Homework 1

September 14, 2022

You are allowed to work in groups of at most 3 people for the following assignment and submit joint responses in ONE pdf file and one .jl (Julia) file. Please write each name and ID clearly in the front page. For Question 1, you should submit a Julia code file as well as copying and pasting the code into the PDF.

1. **Spurious regression problem.**

We verify the spurious regression problem using simulated data.

(a) Use the Random library to fix the seed using *Random.seed*!(1234). This guarantees results are reproducible. Construct two independent random walks x_t and y_t of length $N = 10,000$. To do this, first stimulate two standard normals e_x and e_y. Then set $y_1 = 0, x_1 = 0$ and generate

$$y_t = y_{t-1} + e_{y,t}$$
$$x_t = x_{t-1} + e_{x,t}$$

for $2 \leq t \leq N$. Note, you can generate draws from a standard random normal using *randn*. Plot x and y in the same figure with respect to time. For this, use can use either the Plots or Pyplot libraries.

(b) Using the DataFrame and GLM libraries, regress y on x and a constant:

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

Report the estimate $\hat{\beta}_1$ and its standard error $SE(\hat{\beta}_1)$. Is it significantly different from zero? If so, is this misleading?

(c) Now re-run the regression with a lagged dependent variable:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 y_{t-1} + \varepsilon_t$$

What are the estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ and their respective standard errors? How does the coefficient on x compare in this case to without the lagged dependent variable?

(d) Now, using the same random draws e_{yt} and e_{xt} as before, define AR(1) processes for x and y with $\rho_x = \rho_y = 0.8$:

$$y_t = \rho_x y_{t-1} + e_{y,t}$$
$$x_t = \rho_x x_{t-1} + e_{x,t}$$

and regress y on x (and a constant). What is the slope coefficient and the standard error? How does this compare to part (a)?

(e) Now, maintain the assumption that x_t is a random walk but redefine y_t to depend on x_t as follows:

$$y_t = \beta x_t + e_{y,t}$$

with $\beta = 2$. Repeat the regression of y on x. What is the estimated slope coefficient $\hat{\beta}_1$? How does it compare to 2?

2. **Markov chain**

Consider a 2-state Markov chain and a random variable $y_t = \overline{y}' x_t$, where $\overline{y} = (2, 5)'$. Suppose it is known that $E(y_{t+1}|x_t) = (3, 4)$. *Note: you should do this problem by hand and show your work, but of course you can use the computer to check accuracy.*

(a) Find a transition matrix consistent with the conditional expectations.

(b) Find a stationary distribution for the Markov chain x_t.

(c) What is the probability of moving from state 1 to state 2 in 3 periods?

3. **Impulse response of an ARMA(1, 1) process** Let x_t follow ARMA(1, 1) process–that is, the sum of an AR(1) and MA(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

where $\{\varepsilon_t\}$ is white noise.

(a) Write this in the form $\phi(L)x_t = \theta(L)\varepsilon_t$ for polynomials $\phi(L)$ and $\theta(L)$, being sure to specify the polynomials.

(b) Assuming the roots of the polynomial $\phi(z)$ lie outside the unit circle, the process is covariance stationary, and the series can be represented by a square summable sequence $x_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$. Using $x_t = \phi(L)^{-1}\theta(L)\varepsilon_t$, rearrange terms to solve for $\{\psi_j\}$. You should be able to find the pattern via algebraic rearrangement.

(c) The mapping $t \to \psi_t$ is the impulse response function. Plot the coefficients $\{\psi_t\}$ for $\theta = 0.2$ and (1) either $\rho = 0.8$ or $\rho = -0.8$. How do the responses compare?

Both processes feature declining amplitude over time but $\rho = -0.8$ generates oscillations. The effects of the persistence parameter are modified with θ_1 such that there is an initial upward bump if $\rho = 0.9$ prior to decaying.