Lecture 10 - Seasonal ARIMA Models

MAS 640 - Times Series Analysis and Forecasting

February 14, 2018

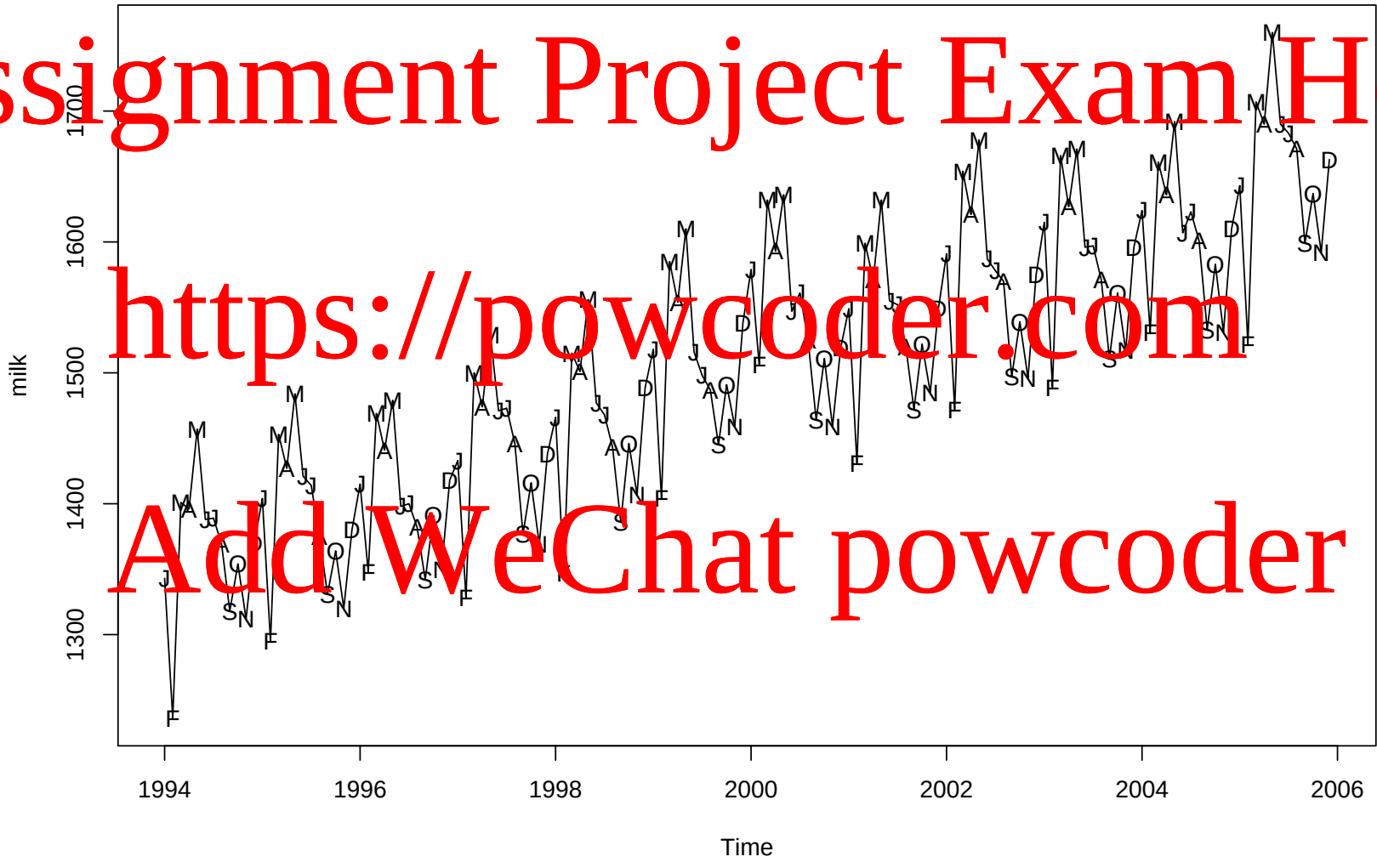
- **Stationarity**
  - Constant mean
  - Constant variance
  - No seasonality

Dealing with nonstationarity

- We have covered two ways for removing trends from a series
  1. Model it and study the residuals
  2. Difference it

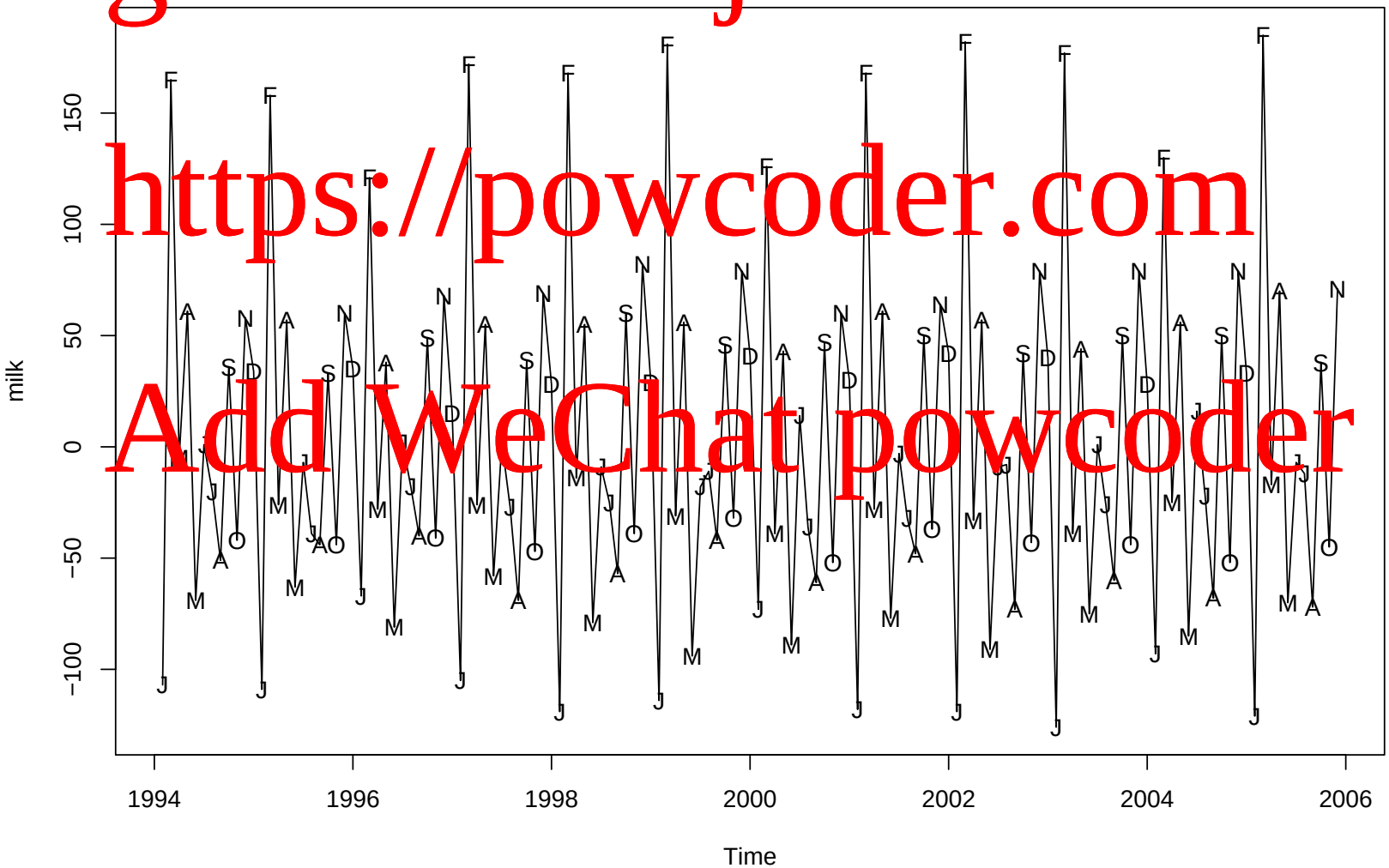
- We have covered how to correct for nonconstant variance
  - Transformation
  - Box-Cox for guidance
- Seasonality is a common problem that needs to be handled as well

Seasonality nonstationarity



Seasonality causing nonstationarity

- 1st order differencing generally used for removing trends
- But seasonality persists...

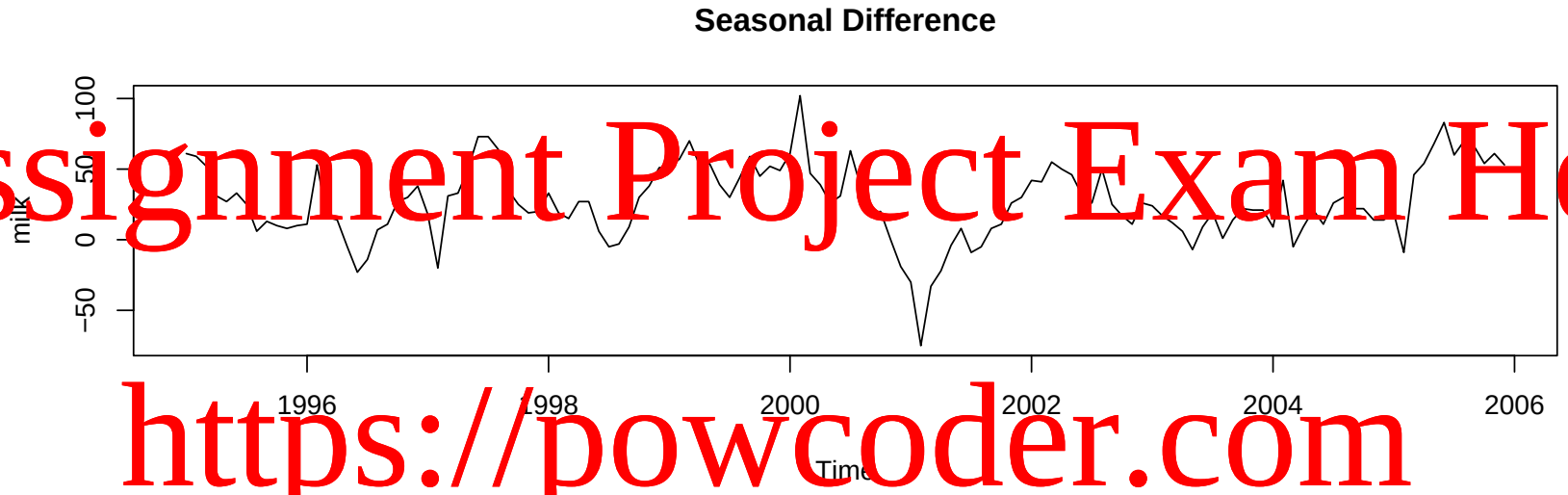


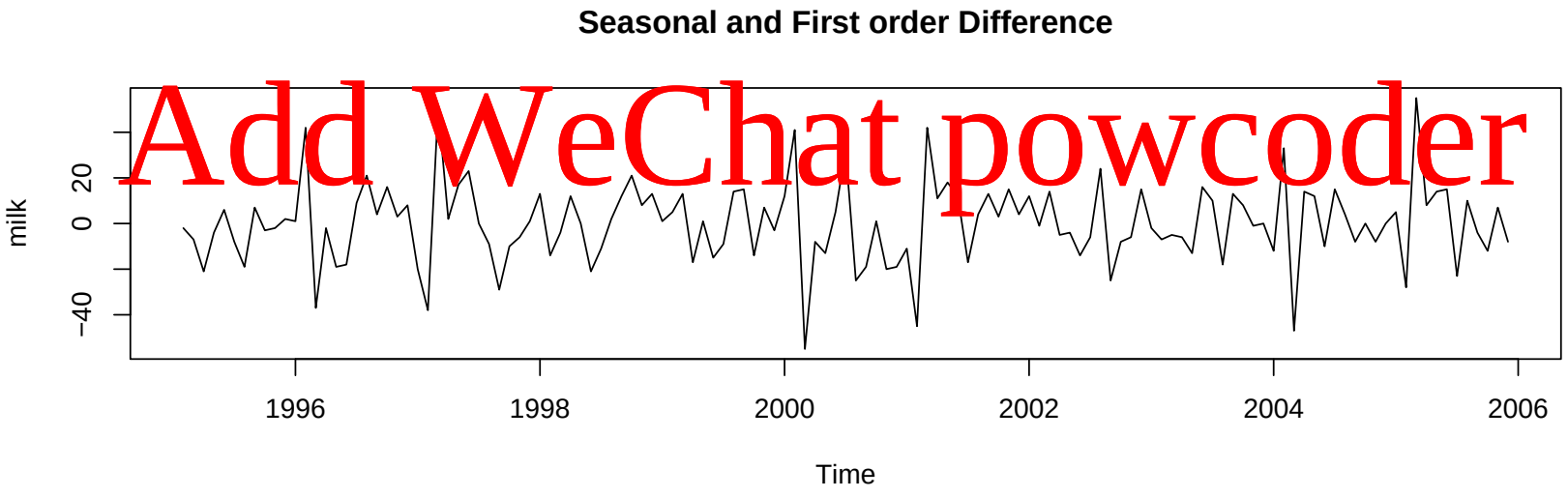
Seasonal Lag Differencing

- To remove seasonality (hopefully) we can try taking seasonal-lagged differences
  - Try both seasonal lag on the differences, and only a seasonal lag
  - Remember, we are just looking for something that looks stationary
- Seasonal-lag difference (with monthly data)

$$\nabla_{12} Y_t = Y_t - Y_{t-12}$$

Seasonal Differencing





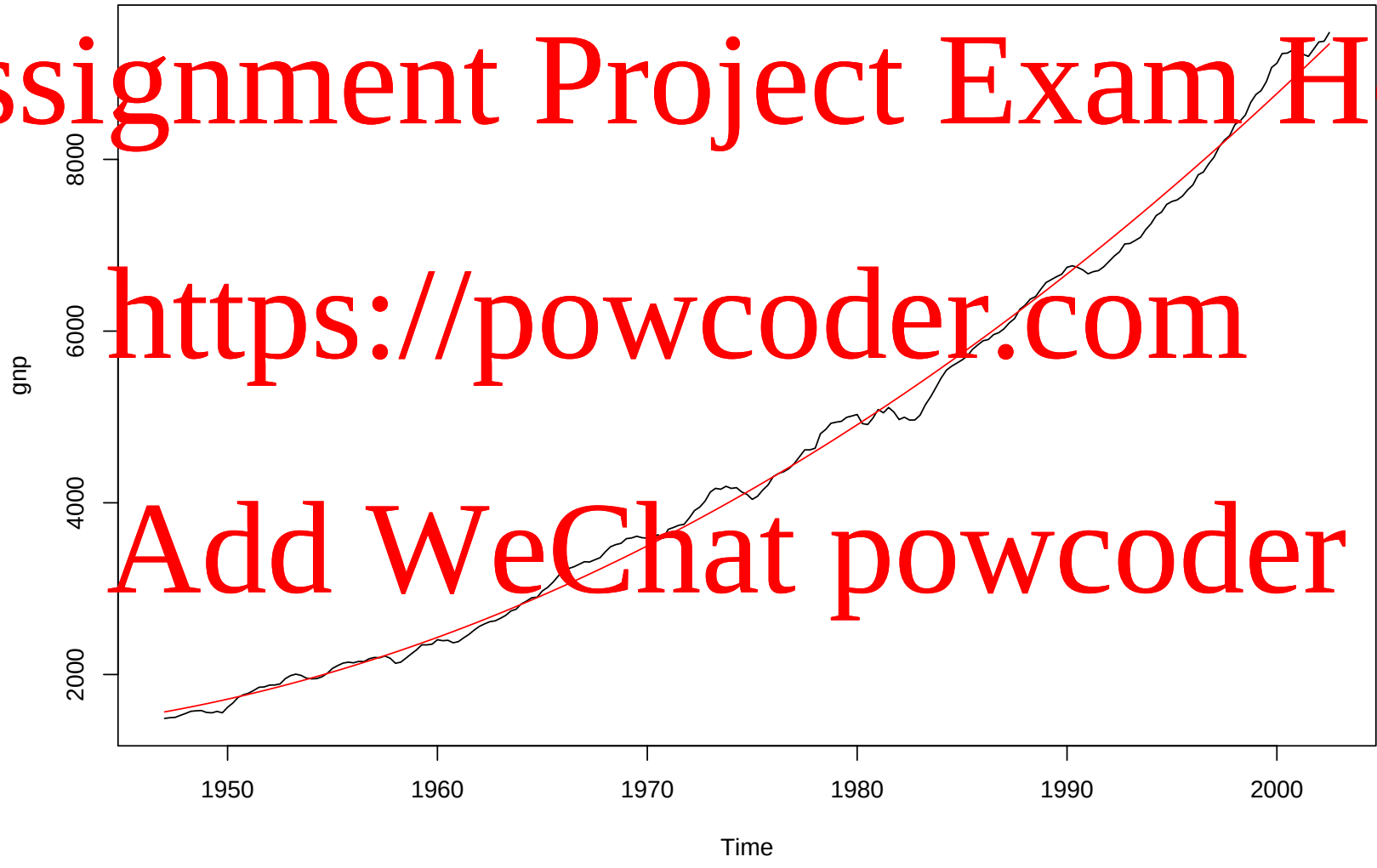
Guidance on Differencing

- Plot the ACF of the original data
  - If it tails TOO slow at recent lags, need first order difference
  - If it tails TOO slow at seasonal lags, need seasonal difference

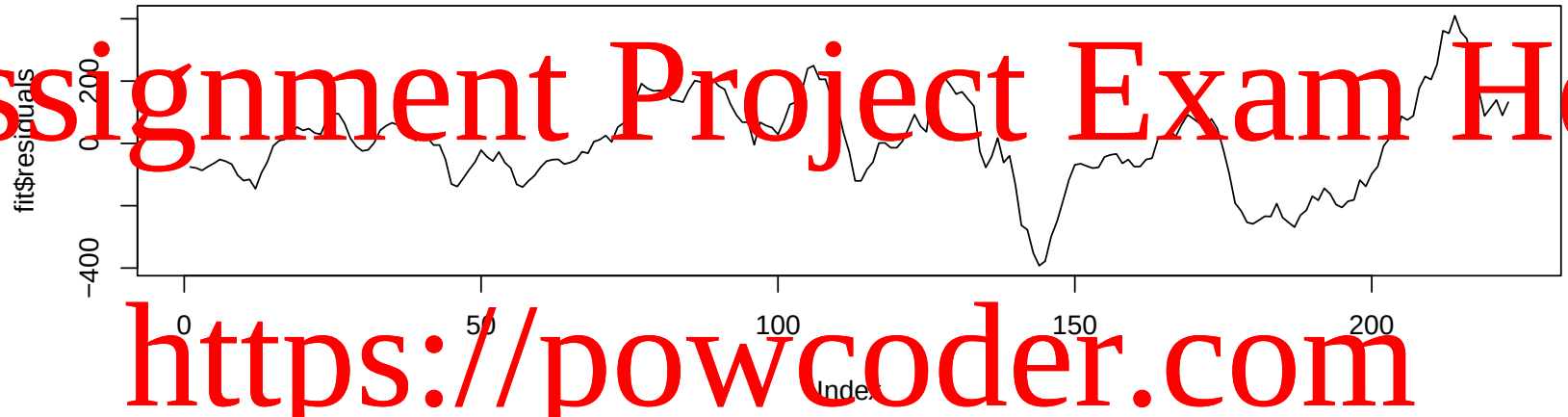
Finding Seasonal Correlation

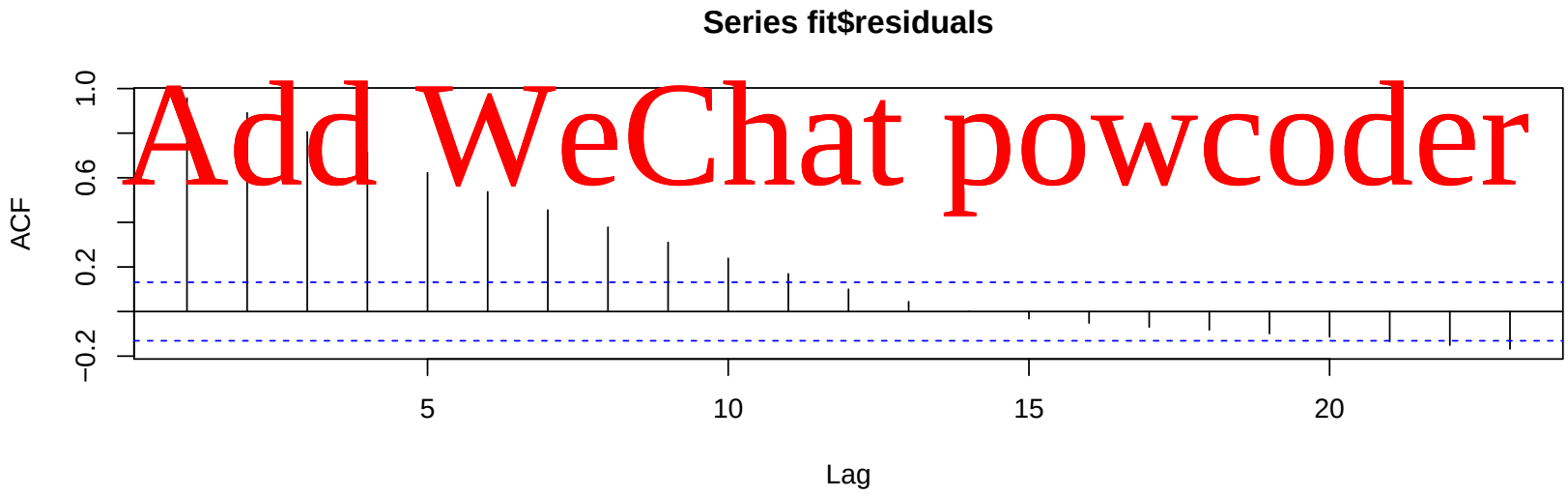
- As before, after converting our series to stationary we hope to model that process with some ARMA model
  - Unlike before, we can now incorporate seasonal ARMA terms

Detrending and Studying Residuals

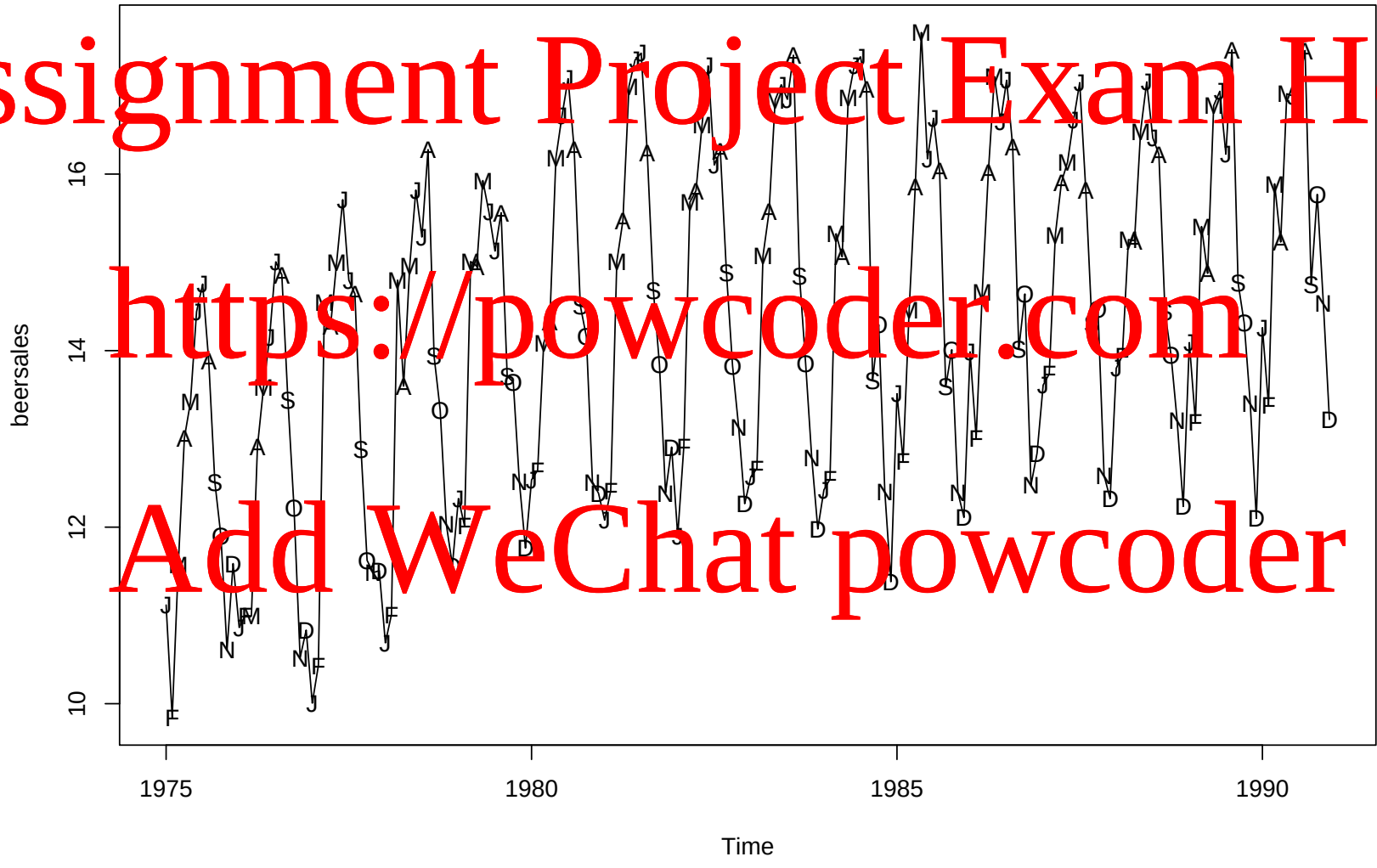


Detrending and Studying Residuals

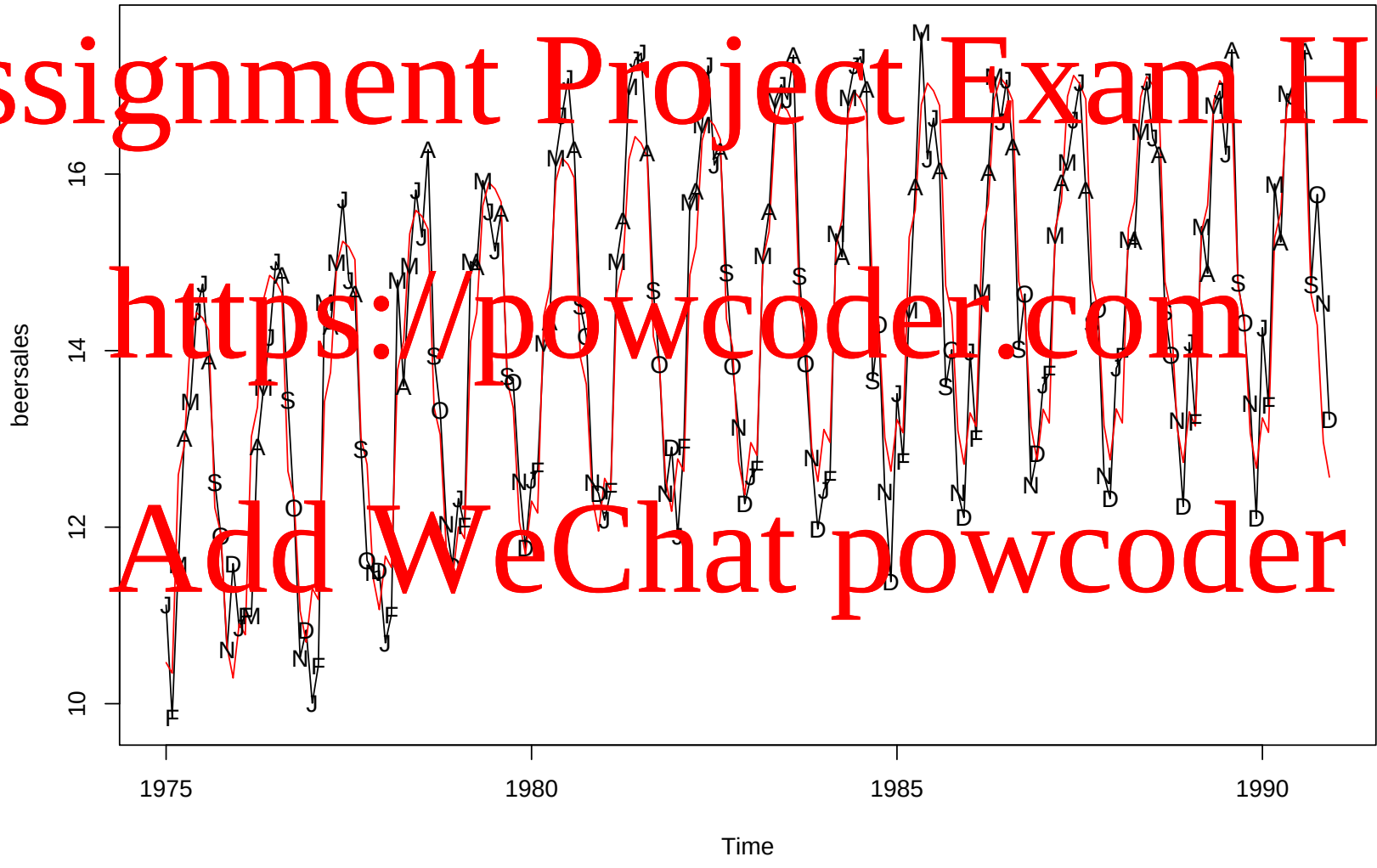




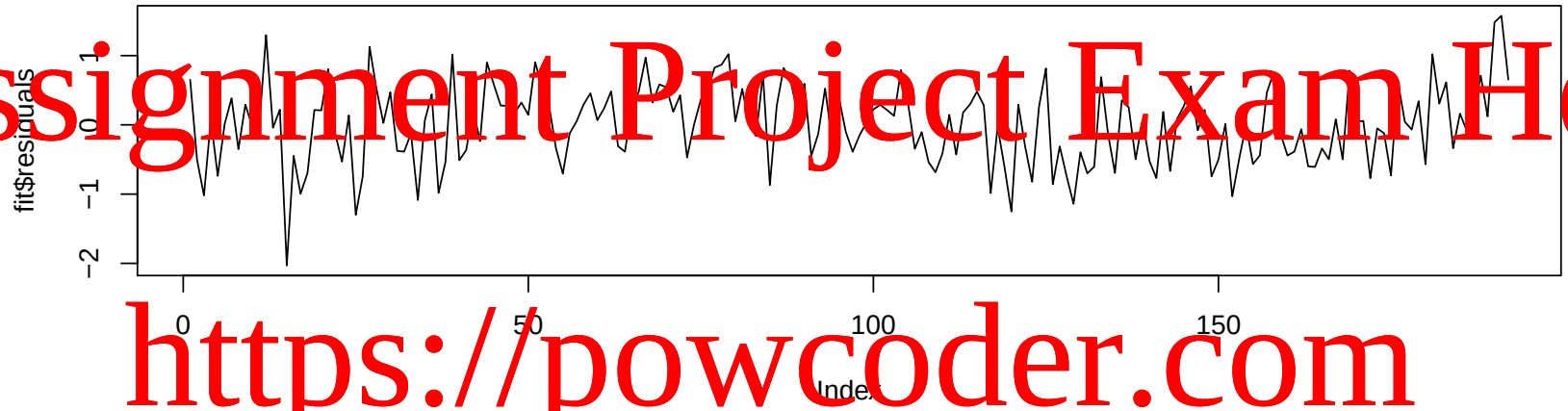
Seasonal Autocorrelations

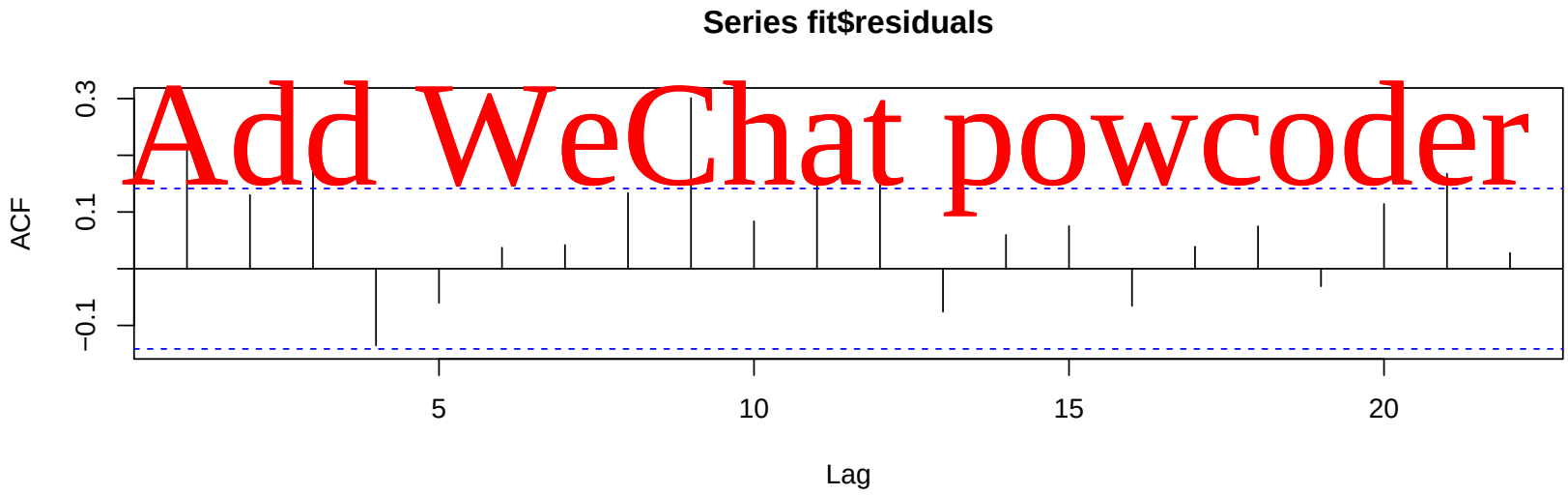


Seasonal Autocorrelations



Plot and ACF of Residuals





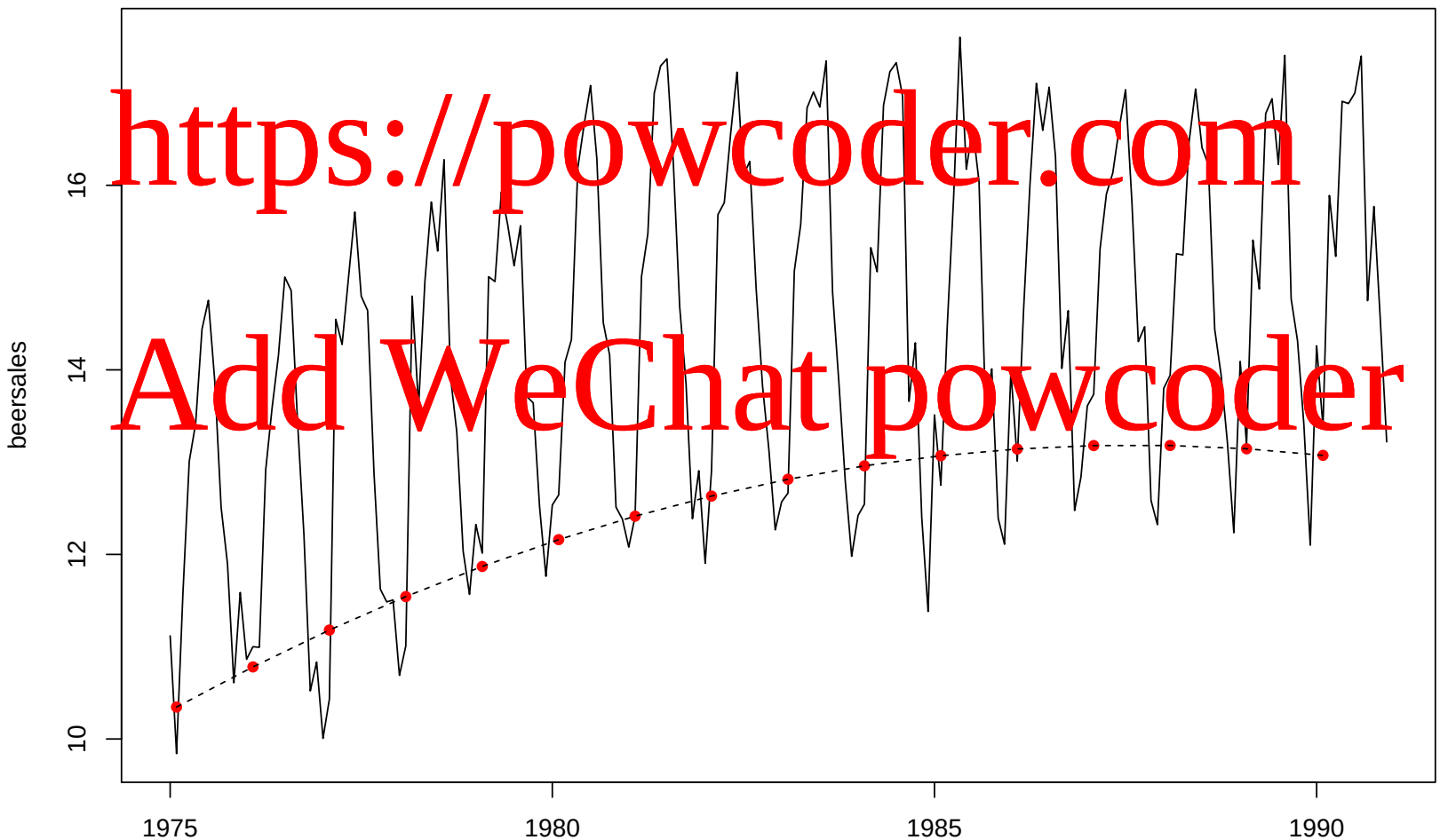
Seasonal Autocorrelations

	Estimate	Std. Error
(Intercept)	-71497.79	8791.41
t	71.96	8.87
I(t^2)	-0.02	0.00
monthFebruary	-0.16	0.21
monthMarch	2.05	0.21
monthApril	2.35	0.21
monthMay	3.54	0.21
monthJune	3.78	0.21
monthJuly	3.68	0.21
monthAugust	3.51	0.21
monthSeptember	1.46	0.21
monthOctober	1.13	0.21
monthNovember	-0.19	0.21
monthDecember	-0.58	0.21

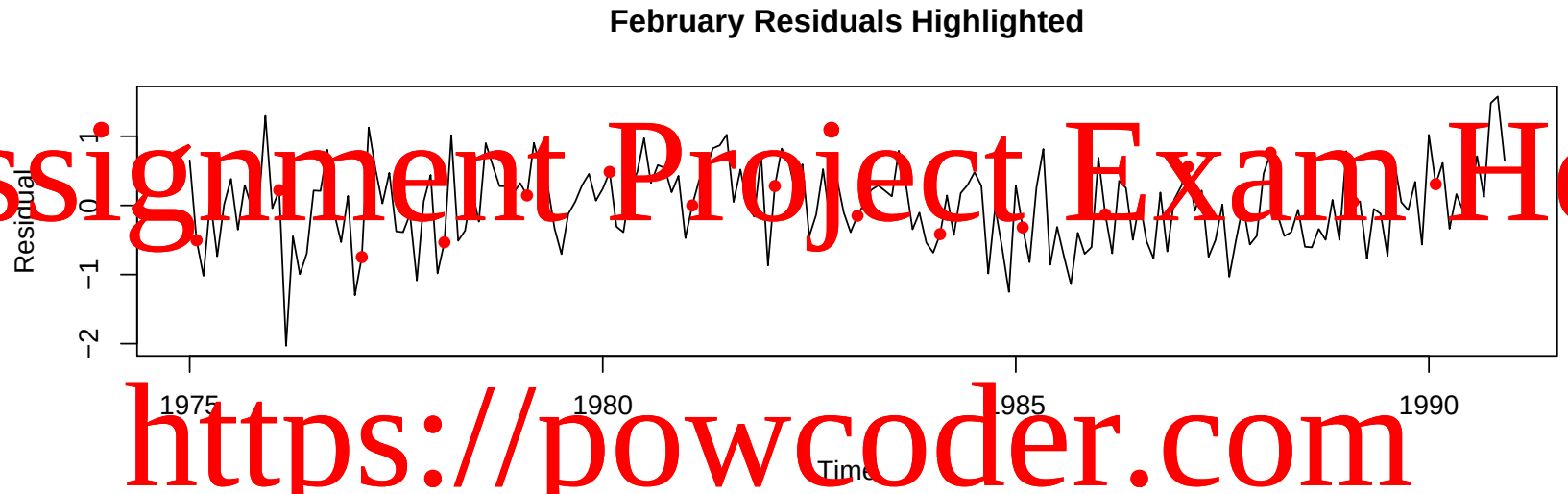
Consider February...

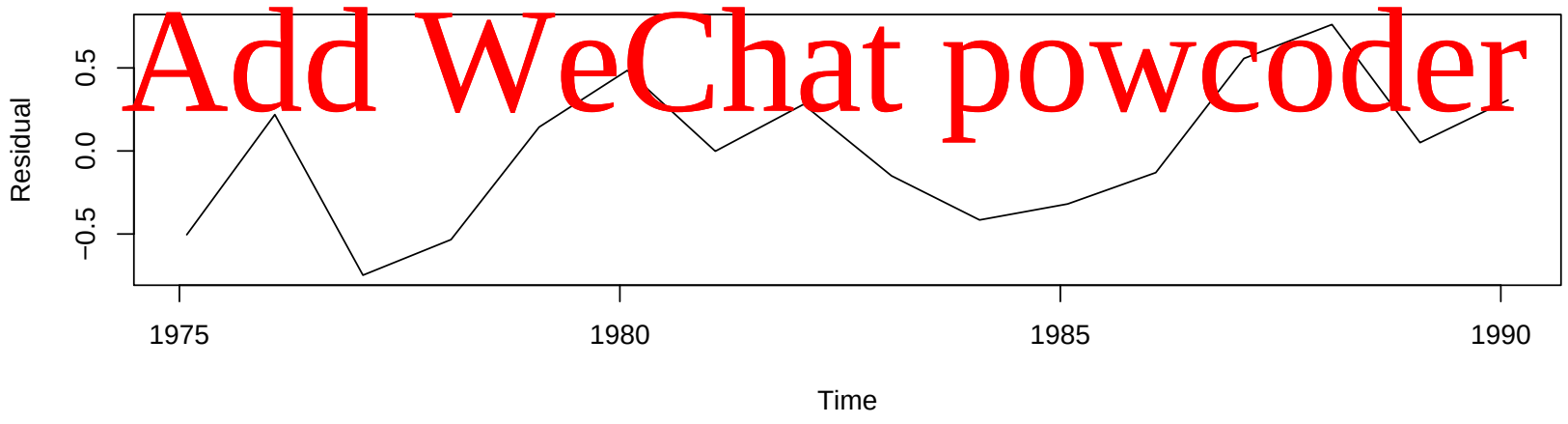
From the regression results, the estimated equation for the month of February is

$$Y_t = (-71497.79 - 0.16) + 71.96t - 0.02t^2$$



Seasonal Autocorrelation





Modeling Process

1. Plot time series
   - Observe trends, variance, seasonality, abrupt changes, outliers
2. Convert to stationary
   - 2.1 Transform for constant variance if needed
     - Box-Cox procedure for guidance
   - 2.2 Remove trend if needed
     - Differencing (lag 1, seasonal lag, both)
     - Build model, use residuals

Modeling Process

3. Investigate autocorrelations via ACF/PACF plots
   - Starting point for p, q, P, and Q
4. Fit model and diagnose
   - Normality, Independence
   - After fitting time series model, no correlation should exist in residuals
   - Residual plots, ACF of residuals, Ljung-Box p-values
5. Overfit until you're happy with model
6. Forecast
   - Always provide interval with your estimate