$\mathcal{L}^{-1}[Y(s)] = y(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} e^{st} Y(s) ds$$

- Region of convergence is  $\Re\{s\} \geq \sigma$  for which

$$|y(t)| < ke^{\sigma t}; \forall t \geq 0 \quad \text{where } \sigma \in \mathbb{R}, k < \infty$$



$f(t) \quad (t \geq 0)$	$\mathcal{L}[f(t)]$	Region of Convergence
1	$\frac{1}{s}$	$\sigma > 0$
$\delta_D(t)$	1	$ \sigma  < \infty$
$t$	$\frac{1}{s^2}$	$\sigma > 0$
$t^n \quad n \in \mathbb{Z}^+$	$\frac{n!}{s^{n+1}}$	$\sigma > 0$
$e^{\alpha t} \quad \alpha \in \mathbb{C}$	$\frac{1}{s - \alpha}$	$\sigma > \Re\{\alpha\}$
$te^{\alpha t} \quad \alpha \in \mathbb{C}$	$\frac{1}{(s - \alpha)^2}$	$\sigma > \Re\{\alpha\}$
$\cos(\omega_o t)$	$\frac{s}{s^2 + \omega_o^2}$	$\sigma > 0$
$\sin(\omega_o t)$	$\frac{\omega_o}{s^2 + \omega_o^2}$	$\sigma > 0$

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



$f(t)$	$\mathcal{L}[f(t)]$	Names
$\sum_{i=1}^l a_i f_i(t)$	$\sum_{i=1}^l a_i F_i(s)$	Linear combination
$\frac{dy(t)}{dt}$	$sY(s) - y(0^-)$	Derivative Law
$\frac{d^k y(t)}{dt^k}$	$s^k Y(s) - \sum_{i=1}^k s^{k-i} \left. \frac{d^{i-1} y(t)}{dt^{i-1}} \right _{t=0^-}$	High order derivative
$\int_{0^-}^t y(\tau) d\tau$	$\frac{1}{s} Y(s)$	Integral Law
$y(t - \tau) \mu(t - \tau)$	$e^{-s\tau} Y(s)$	Delay
$ty(t)$	$-\frac{dY(s)}{ds}$	
$t^k y(t)$	$(-1)^k \frac{d^k Y(s)}{ds^k}$	
$\int_{0^-}^t f_1(\tau) f_2(t - \tau) d\tau$	$F_1(s) F_2(s)$	Convolution
$\lim_{t \rightarrow \infty} y(t)$	$\lim_{s \rightarrow 0} sY(s)$	Final Value Theorem
$\lim_{t \rightarrow 0^+} y(t)$	$\lim_{s \rightarrow \infty} sY(s)$	Initial Value Theorem

Taken from “Control Systems Design”, Goodwin, Graebe & Salgado



# Comments:

- It is useful to understand at least a few proofs of items in the tables of Laplace transforms.  
**Assignment Project Exam Help**
- You need to understand all proofs that are given in slides at the end of this lecture.  
**<https://powcoder.com>**  
**Add WeChat powcoder**
- It is essential to know how to use Laplace tables/rules in solving problems.



# Example:

- Determine the unit step response of the following system, assuming the initial condition  $y(0)=10$ .

Assignment Project Exam Help

<https://powcoder.com>

- Step 1: take Laplace transform of both sides:

$$\underbrace{sY(s) - y(0)}_{\mathcal{L}\{\dot{y}\}} + \underbrace{Y(s)}_{\mathcal{L}\{y\}} = \underbrace{\frac{1}{s}}_{\mathcal{L}\{1\}}$$

(we used the linearity of LT and formulas for the derivative and for the unit step signal)



- Step 2: solve for  $Y(s)$  – this involves algebraic calculations!

$$Y(s) = \frac{y(0)}{s+1} + \frac{1}{s(s+1)}$$

- Step 3: rewrite  $Y(s)$  noting that  $y(0)=10$  and using “partial fraction expansion”:

$$\begin{aligned} Y(s) &= \frac{y(0)}{s+1} + \frac{1}{s(s+1)} \\ &= \frac{10}{s+1} + \frac{1}{s} - \frac{1}{s+1} \\ &= \frac{9}{s+1} + \frac{1}{s} \end{aligned}$$

- Step 4: take the inverse Laplace transform

$$y(t) = \mathcal{L}^{-1}\left\{\frac{9}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 9e^{-t} + 1$$



# Example continued

- Suppose that assuming the same input and initial condition, we now want to calculate

Assignment Project Exam Help

$\lim_{t \rightarrow \infty} y(t)$   
<https://powcoder.com>

- One way to do this is to go through all previous steps, obtain  $y(t)$  and then calculate:

$$\lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} 9e^{-t} + 1 = 1$$



# Example continued

- But this is a place where we can use “Final Value Theorem” and calculate the limit directly in complex domain:

<https://powcoder.com>  
Add WeChat powcoder

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) = \lim_{s \rightarrow 0} s \left[ \frac{9}{s+1} + \frac{1}{s} \right] = 1$$

- Note that you always need to first verify that the limit exists when using Final Value Theorem (e.g. stability).



# Homework

- Consider the system

Assignment Project Exam Help

- Assuming zero initial conditions, find:
  - Response to unit step signal
  - Response to unit impulse signal
  - Response to a sine signal with amplitude 2
  - Limits as time goes to infinity for all the above



# Example revisited

- Recall that in the example in we needed to find the inverse Laplace transform of:

$$\frac{1}{s(s+1)}$$

Assignment Project Exam Help

- This term can not be found in LT tables. We need to transform it to its “partial fraction form”:

<https://powcoder.com>  
Add WeChat powcoder

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

- Note that in new form, we can find all terms in LT tables. We show how to do this in general.



# Partial fraction expansion

(non-repeated poles)

- We can represent strictly proper transfer functions with different poles as follows:

Assignment Project Exam Help  
<https://powcoder.com>

$$G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{a_n s^n + \cdots + a_1 s + a_0} = K \frac{\prod_{k=1}^m (s - \beta_k)}{\prod_{k=1}^n (s - \alpha_k)} = \sum_{k=1}^n \frac{B_k}{s - \alpha_k}$$

where “residues” are computed as follows

Add WeChat powcoder

$$B_k = \lim_{s \rightarrow \alpha_k} (s - \alpha_k) G(s)$$

- Using LT tables, we have

$$g(t) = \sum_{k=1}^n B_k e^{\alpha_k t} \text{ for } t \geq 0$$



# Example:

- If we consider

$$G(s) := \frac{1}{s(s+1)} = \frac{B_1}{s} + \frac{B_2}{s+1}$$

Assignment Project Exam Help

its poles are  $s=0$  and  $s=-1$ . Using the formulas we have just given, we obtain

<https://powcoder.com>

Add WeChat powcoder

$$B_1 = \lim_{s \rightarrow 0} sG(s) = 1$$

$$B_2 = \lim_{s \rightarrow -1} (s+1)G(s) = \frac{1}{-1}$$

which confirms what we already shown earlier by direct computation.



# Complex conjugate poles

- Consider a typical term in FPE:

$$\frac{B_k}{s - \alpha_k}$$

Assignment Project Exam Help

- When the pole is complex, then the residue is too and they appear in conjugate pairs:

https://powcoder.com  
Add WeChat powcoder

$$\frac{B_k}{s - \alpha_k} + \frac{B_k^*}{s - \alpha_k^*}$$

$$\alpha_k = \sigma_k + j\omega_k$$

$$B_k = |B_k|e^{j\phi_k}$$

(if system impulse response is real-valued).



# Complex conjugate pole

- Direct calculations yield:

$$\begin{aligned} B_k e^{\alpha_k t} + B_k^* e^{\alpha_k^* t} &= |B_k| e^{j\Phi_k} e^{(\sigma_k + j\omega_k)t} + |B_k| e^{-j\Phi_k} e^{(\sigma_k - j\omega_k)t} \\ &= |B_k| e^{j\Phi_k} e^{\sigma_k t} e^{j\omega_k t} + |B_k| e^{-j\Phi_k} e^{\sigma_k t} e^{-j\omega_k t} \\ &= |B_k| e^{\sigma_k t} \left\{ e^{j\Phi_k + j\omega_k t} + e^{-(j\Phi_k + j\omega_k t)} \right\} \\ &\quad \text{let } \tilde{\Phi}_k := \Phi_k + \omega_k t \\ &= |B_k| e^{\sigma_k t} \left\{ \cos \tilde{\Phi}_k + j \sin \tilde{\Phi}_k + \cos \tilde{\Phi}_k - j \sin \tilde{\Phi}_k \right\} \\ &= 2|B_k| e^{\sigma_k t} \cos(\Phi_k + \omega_k t) \end{aligned}$$

- In the last line we have a real function!



# Partial fraction expansion

(repeated poles)

- Consider now a transfer function with repeated poles that can be written as follows:

Assignment Project Exam Help

$$G(s) = \frac{b(s)}{a(s)} = \frac{b_ms^m + \dots + b_0}{a_ns^n + \dots + a_0} = \frac{b(s)}{(s-\alpha_1)^{k_1} \dots (s-\alpha_l)^{k_l}}$$

Add WeChat powcoder

$$\alpha_i \neq \alpha_j, \quad i \neq j$$

$$m < n$$



- PFE can be written as follows:

$$\begin{aligned}
 G(s) = & \frac{B_{1,k_1}}{(s - \alpha_1)^{k_1}} + \frac{B_{1,k_1-1}}{(s - \alpha_1)^{k_1-1}} + \dots + \frac{B_{1,1}}{s - \alpha_1} \\
 & + \frac{B_{2,k_2}}{(s - \alpha_2)^{k_2}} + \frac{B_{2,k_2-1}}{(s - \alpha_2)^{k_2-1}} + \dots + \frac{B_{2,1}}{s - \alpha_2} \\
 & \vdots \\
 & + \frac{B_{l,k_l}}{(s - \alpha_l)^{k_l}} + \frac{B_{l,k_l-1}}{(s - \alpha_l)^{k_l-1}} + \dots + \frac{B_{l,1}}{s - \alpha_l}
 \end{aligned}$$

$$B_{i,k_i} = \lim_{s \rightarrow \alpha_i} (s - \alpha_i)^{k_i} G(s), i = 1, 2, \dots, l$$

$$B_{i,k_i-j} = \lim_{s \rightarrow \alpha_i} \frac{1}{j!} \frac{d^j}{ds^j} (s - \alpha_i)^{k_i} G(s), i = 1, 2, \dots, l, j = 1, \dots, k_i - 1$$



# Partial fraction expansion via Matlab

- We can write in Matlab:

```
num=[2 5 3 6]
```

```
den=[1 6 11 6]
```

```
[r,p,k]=residue(num,den)
```

```
r =
```

```
-6.0000
```

```
-4.0000
```

```
3.0000
```

```
p =
```

```
-3.0000
```

```
-2.0000
```

```
-1.0000
```

```
k =
```

```
2
```



Assignment Project Exam Help

$$G(s) = \frac{2s^3 + 5s^2 + 3s + 6}{s^3 + 6s^2 + 11s + 6}$$

<https://powcoder.com>

Add WeChat powcoder



$$G(s) = \frac{-6}{s+3} + \frac{-4}{s+2} + \frac{3}{s+1} + 2$$



# Example 1 (exponential):

Assignment Project Exam Help

Find  $Y_a(s) = \mathcal{L}[y_a](s)$ , where  $y_a(t) \doteq e^{-at}$  and  $a$  is a real constant :

$$Y_a(s) = \int_0^{\infty} e^{-at} e^{-st} dt = \left[ \frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty} = \frac{1}{(s+a)} \quad \text{for } \Re(s) > -a$$

<https://powcoder.com>

Add WeChat powcoder



## Example 2 (powers):

Find  $Y_n(s) = \mathcal{L}[y_n(t)]$  where  $y_n(t) = \frac{t^n}{n!} \int_0^t y_{n-1}(\tau) d\tau$  and  $n \geq 0$  is an integer :

$$Y_n(s) = \int_0^\infty \frac{t^n}{n!} e^{-st} dt = \left[ \frac{t^n e^{-st}}{-s} \right]_0^\infty + \int_0^\infty \frac{t^{(n-1)}}{(n-1)!} \frac{e^{-st}}{s} dt \quad \text{integration by parts}$$

$$= \frac{1}{s} Y_{n-1}(s) = \frac{1}{s^n} Y_0(s) = \frac{1}{s^{(n+1)}} \quad \text{for } \operatorname{Re}(s) > 0$$



# Property 1 (integration):

TRANSFORM POST INTEGRATION: With  $y_{\text{int}}(t) \doteq \int_0^t y(\tau) d\tau$ ,

Assignment Project Exam Help

$$\begin{aligned} Y_{\text{int}}(s) &= \int_0^\infty \left( \int_0^t y(\tau) d\tau \right) e^{-st} dt \\ &= \left[ \left( \int_0^t y(\tau) d\tau \right) \frac{e^{-st}}{s} \right]_0^\infty + \int_0^\infty y(t) \frac{e^{-st}}{s} dt \\ &= \frac{1}{s} Y(s) \quad (s \in \{z : \Re(z) > 0\} \cap \text{ROC}(Y)) \end{aligned}$$

<https://powcoder.com>

Add WeChat powcoder

*integration  
by parts*



# Property 2 (time shift):

Assignment Project Exam Help

TRANSFORM POST SHIFTING: With  $y_\tau(t) = \{y(t - \tau) \text{ if } t \geq \tau; 0 \text{ otherwise}\}$ ,

$$Y_\tau(s) = \int_{\tau}^{\infty} y(t - \tau) e^{-st} dt = \int_0^{\infty} y(\nu) e^{-s(\nu + \tau)} d\nu = e^{-s\tau} Y(s) \quad (s \in \text{ROC}(Y))$$

Add WeChat powcoder

<https://powcoder.com>



# Property 3 (exponential weighting):

TRANSFORM POST-EXPONENTIAL WEIGHTING. With  $y_z(t) = e^{zt}y(t)$ ,

$$Y_z(s) = \int_0^\infty y(t)e^{-(s-z)t} dt = Y(s-z) \quad (s \in \{\Re(z) + w : w \in \text{ROC}(Y)\})$$

Add WeChat powcoder



# Property 4 (differentiation):

TRANSFORM POST-DIFFERENTIATION:

Assignment Project Exam Help  
integration by parts  
<https://powcoder.com>  
Add WeChat powcoder

$$\begin{aligned}\mathcal{L}[\dot{y}](s) &= \int_0^{\infty} \frac{dy}{dt} e^{-st} dt = \left[ y(t) e^{-st} \right]_0^{\infty} + s \int_0^{\infty} y(t) e^{-st} dt; \\ &= sY(s) - y(0) \quad \leftarrow \text{'initial conditions'} \\ \mathcal{L}\left[\frac{d^n y}{dt^n}\right](s) &= s^n Y(s) - s^{n-1} y(0) - s^{n-2} \dot{y}(0) - \dots - \frac{d^{n-1} y}{dt^{n-1}}(0)\end{aligned}$$



# Property 5 (linearity)

Assignment Project Exam Help

LINEARITY OF THE TRANSFORM:

<https://powcoder.com>

$$\mathcal{L}[a_1y_1 + a_2y_2](s) = a_1Y_1(s) + a_2Y_2(s) \quad (\text{because integration is linear})$$

Add WeChat powcoder



# Properties 6 & 7 (limits)

With  $Y(s) = \mathcal{L}[y](s)$ , the following hold whenever the limits exist.

FINAL VALUE:  $\lim_{t \rightarrow \infty} y(t) = \lim_{\sigma \rightarrow 0_+} (sY(s)) \Big|_{s=\sigma+j0}$

INITIAL VALUE:  $\lim_{t \rightarrow 0} y(t) = \lim_{\sigma \rightarrow \infty} (sY(s)) \Big|_{s=\sigma+j0}$

NOTE: You need to always check that limits exist before applying formulas!  
We will see that existence of these limits is related to the notion of “stability”.



# Proof

Suppose there exist constants  $K, a > 0$  and  $y_\infty$  such that  $|y(t) - y_\infty| < Ke^{-at}$ .  
Then  $y_\infty = \lim_{t \rightarrow \infty} y(t)$ ,  $Y(s)$  is defined for  $\Re(s) > 0$ , and

$$\begin{aligned} \lim_{\sigma \rightarrow 0_+} \left| \sigma \int_0^\infty y(t) e^{-\sigma t} dt - y_\infty \right| &= \lim_{\sigma \rightarrow 0_+} \left| \sigma \int_0^\infty (y(t) - y_\infty) e^{-\sigma t} dt \right| \\ &\leq \lim_{\sigma \rightarrow 0_+} \sigma \int_0^\infty K e^{-at} e^{-\sigma t} dt \leq \lim_{\sigma \rightarrow 0_+} \sigma \frac{K}{\sigma + a} = 0. \end{aligned}$$



# Transfer function

- Consider an input-output model:

$$\sum_{k=0}^n a_k \frac{d^k y}{dt^k} = \sum_{k=0}^n b_k \frac{d^k u}{dt^k}$$

Assignment Project Exam Help

- Assuming zero initial conditions:

<https://powcoder.com>

$$\left( \sum_{k=0}^n a_k s^k \right) Y(s) = \left( \sum_{k=0}^n b_k s^k \right) U(s)$$

- This yields:

$$Y(s) = G(s)U(s) \text{ where } G(s) = \frac{B(s)}{A(s)} \leftarrow \text{'transfer function'}$$



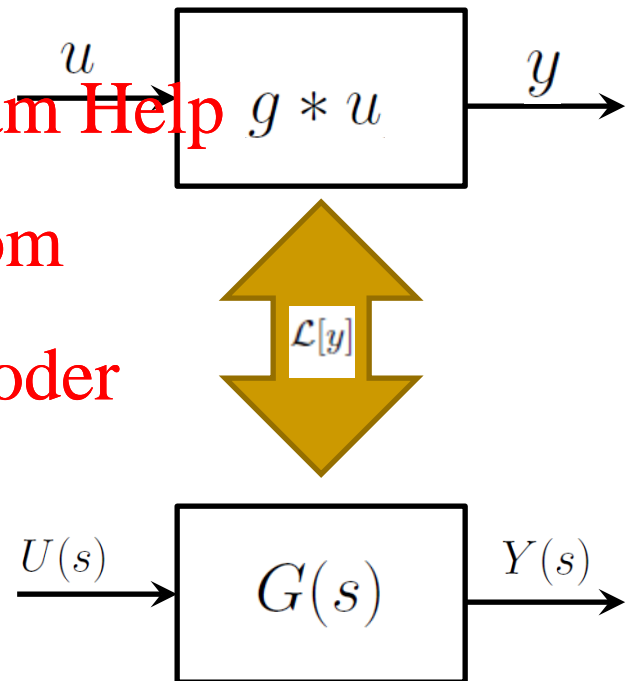
# Property 8 (convolution)

$$\begin{aligned}
 \mathcal{L}[g * u](s) &= \int_0^\infty e^{-st} \left\{ \int_0^t g(t-\tau)u(\tau)d\tau \right\} dt \\
 &= \underbrace{\int_0^\infty \int_0^\infty e^{-st} g(t-\tau)u(\tau)d\tau dt}_{\text{(since } g(t-\tau) = 0, t < \tau)} \\
 &= \underbrace{\int_0^\infty \int_0^\infty e^{-st} g(t-\tau)u(\tau)dtd\tau}_{\text{Fubini's Theorem}} \\
 &= \underbrace{\int_0^\infty \int_{-\tau}^\infty e^{-s(T+\tau)} g(T)u(\tau)dTd\tau}_{T:=t-\tau} \\
 &= \underbrace{\int_0^\infty e^{-s\tau}u(\tau) \left\{ \int_0^\infty e^{-sT}g(T)dT \right\} d\tau}_{\text{(since } g(t-\tau) = 0, t < \tau)} \\
 &= \int_0^\infty e^{-sT}g(T)dT \int_0^\infty e^{-s\tau}u(\tau)d\tau \\
 &= G(s)U(s)
 \end{aligned}$$

Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



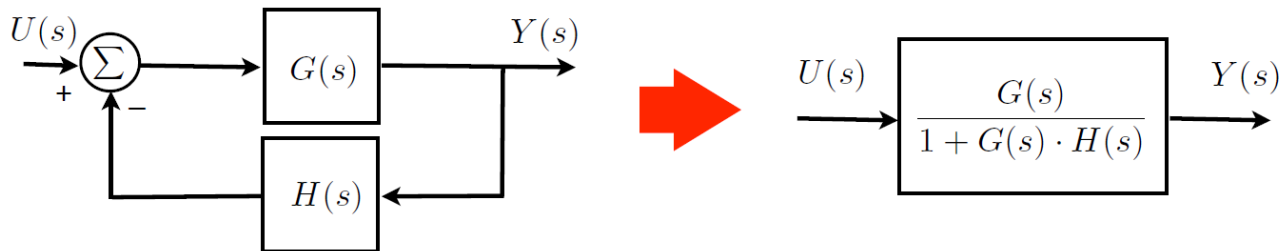
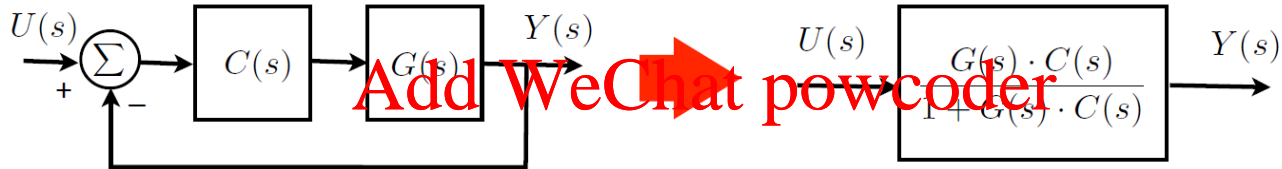
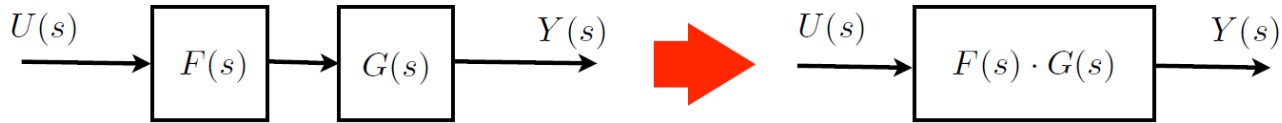


# Comments:

- Transfer functions obtained assuming zero initial conditions.
- Transfer function = Laplace transform of system impulse response.
- Often easier to find the transfer function than the impulse response
- Convolution is difficult to do. But if we go to s-domain it becomes multiplication: easy.



# Algebra of block diagrams



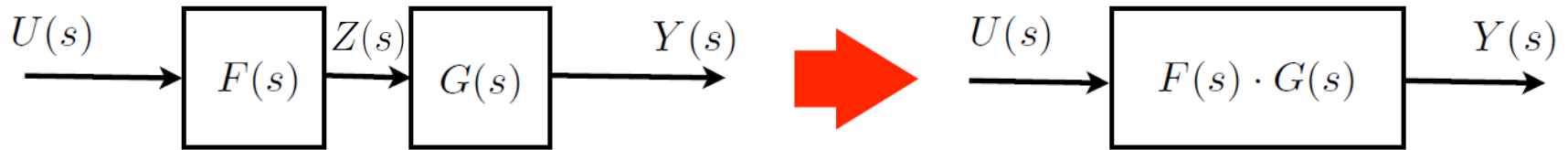
Assignment Project Exam Help

<https://powcoder.com>

Add WeChat powcoder



# Series connection



Assignment Project Exam Help

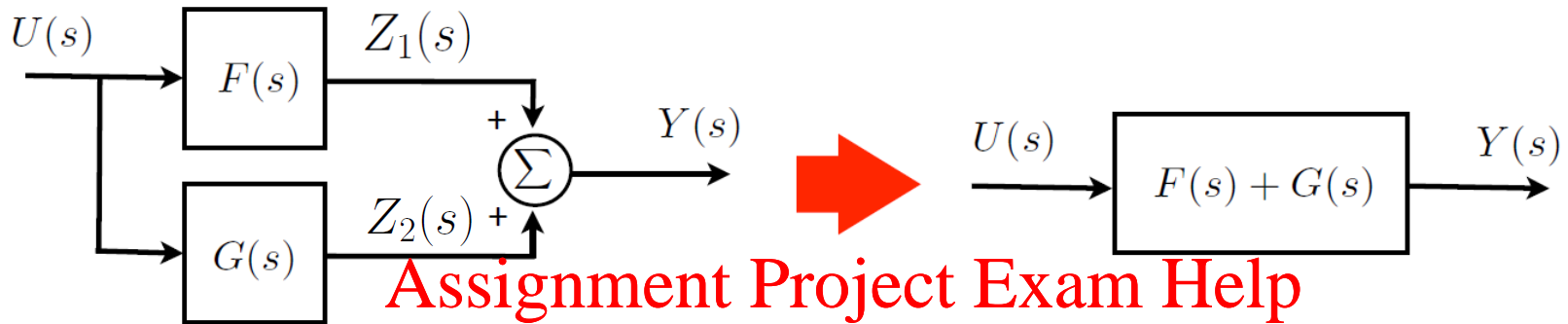
$$\frac{Y(s)}{U(s)} = ? \quad \text{https://powcoder.com}$$

$$\frac{Y(s)}{U(s)} = \frac{Z(s)}{U(s)} \frac{Y(s)}{Z(s)} = F(s)G(s)$$

Add WeChat powcoder



# Parallel connection



Assignment Project Exam Help

$$\frac{Y(s)}{U(s)} = ?$$

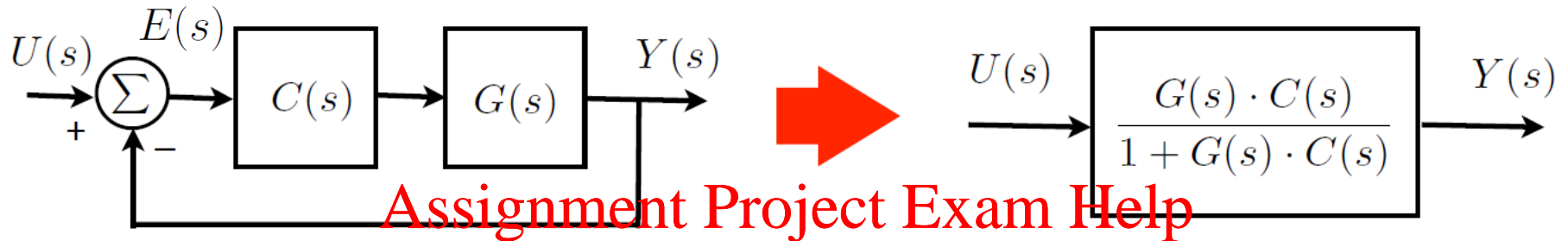
<https://powcoder.com>

Add WeChat powcoder

$$\frac{Y(s)}{U(s)} = \frac{Z_1(s) + Z_2(s)}{U(s)} = \frac{Z_1(s)}{U(s)} + \frac{Z_2(s)}{U(s)} = F(s) + G(s)$$



# Unity feedback connection



$$\frac{Y(s)}{U(s)} = ?$$

<https://powcoder.com>

$$Y(s) = C(s)G(s)E(s) \quad \text{Feedforward transfer function}$$

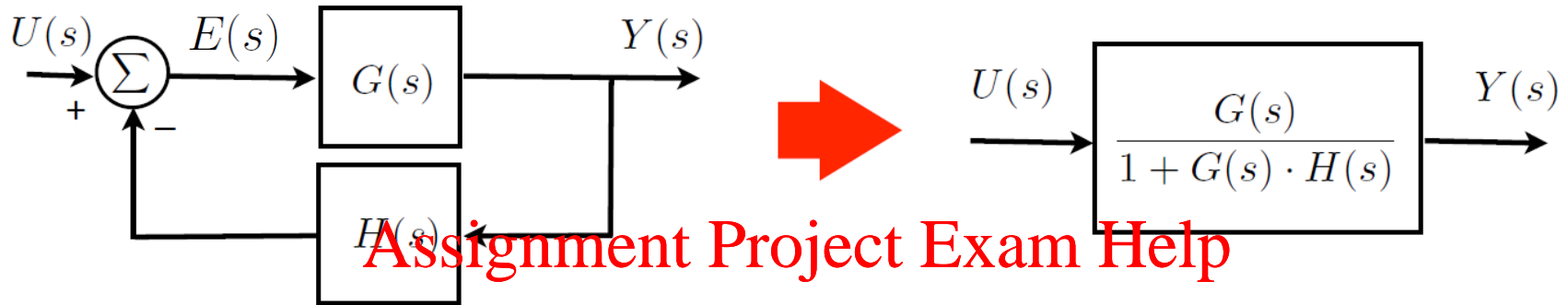
$$E(s) = U(s) - Y(s) = U(s) - C(s)G(s)E(s) \quad \text{Comparator}$$

$$E(s) = \frac{U(s)}{1 + C(s)G(s)}$$

$$\frac{Y(s)}{U(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$



# General feedback connection



Assignment Project Exam Help

<https://powcoder.com>

$$\frac{Y(s)}{U(s)} = ?$$

$$Y(s) = G(s)E(s) \quad \text{Feedforward connection}$$

$$E(s) = U(s) - Y(s) = U(s) - H(s)Y(s) \quad \text{Comparator + feedback}$$

$$E(s) = U(s) - H(s)G(s)E(s)$$

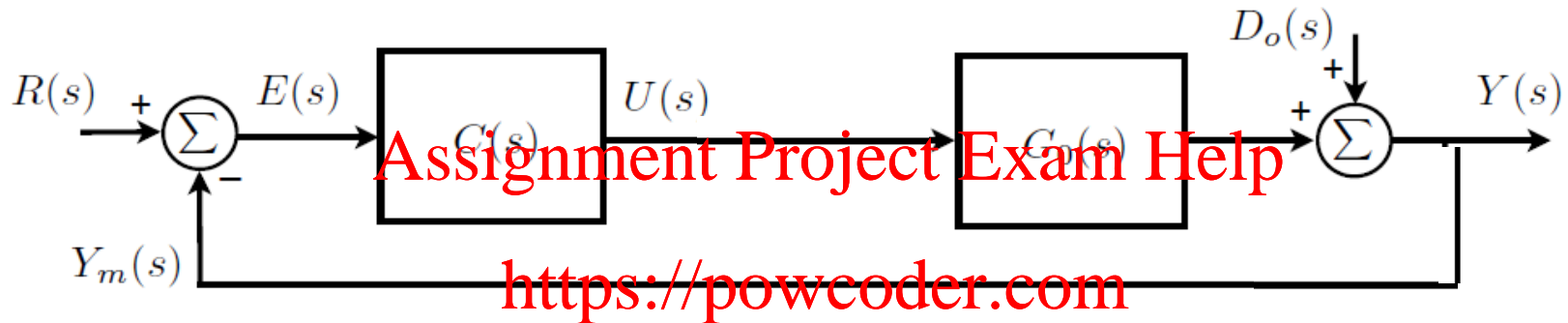
$$E(s) = \frac{U(s)}{1 + H(s)G(s)}$$

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + H(s)G(s)}$$



# Two inputs:

Turn off one input and compute the transfer function in the usual manner for the other input.



$$Y(s) = T_0(s)R(s) + S_0(s)D_o(s)$$

Add WeChat powcoder

$$T_0(s) = \frac{Y(s)}{R(s)} = \frac{G_0(s)C(s)}{1 + G_0(s)C(s)} \quad \text{with only } r \neq 0$$

$$S_0(s) = \frac{Y(s)}{D_o(s)} = \frac{1}{1 + G_0(s)C(s)} \quad \text{with only } d_o \neq 0$$

This is how we will compute “sensitivity functions”.