MAST30001 Stochastic Modelling

Tutorial Sheet 9

1. Show that in in $M/M/1$ queue with arrival rate λ and service rate $\mu > \lambda$, the expected lengths of the idle and busy periods are $1/\lambda$ and $1/(\mu - \lambda)$, respectively. *[Hint: the proportion of time the server is idle is equal to the stationary chance the system is empty.]*

2. A rental car washing facility can wash one car at a time. Cars arrive to be washed according to a Poisson process with rate 3 per day and the service time to wash a car is exponential with mean 7/24 days. It costs the company \$150 per day to operate the facility and the company loses \$10 per day for each car tied up in the washing facility. The company can upgrade the facility to get down to a mean service time of 1/4 days at the cost of \$$C$ per day. What's the largest C can be for this upgrade to make economic sense?

3. (M/G/∞ queue) In a certain communications system, information packets arrive according to a Poisson process with rate λ per second and each packet is processed in one second with probability p and in two seconds with probability $1-p$, independent of the arrival times and other service times. Let N_t be the number of packets that have entered the system up to time t and X_t be the number of packets in the system (including those being served) at time t.

   (a) Is $(X_t)_{t\geq 0}$ a Markov chain? (No detailed argument is necessary here, just think about it heuristically.)

   (b) If $X_0 = 0$, what is the distribution of X_2?

   (c) If $X_0 = 0$, is there a "stationary" limiting distribution $\pi_k = \lim_{t\to\infty} P(X_t = k)$? If so, what is it?

   (d) If $X_0 = N_0 = 0$, what is the joint distribution of X_t and N_t?