MAST30001 Stochastic Modelling

Tutorial Sheet 4

1. Let $p \in (0,1)$ and suppose that a time homogenous Markov chain on $\mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}$ has transition probabilities given by

$$p_{i,i+2} = p,\ p_{i,i-1} = 1-p \text{ and } p_{i,j} = 0 \text{ for all } i \in \mathbb{Z},\ j \neq i+2, i-1.$$

   (a) Using Stirling's approximation, (for $n \geq 1$)

$$1 \leq \frac{n! e^n}{n^n \sqrt{2\pi n}} \leq 2,$$

   determine for what values of p the chain is recurrent.

   (b) Compute the probability of reaching state -1, starting in state 0.

2. Consider a Markov chain with state space $\mathcal{S} = \{1,2,\dots,6\}$ and transition matrix given by

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

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   (a) Find all stationary distributions for this chain

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   (b) Find the probability that we ever reach state 1, starting from state 3

   (c) Find the expected time that we first reach the set of states $A = \{2,5\}$, starting from state 3

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   (d) Starting from state 3, find the limiting proportion of time spent in state 1.

   (e) Starting from state 3, find the expected limiting proportion of time spent in state 1.

   (f) Repeat the above two questions, starting from state 4 instead of state 3.

3. Assume that we have an infinite supply of a certain electrical component and that the lifetimes (measured from the beginning of use) of the components are i.i.d. $\mathbb{N}$-valued random variables (so time is measured in discrete units) with

$$\mathbb{P}(T_i = k) = q_k > 0, \quad k = 1,2,\dots,$$

   and $\mathbb{E}[T_1] < \infty$.

   At time $n = 0$, the first component begins use and when it fails (at time T_1), it is immediately replaced by the second component, and when it fails (at time $T_1 + T_2$), it is replaced, and so on. Let X_n be the age of the component in use at time n and say $X_n = 0$ at times n where there is a failure.

   (a) Show that X_n is a Markov chain and write down its transition probabilities.

   (b) Explain why this chain is positive recurrent.

   (c) Find all equilibrium distributions for this Markov chain.