MAST30001 Stochastic Modelling

Tutorial Sheet 1

1. A box has 3 drawers, one contains two gold coins, one contains two silver coins and the last drawer contains one gold coin and one silver coin. Assume that one drawer is selected randomly and that a randomly selected coin from that drawer turns out to be gold. What is the probability that the chosen drawer is the one that contains the two gold coins?

2. Let $(A_i)_{i\in\mathbb{Z}_+}$ be events, and suppose that $\mathbb{P}(\cap_{i=0}^{n-1} A_i) > 0$. Show that

$$\mathbb{P}(\cap_{i=0}^{n} A_i) = \mathbb{P}(A_0)\prod_{m=1}^{n}\mathbb{P}(A_m | \cap_{i=0}^{m-1} A_i).$$

3. A game involves rolling a ball around a table, starting from a fixed position. Three players, Alex, Bobby and Célia, take turns attempting to grab the ball as it passes them. If Alex is unsuccessful then Bobby attempts to grab the ball and if he is also unsuccessful then Célia attempts to grab the ball. If they are all unsuccessful, the ball is rolled again from the starting position. The game stops as soon as any player is successful (and that player wins!). Suppose that each player is able to grab the passing ball with probability $p \in (0,1)$, independent of previous attempts.

   (a) What is the distribution of the number of grabbing attempts until the game ends?

   (b) Find the probability that all 3 players fail to grab the ball on their first attempt.

   (c) In one game, what is the distribution of the number of times that the ball is rolled from the starting position?

   (d) Find the probability that Alex wins the game.

   (e) Find the probability that Bobby wins the game.

   (f) Find the probability that Célia wins the game.

   (g) If this game is played 3 times, what is the distribution of the number of games that Alex wins?

4. Let $X_1, X_2, \ldots$ be independent random variables with $X_i \sim$geometric(p_i) (each taking values in $\mathbb{N}$).

   (a) Show that $\mathbb{P}(X_1 > n + m | X_1 > m) = \mathbb{P}(X_1 > n)$ (for $n, m \in \mathbb{N}$).

   (b) Find the distribution of $Y_n = \min_{i\le n} X_i$.

   (c) Find the probability mass function of the random vector (Y_n, Y_{n+1}).

   (d) Find $\mathbb{E}[Y_{n+1} | Y_n = m]$.