MAST30001 Stochastic Modelling

Tutorial Sheet 6

You've probably already seen/done some of these before, but it's useful to do them yourselves/see them again!

1. Let $X \sim$Exponential(λ), with $\lambda > 0$. Prove that $\mathbb{P}(X > s + t | X > t) = \mathbb{P}(X > s)$ for every $s, t > 0$.

2. Let $(X_i)_{i \in \mathbb{N}}$ be independent random variables with $X_i \sim$Exponential(λ_i). Find the distribution of $Y_n = \min_{i \leq n} X_i$.

3. Let $(T_i)_{i \in \mathbb{N}}$ be i.i.d. Exponential(λ) random variables, and let N be a Geometric(p) random variable that is independent of the other variables.

   (a) Find the moment generating function $\mathbb{E}[e^{tT_1}]$ of T_1.

   (b) Let $Y = \sum_{i=1}^{N} X_i$. Find the distribution of Y.

4. Let $X \geq 0$ be a random variable satisfying

   $(\star)$ $\quad \mathbb{P}(X > s + t | X > t) = \mathbb{P}(X > s)$, $\quad$ for all $s, t \geq 0$.

   Show that $X \sim$Exponential(λ), for some $\lambda \geq 0$.

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