F70TS2 – Time Series

Exercises 4

1. In the notes it is shown that, under given conditions, the sample mean $\bar{x}$ obtained from a realisation of a stochastic process is asymptotically normal. Is this property important? Why? Assume that $\bar{x} = 1.56$ is obtained from a realisation of a time series with absolutely summable $\gamma(k)$. Furthermore, we also obtained its asymptotic variance Var$(\bar{x}) \approx 0.01$. What is the approximate 95% confidence interval of the unknown expectation μ?

   For a large sample you can use the approximation of the normal quantile $Z_{0.025} = 1.96 \approx 2$.

2. Assume that $\bar{x}$ is calculated from a realisation $x_1, ..., x_{900}$ of the following causal stationary ARMA models with unknown mean:

   (a) $X_t - \mu = 0.5(X_{t-1} - \mu) + 0.2(X_{t-2} - \mu) + 0.4\epsilon_{t-1} + \epsilon_t$,

   (b) $X_t - \mu = 0.2(X_{t-1} - \mu) + 0.6\epsilon_{t-1} + 0.3\epsilon_{t-2} + \epsilon_t$,

   (c) $X_t - \mu = -0.6(X_{t-1} - \mu) + 0.2\epsilon_{t-1} + \epsilon_t$.

   where ϵ_t are i.i.d. $N(0, 1)$ random variables. Calculate the asymptotic Var$(\bar{x})$ in each case and compare your results in all cases. What general conclusions can be drawn?

3. Three causal stationary AR models with unknown mean are given below.

   (a) $X_t - \mu = 0.7(X_{t-1} - \mu) + \epsilon_t$,

   (b) $X_t - \mu = -0.3(X_{t-1} - \mu) + \epsilon_t$,

   (c) $X_t - \mu = 0.45(X_{t-1} - \mu) + 0.3(X_{t-2} - \mu) + \epsilon_t$,

   where ϵ_t are i.i.d. $N(0, 1)$ random variables. Let Y_t denote an i.i.d. process with unknown mean and the same variance as X_t, i.e. Var(Y_t) = Var$(X_t) = \gamma(0)$. Calculate $\gamma(0)$ for each of the models above. Given data $x_1, ..., x_{400}$ and $y_1, ..., y_{400}$, you can obtain $\bar{x}$ and $\bar{y}$. You should calculate the asymptotic Var$(\bar{x})$ for each model and compare them with the corresponding Var$(\bar{y}) = \frac{1}{400}\gamma(0)$. Comment on your results.

4. Suppose you have calculated the first 20 sample autocorrelations $\hat{\rho}(k)$ for $k = 1, ..., 20$, from a time series with $n = 400$ observations. Assume that you know the underlying process $\{X_t\}$ is stationary. You want to check whether X_t could be independent. What are your conclusions in the following cases: a) $|\hat{\rho}(k)| > 0.1$ for at least one k, and b) $|\hat{\rho}(k)| < 0.1$ for all k? Why are the condition '*at least one*' and the bound 0.1 used? Assume now that you calculated the first 40 sample autocorrelations $\hat{\rho}(k)$ for $k = 1, ..., 40$, from a time series with $n = 1600$ observations. How should you formulate and answer similar questions?