Due **Thursday 27 August at 1pm** on blackboard.

Marks will be deducted for sloppy working. Clearly state your assumptions and conclusions, and justify all steps in your work.

The marked questions 4 and 6 are required for MATH7861 students only. However, MATH1061 students are encouraged to try these also!

Q1 Use a truth table to determine whether the following statement is a contradiction, a tautology or neither. If it is a contradiction or a tautology, verify your answer using logical equivalences.

$$((p \to q) \land (\sim r \to \sim q) \land \sim (r \land p)) \longleftrightarrow (\sim (p \lor q) \lor (\sim p \land r))$$

(10 marks)

Q2 Show that the following argument is valid, by adding steps using the rules of inference and/or logical equivalences. Clearly label which rule you used in each step.

1. $p \to q$
2. $\sim (q \lor r)$
3. $s \to r$

$\therefore \sim s$

(5 marks)

Q3 Let $p \not\to q$ be the operation defined by the following truth table:

p	q	$p \not\to q$
T	T	F
T	F	T
F	T	F
F	F	F

Express each of the following statements using *only* the symbols p q $\sim$ $\not\to$ ( ):

(a) $p \lor q$

(b) $p \land q$

Justify your answers, using either logical equivalences or truth tables.

(5 marks)

Q4 [**MATH7861 only**] Give a convincing argument for why it is *not* possible to express $p \land q$ using only the symbols p q $\sim$ $\leftrightarrow$ ( ).

(5 marks)

(Continued on the following page...)

Q5 Let $P(x)$, $Q(x)$, $R(x)$, $S(x)$ and $T(x,y)$ denote the following predicates with domain $\mathbb{Z}$:

$$\begin{aligned} P(x)&:\ x^2 = x,\\ Q(x)&:\ x \leq 0,\\ R(x)&:\ x^2 = x+1,\\ S(x)&:\ x \text{ is even},\\ T(x,y)&:\ (x < y) \wedge (y < x^2) \end{aligned}$$

Determine whether each of the following statements is true or false, and give brief reasons.

(a) $\forall x \in \mathbb{Z}, P(x) \rightarrow Q(x)$

(b) $\forall x \in \mathbb{Z}, P(x) \rightarrow \sim Q(x)$

(c) $\forall x \in \mathbb{Z}, R(x) \rightarrow P(x)$

(d) $\forall x \in \mathbb{Z}, P(x) \rightarrow R(x)$

(e) $\forall x \in \mathbb{Z}, (R(x) \wedge S(x)) \rightarrow Q(x)$

(f) $\forall x \in \mathbb{Z}, (P(x) \wedge Q(x)) \rightarrow S(x)$

(g) $\exists x \in \mathbb{Z}$ such that $R(x)$

(h) $\exists x \in \mathbb{Z}$ such that $S(x) \wedge Q(x)$

(i) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $T(x,y)$

(j) $\forall x \in \mathbb{Z}, (\sim P(x)) \rightarrow \exists y \in \mathbb{Z}$ such that $T(x,y))$

(20 marks)

Q6 **[MATH7861 only]**

(a) Write down the negation of statements (f), (g), (h), (i) and (j) from question 5.

(b) Find a predicate $U(x)$ for which the statement

$$\forall x \in \mathbb{R}, (U(x) \longleftrightarrow \exists y \in \mathbb{R} \text{ such that } (x < y) \wedge (y < x^2))$$

is true, and where the predicate $U(x)$ does not use any other variables (y, z, etc.). Give a brief explanation for why your answer is correct.

Note that, unlike question 5, the domains have now changed from $\mathbb{Z}$ to $\mathbb{R}$.

(10 marks)