

# Spring 2017, MATH 407, Mid-Term Exam 2

Monday, April 17, 2017

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Name: \_\_\_\_\_

Circle the time of your discussion section:    10am    11am

## Instructions:

- No books, notes, or calculators.
- You have 50 minutes to complete the exam.
- **Show your work.**

Problem	Possible	Actual
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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**Problem 1.** A fair die is rolled until the total sum of all rolls exceeds 350. Compute approximately the probability that at most 100 rolls are necessary. Note that, for a single roll of the die, the expected value and variance of the outcome are  $7/2$  and  $35/12$ , respectively. Use the continuity correction. Leave the answer in the form  $P(Z < r)$  or  $P(Z > r)$  (whichever applies), where  $Z$  is a standard normal random variable and  $r$  is a suitable real number.

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**Problem 2.** Customers arrive at a bank according to a Poisson process. Suppose that three customers arrive during the first hour. Compute the probability that at least one arrived during the first 20 minutes. [**To help you out:** the arrival times are order statistics  $U_{(1)}, U_{(2)}, U_{(3)}$  of the uniform distribution on  $[0, 1]$ , with time measured in hours. Your objective is to compute the probability that the smallest of the three is less than  $1/3$ , so it makes sense to go with the complement: compute the probability that the smallest is bigger than  $1/3$ .]

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**Problem 3.** Let  $X, Y$  be independent random variables, both exponential with mean 1.

- (a) Find the joint density of  $U = X + Y$  and  $V = X/(X + Y)$ .
- (b) Are the random variables  $U$  and  $V$  independent? Explain your conclusion.

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**Problem 4.** For a randomly selected group of 50 people, compute the expected number of distinct birthdays (that is, the expected number of the days of the year that are a birthday of at least one person in the group).

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**Problem 5.** The joint probability density function of two random variables  $X$  and  $Y$

$$f_{X,Y}(x,y) = \begin{cases} Cx, & \text{if } x^2 + y^2 \leq 1, x > 0, y \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Are  $X$  and  $Y$  independent? Justify your answer.

(b) Compute  $E(X|Y)$ . Note: there is no need to know  $C$ .

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