

Lecture 3: An Optimization example

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ISyE 6073: Financial Optimization

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Problem setup and model

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A short-term financing problem

Your company has the following short-term financing problem (in thousands of dollars):

Month	Net Cash Flow
Jan	-150
Feb	-100
Mar	200
Apr	-200
May	50
Jun	300

(+ is revenue, - is payment)

You have access to the following funding sources:

- You can access a line of credit of up to \$100k at 1% interest per month.
- You can issue 90-day commercial paper bearing a total interest of 2% over the 3-month period (can only issue in the first three months).
- Each month you can invest excess funds at an interest rate of 0.3% per month.

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Optimization model

- **Step 1:** What are the decision variables?

- x_i is the amount borrowed from the line of credit in month $i = 1, \dots, 5$
- y_i is the amount of commercial paper issued in month $i = 1, 2, 3$
- z_i is the surplus funds available at the end of month $i = 1, \dots, 6$

- **Step 2a:** What are the variable bounds?

- $0 \leq x_i \leq 100$ for $i = 1, \dots, 5$
- $y_i \geq 0$ for $i = 1, 2, 3$
- $z_i \geq 0$ for $i = 1, \dots, 6$

- **Step 2b:** What are the constraints?

(surplus = money in - money out)

- Month 1: $z_1 = x_1 + y_1 - 150$
- Month 2: $z_2 = x_2 + y_2 + 1.003z_1 - 1.01x_1 - 100$
- Month 3: $z_3 = x_3 + y_3 + 1.003z_2 - 1.01x_2 + 200$
- Month 4: $z_4 = x_4 + 1.003z_3 - 1.01x_3 - 1.02y_1 - 200$
- Month 5: $z_5 = x_5 + 1.003z_4 - 1.01x_4 - 1.02y_2 + 50$
- Month 6: $z_6 = 1.003z_5 - 1.01x_5 - 1.02y_3 + 300$

- **Step 3:** What is the objective? $\max z_6$

Optimization model

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$$\begin{array}{llll} \max z_0 & & & \\ \text{s.t.} & -x_1 - y_1 & & +z_1 = -150 \end{array}$$

$$\begin{array}{llll} & -x_2 - y_2 & +1.01x_1 & -1.003z_1 + z_2 = -100 \end{array}$$

$$\begin{array}{llll} & -x_3 - y_3 & +1.01x_2 & -1.003z_2 + z_3 = -200 \end{array}$$

$$\begin{array}{llll} & -x_4 + 1.02y_1 & +1.01x_3 & -1.003z_3 + z_4 = -200 \end{array}$$

$$\begin{array}{llll} & -x_5 + 1.02y_2 & +1.01x_4 & -1.003z_4 + z_5 = 50 \end{array}$$

$$\begin{array}{llll} & +1.02y_3 & +1.01x_5 & -1.003z_5 + z_6 = 300 \end{array}$$

$$x_i \leq 100 \quad \forall i = 1, \dots, 5$$

$$x_i \geq 0 \quad \forall i = 1, \dots, 5$$

$$y_i \geq 0 \quad \forall i = 1, \dots, 3$$

$$z_i \geq 0 \quad \forall i = 1, \dots, 6$$

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Solving the optimization model in Excel

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Excel Solver demo

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Examining the optimal solution

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Optimal solution

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	Line of credit	Commercial paper	Excess funds	Net cash flow	Cash flow requirement
Interest rate	1%	2%	0.50%		
January	0	150	0	-150	-150
February	50.980	49.020	0	-100	-100
March	0	203.434	351.944	200	200
April	0	0	0	-200	-200
May	0	0	0	50	50
June			92.497	300	300

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- The 90-day commercial paper is mostly preferable to the line of credit, hence why we issue a lot in March (last opportunity to do so)
 - Optimal final surplus is \$92,497 and it's impossible to do better
 - Big surplus at the end of March is invested to meet April payment and repay the January 90-day loan.

Sensitivity analysis

You bring the optimal financing plan to your manager
feeling very proud... as you well should!
After all, you found an *optimal* solution!



Curveball:

Your manager asks you:

- What if our January payment was \$200k instead of \$150k?
- What if our March revenue was \$250k instead of \$200k?
- The negative cash flow in January is because we are purchasing a \$100k imaging device. The vendor just called and offered us to defer payment by 6 months at a 4% interest rate. Should we take the deal?



Your reaction

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Sensitivity analysis

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- Possible solution: maybe I can just re-solve the optimization problem for all the possible inputs my manager asks about?
- Yes, but because our problem is a linear program, we can do better!

Pro tip

There is a lot of "what if" information in the *sensitivity report*!

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Sensitivity report
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Shadow prices of constraints

Constraints

Cell	Constraint	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$6	January Net cash flow	-150	1.0373	-150	150	89.1719
\$F\$7	February Net cash flow	-100	1.0302	-100	50.9804	49.0196
\$F\$8	March Net cash flow	200	1.02	200	203.4344	90.6833
\$F\$9	April Net cash flow	-200	1.0169	-200	204.0447	90.9553
\$F\$10	May Net cash flow	50	1.01	50	52	50
\$F\$11	June Net cash flow	300	1	300	1E+30	92.4969

How to read the report:

- One row per constraint $g_i(x, y, z) \leq b_i$
- Final value is the value of $g_i(\cdot)$ ('left-hand side') when we plug in the optimal decision variable values
- Constraint "right-hand side" is b_i
- The "shadow price" tells us the *sensitivity* of the optimum to a change in the constraint right-hand side!

Shadow prices of constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$6	January Net cash flow	-150	1.0373	-150	150	89.1719
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Theorem (Linear programming sensitivity analysis)

If the right-hand side of a constraint with shadow price u changes by an amount Δ , the optimal objective value changes by $u \cdot \Delta$, as long as Δ is within a certain range.

The power of shadow prices

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$6	January Net cash flow	-150	1.0373	-150	150	89.1719
\$F\$7	February Net cash flow	-100	1.0302	-100	50.9804	49.0196
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\$F\$11	June Net cash flow	300	1	300	1E+30	92.4969

Q1: What if our January payment was \$200k instead of \$150k?

- Shadow price of January constraint is 1.0373
- \$50k is less than the allowable decrease (\$89k)
- $50 \cdot 1.0373 = 51.865$
- Answer:** Our June surplus would *decrease* by \$51,865

The power of shadow prices

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$6	January Net cash flow	-150	1.0373	-150	150	89.1719
\$F\$7	February Net cash flow	-100	1.0302	-100	50.9804	49.0196
\$F\$8	March Net cash flow	200	1.02	200	203.4344	90.6833
\$F\$9	April Net cash flow	200	1.0169	-200	104.047	90.9553
\$F\$10	May Net cash flow	50	1.01	50	52	50
\$F\$11	June Net cash flow	300	1	300	1E+30	92.4969

Q2: What if our March revenue was \$250k instead of \$200k?

- Shadow price of March constraint is 1.02
- \$50k is less than the allowable increase
- $50 \cdot 1.02 = 51$
- Answer:** Our June surplus would *increase* by \$51,000

The power of shadow prices

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$5	January Net cash flow	-150	1.0373	-150	150	89.4719
\$F\$7	February Net cash flow	-100	1.0302	-100	50.9804	49.0196
\$F\$8	March Net cash flow	200	1.02	200	203.4344	90.6833
\$F\$9	April Net cash flow	-200	1.0169	-200	204.0447	90.9553
\$F\$10	May Net cash flow	50	1.01	50	52	50
\$F\$11	June Net cash flow	300	1	300	1E+30	92.4969

Q3: The negative cash flow in January is because we are purchasing a \$100k imaging device. The vendor just called and offered us to defer payment by 6 months at a 4% interest rate. Should we take the deal?

- Shadow price of January constraint is 1.0373
- Reducing cash requirements by \$1 in January increases wealth by \$1.0373 in June
- “Break-even” interest rate is 3.73% (and \$100k is allowable increase)
- **Answer:** No, we should not accept the deal (net loss \$270).

Your manager's next query

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- You answered your manager's questions! Good job!
- However, your manager is still dubious.

Our financing problems are similar year-to-year and we usually take out big loans in January. Are you sure we aren't better off withdrawing more from the line of credit in January?

Ugh, another curveball. But it turns out the sensitivity report can answer that question as well!

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Reduced costs of variables

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$6	January Line of credit	0	-0.0033	0	0.0033	1E+30
\$C\$7	February Line of credit	50.9804	0	0	0.0037	
\$C\$8	March Line of credit	0	-0.0071	0	0.0071	1E+30
\$C\$9	April Line of credit	0	-0.0032	0	0.0032	1E+30
\$C\$10	May Line of credit	0	0	0	0	1E+30
\$D\$6	January Commercial paper	150	0	0	0.0040	0.0032
\$D\$7	February Commercial paper	49.0196	0	0	0	0.0032
\$D\$8	March Commercial paper	203.4314	0	0	0.0071	0
\$E\$6	January Excess funds	0	-0.0040	0	0.0040	1E+30
\$E\$7	February Excess funds	0	-0.00714	0	0.0071	1E+30
\$E\$8	March Excess funds	351.9442	0	0	0.0039	0.0032
\$E\$9	April Excess funds	0	-0.0039	0	0.0039	1E+30
\$E\$10	May Excess funds	0	-0.007	0	0.007	1E+30
\$E\$11	June Excess funds	92.969	0	0	1E+30	1

- One row per decision variable
- The “reduced cost” of a zero decision variable tells you by how much the objective changes if the decision variable value increases
- Each dollar we borrow from the line of credit in January decreases our final wealth by \$3.2

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From Excel to Julia

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Julia/JuMP demo

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