Econ/Math C103 - Final

12/18/2020

Instructions: This is a 24-hour open-book take-home exam. You can use any result from the lecture notes, problem sets, and problem set solutions.

The weight of each question is indicated next to it. Please write clearly and explain your answers. Make sure to upload a pdf file of your (scanned or typed-up) solutions to Gradescope by 9am PDT on Saturday, December 19.

Please write the following honor code, sign below it, and include it with your solutions: *"I swear on my honor that: (1) I have not used the internet in relation to this exam other than: accessing the materials in the bCourses website of C103 and uploading my answers to Gradescope, (2) I have neither given nor received aid on this exam."*

Good luck!

1. (20pts) Consider a marriage market with four men $M = \{1, 2, 3, 4\}$ and five women $W = \{a, b, c, d, e\}$ whose preferences are given by:

R_1	R_2	R_3	R_4
b	c	e	c
e	a	b	d
1	b	3	e
c	d	d	4
d	e	c	a
a	2	a	b

R_a	R_b	R_c	R_d	R_e
3	1	c	2	3
2	4	2	d	4
4	3	3	4	1
a	2	4	1	e
1	b	1	3	2

List all the stable matching(s). Explain why there are no other stable matchings.

2. (20pts) Consider the following two-player game where Player 1 chooses one of the four rows and Player 2 chooses one of the four columns:

	C_1	C_2	C_3	C_4
R_1	6,5	0,1	0,1	0,0
R_2	2,0	1,0	1,1	1,0
R_3	1,0	4,5	0,1	3,1
R_4	0,0	0,0	4,0	2,1

(a) What are the strategies that survive IESDS?

(b) At each step of the elimination what were your rationality and knowledge assumptions?

(c) Find all the (possibly mixed) Nash equilibria.

3. (20pts) Let (N, X, R, μ^E) be a housing market, and let μ be a Pareto efficient assignment that is not necessarily in the core. Show that under the assignment μ, there exists an agent who receives his/her top object.

4. (20pts) Consider a quasi-linear model with n agents and m distinct indivisible objects $O = \{o_1, \ldots, o_m\}$. Each agent can consume at most one object and $m < n$. The set K consists of all one-to-one functions $\mu : O \to N$, where $\mu(o_l) = i$ means that object o_l is allocated to agent i at the "project choice" μ. Let $\Theta_i = [0, 1]$ denote the set of possible valuations of agent i for object o_1. We interpret objects with lower indices to be of higher quality: All agents prefer o_1 to o_2 to o_3 etc. To model this, let $a_1, \ldots, a_m \in [0, 1]$ be such that $1 = a_1 > a_2 > \ldots > a_m > 0$, and:

$$u_i(\mu, \theta_i) = \begin{cases} a_l \theta_i & \text{if } \exists l \in \{1, \ldots, m\} \text{ s.t. } i = \mu(o_l) \\ 0 & \text{otherwise} \end{cases}$$

for all $i \in N$, $\mu \in K$, and $\theta_i \in \Theta_i$. Derive the pivotal VCG mechanism.

5. (20pts) Consider the auction of a single indivisible object to two bidders whose valuations are distributed independently. Bidder 1's valuation is distributed uniformly on the interval $[0, 1]$ (i.e., $F_1(x_1) = x_1$ and $f_1(x_1) = 1$). Bidder 2's valuation is distributed on the interval $[2, 3]$ with a continuous nondecreasing density f_2 such that $f_2(2) \in (0, \frac{1}{2})$. Consider a revenue-maximizing feasible direct auction mechanism. Is the object allocated to one of the two bidders with probability one? Is the object always allocated to the bidder with the highest valuation?