

NAME _____ ID# _____

Please show all your work for full credit and **box off your answers.**

Partial credit will be given when you show your work.

Good luck!

Suppose you have a Markov chain with four possible states $\{0, 1, 2, 3\}$. All problems below will deal with different (1 step) transition matrices for those states.

1. **Problem (20 pts total).** (10 pts) Consider the following (one step) transition matrix:

States	0	1	2	3
0	0	1.0	0	0
1	0	0	1.0	0
2	0	0	0	1.0
3	1	0	0	0

- a) (5 pts) Draw a transition diagram.
 b) (5 pts) Indicate how many classes and which states belong to which classes.
 c) (5 pts) What is the period of every state in each class?
 d) (5 pts) What is the expected number of visits to state =1?

2. **Problem (50 pts total).** Now, suppose you have values a_1 and a_2 representing transition probabilities, such that $0 < a_1 < 1$ and $0 < a_2 < 1$. Consider the following transition matrix:

States	0	1	2	3
0	1.0	*	*	*
1	a_1	0	*	0
2	a_2	0	0	*
3	0	1.0	*	*

- a) (5 pts) Fill in the missing entries, marked with *.
 b) (5 pts) Draw a transition diagram. (Label transition probabilities in terms of a_1 and a_2).
 c) (5 pts) Identify number of classes; which elements belong to which class; periodicity of each class.
 d) (5 pts) Looking at your answer from part (b) determine the values of $p_{11}^{(2)}, p_{11}^{(3)}, p_{11}^{(4)}, p_{11}^{(5)}, p_{11}^{(6)}$ (You need to include a short explanation of how you got this answer.)
 e) (5 pts) Same question as (d) for $p_{13}, p_{13}^{(2)}, p_{13}^{(3)}$ (You need to include a short explanation of how you got this answer.)

In the next set of questions, assume $a_1=0.2, a_2=0.1$:

Also, the following formula for the sum of geometric series may be useful to you: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ if $|x| < 1$.

- f) (5 pts) Assuming the process starts in State=1, find the **expected** number of visits to that state.
 g) (10 pts) Find the expected number of visits to State=0.
 h) (10 pts) Does your answer to part (g) depend on where the process starts? Does your answer to part (f) depend on where the process starts?