Forecasting

Forecasting

- Predicting future values is often the main goal of a time series analysis
- Prediction generally a more difficult problem than estimation
  - Parameters are fixed but unknown
  - Future values are random, not fixed

Minimum Mean Square Error Forecasting

- Our goal is to forecast the value Y_{t+l} using the available history of the series $Y_1, Y_2, ..., Y_t$
- We will use Minimum Mean Square Error (MMSE) Forecasting, which minimizes

$$E\left\{[Y_{t+l} - \hat{Y}_{t+l}]^2\right\}$$

- Solution:

$$\hat{Y}_{t+l} = E(Y_{t+l}|Y_1, Y_2, ..., Y_t)$$

Forecasting with Deterministic Trend Models

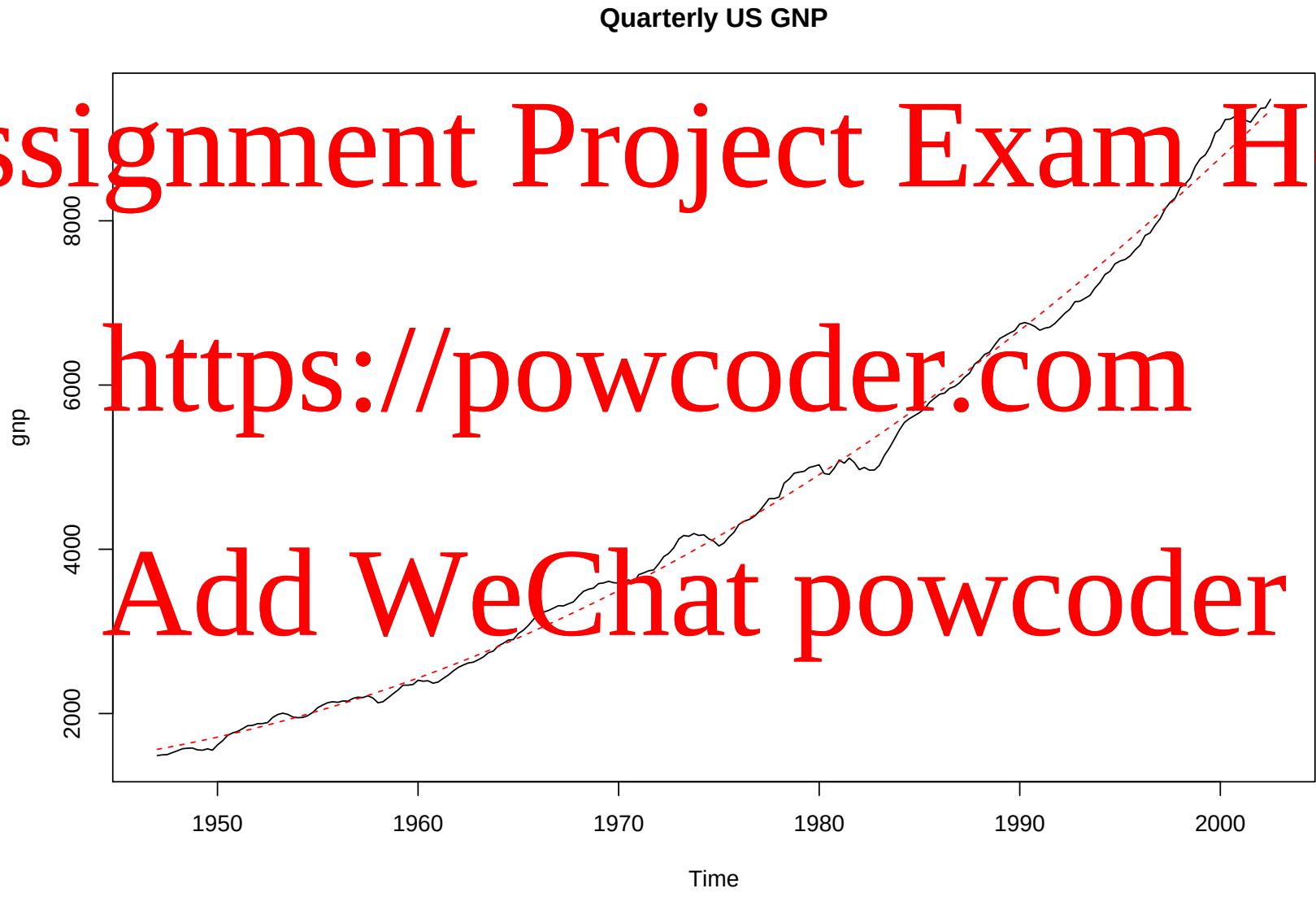
$$Y_t = \mu_t + X_t$$

- μ_t a deterministic trend
- X_t white noise with mean 0 and variance σ^2

To forecast l steps into the future...

$$\hat{Y}_{t+l} = E(Y_{t+l}|Y_1, ..., Y_t) = E(\mu_{t+l} + X_t|Y_1, ..., Y_t) = \mu_{t+l}$$

Example - GNP



Example - GNP


```
(Intercept)             t            t2
6462325.4823   -6680.9788        1.7271
```


- Forecast next quarter - Q4 2002?
- Forecast next year - Q3 2003?
- Forecast for Q1 2013?

Drawbacks

- Forecasts from deterministic trend models based only on least squares fit, ignores potential correlation.
  - Forecast for Y_{t+1} ignores correlation with $Y_1, Y_2, \cdots, Y_t$
- Assumes fitted trend is applicable indefinitely into the future, i.e. "forever trend"

Forecasting with ARIMA models

- Forecasting with ARIMA / time series models is no different than forecasting with your typical regression models
- You estimated a function, simply carry that function forward

Forecasting with ARIMA models

- Y_t is a function of past values of Y and/or past errors.
- When we forecast beyond Y_t, we might need values that haven't yet been observed.
- Consider the following AR(2) model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t$$

Forecasting with ARIMA models

In the forecast for 2 periods ahead, the formula requires Y_{t+1} which has yet to be observed. Simply plug in the forecasted value for Y_{t+1}. Note that the e_t's are gone, why?

- $\hat{Y}_{t+1} = \phi_1 Y_t + \phi_2 Y_{t-1}$
- $\hat{Y}_{t+2} = \phi_1 Y_{t+1} + \phi_2 Y_t$

Example

- Suppose that Y_t follows an AR(1) process with $\phi = 0.5$.
- Suppose we have observed $-.38$, $-.51$, and $.57$ for times 1, 2, and 3.
- What is the forecast for Y_4, Y_5 and Y_6?

Forecasting a Series with Non-zero Mean

- Suppose now that we have an AR(1) process with a non-zero mean, μ.
- Before we fit an AR model to the series, we would remove this mean (either differencing or detrending)

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + e_t$$

Forecasting a Series with Non-zero Mean

- Carrying this forward

$$Y_{t+1} - \mu = \phi(Y_t - \mu) + e_{t+1}$$

- Taking expected value (for the forecast...)

$$\begin{aligned} E(Y_{t+1} - \mu) &= E[\phi(Y_t - \mu) + e_{t+1}] \\ E(Y_{t+1}) - \mu &= \phi(Y_t - \mu) \\ \hat{Y}_{t+1} &= \mu + \phi(Y_t - \mu) \end{aligned}$$

Example

- Suppose Y_t follows an AR(1) with $\phi = .8$ and $\mu = 10$
- Suppose we have observed 9.36, 10.72, and 10.06 for times 1, 2, and 3.
- What is the forecast for Y_4 and Y_5?

Forecasting with MA Models

Suppose we have the following MA(1) model

$$Y_t = \theta e_{t-1} + e_t$$

- MA models have *no correlation* beyond lag q. What does this imply about forecasting?

Forecasting with MA Models

- Y_{t+1}

$$Y_{t+1} = \theta e_t + e_{t+1}$$

- $\hat{Y}_{t+1}$

$$\hat{Y}_{t+1} = \theta e_t + E(e_{t+1}) = \theta e_t$$

Forecasting with MA Models

- Y_{t+2}

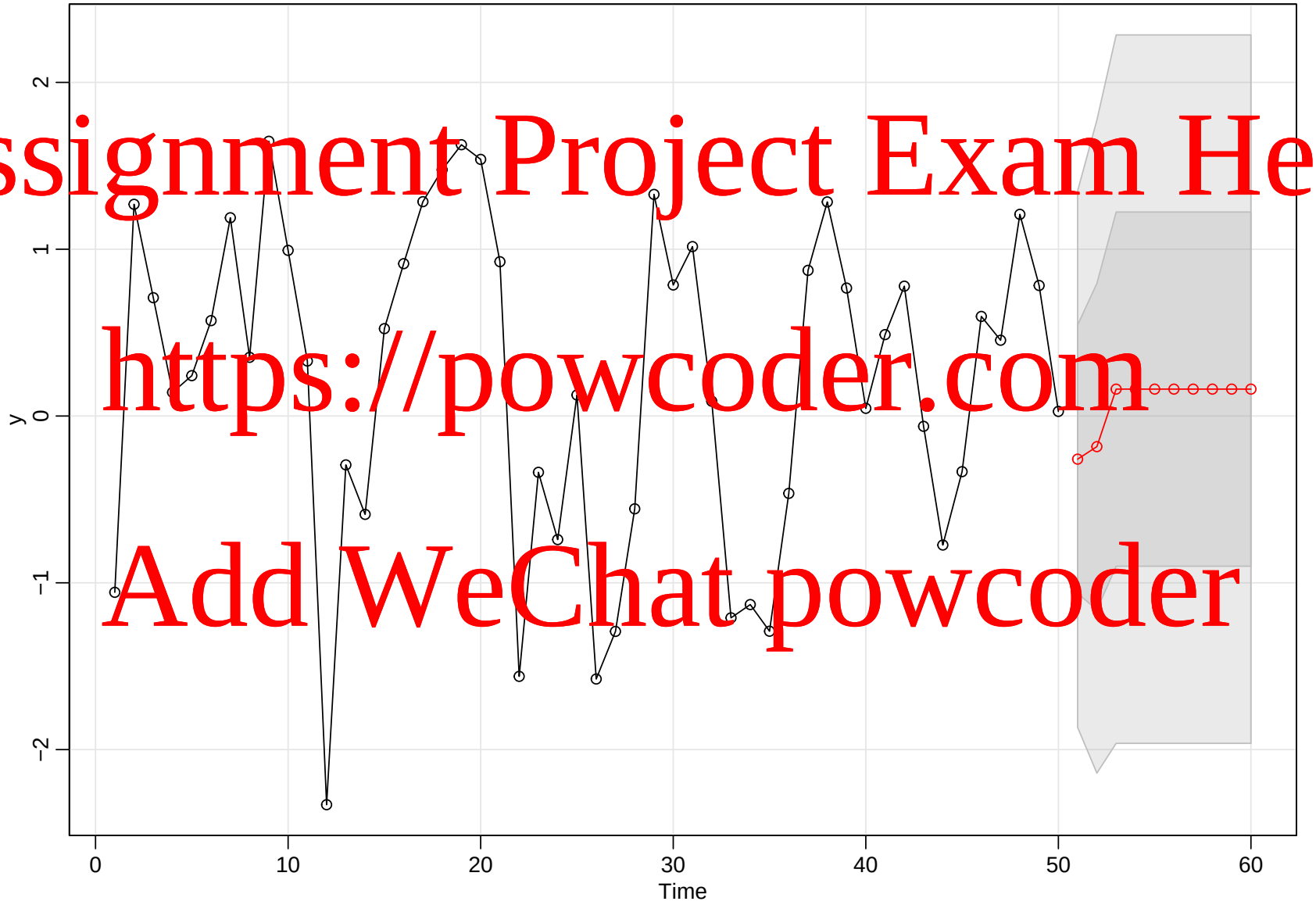
$$Y_{t+2} = \theta e_{t+1} + e_{t+2}$$

- $\hat{Y}_{t+2}$

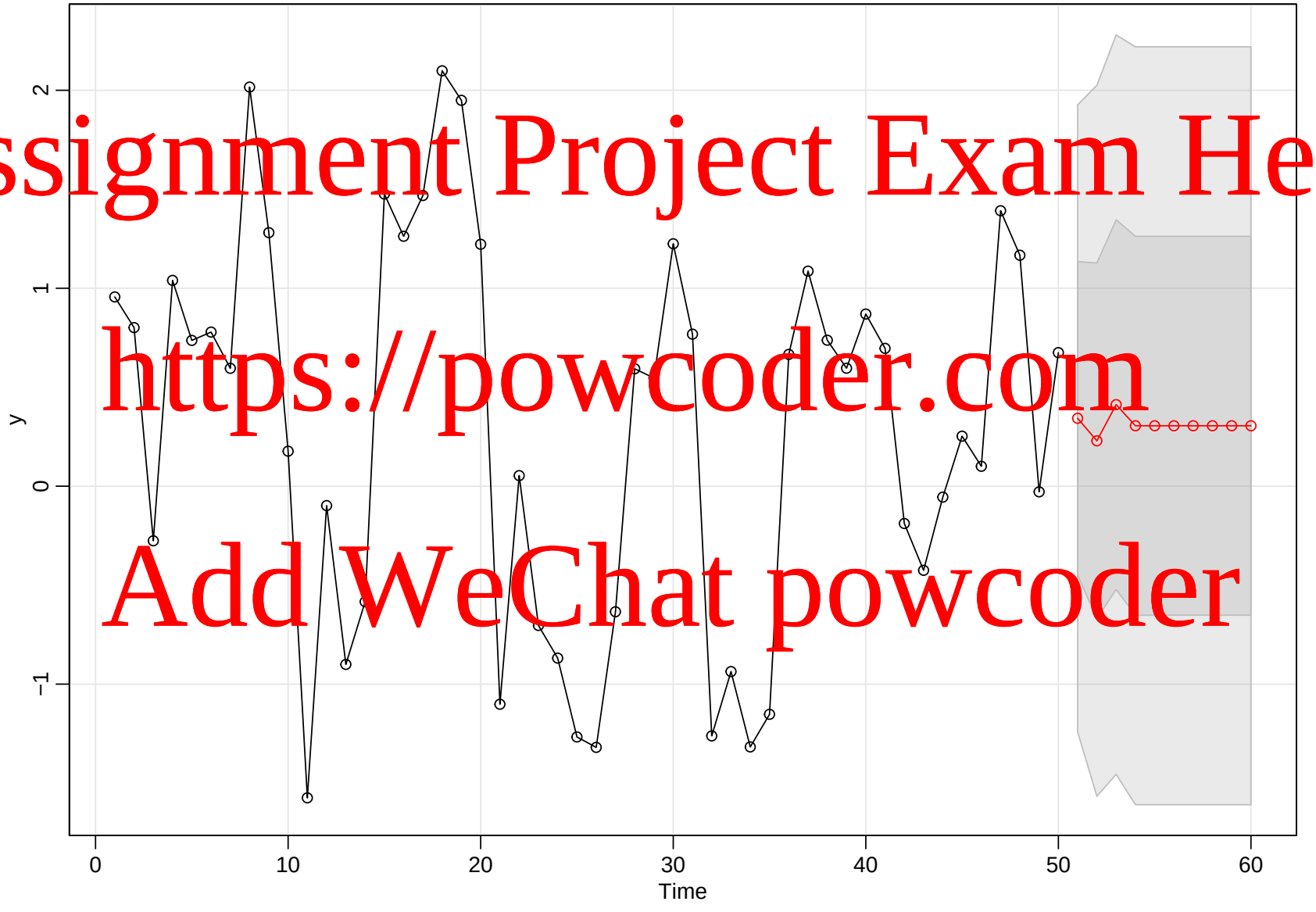
$$\hat{Y}_{t+2} = \theta E(e_{t+1}) + E(e_{t+2}) = 0$$

MA models will flatline after lag q!

Example - Forecasting an MA(2) Model



Example - Forecasting an MA(3) Model



R Code

1. Use `arima()` then `predict()`

```
m <- arima(y, order=c(0, 0, 1))
preds <- predict(m, n.ahead=10)
preds$pred
preds$se
```

2. Use `sarima.for()`

```
m <- sarima.for(y, n.ahead=10, 0, 0, 1)
m$pred
m$se
```


Can construct confidence intervals manually in the usual way if needed...

$$Prediction \pm Z_{\alpha/2} StdError$$

Simulation - Final Plot

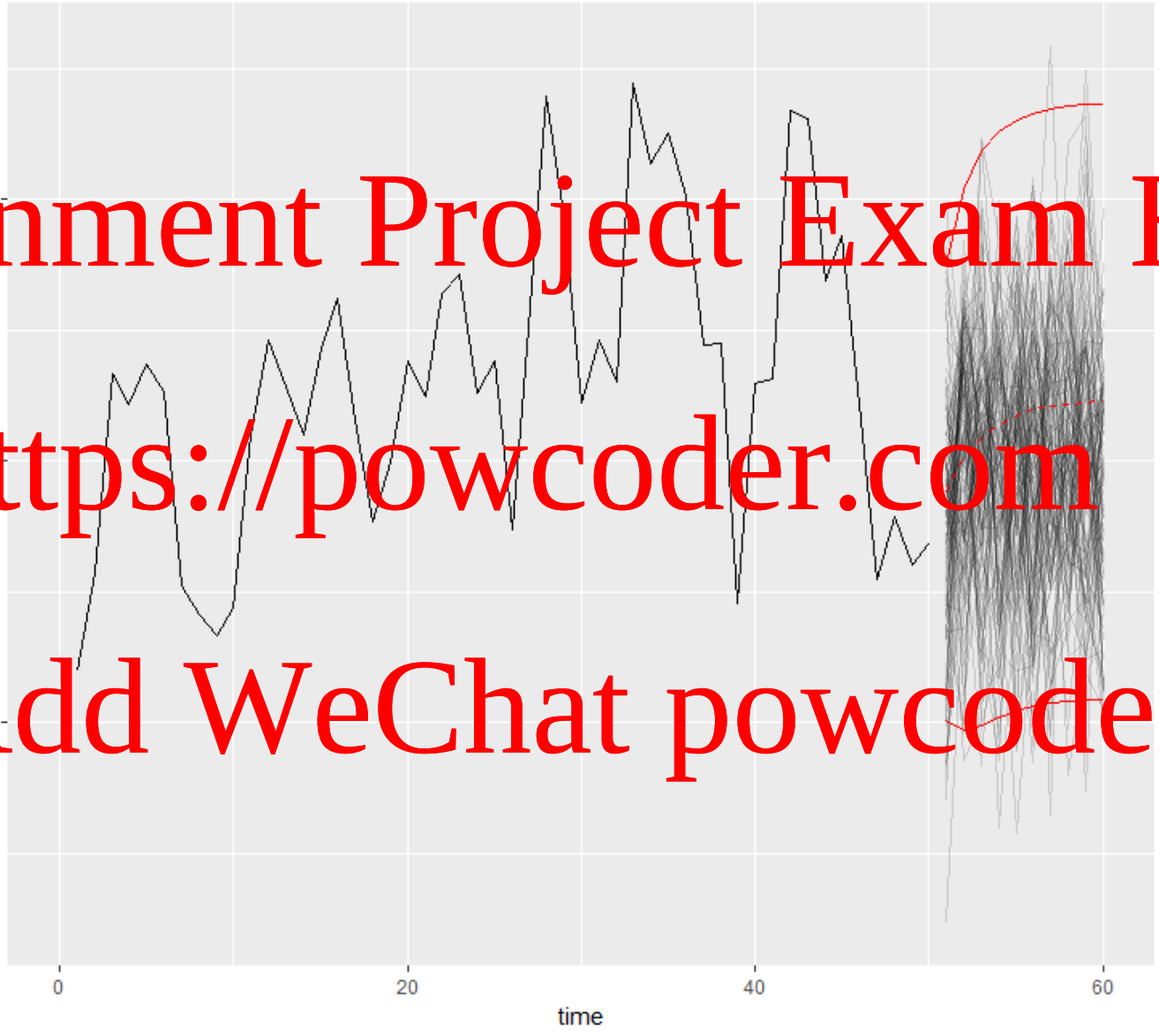


Figure 1: