Economics 403A
Review of MPT and CAPM

Part I
Portfolio Returns with Two Risky Securities

Dr. Randall R. Rojas

Today's Class (Part I)

- Modern Portfolio Concepts
- Portfolio Return and Expected Return

- Covariance and Correlation

- Portfolio Variance and Standard Deviation

- Diversification with Two Assets
  - Fixed Weights
  - Varying Weights

Modern Portfolio Concepts

- Portfolio: Is a combination of N assets, with return $R_1, \ldots, R_N$.

- The long-term goal of a growth-oriented portfolio is long-term price appreciation.

- An income-oriented portfolio is is designed to produce regular dividends and interest payments.

- In general, the ultimate goal of an investor is an efficient portfolio, one that provides the highest return for a given level of risk.

Portfolio Returns and Expected Returns 1 of 2

- Two of the most important characteristics to examine are the returns that each asset might be expected to earn and the uncertainty surrounding that expected return.

- The return on a portfolio is calculated as a weighted average of returns on the assets that make up the portfolio.

- Portfolio weights (ω_i): Represent the percentage of wealth invested in asset i.

$$\omega_i = \frac{\text{\$ value of stock } i\text{'s position}}{\text{total \$ value of the portfolio}}$$

- $\sum_{i=1}^{N} \omega_i = 1$ and $\omega_i < 0$ indicates a short position.

Portfolio Returns and Expected Returns 2 of 2

- The return on portfolio *p*:

$$R_p = \sum_{i=1}^{N} \omega_i R_i$$

- The expected return on portfolio *p*:

$$E(R_p) = \sum_{i=1}^{N} \omega_i E(R_i)$$

Portfolio Example

Stock	Number of Shares	Price per Share	Portfolio Weights	Amount Invested
IBM	1,000	$10	0.1	$10,000
GE	4,000	$20	0.8	$80,000
GM	2,000	$5	0.1	$10,000

Total amount invested = $100,000.

Covariance and Correlation

- Covariance: $Cov(X,Y) = E[(X - E[X])E(Y - E[Y])]$

In general

(after a bit of algebra)

$$Cov\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} Cov(X_i, Y_i)$$

- Correlation: $\rho_{X,Y} = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y}$ (Between 2 RVs)

Measures the strength of the association between X and Y.

- A special case (one RV, X) of the covariance is the variance: $Cov(X,X) = Var(X) = \sigma_X^2$

Portfolio Variance and Standard Deviation

- Variance: In general, for n random variables.

$$Var\left(\sum_{i=1}^{n} \omega_i X_i\right) = \sum_{i=1}^{n} \omega_i^2 Var(X_i) + 2 \sum_{i,j:i<j} \omega_i \omega_j Cov(X_i, X_j)$$

- Portfolio Variance: Assuming N different assets.

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i} \omega_i \omega_j \rho_{i,j} \sigma_i \sigma_j$$

- Portfolio Standard Deviation (Risk): $\sigma_p = \sqrt{\sigma_p^2}$

- Example: Two securities (N=2).

$$\sigma_p^2 = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\omega_1 \omega_2 \rho_{1,2} \sigma_1 \sigma_2$$

Portfolio Variance and Standard Deviation

- **Example**: Portfolio variance with two securities. Assume $\sigma_1 = \sigma_2 = \sigma$ and $\omega_1 = \omega_2 = 1/2$.

$$\sigma_p^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{1,2}\sigma_1\sigma_2$$

$$\sigma_p^2 = \left(\frac{1}{2}\right)^2\sigma^2 + \left(\frac{1}{2}\right)^2\sigma^2 + 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\rho_{1,2}\sigma^2$$

$$= \frac{\sigma^2}{2} + \frac{\sigma^2}{2}\rho_{1,2}$$

$$= \frac{\sigma^2}{2}(1 + \rho_{1,2})$$

$$\rho_{1,2} = -1 \rightarrow \sigma_p^2 = 0$$ Risk-free, no reward

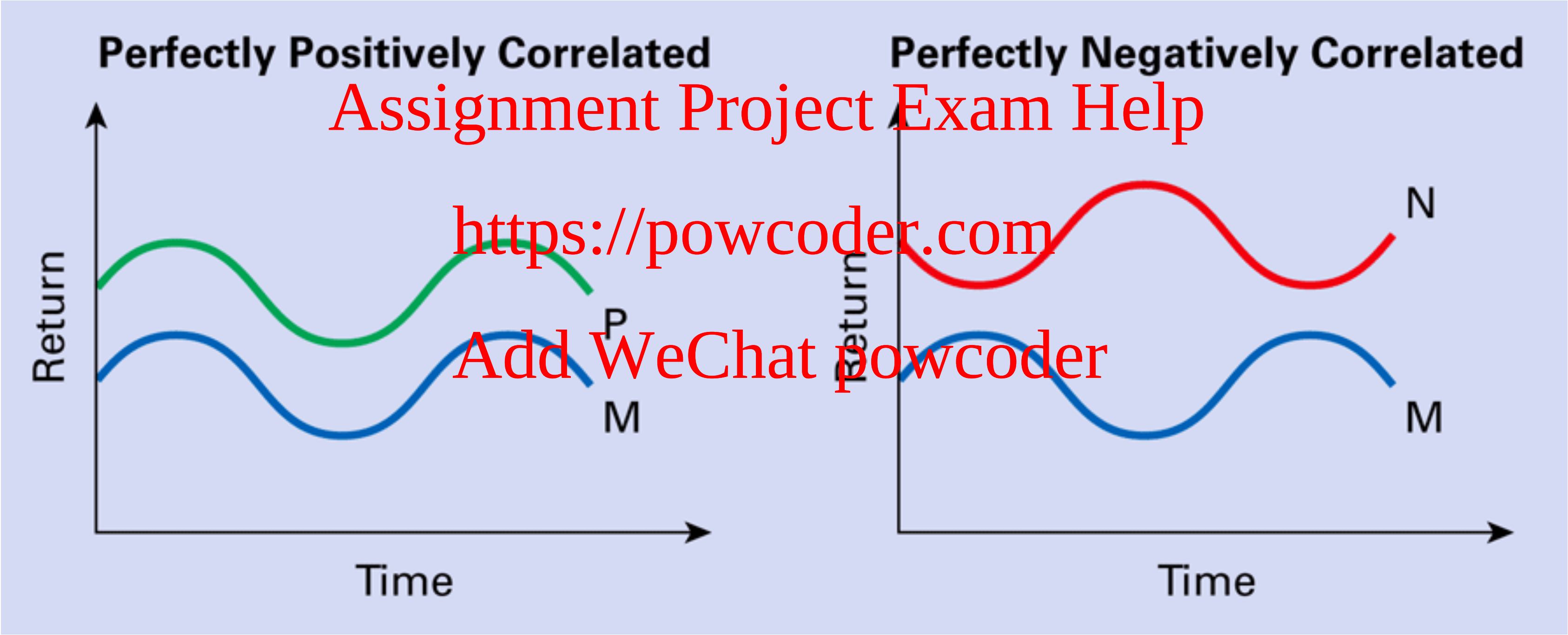
$$\rho_{1,2} = 0 \rightarrow \sigma_p^2 = \frac{1}{2}\sigma^2$$ Reduced risk

$$\rho_{1,2} = +1 \rightarrow \sigma_p^2 = \sigma^2$$ Same risk

The lower the correlation between any two assets, the greater the risk reduction.

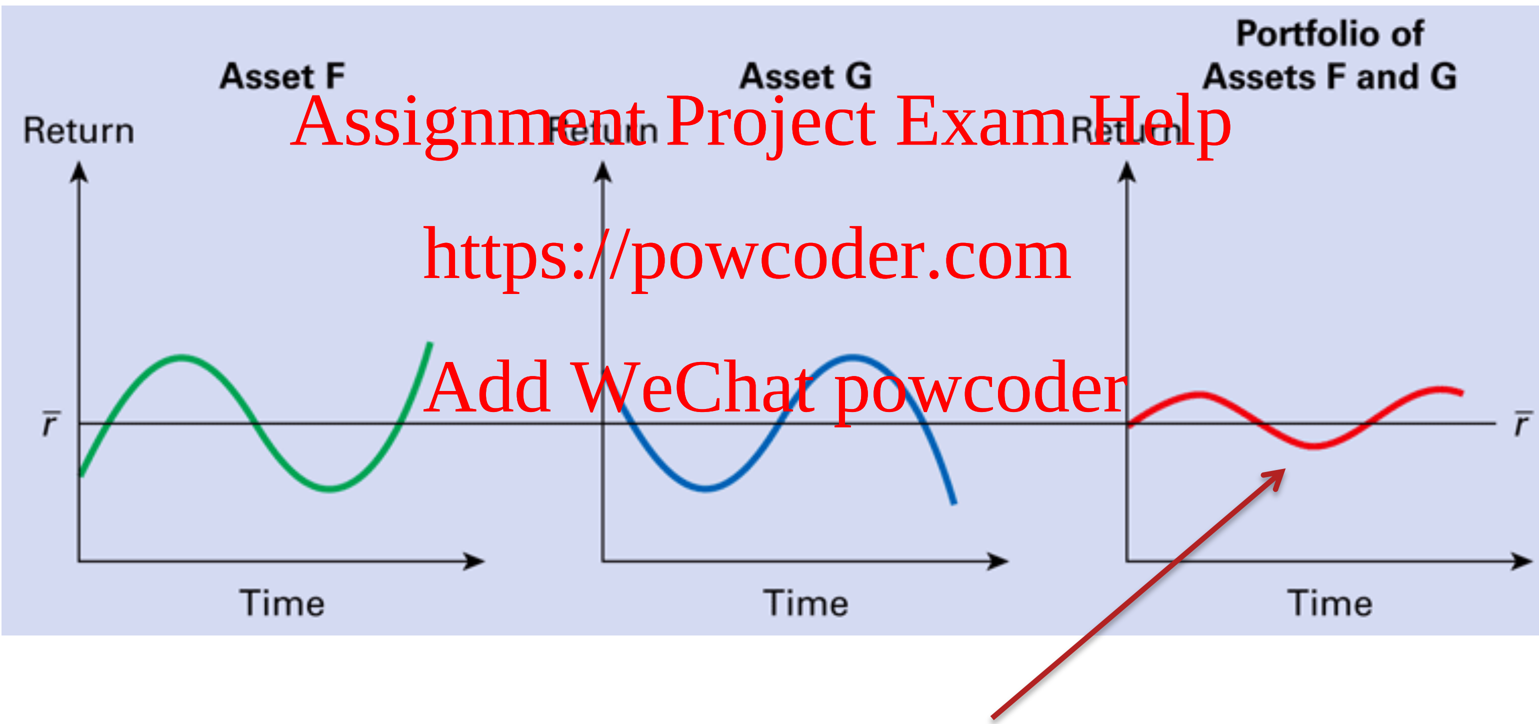
Diversification with Two Assets

The Correlation Between Series M, N, and P



Diversification with Two Assets 2 of 9

Combining Negatively Correlated Assets to Diversify Risk



Notice the reduction in the amplitude of the portfolio return curve with respect to assets F and G.

Diversification with Two Assets

- Assets that are less than perfectly positively correlated tend to offset each others movements, thus reducing the overall risk in a portfolio

- The lower the correlation the more the overall risk in a portfolio is reduced
  - Assets with +1 correlation eliminate no risk
  - Assets with less than +1 correlation eliminate some risk
  - Assets with less than 0 correlation eliminate more risk
  - Assets with -1 correlation eliminate all risk

Diversification with Two Assets

- Given a portfolio comprised of two risky assets (labeled 1 and 2) , we can compute analytically the portfolio's expected return and respective variance:

$$E[R_p] = \omega_1 E[R_1] + \omega_2 E[R_2]$$

- However, since $\omega_1 + \omega_2 = 1$, then $\omega_1 = 1 - \omega_2$. Let $\omega_1 = \omega$:

$$\rightarrow \begin{cases} E[R_p] = \omega E[R_1] + (1-\omega) E[R_2] \\ \sigma_p^2 = \omega^2 \sigma_1^2 + (1-\omega)^2 \sigma_2^2 + 2\omega(1-\omega) cov(R_1, R_2) \end{cases}$$

- Suppose you can invest in the following two risky assets: Stock_1 and Stock_2.

- How should you split your money?

Diversification with Two Assets

- Example: A ExxonMobil and Panera Bread.

	E(R)	Volatility (σ)
Stock$_1$ (XOM)	11.7%	17.2%
Stock$_2$ (PNRA)	44.4%	69.5%
$\rho_{R_1,R_2} = -0.49$		

- Q: What are feasible allocations (weights)?
- Ans: Try different weights: compute E(R_p) and σ_p each time.

Diversification with Two Assets

Fixed Weights

A. Individual and Portfolio Returns

	(1) Historical Returns*	(2)	(3) Portfolio Weights	(4) Portfolio Return
Year (t)	$r_{XOM}\%$	$r_{PNRA}\%$	$w_{XOM} = 0.86$ $w_{PNRA} = 0.14$	$r_p\%$
1999	12.6	14.8	$(0.86 \times 12.6) + (0.14 \times 14.8) =$	12.9
2000	10.2	193.8	$(0.86 \times 10.2) + (0.14 \times 193.8) =$	35.9
2001	−7.6	128.2	$(0.86 \times -7.6) + (0.14 \times 128.2) =$	11.4
2002	−8.9	33.8	$(0.86 \times -8.9) + (0.14 \times 33.8) =$	−2.9
2003	20.6	13.5	$(0.86 \times 20.6) + (0.14 \times 13.5) =$	19.6
2004	28.1	2.0	$(0.86 \times 28.1) + (0.14 \times 2.0) =$	24.4
2005	11.8	62.9	$(0.86 \times 11.8) + (0.14 \times 62.9) =$	18.9
2006	39.1	−14.9	$(0.86 \times 39.1) + (0.14 \times -14.9) =$	31.5
2007	24.3	−35.9	$(0.86 \times 24.3) + (0.14 \times -35.9) =$	15.9
2008	−13.1	45.8	$(0.86 \times -13.1) + (0.14 \times 45.8) =$	−4.9
Average Return	11.7	44.4		16.3

B. Individual and Portfolio Standard Deviations

Standard Deviation Calculation for XOM:

$$S_{XOM} = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{XOM,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(12.6 - 11.7)^2 + \ldots + (-13.1 - 11.7)^2}{10-1}} = \sqrt{\frac{2{,}669.6}{10-1}} = 17.2\%$$

Standard Deviation Calculation for PNRA:

$$S_{PNRA} = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{PNRA,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(14.8 - 44.4)^2 + \ldots + (45.8 - 44.4)^2}{10-1}} = \sqrt{\frac{43{,}403.8}{10-1}} = 69.5\%$$

Standard Deviation Calculation for Portfolio:

$$S_p = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{Port,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(12.9 - 16.3)^2 + \ldots + (-4.9 - 16.3)^2}{10-1}} = \sqrt{\frac{1{,}553.2}{10-1}} = 13.1\%$$

*Historical return is calculated based on end of year prices.

(Source: *End-of-year prices are obtained from Yahoo! Finance (prices are adjusted for dividends and stock splits).*)

$E(R_p) = 16.3$
$s_p = 13.1$

Individual and Portfolio Returns and Standard Deviation of Returns for ExxonMobil (XOM) and Panera Bread (PNRA).

A. Individual and Portfolio Returns

	(1) Historical Returns*	(2)	(3) Portfolio Weights		(4) Portfolio Return
Year (t)	$r_{XOM}\%$	$r_{PNRA}\%$	$W_{XOM} = 0.86$	$W_{PNRA} = 0.14$	$r_p\%$
1999	12.6	14.8	(0.86 × 12.6) + (0.14 × 14.8) =		12.9
2000	10.2	193.8	(0.86 × 10.2) + (0.14 × 193.8) =		35.9
2001	−7.6	128.2	(0.86 × –7.6) + (0.14 × 128.2) =		11.4
2002	−8.9	33.8	(0.86 × –8.9) + (0.14 × 33.8) =		−2.9
2003	20.6	13.5	(0.86 × 20.6) + (0.14 × 13.5) =		19.6
2004	28.1	2.0	(0.86 × 28.1) + (0.14 × 2.0) =		24.4
2005	11.8	62.9	(0.86 × 11.8) + (0.14 × 62.9) =		18.9
2006	39.1	−14.9	(0.86 × 39.1) + (0.14 × –14.9) =		31.5
2007	24.3	−35.9	(0.86 × 24.3) + (0.14 × –35.9) =		15.9
2008	−13.1	45.8	(0.86 × –13.1) + (0.14 × 45.8) =		−4.9
Average Return	11.7	44.4			16.3

B. Individual and Portfolio Standard Deviations

Standard Deviation Calculation for XOM:

$$S_{XOM} = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{XOM,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(12.6 - 11.7)^2 + \ldots + (-13.1 - 11.7)^2}{10-1}} = \sqrt{\frac{2{,}669.6}{10-1}} = 17.2\%$$

Standard Deviation Calculation for PNRA:

$$S_{PNRA} = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{PNRA,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(14.8 - 44.4)^2 + \ldots + (45.8 - 44.4)^2}{10-1}} = \sqrt{\frac{43{,}403.8}{10-1}} = 69.5\%$$

Standard Deviation Calculation for Portfolio:

$$S_p = \sqrt{\frac{\sum_{t=1999}^{2008}(r_{Port,t} - \bar{r})^2}{n-1}} = \sqrt{\frac{(12.9 - 16.3)^2 + \ldots + (-4.9 - 16.3)^2}{10-1}} = \sqrt{\frac{1{,}553.2}{10-1}} = 13.1\%$$

$E(R_p) = 16.3$

$s_p = 13.1$

*Historical return is calculated based on end of year prices.

(Source: *End-of-year prices are obtained from Yahoo! Finance (prices are adjusted for dividends and stock splits).*)

Diversification with Two Assets

Varying Weights

(1) Portfolio Weights	(2)	(3) Portfolio Average Return%	(4) Portfolio Standard Deviations%
W_{XOM}	W_{PNRA}	$\bar{r}_{XOM} = 11.7\%$ $\quad \bar{r}_{PNRA} = 44.4\%$	
1.0	0.0	$(1.0 \times 11.7) + (0.0 \times 44.4) = 11.7$	17.2
0.9	0.1	$(0.9 \times 11.7) + (0.1 \times 44.4) = 15.0$	13.5
0.8	0.2	$(0.8 \times 11.7) + (0.2 \times 44.4) = 18.2$	14.0
0.7	0.3	$(0.7 \times 11.7) + (0.3 \times 44.4) = 21.5$	18.3
0.6	0.4	$(0.6 \times 11.7) + (0.4 \times 44.4) = 24.8$	24.4
0.5	0.5	$(0.5 \times 11.7) + (0.5 \times 44.4) = 28.1$	31.4
0.4	0.6	$(0.4 \times 11.7) + (0.6 \times 44.4) = 31.3$	38.8
0.3	0.7	$(0.3 \times 11.7) + (0.7 \times 44.4) = 34.6$	46.3
0.2	0.8	$(0.2 \times 11.7) + (0.8 \times 44.4) = 37.9$	54.0
0.1	0.9	$(0.1 \times 11.7) + (0.9 \times 44.4) = 41.1$	61.7
0.0	1.0	$(0.0 \times 11.7) + (1.0 \times 44.4) = 44.4$	69.5

Example: Calculation of the Standard Deviation for the Equally Weighted Portfolio

$S_{XOM} = 17.2\%$
$S_{PNRA} = 69.5\%$
$\rho_{XOM/,PNRA} = -0.49$

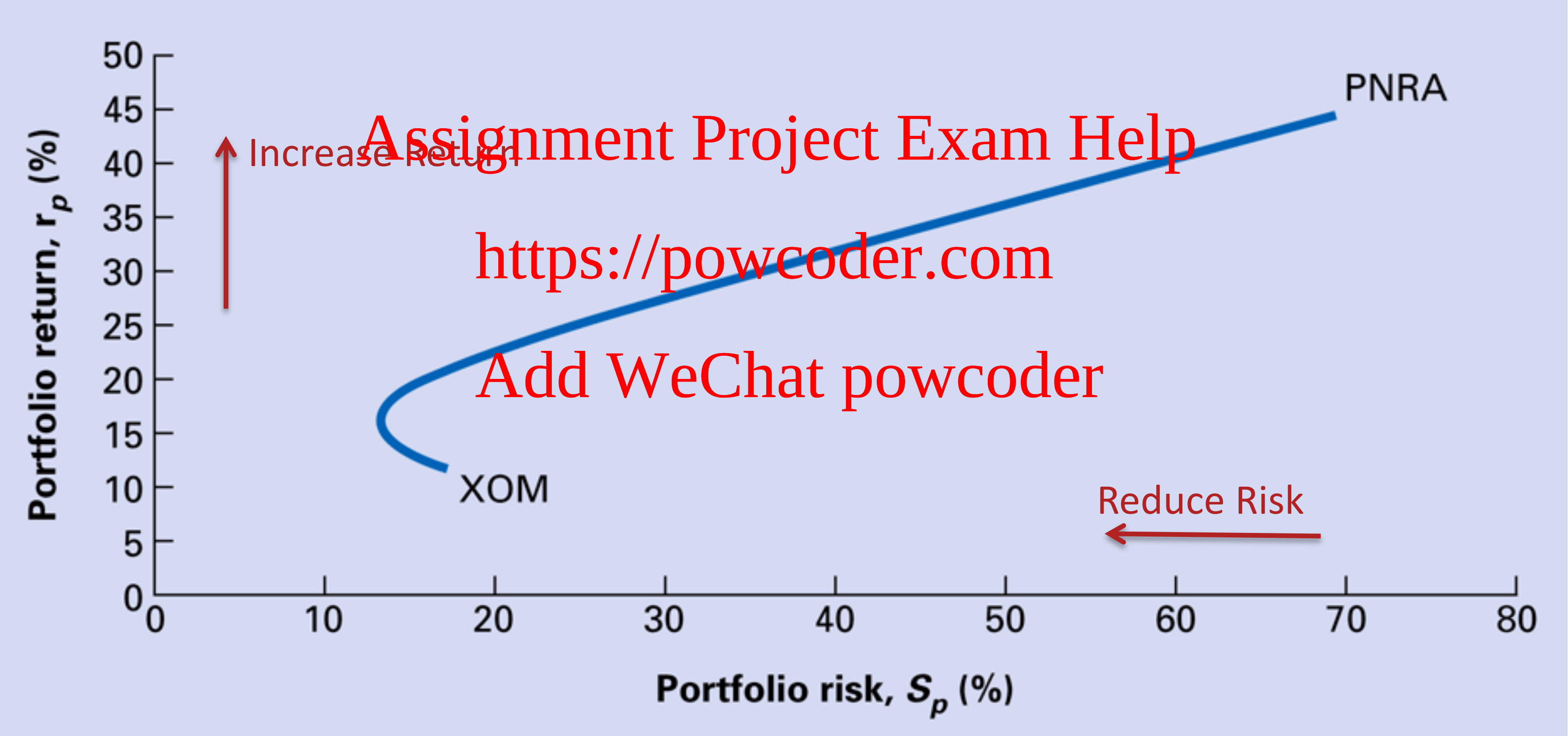
$$S_p = \sqrt{w_i^2 s_i^2 + w_j^2 s_j^2 + 2 w_i w_j \rho_{i,j} s_i s_j}$$

$$S_p = \sqrt{(0.5^2 \times 17.2^2) + (0.5^2 \times 69.5^2) + (2 \times 0.5 \times 0.5 \times -0.49 \times 17.2 \times 69.5)} = 31.4$$

Individual and Portfolio Returns and Standard Deviation of Returns for ExxonMobil (XOM) and Panera Bread (PNRA).

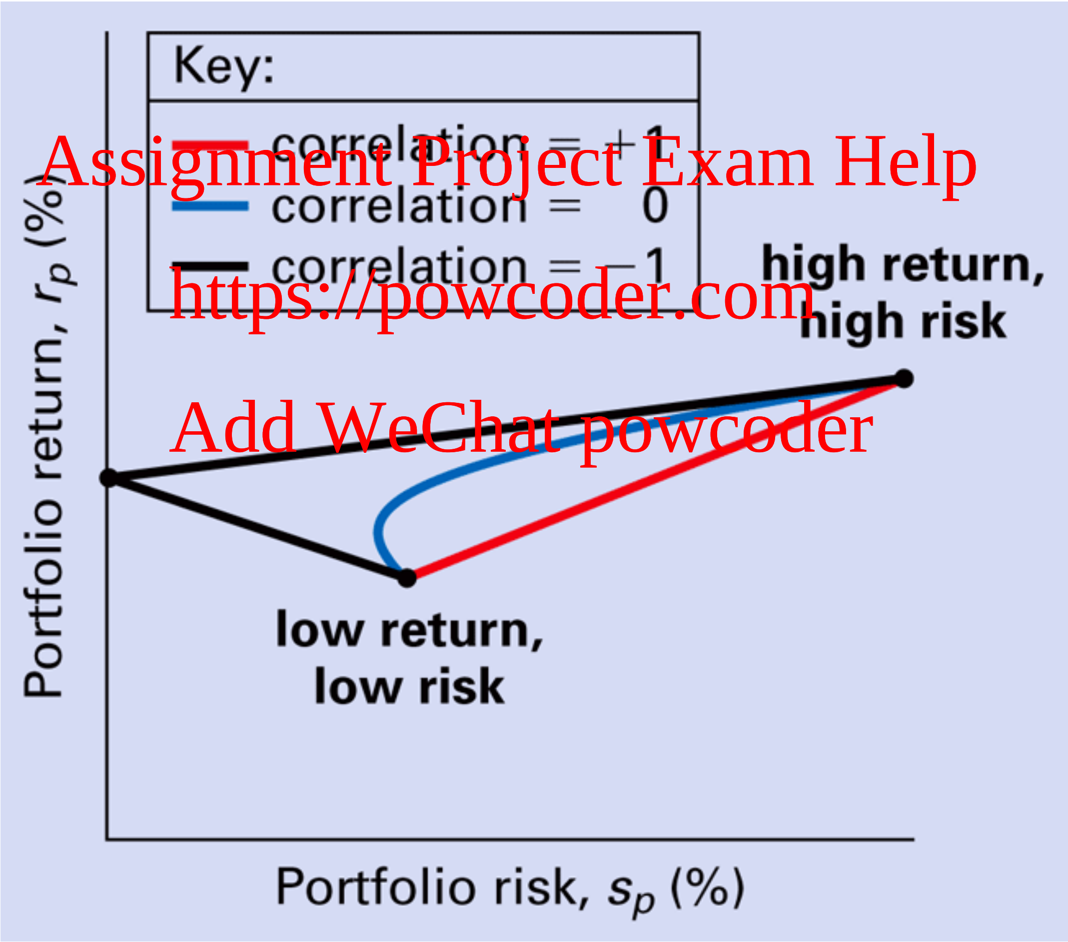
Diversification with Two Assets

Portfolios of ExxonMobil and Panera Bread



Diversification with Two Assets

The investment Opportunity Set



Diversification with Two Assets

- Portfolio Optimizer Demo (see Excel File)
- Mathematical details of portfolio return/std. dev calculations (we will go over this on the blackboard)

Economics 403A
Review of MPT and CAPM

Part II
Investor Preferences and Portfolio Choice with a Riskless Security

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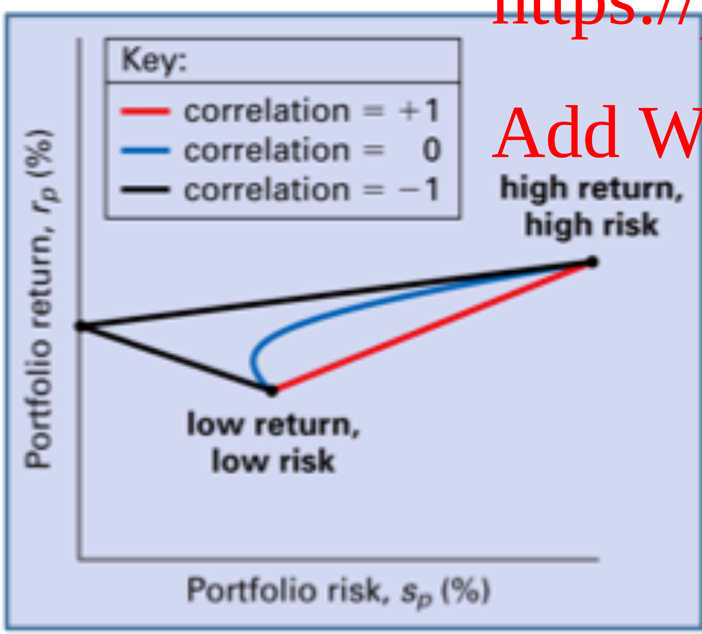
Today's Class (Part IIa)

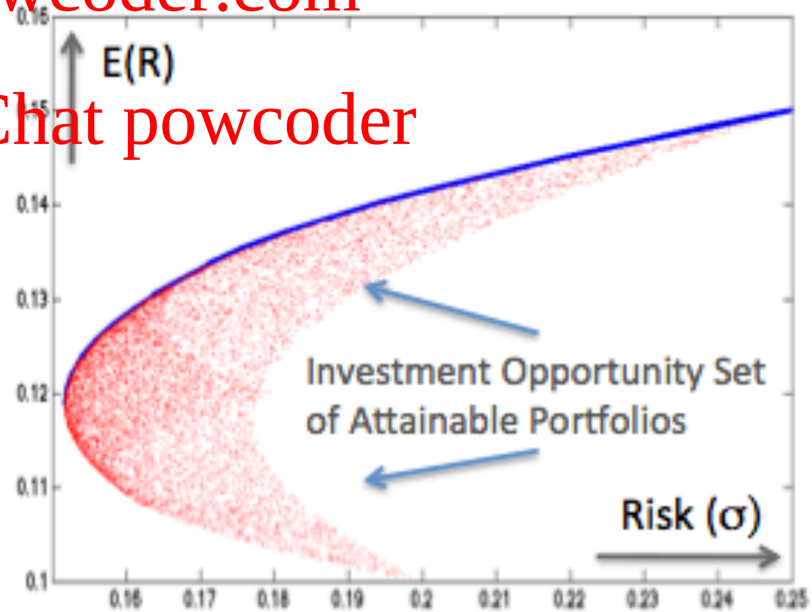
- The Investment Opportunity Set
- The Principle of Dominance
- Portfolio Terminology
- The Efficient Frontier
- Risk-Return Tradeoff
- Indifference Curve
  – Marginal Utility
  – Indifference Curve
  – Indifference Curve in Finance: (σ, E(R)) Diagram

The Investment Opportunity Set

- Investment Opportunity Set: Set of feasible expected return and standard deviation pairs of all portfolios resulting from different values of y (proportion of the investment budget).

Investment Opportunity Set with Varying Correlations





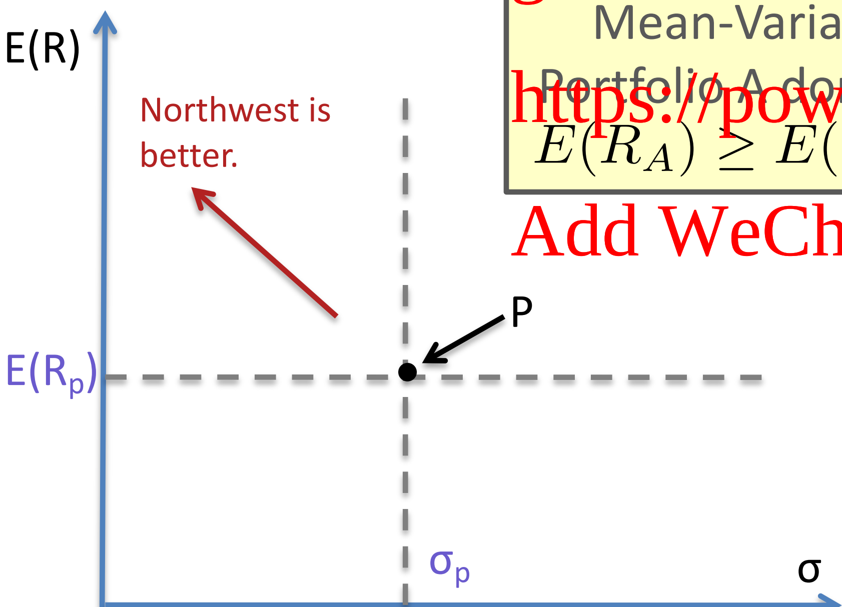
The Dominance Principle

- Among all investments with a given return, the one with the least risk is desirable; or given the same level of risk, the one with the highest return is most desirable.

Mean-Variance (M-V) Criterion

Portfolio A dominates B if:

$E(R_A) \geq E(R_B)$ and $\sigma_A \leq \sigma_B$.



- Same mean, prefer lower std. deviation.
- Same std. deviation, prefer higher mean.

Dominance Principle Example

Security	$E(R_i)$	σ
ATW	7%	3%
GAC	7%	4%
YTC	15%	15%
FTR	3%	3%
HTC	8%	12%

ATW dominates GAC

ATW dominates FTR

Portfolio Terminology

- The investment opportunity set consists of all available risk-return combinations.

- An efficient portfolio is a portfolio that has the highest possible expected return for a given standard deviation (i.e., a portfolio that is not dominated).

- The efficient frontier is the set of efficient portfolios. It is the upper portion of the minimum variance frontier starting at the minimum variance portfolio.

- The minimum variance portfolio (mvp) is the portfolio that provides the lowest variance (standard deviation) among all possible portfolios of risky assets.

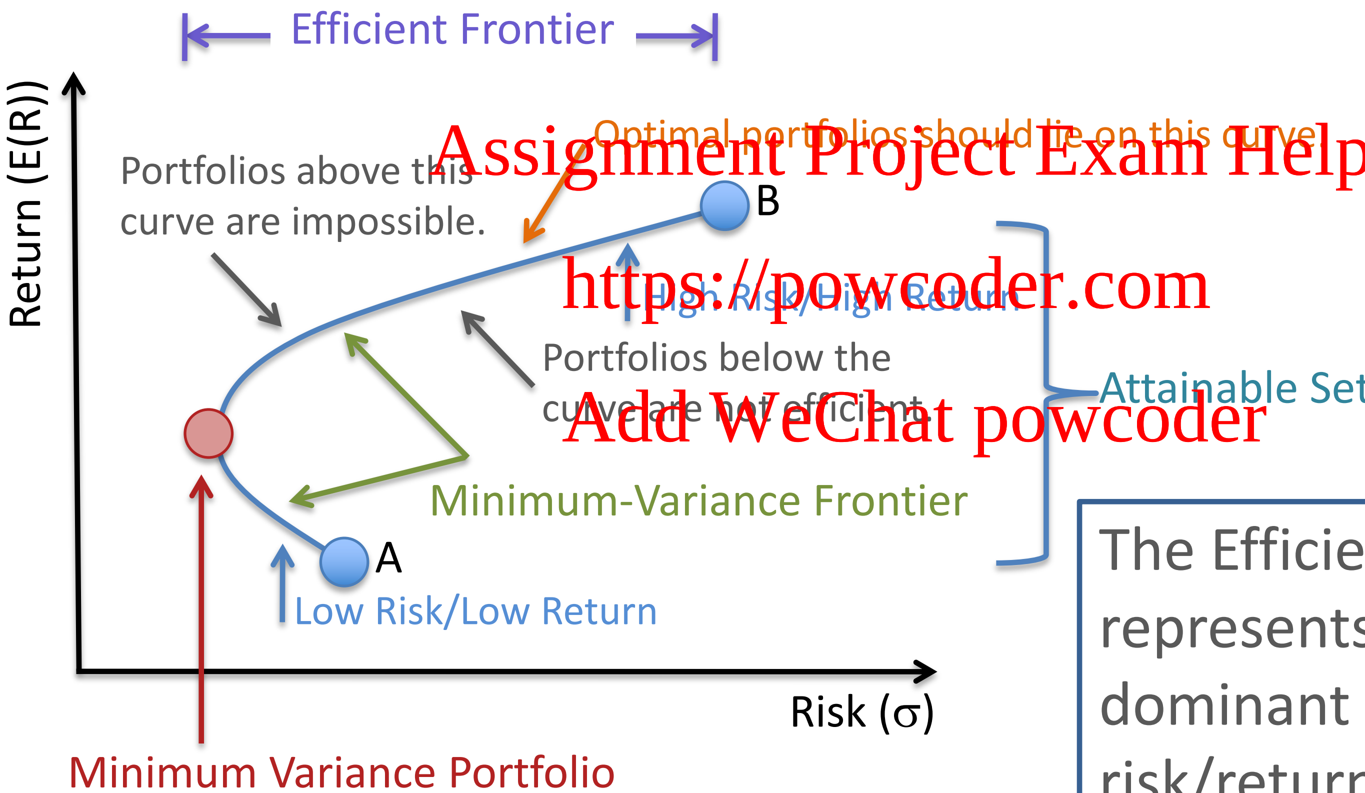
Optimal Portfolio Choice with Two Risky Assets

- Any (mean-variance) investor should choose an *efficient portfolio* to benefit from diversification.

- The specific choice depends on the investor's *risk aversion*.

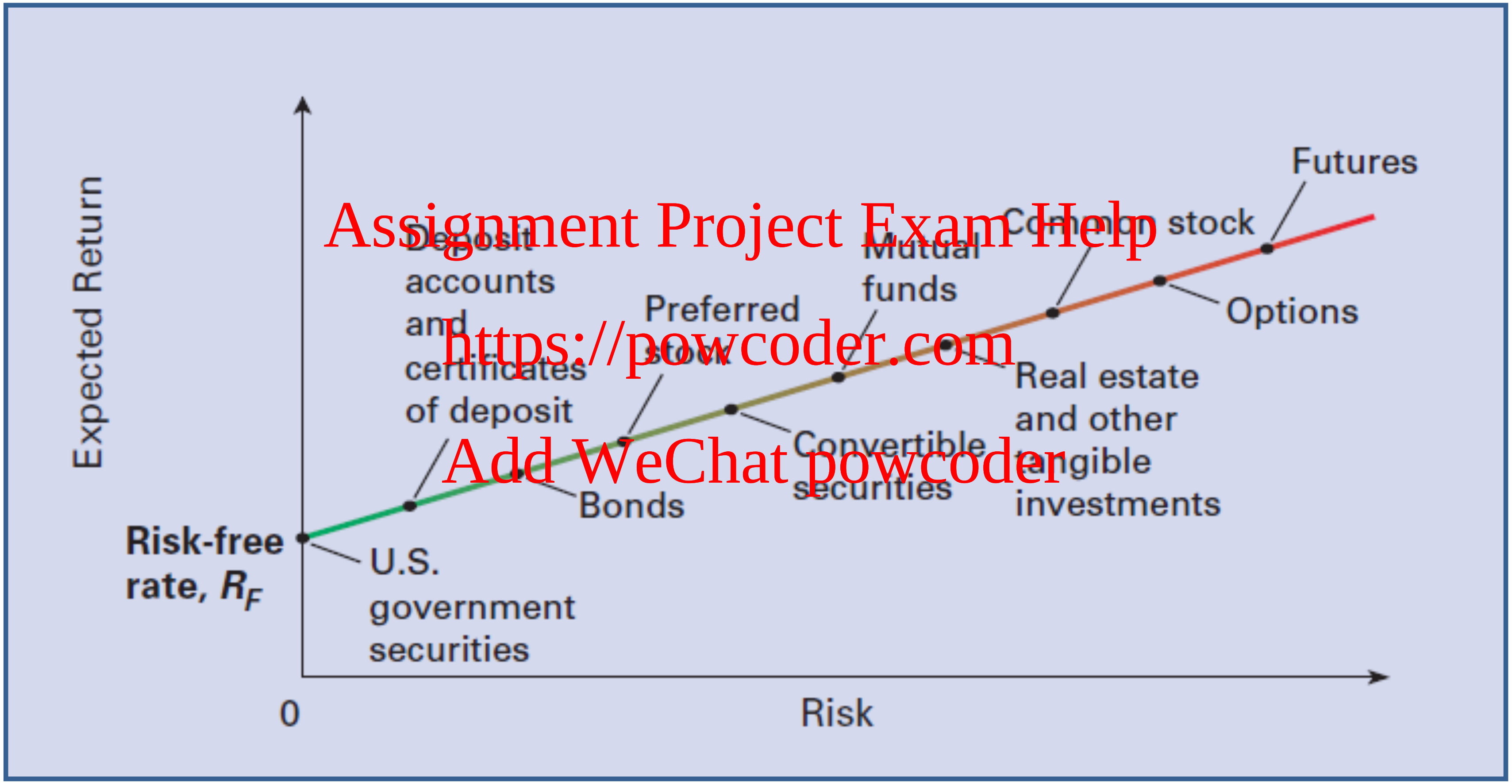
- A more risk-averse investor should choose a portfolio with:
  - Lower risk
  - Lower expected return

Efficient Frontier



The Efficient Frontier represents all the dominant portfolios in risk/return space.

Risk-Return Tradeoff 1 of 3



Risk-Return Tradeoff

- Recall two of the Finance Axioms:
  - Investors prefer more to less
  - Investors are risk-averse

- This means that investors prefer an investment :
  - with a higher expected return $E(R_i)$
  - with a lower variance and standard deviation, σ_i

- Investors must trade-off risk and return in order to *maximize* their expected utility.

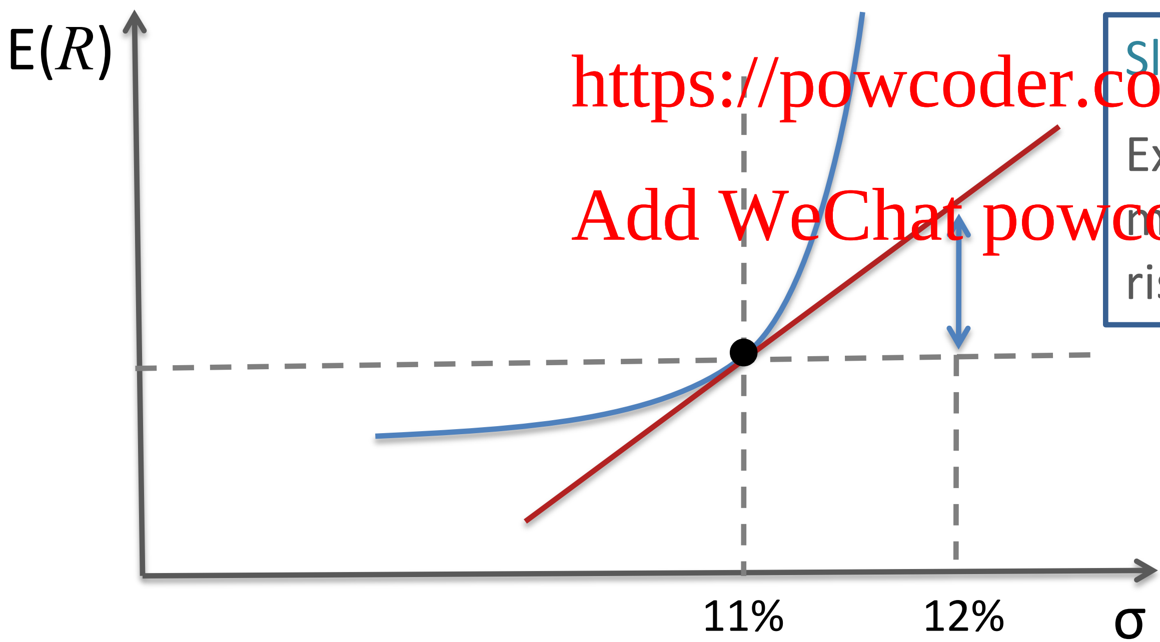
- Utility: Is a measure of satisfaction, referring to the total satisfaction received by a consumer from consuming a good or service.

Today's Class (Part IIb)

- Marginal Rate of Substitution
- Marginal Rate of Transformation
- The Risk-Free Asset
- Portfolio Selection
  - One risk-free security, one risky security
- Capital Allocation Line (CAL)
- Short Sales
- Tangent (or Efficient) Portfolio
- Optimal Portfolio Selection
  - Two Risky Assets and One Risk-Free Asset

Marginal Rate of Substitution (MRS)

- MRS (Slope of the Indifference Curve): How much extra return an investor needs to receive for bearing an extra unit of risk while keeping the same level of utility.

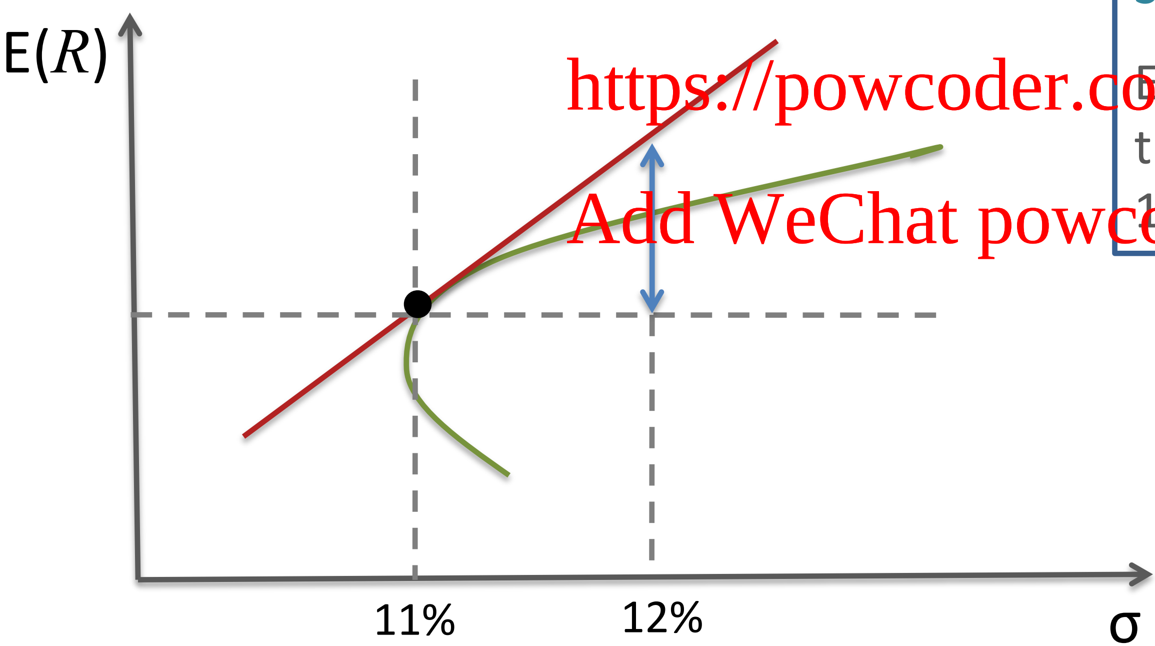


Slope of the Indifference Curve:

Extra return this investment must receive to bear 1% more risk and be as happy as before.

Marginal Rate of Transformation (MRT)

- MRT (Slope of the Efficient Frontier): How much extra return the market gives for bearing an extra unit of risk.

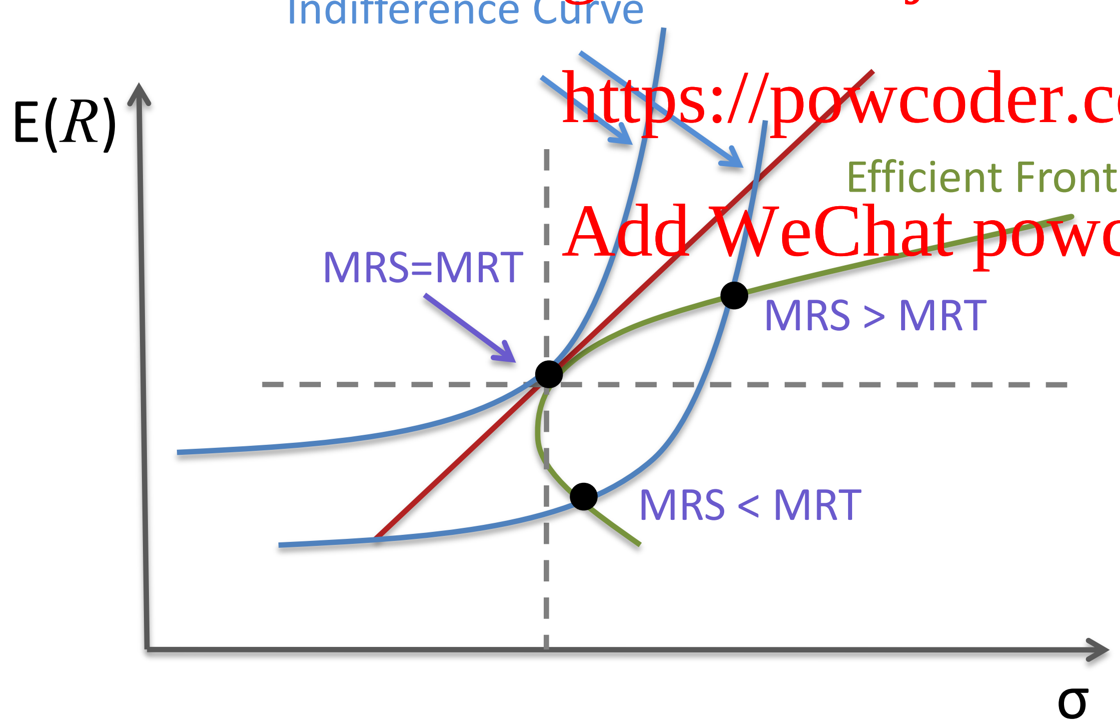


Slope of the Efficient Frontier:

Equals the maximum E(R) that the market gives me if I take 1% more risk.

MRS and MRT Together

- Optimal Portfolio: The optimal portfolio is the portfolio such that MRS=MRT.



Steeper portfolio
→More to the left

Flatter portfolio
→More to the right

Risk-Obsessed
→Vertical

The Risk-Free Asset

- Risk-Free Asset: An asset which has a certain future return. Treasuries (especially T-bills) are considered to be risk-free because they are backed by the U.S. government.

- Risk-Free Return is denoted R_f

- Risk-Free assets usually have a low rate of return.

- The Risk-Free Return is known for sure:
  – $E(R_f) = R_f$
  – $\sigma_f^2 = 0$
  – $Cov(R_f, R_i) = \rho_{f,I} = 0$ for any asset i.

A Portfolio with a Risk-Free Asset and one Risky Asset

- Let ω be the fraction of wealth invested in the risky asset (the rest is invested in the risk-free asset).

- Expected portfolio return:

$$E[R_p] = \omega\, E[R_i] + (1-\omega) R_f = R_f + \omega\, E[R_i - R_f]$$

- Variance of the portfolio return:

$$\sigma_p^2 = \omega^2\, \sigma_i^2$$

- Standard deviation of the portfolio return:

$$\sigma_p = |\omega|\, \sigma_i$$

A Portfolio with a Risk-Free Asset and one Risky Asset 2 of 3

- Since $E(R_f) = R_f$, $\sigma_f^2 = 0$, and $\rho_{f,I} = 0$, when we combine it with R_i in the portfolio, we obtain:

$$R_p = \omega R_i + (1\text{-}\omega)R_f \qquad \text{(Eq. 1)}$$

- Take the expectation on both sides of Eq. 1

$$E[R_p] = \omega E[R_i] + (1-\omega)R_f]$$

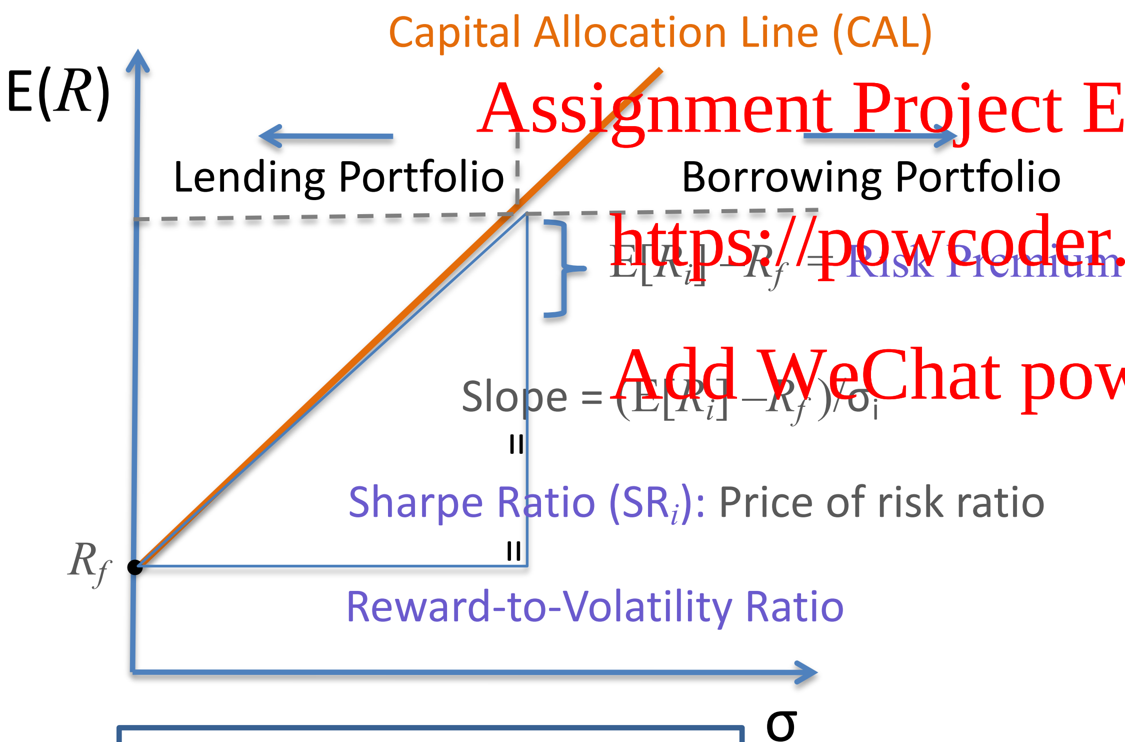
- If $\omega > 0$, then $|\omega| = \omega \rightarrow \sigma_p = \omega\, \sigma_i \rightarrow \omega = \sigma_p / \sigma_i$

- Replace ω into Eq. 1 $\rightarrow$

$$E[R_p] = R_f + \sigma_p \boxed{(E[R_i] - R_f)/\sigma_i} \quad \text{Sharpe Ratio (Eq. 2)}$$

Capital Allocation Line (CAL)

Investment Opportunity Set with
a Risky Asset and a Risk-Free Asset



$$E[R_p] = R_f + \sigma_p\,(E[R_i] - R_f)/\sigma_i$$
$$= R_f + SR_i\,\sigma_p$$

Risk-Return Relationship

Example:

Risky Asset (e.g., US stock market)

$E[R_{US}] = 13.55\%$, $\sigma_{US}=15.35\%$

Risk-Free: $R_f = 7\%$

$E[R_p] = 0.07 + SR\,\sigma_p$

$\rightarrow SR = (E[R_{US}] - R_f)/\sigma_{US}$
$= (0.1355-0.07)/0.1535$
$= 0.427$

You get 0.427 extra return per unit of risk.

Short Sales

- Long Position: Positive investment in a security.
- Short Sale: A transaction in which you sell a stock today that you do not own*, with the obligation to buy it back in the future.
  - Short selling is an advantageous strategy if you expect a stock price to decline in the future.
- Short Position: Negative investment in a security via a short sale.

*For example, you contact your broker, who will try to borrow it from someone who currently owns it. See Berk & DeMarzo page 254 (Mechanics of a Short Sale).

Short Sales 2 of 4

- Example: Suppose you have \$20,000 in cash to invest. You decide to short sale \$10,000 worth of Coca-Cola stock and invest the proceeds from your short sale, plus your \$20,000, in Intel. What is the expected return and volatility of your portfolio?

Stock	Expected Return	Volatility
Intel	26%	50%
Coca-Cola	6%	25%

Think of the short sale as a negative investment of -\$10,000 in Coca-Cola.

Therefore, we invested +\$30,000 in Intel.
→ total net investment =\$30,000-\$10,000=\$20,000

$$\omega_I = \frac{\text{Value of investment in Intel}}{\text{Total value of portfolio}} = \frac{30{,}000}{20{,}000} = 150\%$$

$$\omega_C = \frac{\text{Value of investment in Coca-Cola}}{\text{Total value of portfolio}} = \frac{-10{,}000}{20{,}000} = -50\%$$

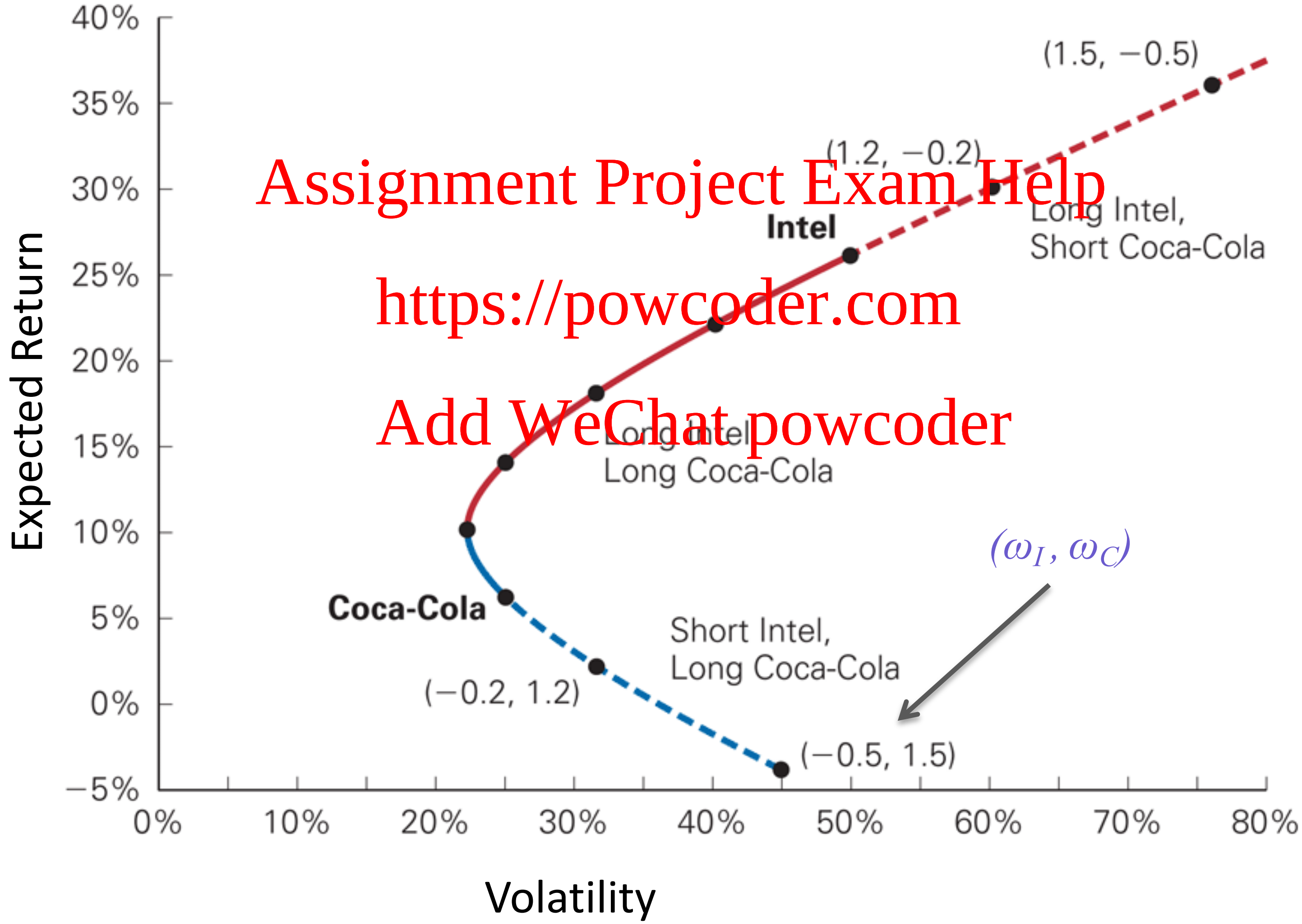
Weights still add up to 100%

$E[R_p] = \omega_I E[R_I] + \omega_C E[R_C] = 1.50\ 0.26 + (-0.5)\ 0.06 = 0.36 = 36\%$ (Increase return)

$\sigma_p = (\omega^2_I \sigma^2_I + \omega^2_C \sigma^2_C + 2\omega_I \omega_C\, cov(R_I, R_C))^{1/2} = 76.0\%$ (Increase volatility)

Short Sales

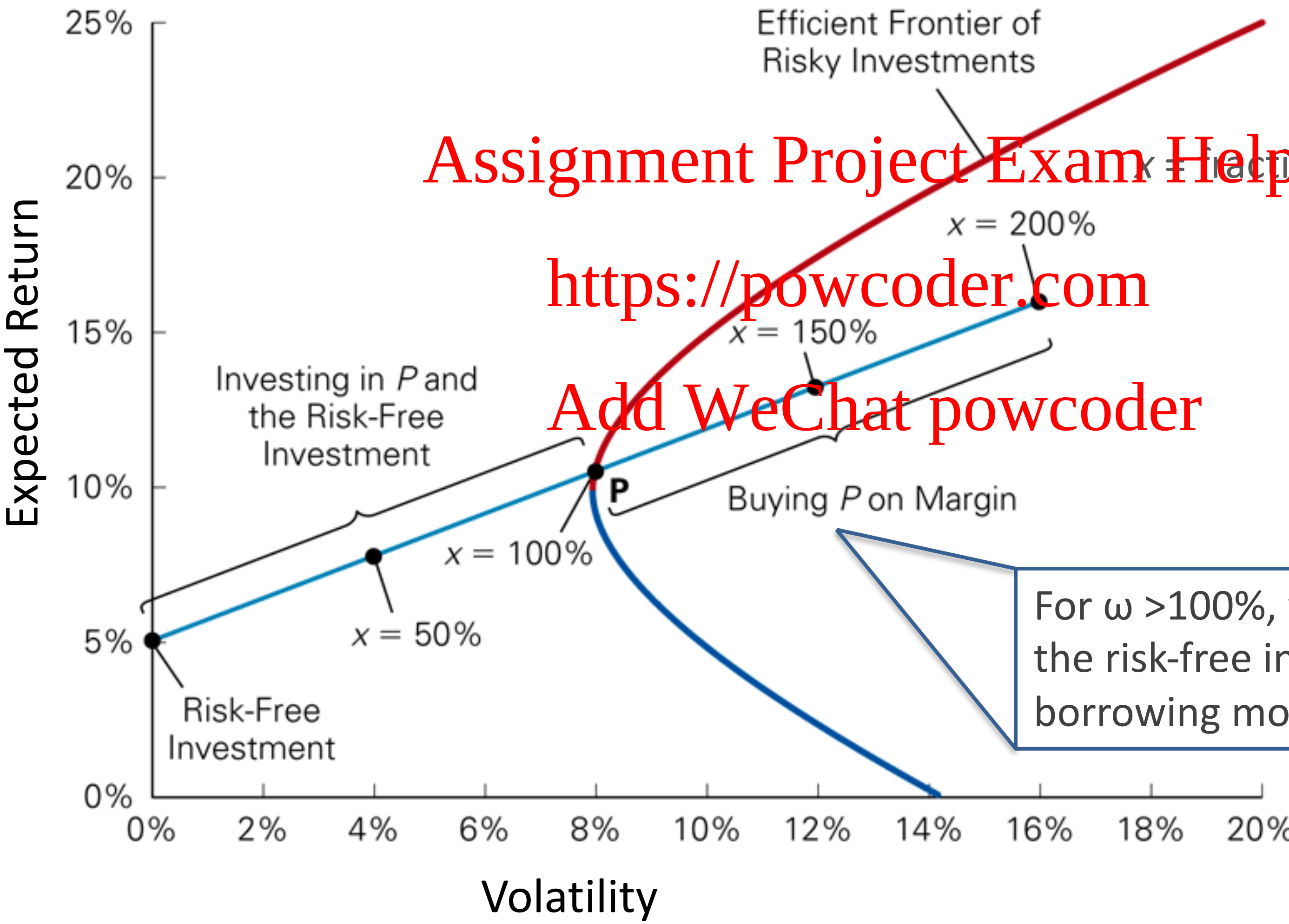
Portfolios of Intel and Coca-Cola Allowing for Short Sales



Short Sales

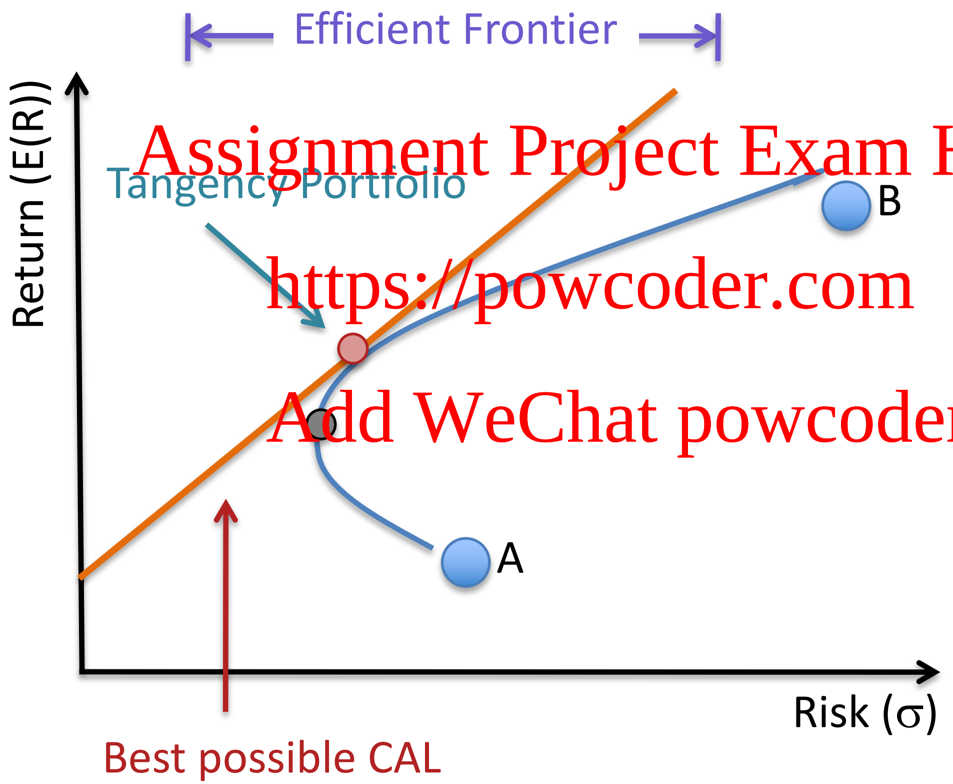
- Buying Stocks on Margin (leverage): Borrowing money to invest in stocks.

x = fraction invested in portfolio P.



For ω >100%, we are short selling the risk-free investment, i.e., borrowing money at the risk-free rate.

Tangent (or Efficient) Portfolio 1 of 10



Tangent (or Efficient) Portfolio

- Tangency Portfolio: Is the portfolio with the highest Sharpe ratio.

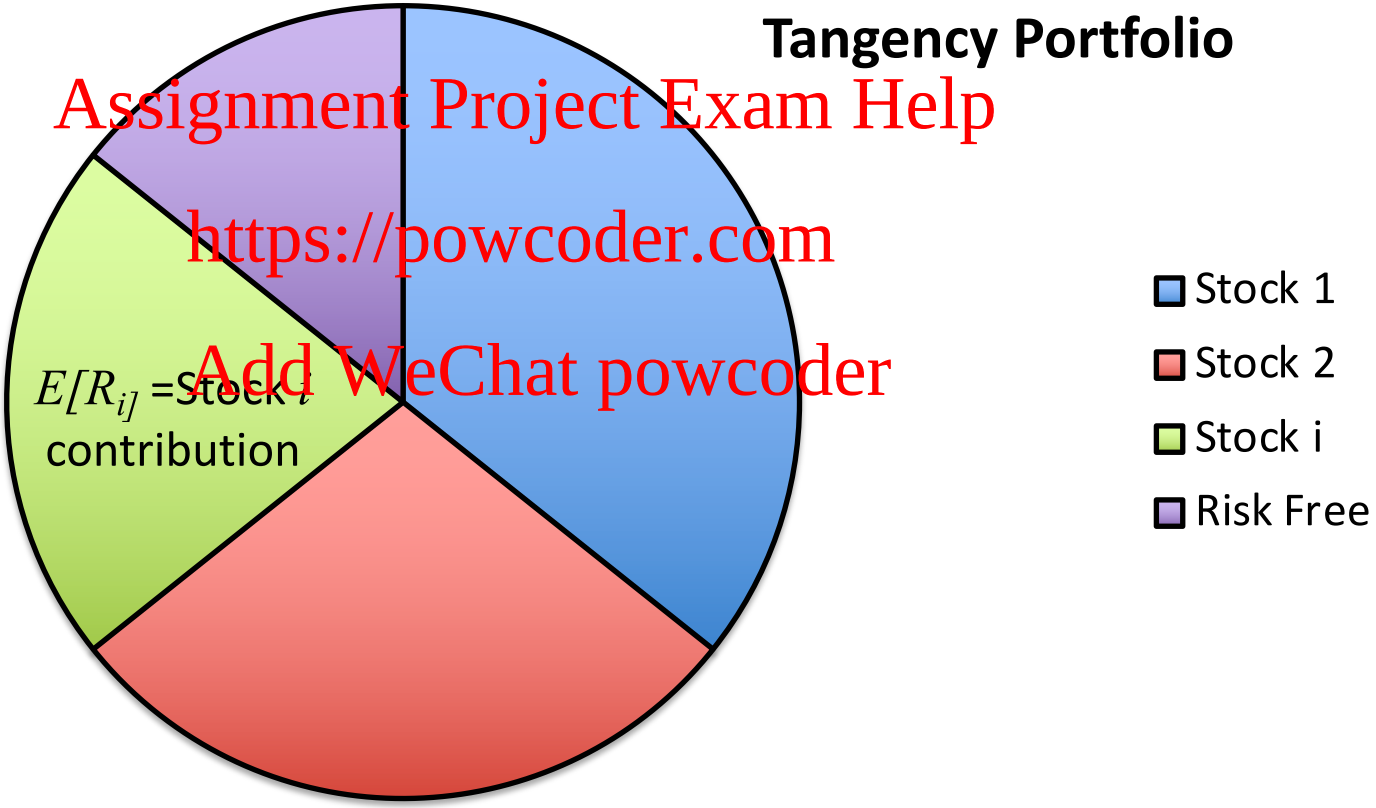
$$SR = \frac{E(R_T) - R_f}{\sigma_T}$$

- The Marginal Sharpe Ratio condition of the tangency portfolio is given by:

$$\frac{E(R_i) - R_f}{cov(R_i, R_T)} = \frac{E(R_j) - R_f}{cov(R_j, R_T)}, \forall i, j$$

- Implications:
  - We find the tangency portfolio by picking the weights so the preceding condition holds.
  - The condition holds for all individual securities and for all portfolios.

Tangent (or Efficient) Portfolio



Tangent (or Efficient) Portfolio

- $Cov(R_i, R_T)$ = Covariance between the portfolio return and the return of one of the underlying assets.

  = the marginal contribution to risk of this asset.

  = the marginal benefit to the marginal cost.

- If the marginal of contribution to portfolio risk is not equal between all the assets in the MVP, then you can do strictly better by putting a little more weight on an asset with lower marginal risk and a little less weight on an asset with higher marginal risk.

Tangent (or Efficient) Portfolio

- The Marginal Sharpe Ratio condition of the tangency portfolio is given by:

$$\frac{E(R_i) - R_f}{cov(R_i, R_T)} = \frac{E(R_j) - R_f}{cov(R_j, R_T)}, \quad \forall\, i\,, j$$

- Since the tangent portfolio has the highest Sharpe ratio, we cannot increase its expected return while keeping the variance constant.

→ Marginal contribution to reward - to - marginal contribution to risk ratios are the same for all assets

Tangent (or Efficient) Portfolio

- Since risk premium to covariance ratio equality holds for all assets, it must also hold for itself.

$$\frac{E(R_i) - R_f}{cov(R_i, R_T)} = \frac{E(R_T) - R_f}{cov(R_T, R_T)}$$

- Recall that $cov(R_T, R_T) = \sigma_T^2$. Therefore,

$$\frac{E(R_i) - R_f}{cov(R_i, R_T)} = \frac{E(R_T) - R_f}{\sigma_T^2} \quad \text{(Eq. 1)}$$

Tangent (or Efficient) Portfolio

- We can use the previous expression (Eq. 1) to derive a relation between an asset's expected return and risk.

$$\frac{E(R_i) - R_f}{cov(R_i, R_T)} = \frac{E(R_T) - R_f}{\sigma_T^2}$$

$$E(R_i) - R_f = cov(R_i, R_T)\frac{E(R_T) - R_f}{\sigma_T^2}$$

$$E(R_i) = R_f + \frac{cov(R_i, R_T)}{\sigma_T^2}[E(R_T) - R_f]$$

- How much an asset covaries with tangency portfolio determines how risky the asset is.

$$\left.\frac{cov(R_i, R_T)}{\sigma_T^2}\right\}$$

Commonly known as Beta (β)

Tangent (or Efficient) Portfolio

- The relation between expected return and risk with the tangency portfolio is typically written as:

$$E(R_i) = R_f + \beta_{i,T}[E(R_T) - R_f]$$

where $\beta_{i,T} = \dfrac{cov(R_i, R_T)}{\sigma_T^2}$

- Differences in expected returns are explained by differences in $\beta_{i,T}$
- The relation between expected return and $\beta_{i,T}$ is linear.
  - The risk-free rate is the y-intercept.
  - The risk premium of the tangency portfolio is the slope.

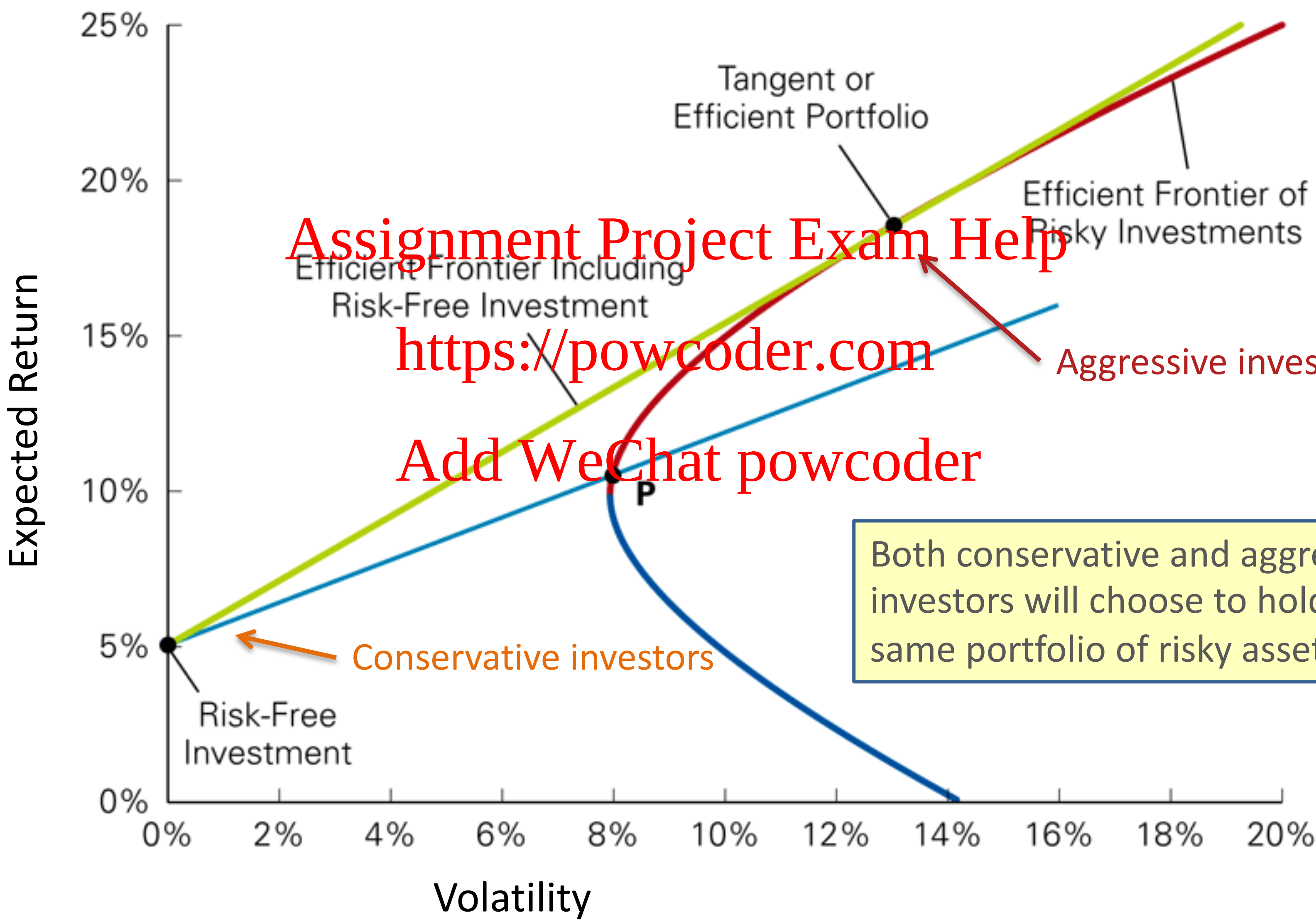
Tangent (or Efficient) Portfolio

- The portfolio with the highest Sharpe ratio is the portfolio where the line with the risk-free investment is tangent to the efficient frontier of risky investments. The portfolio that generates this tangent line is known as the tangent portfolio.

- The tangent portfolio provides the biggest reward per unit of volatility of any portfolio available. Therefore combinations of the risk-free asset and the tangent portfolio provide the best risk and return trade-off available to an investor!

  - The tangent portfolio is efficient.
  - Every investor should invest in the tangent portfolio independent of his or her taste for risk.

Tangent (or Efficient) Portfolio



Both conservative and aggressive investors will choose to hold the same portfolio of risky assets.

Optimal Portfolio Selection with Two Risky Assets and One Risk-Free Asset

- Create the set of possible mean-std. dev combinations from different portfolios of risky assets.

- Find the tangent portfolio, that is, the portfolio with the highest Sharpe ratio:

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

- Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

Economics 403A
Review of MPT and CAPM

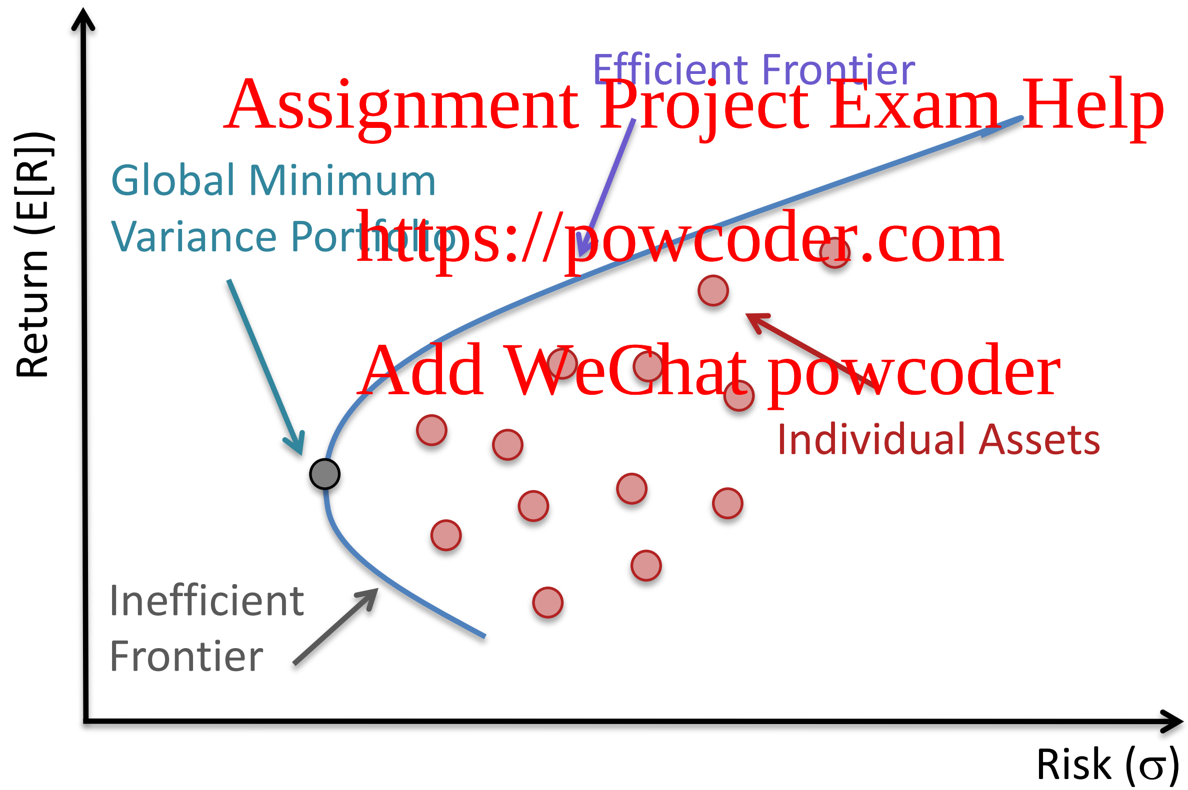
Part III
Portfolio Returns with Multiple Risky Securities

Dr. Randall R. Rojas

Today's Class (Part III)

- Investment Opportunity Set
- Optimal Portfolio Choice and Two Fund Separation
- Risk Reduction in Equally-Weighted Portfolios
  - Independent Returns
- Risk in Equally-Weighted Portfolios
  - The General Case
- Classification of Risk
  - Diversifiable Risk
  - Non-Diversifiable Risk
- Portfolio Optimizer (Review)

Investment Opportunity Set with Many Assets



Optimal Portfolio Selection with Many Risky Assets and a Risk-Free Asset

- Create the set of possible mean-std. dev combinations from different portfolios of risky assets.

- Find the tangent portfolio, that is, the portfolio with the highest Sharpe ratio:

$$SR_i = \frac{E[R_i] - R_f}{\sigma_i}$$

- Choose the combination of the tangency portfolio and the risk-free asset to suit your risk-return preferences.

Two-Fund Separation

- All investors hold combinations of the same two "mutual funds":
  - The risk-free asset
  - The tangency portfolio

- An investor's risk aversion determines the fraction of wealth invested in the risk-free asset

- But, all investors should have the rest of their wealth invested in the tangency portfolio.

Risk Reduction in Equally-Weighted Portfolios: Independent Returns

- Recall that in general, the variance of a portfolio with N assets is given by:

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} \omega_i \omega_j Cov(R_i, R_j)$$

- Suppose we have an equally weighted portfolio (holding weights $\omega_i = 1/N$) of N independent stocks:

Indpendent stocks
$\rightarrow Cov(R_i, R_j) = 0$

$$\sigma_p^2 = \sum_{i=1}^{N} \frac{1}{N^2} \sigma_i^2 + \cancel{2 \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{1}{N^2} Cov(R_i, R_j)}$$

Risk Reduction in Equally-Weighted Portfolios: Independent Returns 2 of 2

- The variance of the portfolio return is:

$$\sigma_p^2 = \sum_{i=1}^{N} \frac{1}{N^2} \sigma_i^2$$

$$\sigma_p^2 = \frac{1}{N} \boxed{\left(\sum_{i=1}^{N} \frac{1}{N} \sigma_i^2 \right)}$$

Average variance

$$\boxed{\sigma_p^2 = \frac{1}{N} \bar{\sigma}^2} \longrightarrow \lim_{N \to \infty} \sigma_p^2 \to 0$$

- As the number of assets increase, the risk is diversified away. (The *insurance principle.)*.

Risk in Equally-Weighted Portfolios: The General Case

- Recall that in general, the variance of a portfolio with N assets is given by:

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} \omega_i \omega_j Cov(R_i, R_j)$$

- Suppose we have an equally weighted portfolio (holding weights $\omega_i = 1/N$) of N independent stocks:

$$\sigma_p^2 = \sum_{i=1}^{N} \frac{1}{N^2} \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j>i}^{N} \frac{1}{N^2} Cov(R_i, R_j)$$

Note that: $\frac{2}{N^2} = \frac{N-1}{N} \times \frac{1}{\frac{N(N-1)}{2}}$

Risk in Equally-Weighted Portfolios: The General Case 2 of 3

- The variance of the portfolio return is:

$$\sigma_p^2 = \frac{1}{N}\overline{\sigma}^2 + \left(1 - \frac{1}{N}\right)\left[\frac{1}{N(N-1)/2}\sum_{i=1}^{N}\sum_{j>i}^{N} Cov(R_i, R_j)\right]$$

$$\boxed{\sigma_p^2 = \frac{1}{N}\overline{\sigma}^2 + \left(1 - \frac{1}{N}\right)\overline{Cov}(R_i, R_j)}$$

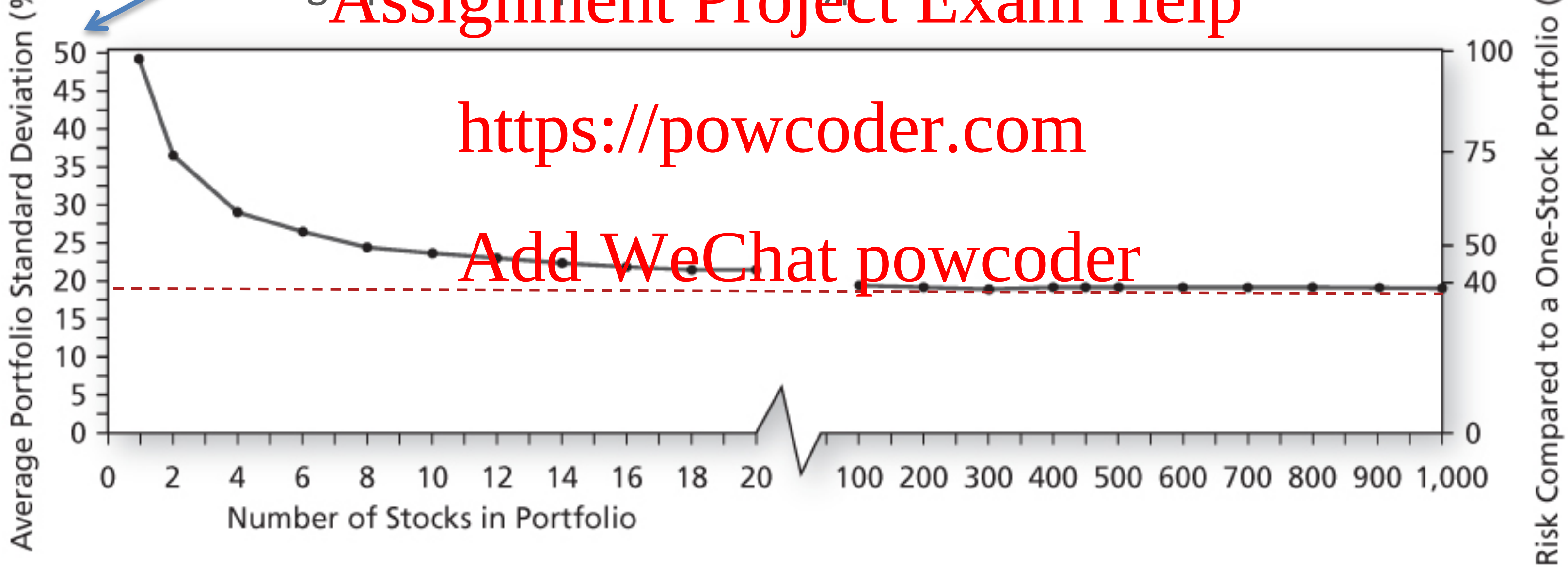
$$\longrightarrow \lim_{N \to \infty} \sigma_p^2 \to \overline{Cov}(R_i, R_j)$$

- As the number of assets increase, the portfolio risk → *non-diversifiable* risk.

Risk in Equally-Weighted Portfolios: The General Case

- What is the percentage reduction in risk we should expect from adding stocks to our portfolio?

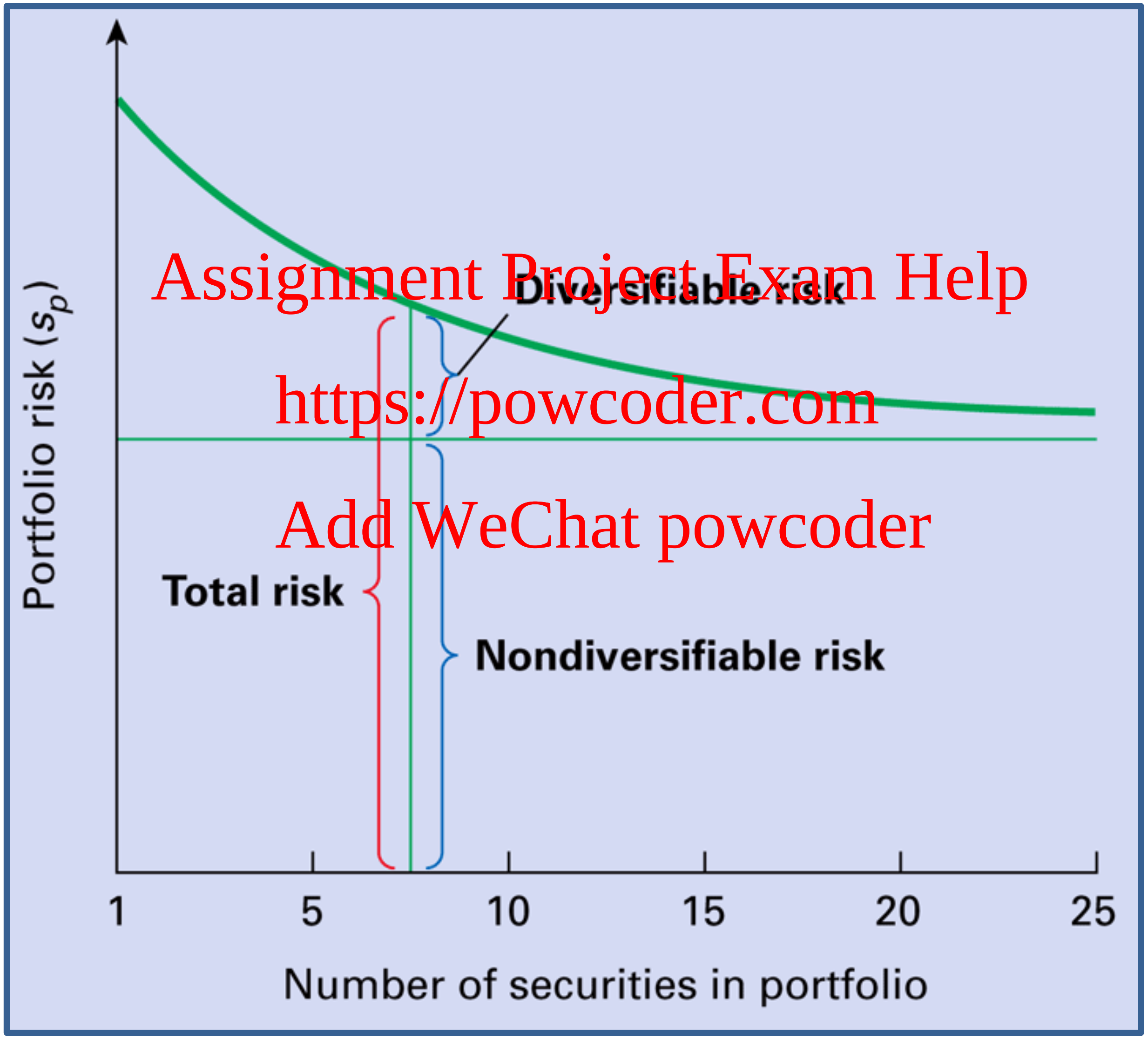
In the graph 50% represents the typical std. dev of a US stock.



Average std. dev of equally weighted portfolios constructed by selecting stocks at random as a function of the the number of stocks in a portfolio.

In the limit, portfolio risk could be reduced to only 19.2%

Classification of Risk

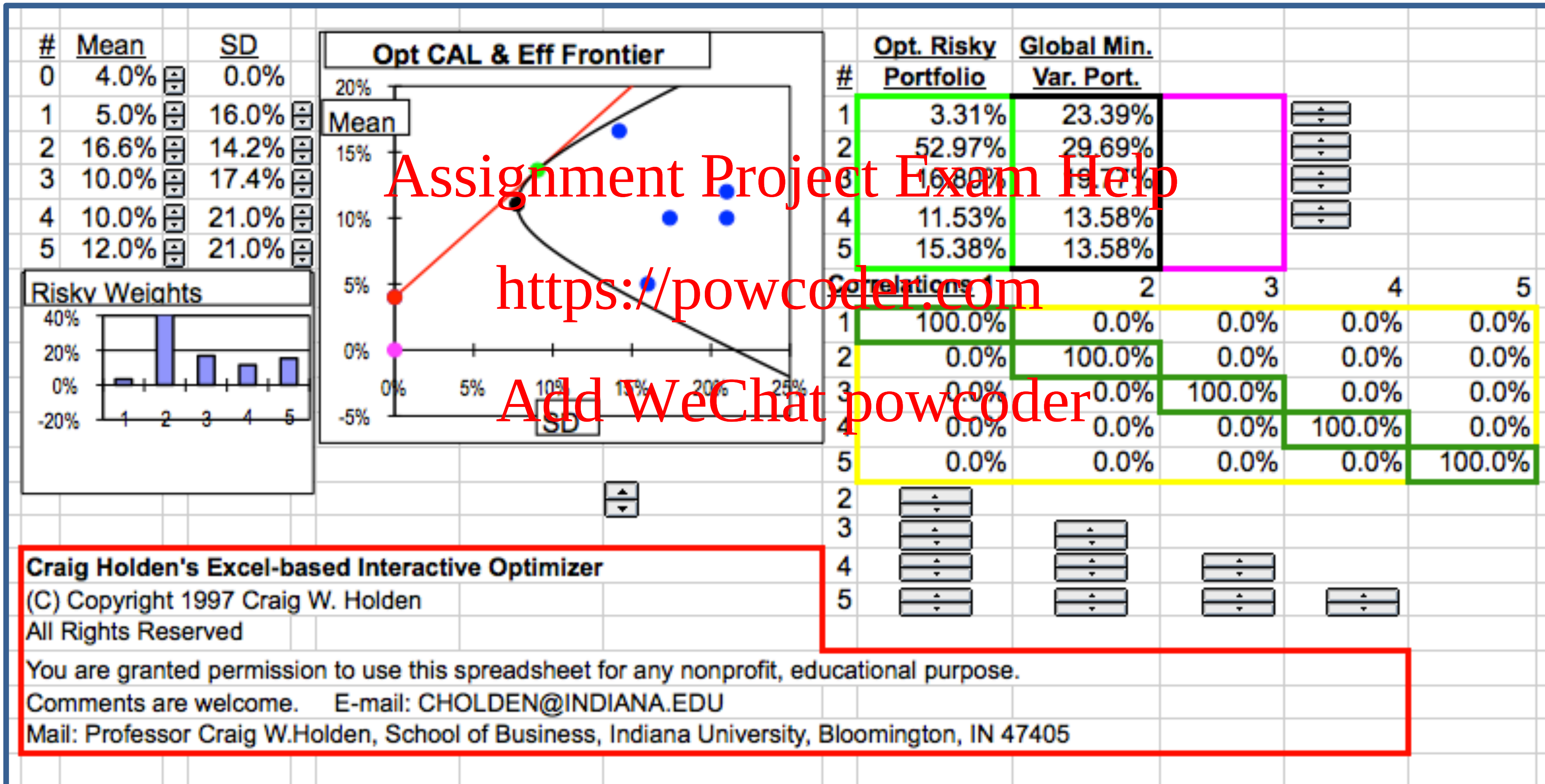


Classification of Risk 2 of 2

- Diversifiable Risk: Risk that can be diversified away (in a large portfolio). Also known as:
  - Idiosyncratic risk
  - Non-systematic risk
  - Unique risk
  - Example: Individual company news.
- Non-Diversifiable Risk: Risk that cannot be diversified away. Also known as:
  - Covariance Risk
  - Systematic Risk
  - Market Risk
  - Example: Market risk, macroeconomic risk, and industry risk
- Total Risk: Diversifiable Risk + Non-Diversifiable Risk

Portfolio Optimizer

Courtesy of Prof. Craig W. Holden



Calculates optimal portfolio with 5 risky assets and 1 riskless asset.

Economics 403A
Review of MPT and CAPM

Part IV

The Capital Asset Pricing Model CAPM

Dr. Randall R. Rojas

Today's Class (Part IV)

- Single Index Model
- The Capital Asset Pricing Model (CAPM)
  - CAPM Assumptions
- The Equilibrium Tangent Portfolio
- The Market Portfolio
- The Capital Market Line (CML)
- The Required Return on Individual Stocks
- Beta (β)

Today's Class (Part IV)

- CAPM Recap
- The Security Market Line (SML)
- Differences Between the CML and SML
- Risk
  - Systematic
  - Non-Systematic
  - Example
- Risk Premium
- Applications of the CAPM
- Stock Selection
- Investment Strategies
- Capital Budgeting
- Summary

The Single-Index Model 1 of 4

- Motivation: The Markovitz procedure used thus far suffers from two main drawbacks:
  - The model requires a huge number of estimates to fill the covariance matrix.
  - The model does not provide any guideline to the forecasting of the security risk premiums.
- Index Models: Simplify estimation of the covariance matrix and greatly enhance the analysis of security risk premiums.
- Number of estimates: For example, for the NYSE (~3000 securities), the Markovitz procedure requires ~4.5million estimates. However, the SIM only requires 9,002.

The Single-Index Model

- How to separate diversifiable from non-diversifiable risk for a security?

- Regression Model: $R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$

  - R_i =Excess return of a security.
  - α_i =Security's expected excess return when the market return is zero (intercept).

  - R_M =Excess return of the market.
  - β_i =cov(R_i , R_M)/σ^2_M
  - $\beta_i R_M$ = Systematic Risk.
  - e_i =Idiosyncratic Risk.

The Single-Index Model

Risk and Covariance in the Single Index Model

- Total Risk = Systematic risk+ Idiosyncratic risk

$$\sigma^2_i = \beta_i^2 \sigma^2_M + \sigma^2(e_i)$$

- Covariance = Product of betas × Market Index Risk

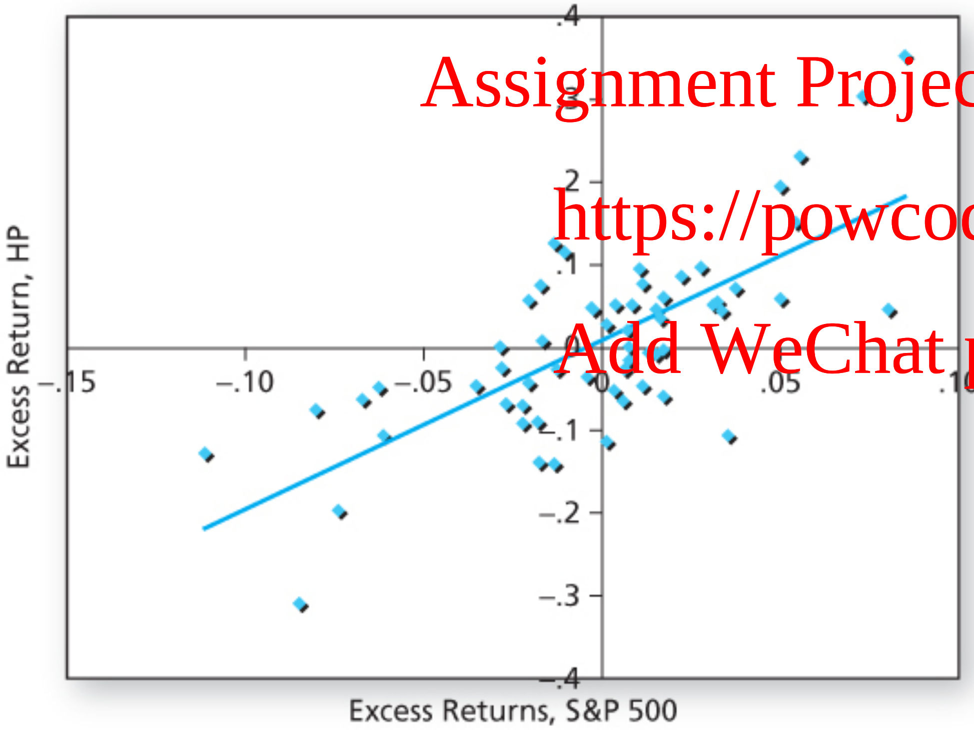
$$\text{cov}(R_i, R_j) = \beta_i \beta_j \sigma^2_M$$

- Correlation = Product of correlations with the market index

$$\text{corr}(R_i, R_j) = \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} = \underbrace{\frac{\beta_i \sigma_M^2 \beta_j \sigma_M^2}{\sigma_i \sigma_M \sigma_j \sigma_M}}_{\text{corr}(R_i, R_j) \times \text{corr}(R_j, R_M)}$$

The Single-Index Model 4 of 4

Scatterplot of HP the S&P 500



1. Collect data on the stock.
2. Collect data on the S&P 500.
3. Compute the regression line.

$$R_{HP}(t) = \alpha_{HP} + \beta_{HP} R_{S\&P500}(t) + e_{HP}(t)$$

The Capital Asset Pricing Model (CAPM)

- Q: What is CAPM?
- A: An Equilibrium model that
  - predicts the relationship between risk and expected return.
  - predicts optimal portfolio choices.
  - underlies much of modern finance theory.
  - underlies most of real-world financial decision making.
- Derived using Markowitz's principles of portfolio theory, with additional simplifying assumptions.
- Sharpe, Lintner and Mossin are the researchers credited with its development.
- William Sharpe won the Nobel Prize in 1990.

CAPM Assumptions 1 of 3

- Assumption 1: The market is in competitive equilibrium.
  - Equilibrium: Supply = Demand
  - Supply of securities is fixed (in the short-run).
  - If Demand > Supply for a particular security, the excess demand drives up price and reduces expected return.
  - (Reverse if Demand < Supply)
- Competitive Market:
  - Investors take prices as given.
  - No investor can manipulate the market.
  - No monopolist

CAPM Assumptions 2 of 3

- Assumption 2-6: Investors face the same investment opportunity set.
- Assumption 2: Single period horizon: Everyone buys and holds for the same period of time.
- Assumption 3: All assets are tradable: Financial assets, real estate, human capital.
- Assumption 4: No frictions: No taxes, no bid-ask spread, no borrowing or short-selling constraints
- Assumption 5-6: Investors are rational mean-variance optimizers with homogeneous expectations: They have the same views about mean, variance and covariances. They pick efficient portfolios.

CAPM Assumptions 3 of 3

- Although these assumptions may seem unrealistically strong.
  - Some can be relaxed, and CAPM still holds.

- If many assumptions are relaxed, generalized versions of CAPM applies. (Topic of ongoing research.)

The Equilibrium Tangent Portfolio

- Recall from portfolio theory that:
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security, and
  - The rest of their wealth is invested in the tangency portfolio.
  - The *tangent portfolio* is the same for all investors (homogeneous expectations).
- In equilibrium, supply=demand, therefore:
  - The tangent portfolio must be the portfolio of all existing risky assets, i.e., the “market portfolio”!

The Equilibrium Tangent Portfolio

- Aggregate Demand of Risky Assets:
  - A large dollar amount of the tangent portfolio (Market Cap).

- Aggregate Supply of Risky Assets:
  - Collection of all risky assets in the world (Market Portfolio)

The Market Portfolio

- Let
  - p_i = price of one share of risky security i.
  - n_i = number of shares outstanding for risky security i.
  - M = Market Portfolio: The portfolio in which each risky security i has the following weight:

$$\omega_{iM} = \frac{p_i \times n_i}{\sum_i p_i \times n_i} = \frac{\text{market capitalization of security } i}{\text{total market capitalization}}$$

- Therefore, the market portfolio is the the portfolio consisting of all assets.

The Market Portfolio

- Example:

	n_i	p_i (\$)	$n_i \times p_i$ (\$)	ω_i
Asset 1	10,000	5	50,000	0.5
Asset 2	5,000	6	30,000	0.3
Asset 3	10,000	2	20,000	0.2

+

= 1

Total Market Cap = $\sum_i n_i \times p_i$ = \$100,000

The Capital Market Line (CML)

- Recall that: The CAL with the highest Sharpe ratio is the CAL with respect to the tangency portfolio.

- In equilibrium, the market portfolio is the tangency portfolio.

- Capital Market Line (CML): The market portfolio's CAL.

The Capital Market Line (CML)

- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

$$E(R_p) = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M}\right)\sigma_p$$

- Q: What determines the market price of risk?
- A: Sharpe ratio:

$$\left(\frac{E(R_M) - R_f}{\sigma_M}\right)$$

Market price of risk

The Capital Market Line (CML) 3 of 3

- Suppose one (small) investor wakes up more risk averse.
  - What happens to his demand for risky assets? $\downarrow$

- Suppose all investors wake up more risk averse
  - What happens to the aggregate demand for risky assets? $\downarrow$
  - What happens to the price of risky assets? $\downarrow$
  - What happens to the market price of risk? $\uparrow$

The Required Return on Individual Stocks 1 of 2

- CAPM is most famous for its prediction concerning the relationship between risk and return for individual securities:

$$\frac{E[R_i - R_f]}{\frac{\partial \sigma_M}{\partial \omega_i}} = \frac{E[R_M] - R_f}{\sigma_M}$$

High return relative to weight in market portfolio.

- The extra return is proportional to the risk contribution of that security.

- This relationship must hold for every security.

The Required Return on Individual Stocks 2 of 2

- What is the practical significance of:

$$\frac{E[R_i - R_f]}{\frac{\partial \sigma_M}{\partial \omega_i}} = \frac{E[R_M] - R_f}{\sigma_M}$$

- To implement it we would need to put a number on that partial derivative…
- How would we do that?

$$\boxed{\frac{1}{\sigma_M}\frac{\partial \sigma_M}{\partial \omega_i}}$$

- It turns out there's an easy way to calculate it using statistical analysis: this partial derivative is, in fact, equal to β.

Beta (β)

- Consider a given stock return.
- Run a regression of that stock return against the market return.

- The beta of the stock is the slope coefficient of this regression:

$$\boxed{\beta_i = \frac{1}{\sigma_M}\frac{\partial \sigma_M}{\partial \omega_i} \quad \text{or} \quad \beta_i = \frac{cov(R_i, R_M)}{\sigma_M^2}}$$

- Beta measures non-diversifiable risk. Reveals how a security responds to market forces.

Beta (β)

- Beta for the overall market is equal to 1.
- $\beta>0 \rightarrow$ The stock moves in the same direction as the market.
- $\beta<0 \rightarrow$ The stock moves in the opposite direction of the market.
- Most stocks have $0.5<\beta<1.75$.
- Careful with 'which' β you use. Recall that they are based on historical data (and their frequency). Different time-series data will yield different betas*.
- Example: Try to estimate β on your own for the Walt Disney Stock. Based on online publically available data, Fernandez (2009) computed: $0.72<\beta_{WD}<1.39$

Beta (β)

- Q: Where does beta come from?
- Take the partial derivative w.r.t ω_i of:

$$\sigma_M = \sqrt{\sum_i \sum_j \omega_i \omega_j cov(R_i, R_j)}$$

- Risk contribution depends on covariances with all other assets.

$$\frac{\partial \sigma_M}{\partial \omega_i} = \frac{1}{\sigma_M} \sum_{j=1}^{N} \omega_j cov(R_i, R_j) = \frac{1}{\sigma_M} cov\left(R_i, \sum_{j=1}^{N} \omega_j R_j\right)$$

$$\beta_i = \frac{1}{\sigma_M^2} cov\left(R_i, \sum_{j=1}^{N} \omega_j R_j\right) = \frac{1}{\sigma_M} \frac{cov\left(R_i, \sum_{j=1}^{N} \omega_j R_j\right)}{\sigma_M}$$

$$\boxed{\beta_i = \frac{1}{\sigma_M} \frac{\partial \sigma_M}{\partial \omega_i}}$$

Beta (β) 4 of 4

- Therefore, since → $\beta_i = \frac{1}{\sigma_M}\frac{\partial \sigma_M}{\partial \omega_i}$

→ $\sigma_M \beta_i = \frac{\partial \sigma_M}{\partial \omega_i}$

→ $\frac{E(R_i - R_f)}{\frac{\partial \sigma_M}{\partial \omega_i}} = \frac{E(R_M) - R_f}{\sigma_M}$

→ $\frac{E(R_i) - R_f}{\sigma_M \beta_i} = \frac{E(R_M) - R_f}{\sigma_M}$

→ $\boxed{E(R_i) = R_f + \beta_i (E(R_M) - R_f)}$

Security Market Line (SML)

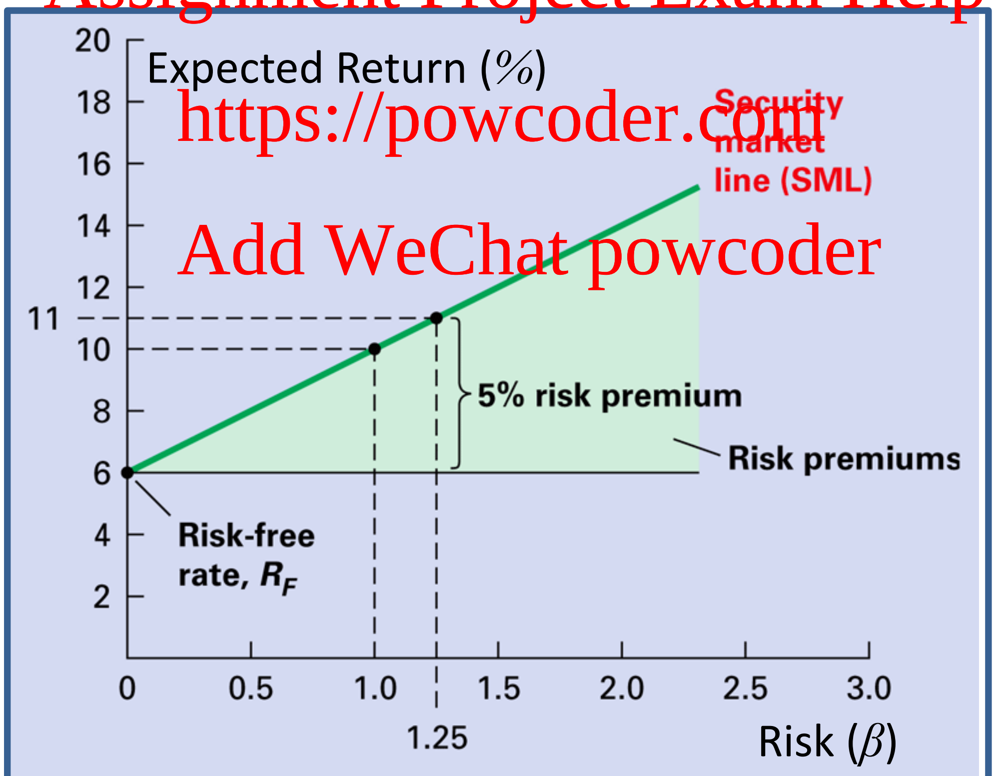
CAPM Recap

- CAPM Assumptions:
  - competitive equilibrium
  - all investors have the same investment opportunity set
  - no frictions
- Demand = Supply means that:
  - Tangency portfolio = Market portfolio
- Capital Allocation Line (CAL): Risk-Return combinations*.
- Capital Market Line (CML): Returns on efficient portfolio.
- Security Market Line (SML): Returns on all stocks.

*Between a risk-free asset and a risky portfolio.

The Security Market Line (SML)

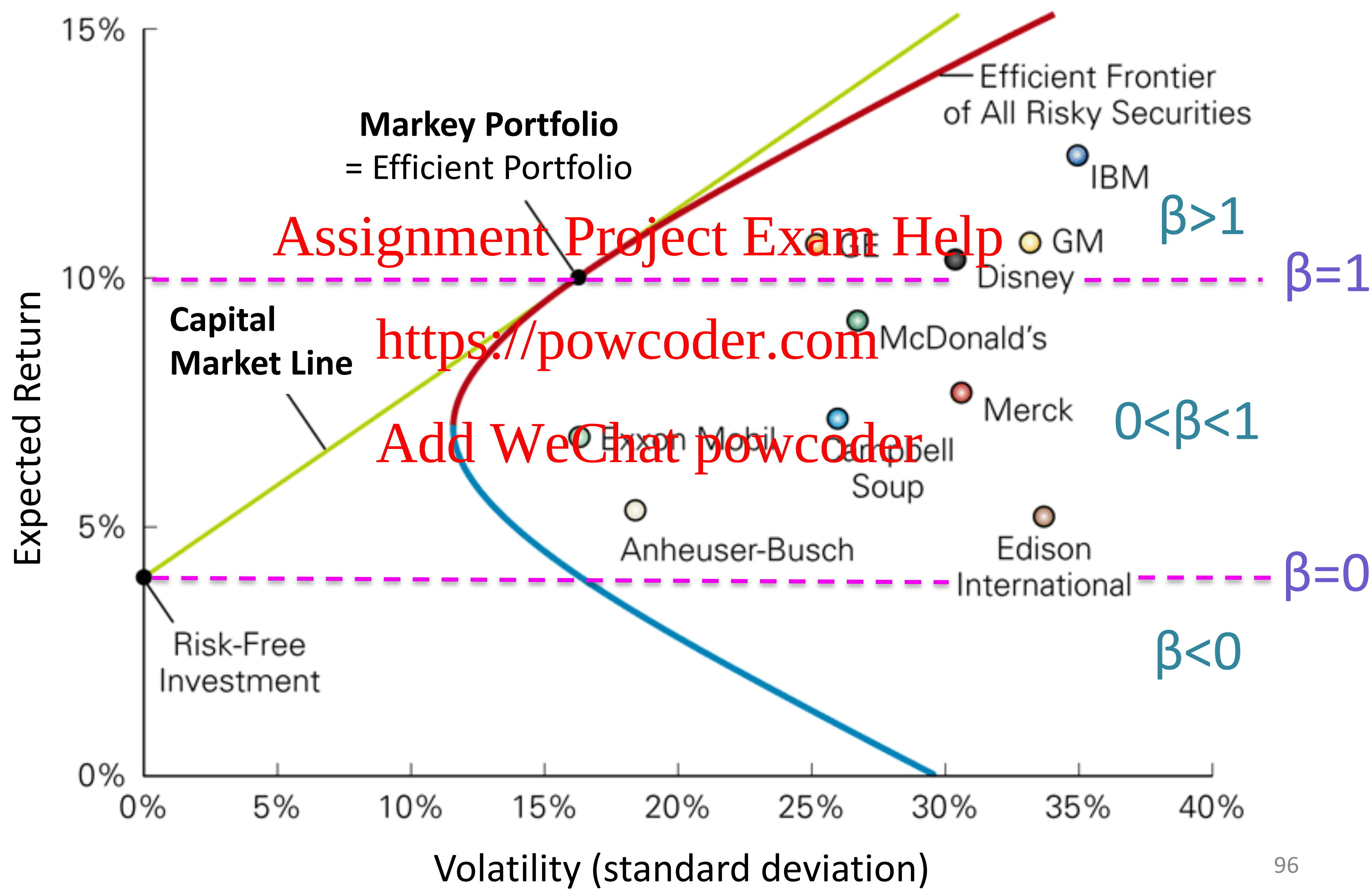
- The Security Market Line (SML) depicts graphically the CAPM.
- The SML is given by: E[R_i]= $R_f + \beta_i$ (E[R_M]-R_f)
- Beta (β_i) measures the security's sensitivity to market movements.



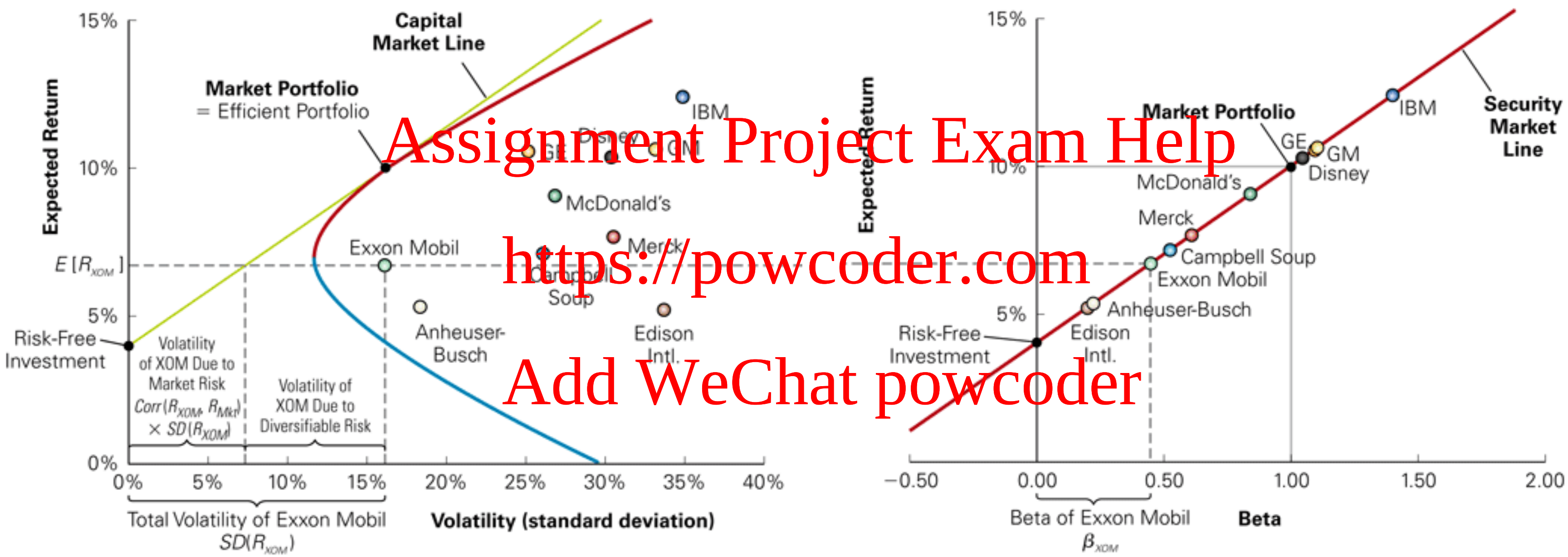
Differences Between the CML and SML

- Capital market line measures risk by standard deviation, or total risk.
- Security market line measures risk by beta to find the security's risk contribution to portfolio M.
- CML graphs only define efficient portfolios.
- SML graphs efficient and inefficient portfolios.
- CML eliminates diversifiable risk for portfolios.
- SML includes all portfolios that lie on or below the CML, but only as a part of M, and the relevant risk is the security's contribution to M's risk.
- Firm specific risk is irrelevant to each, but for different reasons.

Differences Between the CML and SML



Differences Between the CML and SML



Slope of the CML

$$\frac{E[R_M]-R_f}{\sigma_M}$$

Slope of the SML

$$E[R_M]-R_f$$

Risk 1 of 3

- β_i measures security *i*'s contribution of to the total risk of a well-diversified portfolio, namely the market portfolio.

- Hence, β_i measures the non-diversifiable risk of the stock.

- Investors must be compensated for holding non-diversifiable risk. This explains the CAPM equation:
  - $E[R_i] = R_f + \beta_i (E[R_M] - R_f), \; i = 1, \ldots, N$
  - where $R_i(t) = R_f + \beta_i [R_M(t) - R_f] + \boxed{\text{error}_i(t)}$ $\longleftarrow$ (idiosyncratic risk)

Risk

- The CAPM equation can be written as:

$$R_i = R_f + \beta_i(R_M - R_f) + error_i$$

where $\beta_i = \dfrac{cov(R_i, R_M)}{\sigma_M^2}$

- Note that: E[error$_i$(t)]=0 and cov(R_M(t), error$_i$(t))=0. Therefore, the total risk of a security can be partitioned into two components (see Lecture 8):

$$\underbrace{\sigma_i^2}_{\substack{\text{var}(R_i) \\ \text{Total Risk}}} = \underbrace{\beta_i^2 \sigma_M^2}_{\substack{\text{Market} \\ \text{Risk}}} + \underbrace{\overline{\sigma}_i^2}_{\substack{\text{var(error}_i\text{)} \\ \text{Idiosyncratic Risk}}}$$

Risk

- Example: XYZ Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk- free rate is 7%.

- What is the equilibrium expected return on XYZ stock?

From the SML: E[R_i]= R_f + β_i(E[R_M]-R_f)→ E[R] = 0.07 + 3(0.14-0.07)=0.28

- What is the proportion of XYZ Internet's variance which is diversified away in the market portfolio?

Solve for $\overline{\sigma}_i^2$ from: $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \overline{\sigma}_i^2 \longrightarrow \overline{\sigma}^2 = \sigma_i^2 - \beta_i^2 \sigma_M^2$

$\longrightarrow$ $\overline{\sigma}_i^2$= (0.9)² - (3)²(0.15)² = 0.6075 → $\overline{\sigma}_i$ = 0.779

Hence $\frac{0.6075}{(0.9)^2} = 75\%$ of variance is diversified away.

Risk Premium

- Recall the SML: $E(R_i)=R_f+\beta_i\,[E(R_M)-R_f]$
- Stock i's systematic (or market risk) is: β_i

- Investors are compensated for holding systematic risk in form of higher returns.

- The size of the compensation depends on the equilibrium risk premium, $[E(R_M) - R_f]$.

- The equilibrium risk premium increases in:
    1. The variance of the market portfolio.
    2. The degree of risk aversion of the average investor.

Applications of the CAPM

- Portfolio choice

- Shows what a "fair" security return is.

- Provides a benchmark for security analysis.

- Required return used in capital budgeting.

Stock Selection and Active Management

- When computing β via linear regression, the best fit line is given by: $\hat{y} = \alpha + \beta x + \varepsilon$
  - $y = R_i - R_f$ (stock's excess return)
  - $\alpha =$ y-intercept (stock's alpha)
  - $x = R_M - R_f$ (market's excess return)
  - $\beta =$ slope (measure of systematic risk)

CAPM predicts that all α's are zero.

- Therefore, $R_i - R_f = \alpha_i + \beta_i (R_M - R_f) + e_i$ (SCL)

$$\rightarrow E(R_i) = \underbrace{R_f + \beta_i \left(E(R_M) - R_f\right)}_{\text{Expected return for stock } i \text{ from the SML}} + \underbrace{\alpha_i}_{\text{Distance above/below the SML}}$$

Some fund managers try to buy positive-alpha stocks and sell negative-alpha stocks.

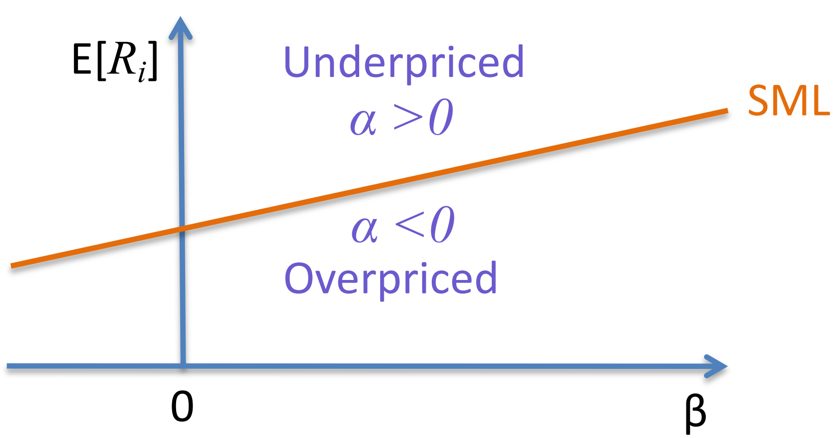
Note: α measures the historical performance of the security relative to the expected return predicted by the SML.

Stock Selection and Active Management

- One possible benchmark for stock selection is to find assets that are cheap relative to CAPM (or more advanced models).
- In the "CAPM world", there is no such thing as overpricing/underpricing.
  - Every asset is correctly priced, and is positioned on the SML.
- For practical real-world purposes, however, we can compare an asset's given price or expected return relative to what it should be according to the CAPM, and in that context we talk about over/under pricing.

Stock Selection and Active Management

- Assets above the SML are underpriced relative to the CAPM.
  - Why? Because the assets' "too" high expected return means their price is "too" low compared to the "fair" CAPM value.
- Assets below the SML are overpriced relative to the CAPM.
  - Why? Because the assets' "too" low expected return means their price is "too" high compared to the "fair" CAPM value



Active and Passive Strategies

- An active strategy tries to beat the market by stock picking, by timing, or other methods.
- But, CAPM implies that
  - security analysis is not necessary
  - every investor should just buy a mix of the risk-free security and the market portfolio, a passive strategy.
- Grossman-Stiglitz Paradox: How can the market be efficient if everyone uses a passive strategy?

Capital Budgeting

- Should the firm undertake a long-term risky project?
- Manager's objective: increase the value of the firm.
- Calculate Net Present Value.
- Use CAPM to calculate discount rate.
- Is this process only appropriate for a well diversified firm?

Capital Budgeting

NPV(Buy security) = PV(All cash flows paid by the security) – Price(Security)

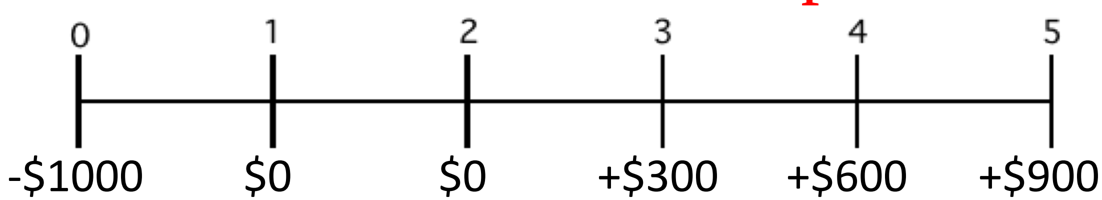
= 0

NPV(Sell security) = Price(Security) – PV(All cash flows paid by the security)

= 0

- The NPV of trading a security in a normal market is zero.

- Example: Given R_f = 0.04, E(R_M) = 0.12, and the cash flows below. Which project should you choose: Project A (β=1.75) or Project B (β=0.5)?



Project A: E(R_i)=0.04 +1.75(0.12-0.04)=0.18

$$NPV = -1000 + \frac{300}{1.18^3} + \frac{600}{1.18^4} + \frac{900}{1.18^5} = -\$114$$

Project B: E(R_i)=0.04 +0.50(0.12-0.04)=0.08

$$NPV = -1000 + \frac{300}{1.08^3} + \frac{600}{1.08^4} + \frac{900}{1.08^5} = \$292$$

Better Choice

Summary

- The CAPM follows from equilibrium conditions in a frictionless mean-variance economy with rational investors.

- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset. (That is, everyone should hold a portfolio on the CML.)
- Prediction 2: The expected return on a stock is a linear function of its beta. (That is, stocks should be on SML.)
- The beta is given by: $\beta_i = \dfrac{cov(R_i, R_M)}{\sigma_M^2}$
- A stock's beta can be estimated using historical data by linear regression. That is, by estimating the Security Characteristic Line (SCL).