MA570 Spring 2021 Stochastic Methods Krigman

Quiz #1

NAME_____ **ID#**_____

Please show all your work for full credit and box off your answers.

Partial credit will be given when you show your work.

Good luck!

Problem 1 (40 pts total). Recall the gambling example we were discussing in class. Suppose that a player has \$1 and with each play of the game wins \$1 with probability $0<p<1$ or loses \$1 with probability $1-p<1$. The game ends when the player either accumulates \$3 or goes broke (he loses all his money). This game is a Markov chain with the states representing the player's current holding of money, that is, 0, \$1, \$2, or \$3.

(a) **(5 pts).** Draw a state transition diagram for the case described above.

Now let's change the rules of this game. Suppose that if a player accumulates \$3, he has an opportunity to play "*double or nothing*" with his money. Meaning that the player can double his \$3 (to \$6) with probability p, or he can lose everything. (No other outcomes are possible.) After the player plays "double or nothing" the game ends. Define the new additional state of the system as state 6. *Everything else about this game remains the same as before*.

(b) **(5 pts)** You need to draw a new state transition diagram.

(c) **(5 pts)** Write down a transition matrix for this new case.

Now let's change the rules of this game again. Suppose that if a player accumulates \$3, he has an opportunity to play "*double or stay the same or nothing*" with his money. Meaning that the player can double his \$3 (to \$6) with probability 1/3, or he can lose everything with the probability 1/3 or he neither win nor lose and keep his \$3 after the play (no other outcomes are possible.) The game ends when the player either accumulates \$6 or goes broke. Nothing else about this game changes.

(d) **(5 pts)** Define all of the possible states of the system and draw a new state transition diagram.

(e) **(5 pts)** Write down a transition matrix for this new case.

(f) **(5 pts)** Looking at your answer from part (a) determine the value of $p_{26}^{(2)}$. (You need to include a short explanation of how you got this answer.)

(g) **(5 pts)** Looking at your answer from part (d) determine the value of $p_{26}^{(3)}$. (You need to include a short explanation of how you got this answer.)

(h) **(5 pts)** Looking at your answer ***from part (b)*** - determine the value of $p_{30}^{(1000)}$. (You need to include a short explanation of how you got this answer.)

Problem 2 (10 pts total). All parts of this problem deal with modifying the original case described in part a of problem 1.

(a) **(5 pts)** Suppose that the casino is tracking how long you have been playing. It modifies your transition probability from state 2 to state 3 depending on how many times you had been in state 2 before. It modifies your transition probability from state 2 to 3 be one half of the value it was on your previous visit to 2. In other words, $p_{23=}p$ if this is your first visit to the state 2. It is p/2 if this is your second visit. It is p/4 if this is your third visit and so on. Does this system represent a Markov Chain? (You need to include a short explanation).

(b) **(5 pts)** Suppose that the casino modifies what happens in state 3 depending on the day of the week. If it's Sunday through Thursday, the game ends when you reach state 3. On Friday and Saturday, it allows you to get to state 4 before ending the game. Does this system represent a Markov Chain? (You need to include a short explanation).

Submission Instructions

Failure to follow these steps may result in your submission not being graded.

1. Write Down Your Name and your BU ID
2. Make a pdf file of your write-up and upload the file to BB Learn
3. Your file name MUST be in the form Last Name_First Name.pdf (I would submit as Krigman_Steven.pdf)
4. Email me a copy in a single file, with the subject: *MA 570; Quiz #1*. → Cut & Paste this subject title.
5. You must upload AND email as attachment in a single file.
a. Do not send multiple files!
b. Attach the single file to your email; do not "drag and drop" it into your email message body.