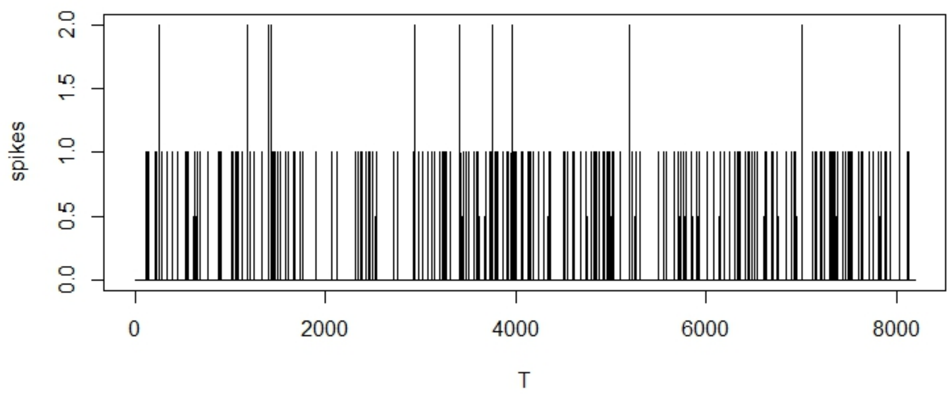
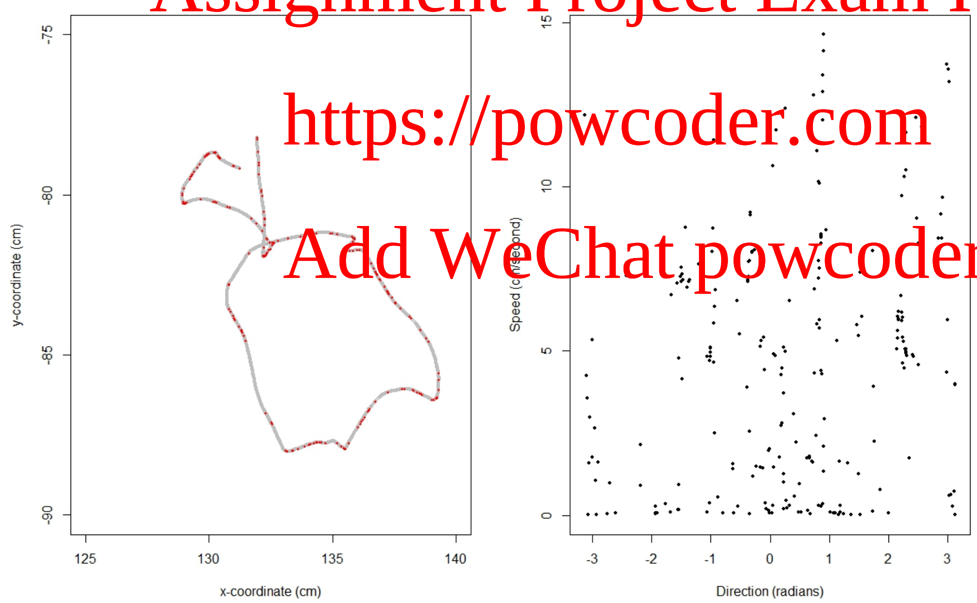
MA 568 Statistical Analysis of Point Process Data
Problem set 2 solution

Long Tao and Uri Eden
Department of Mathematics and Statistics, Boston university

Question 1
Plot the spiking activity as a time series, plot the hand position trajectory (X vs. Y) with the hand position at the spike times overlaid and plot the velocity and direction of movement (V vs. phi) at the spike times as well.



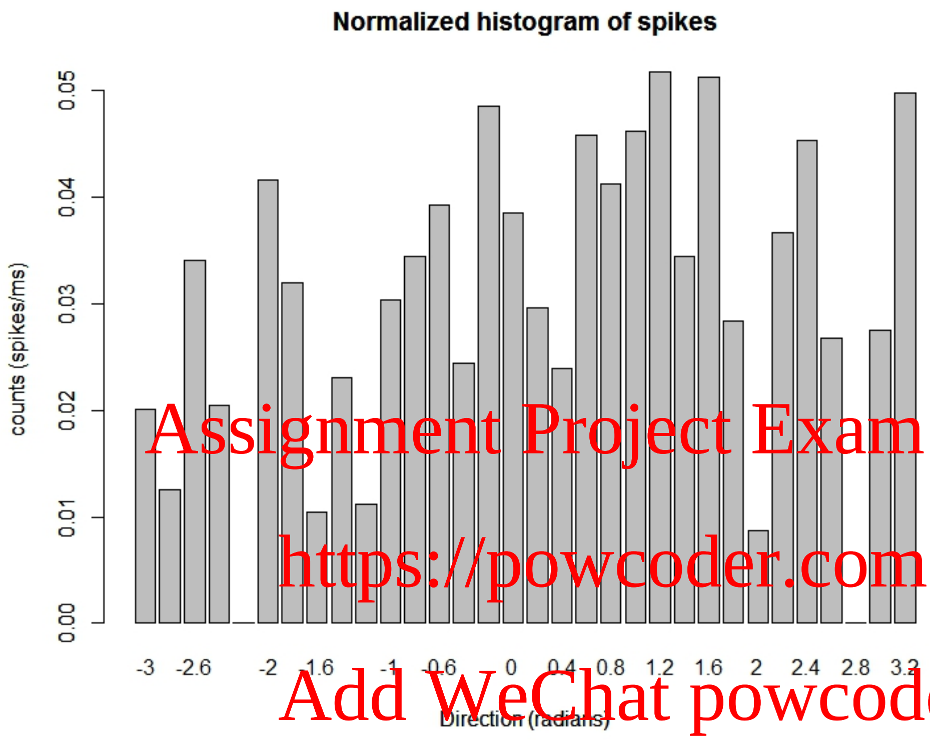



```
library(R.matlab)
data <- readMat('M1_spikes.new.version.mat'); attach(data)
#1.a
par(mfrow=c(1,3))
plot(T, spikes, type='l')
# 1.b
plot(X,Y,col='gray',cex=.5,xlab='x-coordinate (cm)',ylab='y-coordinate (cm)',
xlim=c(125,140),ylim=c(-90,-75))
points(X[spiketimes],Y[spiketimes],col='red',pch=19,cex=.4)
```

```
#1.c
plot(phi[spiketimes],V[spiketimes],cex=.5, pch=19,
xlab='Direction (radians)', ylab='Speed (cm/second)')
```

Question 2

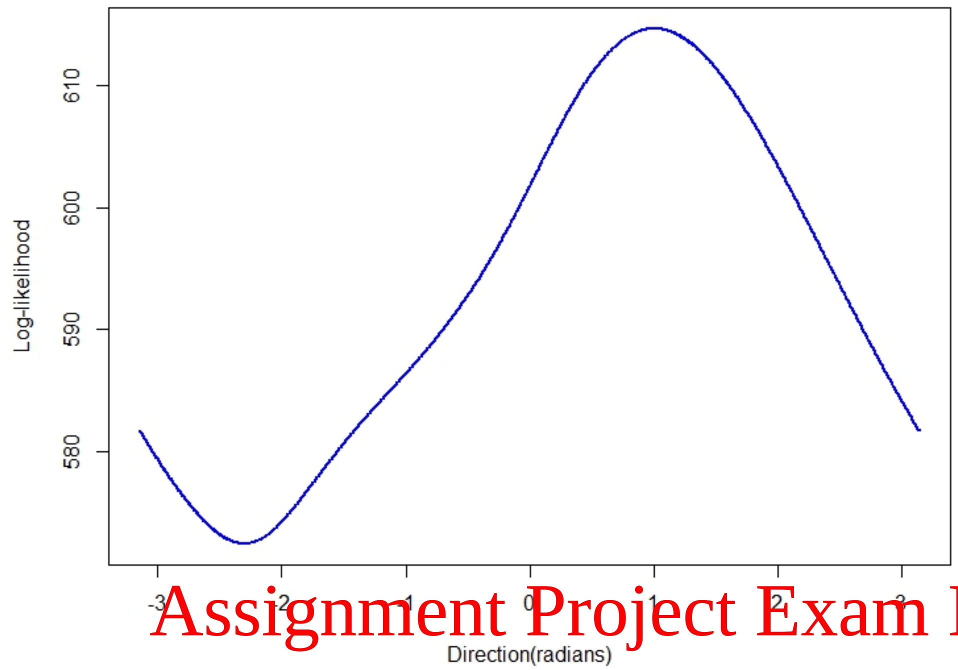
This occupancy normalized histogram suggests that the neuron fires most when the animal is moving approximately 1-1.2 radians relative to movement to the right, at a rate of approximately 55Hz.




```
dirs=seq(-3.2,3.2,by=.2)
a=hist(phi, breaks=dirs, plot=F)
b=hist(phi[spiketimes], breaks =dirs, plot=F)
norm_spike=b$counts/a$counts
names(norm_spike)=dirs[-1]
barplot(norm_spike, main='Normalized histogram of spikes',
xlab='Direction (radians)', ylab="counts (spikes/ms)")
```

Question 3

Here is the log-likelihood function plot, and we find our MLE of $\hat{\phi}_{\text{preferred}} = 1.003$ radian. The 95% confidence interval is [0.634,1.366].



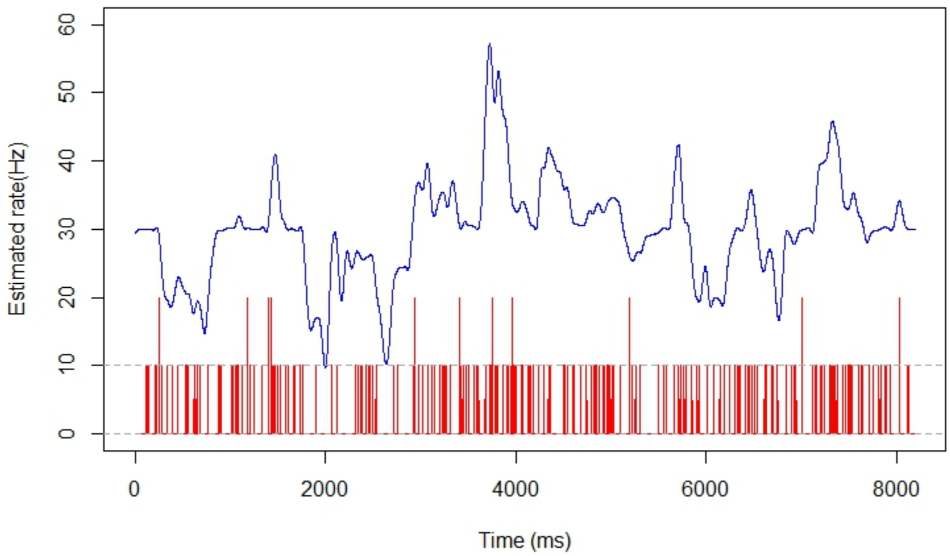

```
# 3.
delta_t = .001
alpha=30; beta=30; v_max =16.1;
phi_max=seq(-3.14,3.14,by=.01)
log_L=numeric(length(phi_max))
for (i in 1:length(phi_max)){
log_L[i]= sum(log((alpha+beta*V[spiketimes]/v_max*cos(phi[spiketimes]-phi_max[i]))*delta_t) ) -
sum(alpha+beta*V/v_max*cos(phi-phi_max[i]) )*delta_t
}
plot(phi_max,log_L,type='l',lwd=2,col='blue',
xlab='Direction(radians)',ylab='Log-likelihood')
phi_MLE = phi_max[which.max(log_L)] # 1.00

idx=which.max(log_L)
se = 1/sqrt( (2*log_L[idx] - log_L[idx-1]-log_L[idx+1])/(.01)^2) #0.187
CI =round( c(phi_MLE - 1.96*se, phi_MLE+1.96*se),3) #(0.634, 1.366)
```

Question 4

Compute $\hat{\lambda}_{\text{ML}}(t)$ by plug $\hat{\phi}_{MLE}$ into the formula

$$\lambda(\nu(t), \phi(t)) = \alpha + \beta \frac{\nu(t)}{\nu_{max}} \cos(\phi(t) - \phi_{\text{preferred}})$$

and plot out $\hat{\lambda}_{ML}(t)$ as a function of time along with the spike times.



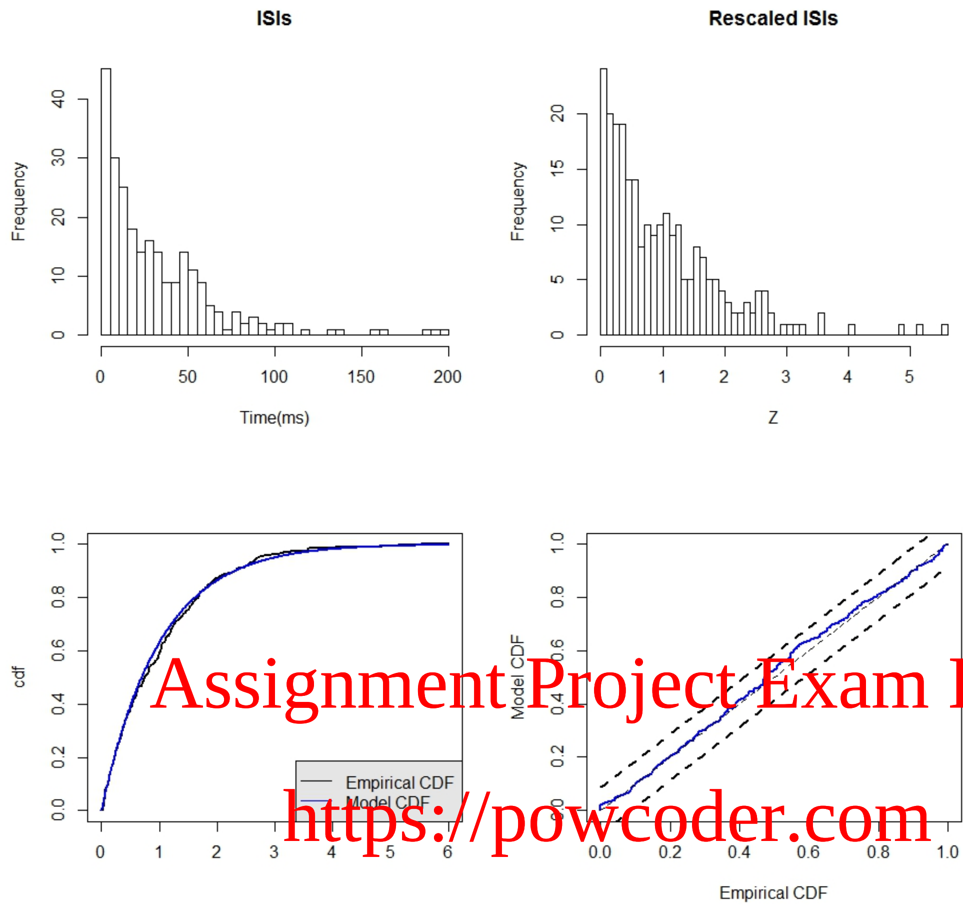

```
# 4
lambda = (alpha+beta*v/v_max*cos(phi-phi_MLE))
plot(T,lambda,type='l',xlab='Time (ms)',
col='blue',ylab='Estimated rate(Hz)', ylim=c(0,60))
lines(T,spikes*10,col='red')
abline(h=c(0,10),lty=2,col='darkgray')
```

Question 5 Goodness-of-fit

We can calculate the set of rescaled waiting times according to

$$Z_i = \int_{S_{i-1}}^{S_i} \hat{\lambda}_{ML}(t)dt$$

where S_i is the ith spike time. We construct KS plot with 95% confidence bound, and this fit model passes the KS plot.




```
#5
spiketimes0=c(0,spiketimes)
n=length(spiketimes)
Z = numeric(n)
for (i in 1:n) {
Z[i] = sum(lambda[spiketimes0[i]:spiketimes0[i+1]])*.001
}
par(mfrow=c(1,2))
hist(diff(spiketimes0), breaks=50, main='ISIs',xlab='Time(ms)')
hist(Z,breaks=50,main='Rescaled ISIs')
# KS plot
lam = 1;w <- seq(0, 6, by=.01)
Femp <- numeric(length(w))
Qs <- Z[order(Z)]
for (i in 1:length(w)){Femp[i] <- sum( Qs<= w[i])/length(Qs)}
plot(w, Femp, type='l', xlab ='', ylab = 'cdf', lwd=2)
lines(w, 1-exp(-w/lam),col='blue', lwd=2)
legend('bottomright', c('Empirical CDF', "Model CDF"), col = c('black','blue'),
lty = c(1, 1),merge = TRUE, bg = "gray90")
N=length(Z)
KSstat <- max(abs(1-exp(-w/lam) - Femp/N)); KSstat
```

```
plot(Femp, 1-exp(-w/lam), type='l',col='blue', ylab='Model CDF', xlab='Empirical CDF', lwd=2)
x=seq(0,1,by=.01);   y=x
lines(x,y,lty=2, lwd=1)
x=seq(0,1,by=.01);   y=x+1.36/sqrt(N)
lines(x,y,lty=2, lwd=2)
x=seq(0,1,by=.01);   y=x-1.36/sqrt(N)
lines(x,y,lty=2, lwd=2)
```

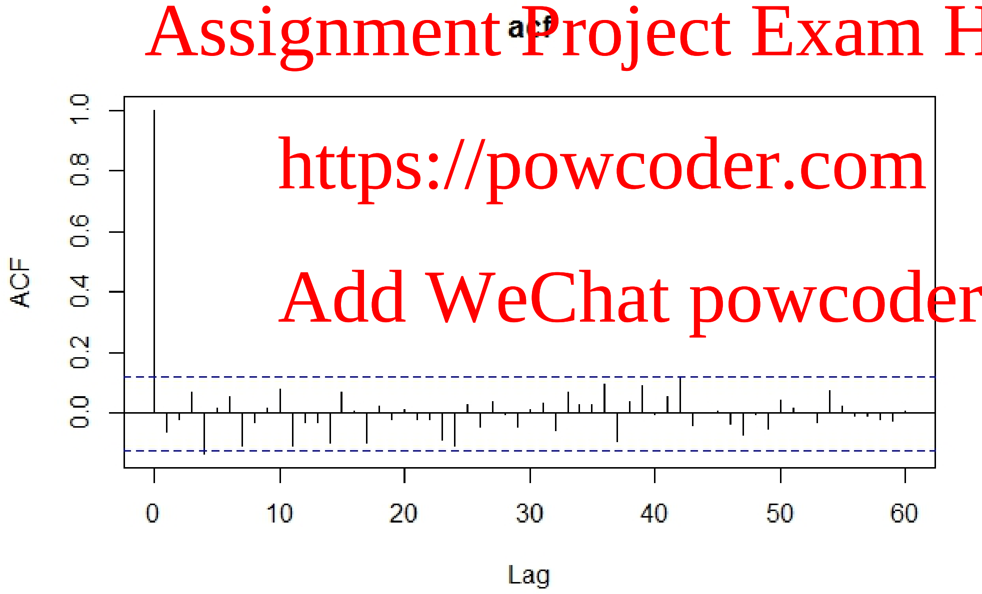
Question 6

ACF plot. We transform the rescaled waiting time to standard normal distribution by

$$X_i = F_{N(0,1)}^{-1}(F_{\exp}(Z_i))$$

And plot out the acf plot for the transformed waiting times. We find that ACF structure is fine.

```
U = qnorm(exp(-Z))
acf(U,lag=60, main ='acf' )
```

Question 7

We can calculate the Fano's Factor based on the rescaled waiting times, FF = 0.8936, lies within the theoretical confidence interval [0.832,1.184], so we cannot preclude an inhomogeneous Poisson model based on this binning of the data.

```
rescaledTimes = cumsum(Z);
n=length(rescaledTimes)
spikes = hist(rescaledTimes,breaks=seq(0,ceiling(rescaledTimes[n]),by=1))
FF = var(spikes$counts)/mean(spikes$counts);FF #0.963
```

```
qgamma(c(0.025,0.975), n/2,scale=2/n) #[0.832,1.184]
```

Question 8

The model passes the KS test and the correlation structure in the rescaled ISIs does not appear to be significant. This suggests that this inhomogeneous Poisson model is able to describe well the statistical structure of this brief segment of data.

The model fit suggest that this neuron is cosine tuned, with a preferred direction about 1.00 radians from the horizontal, and modulated by movement speed.