Fourier Analysis Midterm

Name:

Problem	1	2	3	4	5	**Total:**
Max	20	20	20	20	20	100
Scores						

Instructions

- Time for test: **75 minutes**.
- Do not use any notes or calculators or any textbooks.

1. (a) State the definition of the Fourier coefficients and the Fourier series for a function f defined in the interval $[0, 2\pi]$.

(b) State the definition of Cesaro summability.

(c) State the definition of a family of good kernels on the circle.

(d) State the definition of Fourier transform on $\mathcal{S}(\mathbb{R})$.

2. (a) State a criteria for the uniform convergence of a Fourier series.

(b) State the Riemann-Lebesgue Lemma.

(c) State the uniqueness theorem for a Fourier series.

(d) State the isoperimetric inequality.

3. State and prove the best approximation lemma.

4. Show that the series

$$\sin x + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \frac{\sin(7x)}{7} + ...$$

is constant for all $x \in (0, \pi)$.

Compute this constant and write the identity when $x = \frac{\pi}{2}$.

5. Suppose that f is a continuous function on $\mathbb{R}$ periodic of period 2π and α/π is irrational. Prove that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(x + n\alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)dt$$

for every x.

(*Hint:* Prove this first for trigonometric polynomials.)