**Due next Thursday before class.**

Based on questions today I have made some minor changes. These are marked in red. Do not submit your spread-sheet. I have added a question that can be quickly answered using Solver. Just submit the results of the numerical maximization.

Note that question 3 is very similar to the question you considered in the TA session. I have tried to organize the question so that the method of approach is clear.

### 1. Profit maximization

A firm has a cost function $C(q) = 40q_1 + 60q_2 + (2q_1 + 5q_2)^2 - q_2^{\,2}$ .

The firm is a price take in output markets. The price vector is $p = (168, 372)$.

(a) Solve for the unique critical point $\bar{q} >> 0$ of the profit function.

(b) Explain why this is not the profit-maximizing output vector.



(c) Is $\bar{q}$ a local maximum?

(d) What output vector does maximize the profit of the firm.



(e) Use Solver in Excel to solve the maximization problem numerically. Then choose some number $b$ between 2 and 10 (not necessarily an integer and solve numerically if the cost function is



$$C(q) = 40q_1 + 60q_2 + (2q_1 + 5q_2)^2 + bq_2^{\,2}$$

How does your numerical answer change when $b$ rises?

Note: You are not expected to do part (e) analytically. Simply submit two values of $b$ and the associated outputs.

### 2. Output maximization

A firm has production function $q = F(z) = (\frac{1}{z_1} + \frac{1}{z_2})^{-1}$ .

(a) Show that if inputs are scaled up by $\theta$ , then output is also scaled up by $\theta$ , i.e. $F(\theta z) = \theta F(z)$ .

(b) The input price vector is $p >> 0$. The manager is asked to maximize output given a budget of $B$ dollars. Explain why the solution to this maximization problem is also the solution to the following simpler maximization problem

$$\underset{z \geq 0}{Max}\{-\frac{1}{z_1}-\frac{1}{z_2} \mid B-p\cdot z \geq 0\}$$

(c) Use the Lagrange method for the simpler problem and show that $p_1 z_1 = \frac{p_1^{1/2}}{\lambda^{1/2}}$ . (By an identical argument it follows that $p_2 z_2 = \frac{p_2^{1/2}}{\lambda^{1/2}}$ .)

(d) Solve for $\lambda^{1/2}$ and hence show that $\frac{1}{z_1} = \frac{p_1^{1/2}(p_1^{1/2}+p_2^{1/2})}{B}$

(e) Solve also for $\frac{1}{z_2}$ . Hence obtain an expression for maximized output

**3. Walrasian equilibrium**

Alex has an endowment $\omega^A = (8,8)$ and Bev has an endowment of $\omega^B = (7,12)$ . Alex likes each unit of commodity 1 twice as much as each unit of commodity 2. Bev likes each unit of commodity 1 four times as much as each unit of commodity 2.

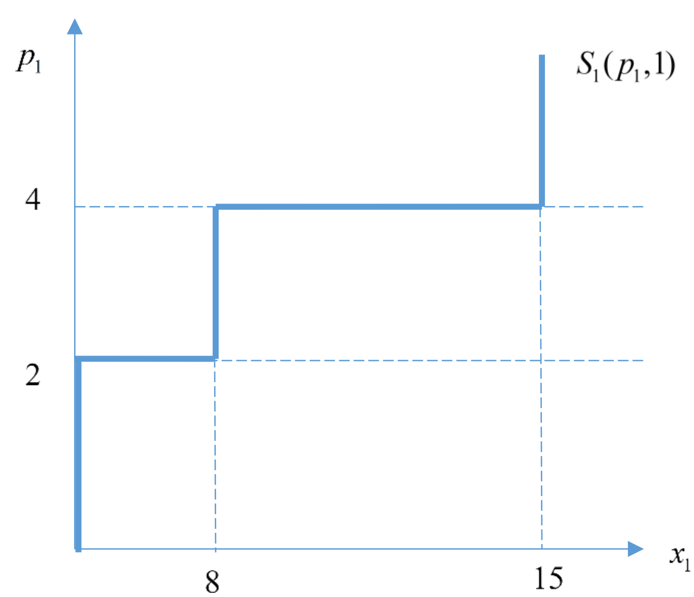


(a) Explain why the optimal choice for each consumer is not affected if the price vector changes from $p = (p_1, p_2)$ to $\theta p = (\theta p_1, \theta p_2)$ for any $\theta > 0$. Thus we can always "normalize" by choosing $\theta = \frac{1}{p_j}$ so that the price of commodity $j$ is equal to 1.





(b) Suppose that $p_2 = 1$. Explain why the market supply of commodity 1, $S_1(p_1,1)$ is as depicted below.



(c) For what price of commodity 1 will (i) neither (ii) one (iii) two consumers have a strictly positive demand for commodity 1.

(d) Fully determine the market demand, $D_1(p_1,1)$ for every price $p_1$ and depict it in a separate figure. Explain any horizontal segments.

(e) What is the equilibrium price of commodity 1.

(f) Let $x_j(p) = x_j^A(p) + x_j^B(p)$ be total demand for commodity $j$ given price vector $p$ and let $\omega = \omega^A + \omega^B$ be the total endowment vector. Write down the budget constraints and hence show that

$$p_1(x_1(p)-\omega_1) + p_2(x_2(p)-\omega_2) = 0$$

Use this result to show that if supply equals demand in the market for commodity 1, then supply must also equal demand in the market for commodity 2.



**4. Equilibrium trades in an exchange economy**



Alex has a utility function $U^A(x^a) = x_1^a x_2^a$ and Bev has a utility function $U^B(x^b) = -\frac{1}{x_1^b} - \frac{1}{x_2^b}$.

(a) Explain why the slope of a level set at $x$ is $-\frac{\partial U^h}{\partial x_1}(x) / \frac{\partial U^h}{\partial x_2}(x)$. Why is $\frac{\partial U^h}{\partial x_1}(x) / \frac{\partial U^h}{\partial x_2}(x)$ called the consumer's marginal rate of substitution?

(b) Suppose both consumers have the same consumption bundle $\overline{x}$.

Show that $MRS^a(\overline{x}) = MRS^b(\overline{x})$ if $\overline{x}_2 = \overline{x}_1$. i.e. the consumption bundle is on the $45^\circ$ line.

Who has the higher $MRS$ at if (i) $\overline{x}_2 > \overline{x}_1$ (ii) $\overline{x}_2 < \overline{x}_1$?

You might try drawing the level sets of Alex and Bev through $\overline{x} = (2,1)$ in a single diagram.

(c) Suppose $\omega^a = \omega^b = (\alpha, \alpha)$ so that the endowment **is on** the $45^\circ$ line.

Explain why demand cannot be equal to supply if $p_1 \neq p_2$. What if $p_1 = p_2$?

(d) Suppose $\omega^a = \omega^b$ and the endowment is above the $45^\circ$ line (more endowment of commodity 2.)

In the Walrasian Equilibrium, which consumer will sell commodity 1?

(e) Suppose $\omega_2^a / \omega_1^a > \omega_2^b / \omega_1^b$ . In the Walrasian Equilibrium, is it clear which consumer will sell commodity 1?

Remark: In parts (d) and (e) you are not expected to solve for the WE price vector.