Solutions to Logic Tutorial 4

Q1a.

1. A∧B	Given
2. A	1, ∧E
3. B	1, ∧E
4. B∧A	2, 3, ∧I

Q1b.

1. A∧B	Given
2. A	1, ∧E
3. A∨B	2, ∨I

Q1c.

1.	P∧Q	Given
2.	P→R	Given
3.	Q→S	Given
4.	P	1, ∧E
5.	Q	1, ∧E
6.	R	2, 4, →E
7.	S	3, 5, →E
8.	R∧S	6, 7, ∧I

Q1d.

1.	P → (Q→R)	Given
2.	P∧Q	Assume
3.	P	2, ∧E
4.	Q	2, ∧E
5.	Q→R	1, 3, →E
6.	R	4, 5, →E
7.	P∧Q→R	2,6, →I

Q1e.
You have to show
(P→Q) → (¬Q→¬P) and
(¬Q→¬P) → (P→Q).
I will just do the first here.The second is similar.

1. (P→Q)	assume	
2. ¬Q	assume	
3. P	assume	
4. Q	1,3, →E	
5. ¬P	RAA, 3,4,2	
6. ¬Q→¬P	→I, 2,5	
7. (P→Q) →(¬Q→¬P)	→I,1,6	

Q2. Using L for “PM loses next vote”, C for “PM’s leadership is challenged”, E for “PM will call a general election”:

i) L→(¬C→E)
ii) (L∧¬C)→E

Showing i |- ii:

1. L→(¬C→E)	Given	
	~~2.~~ L∧¬C	assume
	3. L	2, ∧E
	4. ¬C→E	1, 3, →E
	5. ¬C	2, ∧E
	6. E	4, 5, →E
7. (L∧¬C)→E		2, 6, →I

Showing ii |- i:

1. (L∧¬C)→E	Given		
	~~2.~~ L	assume	
		~~3.~~ ¬C	assume
		4. L∧¬C	2,3, ∧I
		5. E	1, 4, →E
	6. ¬C→E	3, 5, →I	
7. L→(¬C→E)	2, 6, →I		

Q3.

a. Murderer
b. Formalising the information:

1. Murderer∨Blackmailer
2. Murderer→Violent
3. Blackmailer→Rich
4. Rich→Spends∨Account
5. ¬Account
6. ¬Spends

Abbreviate to:

1.	M∨B	Given
2.	M→V	Given
3.	B→R	Given
4.	R→S∨A	Given
5.	¬A	Given
6.	¬S	Given

Deriving M:

	~~7.~~ B	assume
	8. R	3, 7, →E
	9. S∨A	4, 8, →E
	10. S	9, 5, ∨E
11. ¬B		7, 10, 6, RAA
12. M		1, 11, ∨E

Q4

a) Showing $\neg(p \rightarrow q) \equiv p \wedge \neg q$

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q \equiv p \wedge \neg q$

b)

I will use (a) and also

lemma 1 (I leave the proof of lemma 1 to you.):

$A, A \wedge \neg B \rightarrow C \vdash \neg B \rightarrow C$

1. $A \wedge \neg B \rightarrow C$	Given
2. $B \rightarrow C$	Given
3. $C \rightarrow \neg(B \rightarrow A)$	Given
4. $A \vee B \vee C$	Given
5. A	Assume
6. $\neg B \rightarrow C$	1, 5, lemma1
7. C	2, 6, dilemma
8. $\neg(B \rightarrow A)$	3, 7, $\rightarrow$E
9. $B \wedge \neg A$	8, (a)
10. $\neg A$	9, $\wedge$E
11. $\neg A$	5, 10, RAA
12. $B \vee C$	4, 11, $\vee$I
13. B	assume
14. C	2, 13, $\rightarrow$E
15. $B \wedge C$	13, 14, $\wedge$I
16. $B \rightarrow B \wedge C$	13, 15, $\rightarrow$I
17. C	assume
18. $\neg(B \rightarrow A)$	17, 3, $\rightarrow$E
19. B	18, (a), $\wedge$E
20. $B \wedge C$	17, 19, $\wedge$I
21. $C \rightarrow B \wedge C$	17, 20, $\rightarrow$I
22. $B \wedge C$	12, 16, 21, proof by cases