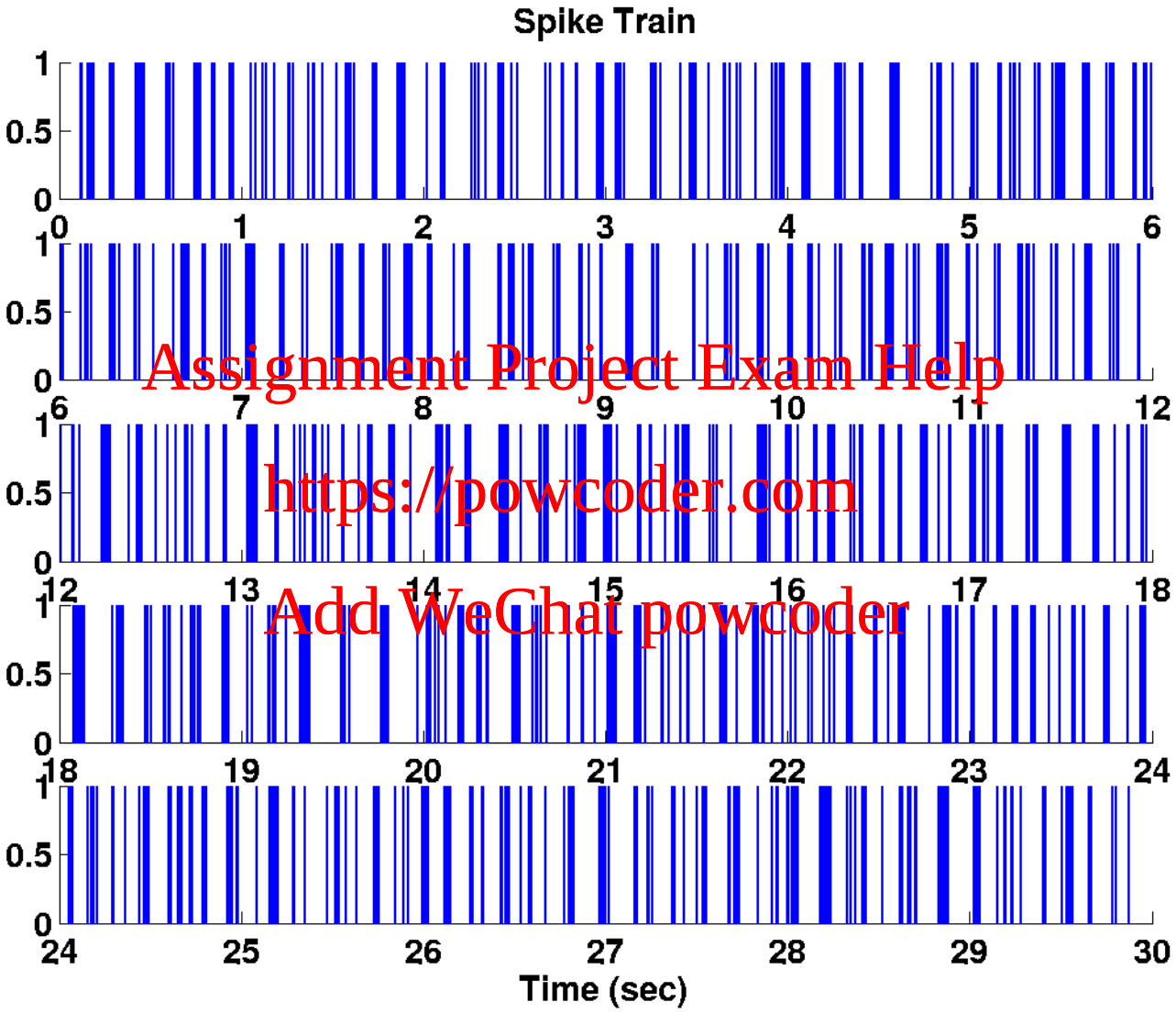
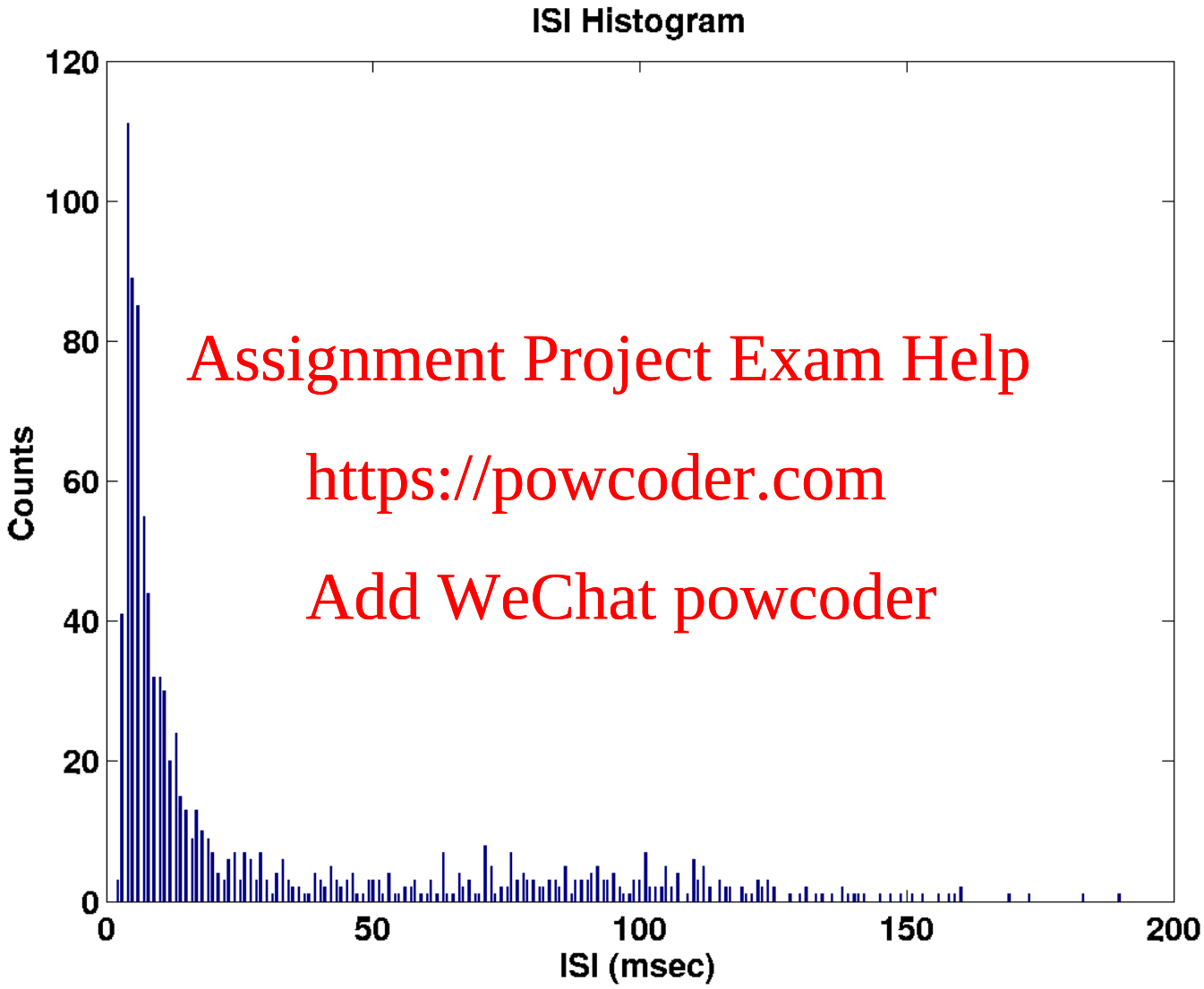
History Dependent GLM Example

An Analysis of the Spiking Activity of Retinal Neurons in Culture (Iyengar and Liu, 1997)

Retinal neurons are grown in culture under constant light and environmental conditions. The spontaneous spiking activity of these neurons is recorded. The objective is to develop a statistical model which accurately describes the stochastic structure of the waiting times, or interspike intervals (ISIs), for this data.



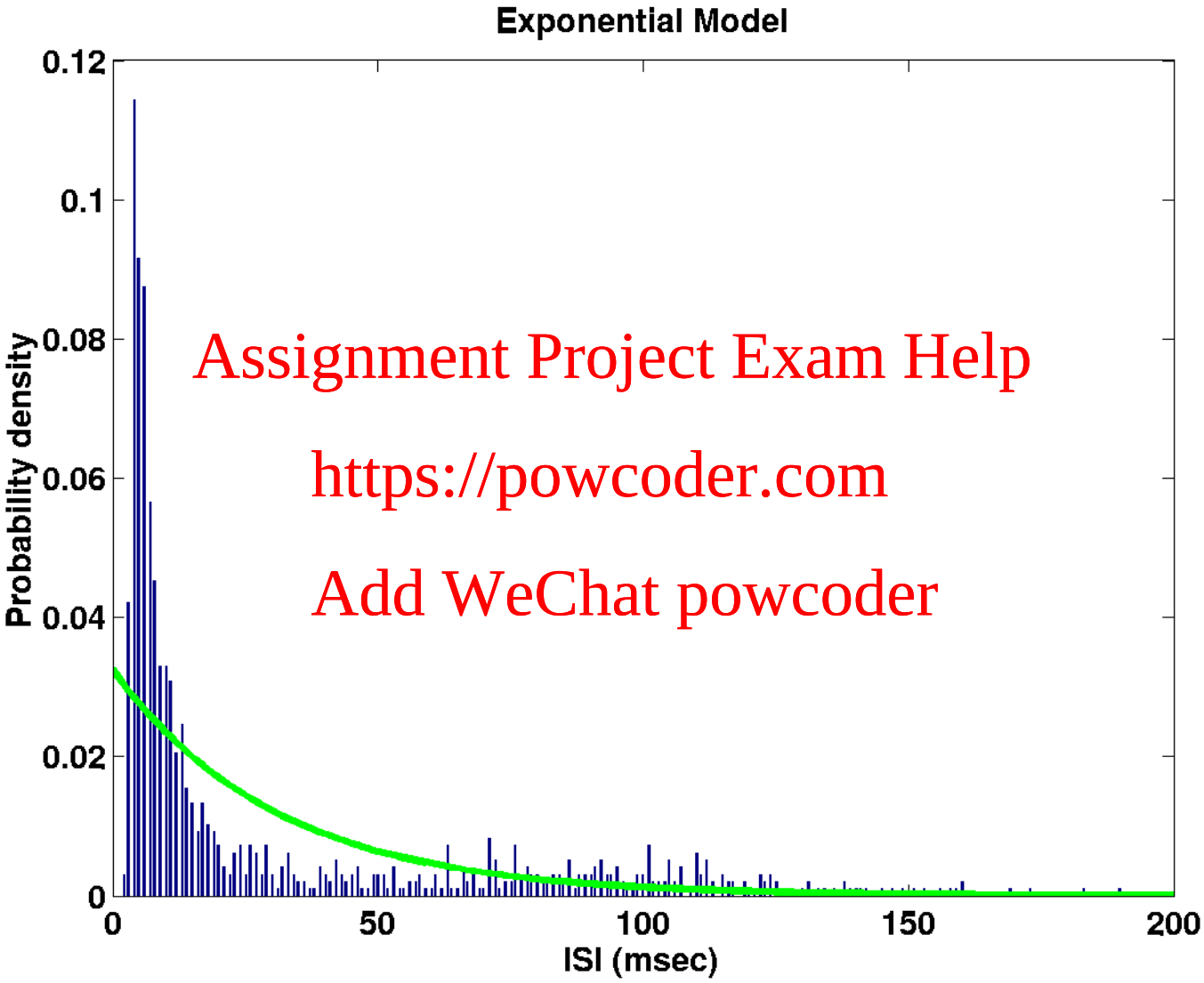


Attempt #1: Poisson Model

- Fit a homogeneous Poisson model to the data:

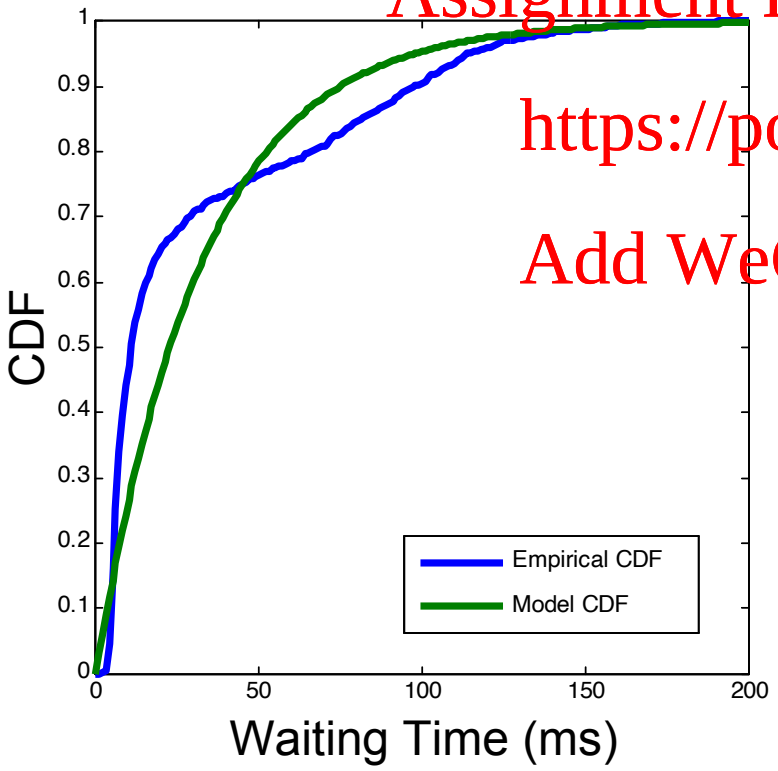
$$\rho_{S_i}(t \mid H_{S_{i-1}}) = \lambda e^{-\lambda(t-S_{i-1})}$$

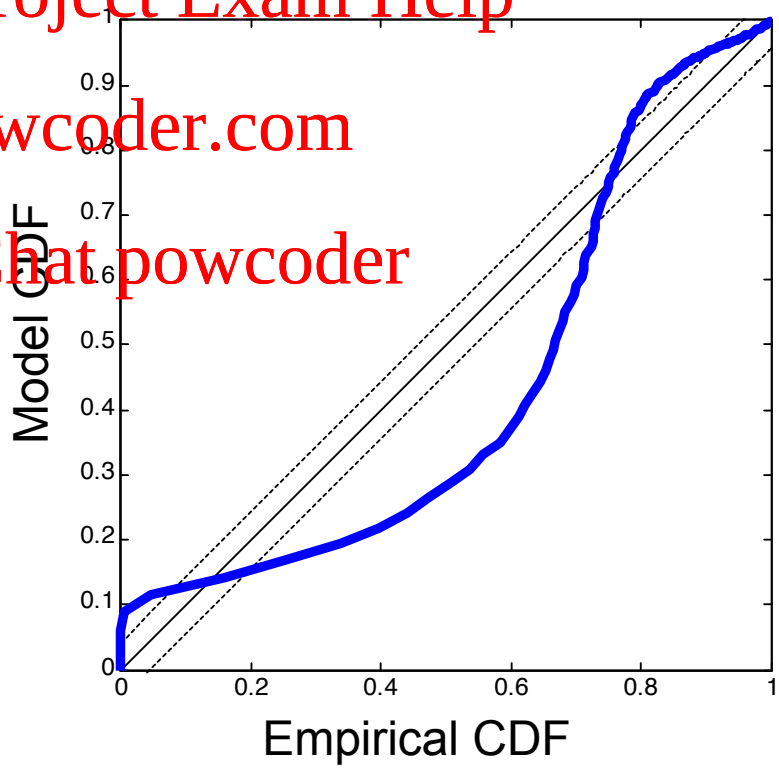
- Estimate rate parameter λ by maximum likelihood.



KS Plot

Graphical comparison of empirical vs model CDFs





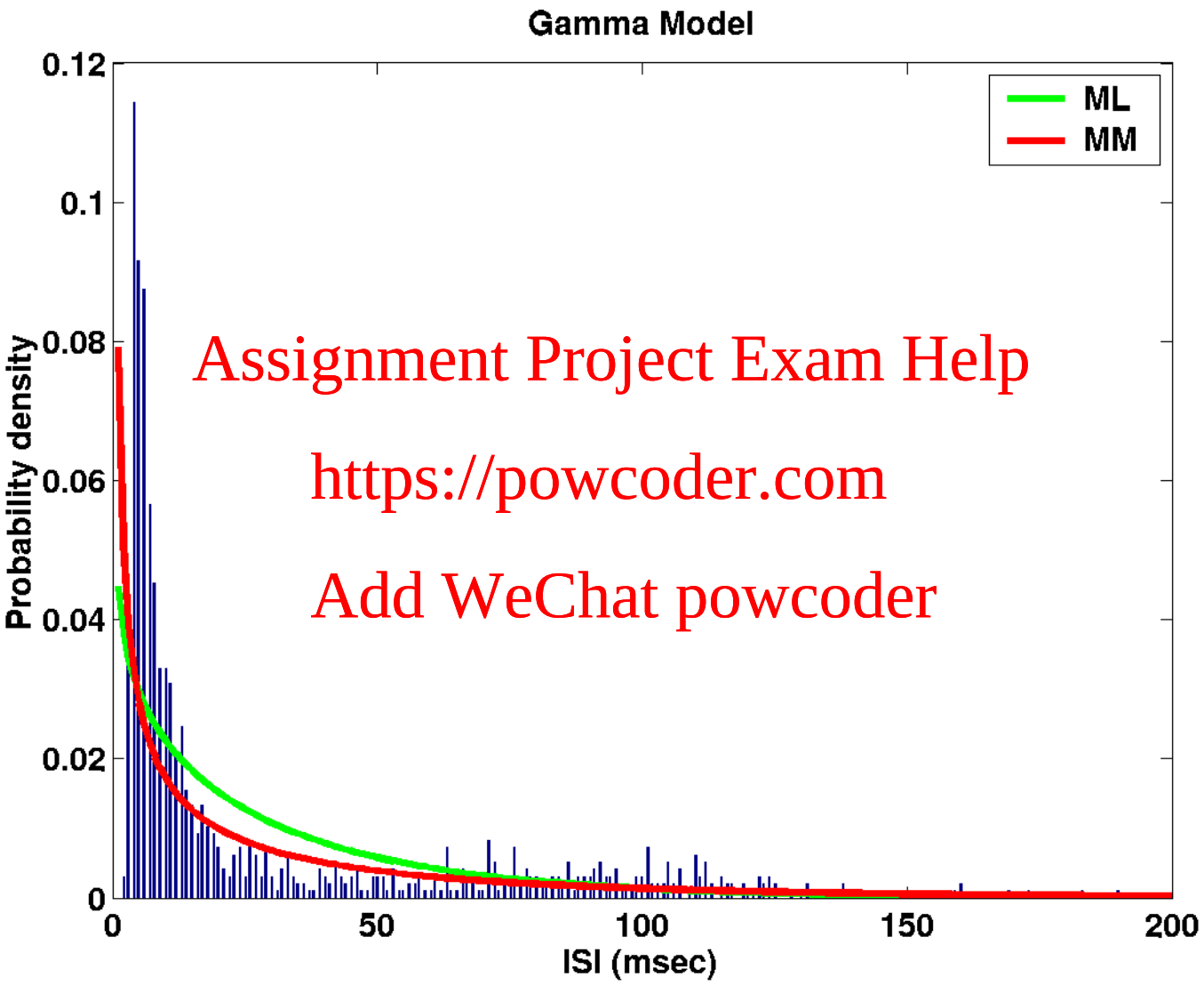
Attempt #2: Renewal Models

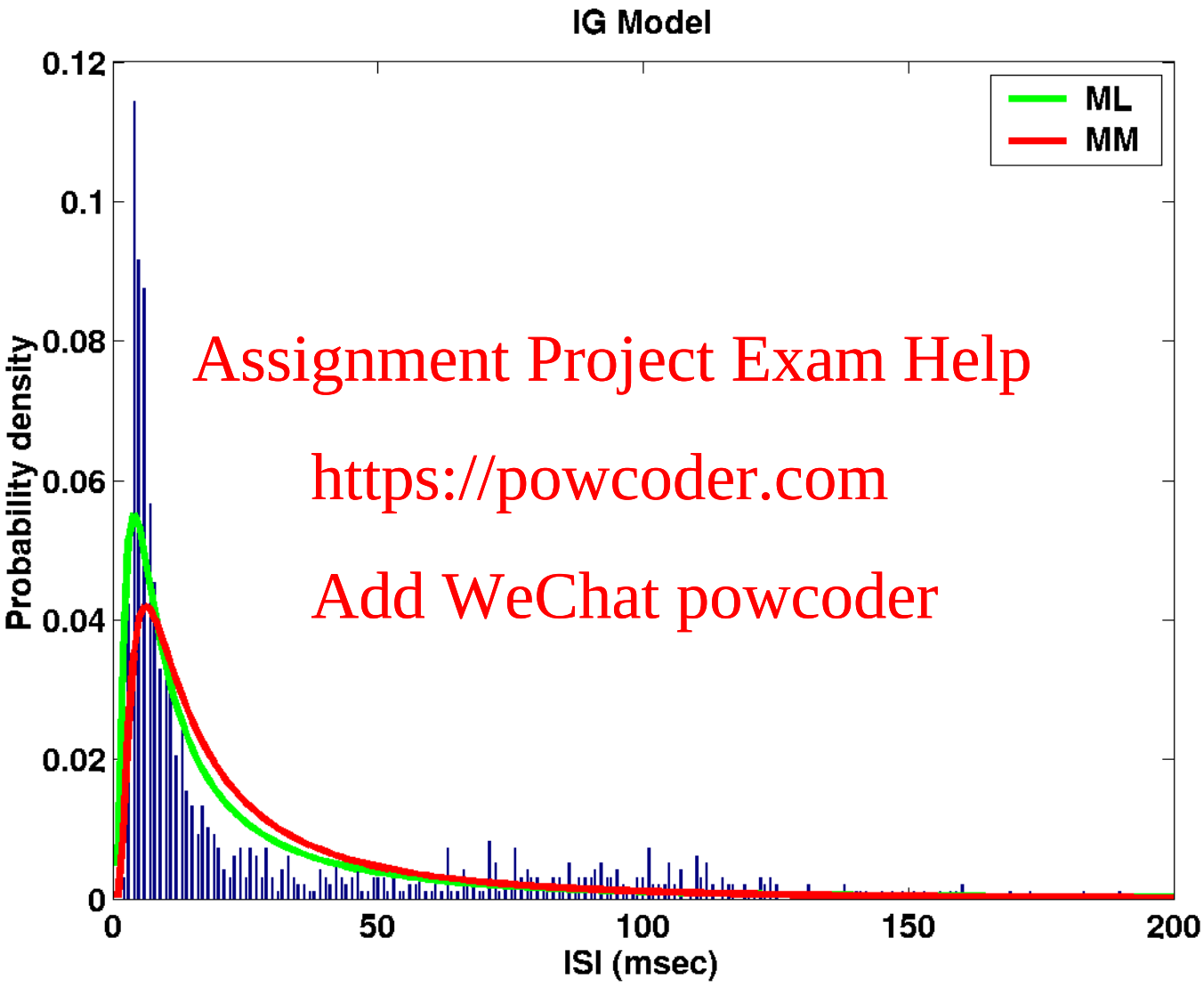
- Fit a variety of renewal models to the data:

$$\rho_{S_i}(t \mid H_{S_{i-1}}) = f(t - S_{i-1})$$

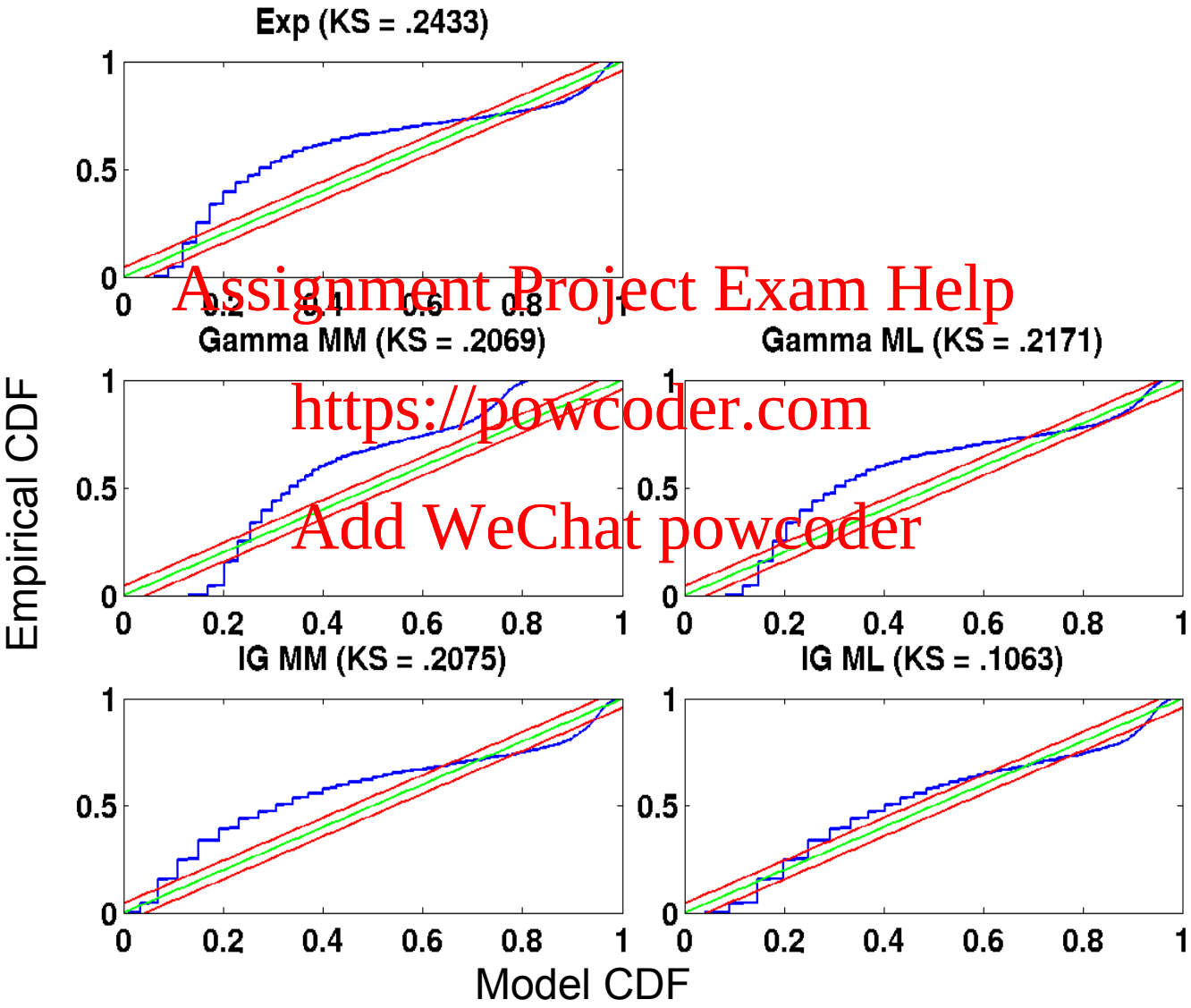
- Candidates:
  - Gamma
  - Inverse Gaussian

- Estimate parameters by Maximum Likelihood or Method of Moments





KS Plots



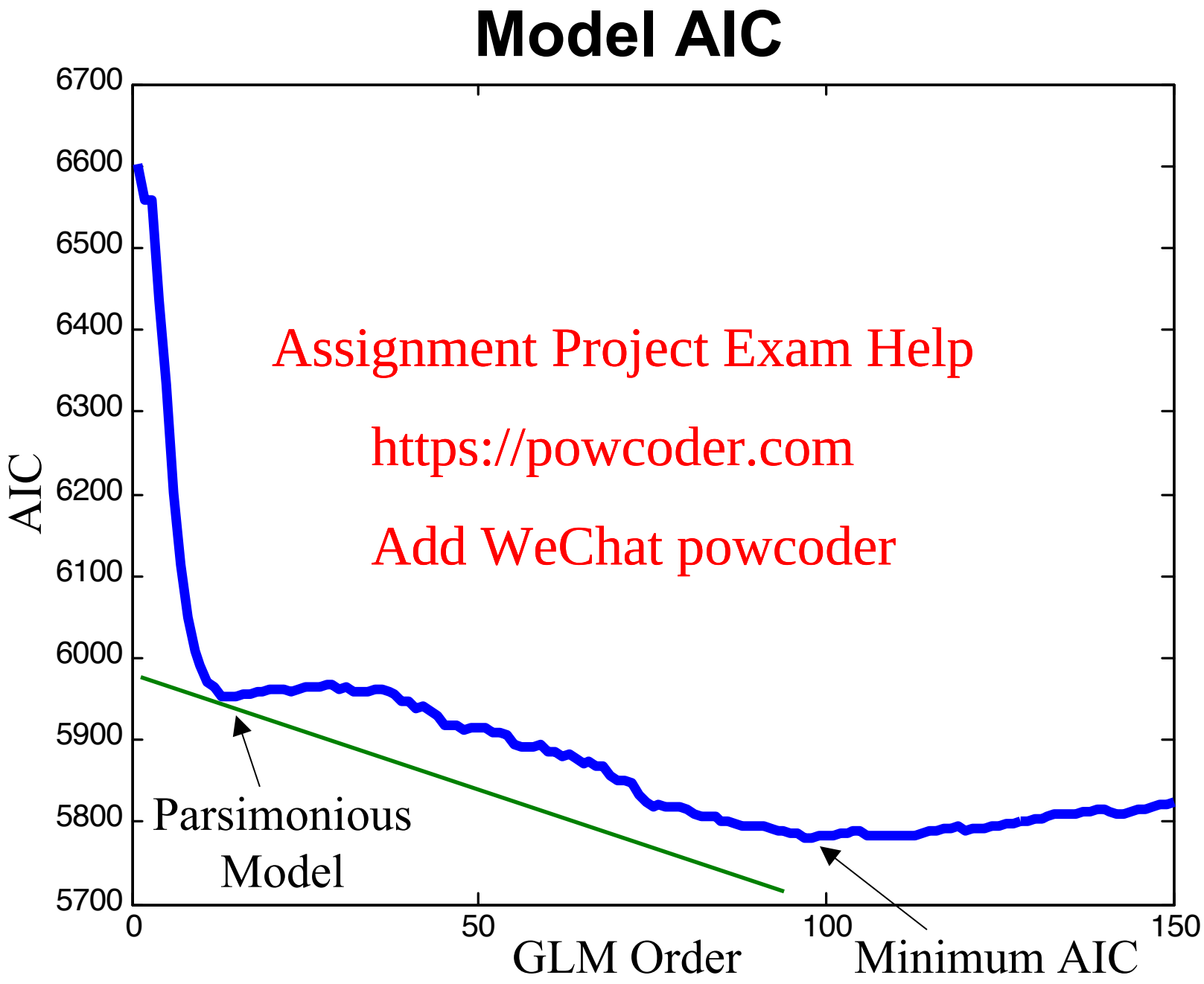
Attempt #3: GLM History Model

The ISI distribution models we constructed previously assume that $p(\mathrm{ISI} \mid H_t) = p(\mathrm{ISI})$

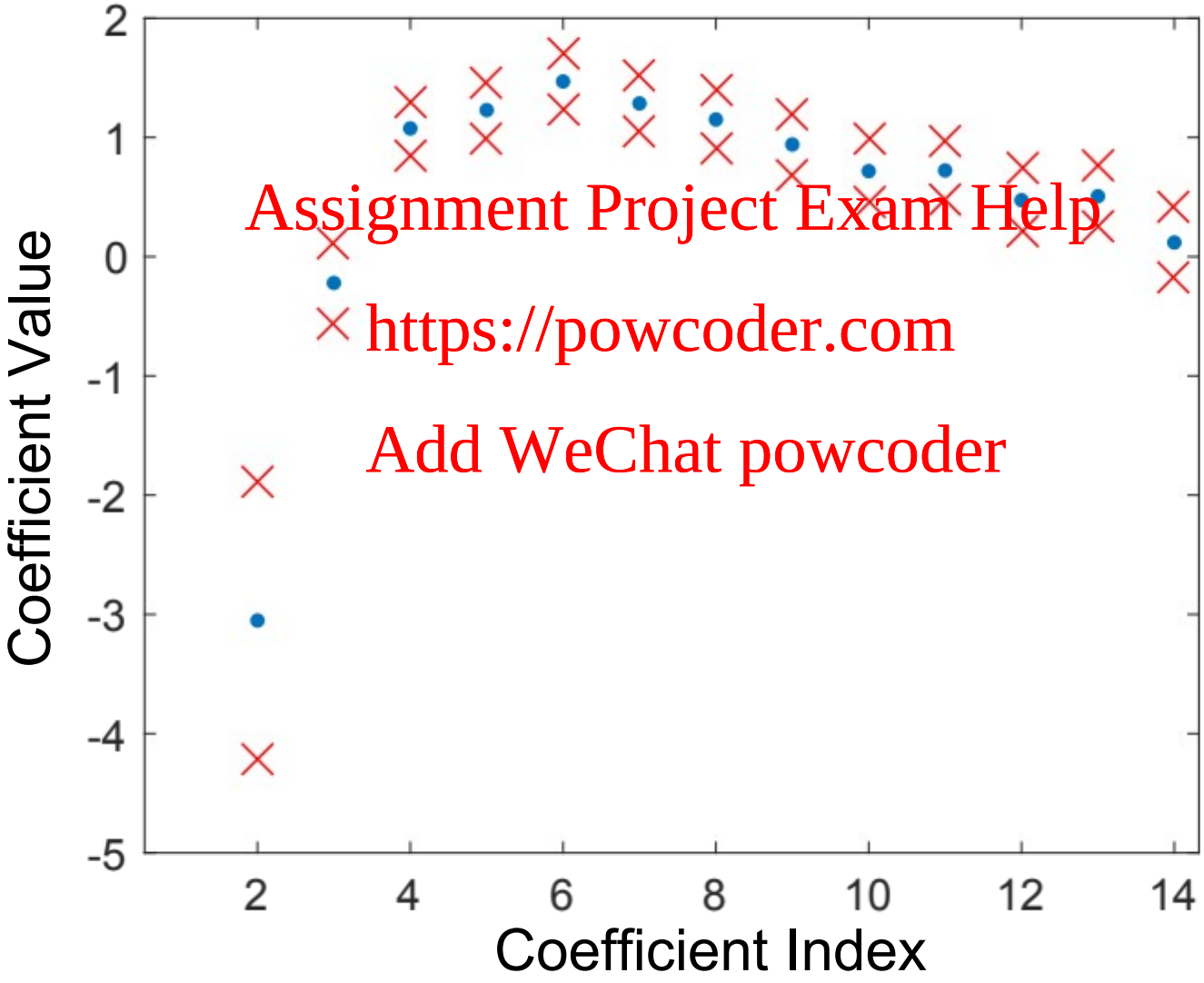
Now, let the conditional intensity be a function of past spiking activity using GLM

$$\lambda(t_k \mid H_k) = \exp\left\{\alpha_0 + \sum_{i=1}^{order} \alpha_i \Delta N_{(t_k - i, t_k - i + 1)}\right\}$$

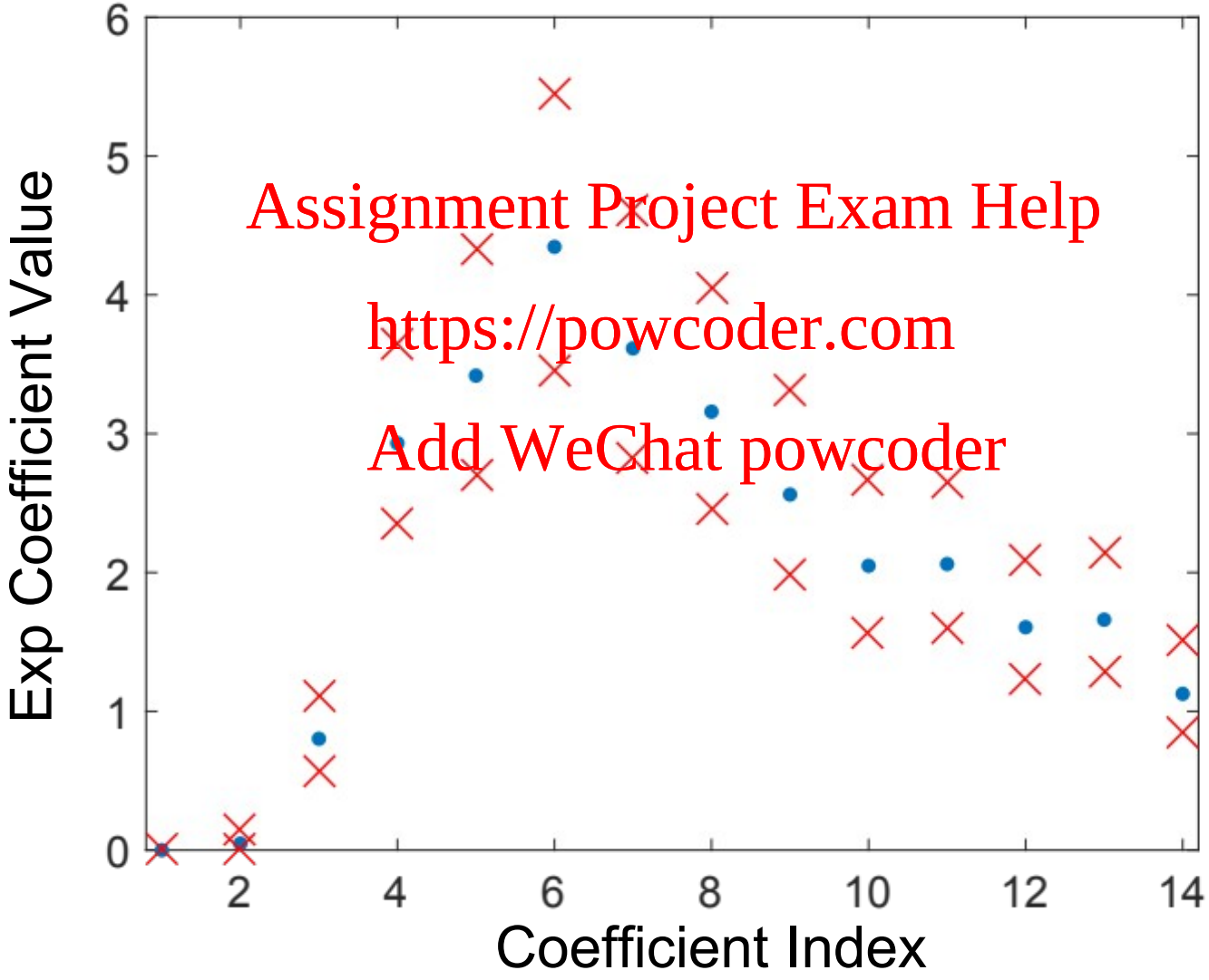
How do we pick a model order?



Maximum Likelihood Model Fit



Maximum Likelihood Model Fit



Goodness-of-Fit

Problem:

Distribution of arbitrary statistics of spike times, S_i, are difficult to compute.

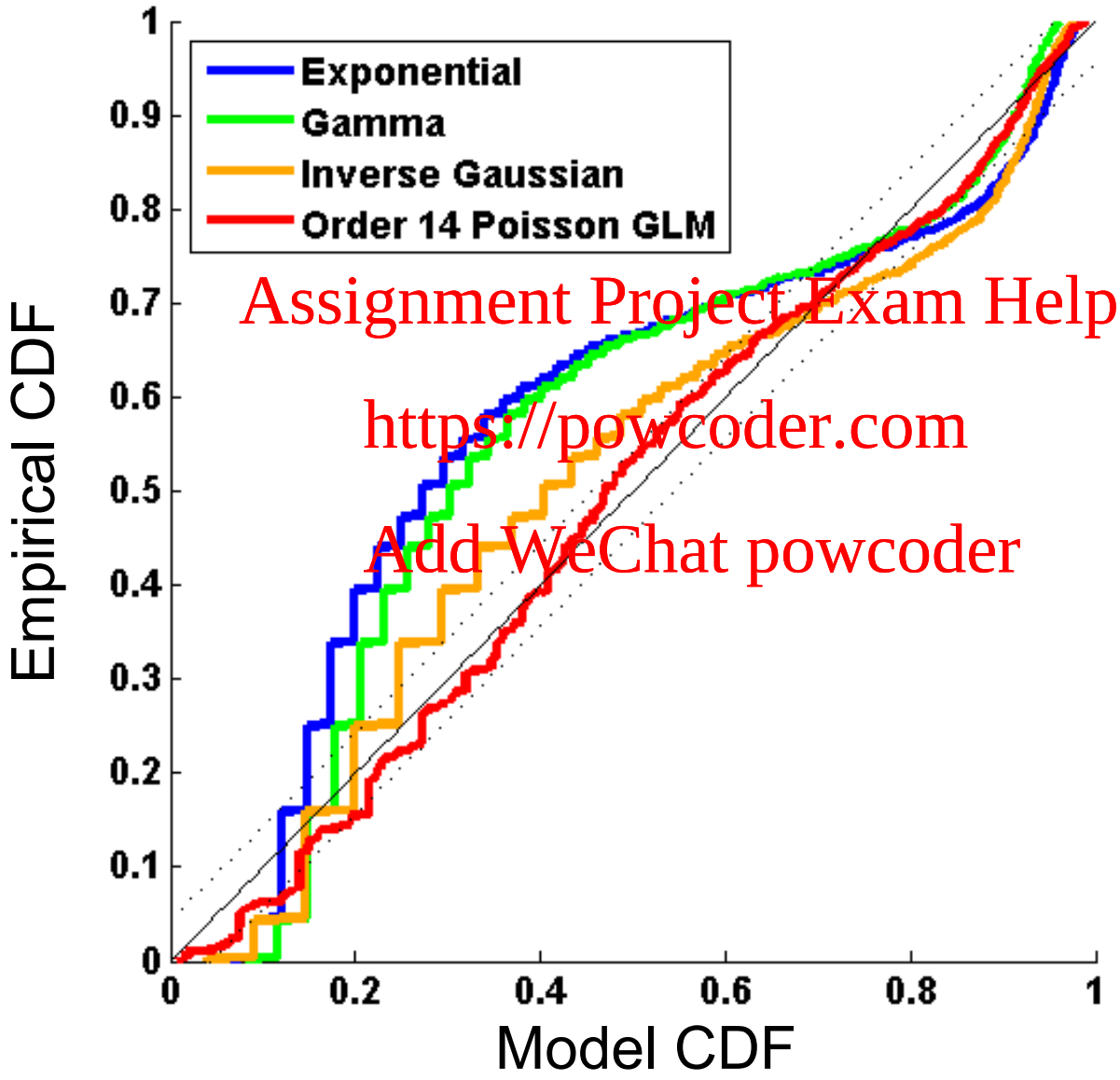
Solution:

Time-rescaling theorem:

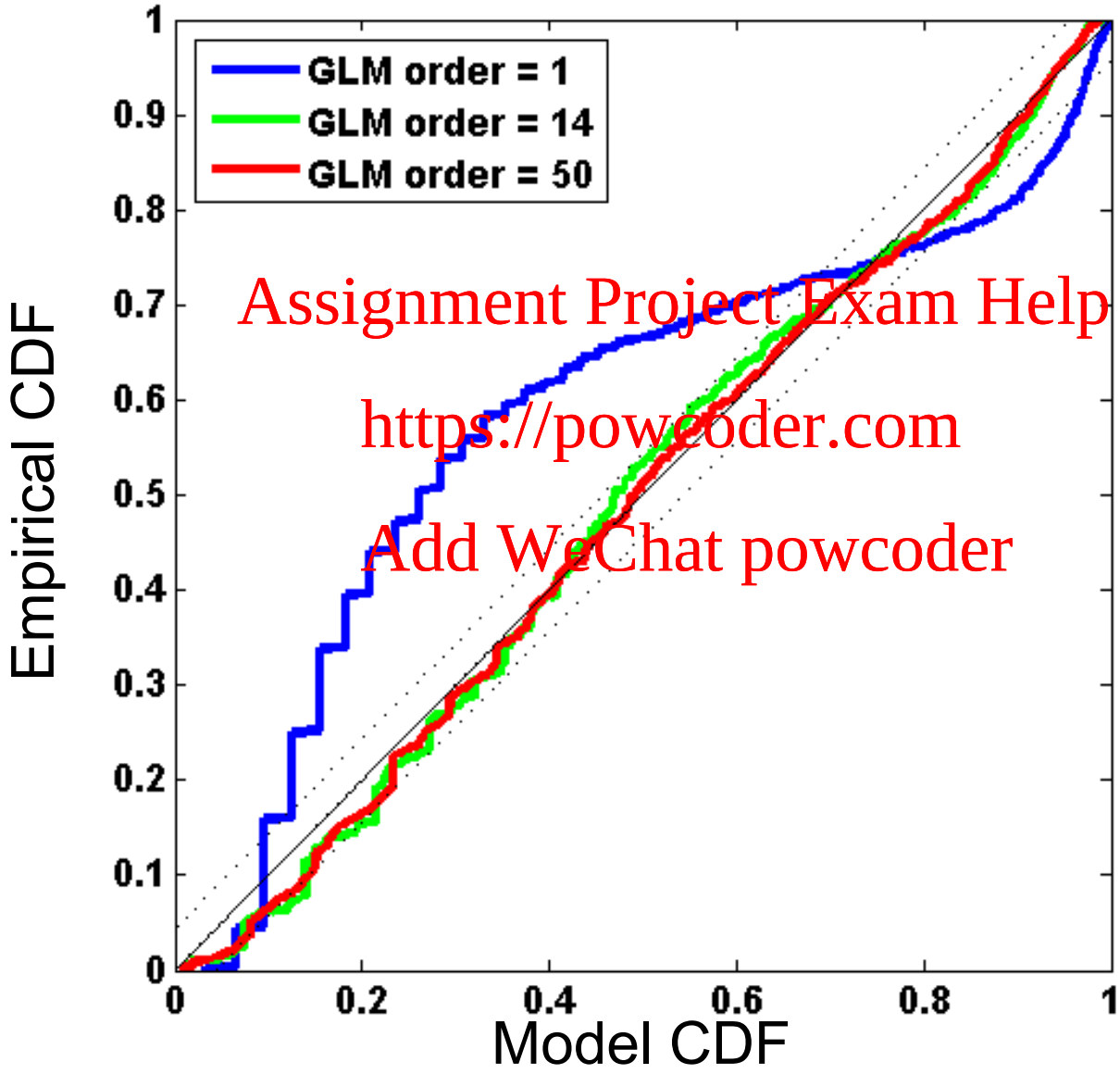
Let S_i where $i = 1,\ldots,k$ be the event times of a point process with conditional intensity $\lambda(t \mid H_t)$.

Then $z_i = \int_{S_i}^{S_{i+1}} \lambda(u \mid H_u)du$ will be i.i.d. exponential random variables.

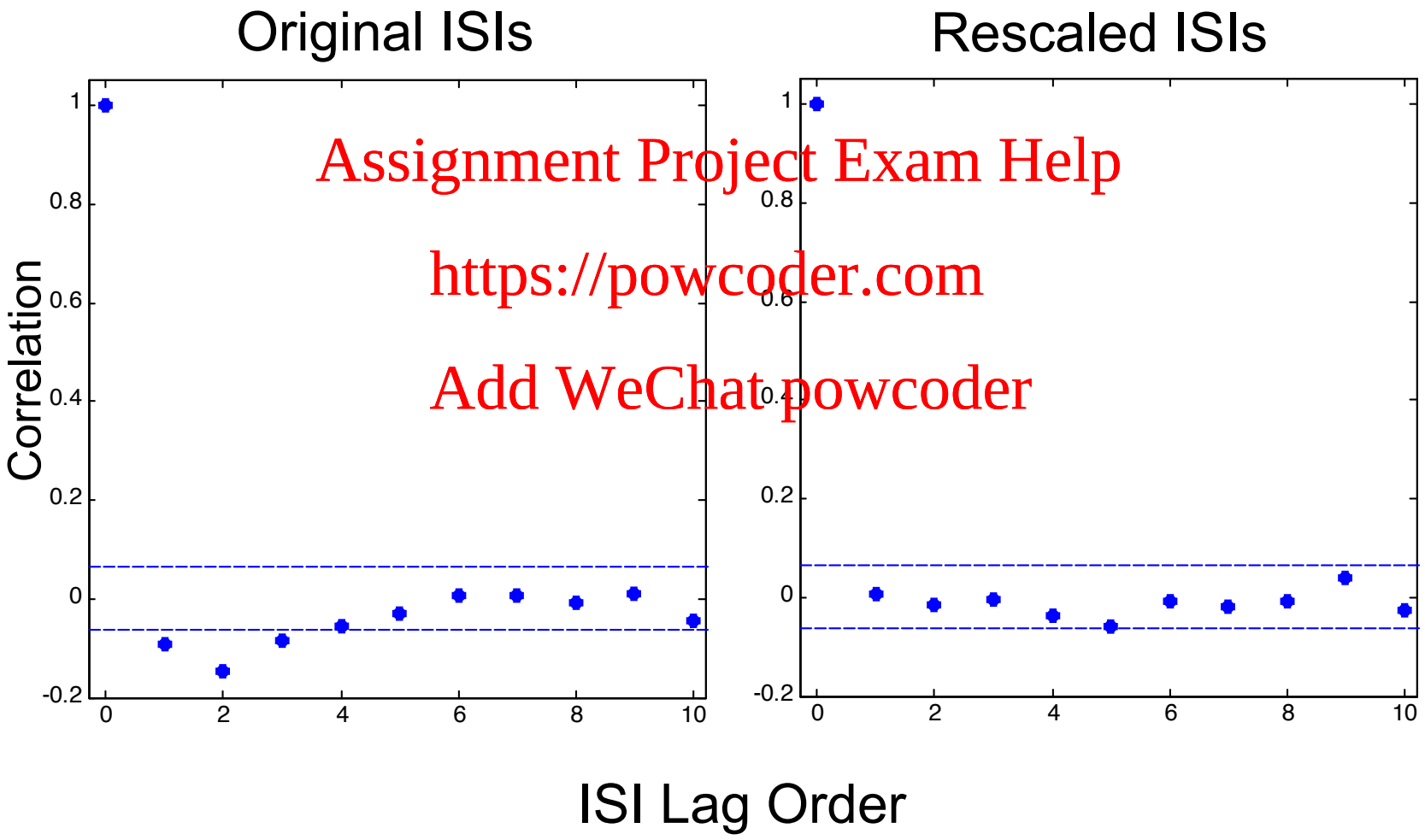
Kolmogorov-Smirnov Plots



KS Plots for Different Order GLMs



Correlation Function for Rescaled ISIs

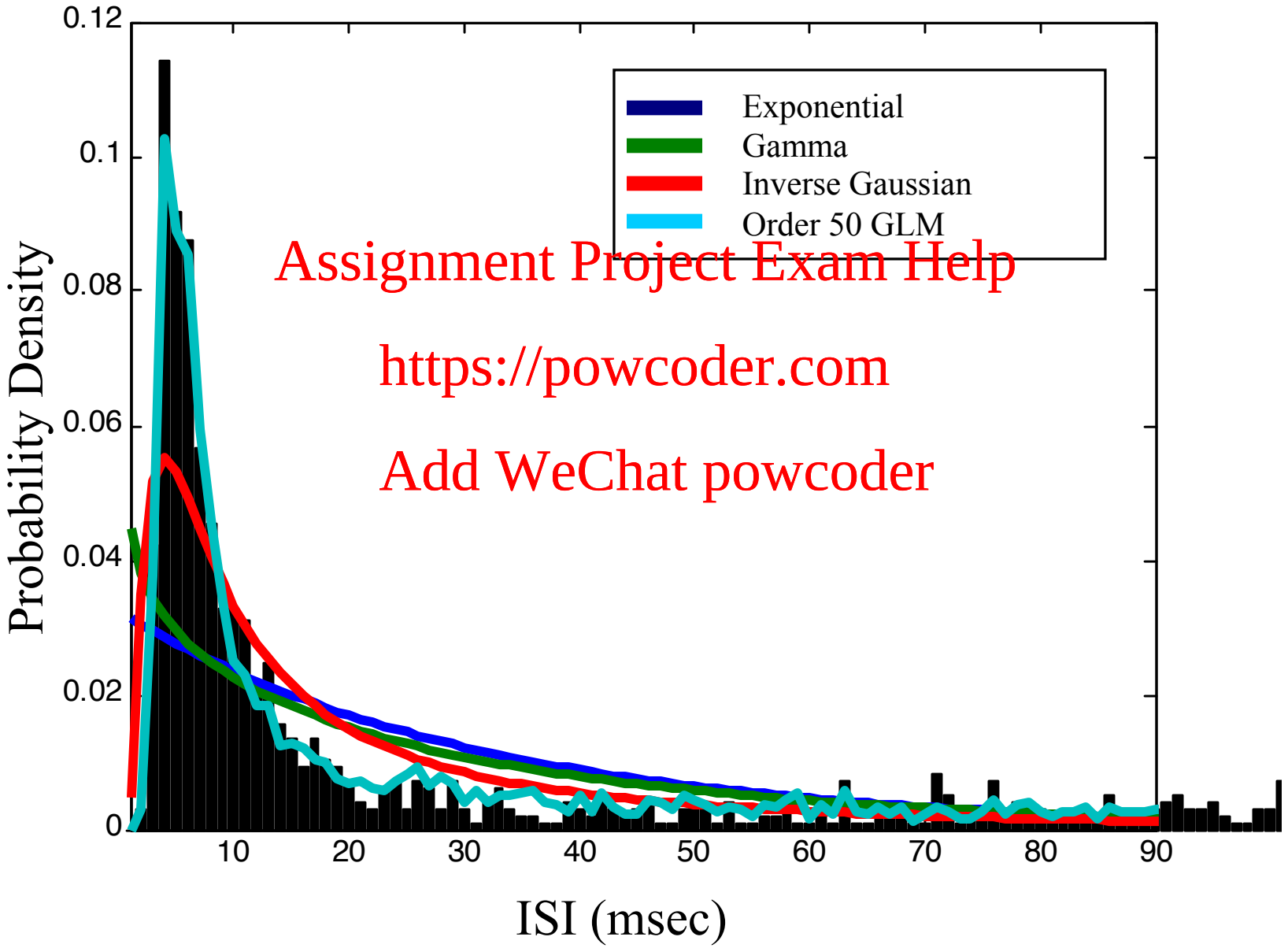


Goodness-of-Fit Summary

	GLM		
Order	1	14	50
AIC	6589	5931	5892
KS	0.2330	0.0657	0.0462

Renewal Models:	Exp	Gamma	Inv. Gauss.
KS Statistic	0.2525	0.2171	0.1063

ISI Histogram

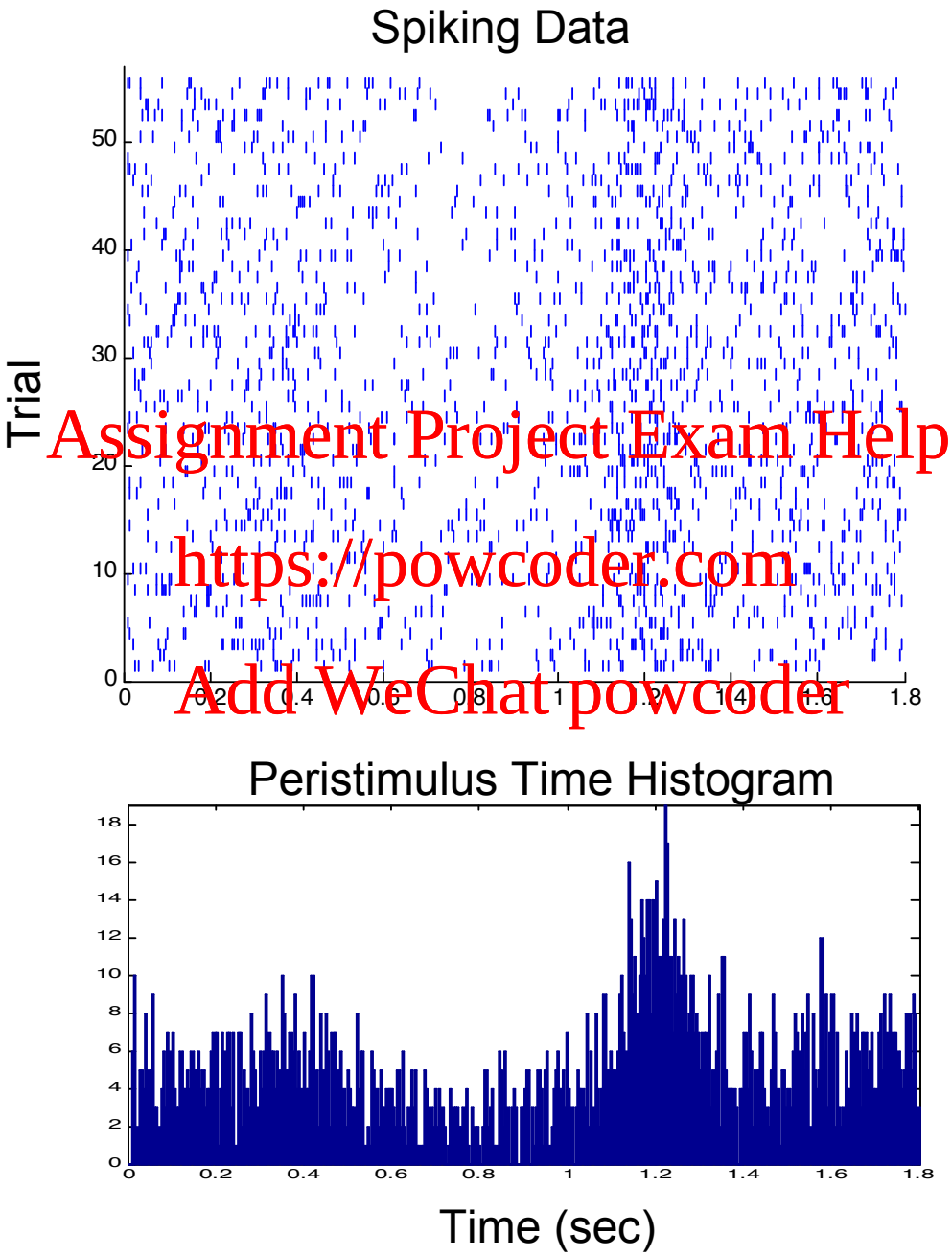


Analysis Summary

- Low order GLMs effectively capture history dependent structure in this data.
- Model order can be selected by AIC.
- Goodness-of-fit can be evaluated by time-rescaling, comparison of empirical to model CDFs, and correlation analyses.

Case 2: Peristimulus Time GLM

- Monkeys were trained to saccade to one of four targets, based on displayed images.
- Single cell recording in monkey hippocampus.

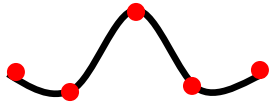


Model

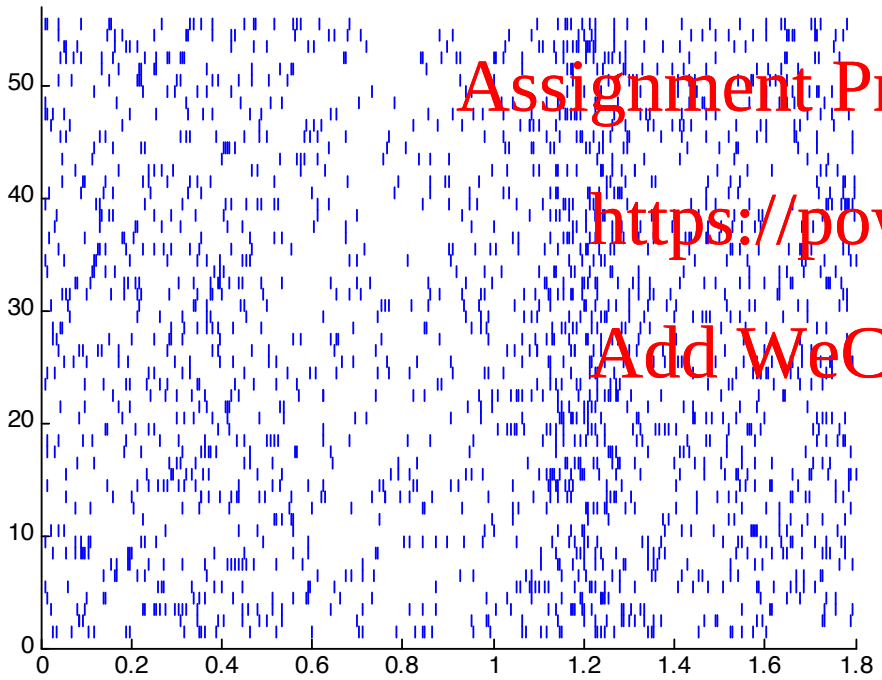
$$\lambda(t \mid H_t) = \exp\left\{\sum_{r=1}^{R} \theta_r g_r(t)\right\}$$

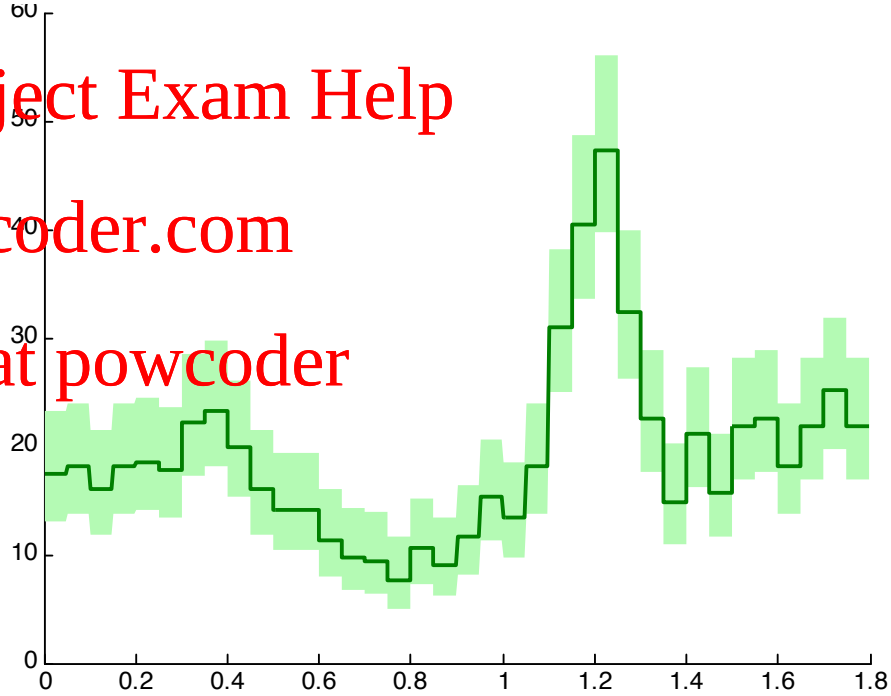
Parameter vector: $\theta = [\theta_1, \ldots, \theta_R]$

Basis functions: $g_r(t)$

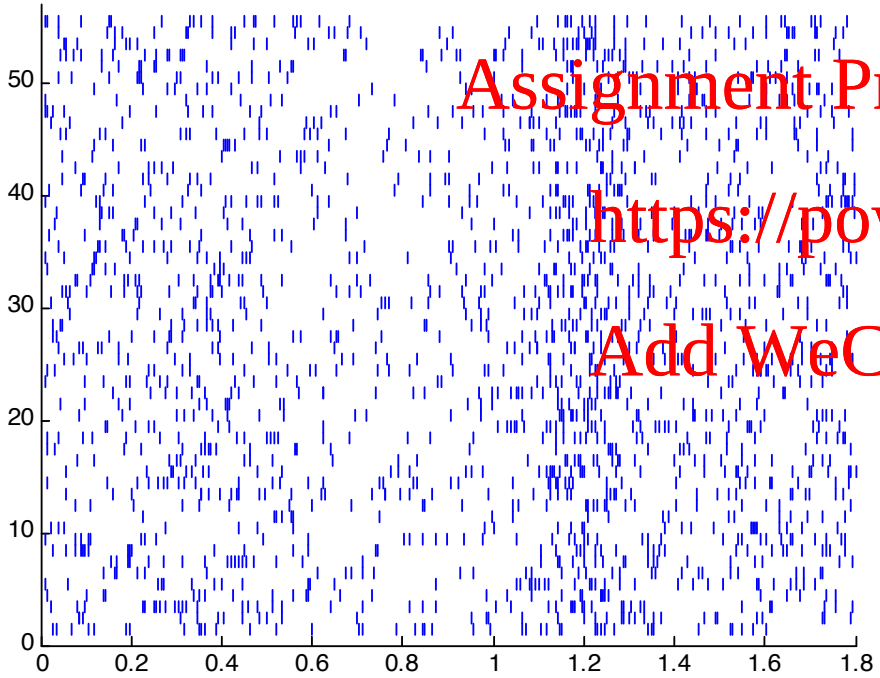
- Indicator Functions: 
- Splines: 

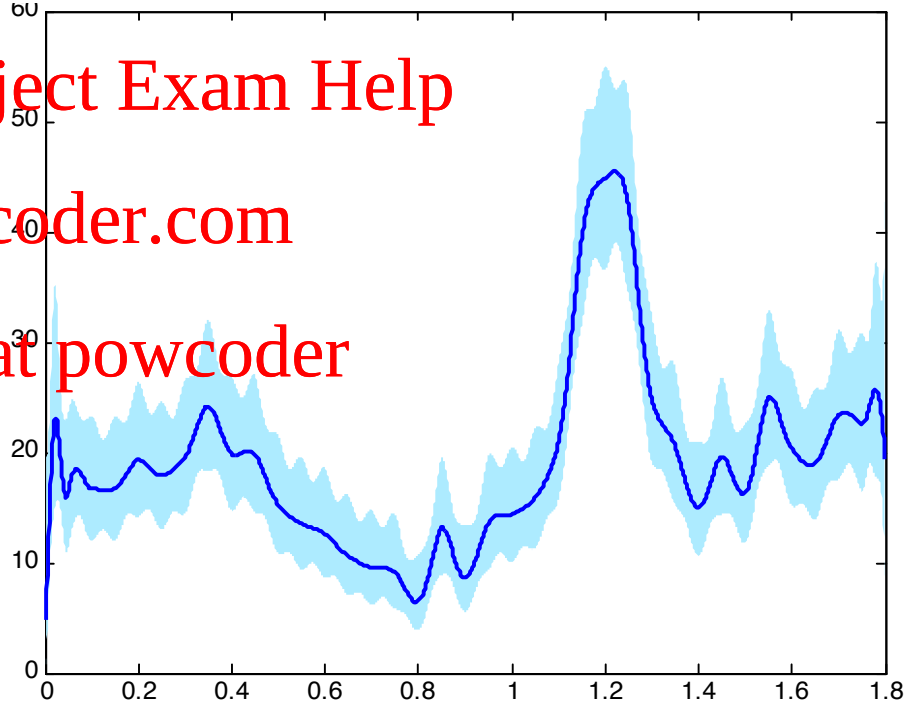
Indicator Function Basis



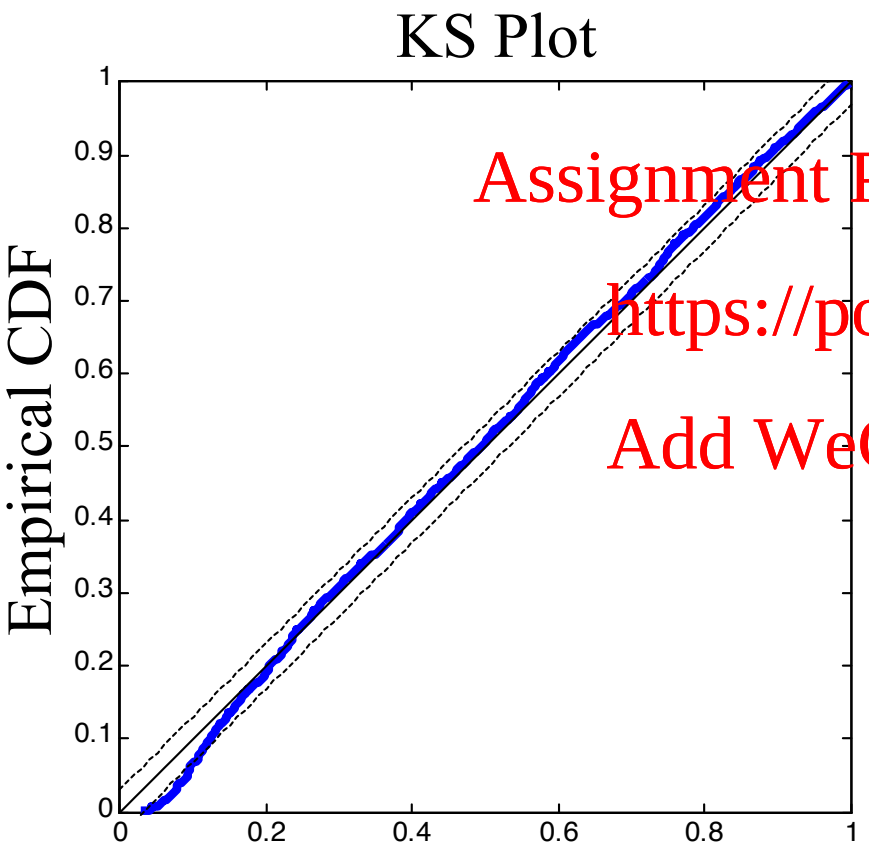


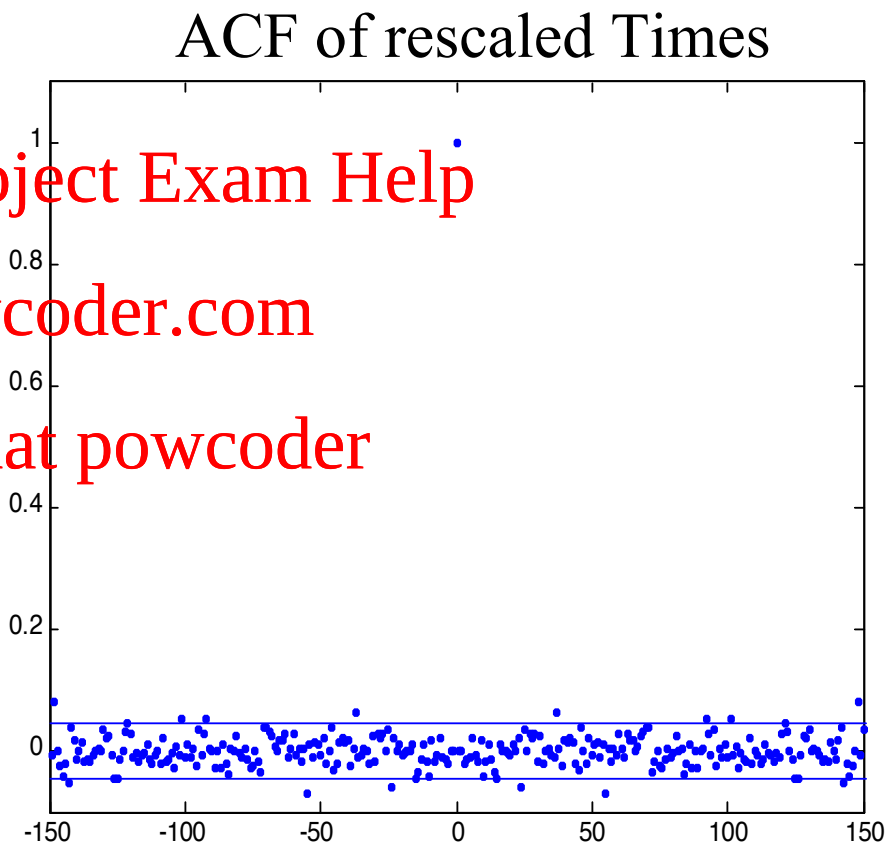
Spline Function Basis





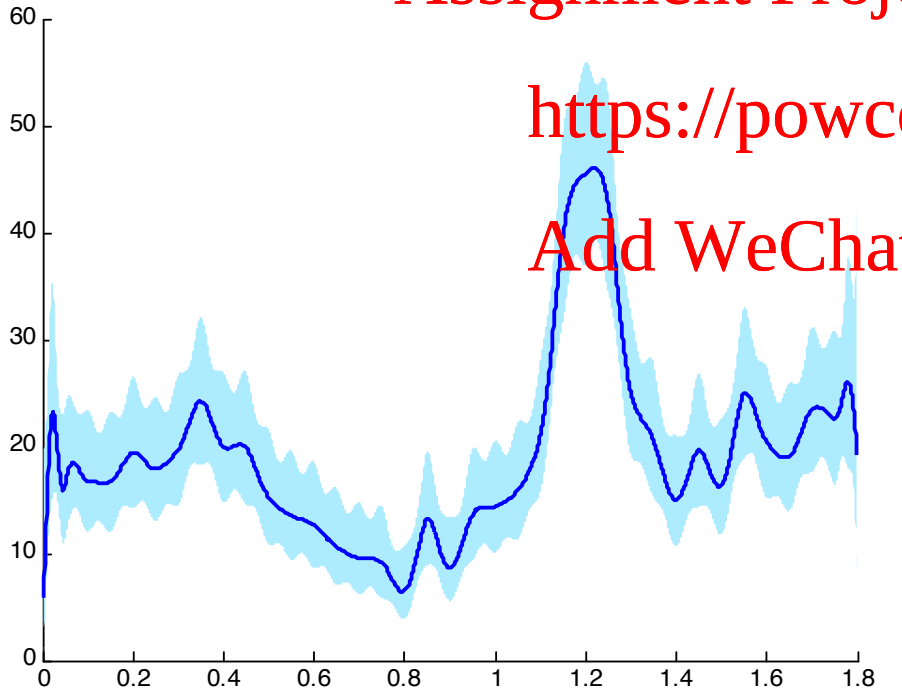
Goodness-of-Fit

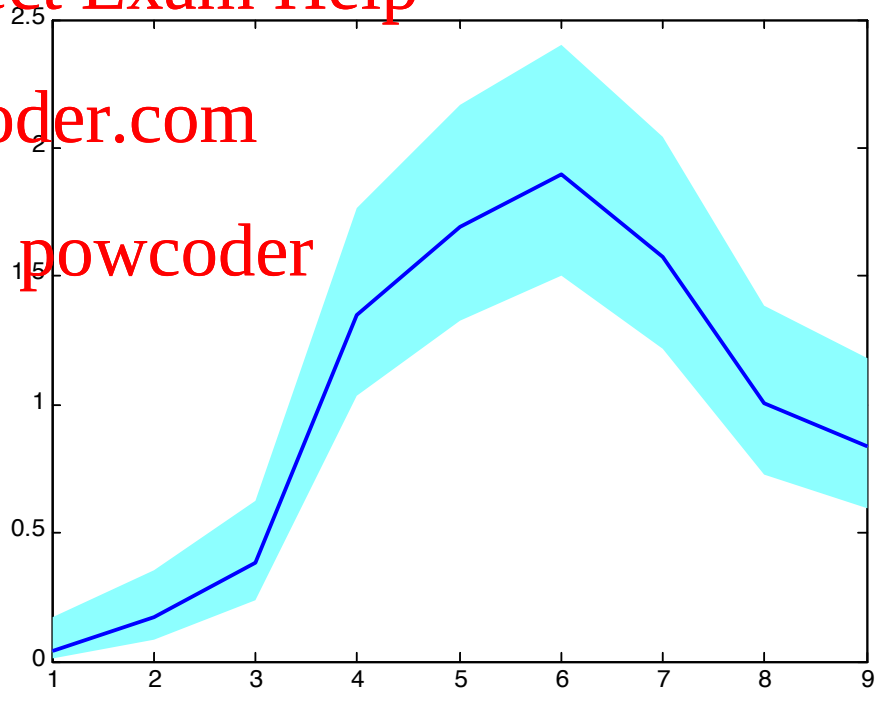




Adding History

$$\lambda_k = \exp\left\{\sum_{r=1}^{R}\theta_{k,r}g_r(t) + \sum_{i=0}^{9}\gamma_i \Delta N_{(t-i-1,t-i)}\right\}$$





Adding History

$$\lambda_k = \exp\left\{\sum_{r=1}^{R}\theta_{k,r}g_r(t) + \sum_{i=0}^{9}\gamma_i \Delta N_{(t-i-1,t-i)}\right\}$$