Econ/Math C103 - Final

12/14/2018

Instructions: This is a closed-book exam. You are allowed to use two two-sided cheat sheets. You have 150 minutes. The weight of each question is indicated next to it. Write clearly, explain your answers, and be concise. You may use any result from class. Good luck!

1. (25pts) Consider the following two-player game where Player 1 chooses one of the four rows and Player 2 chooses one of the three columns:

	C_1	C_2	C_3
R_1	0,0	2,1	4,0
R_2	1,0	1,0	1,5
R_3	0,2	4,2	0,1
R_4	3,5	1,1	-1,2

   (a) What are the strategies that survive IESDS?

   (b) At each step of the elimination what were your rationality and knowledge assumptions?

   (c) Find all the (possibly mixed) Nash equilibria.

2. (20pts) Consider a model of indivisible objects where each agent can consume exactly one object. There are six agents $\{1, 2, 3, 4, 5, 6\}$ and six objects $\{a, b, c, d, e, f\}$. The initial endowment vector μ_E and the preference profile R are given by:

$$\mu_E = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

R_1	R_2	R_3	R_4	R_5	R_6
e	f	e	f	c	e
b	d	a	a	f	c
a	e	c	c	b	a
d	b	b	d	e	f
f	a	d	e	d	b
c	c	f	b	a	d

Find the unique core allocation. Find a price vector that supports it as a Walrasian equilibrium.

3. (10pts) Let $X = \{x_1, x_2, x_3\}$ be the set of prizes. Consider the following preference relation R on $\triangle(X)$:

$$pRq \;\;\Leftrightarrow\;\; [p(x_1) + p(x_3)]^2 \geq [q(x_1) + q(x_3)]^2 \qquad p, q \in \triangle(X).$$

Which of the three conditions: rationality, independence, and solvability, does R satisfy? Explain your answer.

4. (20pts) Consider the quasilinear model with two individuals. Individual 1 lives in city A; individual 2 lives in city B. There is a third city C in between the cities A and B. A public park will be built in one of these three cities, i.e., $K = \{A, B, C\}$. Each individual would prefer having the public park built in her own city than in city C, and receives zero value from having the public park being built in the other individual's city. That is:

$$\Theta_1 = \left\{\theta_1 = (\theta_{1A}, \theta_{1C}) \in \mathbb{R}^2 \,|\, \theta_{1A} \geq \theta_{1C} \geq 0\right\}$$

$$\Theta_2 = \left\{\theta_2 = (\theta_{2B}, \theta_{2C}) \in \mathbb{R}^2 \,|\, \theta_{2B} \geq \theta_{2C} \geq 0\right\}$$

$$v_1(k, \theta_1) = \begin{cases} \theta_{1A} & \text{if } k = A \\ 0 & \text{if } k = B \\ \theta_{1C} & \text{if } k = C \end{cases} \qquad v_2(k, \theta_2) = \begin{cases} 0 & \text{if } k = A \\ \theta_{2B} & \text{if } k = B \\ \theta_{2C} & \text{if } k = C \end{cases}$$

Derive the pivotal VCG mechanism.

5. (25pts) A single indivisible object is auctioned to two bidders whose valuations are distributed i.i.d. uniformly on the interval $[0, 1]$ (i.e., $F(x) = x$ and $f(x) = 1$ for all $x \in [0, 1]$). Consider the auction rule where the highest bidder wins the object and pays the sum of the two bids. The loser does not pay. If both bidders bid the same amount, then one of them is chosen randomly to be the winner. Then, the payoff function is given by:

$$u_i(b_i, b_j; x_i) = \begin{cases} x_i - (b_i + b_j) & \text{if } b_i > b_j \\ \frac{1}{2}\left[x_i - (b_i + b_j)\right] & \text{if } b_i = b_j \\ 0 & \text{otherwise.} \end{cases}$$

where $\{i, j\} = \{1, 2\}$. Derive a symmetric BNE where the bid function $b(\cdot) : [0, 1] \to \mathbb{R}$ is linear and increasing in the valuation, i.e.:

$$b(x) = ax + c \text{ for all } x \in [0, 1].$$

for some constants $a, c \in \mathbb{R}$ such that $a > 0$.[1]

6. *Bonus.* (20pts. *Please provide your answer to this question on separate letter papers.* This is more difficult than the other questions. You are recommended to not spend time on it unless you are done with the other questions.)
   Fix a marriage market (M, W, R) and let μ and ν be stable matchings. Define the function $\lambda : M \cup W \to M \cup W$ by:

$$\lambda(m) = \begin{cases} \mu(m) & \text{if } \mu(m) R_m \nu(m) \\ \nu(m) & \text{otherwise.} \end{cases} \qquad \text{for all } m \in M$$

$$\lambda(w) = \begin{cases} \nu(w) & \text{if } \mu(w) R_w \nu(w) \\ \mu(w) & \text{otherwise.} \end{cases} \qquad \text{for all } w \in W$$

   Prove that λ is a matching and that λ is stable.

[1] Remember that a BNE is symmetric if every bidder i uses the same bid function $b_i(x_i) = b(x_i)$.