Due **Thursday 10 September at 1pm** on blackboard.

Marks will be deducted for sloppy working. Clearly state your assumptions and conclusions, and justify all steps in your work.

The marked question 4 is required for MATH7861 students only. However, MATH1061 students are encouraged to try these also!

Q1 (a) Using unique prime factorisation, find gcd(7260, 5720) and lcm(7260, 5720).

(b) Using the Euclidean algorithm, find gcd(104346, 43095).

(10 marks)

Q2 Determine whether each of the following statements is true or false. If true, give a proof. If false, give a counterexample.

(a) For all $a, b, d \in \mathbb{N}$, if $d \mid a$ and $d \mid b$, then $\frac{ab}{d}$ is an integer that is a common multiple of both a and b.

(b) For all $a, b, d \in \mathbb{N}$, if $a^2 \equiv b^2 \pmod{d}$, then $a \equiv b \pmod{d}$.

(c) For all $a, b \in \mathbb{N}$, if $a^3 \equiv b^3 \pmod{3}$, then $a \equiv b \pmod{3}$.

(d) For all $x \in \mathbb{R}$, if x is irrational and $x > 0$, then $\sqrt{x}$ is also irrational.

(e) For all $x \in \mathbb{Q}$, if $x > 0$ and $x \notin \mathbb{Z}$, then $\sqrt{x}$ is also irrational.

(f) For all $x, y \in \mathbb{R}$, if $x \leq y$ and $\lfloor x \rfloor = \lfloor y \rfloor$, then either $\lfloor 2x \rfloor = \lfloor 2y \rfloor$ or $\lfloor 2x \rfloor = \lfloor 2y \rfloor - 1$.

(25 marks)

Q3 Prove that $\sqrt{10}$ is irrational.

(5 marks)

Q4 **[MATH7861 only]** A *perfect square* is an integer of the form n^2 where $n \in \mathbb{Z}$.

(a) What possible remainders do perfect squares leave when divided by 3?

(b) What possible remainders do perfect squares leave when divided by 4?

(c) What possible remainders do perfect squares leave when divided by 8?

(d) Find all solutions $a, b \in \mathbb{N}$ to the equation $2^a = b^2 - 5$, and prove that there are no more solutions other than the ones that you found.
Hint: Your answers to (a), (b) and/or (c) might come in useful.

(15 marks)