The University of Melbourne — School of Mathematics and Statistics
MAST30012 Discrete Mathematics — Semester 2, 2021

Practice Class 3: Multinomials and Inclusion-Exclusion – Answers

Q1: (a) $\binom{7}{3} = \frac{7!}{3!\,4!} = 7 \cdot 5 = 35.$

(b) See lectures.

(c) $\binom{10}{3,3,1,2} = \frac{10!}{4!\,3!\,1!\,2!} = 12600.$

(d) The multinomial coefficient $\binom{n}{k_1,k_2,n-k_1-k_2}$ counts the number of ways to choose n elements such that k_1 goes into box 1 (labelled s) k_2 goes into box 2 (labelled t) and the remaining $n-k_1-k_2$ elements goes into box 3 (labelled 1).

Q2: (a) Make a bijection such that an element x_j from the set is associated with box j and place a block into box j each time the element x_j is selected.

(b) $\binom{8+4-1}{4} = \frac{11!}{4!\,7!} = \frac{11\times 10\times 9\times 8}{4!} = 11 \times 10 \times 3 = 330.$

Q3: (a) Total $= 11$

(b) Latin only $= 1$.

(c) Russian only $= 3$.

Q4: (a) $\binom{6}{2,2,2} = \frac{6!}{2!2!2!} = 90.$

(b)

(i) # (arrangements with one pair always in succession) $= \binom{5}{2,2,1} = \frac{5!}{2!2!1!} = 30.$

(ii) # (arrangements with two pairs always in succession) $= \binom{4}{2,1,1} = \frac{4!}{2!1!1!} = 12.$

iii) # (arrangements with three pairs always in succession) $= \binom{3}{1,1,1} = \frac{3!}{1!1!1!} = 6.$

(c) #(arrangements with a no pairs of socks in succession) $= 30$.

Q5: (a) $|P_k| = 2! = 2.$

(b) $P_j \cap P_k$ is the set of permutations where both j and k are fixed, so $|P_j \cap P_k| = 1$.

(c) $|P_1 \cap P_2 \cap P_3| = 1.$

(d) Just plug in from above.

(e) $\Pr\left(\begin{array}{c}\text{no one collects}\\ \text{correct umbrella}\end{array}\right) = \frac{1}{3}.$