

F70TS2 – Time Series

Exercise Sheet 3 – $MA(\infty)$, $AR(\infty)$, ARMA and ARIMA

Question 1 Calculate the autocorrelation function of the ARMA(1,2) process

$$Y_t = 0.6Y_{t-1} + Z_t - 0.3Z_{t-1} - 0.1Z_{t-2}.$$

Question 2 By considering the existence of moments show that the process

$$Y_t = Z_t + a(Z_{t-1} + Z_{t-2} + \dots)$$

where a is a constant, is non-stationary. Show, however, that the process $\{V_t\}$ obtained by taking first differences, i.e. $V_t = DY_t = Y_t - Y_{t-1}$, is an $MA(1)$ process and hence stationary. Calculate the autocorrelation function of $\{V_t\}$.

Question 3 Given the following MA processes:

a) $X_t = -0.9\epsilon_{t-1} + 0.2\epsilon_{t-2} + \epsilon_t,$

b) $X_t = 0.3\epsilon_{t-1} - 0.6\epsilon_{t-2} + \epsilon_t,$

c) $X_t = -1.5\epsilon_{t-1} + 0.75\epsilon_{t-2} - 0.125\epsilon_{t-3} + \epsilon_t,$

where $\{\epsilon_t\}$ is a WN. Show that all of these processes are invertible.

Question 4 Find out which of the following ARMA processes are causal stationary and/or invertible, which are neither causal stationary nor invertible.

a) $X_t = 0.3X_{t-1} - 0.4X_{t-2} + 1.3\epsilon_{t-1} + 0.7\epsilon_{t-2} + \epsilon_t,$

b) $X_t = 1.1X_{t-1} - 0.3X_{t-2} + 1.2\epsilon_{t-1} + \epsilon_t,$

c) $X_t = 0.7X_{t-1} + 0.6X_{t-2} - 0.5\epsilon_{t-1} + 0.4\epsilon_{t-2} + \epsilon_t,$

d) $X_t = 0.8X_{t-1} + 0.3X_{t-2} + 0.6\epsilon_{t-1} - 0.5\epsilon_{t-2} + \epsilon_t,$

where ϵ_t are i.i.d. $N(0,1)$ random variables.

Question 5 Given an AR(1) model: $X_t = \phi_1 X_{t-1} + \epsilon_t$ with $|\phi_1| > 1$, where ϵ_t are iid with $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = \sigma_\epsilon^2$. Show that there is a stationary $MA(\infty)$ representation for this process with absolutely summable coefficients. What is the special feature of this process?