Economics 403A

Treatment Effects and Differences-in-Differences

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Today’s Class

- Treatment Effects

- Differences-in-Differences

Treatment Effects

- Avoid the faulty line of reasoning known as **post hoc, ergo propter hoc**
  - One event's preceding another does not necessarily make the first the cause of the second
  - Another way to say this is embodied in the warning that ''correlation is not the same as causation''
  - Another way to describe the problem we face in this example is to say that data exhibit a selection bias, because some people chose (or self-selected) to go to the hospital and the others did not
    - When membership in the treated group is in part determined by choice, then the sample is not a random sample

Treatment Effects

- Selection bias is also an issue when asking:
  - "How much does an additional year of education increase the wages of married women?"
  - "How much does participation in a job-training program increase wages?"
  - "How much does a dietary supplement contribute to weight loss?"
- Selection bias interferes with a straightforward examination of the data, and makes more difficult our efforts to measure a causal effect, or treatment effect

Treatment Effects

- We would like to randomly assign items to a **treatment group**, with others being treated as a **control group**
  - We could then compare the two groups
  - The key is a **randomized controlled experiment**

Treatment Effects

- The ability to perform randomized controlled experiments in economics is limited because the subjects are people, and their economic well-being is at stake.

Treatment Effects

- Define the indicator variable d as:

$$d_i = \begin{cases} 1 & \text{individual in treatment group} \\ 0 & \text{individual in control group} \end{cases}$$

  - The model is then:

$$y_i = \beta_1 + \beta_2 d_i + e_i, \quad i = 1, \ldots, N$$

  - And the regression functions are:

$$E(y_i) = \begin{cases} \beta_1 + \beta_2 & \text{if in treatment group, } d_i = 1 \\ \beta_1 & \text{if in control group, } d_i = 0 \end{cases}$$

The Difference Estimator

- The least squares estimator for β_2, the **treatment effect**, is:

$$b_2 = \frac{\sum_{i=1}^{N}\left(d_i - \bar{d}\right)\left(y_i - \bar{y}\right)}{\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2} = \bar{y}_1 - \bar{y}_0$$

with:

$$\bar{y}_1 = \sum_{i=1}^{N_1} y_i / N_1, \bar{y}_0 = \sum_{i=1}^{N_0} y_i / N_0$$

– The estimator b_2 is called the **difference estimator**, because it is the difference between the sample means of the treatment and control groups

The Difference Estimator

- The difference estimator can be rewritten as:

$$b_2 = \beta_2 + \frac{\sum_{i=1}^{N}\left(d_i - \bar{d}\right)\left(e_i - \bar{e}\right)}{\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2} = \beta_2 + \left(\bar{e}_1 - \bar{e}_0\right)$$

  – To be unbiased, we must have:

$$E\left(\bar{e}_1 - \bar{e}_0\right) = E\left(\bar{e}_1\right) - E\left(\bar{e}_0\right) = 0$$

The Difference Estimator

- If we allow individuals to "self-select" into treatment and control groups, then:

$$E\left(\bar{e}_1\right) - E\left(\bar{e}_0\right)$$

is the selection bias in the estimation of the treatment effect

  - We can eliminate the self-selection bias is we randomly assign individuals to treatment and control groups, so that there are no systematic differences between the groups, except for the treatment itself

Example: Project STAR Kindergarten

Effect of classroom size on student learning

Regular Sized Classroom

Variable	Mean	Std. Dev.	Min	Max
TOTALSCORE	918.0429	73.1380	635	1229
SMALL	0.0000	0.0000	0	0
TCHEXPER	9.0683	5.7244	0	24
BOY	0.5132	0.4999	0	1
FREELUNCH	0.4738	0.4994	0	1
WHITE_ASIAN	0.6813	0.4661	0	1
TCHWHITE	0.7980	0.4016	0	1
TCHMASTERS	0.3651	0.4816	0	1
SCHURBAN	0.3012	0.4589	0	1
SCHRURAL	0.4998	0.5001	0	1

$N = 2005$

Example
Project STAR Kindergarten

Small Sized Classroom

Variable	Mean	Std. Dev.	Min	Max
TOTALSCORE	931.9419	76.3586	747	1253
SMALL	1.0000	0.0000	1	1
TCHEXPER	8.9954	5.7316	0	27
BOY	0.5150	0.4999	0	1
FREELUNCH	0.4718	0.4993	0	1
WHITE_ASIAN	0.6847	0.4648	0	1
TCHWHITE	0.8625	0.3445	0	1
TCHMASTERS	0.3176	0.4657	0	1
SCHURBAN	0.3061	0.4610	0	1
SCHRURAL	0.4626	0.4987	0	1

$N = 1738$

Example
Project STAR Kindergarten

- The model of interest is:

$$TOTALSCORE = \beta_1 + \beta_2 SMALL + e$$

- Adding *TCHEXPER* to the base model we obtain:

$$TOTALSCORE = \beta_1 + \beta_2 SMALL + \beta_3 TCHEXPER + e$$

Example
Project STAR Kindergarten

	(1)	(2)	(3)	(4)
C	918.0429***	907.5643***	917.0684***	908.7865***
	(1.6672)	(2.5424)	(1.4948)	(2.5323)
SMALL	13.8990***	13.9833***	15.9978***	16.0656***
	(2.4466)	(2.4373)	(2.2228)	(2.2183)
TCHEXPER		1.1555***		0.9132***
		(0.2123)		(0.2256)
SCHOOL EFFECTS	*No*	*No*	*Yes*	*Yes*
N	3743	3743	3743	3743
adj. R^2	0.008	0.016	0.221	0.225
SSE	20847551	20683680	16028908	15957534

Standard errors in parentheses
Two-tail p-values: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Example
Project STAR Kindergarten

- The students in our sample are enrolled in 79 different schools
  - One way to account for school effects is to include an indicator variable for each school
  - That is, we can introduce 78 new indicators:

$$SCHOOL\_j = \begin{cases} 1 & \text{if student is in school } j \\ 0 & \text{otherwise} \end{cases}$$

Example
Project STAR Kindergarten

- The model is now:

$$TOTALSCORE_i = \beta_1 + \beta_2 SMALL_i + \beta_3 TCHEXPER_i + \sum_{j=2}^{79} \delta_j SCHOOL\_j_i + e_i$$

  – The regression function for a student in school *j* is:

$$E(TOTALSCORE_i) = \begin{cases} (\beta_1 + \delta_j) + \beta_3 TCHEXPER_i & \text{student in regular class} \\ (\beta_1 + \delta_j + \beta_2) + \beta_3 TCHEXPER_i & \text{student in small class} \end{cases}$$

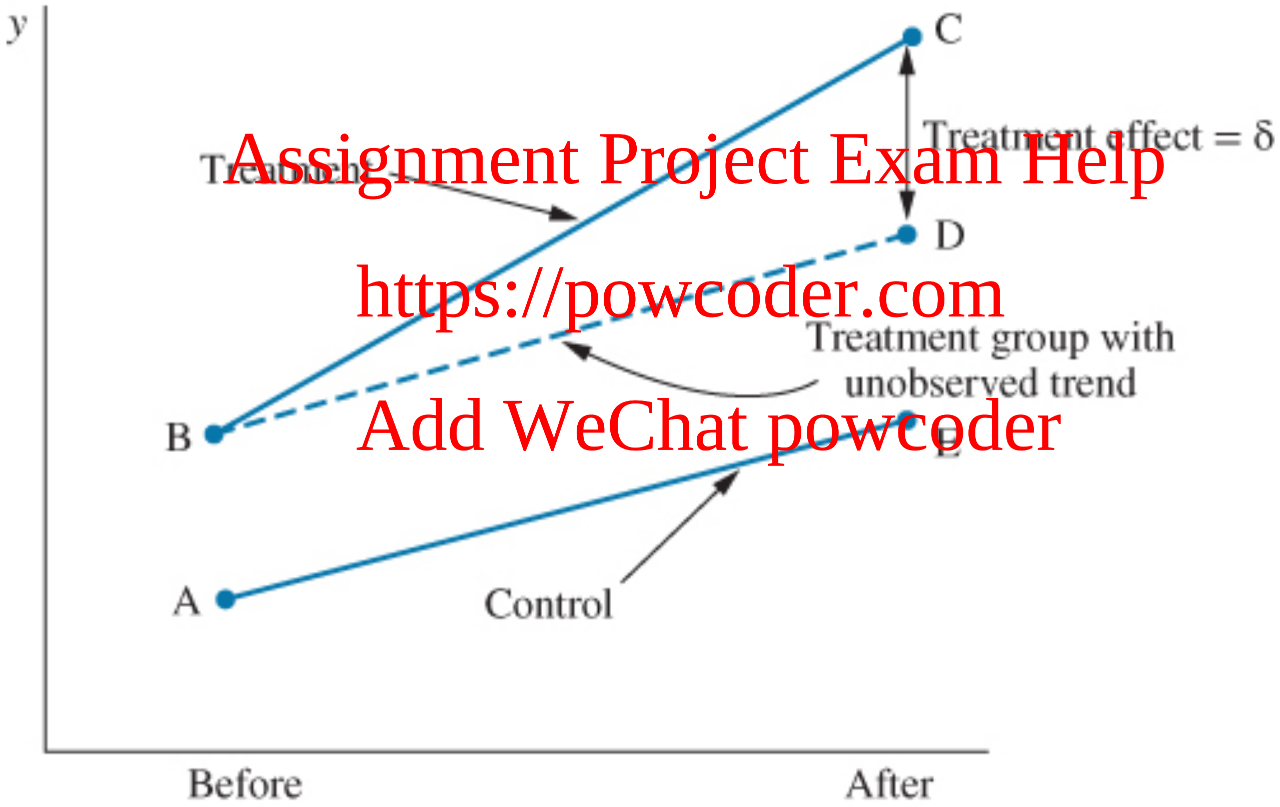
Example
Project STAR Kindergarten

- Another way to check for random assignment is to regress *SMALL* on these characteristics and check for any significant coefficients, or an overall significant relationship
  - If there is random assignment, we should not find any significant relationships
  - Because *SMALL* is an indicator variable, we use the linear probability model

The Differences-in-Differences Estimator

- Randomized controlled experiments are rare in economics because they are expensive and involve human subjects
  - **Natural experiments**, also called **quasi-experiments**, rely on observing real-world conditions that approximate what would happen in a randomized controlled experiment
  - Treatment appears as if it were randomly assigned

The Differences-in-Differences Estimator



The Differences-in-Differences Estimator

- Estimation of the treatment effect is based on data averages for the two groups in the two periods:

$$\hat{\delta} = \left(\hat{C} - \hat{E}\right) - \left(\hat{B} - \hat{A}\right)$$

$$= \left(\bar{y}_{Treatment,After} - \bar{y}_{Control,After}\right) - \left(\bar{y}_{Treatment,Before} - \bar{y}_{Control,Before}\right)$$

  – The estimator $\hat{\delta}$ is called a **differences-in-differences** (abbreviated as *D*-in-*D*, *DD*, or *DID*) estimator of the treatment effect.

The Differences-in-Differences Estimator

- The sample means are:

$$\bar{y}_{Control,Before} = \hat{\text{A}} = \text{mean for control group before policy}$$

$$\bar{y}_{Treatment,Before} = \hat{\text{B}} = \text{mean for treatment group before policy}$$

$$\bar{y}_{Control,After} = \hat{\text{E}} = \text{mean for control group after policy}$$

$$\bar{y}_{Treatment,After} = \hat{\text{C}} = \text{mean for treatment group after policy}$$

The Differences-in-Differences Estimator

- Consider the regression model:

$$y_{it} = \beta_1 + \beta_2 TREAT_i + \beta_3 AFTER_t + \delta\left(TREAT_i \times AFTER_t\right) + e_{it}$$

- The regression function is:

$$E\left(y_{it}\right) = \begin{cases} \beta_1 & TREAT = 0, AFTER = 0 \text{ [Control before = A]} \\ \beta_1 + \beta_2 & TREAT = 1, AFTER = 0 \text{ [Treatment before = B]} \\ \beta_1 + \beta_3 & TREAT = 0, AFTER = 1 \text{ [Control after = E]} \\ \beta_1 + \beta_2 + \beta_3 + \delta & TREAT = 1, AFTER = 1 \text{ [Treatment after = C]} \end{cases}$$

The Differences-in-Differences Estimator

- Using the points in the figure:

$$\delta = (C - E) - (B - A) = \left[(\beta_1 + \beta_2 + \beta_3 + \delta) - (\beta_1 + \beta_3)\right] - \left[(\beta_1 + \beta_2) - \beta_1\right]$$

- Using the least squares estimates, we have:

$$\hat{\delta} = \left[\left(b_1 + b_2 + \mathrm{b}_3 + \hat{\delta}\right) - (b_1 + \mathrm{b}_3)\right] - \left[(b_1 + b_2) - b_1\right]$$
$$= \left(\overline{y}_{Treatment,After} - \overline{y}_{Conrol,After}\right) - \left(\overline{y}_{Treatment,Before} - \overline{y}_{Conrol,Before}\right)$$

Example: Minimum wages PA vs. NJ

- On April 1, 1992 minimum wages were increased in NJ from $4.25/hr to $5.05/hr but remained at $4.25/hr in PA.
- **Q**: What effect did this increase have on full-time employment in fast food restaurants in NJ?

Variable	N	mean	se
Pennsylvania (PA)			
Before	77	23.3312	1.3511
After	77	21.1656	0.9432
New Jersey (NJ)			
Before	321	20.4394	0.5083
After	319	21.0274	0.5203

Full-time Equivalent Employees by State and Period

The Differences-in-Differences Estimator

- We will test the null and alternative hypotheses:

$$H_0 : \delta \geq 0 \text{ versus } H_1 : \delta < 0$$

  - The differences-in-differences estimate of the change in employment due to the change in the minimum wage is:

$$\begin{aligned}\hat{\delta} &= \left(\overline{FTE}_{NJ,After} - \overline{FTE}_{PA,After}\right) - \left(\overline{FTE}_{NJ,Before} - \overline{FTE}_{PA,Before}\right)\\ &= (21.0274 - 21.1656) - (20.4394 - 23.3312)\\ &= \boxed{2.7536}\end{aligned}$$

Example: Minimum wages PA vs. NJ

- Rather than compute the differences-in-differences estimate using sample means, it is easier and more general to use the regression format

  – The differences-in-differences regression is:

$$FTE_{it} = \beta_1 + \beta_2 NJ_i + \beta_3 D_t + \delta\left(NJ_i \times D_t\right) + e_{it}$$

This is the estimate we need

	(1)	(2)	(3)
C	23.3312***	25.9512***	25.3205***
	(1.072)	(1.038)	(1.211)
NJ	−2.8918*	−2.3766*	−0.9080
	(1.194)	(1.079)	(1.272)
D	−2.1656	−2.2236	−2.2119
	(1.516)	(1.368)	(1.349)
D_NJ	2.7536	2.845[illegible]	2.8149
	(1.688)	(1.523)	(1.502)
KFC		−10.4534***	−10.0580***
		(0.849)	(0.845)
ROYS		−1.6250	−1.6934*
		(0.860)	(0.859)
WENDYS		−1.0637	−1.0650
		(0.929)	(0.921)
CO_OWNED		−1.1685	−0.7163
		(0.716)	(0.719)
SOUTHJ			−3.7018***
			(0.780)
CENTRALJ			0.0079
			(0.897)
PA1			0.9239
			(1.385)
N	794	794	794
R^2	0.007	0.196	0.221
adj. R^2	0.004	0.189	0.211

Standard errors in parentheses

Two-tail *p*-values: * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Example: Minimum wages PA vs. NJ

- In our differences-in-differences analysis, we did not exploit one very important feature of the data -namely, that the same fast food restaurants were observed on two occasions
  - We have "before" and "after" data
  - These are called **paired data** observations, or **repeat data** observations, or **panel data** observations

Example: Minimum wages PA vs. NJ

- We previously introduced the notion of a **panel** of data – we observe the same individual-level units over several periods
  - Using panel data we can control for unobserved individual-specific characteristics

Example: Minimum wages PA vs. NJ

- Let c_i denote any unobserved characteristics of individual restaurant *i* that do not change over time:

$$FTE_{it} = \beta_1 + \beta_2 NJ_i + \beta_3 D_t + \delta\left(NJ_i \times D_t\right) + c_i + e_{it}$$

Example: Minimum wages PA vs. NJ

- Subtract the observation for t = 1 from that for t = 2:

$$FTE_{i2} = \beta_1 + \beta_2 NJ_i + \beta_3 1 + \delta\left(NJ_i \times 1\right) + c_i + e_{i2}$$

$$-FTE_{i1} = \beta_1 + \beta_2 NJ_i + \beta_3 0 + \delta\left(NJ_i \times 0\right) + c_i + e_{i1}$$

$$\Delta FTE_i = \beta_3 + \delta NJ_i + \Delta e_i$$

where:

$$\Delta FTE_i = FTE_{i2} - FTE_{i1}$$

$$\Delta e_i = e_{i2} - e_{i1}$$

Example: Minimum wages PA vs. NJ

- Using the differenced data, the regression model of interest becomes:

$$\Delta FTE_i = \beta_3 + \delta NJ_i + \Delta e_i$$

Example: Minimum wages PA vs. NJ

- The estimated model is:

$$\Delta FTE = -2.2833 + 2.7500NJ \qquad R^2 = 0.0146$$

$$(se) \qquad (1.036) \quad (1.154)$$

  - The estimate of the treatment effect $\hat{\delta} = 2.75$ using the differenced data, which accounts for any unobserved individual differences, is very close to the differences-in-differences
  - We fail to conclude that the minimum wage increase has reduced employment in these New Jersey fast food restaurants