MATH 210 Practice Final Exam Questions

December 2019

1. Find all 12 errors in the following code. Indicate the line number and describe the error.

```
1   def euler(f,y0,t)
2       N = len(t)
3       y = np.zeros(N)
4       y[0] = y0
5       for n in range(0,N-1):
6       y[n+1] = y[n] + f(y[n],t[n])*h
7       print y

8   def f(y,t)
9       print y^2 - t

10  T = linspace(0,1,100)
11  Y = euler(f,T,y0)
12  plt.plot(T,Y)
13  plt.show()
```

2. Determine the correct order of the line numbers such that the following code plots the solutions of $y' = t\sin(y)$, $y(0) = 1$, for $t \in [0, 1]$.

```
1    plt.plot(t,y)
2    def f(y,t):
3    import numpy as np
4    y = spi.odeint(f,y0,t)
5    t = np.linspace(0,1,100)
6        return t*np.sin(y)
7    plt.show()
8    import scipy.integrate
9    y0 = 1
```

3. Approximate all solutions of the equations $x^4 + x - 1 = 0$ to 2 decimal places.

4. Write a Python function called `k_sum` which takes 2 input parameters x and N and returns the sum

$$\sum_{k=1}^{N} \frac{x^k}{k}$$

Do **not** import any packages. Use only builtin Python functions and datatypes.

5. Predict the value $y(1)$ for $y' = y^2 - t$ for $y(0) = 1$ using Euler's method with $h = 0.5$.

6. Consider the system of equations

$$\ddot{x} = x - xy$$
$$\ddot{y} = xy - y^2$$

Complete the code below to plot the solution $x(t)$ versus $y(t)$ for $t \in [0, 2]$ with initial conditions $x(0) = 1, \dot{x}(0) = -1, y(0) = 2, \dot{y}(0) = 0$.

```
import numpy as np
import scipy.integrate as spi
importmatplotlib.pyplot as plt

def odefun(u,t):
    # Code required here

    return dudt

# Code required here

plt.show()
```

7. Consider the integrals

$$\int_0^1 e^{-x^2} dx \quad \text{and} \quad \int_0^1 \sin(x) dx$$

If we approximate each integral using the trapezoid rule with the *same* number of subintervals N, which approximation do we expect to have the smallest error? Justify your answer.