

F70TS2 – Time Series

Exercise Sheet 2 – Moving Average and Autoregressive Processes

Question 1 Determine the autocorrelation function for:

(i) the MA(2) process $Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$

(ii) the MA(3) process $Y_t = Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2} + \beta_3 Z_{t-3}$

Plot the autocorrelation for:

(i) MA(2): $\beta_1 = 0.8, \beta_2 = 0.5$

(ii) MA(3): $\beta_1 = 0.8, \beta_2 = -0.4, \beta_3 = -0.3$

Question 2 For the MA(1) process $Y_t = Z_t + \beta Z_{t-1}$, find the maximum and minimum values of ρ_1 and the values of β for which they are attained. Do the same for ρ_1 and ρ_2 (with reference to β_1 and β_2) for the MA(2) process.

Question 3 Check that all of the following AR(2) processes are causal stationary:

a) $X_t = -1.4X_{t-1} - 0.65X_{t-2} + \epsilon_t$,

b) $X_t = 0.45X_{t-1} + 0.25X_{t-2} + \epsilon_t$,

c) $X_t = 1.2X_{t-1} - 0.75X_{t-2} + \epsilon_t$,

where ϵ_t i.i.d. with $E(\epsilon_t) = 0$ and $\text{Var}(\epsilon_t) = \sigma_\epsilon^2$. Calculate and display $\rho(k)$, $k = 0, 1, 2, \dots, 9$.

Question 4 Consider the AR(2) process $Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + Z_t$. Determine ρ_1 and ρ_2 in terms of α_1 and α_2 and vice-versa.

Question 5 (harder question)

1. Show that the AR(2) process $Y_t = -0.5Y_{t-1} + 0.14Y_{t-2} + Z_t$ is stationary, and that the acf $\{\rho_k\}$ is given by:

$$\rho_k = \frac{17}{129}(0.2)^k + \frac{112}{129}(-0.7)^k, \quad k = 0, 1, 2, \dots$$

Plot $\{\rho_k\}$ for $k \geq 0$.

2. Show that the AR(2) process $Y_t = -0.6Y_{t-2} + Z_t$ is stationary, and that the acf $\{\rho_k\}$ is given by:

$$\rho_k = \frac{1}{2}i^k(0.6)^{k/2}\{1 + (-1)^k\} \quad k = 0, 1, 2, \dots$$

Plot $\{\rho_k\}$ for $k \geq 0$.

Hint: for an AR(2) processes with characteristic polynomial with roots z_1 and z_2 outside the unit circle, the solution of the Yule-Walker equations is of the form $\rho_k = az_1^{-k} + bz_2^{-k}$.