H14ERP Coursework 3 - Error and Data Analysis Laboratory Assignment

1. Assignment Learning Outcomes

It is intended that you will demonstrate the following learning outcomes and be able to:

- Formulate and evaluate methods of experimental measurement and error analysis;
- Synthesize experimental data using a variety of software tools.

It is anticipated that you will also develop your skills in:

- Preparation of technical reports and effective communication.
- Effective use of general IT tools.
- Adopting a scientific approach to problem solving.
- Exercising initiative and personal responsibility.

2. Resources

You will need:

- to be enrolled on the Moodle course H14ERP;
- your own memory stick to save files;
- 'bouncy' balls and measuring tapes (all provided);
- your own digital watch or stopwatch phone app;
- a hard floor and next to a wall onto which you can affix the measuring tape;
- a group of 4, 5 or 6 people with whom you will work;
- MS Excel, MATLAB and MS Word or similar; and
- audio recorder (supplied).

3. Requirements

You are required to form groups of 4-6 people and arrange to complete the experiment and data collection in your own time.

Using the equipment provided and the theory below, you are required to determine the Coefficient of Restitution of the 'bouncy' ball as accurately as you can, applying whichever measuring techniques you deem to be most appropriate. Further you are to plot data in MS Excel and MATLAB and present the plots in your report – see later details.

You are required to submit a report which should not need to exceed 6 pages of size 11 text. The report should contain the following:

- a title, including, your name and 'studentID';
- a short summary (one paragraph);
- an introduction to the laboratory (one paragraph);
- a concise but complete account of the selection and justification of measuring techniques, describing your methods and focusing on the more

challenging issues, and outlining how the errors in measurement could be reduced;

- a concise but complete account of the data and error analysis as a commentary to accompany the plots presented within the report;
- main conclusions (one paragraph);
- include appropriate diagrams and screenshots; and
- (additionally) you are to submit the files developed in MS Excel and MATLAB.

4. Marking Scheme

This laboratory is worth 50% of the H14ERP module. A provisional detailed marking scheme can be found on the last page of this document. You must also hand in the MATLAB scripts and the MS Excel file you use.

5. The Laboratory and Theory

The aim of this laboratory is to enable you to apply the techniques for calculating the error in a final result based on the individual errors in measurements recorded. You will then compare the values obtained with theoretical values generated using simulation methods and finally do a comparison of data obtained with these values.

5.1 The Theory

Following each bounce the height to which the ball rises is determined by the coefficient of restitution (COR) which is a fractional value that represents the ratio of velocities before and after impact. For a COR value of 1 the collision is perfectly elastic, there is no loss of energy and the ball will continue to bounce indefinitely; a value of COR=0 and the ball stop on the first impact; for values in between there is a decay in bounce height (there is a theoretical idea of COR>1 but we will not consider this!).

$$C_R = \frac{v_b - v_a}{u_a - u_h} \qquad (1)$$

for two colliding objects, where

- v_a is the final velocity of the first object after impact
- v_b is the final velocity of the second object after impact
- u_a is the initial velocity of the first object before impact
- u_b is the initial velocity of the second object before impact.

Even though the equation does not reference mass, it is important to note that it still relates to momentum since the final velocities are dependent on mass.

For an object bouncing off a stationary object, such as a floor:

$$C_R = -\frac{v}{u} \qquad (2)$$

where,

- v is the scalar velocity of the object after impact, and
- u is the scalar velocity of the object before impact.

The coefficient can also be found with:

$$C_R = \sqrt{\frac{h}{H}} \tag{3}$$

for an object bouncing off a stationary object, such as a floor, where

- h is the bounce height, and
- H is the drop height.

For two- and three-dimensional collisions of rigid bodies, the velocities used are the components perpendicular to the tangent line/plane at the point of contact.

5.2 Speeds after impact

The equations for collisions between elastic particles can be modified to use the COR, thus becoming applicable to inelastic collisions as well, and every possibility in between.

$$v_a = \frac{m_a u_a + m_b u_b + m_b C_R(u_b - u_a)}{m_a + m_b} \tag{4}$$

and

$$v_b = \frac{m_a u_a + m_b u_b + m_a C_R(u_a - u_b)}{m_a + m_b} \tag{5}$$

where

- v_a is the final velocity of the first object after impact,
- v_b is the final velocity of the second object after impact,
- u_a is the initial velocity of the first object before impact,
- u_b is the initial velocity of the second object before impact,
- m_a is the mass of the first object, and
- m_b is the mass of the second object.

5.3 Derivation

The above equations can be derived from the analytical solution to the system of equations formed by the definition of the COR and the law of the conservation of momentum (which holds for all collisions). Using the notation from above where *u* represents the velocity before the collision and *v* after, we get:

$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$
$$C_R = \frac{v_2 - v_1}{u_1 - u_2} \qquad (6)$$

Solving the momentum conservation equation for v_a and the definition of the coefficient of restitution for v_b yields:

$$\frac{m_a u_a + m_b u_b - m_b v_b}{m_a} = v_a$$
$$v_b = C_R(u_a - u_b) + v_a \qquad (7)$$

Next, substitution into the first equation for v_b and then re-solving for v_a gives:

$$\frac{m_a u_a + m_b u_b - m_b C_R(u_a - u_b) - m_b v_a}{m_a} = v_a$$

$$\frac{m_a u_a + m_b u_b + m_b C_R(u_b - u_a)}{m_a} = v_a\left[1 + \frac{m_b}{m_a}\right]$$

$$\frac{m_a u_a + m_b u_b + m_b C_R(u_b - u_a)}{m_a + m_b} = v_a \qquad (8)$$

A similar derivation yields the formula for v_b.

5.4 Technique

You will drop one of the provided balls from a height of your choice. You need to record the time between bounces, the initial drop height and the height to which the ball rises following each bounce; this is shown in Figure 1. This should be done for five bounces; you should also record the time taken for bounce number 10.

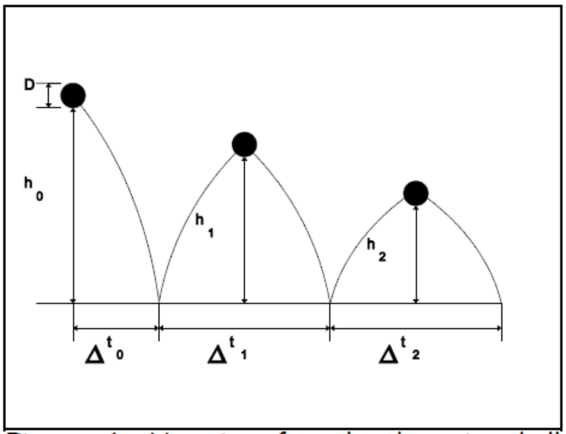

Figure 1. Notation for the bouncing ball experiment.

You should estimate the accuracy with which each measurement can be taken. This will depend on many factors, one of which is your reaction speed. As well as taking manual measurements, record the sound of the ball dropping using the audio recorder, for later analysis.

6. Data Analysis (using Excel)

Using the data you took by hand, you are required to plot graphs showing for each bounce:

- the heights returned to (H_0, H_1, H_2 etc.); and
- the time taken from initial release to each bounce (i.e. Δt_0, $\Delta t_0 + \Delta t_1$, $\Delta t_0 + \Delta t_1 + \Delta t_2$,).

For each graph, plot a line of best fit and determine the value of COR – you may need to research how to do this yourself. (Hint: This will not be a straight line – think about equation 3!)

Now calculate the average and standard deviation the COR values within each ball drop experiment. Also calculate the average value of COR and standard deviation across all experiments, which value do you think will be more accurate?

6. Simulations based on determined value for COR

6.1 Using MS Excel

Knowing the COR and initial height dropped in the experiment, simulate ten bounces of the ball displaying both the height returned to following each bounce and the total time to each impact.

Compare the value for the time taken for bounce ten and compare this with the value observed during the experimental work.

6.2 Using MATLAB to simulate the ball bounce

You are required to develop a simulation that prompts for the initial drop height and COR. It should then produce a plot showing the bounce pattern of the ball (it should resemble Figure 1).

Compare the value for the time taken for bounce ten and compare this with the value observed during the experimental work and from the simulation from 6.1. Comment on any dependencies.

Once you have done this edit your code to produce an animation of the bouncing ball. Save this as an *.avi* file. Make sure you submit it to Moodle in the *.zip* file with the rest of your work.

6.3 Using MATLAB to more accurately analyze the ball bounce data

You should have used the audio recorder to record the sounds of the ball dropping and bouncing. Use the following code to plot the audio data as a function of time. Make sure the graph has correctly labeled axes and a title. Now adjust the code to remove the background noise. This should only require one ***for loop*** and one ***if*** statement, make sure you place the code in your final report. Replot this graph of processed data and include it in the report. Choosing just one experiment, use this program to help you calculate the error in the manual measurements. Calculate by how much this error could have changed the value of COR? Comment on how accurate the audio recorder and MATLAB script are in determining the time between bounces.


```
a=audioread('file_name.WMA');
l=size(a);
l=l(1);
sound(a,44000);

b=a(1:l);
x=linspace(0,1,l);
plot(x,b);
```

7. Submission

You should submit, via electronic submission, the following in a **single** *.zip* file:

- The report file as a pdf. This pdf should include plots from MS Excel and MATLAB.
- The MS Excel file
- The MATLAB files
- The animation.

The zipped folder should be entitled as *<surname>_<firstname>_<studentID>*

e.g. Smith_John_00000000.zip

The assignment should be submitted in Moodle as follows:

H14ERP > Coursework Information > Coursework 3 - Submission

Note: The main text must be submitted as a pdf, docx/doc files will not be marked and awarded a mark of zero. Also, only zip files should be submitted. If our computers cannot open the archive file because it's in a strange formation (*.rar*, *.tar*, *.gz*, *.7z*, etc.) we will also award a mark of zero for the work.

N.B.: Although you are collecting the experimental data as a group, you should note that the analysis of the data and all subsequent written work must be individual work not group work. For this assignment, all work submitted to Moodle or the student office must be individual work not group work. The standard university plagiarism rules apply.
Please keep a backup of your submission.