Econ/Math C103 - Fall 2020

Non-Cooperative Game Theory II:

Static Bayesian Games

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1. Overview
2. Bayes' rule
3. Definitions
    1. Bayesian Game
    2. Bayesian Nash Equilibrium
4. Application: Cournot Duopoly with Incomplete Information

Bayesian Games

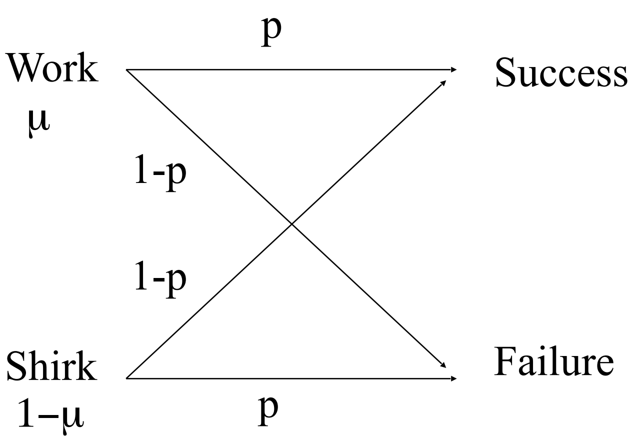
There is incomplete (or asymmetric) information if some players have payoff relevant information that the other players do not know.

These games are also sometimes refereed to as incomplete information games, asymmetric information games, or games with payoff uncertainty.

Bayes' Rule

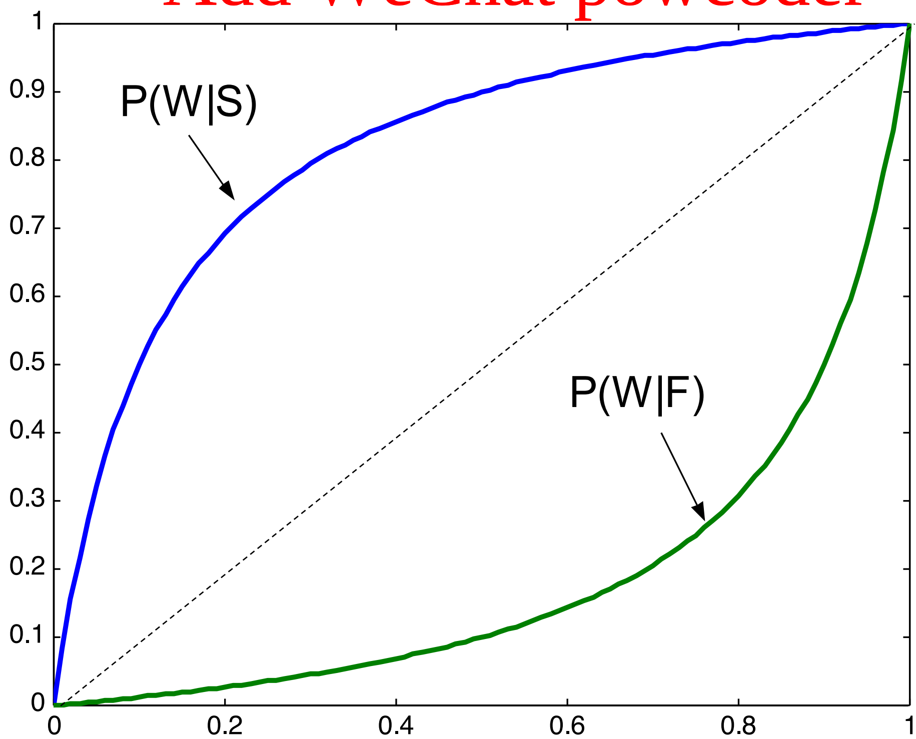
- $\text{Prob}(A|B) = \dfrac{\text{Prob}(A \text{ and } B)}{\text{Prob}(B)}$
- $\text{Prob}(A \text{ and } B) = \text{Prob}(A|B)\text{Prob}(B) = \text{Prob}(B|A)\text{Prob}(A)$
- $\text{Prob}(A|B) = \dfrac{\text{Prob}(B|A)\text{Prob}(A)}{\text{Prob}(B)}$

Example



- Prob(Work|Success) = $\mu p/[\mu p + (1-\mu)(1\text{-}p)]$
- Prob(Work|Failure) = $\mu(1\text{-}p)/[\mu(1-p) + (1-\mu)p]$

$\frac{1}{2}<p<1$



p=0.9

(Static) Bayesian Games

A Bayesian game is a list

$G = \{A_1,\ldots,A_n;T_1,\ldots,T_n;p;u_1,\ldots,u_n\}$, where

- $N=\{1,\ldots,n\}$ is the set of players
- A_i is the action space of i (a_i in A_i)
- T_i is the type space of i (t_i in T_i)
- $u_i(a_1,\ldots,a_n;t_1,\ldots,t_n)$ is i's payoff.
- p is a probability distribution over type profiles $T= T_1\times T_2\times\ldots\times T_n$. (t in T)

$p(t_{-i}|t_i)$ is used to denote i's belief about the other players types when his type is t_i.

Bayesian Nash Equilibrium (BNE)

Given any Bayesian game, a **strategy** of player i is a function $s_i:T_i \to A_i$, where $s_i(t_i)$ corresponds to the action taken by player i of type t_i.

A strategy profile $s^* = (s_1^*,\ldots, s_n^*)$ is a **Bayesian Nash Equilibrium (BNE)** if $s_i^*(t_i)$ solves:

$$\max_{a_i\in A_i} \sum_{t_{-i}\in T_{-i}} u_i\left(s_1^*(t_1),\ldots,s_{i-1}^*(t_{i-1}),a_i,s_{i+1}^*(t_{i+1}),\ldots,s_n^*(t_n),t\right)p(t_{-i}\,|\,t_i)$$