

Predicate Logic
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Part 2
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Recall Syntax of a grammatically correct sentence (wff) in predicate logic

- $p(t_1, \dots, t_n)$ is a wff if p is an n -ary predicate symbol and the t_i are terms.
- If W , $W1$, and $W2$ are wffs then so are the following:
 - $\neg W$
 - $W1 \wedge W2$
 - $W1 \vee W2$
 - $W1 \rightarrow W2$
 - $W1 \leftrightarrow W2$
 - $\forall X(W)$
 - $\exists X(W)$
 where X is a variable symbol.
- There are no other wffs.

From the description above you can see that propositional logic is a special case of predicate logic.

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Predicate Logic: the predicates are n -ary, $n \geq 0$, and we have terms and quantifiers

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Propositional Logic:
all the predicates are
nullary

Some useful equivalences

All propositional logic equivalences hold for predicate logic wffs.

E.g. $A \rightarrow B \equiv \neg A \vee B$

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So

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$\text{able_to_work}(\text{john}) \rightarrow \text{employed}(\text{john}) \equiv$
 $\neg \text{able_to_work}(\text{john}) \vee \text{employed}(\text{john})$
 $\forall X (\text{able_to_work}(X) \rightarrow \text{employed}(X)) \equiv$
 $\forall X (\neg \text{able_to_work}(X) \vee \text{employed}(X))$

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Some useful equivalences cntd.

E.g. $\neg(A \wedge B) = \neg A \vee \neg B$

So

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$\neg (\text{academic}(\text{john}) \wedge \text{rich}(\text{john})) \equiv$
 $\neg \text{academic}(\text{john}) \vee \neg \text{rich}(\text{john})$

Another instance of the same equivalence:

$$\neg(A \wedge B) \equiv \neg A \vee \neg B$$

A

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$$\neg(\forall X (\text{able_to_work}(X) \rightarrow \text{employed}(X)) \wedge \text{inflation}(\text{low})) \equiv$$

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B

$$\neg (\forall X (\text{able_to_work}(X) \rightarrow \text{employed}(X))) \vee \neg \text{inflation}(\text{low})$$

Some other equivalences in predicate logic

- $\forall X p(X) \equiv \neg \exists X \neg p(X)$

all true, none false

- $\forall X \neg p(X) \equiv \neg \exists X p(X)$

all false - none true

- $\exists X p(X) \equiv \neg \forall X \neg p(X)$

at least one true - not all false

- $\exists X \neg p(X) \equiv \neg \forall X p(X)$

at least one false - not all true

Equivalence exercises

$$\begin{aligned} & \forall X (\text{cautious}(X) \vee \text{normal}(X) \rightarrow \\ & \quad \exists Y \text{shelter}(Y, X)) \equiv \\ & \neg \exists X ((\text{cautious}(X) \vee \text{normal}(X)) \wedge \\ & \quad \neg \exists Y \text{shelter}(Y, X)) \\ & \forall X \forall Y (\text{aggressive}(X) \wedge \text{sees}(X, Y) \rightarrow \\ & \quad \text{fights}(X, Y)) \equiv \\ & \forall X \neg \exists Y (\text{aggressive}(X) \wedge \text{sees}(X, Y) \wedge \\ & \quad \neg \text{fights}(X, Y)) \end{aligned}$$

Some other equivalences in predicate logic

Suppose $W1$, $W2$ are wffs.

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If $W1$ can be transformed to $W2$ by a consistent renaming of variables, then $W1$ and $W2$ are equivalent.

E.g.

$$\forall X p(X) \equiv \forall Y p(Y)$$

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$$\forall X \exists Y (p(X, Y) \rightarrow q(Y, X)) \equiv$$

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$$\forall Z \exists W (p(Z, W) \rightarrow q(W, Z))$$

Some other equivalences in predicate logic

If two wffs differ only in the order of two adjacent quantifiers of the same kind, then they are equivalent. E.g.

$$\forall X \forall Y p(X, Y) \equiv \forall Y \forall X p(X, Y)$$
$$\exists X \exists Y p(X, Y) \equiv \exists Y \exists X p(X, Y)$$

But

$$\forall X \exists Y p(X, Y) \quad \text{is not equivalent to}$$
$$\exists Y \forall X p(X, Y)$$

More Equivalences

$$\exists X(A \vee B) \equiv \exists XA \vee \exists XB$$

E.g.

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$$\exists X(\text{male}(X) \vee \text{female}(X)) \equiv$$

$$\exists X \text{ male}(X) \vee \exists X \text{ female}(X)$$

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More Equivalences

$$\forall X (A \wedge B) \equiv \forall X A \wedge \forall X B$$

E.g.

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$$\forall X (\text{mscDegree}(X) \rightarrow \text{duration}(X, 12\text{months}) \wedge \text{phdDegree}(X) \rightarrow \text{duration}(X, 42\text{months})) \equiv$$

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$$\forall X (\text{mscDegree}(X) \rightarrow \text{duration}(X, 12\text{months})) \wedge \forall X (\text{phdDegree}(X) \rightarrow \text{duration}(X, 42\text{months}))$$

Some notes on quantifiers

1. Free and Bound variables:

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An occurrence of a variable in a wff is
bound if it is within the scope of a quantifier
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in that wff. It is free if it is not within the
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scope of any quantifier in that wff.

Examples

$$\forall X (p(X) \rightarrow q(Y, X))$$

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Both occurrences of X in the above wff are bound (they are both within the scope of the \forall .)

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The occurrence of Y is free (it is not within the scope of any quantifier.)

$$(\forall X p(X)) \wedge (\exists X q(X))$$

In the wff above, both occurrences of X are bound, the first by the \forall , the second by the \exists .

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$$(\forall X p(X)) \wedge (\exists Y q(X, Y))$$

In the wff above, the first occurrence of X is bound, the second is free. The occurrence of Y is bound.

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Definition.

If a wff contains no free occurrences of variables it is said to be **closed**, otherwise it is said to be **open**.

A wff with no free occurrences of variables is also called a **sentence**.

E.g.

$\text{Bird}(X) \rightarrow \text{has_beak}(X)$

is a wff but not a sentence.

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$\forall X (\text{Bird}(X) \rightarrow \text{has_beak}(X))$

is a wff and a sentence.

Back to equivalences

2. A particular occurrence of a variable is bound by the closest quantifier which can bind it.

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E.g.

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$$\forall X (p(X) \rightarrow \forall X q(X)) \equiv$$

$$\forall X (p(X) \rightarrow \forall Y q(Y))$$

3. Law of vacuous quantification

$\forall X W \equiv W$ if W (a wff) contains no free occurrences of X .

$$\forall X (p(a) \rightarrow q(a)) \equiv p(a) \rightarrow q(a)$$

$$\forall X \exists X p(X) \equiv \exists X p(X)$$

$$\forall X \exists Y (p(X) \rightarrow \exists Y q(X, Y))$$

which quantification can we drop?

More Equivalences

If X does not occur free in A then

$\forall X(A \rightarrow B) \equiv A \rightarrow \forall X B$, and

$\exists X(A \rightarrow B) \equiv A \rightarrow \exists X B$.

E.g.

$\forall X(\text{funny}(\text{john}) \rightarrow \text{loves}(X, \text{john})) \equiv$
 $\text{funny}(\text{john}) \rightarrow \forall X \text{ loves}(X, \text{john})$

More Equivalences

If X doesn't occur free in A , then

$\exists X(A \wedge B) \equiv A \wedge \exists X B$, and

$\forall X(A \vee B) \equiv A \vee \forall X B$.

E.g.

$\exists X(\text{station}(\text{victoria}) \wedge \text{tubeLine}(X, \text{victoria}))$
 $\equiv \text{station}(\text{victoria}) \wedge \exists X \text{tubeLine}(X, \text{victoria})$

More Equivalences

If X does not occur free in B then

$\forall X(A \rightarrow B) \equiv \exists X A \rightarrow B$, and

$\exists X(A \rightarrow B) \equiv \forall X A \rightarrow B$.

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Be careful:

The quantifier changes.

$\forall X(A \rightarrow B)$ is equivalent to $\exists X A \rightarrow B$, and
 $\exists X(A \rightarrow B)$ is equivalent to $\forall X A \rightarrow B$

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E.g.

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$\forall X(\text{loves}(X, \text{john}) \rightarrow \text{happy}(\text{john})) \equiv$
 $(\exists X \text{ loves}(X, \text{john})) \rightarrow \text{happy}(\text{john})$

Warning: non-equivalences

The following are *NOT* logically equivalent
(though always, the first \models the second):

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$\forall X(A \rightarrow B)$ and $\forall XA \rightarrow \forall XB$

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$\exists X(A \wedge B)$ and $\exists XA \wedge \exists XB$

$\forall XA \vee \forall XB$ and $\forall X (A \vee B)$

Can you find a ‘counter-example’ for each one?

Counter-example for

$\forall X(p(X) \rightarrow q(X))$ and $\forall Xp(X) \rightarrow \forall Xq(X)$

Take

$p(a)$ $p(b)$ $\neg p(c)$

$q(a)$ $\neg q(b)$

Then RHS is true, but LHS is not.

Examples for slide 26

$\exists X(A \wedge B)$ and $\exists XA \wedge \exists XB$

Not equivalent

$\exists X(\text{male}(X) \wedge \text{female}(X))$ and

$\exists X \text{ male}(X) \wedge \exists X \text{ female}(X)$

$\forall XA \vee \forall XB$ and $\forall X (A \vee B)$

$\forall X \text{ msc}(X) \vee \forall X \text{ meng}(X)$ and

$\forall X (\text{msc}(X) \vee \text{meng}(X))$