MA 568 – Statistical Analysis of Point Process Data

Problem Set #2
Due October 15, 2018

In Problem Set #1, we modeled baseline neural spiking as a simple Poisson process, based on the assumption that the data was stationary and independent of past history. More typically, we want to characterize how a point process varies as a function of other time-varying signals. Inhomogeneous Poisson process models provide one approach to relating point process data to these other observed signals.

In this problem we construct, fit, and evaluate a model of neural spiking activity in primate primary motor cortex. Neurons in this region of the brain have been shown to relate to limb movement kinematics. Recently there has been a lot of research into characterizing the coding properties of these neurons for use in driving brain computer interfaces, for paralyzed and movement-disabled individuals.

1. Please download the **`M1_spikes.mat`** file from the course website. The data consists of simulated spiking activity as a primate performs an 8 second hand movement in two dimensions. Loading this dataset in MATLAB creates the following variables:

`T`	-	Time index in ms.
`X,Y`	-	(x,y) coordinates of hand during a two-dimensional movement task
`V`	-	Speed of hand movement
`phi`	-	Direction of hand movement in radians (0 = movement to the right)
`spiketimes`	-	Time index of spikes (in ms.)
`spikes`	-	Time series of spikes binned at 1 ms. resolution

Plot the spiking activity as a time series. Plot the hand position trajectory (**`X`** vs. **`Y`**) with the hand position at the spike times overlaid. Plot the velocity and direction of movement (**`V`** vs. **`phi`**) at the spike times as well.

2. An occupancy-normalized histogram is a tool to visualize the firing rate as a function of another covariate. Construct an occupancy-normalized histogram of the spiking activity as a function of movement direction by first dividing the domain of possible directions into discrete bins. For each bin, divide the total number of spikes observed during movement in that range of directions by the total amount of time spent moving in those directions. From your histogram, estimate the direction of movement in which this neuron is most likely to fire.

The spiking properties of these neurons have previously been described by the following cosine tuning model:

$$\lambda(v(t),\phi(t)) = \alpha + \beta \frac{v(t)}{v_{\max}} \cos(\phi(t) - \phi_{\text{preferred}}).$$

Under this model, for a particular hand velocity, the firing rate is maximal when the hand is moving in the preferred direction, $\phi_{\text{preferred}}$, and drops off according to a cosine function in other directions.

3. Assume that $\alpha = 30$, $\beta = 30$, and $v_{\max} = 16.1\ \mathrm{cm/sec}$ are known. Compute the likelihood of the data as a function of preferred direction, $L(\phi_{\mathrm{preferred}})$, for a range of values for $\phi_{\mathrm{preferred}}$. Find the maximum likelihood estimate $\hat{\phi}_{\mathrm{preferred,ML}}$. Compute the observed Fisher information and construct a 95% confidence interval for this parameter.

4. Compute $\hat{\lambda}_{\mathrm{ML}}(t)$, the time series of the estimated firing rate using the maximum likelihood estimate for the preferred direction. Plot $\hat{\lambda}_{\mathrm{ML}}(t)$ as a function of time along with the spike times.

Goodness-of-Fit:

5. Calculate a set of rescaled waiting times according to:

$$Z_i = \int_{S_{i-1}}^{S_i} \hat{\lambda}_{\mathrm{ML}}(t)dt,$$

where $S_0 = 0$ and S_i is the i$^{\text{th}}$ spike time. Plot histograms of the original interspike intervals (ISIs), and the rescaled intervals. Construct a KS plot with 95% confidence bounds for this data. Does the fit model pass the KS test?

6. Compute and plot the autocorrelation function of the rescaled intervals. What does this suggest about the independence assumption for the inhomogeneous Poisson model?

7. Bin the rescaled event times into time bins of size 1. Compute the Fano Factor for this rescaled and binned data, and 95% confidence bounds for this statistic under the assumption that the data comes from an inhomogeneous Poisson (the sample Fano factor is distributed according to a Gamma(n/2,2/n)). What, if anything, can you conclude about the data from this result?

8. What conclusions can you draw about the firing properties of this neuron from the data?