Homework 4 Due on Tuesday in class.

Mengshan has reviewed the questions and answer key.

1. **Asset market prices with three states**

There are three possible contingencies (state of nature.) All consumers have the same utility function $U(x^h)=\pi_1 v(x_1^h)+\pi_2 v(x_2^h)+\pi_3 v(x_3^h)$ where $\pi=(\pi_1,\pi_2,\pi_3)=(\frac{1}{2},\frac{1}{4},\frac{1}{4})$. The VNM utility function is $v(x)=x^{1/2}$.

There are three assets. (You may wish to think of them as coconut plantations). Asset returns (dividends) are as follows:

$z_1=(50,10,100)$, $z_2=(20,10,200)$, $z_3=(30,5,100)$

Consumer h, $h=1,...,H$ has an initial portfolio of $\bar{q}^h=(\bar{q}_1^h,\bar{q}_2^h,\bar{q}_3^h)$ (shares in the three assets.)

Initially suppose that there are markets for every contingency (state claims markets).

(a) Solve for the WE prices. (Choose them so that the price of a claim to state 1 is 1.

(b) What is the equilibrium value of each asset?

(c) Suppose instead that $v(x)=\ln(x)$. Solve again for the WE state claims prices with $p_1=1$.

(d) Provide some intuition for the difference (if any) in the WE state 2 claims prices.

(e) Show that there is a portfolio $q^1=(0,q_2^1,q_3^1)$ that has a dividend vector $z^1=(1,0,0)$ and is therefore equivalent to a state 1 claim.

Hint: Consider the portfolio $q^*=(0,1,-2)$

(f) Hence show that there is a portfolio that has a dividend of $(0,5,100)$ and another portfolio with a dividend of $(0,10,100)$.

(g) Explain briefly why it is also true that there are portfolios paying off only in state 2 and state 3. Hence explain why a consumer can achieve the state claim WE outcome by trading in the three assets.

2. **Bidding on "the game"**

Individual h, with wealth W^h thinks that the Bruins will win "the game" with probability π_1 and so lose with probability $\pi_2=1-\pi_1$. Her VNM utility function is $v(x)=\ln(x-\gamma^h)$ where the parameter $\gamma^h < W^h$. The competitive book-maker market odds on the Bruin victory are $\dfrac{p_1}{p_2} < \dfrac{\pi_1}{\pi_2}$.

(a) What is the degree of absolute risk aversion for this individual? Show that it is an increasing function of γ^h.

(b) Solve for this individual's demand, $\bar{x}_1^h$ for state 1 claims (Bruins wins) is

$$\bar{x}_1^h = \gamma^h + \pi_1(1+\frac{p_2}{p_1})(W^h - \gamma^h).$$

HINT: Note that the budget constraint can be rewritten as follows:

$$p_1(x_1 - \gamma^h) + p_2(x_2 - \gamma^h) \le (p_1 + p_2)(W^h - \gamma^h)$$

Hence solve for $\bar{x}_1^h - \gamma^h$ and then show that her net demand for state 1 claims is

$$\bar{n}_1^h = \bar{x}_1^h - W^h = [\pi_1 \frac{p_2}{p_1} - \pi_2](W^h - \gamma^h)$$

and confirm that under the above assumptions, her net demand for state 1 claims is strictly positive. (Then she is a net supplier of state 2 claims.)

(c) Suppose that there is a population of individuals who are Bruin fans. Each believes that the probability of a Bruin victory is π_1^B. Define $\gamma^B = \sum_{h=1}^{H} \gamma^h$ and $W^B = \sum_{h=1}^{H} \gamma^h$. Explain why the net demand for state 1 claims by Bruin fans is

$$\bar{n}_1^B = \bar{x}_1^B - W^B = [\pi_1^B \frac{p_2}{p_1} - \pi_2^B](W^B - \gamma^B)$$

(d) There is also a population of Trojan fans who believe that the Bruins will win with probability $\pi_1^T < \pi_1^B$. For this population of J individuals define $\gamma^T = \sum_{h=1}^{J} \gamma^h$ and $W^T = \sum_{h=1}^{J} W^h$.

Explain very briefly why the net demand for state 1 claims by Trojan fans is

$$\bar{n}_1^T = \bar{x}_1^T - W^T = [\pi_1^T \frac{p_2}{p_1} - \pi_2^T](W^T - \gamma^T).$$

(e) Suppose $\gamma^T = \gamma^B$ and $W^B = W^T$. Show that the equilibrium market odds are

$$\frac{p_1}{p_2} = \frac{\frac{1}{2}\pi_1^B + \frac{1}{2}\pi_1^J}{\frac{1}{2}\pi_2^B + \frac{1}{2}\pi_2^T}.$$

(f) By considering the effect on the Bruin fans net demand, does an increase in γ^B increase or decrease the equilibrium market odds? What is the intuition?

3. Nash equilibrium with two firms

The demand price functions and cost functions of two firms are as follows:

$$p_1(q) = 160 - 5q_1 - 4q_2 \ , \ p_2(q) = 80 - q_1 - 2q_2 \ , \ C_1(q_1) = 20q_1 \ , \ C_2(q_2) = 30q_2$$

(a) Solve for the Nash Equilibrium of the simultaneous move game in which each firm chooses its output.

Solve for the Nash Equilibrium of the alternating move game in which each form chooses its output and firm 1 moves first.

(b) Explain why firm 1 chooses a higher output in the alternating move game.

(c) If the two firms were to collude and maximize total profit, what would be the profit of each firm? (Any cash transfers between the firms is both illegal and observable so there are no such transfers.)

(d) Does this seem a plausible outcome to the bargaining between the two firms attempting to reach a collusive agreement? Explain.

HINT: If they do not reach a collusive agreement, what is the outcome?

4. Sealed second and third price auctions when there are two identical items for sale

Two identical items for sale. Each buyer wants one unit.

There are four buyers. The buyers' values are $v_1 = 100, \ v_2 = 80, \ v_3 = 60, \ v_4 = 40$. All of the values are public information. (This makes the problem easier.) Buyers each submit a sealed bid (an integer). The two highest bids are winning bids. If there is a tie the winner is selected randomly among the tying bidders.

(a) The two items will be sold to the buyers submitting the two highest bids and each will pay the **second** highest bid. Are the following strategies mutual best responses?

$$b_1 = 100, \ b_2 = 80, \ b_3 = 60, \ b_4 = 40$$

If so explain. If not what four strategies are mutual best responses?

(b) The two items will be sold to the buyers submitting the two highest bids and each will pay the **third** highest bid. What are equilibrium bidding strategies?

(c) Is it a dominant strategy for each buyer to bid $b_i = v_i$ in either of these auctions?

Two identical items for sale. Buyer 1 values two units

Henceforth, suppose that buyer 1 is interested in buying two items. She is willing to pay 120 for one item and 210 for both items so her value of receiving a second item is 90. The other three buyers only want one unit. Their values are as before, i.e. $v_2 = 80,\ v_3 = 60,\ v_4 = 40$.

(d) The rules of the auction are as in (b). Explain why it is a dominant strategy for buyers 2, 3 and 4 to bid their values. What should buyer 1 do?

(e) 1 BONUS POINT

Finally suppose that the item is sold using an electronic auction. The asking price p starts at zero and rises one dollar at a time. If a buyer has x light switched to green, that buyer is willing to pay the price p for x units. If a buyer switch a light to red that indicates this buyer has permanently stopped bidding on that item. Each person has two switches as there are two items for sale. Of course buyer 2, 3 and 4 will immediately flip one switch to red so that at the beginning of the auction there are 5 green lights on. What should buyer 2 do in this auction? What should buyer 1 do in this auction?