School of Mathematics and Statistics

MAST30012 Discrete Mathematics

Answers to the 2019 Exam

Q1: (a) (i) 2^{19}

(ii) $\binom{19}{2}$

(iii) $\sum_{k=3}^{10} \binom{19}{k}$

(b) $\binom{6}{2} = 15$

(c) 5^8

Q2: (a) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

(b) $|M| = 14$, $|G| = 11$, $|P| = 11$, $|M \cap G| = 6$, $|M \cap P| = 5$, $|G \cap P| = 5$, $|M \cap G \cap P| = 3$

(i) $|M \cup G \cup P| = 14 + 11 + 11 - 6 - 5 - 5 + 3 = 23$

(ii) $|(M \cap P) \cup (M \cap G) \cup (P \cap G)| = |M \cap P| + |M \cap G| + |P \cap G| - |M \cap P \cap G| - |M \cap P \cap G| - |M \cap P \cap G| + |M \cap P \cap G| = 6 + 5 + 5 - 2 \cdot 3 = 10$

(iii) 13

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Q3: (a) Each path bijects to a binary word in $\{E, N\}^*$ of length $n + m$. Complete the proof by yourself.

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(b) $\binom{2n+m}{2n}\binom{3n+3m}{3m}$

(c) See the lecture notes.

(d) Note that you are required to give a bijective proof. Complete the proof by yourself.

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Q4: (a) See the lecture notes.

(b) No.

(c) Complete by yourself.

Q5: (a) Recurrence relation: $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$. Using this and induction on n, prove the binomial theorem by yourself.

(b) Set $x = -1, y = 2$ in (a) and evaluate LHS and RHS.

Q6: (a) $a_2 = 2$, $a_3 = -10$

(b) Use the given recurrence relation and initial conditions to work out a functional equation for $G(x)$. Solve this equation to obtain the required expression of $G(x)$. You will see that $P(x)$ is a linear function of x. Complete the proof by yourself.

(c) $a_n = 2 \cdot 3^n - 4^n$

Q7: (a) (i) $\frac{1}{10}$

(ii) $\frac{1}{5}$

(b) Draw all these by yourself.

Q8: (a) (i) (1 5 4 3)

(ii) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 4 & 6 & 3 & 1 \end{bmatrix}$

(iii) $\sigma^{-1} = (1\ 3\ 4\ 5)$

(iv) See the lecture notes for the definition of an inversion.

$$I_\sigma = \{(1,2), (1,3), (1,4), (1,5), (2,3)\}$$

(v) $\text{sign}(\sigma) = (-1)^{|I_\sigma|} = (-1)^5 = -1$

(b) See the lecture notes for the definition of $S_1(n,k)$. Note that you are required to give a bijective proof of the identity. For this purpose, you need to define two sets whose cardinalities are given by LHS and RHS, respectively, and establish a bijection between them. Complete the proof by yourself.

Q9: (a) (i) (1 3 2)(2 4 5)

(ii) See the lecture notes for these basic algebraic relations.

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$$s_3s_1s_2s_1s_4s_2 = s_3s_2s_1s_2s_4s_2 = s_3s_2s_1s_4s_2s_2 = s_3s_2s_1s_4$$

(b) The two board positions can be related by sliding moves as their corresponding permutations have the same parity (odd).

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Q10: (a) Draw these link diagrams by yourself.

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(b) Consider the last node and the link connected to it. Complete the proof by yourself.

(c) Complete the proof by yourself.