F70TS2: Time Series

Multiple Choice Revision Questions: Solutions

1. **D** Under H_0, number of TPs $P \sim N(132,\ 3171/90)$ approx.
P-value = $P(P \geq 145|H_0) = P[Z > (144.5 - 132)/5.936)] = P(Z > 2.106) = 0.018$

2. **C** $p = 3,\ m = 2$ The equations are: $5a_0 + 10a_2 = \sum y_t$ and $10a_0 + 34a_2 = \sum t^2 y_t$
$\Rightarrow 7\tilde{a}_0 = 3.4\sum y_t - \sum t^2 y_t \Rightarrow \tilde{a}_0 = \frac{1}{35}[-3,\ 12,\ \underline{17}]$

3. **A** $\gamma_4 = \gamma_0 \times \rho_4 = 3 \times 0.6^2 = 1.08$

4. **A** $Y_t = (1 - 0.9B + 0.2B^2)Z_t = (1 - 0.5B)(1 - 0.4B)Z_t$
$\Rightarrow Z_t = (1 - 0.5B)^{-1}(1 - 0.4B)^{-1}Y_t$
$= (1+0.5B+0.25B^2+\ldots)(1+0.4B+0.16B^2+\ldots)Y_t = (1+0.9B+0.61B^2+\ldots)Y_t$
$\Rightarrow Y_t = -0.9Y_{t-1} - 0.61Y_{t-1} + \ldots + Z_t$

5. **B** $Y_t = A(B)Z_t$ where $A(B) = \frac{1}{5}(-B^2 + 2B + 3 + 2B^{-1} - B^{-2})$
$\gamma_0^{(Y)} = \frac{1}{25}(1 + 4 + 9 + 4 + 1)\sigma_Z^2 = \frac{19}{25}\sigma_Z^2$
$C_Y(z) = \frac{1}{25}(-z^2 + 2z + 3 + 2z^{-1} - z^{-2})\sigma_Z^2$
$\Rightarrow G_Y(z) = \frac{1}{19}\{19 + 8(z + z^{-1}) - 2(z^2 + z^{-2}) - 4(z^3 + z^{-3}) + (z^4 + z^{-4})\}$
$\Rightarrow f^*(\omega) = 1 + \frac{16}{19}\cos\omega - \frac{4}{19}\cos 2\omega - \frac{8}{19}\cos 3\omega + \frac{2}{19}\cos 4\omega$

6. **D**

7. **B** area is $\pi\sigma_Y^2 = \pi(1 + 0.8^2 + 0.1^2)\sigma_Z^2$

8. **A** we require $\alpha \pm 0.6 < 1,\ \alpha > -1$

9. **A** model is $(1 - B)(1 + 0.4B)Y_t = (1 - B)Z_t$, i.e. $(1 + 0.4B)Y_t = Z_t$

10. **D**

11. **C** $\rho_2 = \gamma_2/\gamma_0 = 5/22.5$

12. **A** model is $Y_t = \{1 + (\alpha + \beta)B(1 - \alpha B)^{-1}\}Z_t$ i.e. $(1 - \alpha B)Y_t = (1 + \beta B)Z_t$

13. **A** MA(2) part: $Z_t + \beta_1 Z_{t-1} + \beta_2 Z_{t-2}$ we require $\beta_2 \pm \beta_1 > -1,\ \beta_2 < 1$

14. **D** $\{Y_t\}$ is MA(2)

15. **C** model is $Y_t = (1 - B)^{-2}(1 + \beta B)Z_t = [1 + (2 + \beta)B + (3 + 2\beta)B^2 + \ldots]Z_t$

16. **D** $y_{50}(2) = 50.4 = 0.1y_{50}(1) + 0.8y_{50} \Rightarrow y_{50} = 56.7875$
$\Rightarrow y_{51}(1) = 0.1y_{51} + 0.8y_{50} = 50.44$

17. **B** $Y_t = (1 - 0.6B - 0.2B^2)^{-1}(1 - B)^{-1}(1 - 0.8B)Z_t$
$\Rightarrow Y_t = (1 + 0.6B + \cdots)(1 + B + \cdots)(1 - 0.8B)Z_t = Z_t + (0.6 + 1 - 0.8)Z_{t-1} + \cdots$
$\Rightarrow \psi_1 = 0.6 + 1 - 0.8 = 0.8$
The prediction limits are $57.9 \pm 1.6449\sqrt{(1 + 0.8^2)1.65} = (55.19, 60.61)$