School of Mathematics and Statistics

MAST30012 Discrete Mathematics 2021

Assignment 3

Due 23:59pm Monday 18 October 2021

Student Name	*Student Number*
Tutor's Name	*Practice Class Day/Time*

Submit your assignment online in Canvas LMS.

Please attach this cover sheet to your assignment or use a blank sheet of paper as the first page of your assignment with Student Name, Student Number, Tutors Name, Practice Class Day/Time clearly stated.

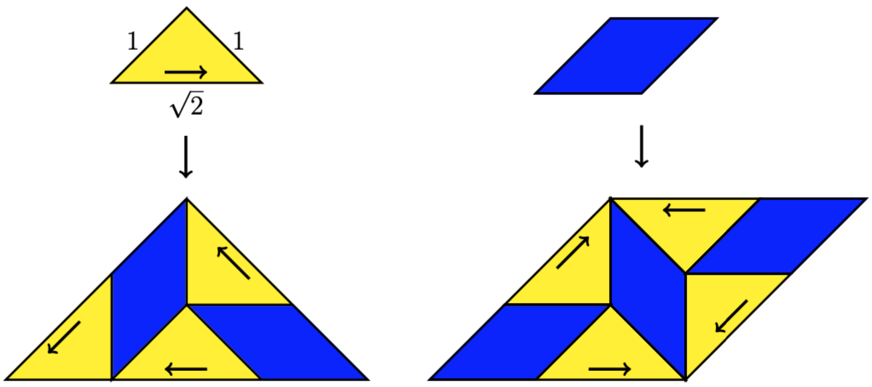
- Late submission will not be accepted unless accompanied by a medical certificate (or a similar special consideration). If there are extenuating circumstances you need to contact your lecturer, preferably prior to the submission deadline. Medical certificates are usually required.

- Information on how to submit assignments can be found in the Canvas LMS.
- Full working must be shown in your solutions.

- Marks will be deducted for incomplete working, insufficient justification or incorrect notation.
- Unless otherwise stated, proofs of identities etc. must use combinatorial arguments.

- There are 3 problems (on three pages) each worth 10 marks.

Q1: The Ammann-Beenker tiling can be specified by the substitution rules:



(a) Further specify the tiling by giving the side lengths and angles of the rhombus and the factor by which the tiling is expanded in each iteration.

(b) Why does the triangle require an arrow?

(c) Let R_n denote the number of rhombi at step n, and let T_n denote the number of triangles at step n. Give equations that express (T_{n+1}, R_{n+1}) in terms of (T_n, R_n) and write these equations in matrix form.

(d) Using your results in (c), calculate the total number of tiles after 2 applications of the substitution rules, starting with a single triangle.

(e) The problem of enumerating the number of rhombi and triangles can be viewed as counting the number of walks on a weighted directed pseudo-graph G. Draw this pseudo-graph G.

(f) Use a theorem from lectures to find the generating function $S(x)$ for the total number of tiles $S_n = T_n + R_n$ after n iterations when we start with a single triangle.

Q2: You are asked to distribute n apples and n oranges to n children such that each child receives two pieces of fruit. Let a_n be the number of ways the fruit can be distributed.

(a) Prove that

$$a_n = [x^n](1 + x + x^2)^n.$$

(b) Using (a), prove that

$$a_n = \sum_{k=0}^{n} \binom{n}{k}\binom{n-k}{k} = \sum_{k=0}^{n} \binom{n}{k}\binom{k}{n-k},$$

where we use the fact that $\binom{a}{b} = 0$ when $a < b$.

(c) Show that there is a bijection from the set of ways of fruit distribution to the set of lattice paths from $(0, 0)$ to $(n, 0)$ with step set $S = \{(1, 1), (1, -1), (1, 0)\}$.

(d) Show that the generating function $G(x) = \sum_{n=0}^{\infty} a_n x^n$ is given by

$$G(x) = \frac{1}{\sqrt{1 - 2x - 3x^2}}.$$

You may wish to consider the problem with no $(1, 0)$ steps first and then make use of results from Practice Classes and/or Lectures.

Q3: Consider the following permutations:

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 5 & 8 & 1 & 4 & 2 & 6 \end{pmatrix}$$

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 7 & 1 & 6 & 3 & 8 & 2 & 4 \end{pmatrix}$$

(a) What is the relationship between τ and σ?

(b) Write τ in standard cycle format.

(c) What is the parity of τ?

(d) What is the smallest value of n such that τ^n is equal to the identity permutation σ_{I}?

(e) Write τ as a product of transpositions.

(f) Represent σ as a bipartite graph.

(g) How is the number of inversions related to this bipartite graph? How many inversions does σ have?

(h) Write down the set of inversions, I_σ, of σ.