MAST30001 Stochastic Modelling

Tutorial Sheet 7

1. Let $(N_t)_{t\geq 0}$ be a Poisson process with rate λ and let $0 < T_1 < T_2 < \cdots$ be the times of "arrivals" or jumps of $(N_t)_{t\geq 0}$. Compute:

   (a) $\mathbb{P}(N_3 \leq 2, N_1 = 1)$,

   (b) $\mathbb{P}(N_3 \leq 2, N_1 \leq 1)$,

   (c) $\mathbb{P}(N_2 = 2, N_1 = 2, N_{1/2} = 0)$,

   (d) $\mathbb{P}(N_7 - N_3 = 2 | N_5 - N_2 = 2)$,

   (e) the (joint) distribution function of (T_1, T_2),

   (f) the joint density of (T_1, T_2),

   (g) the distribution of $T_1 | \{T_2 = t_2\}$.

2. Yeast microbes from the air outside of a culture float by according to a Poisson process with rate 2 per minute. Each microbe that floats by joins the population of the culture with probability p and with probability $1 - p$ the microbe doesn't join the culture, and this choice is made independent from the times of arrival and choice to join of all other microbes.

   (a) Find the probability that exactly four outside microbes float by in the first 3 minutes.

   (b) Find the probability that exactly four outside microbes join the culture in the first 3 minutes.

   (c) Given that 7 outside microbes have floated by the culture in first 3 minutes, what is the probability that at least two of the seven join the culture?

   (d) Given that 7 outside microbes have floated by the culture in first 3 minutes, what is the probability that exactly 3 float by in the first 1 minute?

   (e) What is the probability that in the first 3 minutes, exactly four microbes join the culture and 3 float by that don't join the culture?

   Assume now that a second strain of yeast microbes independently float by the culture according to a Poisson process with rate 1, and each microbe joins the culture with probability q, analogous to the previous process.

   (f) What is the probability that exactly four yeast microbes (from either strain) float by in the first 3 minutes?

   (g) What is the probability that exactly four yeast microbes (from either strain) join the culture in the first 3 minutes?

3. Let $U_{(1)}, \ldots, U_{(n)}$ be order statistics of independent variables, uniform on the interval $(0, 1)$. For $0 < x < y < 1$ find:

   (a) $\mathbb{P}(U_{(1)} > x, U_{(n)} < y)$,

   (b) $\mathbb{P}(U_{(1)} < x, U_{(n)} < y)$,

   (c) $\mathbb{P}(U_{(k)} < x, U_{(k+1)} > y)$.