The University of Melbourne — School of Mathematics and Statistics
MAST30012 Discrete Mathematics — Semester 2, 2021

Practice Class 4: Pigeon Holes and Ramsey Numbers – Answers

Q1: (a) Taking into account the number of days in each month and the fact that 28 days equals 4 weeks we find that the 13th day of each month corresponds to days of the week as

$$\text{Jan} \leftrightarrow 0, \text{Feb} \leftrightarrow 3, \text{Mar} \leftrightarrow 3, \text{Apr} \leftrightarrow 6, \text{May} \leftrightarrow 1, \text{Jun} \leftrightarrow 4,$$
$$\text{Jul} \leftrightarrow 6, \text{Aug} \leftrightarrow 2, \text{Sep} \leftrightarrow 5, \text{Oct} \leftrightarrow 0, \text{Nov} \leftrightarrow 3, \text{Dec} \leftrightarrow 5.$$

(b) There is at least one and no more than three months with the 13th day any given type. So there must be at least one, and no more than 3, Friday the 13ths in any one year.

Q2: In each case there is n people and $n-1$ possible number of friends.

Q3: Divide the circle into 6 sectors of equal size and choose one of the boundaries to pass through one the points. Each sector contains an equilateral triangle and it follows that two points inside a sector are at most a unit distance apart. Then apply the PHP.

Q4: (a) Number of games: $\binom{10}{2} = 45$.

(b) Number of draws is at least $45 \times 0.7 = 31.5$ so not less than 32.

(c) By the generalised pigeonhole principle there are at least 5 players with a positive score or at least 5 players with a negative score.

(d) Suppose at least 5 players have positive scores which are all different. Then there must be a minimum of $1+2+3+4+5 = 15$ wins. This contradicts the result from (ii).

Q5: $R(a,b)$ is the minimum number of points, which when connected with red or blue lines, will always produce a complete graph K_a in red or a complete graph K_b in blue.

Q6: Identify sending a message about relationship gossip as 'friend' and sending a message about what's on next week as 'stranger' and use $R(3,3) = 6$.

Q7: (a) After singling out person 1 there are 9 people to distribute too the pigeonholes 'friend of 1' or 'stranger to 1'. Hence one or the other contains at least 5 people.

(b) Suppose 5 or more mutual strangers to person 1. If 4 or more of these are mutual friends we are done. Otherwise there must be at least one pair of mutual strangers. Combining such a pair with person 1 produces a group of 3 mutual strangers.

(c) The above argument still holds with 4 mutual strangers to person 1.

(d) There are 6 or more mutual friends of person 1. In any group of 6 people there are at least 3 mutual friends or 3 mutual strangers. In the latter case we are done, while in the former case combining with person 1 we have a group of 4 mutual friends.

Q8: (a) K_9 consists of 9 points (vertices) each connected to all the other points by an edge.

(b) The number of edges is $\binom{9}{2} = 36$.

(c) The number of possible colourings is 2^{36}.

(d) We know that $R(3,4) = 9$. This means that in any two-colouring of K_9 there must be a K_3 in one of the colours or a K_4 in the other colour.

It is not true for K_8.