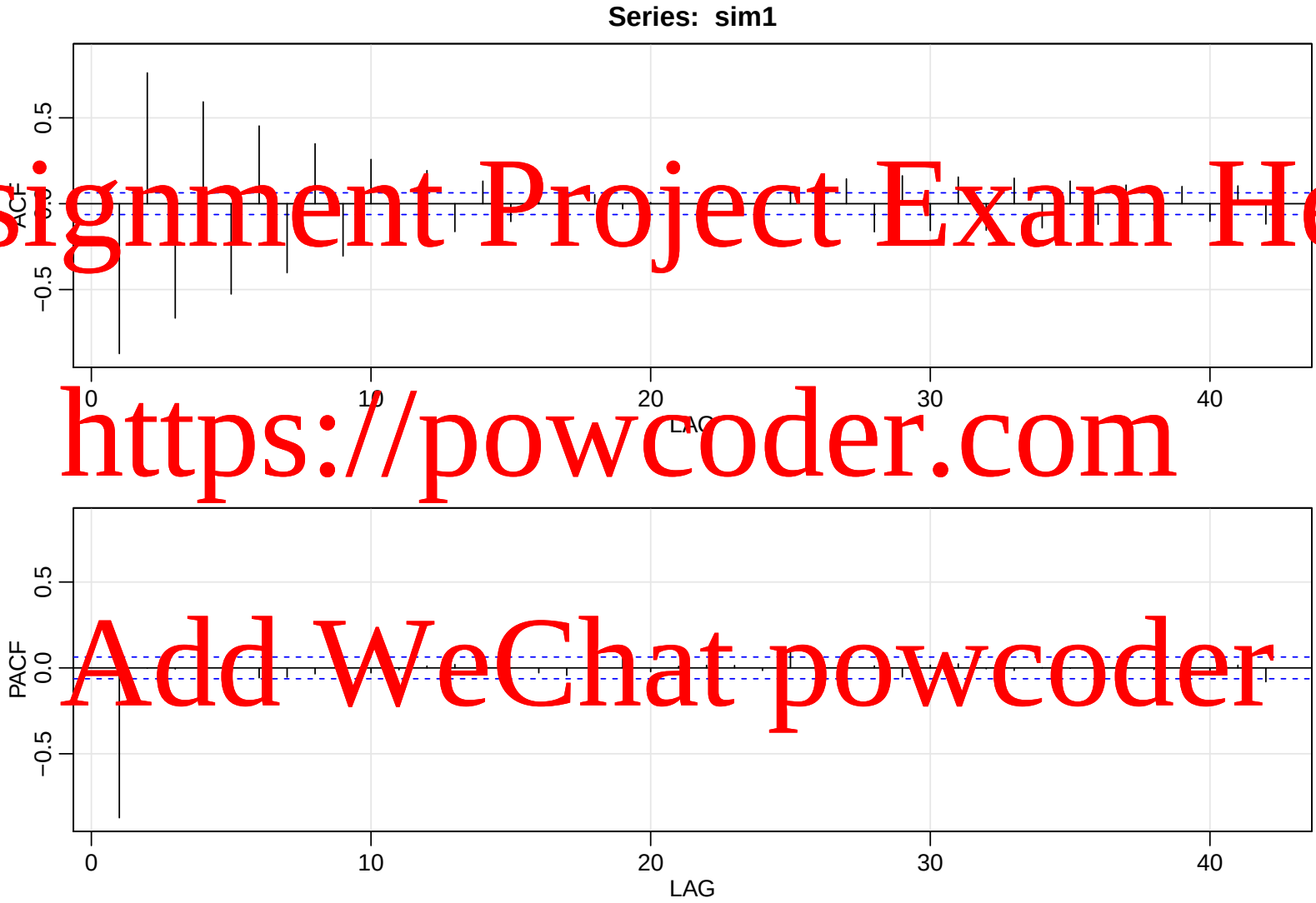
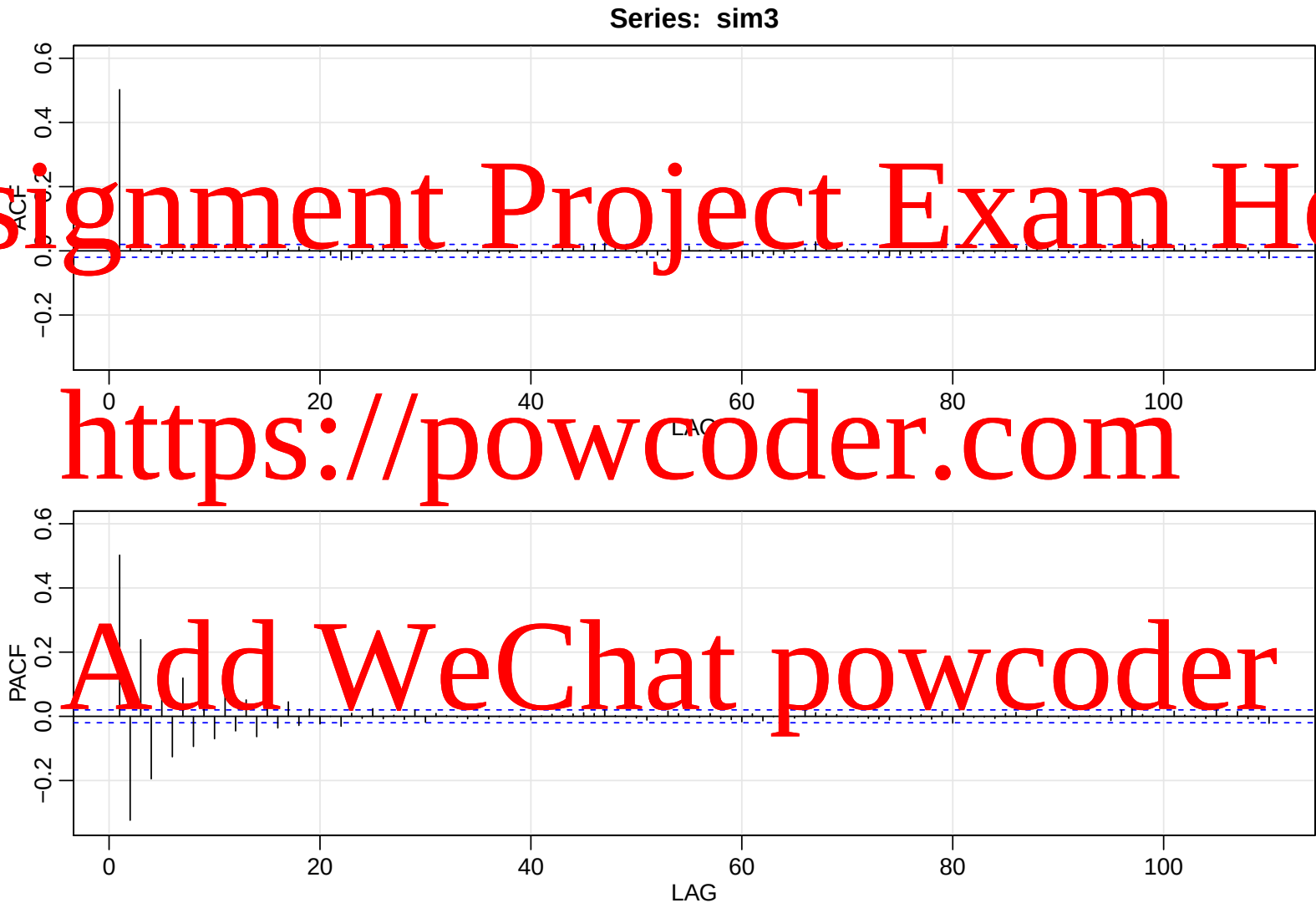
Models for Nonstationary Time Series

Determining Model Order



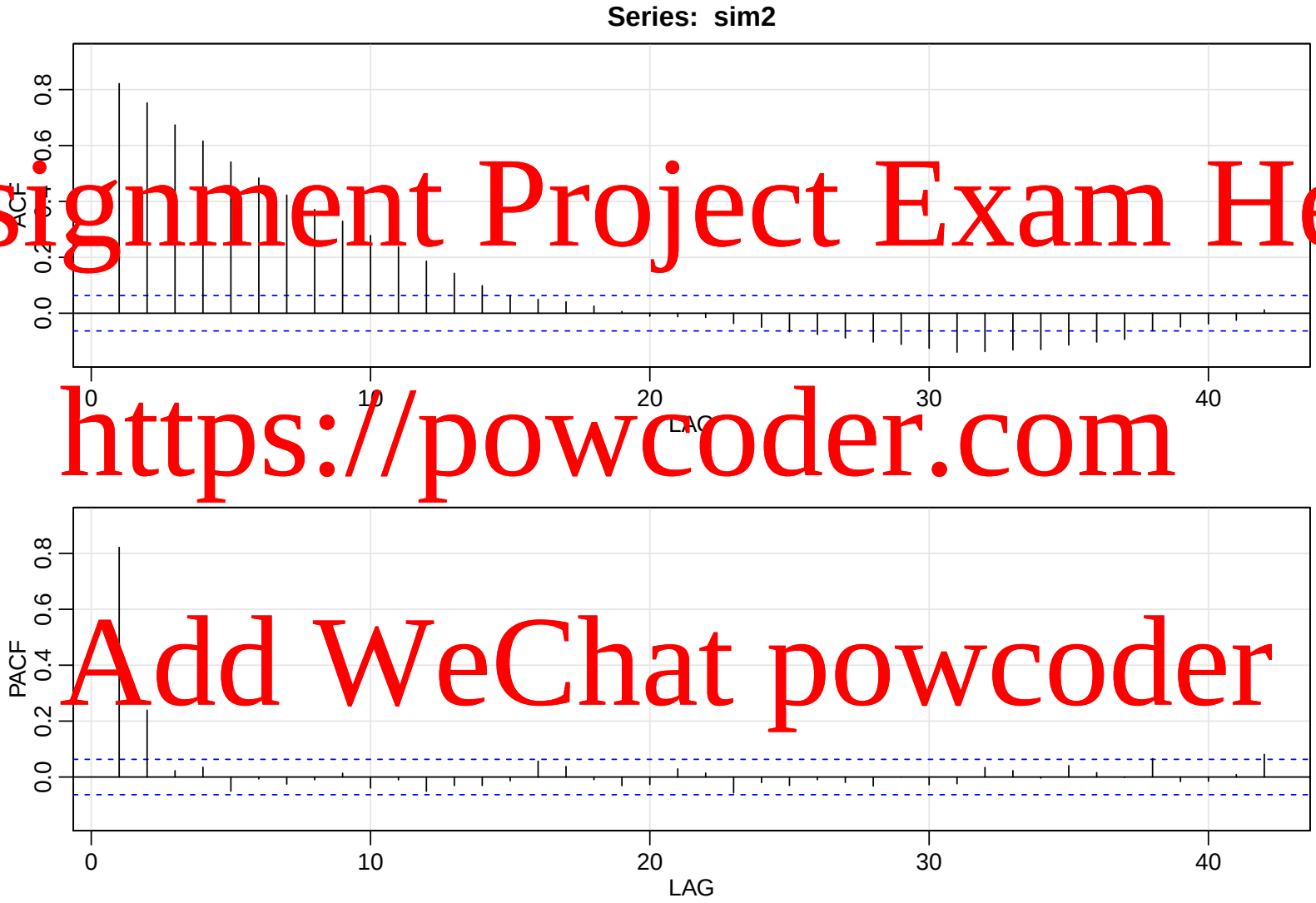

```
      ACF  PACF
[1,] 0.87  0.87
```

Determining Model Order



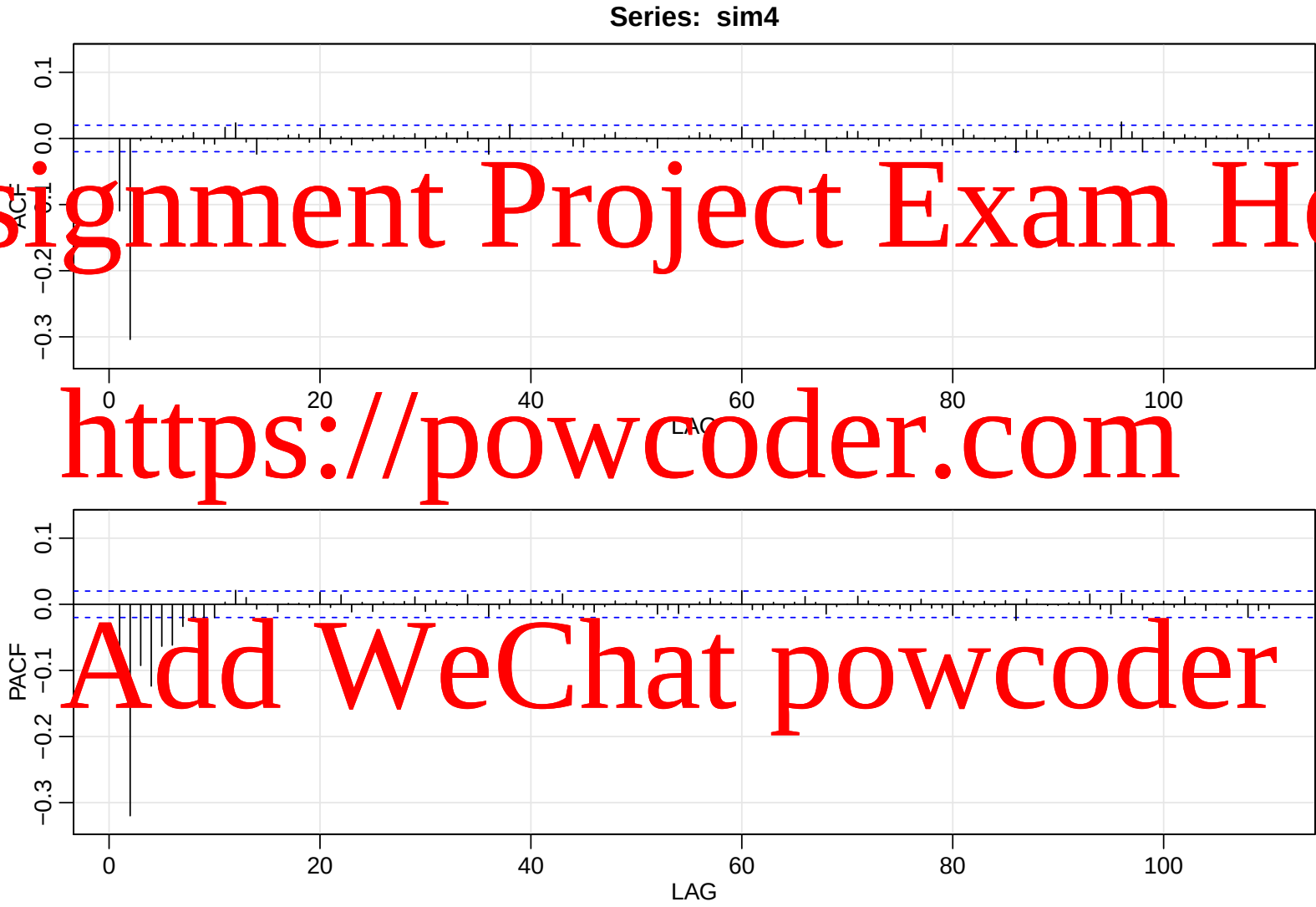
	ACF	PACF
[1,]	0.50	0.50

Determining Model Order




```
      ACF  PACF
[1,] 0.82  0.82
```

Determining Model Order



	ACF	PACF
[1,]	0.11	0.11

Outline

- Models for Nonstationary Time Series
  - Dealing with nonconstant mean functions
  - Dealing with nonconstant variance
- Chapter 5 from text

Detrending and Stationarity

Any time series without a constant mean over time is nonstationary.

$$Y_t = \mu_t + e_t$$

If μ varies across t, the series is nonstationary

Detrending and Stationarity

- We fixed this issue by building a model for μ_t and studying the residuals
  - "Detrending"
- We called this modeling "deterministic" trends
- Only reasonable if we assume this trend is an intrinsic property of the time series
- Implicitly assumes that the trend is "forever"
  - Which is often difficult to believe

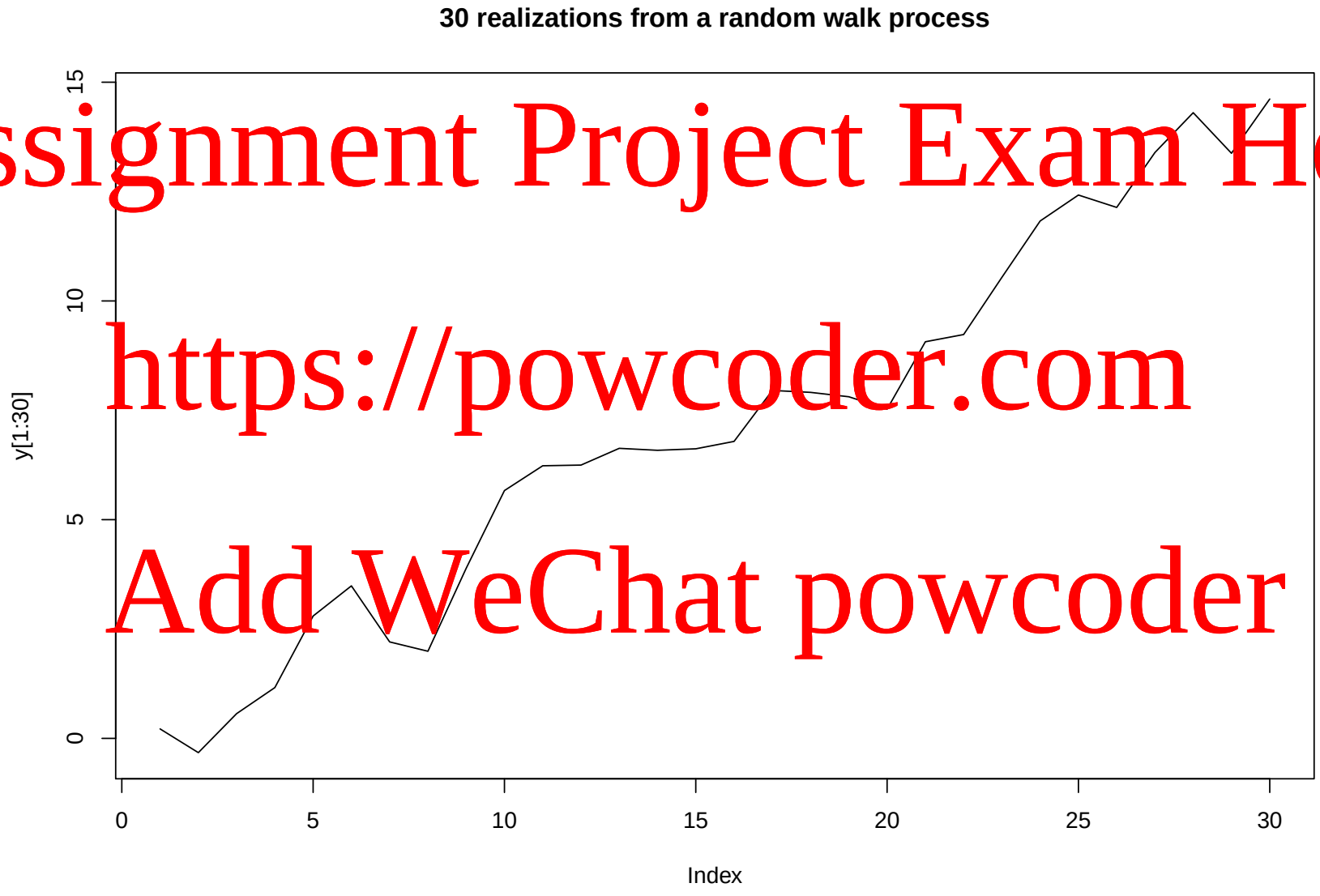
Random Walk

- Consider the random walk process

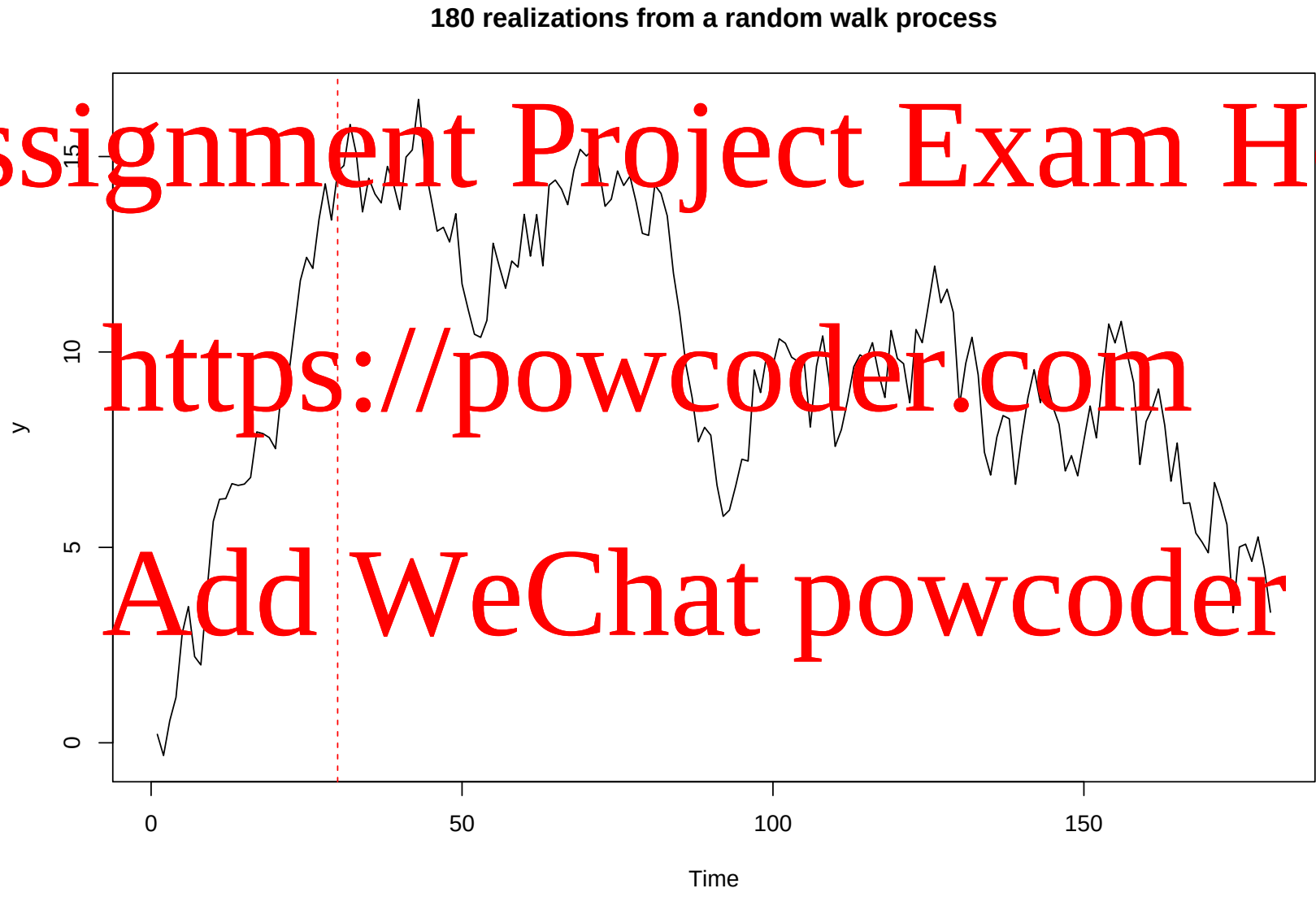
$$Y_t = Y_{t-1} + e_t$$

- By definition, it has constant mean $\mu_t = 0$
- Mistaking a trend seen from a random walk as deterministic ("forever") wouldn't be appropriate

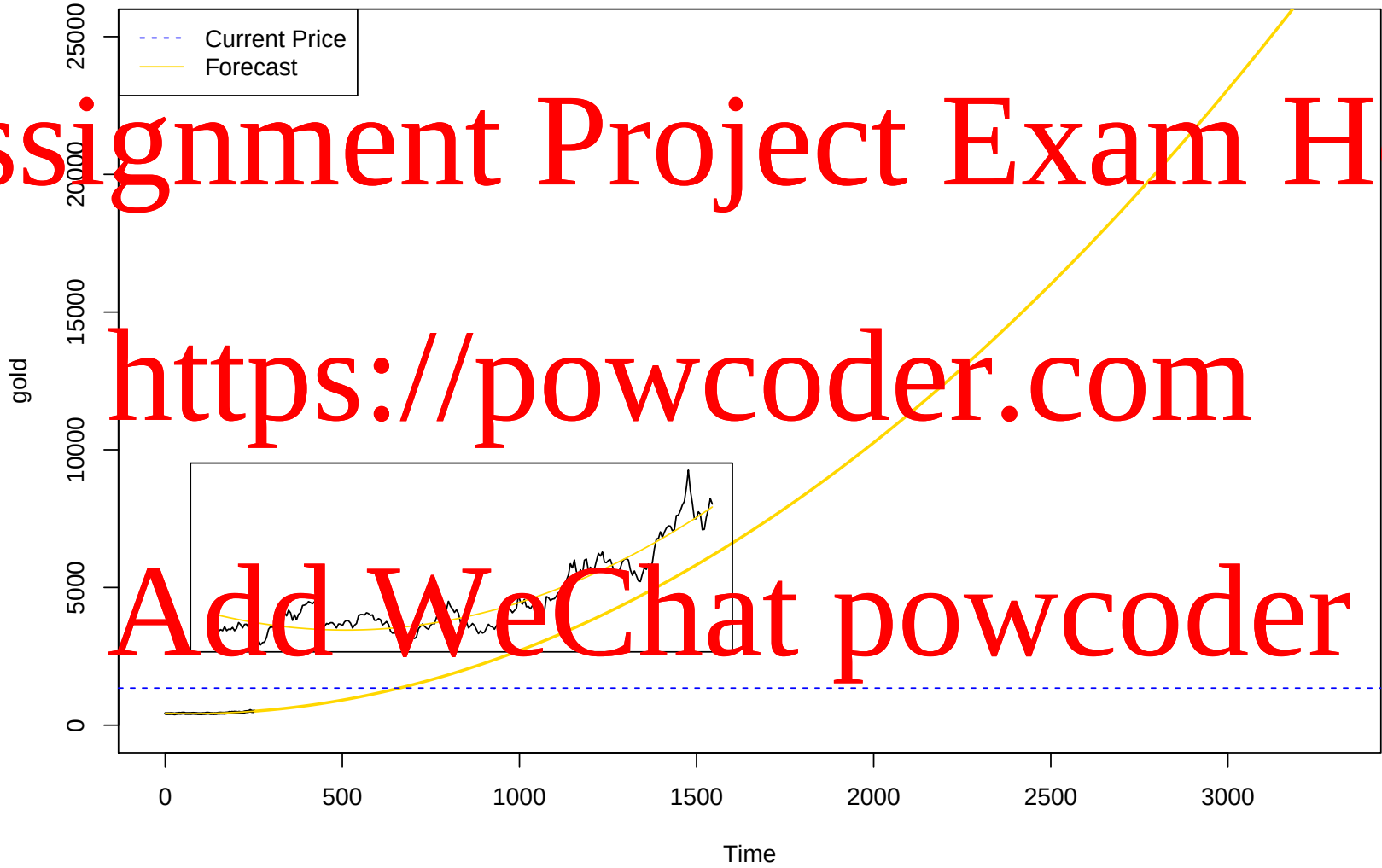
Random Walk - 30 Days of Data



Random Walk - 180 Days of Data

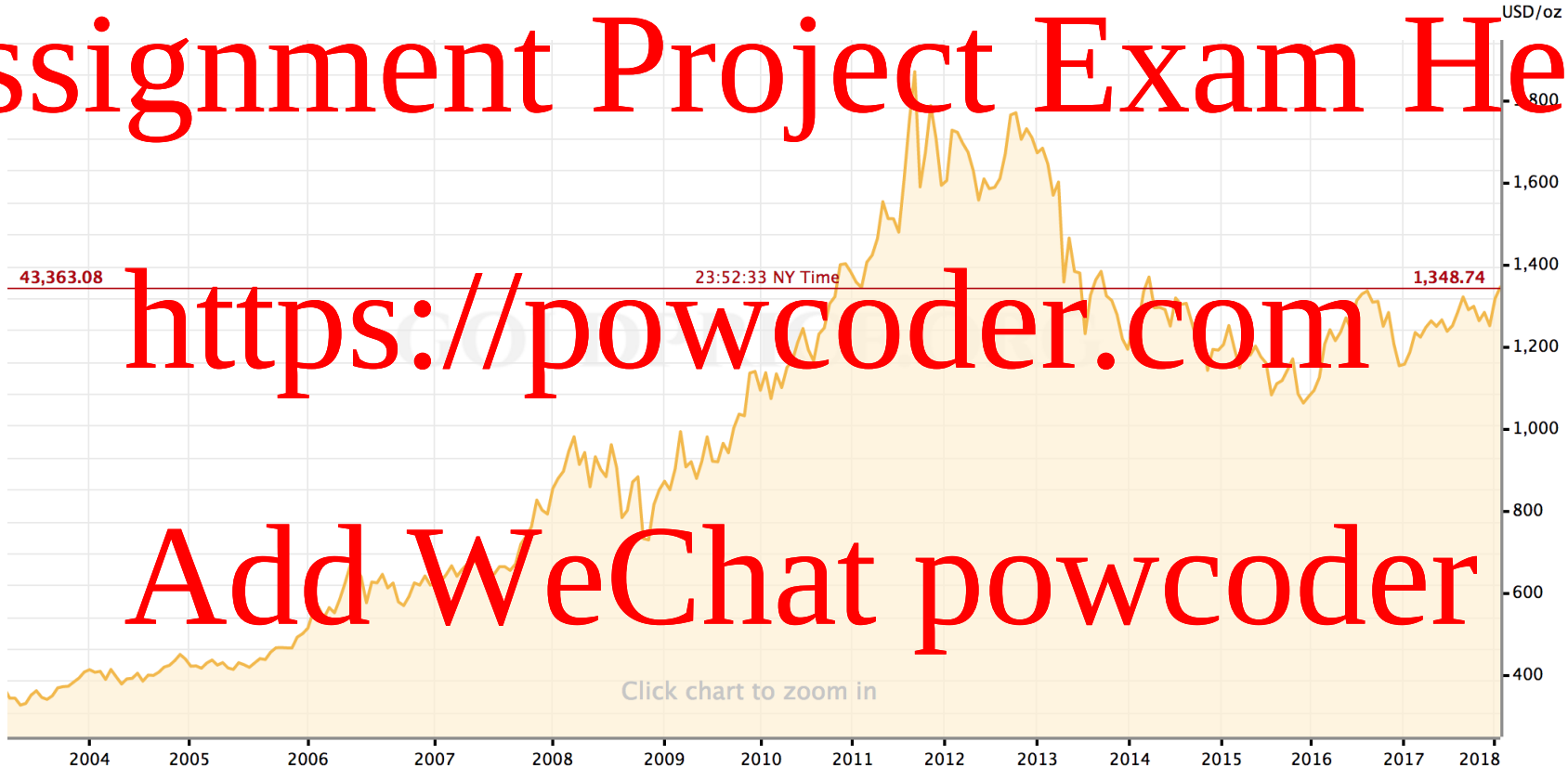


Price of Gold

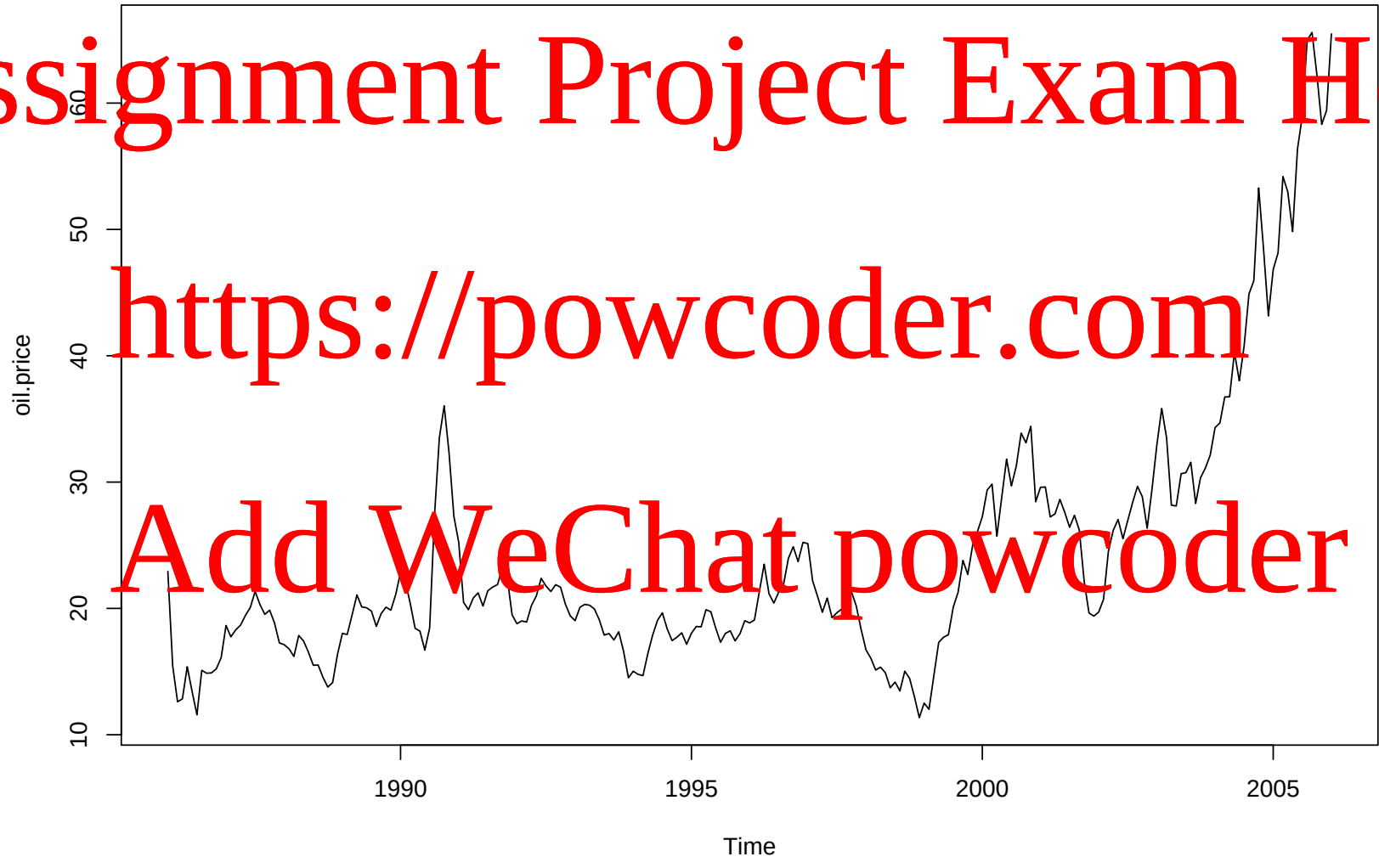



```
null device
          1
```

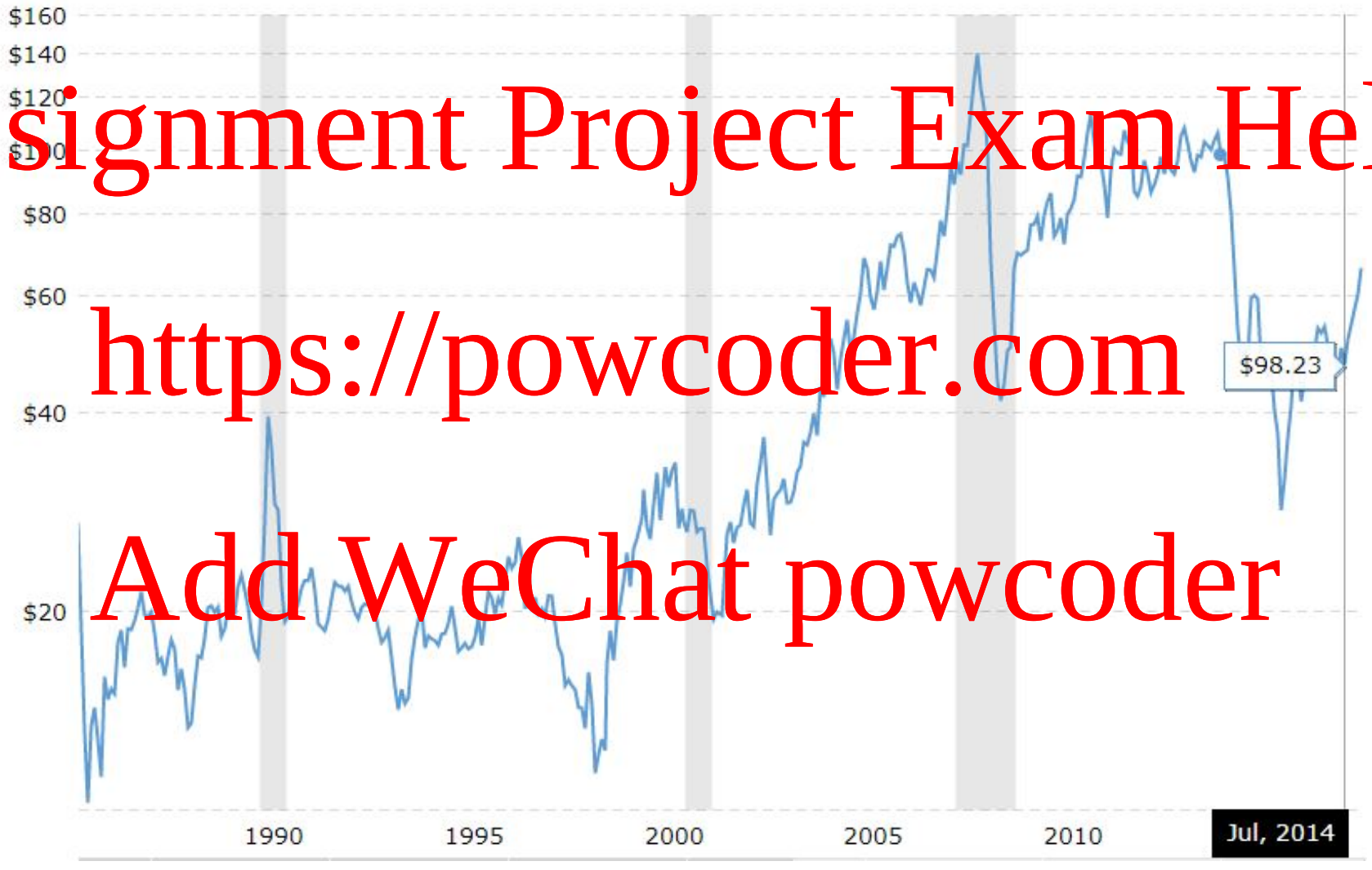
Price of Gold



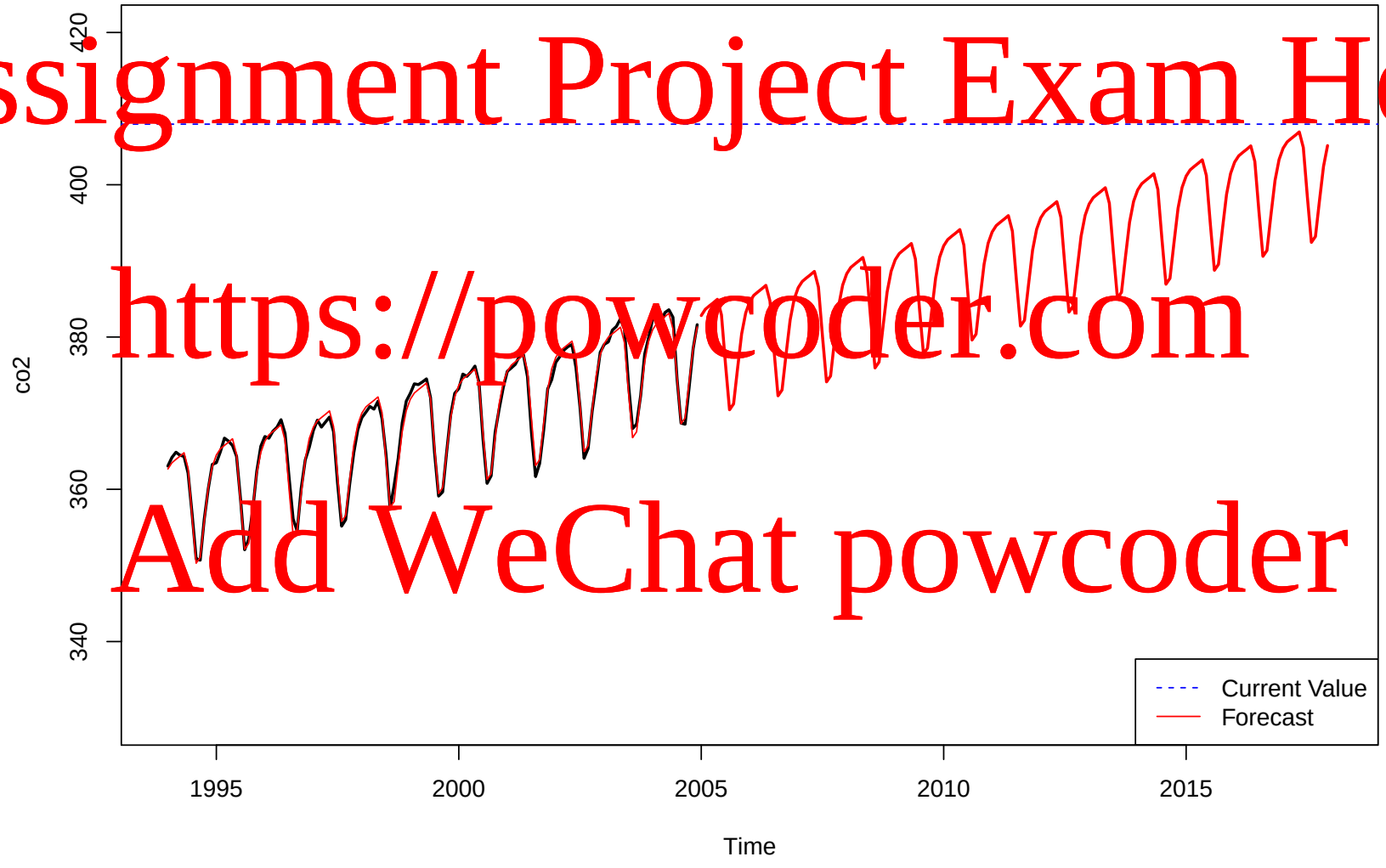
Oil Price



Oil Price



CO2 Levels



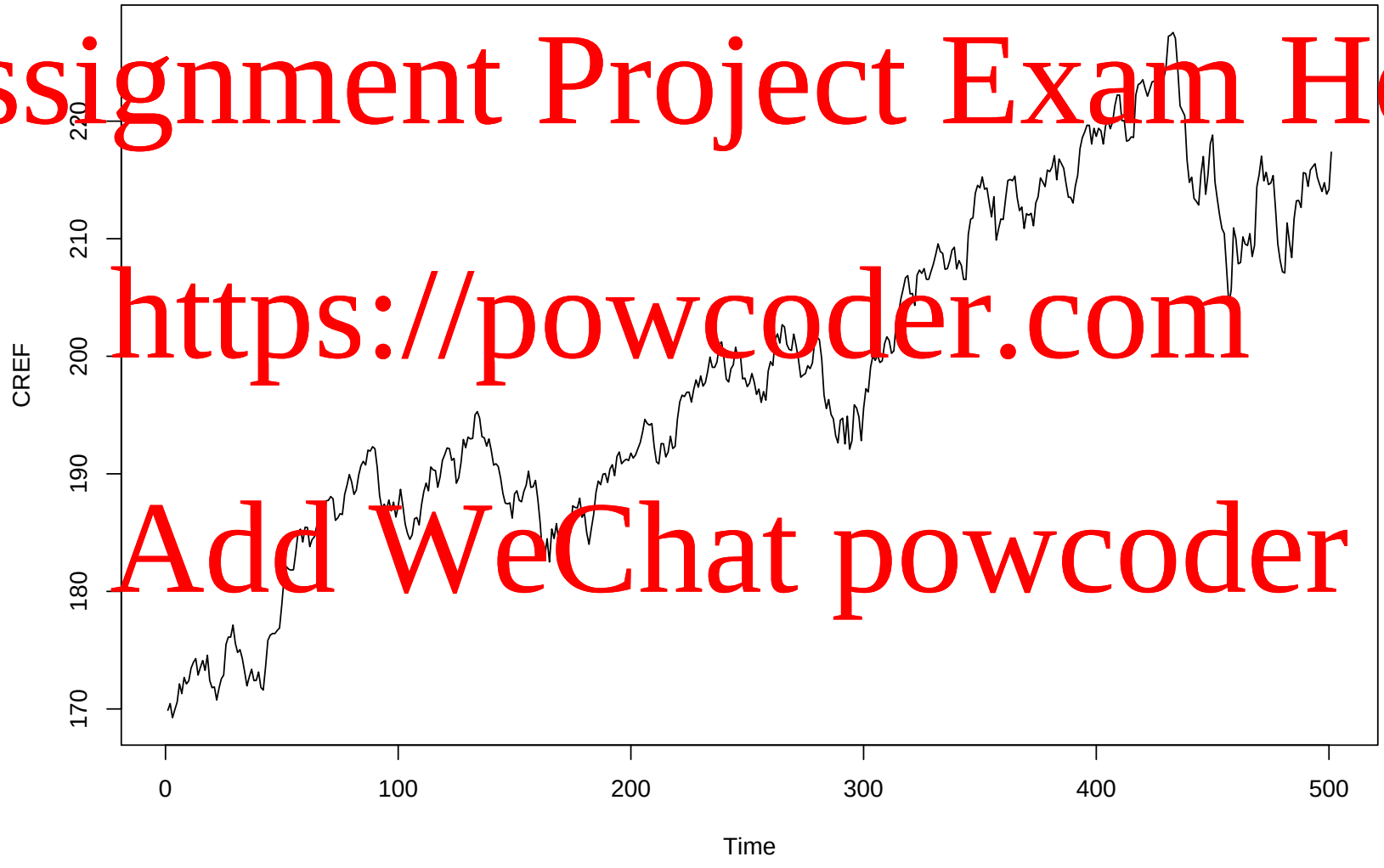
Differencing

- An alternative approach is to study the **differenced** time series

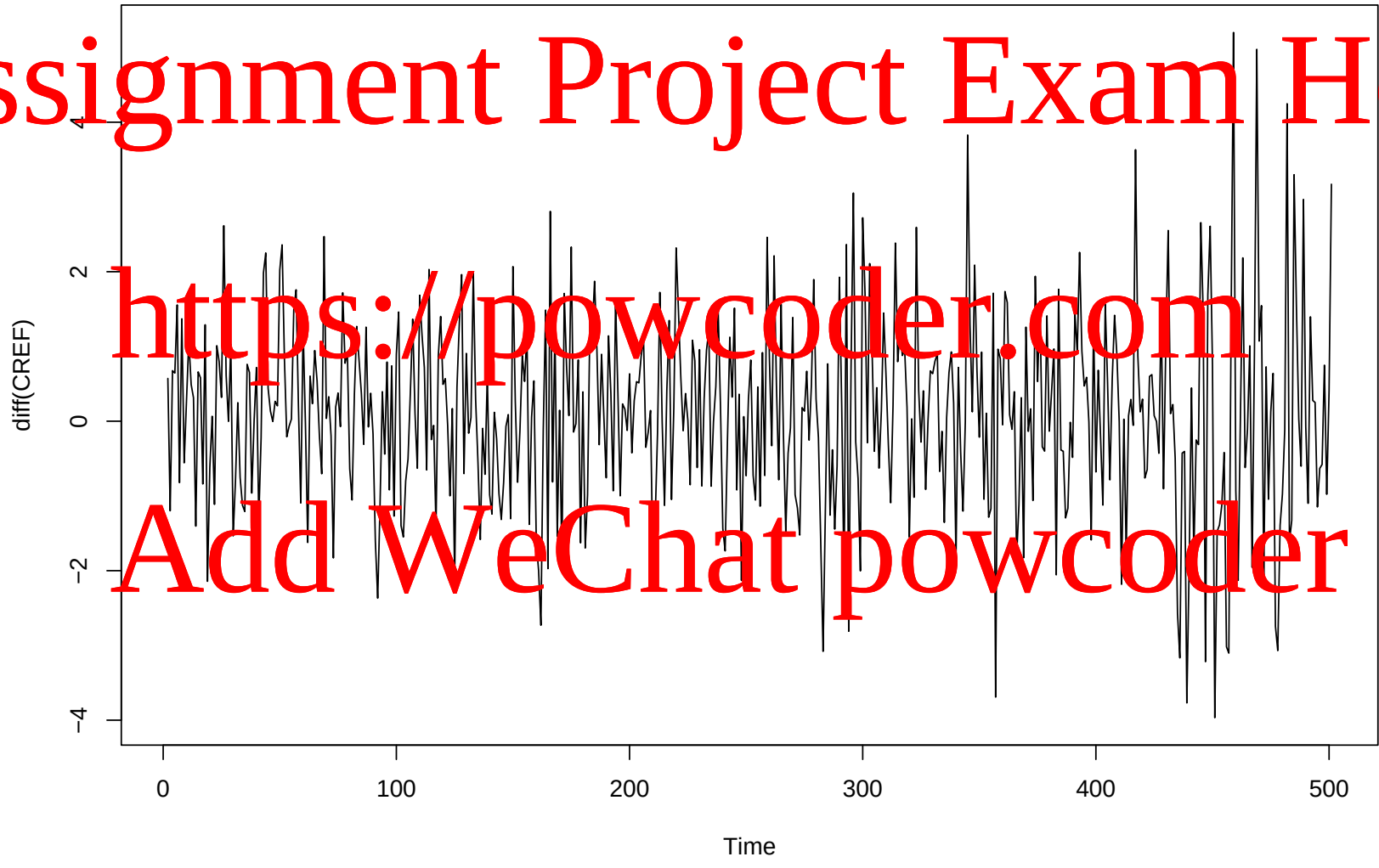
$$\nabla Y_t = Y_t - Y_{t-1}$$

- No assumptions on trend through time
- No model to fit or parameters to estimate
- Tends to work well in practice
  - Good, quick, simple for forecasts

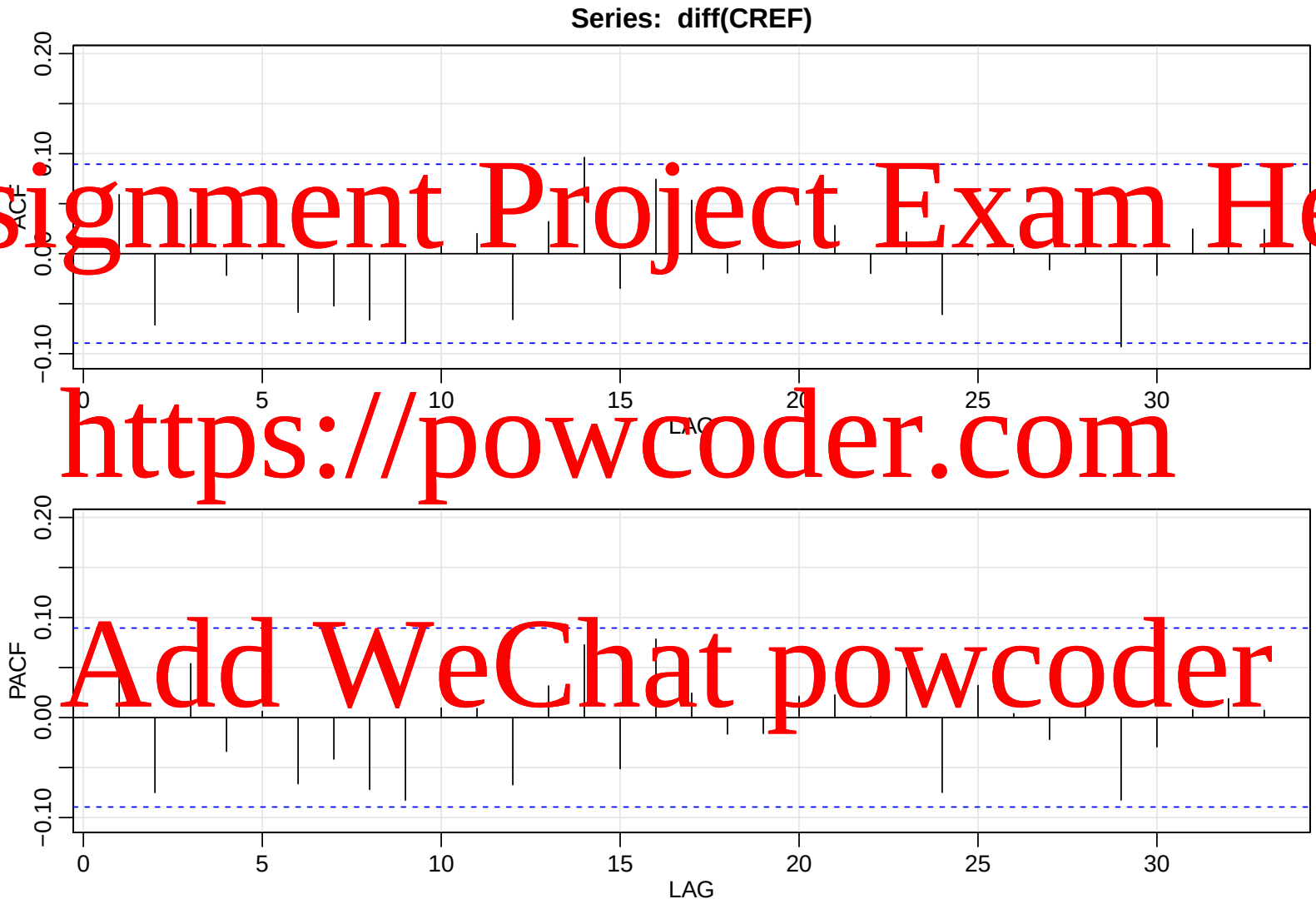
CREF Time Series



CREF Differenced

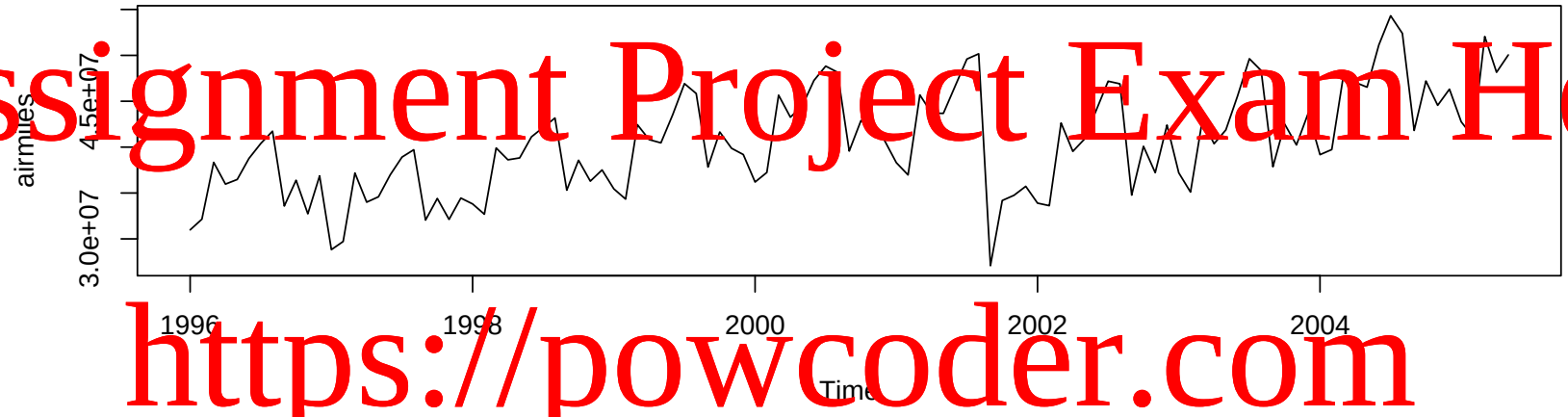


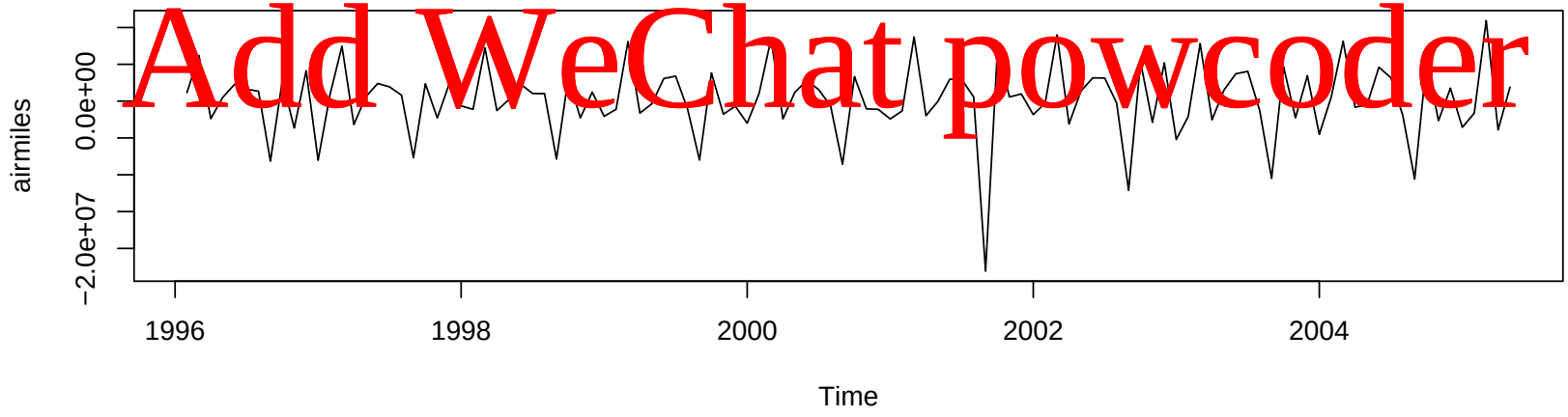
Autocorrelations for Differenced CREF Data



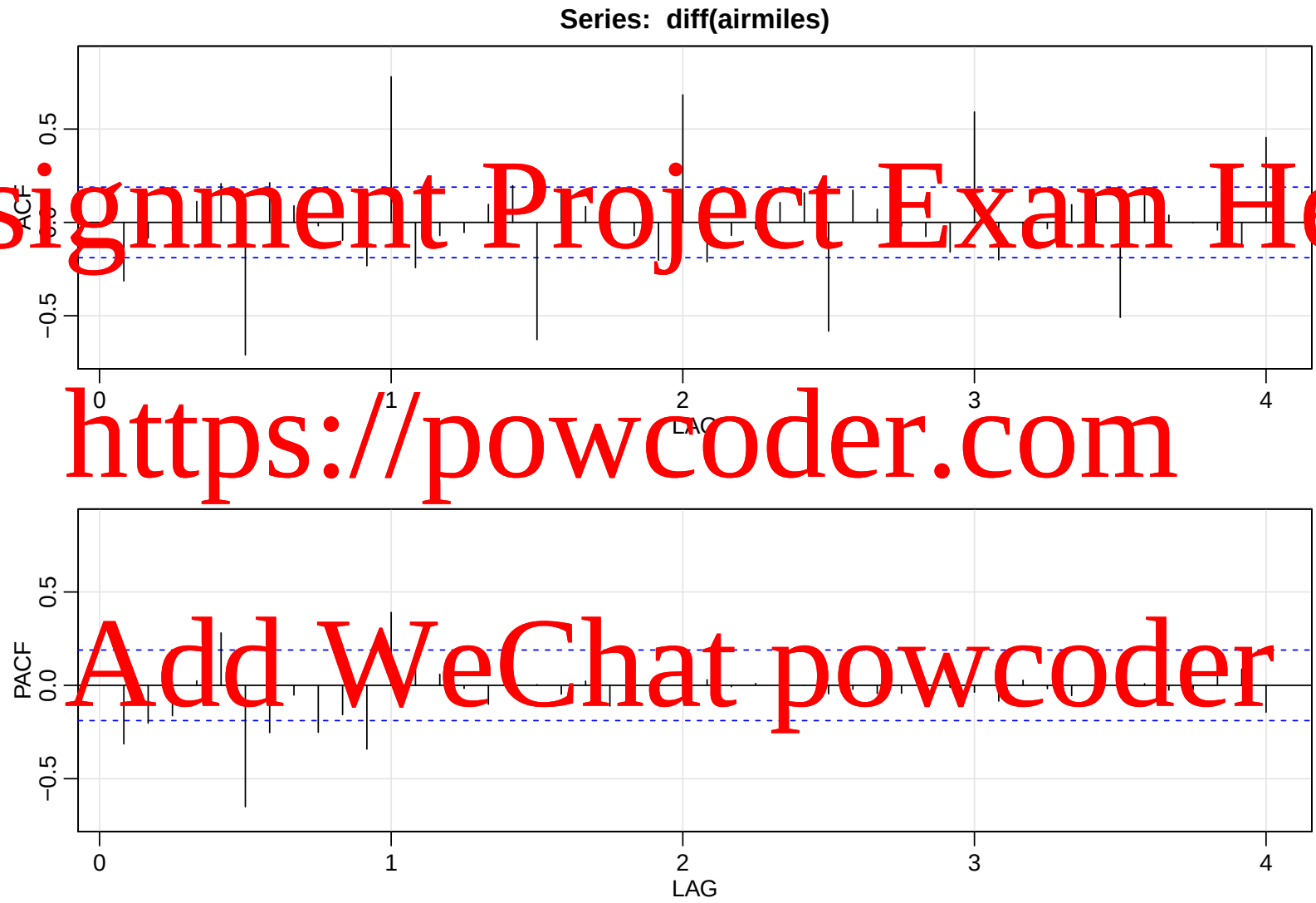
	ACF	PACF
[1,]	0.06	0.06

Air Passenger Miles



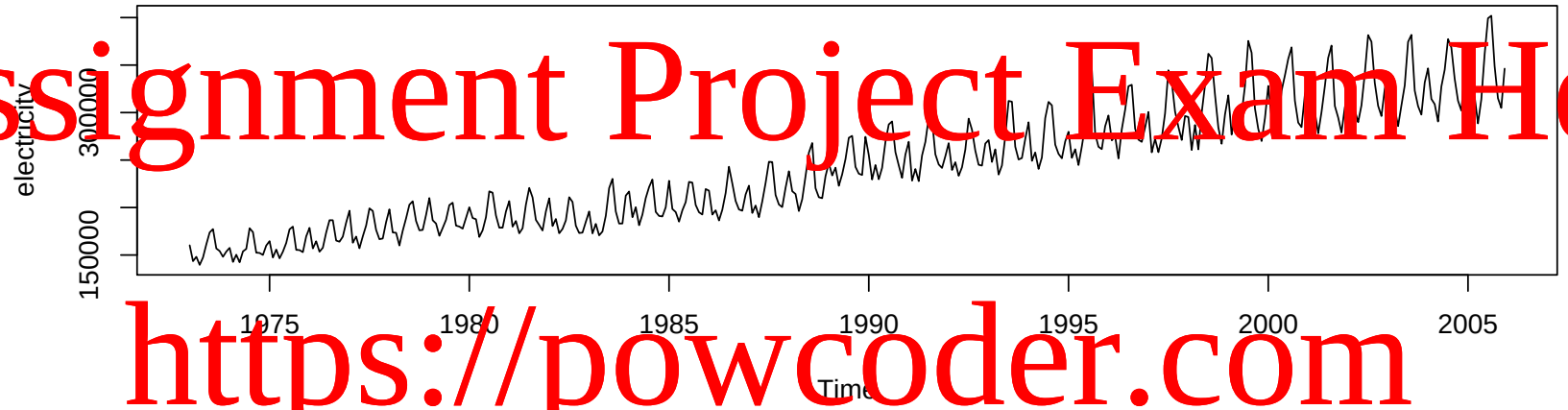


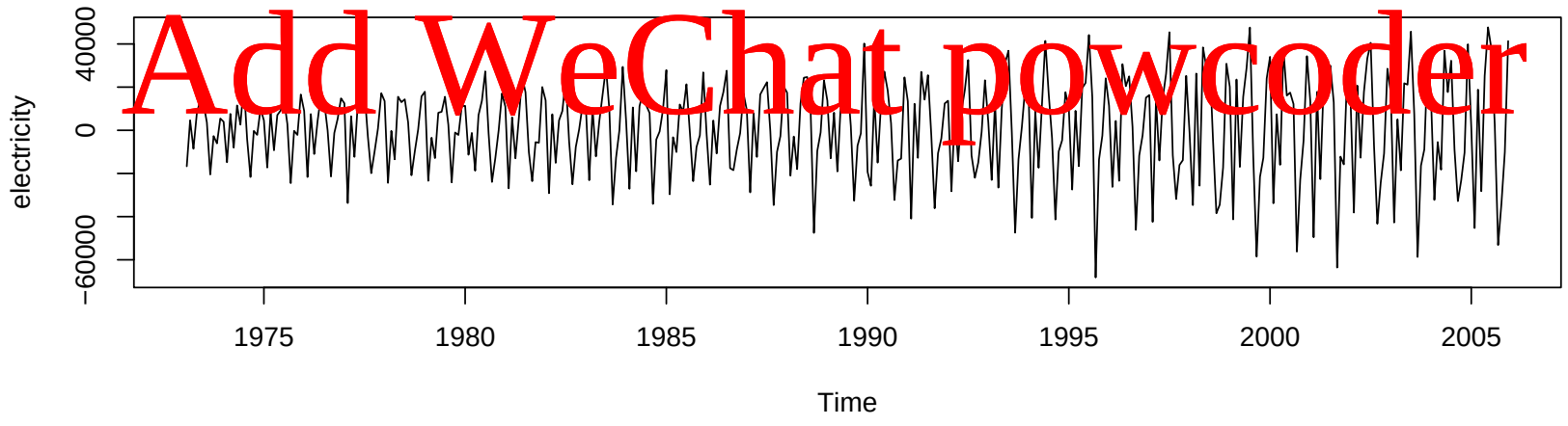
Autocorrelations for Differenced `airmiles` Data



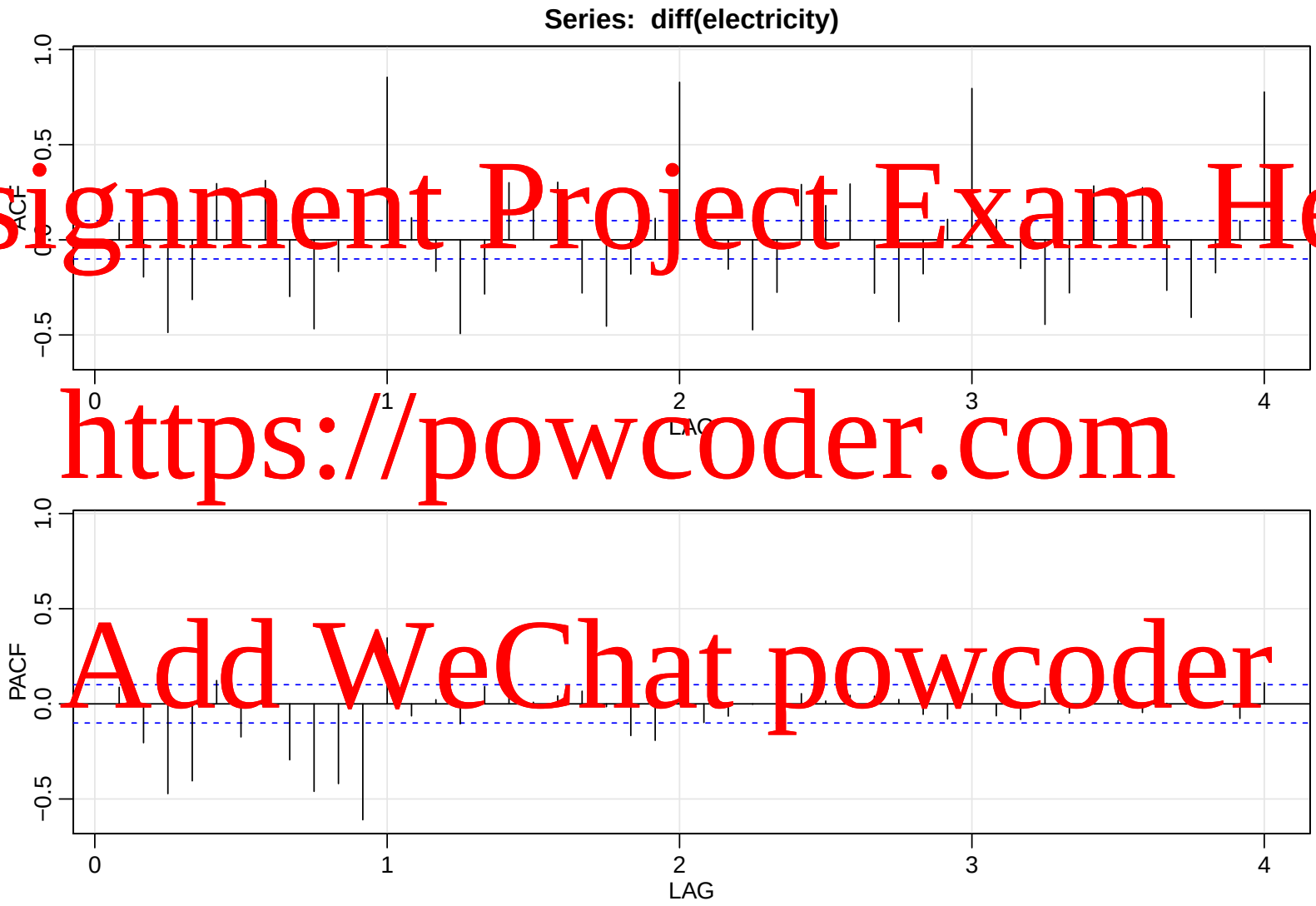

```
      ACF  PACF
[1,] 0.31  0.31
```

Electricity





Autocorrelations for Differenced `electricity` Data




```
      ACF  PACF
[1,] 0.00  0.00
```

Differencing

- Moral of the story: differencing can effectively remove the nonconstant mean in a time series
  - Rather than build a model for the trend and study residuals, study the differenced data
- Note: For exponential trends, you may need to take two differences!
  - `diff(diff(DATA))`

Implementation of Differencing

- Since differencing is so standard for time series analyses and forecasting, most software will allow you to specify it directly.
  - So rather than building a model and studying the residuals, we can simply pass the original data and say "difference it"


```
sarima(x, p, d, q)
```


- p = AR order
- d = Number of differences for stationarity
- q = MA order

ARIMA(p, d, q)

ARMA models that need to be differenced are referred to as ARIMA models

- Autoregressive Integrated Moving Average
  - Denoted ARIMA(p, d, q)

ARIMA(p, d, q)

ARIMA(p, d, q) models encompass every class of models we have encountered up to this point

- ARIMA(p, 0, q) =
- ARIMA(p, 0, 0) =
- ARIMA(0, 0, q) =
- ARIMA(0, 0, 0) =

Stationarity

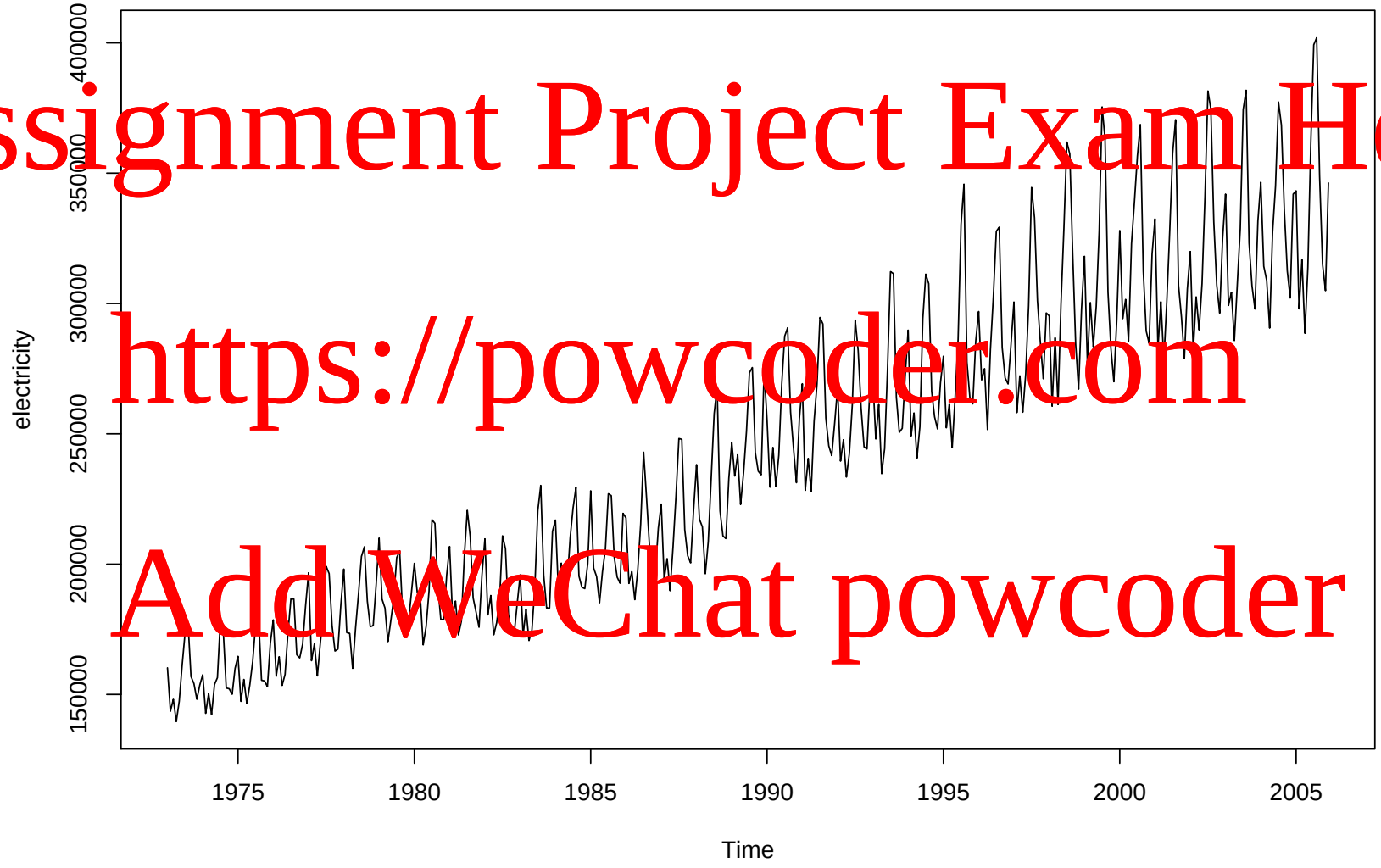
- Stationarity is a very important concept in time series and one that you will often hear. Broadly speaking, a time series is called **stationary** if...
  1. No systematic change in the mean (no trend),
  2. No systematic change in the variance,
  3. No noteable seasonal patterns exist

In other words, the properties of one section of the data are the same as any other section.

Transformations

- If we have clear evidence of nonconstant variance over time, a suitable transformation might fix (or lessen the impact of) the nonconstant variance pattern.
- Any transformations we apply to the data should be a **first** step.
  - Transform the data before looking at differences or modeling the trend.

Monthly Electricity Usage in the US

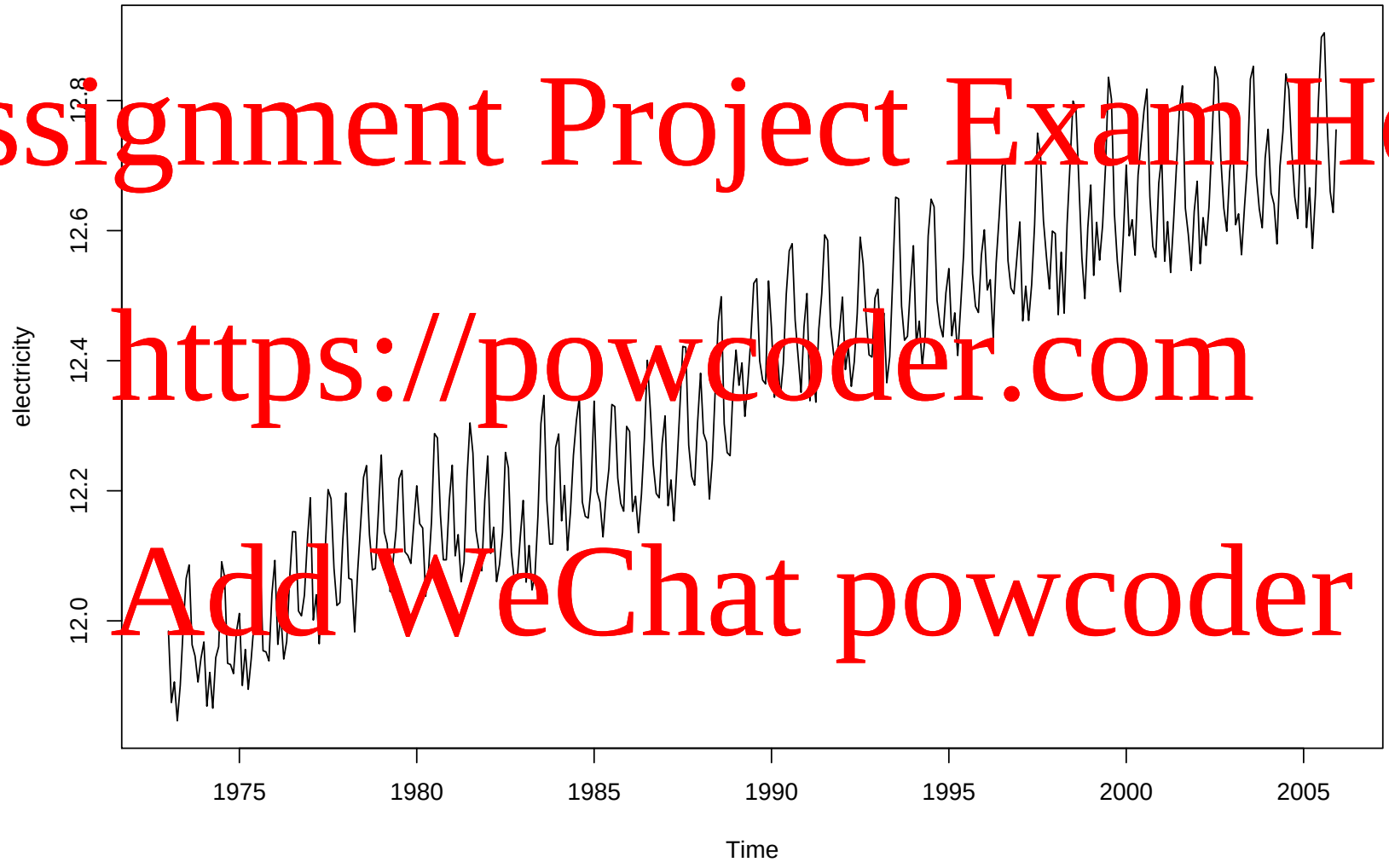


What do we learn from this time series plot?

Monthly Electricity Usage in the US

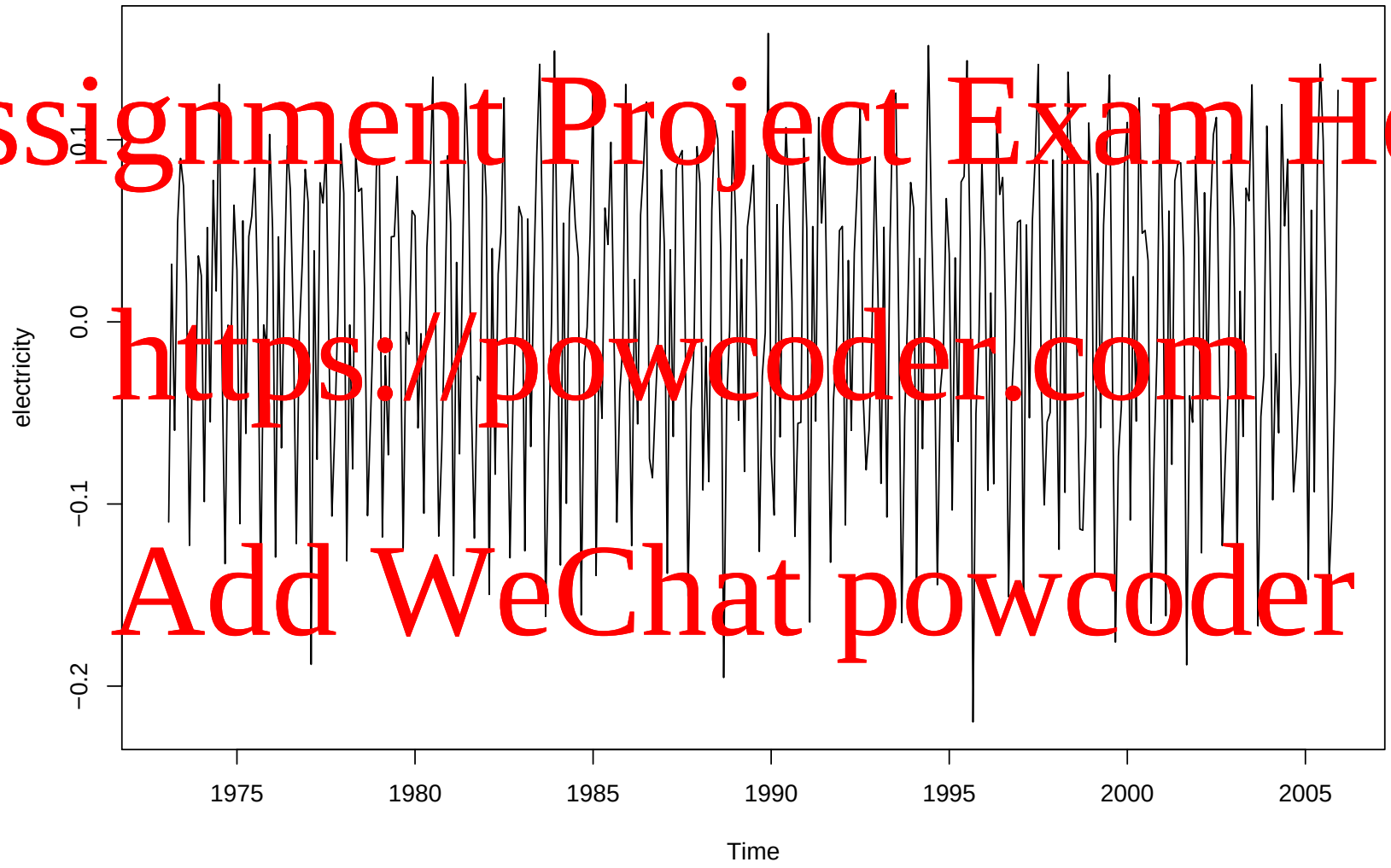
- Variance is increasing over time
- Time series that exhibit a "fanning-out" shape are not stationary because the variance changes over time.
- Before modeling, we should transform the data to *stabilize* the variance.

log(Electricity) in the US



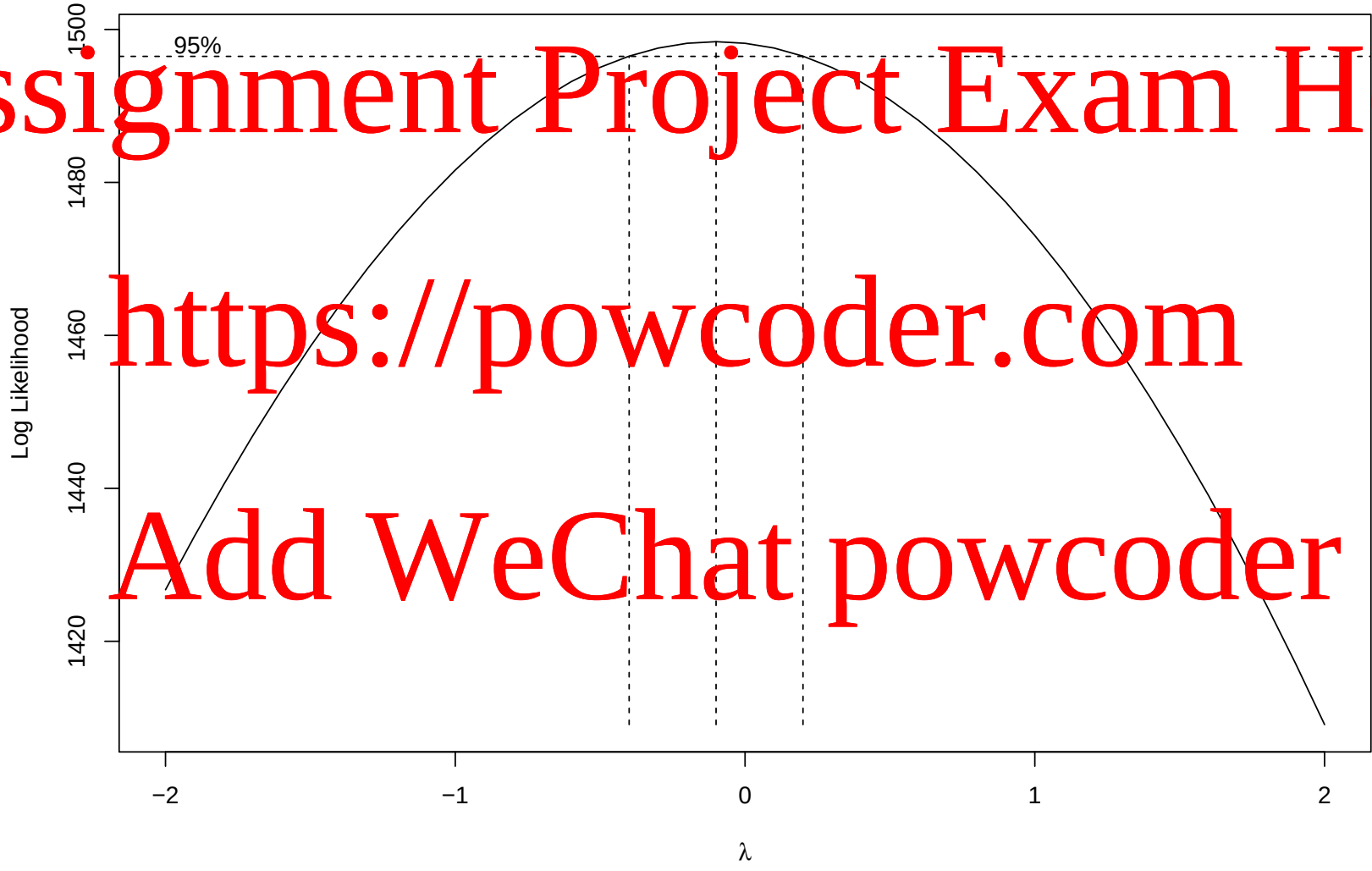
Variance looks ok! Let's plot the differences next.

diff(log(Electricity)) in the US



Remember: transform first, them difference

Box-Cox for Power Transformations



Power Transformations

λ	$T(Y)$	Description
-2.0	$1/Y^2$	Inverse Square
-1.0	$1/Y$	Inverse or Reciprocal
-0.5	$1/\sqrt{Y}$	Inverse square root
0.0	$ln(Y)$	Logarithm
0.5	$\sqrt{Y}$	Square root
1.0	Y	Identity (No transformation)
2.0	Y^2	Square

Comments on using BoxCox procedure

- In the electricity example, the optimal λ was found to be about -0.1. However, this transformation makes little sense and isn't interpretable.
- Interval suggested values between about -0.4 and 0.2.
- So log transformation would be appropriate.

Want to find a reasonable transformation, not necessarily an optimal..

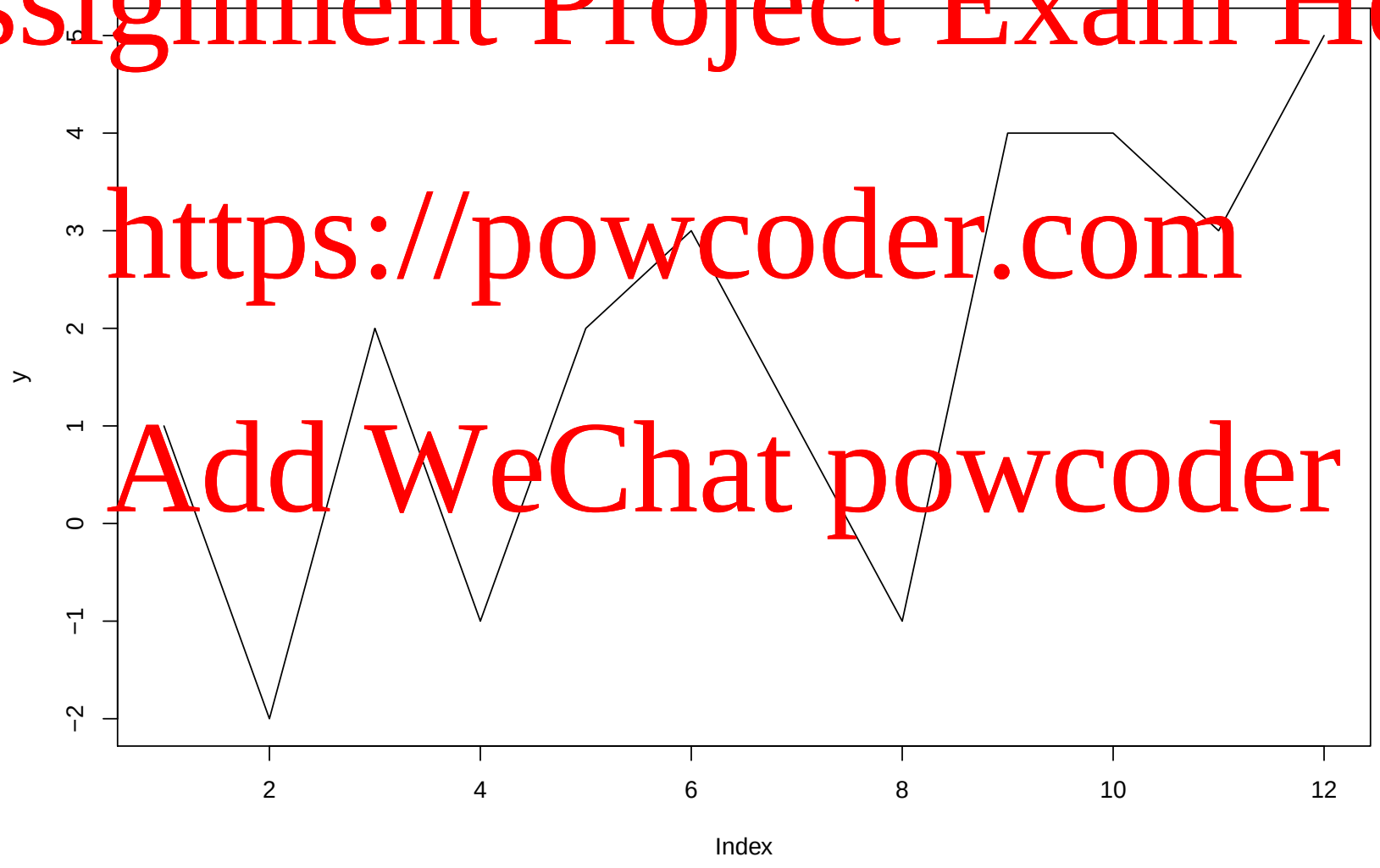
Variance Stabilizing Transformations

- Can only perform variance stabilizing transformations on positive time series
  - All values > 0

- However, if some or all Y are negative, we can simply add the same positive constant c to every observation so that every value becomes positive
  - This does not affect anything

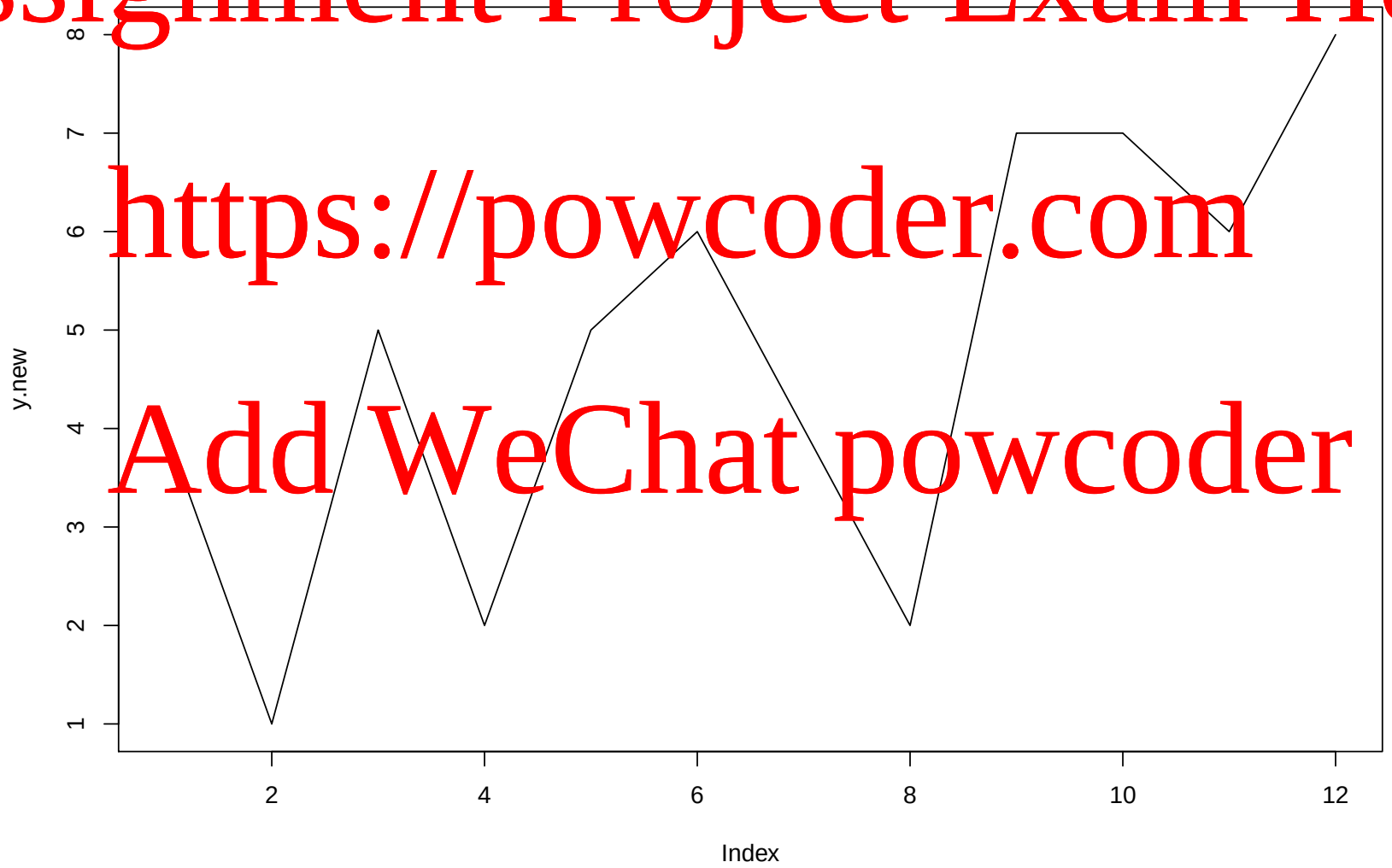
Adding Constant to Obtain a Positive Time Series

Consider the hypothetical time series below. Note that observations 2, 4, and 8 are negative, reach as low as -2.



Adding Constant to Obtain a Positive Time Series

Simply add 3 to it. Shape is maintained, but now it's entirely positive.



Relationship with Returns

- Suppose that Y_t has relatively stable percent changes from period to period. That is, suppose that

$$Y_t = (1 + r_t)Y_{t-1}$$

- where $100r_t$ represents the percent change from Y_{t-1} to Y_t.

Relationship with Returns

- Suppose now that we take the log of this time series, and then the difference.

$$log(Y_t) - log(Y_{t-1}) = log\left(\frac{Y_t}{Y_{t-1}}\right) = log(1 + r_t)$$

- If r_t is relatively low (<20% returns), then $log(1 + r_t) \approx r_t$
- Consequently,

$$\nabla log(Y_t) \approx r_t$$

- Common in time series studies of financial data where returns are important and meaningful