Solutions to Predicate Logic Tutorial 3

Q1.

i) c and d.

ii) You have to show $\vdash c \to d$ and $\vdash d \to c$. I will show the first.

showing $\vdash c \to d$:

~~1~~. $\forall X\,(\text{banker}(X) \lor \text{estate\_agent}(X) \to \text{unpopular}(X))$	assume
~~2~~. $\text{banker}(a)$	assume
3. $\text{banker}(a) \lor \text{estate\_agent}(a)$	2, $\lor$I
4. $\text{banker}(a) \lor \text{estate\_agent}(a) \to \text{unpopular}(a)$	1, $\forall$E
5. $\text{unpopular}(a)$	3,4, $\to$E
6. $\text{banker}(a) \to \text{unpopular}(a)$	2,5, $\to$I
7. $\forall X\,(\text{banker}(X) \to \text{unpopular}(X))$	6,$\forall$I

In an almost identical way you can show

$\forall X\,(\text{estate\_agent}(X) \to \text{unpopular}(X))$

Then use $\land$I to derive

$\forall X\,(\text{banker}(X) \to \text{unpopular}(X)) \land \forall X\,(\text{estate\_agent}(X) \to \text{unpopular}(X))$

Then by $\to$I, you get $c \to d$, discharging 1.

Showing $\vdash d \to c$:

1. $\forall X\,(\text{banker}(X) \to \text{unpopular}(X)) \land \forall X\,(\text{estate\_agent}(X) \to \text{unpopular}(X))$	assume
2. $\forall X\,(\text{banker}(X) \to \text{unpopular}(X))$	1, $\land$E
3. $\forall X\,(\text{estate\_agent}(X) \to \text{unpopular}(X))$	1, $\land$E
4. $\text{banker}(a) \lor \text{estate\_agent}(a)$	assume
5. $\text{banker}(a) \to \text{unpopular}(a)$	2, $\forall$E
6. $\text{estate\_agent}(a) \to \text{unpopular}(a)$	3, $\forall$E
7. $\text{unpopular}(a)$	Proof by cases, 4, 5, 6
8. $\text{banker}(a) \lor \text{estate\_agent}(a) \to \text{unpopular}(a)$	$\to$I, 4, 7
9. $\forall X\,(\text{banker}(X) \lor \text{estate\_agent}(X) \to \text{unpopular}(X))$	$\forall$I, 8

Then by $\to$I you get $d \to c$, discharging 1.

Q2.

a.

1. $\forall X\,(p(X) \to q(X) \land r(X))$	given
2. $p(a)$	assume
3. $p(a) \to q(a) \land r(a)$	1, $\forall$E
4. $q(a) \land r(a)$	3, $\to$E
5. $q(a)$	4, $\land$E
6. $p(a) \to q(a)$	2,5, $\to$I
7. $\forall X\,(p(X) \to q(X))$	6, $\forall$I

Similarly we prove

$\forall X\,(p(X) \to r(X))$

And then apply $\land$I to get:

$\forall X\, (p(X) \rightarrow q(X)) \wedge \forall X\, (p(X) \rightarrow r(X))$

b.

1. $\forall X\, (p(X) \rightarrow (q(X) \rightarrow r(X)))$	given
2. $p(a) \wedge q(a)$	assume
3. $p(a)$	2, $\wedge E$
4. $q(a) \rightarrow r(a)$	1,3, $\forall\!\rightarrow\! E$
5. $q(a)$	2, $\wedge E$
6. $r(a)$	4, 5,$\rightarrow E$
7. $p(a) \wedge q(a) \rightarrow r(a)$	2,6, $\rightarrow I$
8. $\forall X\, (p(X) \wedge q(X) \rightarrow r(X))$	7, $\forall I$

c.

1. $\forall X\, (p(X) \rightarrow \neg q(X))$	given
2. $p(a)$	given
3. $\forall Y (q(Y) \vee s(Y))$	given
4. $\neg q(a)$	1,2,$\forall\!\rightarrow\! E$
5. $q(a) \vee s(a)$	3, $\forall E$
6. $s(a)$	4,5, $\vee E$

d. Hint: Think of using proof by cases. Then it is easy!