# ELEN90055 Control Systems

## Midsemester Test

Semester 2, 2021

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## Instructions

This test consists of 2 questions, with marks as indicated, summing to 27. You have one (1) hour to complete this test, including reading, writing, scanning and uploading. Upload your answers through Gradescope by 1.05pm Melbourne time. Aim to finish writing by 12.50pm, so you have time for scanning and upload. For full marks, complete all questions and show your working.

This test is 'open book' and you may refer to any subject materials. The lecturer will be available on the usual zoom channel until 1.05pm if you need clarification - use the chat function in zoom to avoid distracting others. You are **not** allowed to communicate or collaborate with, or seek/provide assistance from/to, any other persons, from the start time of the test until the late submission time is over. The questions are randomised.



Late submissions are permitted until 1.30pm Melbourne time but a penalty may apply. Submissions after that time will not be accepted. In case of potential technical issues, download a copy of this question sheet onto your computer or device as soon as you start. If you have technical issues, take a screenshot of the error message.



1. (2 + 5 + 4 + 3 = 14 marks) Consider a system given by the time-domain differential equation



$$\ddot{y} - \frac{\dot{y}}{y} + 6\dot{y} - 6\ln|y| = \dot{u} - u,$$

where $y \neq 0$ is the output and $u$ is the input. Let $(\bar{y}, \bar{u})$ denote an equilibrium point.

(a) Find an expression for $\bar{y}$ in terms of $\bar{u}$.

(b) Find the linearised (i.e. incremental) model of this system if $\bar{u} = 0$ and $\bar{y} = 1$. Express your answer as a differential equation involving the incremental variables $\delta_y$, $\delta_u$ and their time-derivatives.

(c) Find the simplified transfer function $G(s)$ for your linearised model as a ratio of two coprime polynomials. Plot any poles and zeros (using x's and o's respectively) in the complex plane, and check that $G(s)$ is BIBO stable (do not define BIBO stability).

(d) Suppose $\bar{u}$ and $\delta_u$ are both $= 0$ for all $t \geq 0^-$. It is observed that when the incremental initial condition $\delta_y(0^-)$ is not exactly zero, the output $y(t)$ does not approach the equilibrium value $\bar{y} = 1$ as $t \to \infty$. Explain briefly why this happens even though $G(s)$ is BIBO stable.

2. (3 + 2 + 5 + 3 = 13 marks) Consider a plant with transfer function

$$G(s) = \frac{1}{s^2 + 4}$$

You wish to use a controller $C(s) = q(s+1) + k/s$ to stabilise this plant in a unity feedback loop, where $k$ and $q$ are real parameters to be chosen.

(a) Find a simplified expression for the closed-loop transfer function $H(s)$ from the reference signal to the plant output. Express your answer as a ratio of two polynomials.

(b) Assuming closed-loop stability, show that this controller achieves a steady-state error of zero when the reference is a step function.

(c) The characteristic equation is

$$s^3 + qs^2 + (4+q)s + k = 0.$$

Find conditions on $q$ and $k$ for the closed-loop system to be stable.



(d) Referring to suitable transfer functions, explain briefly why making $q$ large would improve how well the output follows the reference, but also increases the sensitivity of the output to high-frequency measurement noise.