Econ/Math C103 - Final

5/14/2020

Instructions: This is a 36-hour open-book take-home exam. Make sure to upload a pdf file of your (scanned or typed-up) solutions to Gradescope by 9pm PST on Friday, May 15th. Please write the following honor code, sign below it, and include it with your solutions: *"I swear on my honor that: (1) I have not used the internet in relation to this exam other than: accessing the materials in the bCourses website of C103 and uploading my answers to Gradescope, (2) I have neither given nor received aid on this exam."* The weight of each question is indicated next to it. You can use any result from class. Please write clearly and explain your answers. Good luck!

1. (16pts) Consider a model of indivisible objects where each agent can consume exactly one object. There are six agents $\{1, 2, 3, 4, 5, 6\}$ and six objects $\{a, b, c, d, e, f\}$. The initial endowment vector μ_E and the preference profile R are given by:

$$\mu_E = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & b & c & d & e & f \end{pmatrix}$$

R_1	R_2	R_3	R_4	R_5	R_6
e	e	d	a	a	e
.	d	f	e	.	d
.	a	.	d	.	a
.	c	.	.	.	b
.	.	.	.	.	.

Find the unique core allocation. Find a price vector that supports it as a Walrasian equilibrium. The unspecified parts of the preferences do not change the answers.

2. (16pts) Consider the following two-player game where Player 1 chooses one of the three rows and Player 2 chooses one of the three columns:

	L	m	R
U	3,0	2,4	3,2
M	7,1	0,2	7,4
D	0,5	5,0	0,0

   (a) What are the strategies that survive IESDS?

   (b) At each step of the elimination what were your rationality and knowledge assumptions?

(c) Find all the (possibly mixed) Nash equilibria.

3. (16pts) Two identical indivisible objects are to be allocated among three agents. Each agent can consume zero, one, or two objects. The set of types of each agent i is given by $\Theta_i = \{(v_1, v_2) \in \mathbb{R}^2_+ | 0 \leq v_1 \leq v_2\}$ where v_1 denotes utility of consuming just one object and v_2 denotes the utility of consuming two objects. The utility of consuming no object is normalized to be zero.

   (a) Find the allocations of objects and transfers under the pivotal VCG mechanism for the following type profiles:

	θ_1	θ_2	θ_3
v_1	0	4	4
v_2	10	4	4

	θ_1	$\hat{\theta}_2$	$\hat{\theta}_3$
v_1	0	9	9
v_2	10	9	9

   (b) Briefly interpret your findings from part (a).

4. (16pts) X is a finite set of potential job candidates such that $|X| \geq 3$. For each candidate x, let $e(x)$ denote the years of experience that candidate x has in similar jobs, and let $g(x)$ be the undergraduate GPA of the candidate. Assume that no two candidates have the same GPA. For any set of applicants $A \in \mathcal{P}(X)$, if there is a candidate $x_e \in A$ such that:

$$\forall y \in A \setminus \{x_e\} : \quad e(x_e) \geq e(y) + 1, \tag{1}$$

then the employer choses x_e. If there is no candidate $x_e \in A$ satisfying (1), then the employer choses the candidate $x_{gpa} \in A$ with the highest GPA in A. Let c denote the employer's choice function. Does there exist a rational preference R on X such that $c = c^R$? If your answer is yes show why, otherwise give a counterexample.[1]

5. (16pts) Consider the auction of a single indivisible object to two bidders whose valuations are distributed independently. Bidder 1's valuation is distributed uniformly on the interval $[0, 1]$ (i.e., $F_1(x_1) = x_1$ and $f_1(x_1) = 1$). Bidder 2's valuation is distributed uniformly on the interval $[0, 2]$ (i.e., $F_2(x_2) = \frac{1}{2}x_2$ and $f_2(x_2) = \frac{1}{2}$). Derive the probabilities of allocating the object to the two bidders in a revenue-maximizing feasible direct auction mechanism. Is the object allocated to one of

[1] To give a counterexample, you should specify a set X, and functions e, g (such that $|X| \geq 3$ and g is one-to-one) for which there is no rational preference R on X such that $c = c^R$.

the two bidders with probability one? Is the object always allocated to the bidder with the highest valuation?

6. (20pts) Consider a marriage market with three men $M = \{1, 2, 3\}$ and three women $W = \{a, b, c\}$ whose preferences are as follows:

R_1	R_2	R_3
a	b	c
b	c	a
c	a	b

R_a	R_b	R_c
2	3	1
3	1	2
1	2	3

where everyone finds those on the other side of the acceptable. List all the stable matchings. Explain why the matchings you listed are stable, and why there are no other stable matchings.