F70TS2: Time Series

Multiple Choice Revision Questions

1. A series of length 200 has 145 turning points. Consider a test of the hypotheses:

   H_0: the series is noise ("random") v H_1: the series is more oscillatory than noise.

   With respect to this test, which of the following statements is true?

   A: The *P-value* of the observation "145 TPs" is 1 (100%) and H_0 can stand

   B: The *P-value* of the observation "145 TPs" is 0.180 (18.0%) and H_0 can stand

   C: The *P-value* of the observation "145 TPs" is 0.047 (4.7%) and H_0 is rejected at 5%

   D: The *P-value* of the observation "145 TPs" is 0.018 (1.8%) and H_0 is rejected at 2.5%

2. The fitted value at the middle of the range for the 5-point least-squares fit of cubic accuracy is:

   A: $\frac{1}{5}[-1, 2, \underline{3}]$ B: $\frac{1}{21}[3, 4, \underline{9}]$ C: $\frac{1}{35}[-3, 12, \underline{17}]$ D: $\frac{1}{39}[-1, 10, \underline{21}]$

3. A stationary time series process $\{Y_t\}$ has variance $\gamma_0 = 3$ and autocorrelation function given by:

   $$\rho_k = \frac{1}{2} i^k (0.6)^{k/2} \{1 + (-1)^k\} = (0.6)^{k/2} \cos(k\pi/2), \quad k = 0, 1, 2, \ldots$$

   The autocovariance between variables with time lag 4 is:

   A: 1.08 B: 1.8 C: 10.8 D: 18

4. Consider the process $\{Y_t\}$ defined by $Y_t = Z_t - 0.9Z_{t-1} + 0.2Z_{t-2}$ where $\{Z_t\}$ is a noise process. By using operators, the process can be rewritten in the form $Y_t = \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \pi_3 Y_{t-3} + \cdots + Z_t$.

   The values of π_1 and π_2 are, respectively:

   A: –0.9, –0.61 B: –0.9, 0.61 C: 0.9, –0.61 D: 0.9, 0.61

5. The process $\{Y_t\}$ is produced as the result of applying the linear filter $\frac{1}{5}[-1, 2, \underline{3}]$ to a noise process $\{Z_t\}$. Suppose we represent the (normalised) spectrum of $\{Y_t\}$, $f^*(\omega)$, by $f^*(\omega) = 1 + \sum_{k=1}^{\infty} a_k \cos k\omega$.

Which of the following statements is true?

A: $a_3 = 8/19$ and $a_k = 0$ for $k \geq 5$ B: $a_3 = -8/19$ and $a_k = 0$ for $k \geq 5$

C: $a_3 = 6/19$ and $a_k = 0$ for $k \geq 4$ D: $a_3 = -6/19$ and $a_k = 0$ for $k \geq 4$

6. Let $\{Z_t\}$ and $\{V_t\}$ be independent noise processes, and let $\{Y_t\}$ be defined by $Y_t = Z_t + V_t$. $\{Y_t\}$ is a:

A: AR(1) process B: MA(1) process C: MA(2) process D: noise process

7. The area under the unnormalised spectrum $f(\omega)$ (from 0 to π) for the process $\{Y_t\}$ defined by $Y_t = Z_t + 0.8Z_{t-1} + 0.1Z_{t-2}$, where $\{Z_t\}$ is a noise process with $\sigma_Z^2 = 4$ is:

A: 1 B: 6.6π C: π D: 6.6

8. Consider the process $\{Y_t\}$ defined by $Y_t - 0.6Y_{t-1} - \alpha Y_{t-2} = Z_t$. For what range of values of α is the process stationary?

A: $-1 < \alpha < 0.4$ B: $-1 < \alpha < 0.6$ C: $\alpha > -1$ D: $\alpha < 1$

9. The process $\{Y_t\}$ defined by $Y_t = 0.6Y_{t-1} + 0.4Y_{t-2} + Z_t - Z_{t-1}$ is over-parameterised and can be expressed in simpler terms. It is in fact:

A: a stationary AR(1) process B: a stationary AR(2) process

C: an invertible MA(1) process D: an invertible MA(2) process

10. How do you get from the autocovariance generating function for a time series process $\{Y_t\}$ to the normalised spectrum for the process?

A: divide by σ_Y^2 B: replace z by $e^{-i\omega}$

C: replace z by $\cos\omega$ D: replace z by $e^{-i\omega}$ and divide by σ_Y^2

11. A time series process $\{Y_t\}$ of the ARMA class has autocovariance generating function $C_Y(z) = 22.5 - 15(z + z^{-1}) + 5(z^2 + z^{-2})$.

The value of ρ_2, the autocorrelation coefficient of lag 2, is:

A: 0 B: 1/9 C: 2/9 D: 1/3

12. A process in general linear process form is given by

$$Y_t = \left(1 + (\alpha + \beta) \sum_{k=1}^{\infty} \alpha^{k-1} B^k\right) Z_t\,, \quad |\alpha| < 1, |\beta| < 1$$

This process is:

A: ARIMA(1,0,1) B: ARIMA(1,1,1) C: ARIMA(1,0,0) D: ARIMA(1,1,0)

13. Which of the following processes are invertible?

Process 1: $Y_t = Z_t + 0.5Z_{t-1} + 0.8Z_{t-2}$

Process 2: $Y_t = 0.6Y_{t-1} + Z_t + 0.9Z_{t-2}$

Process 3: $Y_t = Z_t + 0.5Z_{t-1} - 0.5Z_{t-2}$

A: 1,2 only B: 1 only C: all three D: 2 only

14. A process $\{W_t\}$ is defined by $W_t = Z_t + \beta Z_{t-1}$ where $0 < \beta < 1$ and $\{Z_t\}$ is a noise process with $\sigma_Z^2 = 1$. The process $\{Y_t\}$ is defined by $Y_t = W_t - \gamma W_{t-1}$ where $0 < \gamma < 1$.

The value of ρ_3, the autocorrelation coefficient of lag 3 of the $\{Y_t\}$ process is:

A: $\beta\gamma$ B: $\beta^2\gamma$ C: $\beta\gamma^2$ D: 0

15. Consider the process $\{Y_t\}$ defined by $(1 - B)^2 Y_t = Z_t + \beta Z_{t-1}$.

The coefficient a_k in the representation of $\{Y_t\}$ as a general linear process, namely

$$Y_t = \sum_{k=0}^{\infty} a_k Z_{t-k}$$

is given by:

A: $1 + k\beta$ B: $1 + k\beta^2$ C: $1 + k(1 + \beta)$ D: $1 + k(1 + \beta^2)$

16. An AR(2) model is fitted to the time series $y_1, \ldots, y_{50}$. It has forecast function

$$Y_t(k) = 0.1Y_t(k-1) + 0.8Y_t(k-2)\,, \qquad k \geq 1$$

Based on this model, the forecasts $y_{50}(1) = 49.7$ and $y_{50}(2) = 50.4$ are made. Suppose we later observe $y_{51} = 50.1$. What is the updated forecast $y_{51}(1)$?

A: 49.96 B: 50.1 C: 50.4 D: 50.44

17. The model

$$(1 - 0.6B - 0.2B^2)(1 - B)Y_t = (1 - 0.8B)Z_t$$

is fitted to a time series $y_1, \ldots, y_{100}$, where $\{Z_t\}$ is a noise process with variance estimated to be $\widehat{\sigma}_Z^2 = 1.65$. The forecast $y_{100}(2) = 57.9$ is obtained. What are the 90% prediction limits for this forecast?

A: $(55.07, 60.73)$ B: $(55.19, 60.61)$ C: $(55.79, 60.01)$ D: $(55.38, 60.42)$