Logic Tutorial 3 Solutions

1.

a) Proving $P \vee Q \equiv (P \to Q) \to Q$:

$(P \to Q) \to Q \equiv \neg(P \to Q) \vee Q$	Implication rule
$\equiv \neg(\neg P \vee Q) \vee Q$	Implication rule
$\equiv (\neg\neg P \wedge \neg Q) \vee Q$	De Morgan
$\equiv (P \wedge \neg Q) \vee Q$	Double negation rule
$\equiv (P \vee Q) \wedge (\neg Q \vee Q)$	Distributive rules
$\equiv P \vee Q$	$\neg Q \vee Q$ is a tautology

b) Proving $P \wedge Q \to R \equiv (P \to R) \vee (Q \to R)$:

$(P \to R) \vee (Q \to R) \equiv (\neg P \vee R) \vee (\neg Q \vee R)$	Implication rule
$\equiv \neg P \vee (R \vee (\neg Q \vee R))$	Associative rules
$\equiv \neg P \vee (R \vee (R \vee \neg Q))$	Commutative rules
$\equiv \neg P \vee ((R \vee R) \vee \neg Q)$	Associative rules
$\equiv \neg P \vee (R \vee \neg Q)$	$R \vee R \equiv R$
$\equiv \neg P \vee (\neg Q \vee R)$	Commutative rules
$\equiv (\neg P \vee \neg Q) \vee R$	Associative rules
$\equiv \neg(P \wedge Q) \vee R$	De Morgan
$\equiv P \wedge Q \to R$	Implication rule

c) Proving $P \to (Q \to R) \equiv (P \to Q) \to (P \to R)$:

$(P \to Q) \to (P \to R)$	Implication rule
$\equiv \neg(P \to Q) \vee (P \to R)$	Implication rule
$\equiv \neg(\neg P \vee Q) \vee (P \to R)$	Implication rule
$\equiv (\neg\neg P \wedge \neg Q) \vee (\neg P \vee R)$	Implication rule and De Morgan
$\equiv (P \wedge \neg Q) \vee (\neg P \vee R)$	Double negation rule
$\equiv (\neg P \vee R) \vee (P \wedge \neg Q)$	Commutative rules
$\equiv ((\neg P \vee R) \vee P) \wedge ((\neg P \vee R) \vee \neg Q)$	Distributive rules
$\equiv ((\neg P \vee P) \vee R) \wedge ((\neg P \vee R) \vee \neg Q)$	Commutative and Associative rules
$\equiv (\neg P \vee R) \vee \neg Q$	$\neg P \vee P$ is a tautology
$\equiv \neg P \vee (R \vee \neg Q) \equiv \neg P \vee (\neg Q \vee R)$	Commutative and Associative rules
$\equiv P \to (Q \to R)$	Implication rule

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a. I will use the following propositional symbols:
D: to stand for “capital punishment deters capital crime”
J: to stand for “capital punishment is justified”

Premise:
$D \rightarrow J$
$\neg D$

Conclusion
$\neg J$

D	J	$D \rightarrow J$	$\neg D$	Premise	$\neg J$
T	T	T	F	F	F
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	T	T

The third row shows that there is an interpretation in which the premise is true but the conclusion is not. So the conclusion is a not a semantic consequence of the premise.

b. I will use the following propositional symbols:
W: to stand for “we conduct a war”
S : to stand for “we solve our domestic problems”

Premise:
$\neg(W \wedge S)$

Conclusion:
$S \rightarrow \neg W$

			Premise		
W	S	$W \wedge S$	$\neg(W \wedge S)$	$\neg W$	$S \rightarrow \neg W$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	F	T	T	T

The conclusion follows from the premise.

3.
I will use the following propositional symbols:

L: to stand for “lung cancer is more common among male smokers”
S: to stand for “smoking causes lung cancer”
M: to stand for “lung cancer is caused by something in the male makeup”

Premise:
L
$S \rightarrow \neg L$
$L \rightarrow M$

Conclusion
$\neg S \wedge M$

L	S	M	$\neg L$	$S \rightarrow \neg L$	$L \rightarrow M$	$\neg S$	Premise	$\neg S \wedge M$
T	T	T	F	F	T	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F
F	T	T	T	T	T	F	F	F
F	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	F

The third row is the only interpretation in which all the wffs in the premise are true, and there the conclusion is also true. So the conclusion is semantically entailed by the premise.