The University of Melbourne — School of Mathematics and Statistics
MAST30012 Discrete Mathematics — Semester 2, 2021

Practice Class 2: Arrangements and Combinations – Answers

Q1: (a) $\binom{20}{3} = \dfrac{20!}{3!\,17!} = 1140.$

(b) $7 \cdot 5 \cdot 3 \cdot 1 = 105.$

(c) $\dfrac{\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}}{4!} = \dfrac{8!}{2^4\,4!} = \dfrac{8!}{8 \cdot 6 \cdot 4 \cdot 2} = 7 \cdot 5 \cdot 3 = 105.$

(d) Total number of possible configurations is 2^{10}. 5 heads can be chosen in $\binom{10}{5}$ ways so

$$\Pr(\text{5 heads}) = \frac{\binom{10}{5}}{2^{10}} = \frac{63}{256} = 0.246073\ldots$$

At least 5 heads in a row. Number of favourable outcomes is $2^5 + 5 \times 2^4 = 112$ and hence

$$\Pr(\text{at least 5 heads in a row}) = \frac{112}{1024} = \frac{7}{64} = 0.109375.$$

Exactly 5 heads in a row. N umber of favourable outcomes is 64 and therefore

$$\Pr(\text{exactly 5 heads in a row}) = \frac{1}{16} = 0.0625.$$

Q2: (a) *aaa*, *aab*, *abb*, *bbb* total of $4 = \binom{2+3-1}{3} = \binom{4}{3}$.

(b) $\binom{20+3-1}{3} = \binom{22}{3} = \dfrac{22!}{3!\,19!} = 1540.$

(c) Number of ways to arrange $n-1$ symbols I, and r symbols in a line is $\binom{n+r-1}{r}$

(d) Same as (c).

Q3: (a) Derivation required.

(b) Derivation required.

(c) Arguments required.

Q4: (a) Derivation required.

(b) Derivation required.

Q5: (a) There are $\binom{n}{r}$ ways to order r '1's and $n-r$ '2's in a line.

(b) By the stated correspondence (bijection) the two counting problems are the same.